Graph Theory →

vertex V → non-empty finite Set

edges E → $(1,2)\ (1,3)\ (2,3)$

↑ paired element of V

$V = \{1,2,3\}$

$V = \{v_1, v_2, v_3 \ldots\}$

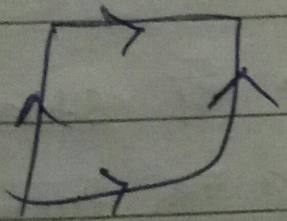
$G(V, E)$

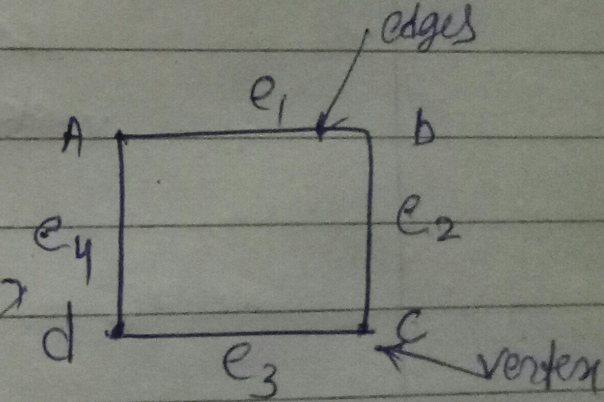
Graph →

$G(V, E)$

Directed G.

undirected G.

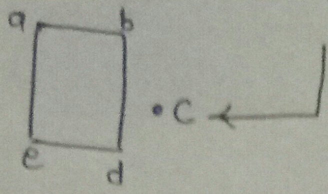




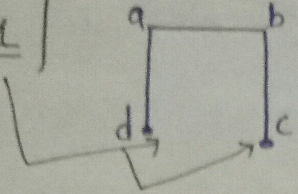
$V = \{a, b, c, d\}$

$E = \{(a,b), (b,c), (c,d), (d,a)\}$
$= \{e_1, e_2, e_3, e_4\}$

Isolated Vertex → is a vertex which has no connected edge.
↳ degree is zero.



Pendent Vertex



NULL Graph → G (V, E)

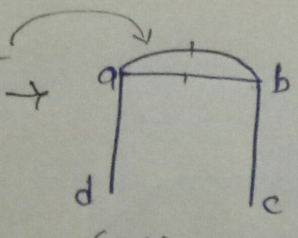
→ Single vertex NULL Graph → •a

→ Double vertex NULL Graph → •a •b

↳ Compulsory Condition for graph is a vertex must be non-empty set.

Non-Empty Set → set.

Parallel Edge →

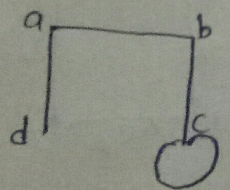


(a, b) (a, d) (b, c)

more than one edges exist b/w 2 vertexes.

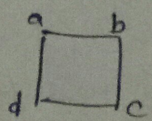
vertices (कोने, सिरा, शिखर, चोटी)

Loop



a edge start from a vertex & ends to same vertex, is k/n as loop.

Simple Graph → G (V, E)



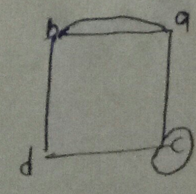
→ No loop

→ No parallel edges

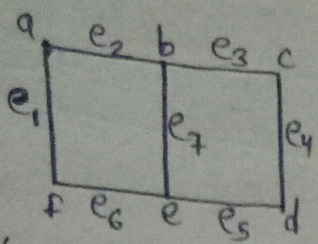
Multi Graph →
G (V, E)

→ loop

→ having parallel edges also



Finite Graph →

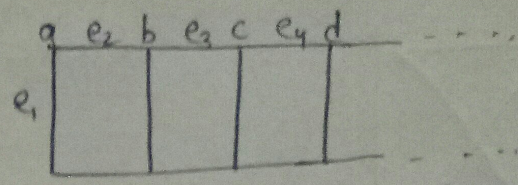


Finite vertices & finite edges.

$V = \{a, b, c, d, e, f\}$

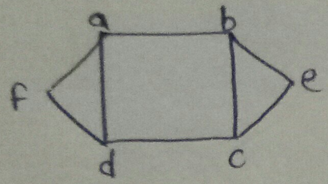
$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$

Infinite Graph →



infinite vertices & edges.

Order of Graph → The number of vertices in a graph is called the order of graph.



G (V, E)

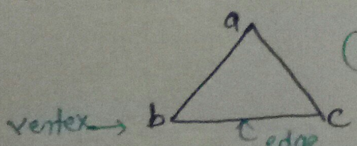
order = 6

Size = 8

Size of a Graph → The number of edges in a graph is called size.

