PAGE NO.:
DATE: / /

19/01/22

Preposition

Preposition → True, False

I Compound Preposition

i) logical connectives

1. conjunction → and ($\wedge$)
2. Disjunction → or ($\vee$)
3. Negation ($\sim$) →

r	p	q	$(p \wedge q)$	$(p \vee q)$
T	T	T		
F	T	T		
T	T	F		
F	T	F		
T	F	T		
F	F	T		
T	F	F		
F	F	F		

p	q	$(p \wedge q)$	$(p \vee q)$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

p	$\sim p$
T	F
F	T

Q1 $\sim p \vee \sim q$

p	q	$\sim p$	$\sim q$	$(\sim p \vee \sim q)$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Q2 $\sim (p \wedge q)$

p	q	$(p \wedge q)$	$\sim (p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

Q $\sim(\sim p \land q) \lor p$

p	q	$\sim p$	$(\sim p \land q)$	$(\sim p \land q) \lor p$
T	T	F	F	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F

statement / variable → cases

1 → 2

2 → 4 2^n where n = no. of inputs

3 – 8

4 – 16

(+) conditional connectives / implications → (if p then q)

$(p \Rightarrow q)$ $(p \rightarrow q)$

(p implies q)

p	q	$p \Rightarrow q$ (T F)	$q \Rightarrow p$ (T F)
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

{False} =

p	q	
T	F	$(p \rightarrow q)$
F	T	$(q \rightarrow p)$

otherwise it's true

Q $(p \Rightarrow q) \equiv (\sim p \lor q)$

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

$(p \rightarrow q)$

p	q	$\sim p \lor q$
T	T	T
T	F	F
F	T	T
F	F	T

$(\sim p \lor q)$

logically equivalent

PAGE NO.:
DATE: / /

4) $\sim(p \to q) \equiv (p \wedge \sim q)$

p	q	$p \to q$	$\sim(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

p	q	$p \sim q$	$(p \wedge \sim q)$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

$\sim(p \to q)$ logically equivalent

\# Converse → $q \to p$
\# inverse $\sim p \to \sim q$
\# Contra positive $\sim q \to \sim p$

p	q	$\sim p$	$\sim q$	$p \to q$	$q \to p$	$\sim p \to \sim q$	$\sim q \to \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

(converse) (inverse) (contra +ve)

5) **Bi- Conditional Connectives-** $(p \leftrightarrow q)$

// False when either of the input is false ~~or~~

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

{ p iff q or
p if and only if q }

$(p \leftrightarrow q) \to ((p \to q) \wedge (q \wedge p))$

Truth table -

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

(equal)

Q $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$q \wedge \sim p$	$\sim(p \leftrightarrow q)$	
T	T	F	F	F	F	T	F
T	F	F	T	T	F	F	T
F	T	T	F	F	T	F	T
F	F	T	T	F	F	T	F

$(p \wedge \sim q) \vee (q \wedge \sim p)$
F
T
T
F

Q $p \leftrightarrow q = (\sim p \vee q) \wedge (p \vee \sim q)$

p	q	$p \leftrightarrow q$	$\sim p$	$\sim q$	$\sim p \vee q$	$\sim q \vee p$	$(\sim p \vee q) \wedge (p \vee \sim q)$
T	T	T	F	F	T	T	T
T	F	F	F	T	F	T	F
F	T	F	T	F	T	F	F
F	F	T	T	T	T	T	T

Statements -

$$p \leftrightarrow q = (\sim p \vee q) \wedge (p \vee \sim q)$$
$$\sim (p \leftrightarrow q) = (\sim p \wedge q) \vee (p \wedge \sim q)$$

$p = (P_1, P_2, P_3 \ldots P_n)$

(compound statements) ↑ variables / statements

Tautology

If a proposition 'p' contains only true cases in the last column or if the result contains only True cases then it is tautology.

p	$\sim p$	$p \vee (\sim p)$
T	F	T
F	T	T

$(p \wedge q) \Rightarrow p$

p	q	$(p \wedge q)$	$(p \wedge q) \rightarrow p$	$\sim[(p \wedge q) \rightarrow p]$
T	T	T	T	F
T	F	F	T	F
F	T	F	T	F
F	F	F	T	F
			Tautology	contradiction

Contradiction

— when the resultant / last column contains only false cases, then it is contradiction, (opposite of tautology).

