Matrix

Importance of matrix →

- In d/f areas of business like budgeting, sales projection, cost estimatn, industrial mgmt.

Matrix → suppose we wish to express the infoⁿ that aman has 12 books & 5 pens.

[12] [5]

$$\begin{array}{c c} \text{Books} & \text{Pens} \end{array}$$

eg → $\begin{array}{c} \\ \text{Aman} \end{array} \begin{array}{c} \begin{array}{cc} \text{Books} & \text{Pens} \end{array} \\ \begin{bmatrix} 12 & 5 \end{bmatrix} \end{array}$

eg →

$$\begin{array}{c|cc|l} & \text{Books} & \text{Pens} & \\ \text{Aman} & 12 & 5 & \rightarrow R_1 \\ \text{Anil} & 8 & 4 & \rightarrow R_2 \\ \text{Amit} & 1 & 2 & \rightarrow R_3 \\ & \uparrow & \uparrow & \\ & C_1 & C_2 & \end{array}$$

3×2 ← order

Def → A matrix is an ordered rectangular array of no.s or fns. The no.s & fns are called the elements / entries of the matrix.

→ we denote the matrix by capital letters A, B

$\boxed{\text{order of a matrix}}$ → If a matrix have m rows & n columns, then m×n is called order of that matrix.

$$\begin{bmatrix} 12 & 5 \\ 8 & 4 \end{bmatrix}_{2\times 2} \leftarrow \text{order}$$

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} \text{Books} \\ \text{Pens} \end{array} & \begin{bmatrix} 12 & 8 & 1 \\ 5 & 4 & 2 \end{bmatrix}_{2\times 3} \end{array} \leftarrow \text{order}$$

$\boxed{\text{Types of matrix}}$ →

1. Column Matrix → A matrix which has only one column.

eg → $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1}$

2. Row Matrix → A matrix which has only 1 row.

eg $A \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}_{1\times 4}$

3. Square Matrix → A matrix in which no. of rows is equal to no. of columns.

eg $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}_{2\times 2}$ $A = \begin{bmatrix} 1 \end{bmatrix}_{1\times 1}$ $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}_{3\times 3}$

(4) Diagonal Matrix → A square matrix $A = [a_{ij}]_{m\times m}$ is said to be Diagonal Matrix $A = [a_{ij}]_{m\times m}$ if all its non-diagonal elements are zero.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times 3} \qquad [a_{ij}]_{3\times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}_{2\times 2}$$

(5) Scalar Matrix → A Diagonal matrix is said to be a scalar matrix if its Diagonal elements are equal.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}_{3\times 3} \qquad B = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}_{2\times 2}$$

(6) Identity Matrix → A square matrix in which elements in the diagonal are all 1, & rest are zero.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3} \qquad A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2} \qquad a_{11} = [1]_{1\times 1}$$

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(7) $\boxed{\text{Zero matrix}}$ → A matrix is said to be null / zero matrix if all its elements are zero.

eg → $A = [0]_{1\times1}$, $B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times2}$, $C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times3}$ $= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times3}$

22-3-22 (8) $\boxed{\text{Equal Matrix}}$ $\boxed{\text{and Equality of matrices}}$ →

→ 2 matrix $A = [a_{ij}]$
$B = [b_{ij}]$
are said to be equal if (1) they are of the same order.
(2) Each element of A is equal to the corresponding element of B.

i.e. $a_{ij} = b_{ij}$ for $\forall$ i and j

eg → $A = \begin{bmatrix} 0 & 8 & 5 \\ 2 & 9 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 8 & 5 \\ 2 & 9 & 0 \end{bmatrix}$

$\boxed{\text{Que.}}$ Find the values of a, b, c, x, y & z if

$$\begin{bmatrix} x+3 & z+4 & 2y-7 \\ -6 & a-1 & 0 \\ b-3 & -21 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ -6 & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$$

Sol →

$a_{11} = b_{11}$

$x + 3 = 0$

$\boxed{x = -3}$

$a_{22} = b_{22}$

$a - 1 = -3$

$\boxed{a = -2}$

$0 = 2c + 2$

$\boxed{c = -1}$

$b - 3 = 2b + 4$

$\boxed{-7 = b}$

$z + 4 = 6$

$\boxed{z = 2}$

$2y - 7 = 3y - 2$

$\boxed{-5 = y}$

Algebra of Matrices →

① Addition of matrix :

→ The matrix should be same order.

→ The sum of 2 matrix is a matrix obtain by adding the corresponding elements of the given matrices.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3} \quad B = \begin{bmatrix} 0 & 5 & 8 \\ 1 & 9 & 7 \end{bmatrix}_{2\times 3}$$

$$C = A + B = \begin{bmatrix} 1 & 7 & 11 \\ 5 & 14 & 13 \end{bmatrix}$$

② Difference of matrix / Subtraction of matrix →

→ IF $A = [a_{ij}]$ and $B = [b_{ij}]$ are of the same order say $m \times n$ matrices, then the difference $A - B$ is define as a matrix

$D = [d_{ij}]$

where $= d_{ij} = a_{ij} - b_{ij}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 5 & 8 \\ 1 & 9 & 7 \end{bmatrix}$$

$$D = A - B = \begin{bmatrix} 1-0 & 2-5 & 3-8 \\ 4-1 & 5-9 & 6-7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & -5 \\ 3 & -4 & -1 \end{bmatrix}$$

(3) **Multiplication of a Matrix by a Scalar** →
if $A = [a_{ij}]_{m \times n}$ is a matrix & k is a Scalar, then kA is another matrix which is obtain by multiplying each element of A by the Scalar k.