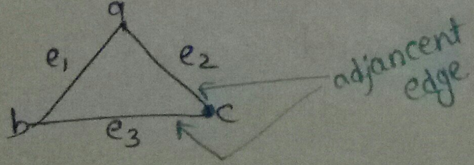
Adjacent vertex → 2 Vertex (u, v) Connected by an edge is called adjancent vertex in a graph.

(a,b) (b,c) (a,c)

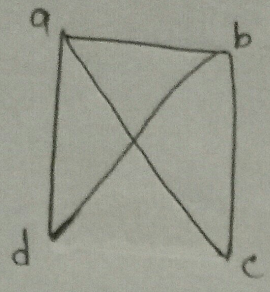


↳ Loop is not consider as adjancent vertex.

Adjancent Edge → 2 non-parallel edges are said to be adjancent if they both are connected to a same vertex.



Degree of a vertex → number of connected edges.



G (V, E)

degree of a ⇒ deg (a) = 3

deg (b) = 3

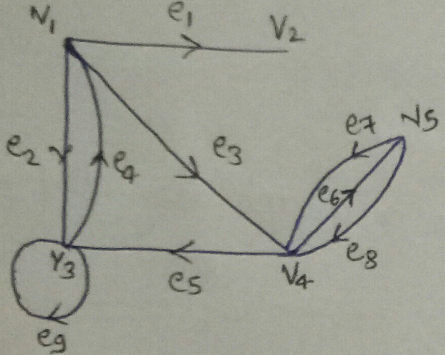
deg (c) = 2

deg (d) = 2

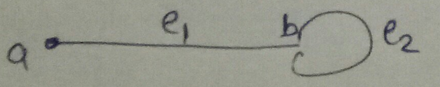
Directed Graph

(I.V.) a —e_1→ b (T.V.)

initial vertex   Terminal vertex



Note → A Loop contribute 2 count in finding degree of a graph.

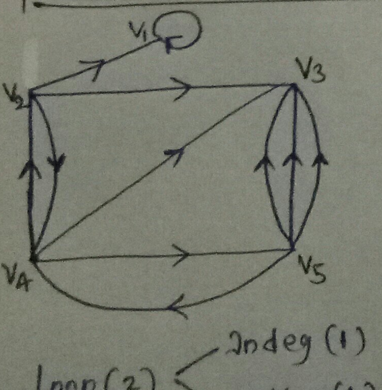


deg (a) = 1

deg (b) = 3

⇒ In directed graph, same directed parallel edge → (in ex.) $e_7 \parallel e_8$

Degree of Vertex in directed graph →



Loop (2) < Indeg (1), Outdeg (1)

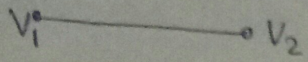
deg(V_1): indegree (−) $deg^-(V_1)$ | outdegree (+) $deg^+(V_1)$

indegree	outdegree
$deg^-(V_1) = 2$	$deg^+(V_1) = 1$
$deg^-(V_2) = 1$	$deg^+(V_2) = 3$
$deg^-(V_3) = 5$	$deg^+(V_3) = 0$
$deg^-(V_4) = 2$	$deg^+(V_4) = 3$
$deg^-(V_5) = 1$	$deg^+(V_5) = 5$

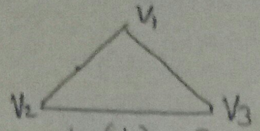
Regular Graph

2 condⁿ
1. simple Graph
2. every vertex has same degree

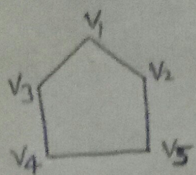
Let $G(V,E)$ is a Simple graph.



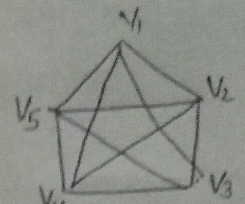
$\deg(V_1) = 1$
$\deg(V_2) = 1$



$\deg(V_1) = 2$
$\deg(V_2) = 2$
$\deg(V_3) = 2$



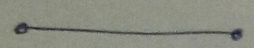
every vetex deg = 2



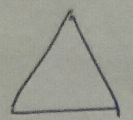
every V deg = 4

r - regular Graph

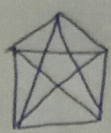
↓ degree of vertex



1 - regular Graph



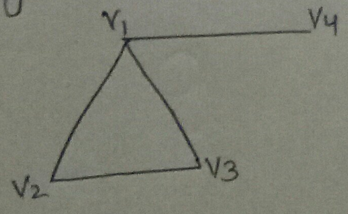
2- RG



4-RG

Even Vertex In a graph if the degree of a vertex is an even integer then it's called even vertex.

odd vertex If in a graph the degree of a vertex is a odd integer then it's called odd vertex.



$\deg(V_1) = 3$
$\deg(V_4) = 1$
→ odd vertex O.V.

$\deg(V_2) = 2$
$\deg(3) = 2$
→ Even vertex E.V.