Logical Equivalence → (identical)

$P(P_1, P_2, P_3 \ldots P_n)$
$Q(~~Q~~ P_1, P_2, P_3 \ldots P_n)$

$\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

p	q	p ^ q	~(p^q)	~p	~q	(~p)v(~q)
T	T	T	F	F	F	F
T	F	F	T	F	T	F
F	T	F	T	T	F	F
F	F	F	T	T	T	T

identical

Q $(p \vee q) \leftrightarrow (q \vee p)$ is a tautology.

p	q	p v q	q v p	(pvq) ⟷ (qvp)
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

tautology.

Laws

① De-morgan's law -

$\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$
$\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

② Idempotent law —

$p \wedge p \equiv p$
$p \vee p \equiv p$

DATE: / /

3) Commutative law -

$$p \wedge q \equiv q \wedge p$$
$$p \vee q \equiv q \vee p$$

4) Associative law -

$$(p \wedge q) \wedge r = p \wedge (q \wedge r)$$
$$(p \vee q) \vee r = p \vee (q \vee r)$$

5) Distributive law -

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$
$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

6) Absorption law -

$$p \wedge (p \vee q) \equiv p$$
$$p \vee (p \wedge q) \equiv p$$

7) Involution law -

$$\sim(\sim p) \equiv p.$$

8) Complement law -

$$p \wedge (\sim p) = F \quad \text{(contradiction)}$$
$$p \vee (\sim p) = T \quad \text{(tautology)}$$

NORMAL FORM -

Conjunction → and $\wedge$ product

Disjunction → or $\vee$ sum

① Elementary Product

$p \quad q$
$\sim p \quad \sim q$

$p \wedge q \qquad p \wedge \sim p \wedge q$

$\sim p \wedge q \qquad \sim p \wedge \sim q$

$p \wedge \sim q$

② Elementary Sum

$p \vee q \qquad p \vee \sim q \qquad p \vee q \vee \sim q$

$\sim p \vee \sim q \qquad \sim p \vee q$

Types of Normal form

① CNF — Conjunctive Normal form
② DNF — Disjunctive Normal form
③ PCNF — Principle Conjunctive NF
④ PDNF — " Disjunctive NF

I CNF ⇒ product of elementary sum

$(p \vee q) \wedge (\sim p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee \sim q)$

element sum product

Q $\sim(p \leftrightarrow q)$ CNF

p	q	$p \leftrightarrow q$	$\sim p \leftrightarrow q$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	T	F

$$\left.\begin{array}{c}(p \vee q) \\ \wedge \\ (\sim p \vee \sim q)\end{array}\right] \Rightarrow (\sim p \vee \sim q) \wedge (p \vee q)$$

False terms forms CNF

PAGE NO. :
DATE : / /

Q $(p \leftrightarrow q) \equiv (p \to q) \wedge (q \to p)$

p	q	$p \to q$	$q \to p$	$p \leftrightarrow q$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

identical

Q $\boxed{p \to q \equiv \sim p \vee q}$ $\boxed{q \to p \equiv p \vee \sim q}$

p	q	$p \to q$	$\sim p$	$\sim p \vee q$	$q \to p$	$p \vee \sim q$
T	T	T	F	T	T	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

Q form CNF of $\sim (p \leftrightarrow q)$

$= \sim (p \leftrightarrow q)$

$= \sim [(p \to q) \wedge (q \to p)]$

$= \sim [(\sim p \vee q) \wedge (\sim q \vee p)]$

$= \sim (\sim p \vee q) \vee \sim (\sim q \vee p)$

$= (p \wedge \sim q) \vee (q \wedge \sim p)$ (By distributive law) ×

$= [p \vee (q \wedge \sim p)] \wedge (\sim q \vee (q \wedge \sim p))$

$= [(p \vee q) \wedge (p \vee \sim p)] \wedge [(\sim q \vee q) \wedge (\sim q \vee \sim p)]$ ✓

$= [(p \vee q) \wedge T] \wedge [T \wedge (\sim p \vee \sim q)]$

$= (p \vee q) \wedge (\sim p \vee \sim q)$

$= \text{CNF}$

product of elementary sum.