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 8 & 9 & 0 \\ 1 & 3 & 6 \end{bmatrix}_{3 \times 3}$$

$$2A = \begin{bmatrix} 2\times1 & 2\times2 & 2\times5 \\ 2\times8 & 2\times9 & 2\times0 \\ 2\times1 & 2\times3 & 2\times6 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 10 \\ 16 & 18 & 0 \\ 2 & 6 & 12 \end{bmatrix}$$

4 **Multiplication of 2 Matrix** → A & B are 2 matrix
i) IF the no. of column of A matrix is equal the no. of rows of B matrix.
ii) $(i, k)^{th}$ element C_{ik} of the matrix is obtained by multiplying ith row of A & k^{th} column of B matrix element wise and

$x+y \quad 2y+2$

take the sum of all this products.

$$C = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 4 \end{bmatrix}_{2\times 3} \qquad D = \begin{bmatrix} 2 & 7 \\ -1 & 1 \\ 5 & -4 \end{bmatrix}_{3\times 2}$$

$$C\times D = \begin{bmatrix} 1\times 2 + -1\times -1 + 2(5) & 1(7)+(-1)1+2(-4) \\ 0(2)+3(-1)+4(5) & 0(7)+3(1)+4(-4) \end{bmatrix}$$

$$= \begin{bmatrix} 2+1+10 & 7-1-8 \\ 0+(-3)+20 & 0+3-16 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & -2 \\ 17 & -13 \end{bmatrix}$$

$$\boxed{C\times D \neq D\times C}$$

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix}_{2\times 2} \qquad \begin{bmatrix} 2 & -1 \\ 0 & 5 \\ 7 & 8 \end{bmatrix}_{3\times 2}$$

$2 \neq 3$

∴ multiplicatn not possible

(8) Transpose of a Matrix →

→ If $A = [a_{ij}]_{m\times n}$ be a matrix then matrix obtained by interchanging rows & column of A it is denoted by A^T or A'.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \end{bmatrix}_{2\times 3} \Rightarrow A' = \begin{bmatrix} 1 & 5 \\ 2 & 6 \\ 3 & 7 \end{bmatrix}$$

Properties of Transpose of the matrix →

① $(A')' = A$

② $(KA)' = KA'$

③ $(A+B)' = A' + B'$

④ $(AB)' = B'A'$

examples of these properties →

I property $A = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow (A')' = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Property II

$K = 2$

$KA = \begin{bmatrix} 0 & 4 & 0 \\ 2 & 6 & 2 \\ 2 & 2 & 2 \end{bmatrix} \Rightarrow (KA)' = \begin{bmatrix} 0 & 2 & 2 \\ 4 & 6 & 2 \\ 0 & 2 & 2 \end{bmatrix}$

$KA' = \begin{bmatrix} 0 & 2 & 2 \\ 4 & 6 & 2 \\ 0 & 2 & 2 \end{bmatrix}$

Property III

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 0 & 1 & 3 \\ 4 & 1 & 4 \\ 4 & 2 & 2 \end{bmatrix} \qquad (A+B)' = \begin{bmatrix} 0 & 4 & 4 \\ 1 & 1 & 2 \\ 3 & 4 & 2 \end{bmatrix}$$

$$A' = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 0 & 0 \\ 2 & 1 & 1 \end{bmatrix} \qquad B' = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} 0 & 4 & 4 \\ 1 & 1 & 2 \\ 3 & 4 & 2 \end{bmatrix}$$

Property IV

$$AB = \begin{bmatrix} 0+2+2 & 0+1+4 & 0+3+2 \\ 0+0+1 & 0+0+2 & 2+0+1 \\ 0+0+1 & 0+0+2 & 3+0+1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & 5 & 5 \\ 1 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix} \qquad (AB)' = \begin{bmatrix} 4 & 1 & 1 \\ 5 & 2 & 2 \\ 5 & 3 & 4 \end{bmatrix}$$

$$B'A' = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 3 \\ 1 & 0 & 0 \\ 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0+2+2 & 0+0+1 & 0+0+1 \\ 0+1+4 & 0+0+2 & 0+0+2 \\ 0+3+2 & 2+0+1 & 3+0+1 \end{bmatrix}$$

$$B'A' = \begin{bmatrix} 4 & 1 & 1 \\ 5 & 2 & 2 \\ 5 & 3 & 4 \end{bmatrix}$$

Symmetric & Skew symmetric Matrix →

→ A square matrix $A = [a_{ij}]$ is said to be symmetric if

$$A^T = A$$

$$A = \begin{bmatrix} \sqrt{3} & 2 & 3 \\ 2 & -1.5 & -1 \\ 3 & -1 & 1 \end{bmatrix}_{3\times 3}$$

$$A^T = \begin{bmatrix} \sqrt{3} & 2 & 3 \\ 2 & -1.5 & -1 \\ 3 & -1 & 1 \end{bmatrix}_{3\times 3}$$

$$A = A^T$$

→ A square matrix $A = [a_{ij}]$ is said to be skew symmetric matrix if

$$A' = -A$$

$$A = \begin{bmatrix} 0 & e & f \\ -e & 0 & g \\ -f & -g & 0 \end{bmatrix} \qquad A^T = \begin{bmatrix} 0 & -e & -f \\ e & 0 & -g \\ f & g & 0 \end{bmatrix} \text{ — (1)}$$

$$A = -A'$$

$$-A = (-1) \times A = \begin{bmatrix} 0 & -e & -f \\ e & 0 & -g \\ f & g & 0 \end{bmatrix} \text{ — (2)}$$

$$(1) = (2)$$

$$A^T = -A$$

* A square matrix can be expressed as the sum of a symmetric matrix and a skew symmetric matrix.