Theorem I → Hand-shaking Theorem

$\boxed{\deg(V) = 2 \times edge}$ $\boxed{\sum_{v=V} \deg(v) = 2e}$

Statement → The sum of the degree of all vertices in a graph is equal to the twice the number of edges in a graph.

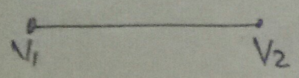
proove → let $G(V,E)$ be a graph.

where $V = \{v_1, v_2, v_3 \ldots v_n)$ set of vertices

and $E = \{e_1, e_2, e_3, \ldots e_n)$ set of edges.

We know that every edge line b/w 2 vertex, so it provide degree 1 to each vertex. hence every edge is contribute degree 2 for the graph.

So sum of the degree of all vertices is equal to the twice of the no. of edges in a graph.



edge = 1

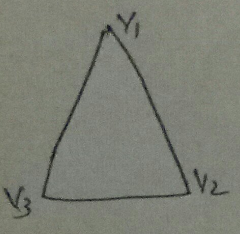
$\deg(v_1) = 1$

$\deg(v_2) = 1$

$$\deg(V) = \deg(v_1) + \deg(v_2)$$
$$= 1+1$$
$$= 2$$

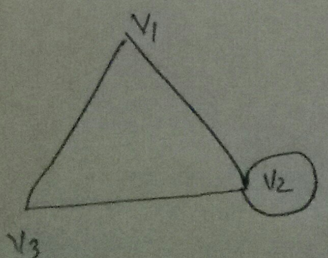
$\Rightarrow 2 \times 1 = 2$

$2 \times edge = \deg(V)$



$\deg(V) = 2+2+2 = 6$

$\Rightarrow 2 \times 3 = 6$



$\deg(V) = 2+2+4 = 8$

$\Rightarrow 2 \times 4 = 8$

Theorem - II

Statement → The number of vertices of odd degree in a graph is always even.

$$\boxed{\sum_{v \in odd\,v} deg(v) = even}$$

prove → $G(V,E)$

no. of edges → e

$$\sum_{v=V} deg(v) = 2e \longleftarrow \text{Theorem 1}$$

V_1 = degree odd no. vertex $= \{v_1, v_3, v_4\}$

V_2 = degree even no. vertex $= \{v_2, v_5, v_6\}$

$$V = V_1 \cup V_2$$

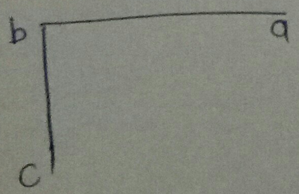
$$\sum_{v \in V_1} deg(v) + \sum_{v \in V_2} deg(v) = 2e$$

$$\sum_{v \in V_1} deg(v) = 2e - \sum_{v \in V_2} deg(v)$$

$$\boxed{\sum_{v \in V_1} \underset{\uparrow \text{odd}}{deg}(v) = even}$$

Degree sequence of a graph →

→ The ascending order of degrees of vertices of $G(V,E)$ is called degree sequence.



$deg(a) = 1$

$deg(b) = 2$

$deg(c) = 1$

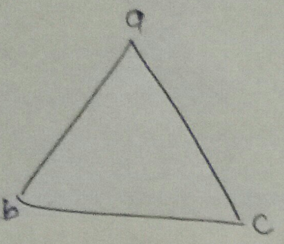
Degree Sequence $= (1, 1, 2)$

ascending order

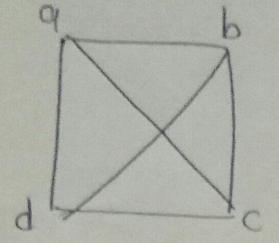
Small → big

$\boxed{\text{Complete Graph}}$

A Graph $G(V,E)$ which is simple Graph is said to be complete graph if that contain fully connected edges b/w each pair of vertices.



K_3



K_4

$\boxed{K_n}$ n = number of vertex

Universal Graph

number of edges in complete graph $= \dfrac{n(n-1)}{2}$

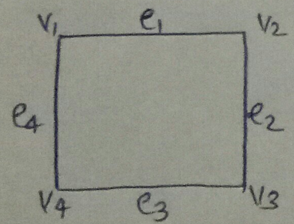
vertex $= n = 3$

no. of edges $= \dfrac{3(2)}{2} = 3$

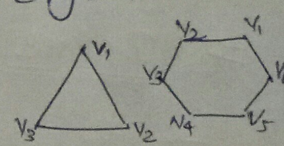
$\boxed{\text{Cycle}}$

→ A graph $G(V,E)$ with $\{V_1, V_2, V_3 \dots V_n\}$ vertices & $\{e_1, e_2, e_3 \dots e_n\}$ edges is called cycling & denoted by C_n where $n >= 3$.

If it is a continuous chain of vertices & edges.