PAGE NO.:
DATE: / /

By truth table

$\boxed{(p \vee q) \wedge T \equiv (p \vee q)}$

p	Q	p∨q	T	(p∨q)∧T
T	T	T	T	T
T	F	T	T	T
F	T	T	T	T
F	F	F	T	F

identical

II $\boxed{\text{DNF}}$ - disjunctive normal form
(Sum of products)

A formula which is equivalent to a given formula & which consist of a sum of elementary product

↳ $(p \wedge \sim q) \vee (\sim p \wedge q)$

$\boxed{p \wedge (p \to q)} = \text{DNF}$

p	q	p→q	p∧(p→q)
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

$(p \wedge q)$

$$(\sim p \vee q) \equiv p \rightarrow q$$

$= p \wedge (p \rightarrow q)$

$= p \wedge (\sim p \vee q)$

$= (p \wedge \sim p) \vee (p \wedge q)$

$= F \vee (p \wedge q)$

$= p \wedge q$

Principle of Conjunctive Normal form (PCNF)

CNF $\rightarrow$ F   POS $\rightarrow$ product of elementary sum

DNF $\rightarrow$ T   SOP $\rightarrow$ sum of elementary product

SOP $\rightarrow$ Minterm   POS $\rightarrow$ Maxterm

$(p \wedge \sim p) \vee (p \wedge q)$ $(p \vee \sim p) \wedge (p \vee q)$

$\therefore$ p and q, both should exist

if (p, q, r) $\rightarrow$ minterm of 3 variable $\rightarrow$ $(p \wedge q \wedge r)$
$(p \wedge \sim q \wedge r)$, etc.

if (p, q, r) $\rightarrow$ maxterm of 3 variable $\rightarrow$ $(p \vee q \vee r)$,
$(p \vee q \vee \sim r)$, etc.

Let p and q are 2 propositional variables. All possible formulas which consist of product of p & its negation & product of q & its negation

PCNF - For a given formula of equivalent formula, consisting of conjunctions of the maxterms, is called PCNF.

CNF = Product of elementary sum

PCNF = Product of Maxterm

$$\underbrace{(p \vee q)}_{\text{sum}} \overset{\text{product}}{\wedge} (\sim p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee \sim q)$$

let (p, q, r) be 3 variables

$(\sim p \vee q) \wedge (q \vee r)$ — CNF ✓, PCNF ✗

$(\sim p \vee q \vee r) \wedge (q \vee r \vee p)$ — CNF ✓, PCNF ✓

$\boxed{\boxed{\text{PCNF}} \rightarrow F}$

$p \longleftrightarrow q \xrightarrow{\text{PCNF}} (\sim p \vee q) \wedge (p \vee \sim q)$

p	q	$p \longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$(p \vee \sim q) \wedge (\sim p \vee q)$

Formula -

$p \longleftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$p \rightarrow q \equiv \sim p \vee q$

$q \rightarrow p \equiv \sim q \vee p$

$p \wedge q = q \wedge p$

$p \vee q = q \vee p$

Q Find PCNF $(p \wedge q) \vee (\sim p \wedge q)$

$= (p \wedge q) \vee (\sim p \wedge q)$

$= [p \vee (\sim p \wedge q)] \wedge [q \vee (\sim p \wedge q)]$

$= [(p \vee \sim p) \wedge (p \vee q)] \wedge [(q \vee \sim p) \wedge (q \vee q)]$