$$\boxed{A = \underbrace{\frac{1}{2}(A+A')}_{\text{Sym. matrix}} + \underbrace{\frac{1}{2}(A-A')}_{\text{Skew Sym. matrix}}}$$

eg Explain the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of the symmetric & a skew symmetric matrix.

$$B = \underbrace{\frac{1}{2}(B+B')}_{P \text{ Symmetric}} + \underbrace{\frac{1}{2}(B-B')}_{Q \text{ Skew symmetric}}$$

$P = \frac{1}{2}(B+B')$

$$B' = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$$

$$B+B' = \begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}$$

$$\frac{1}{2}(B+B') = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} = P$$

$$P' = \begin{bmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{bmatrix}$$

$P = P'$

$$Q = \frac{1}{2}(B - B')$$

$$B - B' = \begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$$

$$\frac{1}{2}(B - B') = \begin{bmatrix} 0 & \frac{-1}{2} & -\frac{5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix} = Q \qquad Q^T = \begin{bmatrix} 0 & \frac{1}{2} & \frac{5}{2} \\ -\frac{1}{2} & 0 & -3 \\ -\frac{5}{2} & 3 & 0 \end{bmatrix}$$

$$Q = -Q^T$$

$$P + Q$$

$$\Rightarrow \begin{bmatrix} 2 & -\frac{3}{2} - \frac{1}{2} & -\frac{3}{2} - \frac{5}{2} \\ -\frac{3}{2} + \frac{1}{2} & 3+0 & 4 \\ -\frac{3}{2} + \frac{5}{2} & -2 & -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = B$$

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$\boxed{\text{Determinant}}$ →

→ to every square matrix $A = [a_{ij}]$ of order n, we associate a no. called determinant of square matrix A.

→ It is denoted by $|A| = \det A = \Delta$

$$A = [a]_{1\times1} \Rightarrow |A| = a$$

$$B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2} = a_{11}a_{22} - a_{12}a_{21}$$

$$C = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$$

row wise →

$$\Rightarrow + a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$+ a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

eg → $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$

$$\Rightarrow 1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} + (-2)\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$$

$$\Rightarrow 1(-9+12) - 1(-18+15) - 2(8-5)$$

$$\Rightarrow 3 + 3 - 6$$

$$\Rightarrow 0$$

(1) The value of the determinant remains unchanged if its rows & columns are interchanged.

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = A^T = \begin{bmatrix} 2 & 6 & 1 \\ -3 & 0 & 5 \\ 5 & 4 & -7 \end{bmatrix}$$

$$2 \begin{vmatrix} 0 & 5 \\ 4 & -7 \end{vmatrix} - 6 \begin{vmatrix} -3 & 5 \\ 5 & -7 \end{vmatrix} + 1 \begin{vmatrix} -3 & 0 \\ 5 & 4 \end{vmatrix}$$

$$2(-20) - 6(21 - 25) + (-12)$$

$$-40 + 24 - 12 \Rightarrow 24 - 52 \Rightarrow -28$$

(2) → If any 2 rows or columns of a determinant are interchanged, then sign of determinant changes,

$$R_2 \leftrightarrow R_3$$

$$\Delta_1 = \begin{vmatrix} 2 & -3 & 5 \\ 1 & 5 & -7 \\ 6 & 0 & 4 \end{vmatrix}$$

$$40 + 138 - 150 = 28$$

$$2(20) + 3(4 + 42) + 5(-30) \Rightarrow \cancel{40 + 150} - 150 \Rightarrow 1\cancel{8}2 \Rightarrow 28$$

44 × 3 = 132   46 × 3 = 138   178 − 150 = 28

(3) if any 2 rows or column of a determinant are identical, (all corresponding elements are same), then value of determinant is zero

$$\Delta = \begin{vmatrix} 3 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix} \quad \boxed{R_1 = R_3}$$

$$3(6-6) - 2(6-9) + 3(4-6)$$

$$6 - 6 = 0$$

(4) If each element of a row or a column of a determinant is multiplied by a constant, (k) then its value gets multiplied by k.

$$\Delta = \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

$$\times\ 102(18-12) - 18(6-58) + 36(3-1)$$

$$\begin{vmatrix} 6\times17 & 6\times3 & 6\times6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$$

$$= 6 \begin{vmatrix} 17 & 3 & 6 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} \Rightarrow \quad 6\times0 = 0$$

(5) If some or all elements of a row or a column of a determinant are expressed as sum of 2 or more terms then the determinant can be expressed as sum of 2 or more determinants.