$V_1 e_1 V_2 e_2 V_3 e_3 V_4 e_4 V_1$

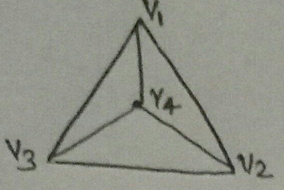


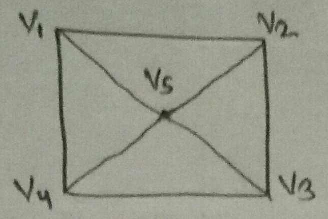
Cond^n For cycle
- → No loop
- → No parallel edges
- → same starting & ending vertex point in walking of continuous cycle

$\boxed{\text{Note}}$ → A cycle is a 2-Regular Graph (2-RG).

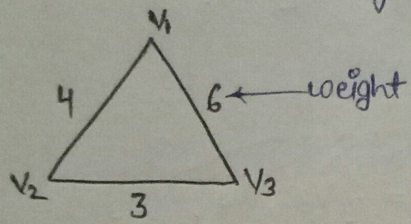
(5)

* **Wheel** → To Find wheel include a new vertex in a cycle & connect this new vertex to each of the vertex of cycle, we get wheel.





Weighted Graph → In a graph, if the every edge of the graph is assign by an positive integer number then the graph is called weighted graph.



Bipartite Graph → A simple $G(V,E)$ graph is said to be bipartite, if the set of vertices (V) can be divided into 2 non-empty subset V_1 & V_2 be such that :→

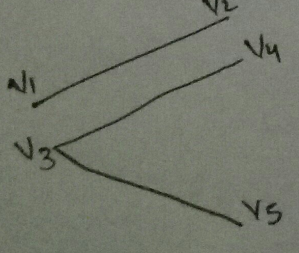
I every vertex of V_1 is connected with some vertex of V_2 or every vertex of V_2 is connected with some vertex of V_1.

↑ Property

II There is no edge b/w the vertices of set V_1 itself, & V_2 itself.

$\boxed{V_1 \cap V_2 = \phi}$

$V_1 = \{a, b, c\}$
$V_2 = \{e, F\}$



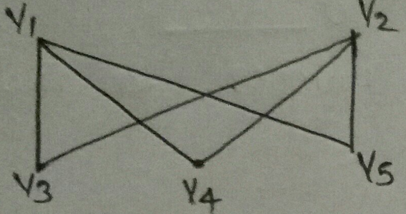
$\boxed{V = \{V_1, V_2, V_3, V_4, V_5\}}$

$V_1 = \{V_1, V_3\}$ $V_2 = \{V_2, V_4, V_5\}$

Complete Bipartite Graph → In a bipartite graph, $G(V_1, V_2, E)$ if every vertex of V_1 is connected with every vertex of V_2 then it is called complete Bipartite Graph.

$$K_{m,n}$$

where, m = is the no. of vertex in set V_1
n = is the no. of vertices in set V_2

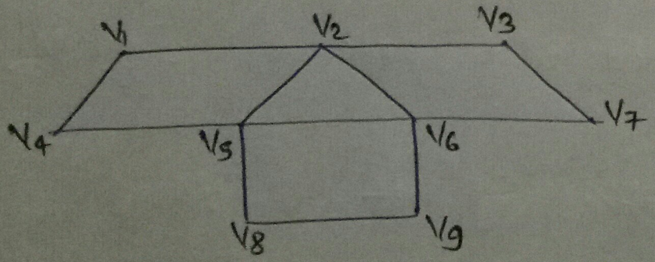


$V_1 \cap V_2 = \phi$ ⇒ $K_{2,3}$

$V = \{V_1, V_2, V_3, V_4, V_5\}$

$V_1 = \{V_1, V_2\}$ $V_2 = \{V_3, V_4, V_5\}$

Que. check it is a complete Bipartite graph or not?



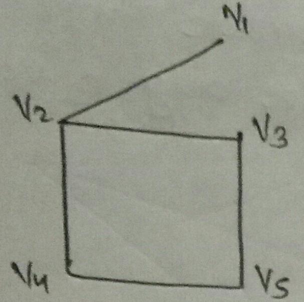
$V_1 = \{V_1 \quad V_3 \quad V_5 \quad \boxed{V_6 \quad V_9}\}$

$V_2 = \{V_2 \quad V_4 \quad V_7 \quad \boxed{V_8 \quad V_6}\}$

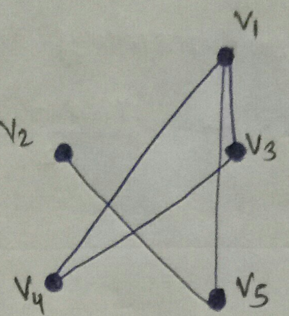
X

⇒ it is not a complete Bipartite graph.