$= [T \wedge (p \vee q)] \wedge [(q \vee \sim p) \wedge q]$

$= (p \vee q) \wedge [(q \wedge q) \vee (\sim p \wedge q)]$

$= (p \vee q) \wedge [q \vee (\sim p \wedge q)]$

$= (p \vee q) \wedge [(q \vee \sim p) \wedge (q \vee q)]$

$= (p \vee q) \wedge [(\sim p \vee q) \wedge q)$

$= (p \vee q) \wedge [(\sim p \vee q) \wedge (q \vee F)$

$= (p \vee q) \wedge (\sim p \vee q) \wedge (q \vee (p \wedge \sim p))$

$= (p \vee q) \wedge (\sim p \vee q) \wedge [(q \vee p) \wedge (q \vee \sim p)]$

$= (p \vee q) \wedge (\sim p \vee q) \wedge (q \vee p) \wedge (q \vee \sim p)$

$(p \vee q) \wedge (\sim p \vee q)$

$=$ PCNF

p	q	$p \wedge q$	$\sim p \wedge q$	$(p \wedge q) \vee (\sim p \wedge q)$
T F	T	T	F	T
T F	F	F	F	F
F T	T	F	T	T
F T	F	F	F	F

$(p \vee q) \wedge (\sim p \vee q)$

= PCNF

PAGE NO.:
DATE: / /

Q $T \wedge (\cdot q \vee \sim p)$

p	q	$\sim p$	$q \vee \sim p$	T	$T \wedge (q \vee \sim p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

identical

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

Q Find PCNF of $[(\sim p \to q) \wedge (q \leftrightarrow p)]$

$= (\sim p \to q) \wedge (q \leftrightarrow p)$

$= [\sim(\sim p) \vee q] \wedge [(p \to q) \wedge (q \to p)]$

$= [p \vee q] \wedge [(\sim p \vee q) \wedge (\sim q \vee p)]$

$= (p \vee q) \wedge (\sim p \vee q) \wedge (\sim q \vee p)$

= Maxterms

PDNF — Sum of Minterms

$= T$

of $(p, q) \Rightarrow (p \wedge q) \vee (\sim p \wedge \sim q)$ etc.

Q Find PDNF of $p \to q$

$= p \to q$

$= (\sim p \vee q)$

$= (\sim p \wedge T) \vee (q \wedge T)$

$= [\sim p \wedge (q \vee \sim q)] \vee (q \wedge (p \vee \sim p))$

$\overline{A'+B'} = A''.B''$

PAGE NO.:
DATE: / /

$= [(\sim p \wedge q) \vee (\sim p \wedge \sim q)] \vee ((q \wedge p) \vee (q \wedge \sim p))$

$= (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (q \wedge p) \vee (q \wedge \sim p)$

$=$ Minterm $(q \wedge \sim p) \vee (q \wedge p) \vee (\sim p \wedge \sim q)$

$p \rightarrow q$

p	q	$p \rightarrow q$	
T	T	T	$= (p \wedge q) \vee$
T	F	F	$= (\sim p \wedge q) \vee$
F	T	T	$= (\sim p \wedge \sim q)$
F	F	T	$\equiv (p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

Q Find PDNF $\rightarrow$ $p \rightarrow [(p \rightarrow q) \wedge \sim (\sim q \vee \sim p)]$

$p \rightarrow q = (\sim p \vee q)$

$= \sim p \vee [(p \rightarrow q) \wedge \underline{\sim (\sim q \vee \sim p)}]$ by demorgans law

$= \sim p \vee [(\sim p \vee q) \wedge (q \wedge p)]$

$= \sim p \vee [[(\sim p \vee q) \wedge q] \wedge (p \wedge (\sim p \vee q))]$

$= \sim p \vee [(q \wedge \sim p) \vee (q \vee q)] \wedge (p \wedge \sim p) \vee (p \wedge q)$