$$\Delta = \begin{vmatrix} a_1 + d_1 & a_2 + d_2 & a_3 + d_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} d_1 & d_2 & d_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

eg →

$$\Delta = \begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix} = 0$$

$$\begin{vmatrix} a & b & c \\ a & b & c \\ x & y & z \end{vmatrix} + \begin{vmatrix} a & b & c \\ 2x & 2y & 2z \\ x & y & z \end{vmatrix}$$

$$0 + 2\begin{vmatrix} a & b & c \\ x & y & z \\ x & y & z \end{vmatrix} \Rightarrow 0 + 2(0)$$

$$\Rightarrow 0$$

(6) 1 row / 1 col = 0 , $\Delta = 0$

$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = 0$$

(7)
$$= \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$R_2 \to R_2 - 4R_1$

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 7 & 8 & 9 \end{vmatrix}$$

$C_2 \to C_2 - 2C_1$

$$\begin{vmatrix} 1 & 0 & 3 \\ 4 & -3 & 6 \\ 7 & -6 & 9 \end{vmatrix}$$

2 − 2
5 − 8
8 − 14

$$A \xrightarrow[\text{operation apply}]{\text{Row or column}} A$$

$$\Delta \Longleftarrow \Delta$$

eg →

$$\Delta = \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$$

$$x\left[b(z+c) - c(y+b)\right] - a\left[y(z+c) - z(y+b)\right] + (x+a)\left[yc - zb\right]$$

$\boxed{\text{or}}$

$$\begin{vmatrix} x & a & x \\ y & b & y \\ z & c & z \end{vmatrix} + \begin{vmatrix} x & a & a \\ y & b & b \\ z & c & c \end{vmatrix}$$

$$0 + 0 = 0$$

or $\quad C_1 \to C_1 + C_2$

$$\Downarrow$$

$$C_1 = C_3 \Rightarrow 0 = \Delta$$

$$\begin{vmatrix} x+a & a & x+a \\ y+b & b & y+b \\ z+c & c & z+c \end{vmatrix} = 0$$

eg $\rightarrow$

$$\Delta = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

$R_1 \rightarrow R_1 + R_2$

$$\Delta = \begin{vmatrix} a-c & b-a & c-b \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

$$(-1) \begin{vmatrix} c-a & a-b & b-c \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

$C_1 = C_3$

$\Rightarrow (-1)(0) \quad \Rightarrow 0$

matrix

$\boxed{\text{common element}}$ #

if all rows & colⁿ having it, multiply → to whole matrix

Determinant

any one row / colⁿ having it. to one col or row

$$\Delta = \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$$

$$(a)(b)(c) \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$

$R_1 \rightarrow R_1 + R_2$

$$abc \begin{vmatrix} 0 & 0 & 2c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$

$$abc \left[2c \left(ab + ab \right) \right]$$

$$abc \left[2c \left(2ab \right) \right]$$

$$\Rightarrow \underline{\underline{4 a^2 b^2 c^2}}$$

Q $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

$R_1 \to R_1 - R_2$

$$\begin{vmatrix} 0 & a-b & a^2-b^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$R_2 \to R_2 - R_3$

$$(a-b)\begin{vmatrix} 0 & 1 & a+b \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$(a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

$(a-b)(b-c)(\cancel{b}+c) - (a+\cancel{b})$

$\Rightarrow (a-b)(b-c)(c-a)$

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$\boxed{\text{Rank of a Matrix}}$ → A non-zero number r is said to be rank of matrix A if (i) there is atleast a minor of A of order r
(ii) Every minor of higher order then r is Zero.

The rank of A is denoted by $\rho(A)$ ← rank of A

Q ~~Min~~ Find Rank of below matrix :

$$A \begin{bmatrix} -1 & -2 & 3 \\ -2 & 4 & -1 \\ -1 & 2 & 7 \end{bmatrix}_{3\times3}$$

Sol $|A| = \begin{vmatrix} -1 & -2 & 3 \\ -2 & 4 & -1 \\ -1 & 2 & 7 \end{vmatrix}$

$-1(28+2) + 2(-14-1) + 3(-4+4)$

$-30 + 2(-15) + 0$

$-30 - 30 \Rightarrow -60$

det A $\neq 0$.

3×3 rank 3, 2 or 1 det A $\neq 0$ rank = 3	2×2 rank 2 or 1 2 → det $\neq 0$ 1 → det $= 0$

Determinant Method of finding rank of Matrix

$$A = \begin{vmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{vmatrix}_{3\times 3}$$

$\det A \Rightarrow 2(-9+8) - 1(0+4) - 1(0-6)$

$-2 - 4 + 6 \Rightarrow -6+6 \Rightarrow 0$

rank of $A \neq 3$

rank = 2

$$\begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} \Rightarrow 6 \neq 0$$

if we findout non-zero minor, then rank = 2
if non-zero minor doesn't find, then rank = 1

Q → $\begin{bmatrix} 1 & 2 & 3 \\ 7 & 5 & 8 \\ 2 & 4 & 8 \end{bmatrix}$ Rank = ?