Complementary Graph → Let $G(V,E)$ be a simple graph with n vertex & E edges, then the complementary graph $\bar{G}(V,E')$ is a graph, which contain all the vertices of G & all those edges which are not in G but exist in K_n. ⑥



$G(V,E)$
Simple Graph

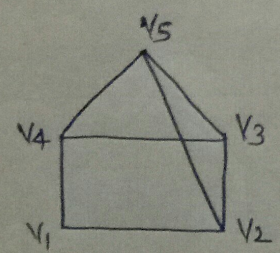


$\bar{G}(V,E')$
Complementary Graph

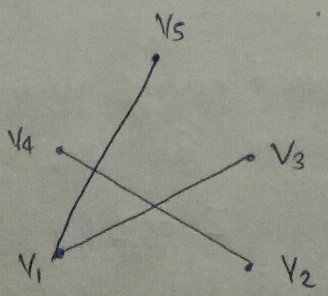
Property I → no. of edges in complementary graph $= E' = \frac{n(n-1)}{2} - e$

$= \frac{5(4)}{2} - 5$

$E' = 5$



$G(V,E)$



$\bar{G}(V,E')$

Property III

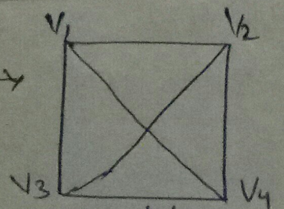
in G, $\deg(V_1) = 2$

$r = 2$

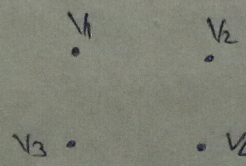
in $\bar{G}$, $\deg(V_1) = n - r - 1$

$= 5 - 2 - 1 = 2$

Property II →



Complete Graph



$\bar{G}$
Complementary Graph

← $E' \Rightarrow$ no edge

Note

① **Property I** → Let $G(V,E)$ is a graph of n vertices & e edges then the no. of edges in complementary graph $\bar{G}$ is $\left(\frac{n(n-1)}{2} - e\right)$.

② **Property II** → The complementary Graph of a Complete Graph K_n with n vertices & $\frac{n(n-1)}{2}$ edges, have only n vertices & no edges.

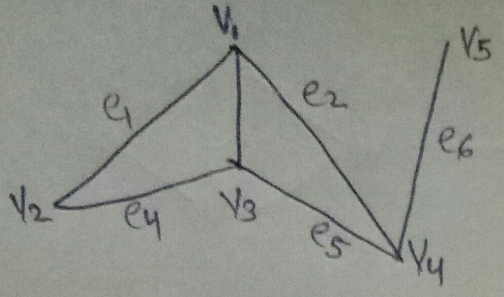
③ **Property III** → If in a graph r be the degree of any vertex V then the degree of this vertex in complementary graph is $(n-r-1)$.

Isomorphic Graph →

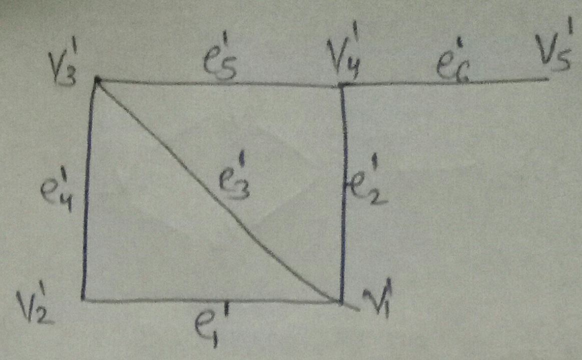
$$G_1(V_1, E_1) \cong G_2(V_2, E_2)$$

Cond^{n}s	G_1	G_2
I	(G_1) no. of vertices are same in	no. of vertices (G_2)
II	no. of edges are	same in G_2 no. of edges
III	Sum of deg of v $\Sigma \deg(v)$	$\Sigma \deg(v)$
IV	deg sequence of G_1 same as	deg sequence of G_2
V	any vertex degree — also in deg $(v_1) = 1$ v_1, v_2, v_3 = deg ③	$v_4\ v_2\ v_3$ deg 3 any vertex degree deg $(v_1{*}) = 1$
VI	Adjacent matrix of G_1 is	$Adj(G_2)$ are same

2 graph are called Isomorphic, if these Cond^{ns} are fullfilled.



G



G'

Q Show that G & G' are isomorphic or not?