$= \sim p \vee [(q \wedge \sim p) \vee q] \wedge \{F \vee (p \wedge q)$

$= \sim p \vee (q \vee q) \wedge (\sim p \vee q) \wedge F \vee (p \wedge q).$

$= \sim p \vee (q \wedge \sim p) \vee (q \wedge q) \vee (p \wedge q)$

$= \sim p \vee (q \wedge \sim p) \vee (q) \vee (p \wedge q)$

$= (\sim p \wedge T) \vee (q \wedge \sim p) \vee (q \wedge T) \vee (p \wedge q)$

$= [\sim p \wedge (q \vee \sim q)] \vee (q \wedge \sim p) \vee (q \wedge (p \vee \sim p)) \vee (p \wedge q)$

$= (\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (q \wedge \sim p) \vee (q \wedge p) \vee$
$(q \wedge \sim p) \vee (p \wedge q)$

DATE : / /

$(\sim p \wedge q) \vee (p \wedge q) \vee (\sim p \wedge \sim q)$

By truth table.

$p \rightarrow [(p \rightarrow q) \wedge \sim(\sim q \vee \sim p)]$

p	q	$p \rightarrow q$	$\sim p$	$\sim q$	$\sim q \vee \sim p$	$\sim(\sim q \vee \sim p)$
T	T	T	F	F	F	T
T	F	F	F	T	T	F
F	T	T	T	F	T	F
F	F	T	T	T	T	F

$(p \rightarrow q) \wedge \sim(\sim q \vee \sim p)$	$p \rightarrow [(p \rightarrow q) \wedge \sim(\sim q \vee \sim p)]$
T	T
F	F
F	T
F	T

$= (p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$ = Minterms

Logical inference -

problem → valid → Tautology
or
problem → invalid

Rules	Statement	Other forms
① Addition Rule	p $\therefore p \vee q$	$p \rightarrow (p \vee q)$
② Simplifications	$p \wedge q$ $\therefore p$	$(p \wedge q) \rightarrow p$

DATE: / /

3 Modus Ponens	p $p \rightarrow q$ $\therefore q$	$[p \wedge (p \rightarrow q)] \rightarrow q$
4 Modus ~~Ponens~~ Tollens	$\sim q$ $p \rightarrow q$ $\therefore \sim p$	$[(\sim q) \wedge (p \rightarrow q)] \rightarrow \sim p$
5 Disjunctive Syllogism	$p \vee q$ $\sim p$ $\therefore q$	$[(p \vee q) \wedge \sim p] \rightarrow q$
6 Transitive Rule	$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
7) Conjunction	p q $\therefore p \wedge q$	$(p \wedge q) \rightarrow (p \wedge q)$
8 Constructive Dilemma	$(p \rightarrow q) \wedge (r \rightarrow s)$ $(p \vee r)$ $\therefore (q \vee s)$	$[\{(p \rightarrow q) \wedge (r \rightarrow s)\} \wedge (p \vee r)] \rightarrow (q \vee s)$
9 Destructive Dilemma	$(p \rightarrow q) \wedge (r \rightarrow s)$ $\sim q \vee \sim s$ $\therefore \sim p \vee \sim r$	$[\{(p \rightarrow q) \wedge (r \rightarrow s)\} \wedge (\sim q \vee \sim s)] \rightarrow (\sim p \vee \sim r)$

If you invest in stock market, then you will get rich, if you get rich, then you will be happy. Therefore, if you invest in the stock market, then you will be happy

Transitive Rule.

$p \to q$
$q \to s$
$\therefore p \to s$

$[(p \to q) \wedge (q \to s)] \to (p \to s)$

p: you will invest in stock market
q: you will get rich
s: you will be happy.

p	q	s	$p \to q$	$q \to s$	$(p \to q) \wedge (q \to s)$	$p \to s$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	F	T
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

$(p \to q) \wedge (q \to s) \to (p \to s)$

T
T
T
T
T $\to$ Tautology.
T +
T valid

DATE: / /

Q Determine the validity of the arguments:
It is below freezing now. Therefore, it is either below freezing or raining now.

p: It is below freezing now
q: It is raining now

$$\frac{p}{\therefore p \vee q} \qquad p \to (p \vee q) \text{ (Addition Rule)}$$

p	q	p∨q	p → (p∨q)
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Tautology + valid.

