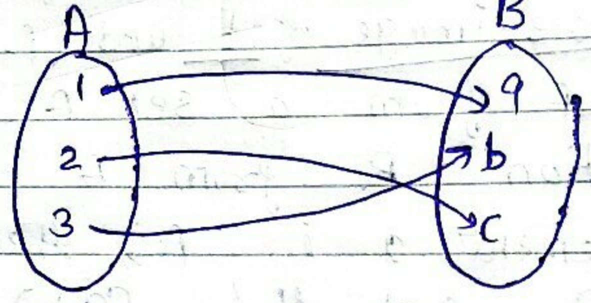
Function → Let A and B are 2 sets suppose that ~~to~~ each element of the set A we assign a unique element of the set B then the collectn of such assignment is called a Fn from set A to B.

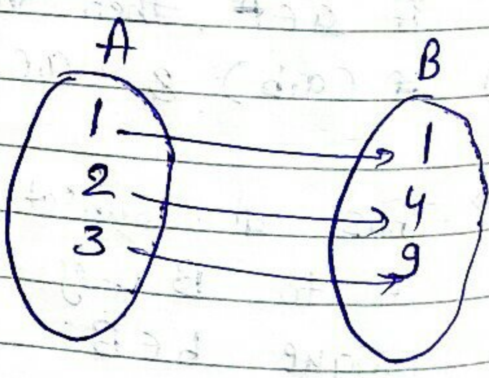


$$f : A \to B$$

eg → Consider the function which sends each real numbes to 1 of its square.

$$f(1) = a$$
$$f(2) = c$$
$$f(3) = b$$

$$\begin{array}{ccc} f(x) & = & x^2 \\ \downarrow & & \downarrow \\ A & & B \end{array}$$



$f(1) = 1$
$f(2) = 4$
$f(3) = 9$

Function

$f: A \to B$ when $f(a) = b$

→ f is a function from set A to B,
→ b is f-image of a, under f.
→ Set A is called domain for function f, (i/p)
→ Set B, codomain
→ $f(A)$ is called range. [The set of all images] (o/p)
→ a is called pre-image of b under f.

Def → A function f from a set A to B is a binary relation R from A to B such that for every element a in A, there exists unique element in B such that (a,b) is an element of relation R.

or

Function (mapping) →
Let A and B be two nonempty sets.
A function f from A to B is a subset of $A \times B$ such that,

(i) if $a \in A$ then there exists $b \in B$ such that $(a,b) \in f$
(ii) if (a,b) & (a,c) are in f then $b = c$

⇒ Thus a subset of $A \times B$ is called a function from A to B if to each $a \in A$ there exists a unique $b \in B$ such that $(a,b) \in f$.

range, $f(A) = \{f(a) \mid a \in A\}$

eg → $f: N \to N$, $f(x) = 2x$, $\forall x \in N$, is the set of even natural no.s $\{2, 4, 6, \_\_\}$

→ x^2 is a function, $f(x) = x^2$

Fn name, i/p, o/p

CoDomain, Domain, Range of a Function

Domain → Let f is defined from set A to B, then the set A is K/n as the domain of a f. Ie

Co-domain Set B is the Codomain of f.

eg → $A = \{2, 3, 4\}$ $B = \{1, 4, 9, 16, 25\}$

$$f(x) = x^2$$

$x = 2$ $f(x) = 4$

$x = 3$ $f(x) = 9$

$x = 4$ $f(x) = 16$

Domain $= A = \{2, 3, 4\}$

Co-Domain $= B = \{1, 4, 9, 16, 25\}$

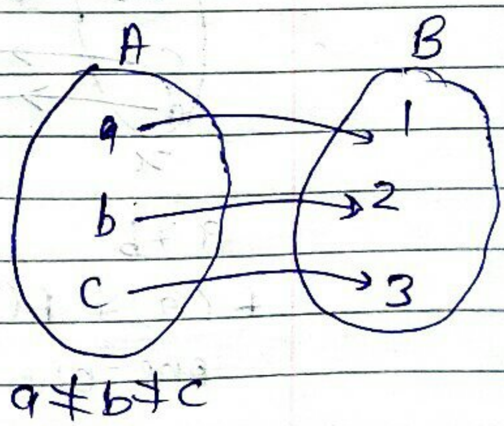
Range $= f(x) = \{4, 9, 16\}$

Range → Range of f = set of f-images of A.

I **One-One Function or Injection** →

$f : x \to y$

A function f from $x \xrightarrow{to} y$ is said to be one-one if d/f element of x have d/f images in y.



$x_1, x_2 \in X$

$f(x_1), f(x_2) \in Y$

$x_1 \neq x_2$

$f(x_1) \neq f(x_2)$

$a \neq b \neq c$

Or

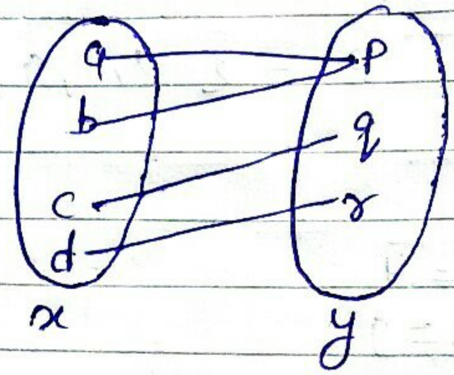
$x_1 = x_2$

$f(x_1) = f(x_2)$

II $\boxed{\text{onto Function} \mid \text{sur-jection}}$ →

A f from $x \to y$ is said to be onto if every element of y is an image of some element of x.