S I $|V| = 5$ ✓ $|V| = 5$

II $|E| = 6$ ✓ $|E| = 6$

III $\Sigma \deg(V) = 3+2+3+3+1 = 12$ ✓ $\Sigma \deg(V) = 3+2+3+3+1 = 12$

IV degree sequence $\Rightarrow$ $(1,2,3,3,3)$ ✓ deg seq $\Rightarrow$ $(1,2,3,3,3)$

$\deg(V_1) = 3$
$\deg(V_2) = 2$
$\deg(V_3) = 3$
$\deg(V_4) = 3$
$\deg(V_5) = 1$
$\Sigma \deg(V) = 12$

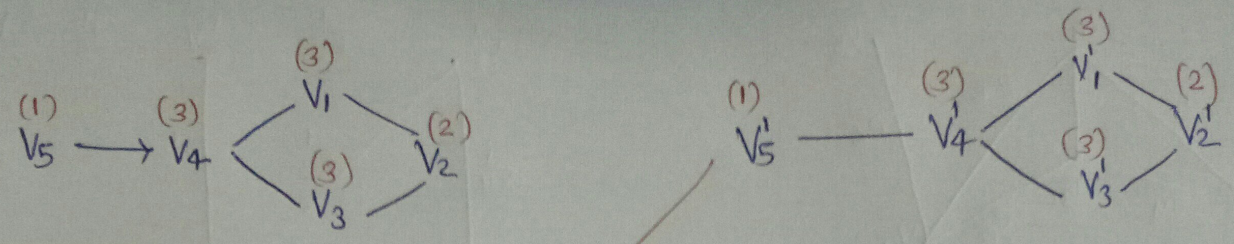
$\deg(V_1') = 3$
$\deg(V_2') = 2$
$\deg(V_3') = 3$
$\deg(V_4') = 3$
$\deg(V_5') = 1$
$\Sigma \deg(V) =$

V Adjacent Matrix $\rightarrow$ Cond^n $\rightarrow$ No parallel edge ✓

	V_1	V_2	V_3	V_4	V_5
V_1	0	1	1	1	0
V_2	1	0	1	0	0
V_3	1	1	0	1	0
V_4	1	0	1	0	1
V_5	0	0	0	1	0

	V_1'	V_2'	V_3'	V_4'	V_5'
V_1'	0	1	1	1	0
V_2'	1	0	1	0	0
V_3'	1	1	0	1	0
V_4'	1	0	1	0	1
V_5'	0	0	0	1	0

VI

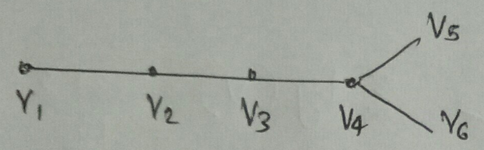


degree walking sequence

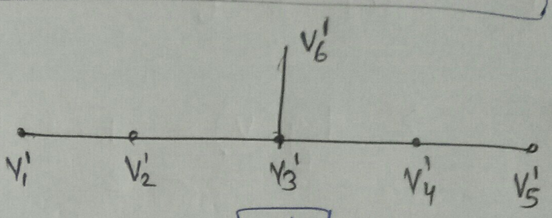
(3) deg < V_1 → 2 → 3 (deg) (deg), V_2, V_4 — deg walking sequence same ↓ then isomorphic

V_3' — (2) — (3) deg deg
(3) deg < V_4', V_1'

Q



$\boxed{G}$



$\boxed{G'}$

Not a isomorphic Graph

I $|V| = 6$ $|V| = 6$

II $|E| = 5$ $|E| = 5$

III
$deg(V_1) = 1$ $deg(V_1') = 1$
$deg(V_2) = 2$ $deg(V_2') = 2$
$deg(V_3) = 2$ $deg(V_3') = 3$
$deg(V_4) = 3$ $deg(V_4') = 2$
$deg(V_5) = 1$ $deg(V_5') = 1$
$deg(V_6) = 1$ $deg(V_6') = 1$

VI deg seq (1, 1, 1, 2, 2, 3) deg seq (1, 1, 1, 2, 2, 3)

IV Adj matrix

	V_1	V_2	V_3	V_4	V_5	V_6
V_1	0	1	0	0	0	0
V_2	1	0	1	0	0	0
V_3	0	1	0	1	0	0
V_4	0	0	1	0	1	1
V_5	0	0	0	1	0	0
V_6	0	0	0	1	0	0

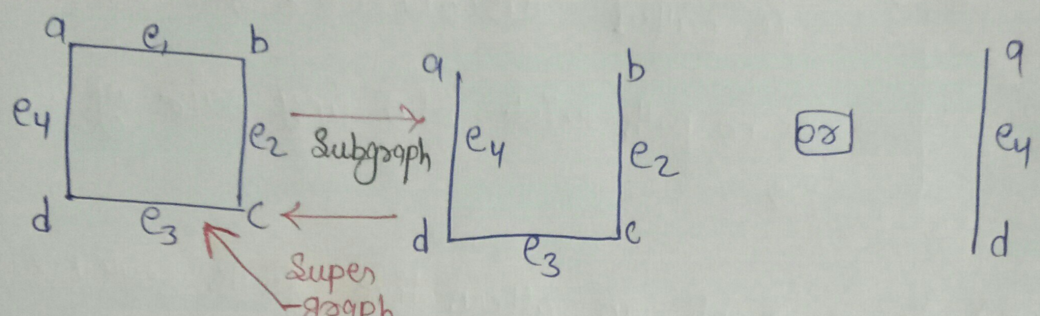
X

	V_1'	V_2'	V_3'	V_4'	V_5'	V_6'
V_1'	0	1	0	0	0	0
V_2'	1	0	1	0	0	0
V_3'	0	1	0	1	0	1
V_4'	0	0	1	0	1	0
V_5'	0	0	0	1	0	0
V_6'	0	0	1	0	0	0