Q If 2 sides of a triangle are equal, then the opposite angles are equal.
If two sides are not equal, therefore the opposite angles are not equal

$$\begin{array}{l} p \to q \\ \underline{\sim p} \\ \therefore \sim q \end{array} \qquad [(p \to q) \wedge (\sim p)] \to \sim q$$

p	q	~q	p → q	~p	(p→q) ∧ (~p)	→ ~q
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	T	T	F
F	F	T	T	T	T	T

invalid

Predicates

Let a statement :

'$\underbrace{x}_{\text{subject of statement}}$ is $\underbrace{\text{a positive integer}}_{\text{predicate (property)}}$'

$P(x) \rightarrow$ predicate on x

proportional function

value $= x$

Q Let $P(x)$ denotes the statement "$x > 3$"

What are the truth values of $P(2)$ & $P(4)$

$P(x) = x > 3$

$P(2) = 2 > 3, \Rightarrow F$

$P(4) = 4 > 3, \rightarrow T$

Q Let $q(x,y)$ be the statement

$x = y + 3$

What are the truth values of the statements

$q(1,3)$ $q(3,0)$

~~P(q~~ $q(1,3)$, $x = 1$

$y = 3$

$q(1,3) = 1 = 3 + 3$

$1 \neq 6$ F

$q(3,0) =$ $3 = 0 + 3$

$3 = 3$ T

Quantifiers (indicating quantity)

indicate quantity

- **All** birds have wings
- **Some** students are intelligent
- **For every** integer x, x^2 is non-negative integer
- **There exist** real number whose sequence is equal to itsef

2 types
- Universal
- Existential

i) Universal Quantifiers – $\forall$ for all

→ represented by '$\forall$' (for all). The word all is used as universal quantifier

→ all, for all, for every, for every each, everything, each thing

→ The universal quantification of $P(x)$ is the statement
" $P(x)$ is true for all the values of x in the universe of discourse"

universe of discourse → is the domain that specifies the possible values of the variable 'x'.

→ The notation for all x $P(x)$
$\boxed{\forall x\, P(x)}$ denotes the universal quantification of $P(x)$.

DATE: / /

Q "Every" $\boxed{\text{student}}$ in the class has studied calculas". Express the statement as universal predicate quantification.

UQ (S) UOD

$P(x)$ x has studied calculas

$\forall x \, P(x)$

$P(x)$ is true for all the value of x in universe of discourse.

Q What is the truth value of (for all) $\forall x \, P(x)$ where $P(x)$ is the statement ~~$x^2 > 10$~~ $x^2 < 10$ and UOD consists the positive integer not exceeding 4.

$P(x) = x^2 < 10$

$P(1) = 1^2 < 10$ T

$P(2) = 2^2 < 10$ T

$P(3) = 3^2 < 10$ T

$P(4) = 4^2 < 10$ F (not an universal quantifier)

ii) Existential Quantifier → $\exists$ (there exists)

→ Some, there exist, there is atleast, there is on

→ The existential quantifier is represented by letter '$\exists$'.

→ The existential quantifier of $P(x)$ is the statement,

" there exists an element 'x' in the UOD for which $P(x)$ is true".

PAGE NO.:
DATE: / /

→ The notation $\boxed{\exists x\ P(x)}$ denotes the existential Quantification of $P(x)$

Statement	When True?	When False?
$\forall x\ P(x)$	$P(x)$ is true for all the value of x	There is atleast one 'x' for which $P(x)$ is false
$\exists x\ P(x)$	There is atleast one x for which $P(x)$ is true	$P(x)$ is false for every x.

Q. Express the statement if (neglect → connector) somebody (S) is a female (predicate), & is a parent, then this person is someone's mother as a logical form

$F(x)$ = is a female
$P(x)$: is a parent
$M(x,y)$: x is the mother of y.

$$\forall x\,[\,F(x) \wedge P(x)\,] \Longrightarrow M(x,y)$$