$1(40-32) - 2(56-16) + 3(28-10)$

$8 - 80 + 54$

$62 - 80$

$\Rightarrow -18 \rightarrow \underline{\text{rank} = 3}$

Q $\begin{vmatrix} 1 & 2 & 3 \\ 7 & 5 & 8 \\ 2 & 4 & 6 \end{vmatrix}$

42
16
26

$1(30-32) - 2(42-16) + 3(28-10)$

$-2 - 2(26) + 54$

$-2 - 52 + 54$

$-54 + 54 = 0$

rank = 2

$\begin{vmatrix} 1 & 2 \\ 7 & 5 \end{vmatrix} = 5 - 14 = -9$

Minors & Co-Factors →

Minor → minor of an element a_{ij} of a determinant is the determinant obtained by deleting its i^{th} row & j^{th} column, in which element a_{ij} lies.

→ Minor of an element a_{ij} is denoted by M_{ij}.

Cofactor → Co-factor of an element a_{ij} denoted by A_{ij} is defined by $\boxed{A_{ij} = (-1)^{i+j} M_{ij}}$

eg → Find minors & Co-factors of all the elements of the determinant

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

Sol → $a_{11} = 2 \Rightarrow M_{11} = \begin{vmatrix} 0 & 4 \\ 5 & -7 \end{vmatrix}$

$\Rightarrow 0 - 20 = -20$

$a_{12} = -3 \Rightarrow M_{12} = \begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} = -42 - 4$
$= -46$

$A_{ij} = (-1)^{i+j} M_{ij}$

$A_{11} = (-1)^{1+1} (-20) = 1(-20) = -20$

$A_{12} = (-1)^{1+2} (-46) = (-1)^3 (-46) = 46$

$$a_{13} = \begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix} = M_{13} = 30 \quad , \quad A_{13} = 30$$

$$a_{21}, \; M_{21} = \begin{vmatrix} -3 & 5 \\ 5 & -7 \end{vmatrix} = 21 - 25 = -4 \quad , \quad A_{21} = 4$$

$$a_{22}, \; M_{22} = \begin{vmatrix} 2 & 5 \\ 1 & -7 \end{vmatrix} = -14 - 5 = -19 \quad , \quad A_{22} = -19$$

$$a_{23}, \; M_{23} = \begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} = 10 + 3 = 13 \quad , \quad A_{23} = -13$$

$$a_{31}, \; M_{31} = \begin{vmatrix} -3 & 5 \\ 0 & 4 \end{vmatrix} = -12 \quad , \quad A_{31} = -12$$

$$a_{32}, \; M_{32} = \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} = 8 - 30 = -22 \quad , \quad A_{32} = 22$$

$$a_{33}, \; M_{33} = \begin{vmatrix} 2 & -3 \\ 6 & 0 \end{vmatrix} = 18 \quad , \quad A_{33} = 18$$

$$M_{ij} = \begin{bmatrix} -20 & -46 & 30 \\ -4 & -19 & 13 \\ -12 & -22 & 18 \end{bmatrix} \qquad A_{ij} = \begin{bmatrix} -20 & 46 & 30 \\ 4 & -19 & -13 \\ -12 & 22 & 18 \end{bmatrix}$$

Adjoint of a Matrix ⇒

① matrix should be square matrix.

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$\text{adj } A = \begin{bmatrix} -20 & 4 & -12 \\ 46 & -19 & 22 \\ 30 & -13 & 18 \end{bmatrix}$$

Find determinant of 4×4 matrix

4×4 ⇁

Q

$$\Delta = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 0 & 1 & 2 & 3 \\ 5 & 6 & 8 & 9 \end{vmatrix}_{4\times4}$$

$$\Rightarrow 1\begin{vmatrix} 6 & 7 & 8 \\ 1 & 2 & 3 \\ 6 & 8 & 9 \end{vmatrix} - 2\begin{vmatrix} 5 & 7 & 8 \\ 0 & 2 & 3 \\ 5 & 8 & 9 \end{vmatrix} + 3\begin{vmatrix} 5 & 6 & 8 \\ 0 & 1 & 3 \\ 5 & 6 & 9 \end{vmatrix} - 4\begin{vmatrix} 5 & 6 & 7 \\ 0 & 1 & 2 \\ 5 & 6 & 8 \end{vmatrix}$$

$$\Rightarrow 1\left[6\begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} - 7\begin{vmatrix} 1 & 3 \\ 6 & 9 \end{vmatrix} + 8\begin{vmatrix} 1 & 2 \\ 6 & 8 \end{vmatrix}\right]$$

$$-2\left[5\begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} - 7\begin{vmatrix} 0 & 3 \\ 5 & 9 \end{vmatrix} + 8\begin{vmatrix} 0 & 2 \\ 5 & 8 \end{vmatrix}\right]$$

$$+3\left[5\begin{vmatrix} 1 & 3 \\ 6 & 9 \end{vmatrix} - 6\begin{vmatrix} 0 & 3 \\ 5 & 9 \end{vmatrix} + 8\begin{vmatrix} 0 & 1 \\ 5 & 6 \end{vmatrix}\right]$$

$$-4\left[5\begin{vmatrix} 1 & 2 \\ 6 & 8 \end{vmatrix} - 6\begin{vmatrix} 0 & 2 \\ 5 & 8 \end{vmatrix} + 7\begin{vmatrix} 0 & 1 \\ 5 & 6 \end{vmatrix}\right]$$

$$\Rightarrow \big(6(18-24) - 7(9-18) + 8(8-12)\big)$$
$$-2\big(5(18-24) - 7(-15) + 8(-10)\big)$$
$$+3\big(5(9-18) - 6(-15) + 8(-5)\big)$$
$$-4\big(5(8-12) - 6(-10) + 7(-5)\big)$$

$$\Rightarrow \big[6(-6) - 7(-9) + 8(-4)\big] - 2\big[5(-6) - 7(-15) + 8(-10)\big]$$
$$+3\big[5(-9) - 6(-15) + 8(-5)\big] - 4\big[5(-4) - 6(-10) + 7(-5)\big]$$