V (2) V_2 —— (2) V_3 —— (3) V_4 < V_5(1), V_6(1) X (2) V_2 — (3) V_3 <

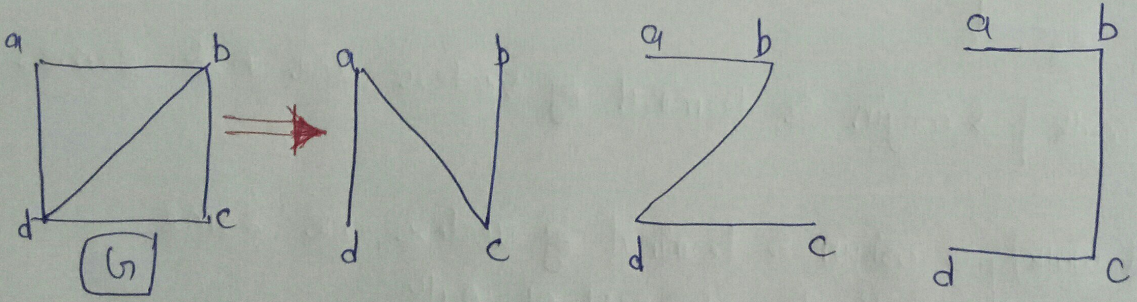
* **Subgraph** → Let $G(V, E)$ be a graph then $S(V_1, E_1)$ is said to be a subgraph of G, if the V_1 is the subset of V & E_1 is the subset of E,

$$V_1 \subset V \neq \phi \qquad E_1 \subset E \neq \phi$$



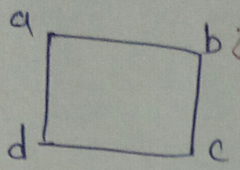
* **Supergraph** → If $G(V, E)$ be a graph & $S(E_1, V_1)$ be the subgraph of G then G is called supergraph of $S(V_1, E_1)$.

* **Spanning Subgraph** → A subgraph $S(V, E_1)$ containing all the vertices of Graph G, w/o circuit is called spanning subgraph of G.



Cond^n → No closed structure
→ All vertices exist

Connected Graph → A $G(V,E)$ is said to be Connected if there exist a path b/w every pair of its vertices.

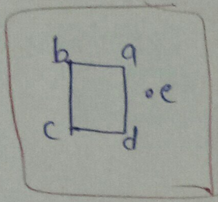


ab
a → d
a → b → c
a → d → c

b → c
b → a
b → c → d
b → a → d

Disconnected Graph → A graph which is not connected is called disconnected graph.

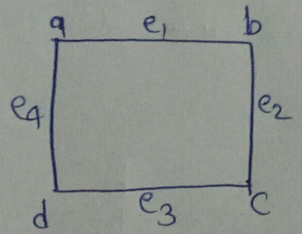
→ no path exist b/w any pair of Vertices



Walk → $G(V,E)$ In a graph an alternative finite sequence of vertex & edges is called walk.

→ It's denoted by 'W' and start from vertex and end on vertex.

→ Starting vertex is called origin & end vertex is called terminal vertex and no. of edges in walk is called length of walk.



$a\, e_4\, d \rightarrow w_1$ (walk) $\Rightarrow$ length $(l) = 1$ ← open walk

$a e_1 b e_2 c e_3 d e_4 a \rightarrow w_2 \Rightarrow l = 4$ ← closed walk

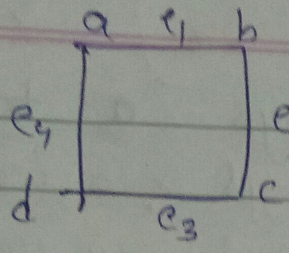
open walk → origin & terminal vertex are not same.

closed walk
→ origin & terminal vertex are same.
→ indicate closeness of walk
→ No repetation

⇒ vertex & edge repetation possible → $a e_1 b e_1 a$

$\boxed{\text{Trivial Walk}}$ → No any edge exist.