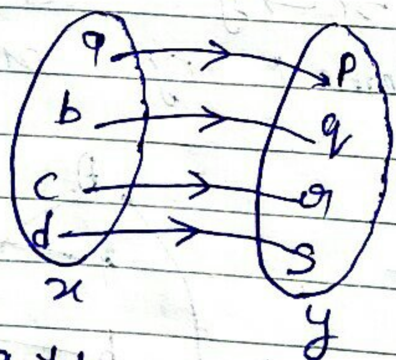
eg →



one-one ✗

III $\boxed{\text{Bi-jection Function}}$ A $f: A \to B$ is a bijection if it's one-one as well as onto.
(one-one - onto Function)

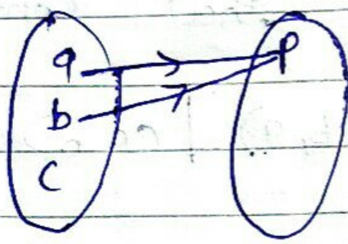
eg →
$x = \{a, b, c, d\}$
$y = \{p, q, r, s\}$
$f: x \to y$
let $f: x \to y$ is defined as $f(a) = p$
$f(b) = q$
$f(c) = r$
$f(d) = s$



$a \neq b$
$f(a) \neq f(b)$
one-one ✓

Note :

$\boxed{\text{Different Element of } x}$ may be associated with the same element of y.



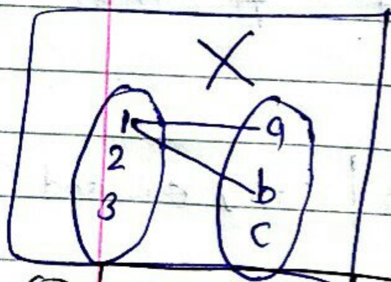
There may exist some element of y which are not associated with any element of x.

to each element x in X there exist a unique element y in Y.

$x \in X$

$y \in Y$

Such that $y = f(x)$



(4) Inverse Function → If $f: A \to B$ is one one-onto fn then we can define a new fn from $B \to A$ in which every element of B is related by its pre image in set A. This type of fn is called inverse fn of f [f^{-1}].

$\boxed{f^{-1} : B \to A}$ as $\boxed{f^{-1}(y) = x} \Leftrightarrow \boxed{y = f(x)}$ $\forall\, y \in y, x \in x$

① $\boxed{\text{one-one Function}} \rightarrow$

eg → $F: R \to R$

Such that

$$f(x) = x^3 + 2$$

$$x \in R$$

$x_1, x_2 \in R$

$$f(x_1) = f(x_2)$$

$$x_1^3 + 2 = x_2^3 + 2$$

$$x_1 = x_2$$

eg $f: Z \to Z$

$$f(x) = x^2 + 1 \quad \forall x \in Z$$

$x_1, x_2 \in Z$

$$f(x_1) = f(x_2)$$

$$x_1^2 + 1 = x_2^2 + 1$$

$$x_1^2 = x_2^2, \quad x_1 = \pm x_2$$

~~Hence proved~~ - it is not a one-one function.

$2 \in Z$

$$f(2) = 4 + 1 = 5$$

$$f(-2) = 4 + 1 = 5$$

$x_1 \neq x_2 \longrightarrow f(x_1) = f(x_2)$ Not possible.

$$f(x_1) \neq f(x_2)$$

② onto (Surjection) →

A Fn $F: x \to y$ is said to be onto if every element of y is an image of some element of x.

③ one-one onto Function (Bijection) →

A fn F is a bijection if it's one-one as well as onto.

⑤ $\boxed{\text{Identity Function}}$ → Any function $f: A \to A$ is an identity function I_A if & only if

$$f(x) = x \quad \forall x \in A$$

⑥ $\boxed{\text{Composition of Function}}$ →

If function $f: A \to B$ &
function $g: B \to C$ are 2 fn then
fn $(g o f): A \to C$ where

$$g o f(a) = g[f(a)], \forall a \in A$$

is called composition of fns f & g.

→ Composite fn gof is defined when range of the fn f is subset of domain of fn g ie. Range f ⊆ Domain g.

eg → $f: R \to R$, $f(x) = x^2$, $g: R \to R$, $g(x) = x+7$

Composite fn $g o f: R \to R$ is defined as

$$g o f(x) = g[f(x)]$$
$$= g(x^3) = x^3 + 7$$

& $(f o g): R \to R$

$$f o g(x) = f[g(x)]$$
$$= f(x+7)$$
$$= (x+7)^3$$

⇒ $g o f \neq f o g$.