3 15 3
10 5
90

$$\Rightarrow [-36+63-32]-2[-30+105-80]+3[-45+90-40]-4[-20+60-35]$$

$$\Rightarrow [63-68]-2[105-110]+3[90-85]-4[60-55]$$

$$\Rightarrow (-5)-2(-5)+3(5)-4(5)$$

$$\Rightarrow -5+10+15-20$$

$$\Rightarrow 25-25$$

$$\Rightarrow \underline{\underline{0}}$$

Rank of matrix →

⇒ $\rho(A)$

4×4

1, 2, 3, 4 → det = non-zero
$\neq 0$

det = 0
3×3 – minor $\neq 0$

det = 0
3×3 – minor = 0

2×2
$\neq 0$ →

Another method to find rank of matrix →

Echelon form of matrix → Row operation
diagonal element → 1
under → 0 make
with help of R_1

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 2 & 2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$

make zero → lower row with just upper row of its

$$A \sim \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$\downarrow R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \to (-R_2)} \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{Rank} = 2}$

2 rows nonzero element

8

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 5 & 7 & 5 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - 3R_1$
$R_4 \rightarrow R_4 - 6R_1$

$$\longrightarrow \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -7 & -11 & 5 \end{bmatrix}$$

$R_2 \longleftrightarrow R_4$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -7 & -11 & 5 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

Row wise no. of zeros increase

$R_2 \rightarrow \left(\frac{R_2}{-7}\right)$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 11/7 & -5/7 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_3 \rightarrow R_3 + 4R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 11/7 & -5/7 \\ 0 & 0 & -12/7 & 1/7 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$-8 + \frac{44}{7}$ $\quad 3 - \frac{20}{7}$ $\quad \frac{21-20}{7}$

$-56 + 44 = 12$

$R_3 \rightarrow R_3 * 7$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 11/7 & -5/7 \\ 0 & 0 & -12 & 1 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_3 \rightarrow \frac{R_3}{-12}$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 11/7 & -5/7 \\ 0 & 0 & 1 & 1/-12 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 \rightarrow R_4 + 3R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 11/7 & -5/7 \\ 0 & 0 & 1 & 1/-12 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$R_4 = \frac{R_4}{5}$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 11/7 & -5/7 \\ 0 & 0 & 1 & 1/-12 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rank = 4

$$A = \begin{bmatrix} 1 & 4 & 3 & 2 \\ 1 & 2 & 3 & 4 \\ 2 & 6 & 7 & 5 \end{bmatrix}$$

$\Rightarrow$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - 2R_1$

$$\longrightarrow \begin{bmatrix} 1 & 4 & 3 & 2 \\ 0 & -2 & 0 & 2 \\ 0 & -2 & 1 & 1 \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} 1 & 4 & 3 & 2 \\ 0 & -2 & 1 & 1 \\ 0 & -2 & 0 & 2 \end{bmatrix}$$

$R_2 \to \dfrac{R_2}{-2}$

$$\longrightarrow \begin{bmatrix} 1 & 4 & 3 & 2 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -2 & 0 & 2 \end{bmatrix}$$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$R_3 \to \dfrac{R_3}{-1}$

$-\frac{2}{2}$ $2-1$

$-\frac{2}{2}$

$$\begin{bmatrix} 1 & 4 & 3 & 2 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\boxed{\text{Rank} = 3}$$

Q

$$B = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - R_1$

$$\longrightarrow \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$$\Big\downarrow R_3 \to \frac{R_3}{-2}$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

~~Rank~~

$\boxed{\text{Rank} = \cancel{3}\ 2}$

$$\Big\downarrow R_2 \to R_2 + 6R_3$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

29 3 22

Singular & Non-Singular Matrix →

→ A square matrix A is said to be singular matrix if determinant of $|A| = 0$.

→ A square matrix A is said to be non-singular matrix if $\det A \neq 0$.

Inverse of a square matrix →

→ matrix should be non-singular.

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

Find inverse.

Q $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$ $\qquad B^{-1} = \frac{\text{adj } B}{|B|}$

$|B| = 3 - 2 = 1$ $\qquad \text{adj } B = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

~~$B^T = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$~~

$(\text{Cofactor})^T \Rightarrow$

$M_{11} = 3$ $\qquad A_{11} = 3$

$M_{12} = -1$ $\qquad A_{12} = 1$

$M_{21} = -2$ $\qquad A_{21} = 2$

$M_{22} = 1$ $\qquad A_{22} = 1$

A^T

$\Downarrow$

$\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}^T$

$\Downarrow$

$\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

Q $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$

find $(AB)^{-1}$

$$AB = \begin{bmatrix} 2-3 & -4+9 \\ 1+4 & -2-12 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 5 & -14 \end{bmatrix}$$

$M_{11} = -14$ $A_{11} = -14$

$M_{12} = 5$ $A_{12} = -5$

$M_{21} = 5$ $A_{21} = -5$

$M_{22} = \cancel{2} -1$ $A_{22} = \cancel{2} -1$

$$\begin{bmatrix} -14 & -5 \\ -5 & \cancel{2}\,-1 \end{bmatrix}$$

A^T

$\Downarrow$

$$\begin{bmatrix} -14 & -5 \\ -5 & \cancel{2}\,-1 \end{bmatrix}$$

$|AB| \Rightarrow 14 - 25 = -11$

$$(AB)^{-1} = \frac{\text{adj } AB}{|AB|}$$

$$\Rightarrow \frac{1}{-11} \begin{bmatrix} -14 & -5 \\ -5 & \cancel{2}\,-1 \end{bmatrix}$$

$$(AB)^{-1} \Rightarrow \begin{bmatrix} \frac{14}{11} & \frac{5}{11} \\ \frac{5}{11} & \frac{+1}{11} \end{bmatrix}$$

homogenous

$2x+y=0$
$7x+y=0$

non-homogenous

$\neq 0$

Simultaneous Linear Equⁿ →

$$\left.\begin{aligned} 2x+3y &= 8 \\ 5x+y &= 7 \end{aligned}\right] \begin{array}{l} x \,\&\, y \\ \text{power } 1 \end{array}$$

- **Consistent system** → A system of equⁿ is said to be consistent if its solution exist [one or many].

- **In-consistent system** → A system of equⁿ is said to be inconsistent if its solution doesn't exist.

Consistency & Non-consistency of non-homogenous System of Linear Equⁿ →

System of eqⁿ is :

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

matrix representation →

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$\boxed{AX = B}$$

Augmented matrix: $C \sim \begin{bmatrix} A & : & B \end{bmatrix}$

Working Sequence →

① Draw a augmented matrix. $C \simeq [A : B]$

$$C = \begin{bmatrix} a_1 & b_1 & c_1 & : & d_1 \\ a_2 & b_2 & c_3 & : & d_2 \\ a_3 & b_3 & c_3 & : & d_3 \end{bmatrix}$$

② Reduce the matrix C into Echelon form.

③ if Rank of A = Rank of C

$$\rho(A) = \rho(C)$$

(i) if $\boxed{\text{rank of } A = \rho(C) = n}$

where n is the no. of variables in linear eqn. ~~matrix,~~ then the system is consistent & $\boxed{\text{unique}}$ solution.

(ii) if $\boxed{\rho(A) = \rho(C) < n}$

then the system of linear eqn has $\boxed{\text{infinite}}$ solution. & consistent.

(iii) if $\boxed{\rho(A) \neq \rho(C)}$

then the system is inconsistent & has $\boxed{\text{no solution}}$ it mean we're unable to find the soluth of linear eqn. / system.

eg → show that the eqn

$$x + 2y - z = 3$$
$$3x - y + 2z = 1$$
$$2x - 2y + 3z = 2$$
$$x - y + z = -1$$

are consistent & solve them.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}$$

$$AX = B$$

$$(AX = 0 \rightarrow \text{homogenous})$$

① augmented matrix :

$$C = \left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 2 & -2 & 3 & 2 \\ 1 & -1 & 1 & -1 \end{array}\right]$$

$R_2 \rightarrow R_2 - 3R_1$
$R_3 \rightarrow R_3 - 2R_1$
$R_4 \rightarrow R_4 - R_1$

$$\longrightarrow \left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{array}\right]$$

$R_2 \rightarrow (R_2 - R_3) \times -1$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{array}\right]$$

$R_3 \rightarrow R_3 + 6R_2$
$R_4 \rightarrow R_4 + 3R_2$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & 2 & 8 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & 2 & 8 \end{array}\right] \quad R_3 \rightarrow \frac{R_3}{5}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 2 & 8 \end{array}\right] \quad R_4 \rightarrow R_4 - 2R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Rank of C = 3

$\rho(A) = \rho(c) =$ No. of Var

$3 = 3 = 3$

Consistent & unique ~~consistent~~ Solution.

$x + 2y - z = 3$

$\boxed{y = 4}$

$\boxed{z = 4}$

$\cancel{4 + 2(4) - z = 3}$

$\cancel{-z = 3 - 12}$

$\cancel{-z = -9}$

$\cancel{\boxed{z = 9}}$

$\cancel{4 + 2y - 4 = 3}$

$\cancel{\boxed{y = \frac{3}{2}}}$

$x + 2(4) - 4 = 3$

$x = 3 - 4$

$\boxed{x = -1}$

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$$C \sim \left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

$x \quad y \quad z : D$

$$\rho(C) \neq \rho(A)$$

$$4 \neq 3$$

$\Rightarrow$ so the system is incosisfent

So no soluth

Q

$$C \sim \left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rho(C) = \rho(A) <$ no. of variables

$$2 = 2 < 3$$

$\Rightarrow$ system is consistent & infinite Soluth exist.

$\Rightarrow$ Consistent But have many solution.

$$x + 2y - z = 3$$

$$y = 4$$

$$x - 8 - z = 3$$

$$x - z = -5$$

Let z equals to a Scalar. $\boxed{z = K}$

where $K = 0$ to ∞

$$x - K = -5$$

$$\boxed{x = K - 5}$$

as value of z is change ie. value of x is also change $\rightarrow$ so equation has many solution.

$z = K = 0$

$x = -5$

Solution of homogeneous system of linear equⁿs

→ Suppose the system of eqⁿ is $AX = B$
here if $B \neq 0$ then. this is non-homogeneous eqⁿ & if $B = 0$ then this is homogeneous eqⁿ.

→ So the system of homogenous eqⁿ is

$$\boxed{AX = 0} \quad \text{——— (1)}$$

→ the nature of solution depend on $\rho(A)$.

Case - I If rank (A) = no. of variables, then the system is consistent and have trivial solution.

$$X = 0$$

$$x = 0 \quad y = 0 \quad z = 0$$

If $\rho(A) <$ no. of variables
system is consistent and have infinite solution.

8 $x_1 + x_2 + x_3 = 0$

$x - 2x_2 - x_3 = 0$

$2x_1 + x_2 + 3x_3 = 0$

$AX = 0$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -1 \\ 2 & 1 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad B = 0$$

$R_2 \to R_2 - R_1$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -3 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$R_1 \to R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & -3 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$R_2 \to \frac{R_2}{-3}$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2/3 \\ 0 & -1 & 1 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2/3 \\ 0 & 0 & 5/3 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$ $\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & -3 & -2 \end{bmatrix}$

$R_2 = -R_2$ $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & -3 & -2 \end{bmatrix}$

$R_3 \to R_3 + 3R_2$ $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & -5 \end{bmatrix}$

$R_3 \to \frac{R_3}{-5}$

$\rho(A) = 3$

system is consistent & trivial solution.

$x_1 = 0$

$x_2 = 0$

$x_3 = 0$

Q//

$$4x + 2y + z + 3u = 0$$
$$6x + 3y + 4z + 7u = 0$$
$$2x + y + u = 0$$

$$A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 0 & 1 \\ 6 & 3 & 4 & 7 \\ 4 & 2 & 1 & 3 \end{bmatrix}$$

$\Rightarrow R_1 \to \frac{R_1}{2}$

$$\begin{bmatrix} 1 & 1/2 & 0 & 1/2 \\ 6 & 3 & 4 & 7 \\ 4 & 2 & 1 & 3 \end{bmatrix}$$

$\Rightarrow R_2 \to R_2 - 6R_1$

$$\begin{bmatrix} 1 & 1/2 & 0 & 1/2 \\ 0 & 0 & 4 & 4 \\ 4 & 2 & 1 & 3 \end{bmatrix}$$

$\Rightarrow R_3 - 4R_3$

$$\begin{bmatrix} 1 & 1/2 & 0 & 1/2 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$\rho(A) = 2$

$2 < 4$

consistent infinite solution

$R_3 \to R_2 - 4R_3$

$$\begin{bmatrix} 1 & 1/2 & 0 & 1/2 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$x + \frac{y}{2} + \frac{u}{2} = 0$$

$$\boxed{2x + y + u = 0}$$

$$4z + 4u = 0$$

$$\boxed{z + u = 0}$$

$$x + \frac{y}{2} + \frac{z}{4} + \frac{3u}{4} = 0$$

$u = k$

$z = -k$

$y = -k - 2p$

$x = p$

$$4x + 2y + z + 3u = 0$$

$$z + u = 0$$

$$z = -k$$

$$\boxed{u = k}$$

$$4x + 2y - k + 3k = 0$$

$$4x + 2y + 2k = 0$$

$$4p + 2y + 2k = 0$$

Variable − rank

$4 - 2 = 2$

$$2y = -2k - 4p$$

$$y = -k - 2p$$

$$x = p$$

$$\begin{array}{cccc} x & y & z & u \\ \end{array}$$

$$\begin{bmatrix} 1 & 1/2 & 1/4 & 3/4 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

t

tion

$$\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

$\frac{R_1}{4}$

$$\begin{matrix} 1 & 1/2 & 1/4 & 3/4 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{matrix}$$

$R_2 \to R_2 - 3R_3$

$R_2 - 6R_1$

$4 - \frac{2}{3}$

$$\begin{matrix} 1 & 1/2 & 1/4 & 3/4 \\ 0 & 0 & 10/3 & 5/2 \\ 2 & 1 & 0 & 1 \end{matrix}$$

$7 - \frac{3.6\ 3}{4\ 2}$

$R_3 \to R_3 - 2R_1$

$7 - \frac{9}{2}$

$$\begin{bmatrix} 1 & 1/2 & 1/4 & 3/4 \\ 0 & 0 & 10/3 & 5/2 \\ 0 & 0 & -1/2 & -1/2 \end{bmatrix}$$

$\frac{5}{2}$

$R_2 \to R_2 - 3R_3$

$0 - \frac{2}{4}$

$$\begin{bmatrix} 1 & 1/2 & 1/4 & 3/4 \\ 0 & 0 & 4 & 4 \\ 2 & 1 & 0 & 1 \end{bmatrix}$$

$1 - \frac{3}{2}$

$R_3 \to R_3 - 2R_1$

$$\begin{pmatrix} 1 & 1/2 & 1/4 & 3/4 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & -1/2 & -1/2 \end{pmatrix}$$

$0 - \frac{1}{2}$

$1 - \frac{3}{2}$

$R_3 \to 2R_3 - R_2$

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

$R_2 \to \frac{R_2}{4}$

$R_3 \to R_2 - 4\left(\frac{R_3}{-2}\right)$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{4} & \frac{3}{4} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

End