$\boxed{\text{Trail}}$ →



open walk
No edge repetation
Vertex can be reapet

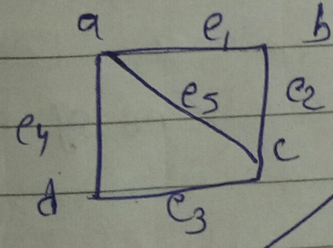
→ $a e_1 b e_2 c e_3 d$

$\boxed{\text{Circuit}}$ → A closed trail is called circuit

circuit → $a e_1 b e_2 c e_3 d e_4 a$ ← closed → W_1

No trail → $a e_1 b e_2 c e_3 d e_4 a e_1 b$ → W_2 → open walk
No circuit ↗

$\boxed{\text{Path}}$ → open walk
No edge & No vertex repeat



$a e_1 b e_2 c e_5 a$ → W_1 ✗
$a e_1 b e_2 c e_3 d e_4 a e_1 b$ → W_2 ✗ ✗
$a e_5 c e_3 d e_4 a$ → W_3

$a e_1 b e_2 c$ → Path

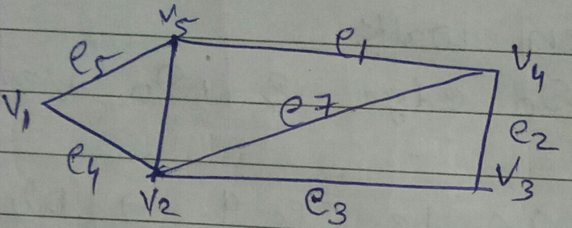
$\boxed{\text{Cycle}}$ → A closed path is called a cycle.

Q

Note ① A walk is always a trail. F
② A trail is always a walk. T
③ A path is always a walk. T
④ A walk is always a path. F

Distance b/w Vertices → The length of the shortest path b/w 2 vertex of connected graph is called the Distance.

& its denoted by $d(u,v)$
$d(v_1, v_2)$



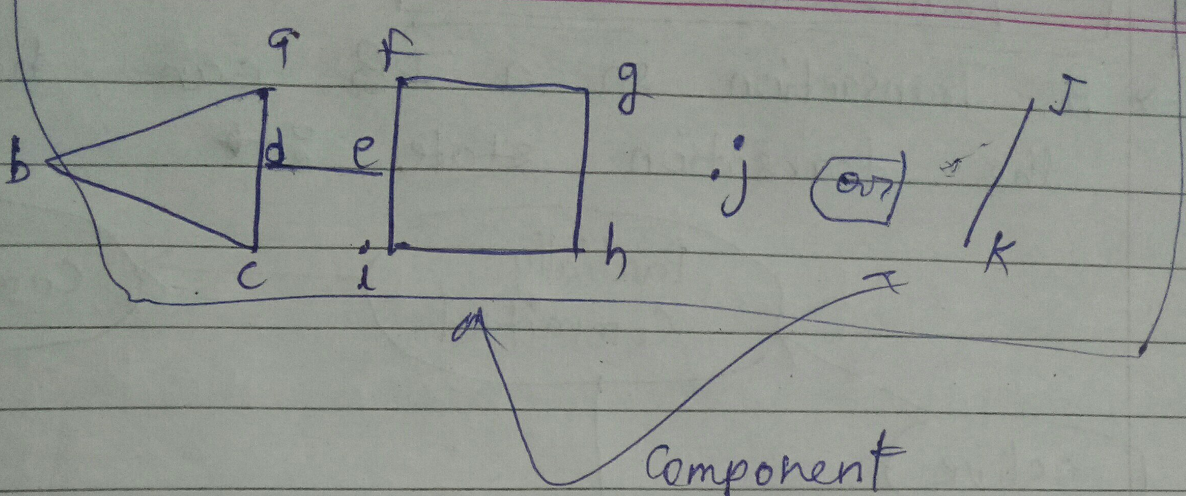
(no of edges → length)

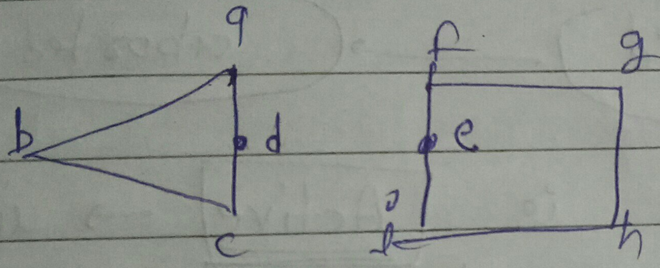
$d(v_1, v_2) \rightarrow 1$ e_4
$d(v_1, v_3) \rightarrow 2$ $e_4\ e_3$
$d(v_1, v_4) \rightarrow 2$ $e_5\ e_1$
$d(v_1, v_5) \rightarrow 1$ e_5

$d(v_2\ v_1) = 1$
$d(v_2\ v_3) = 1$
$d(v_2\ v_4) = 1$
$d(v_2\ v_5) = 1$

Bridge → An edge is called bridge if removal of it convert a connected graph into a disconnected graph. or into components

disconnected graph

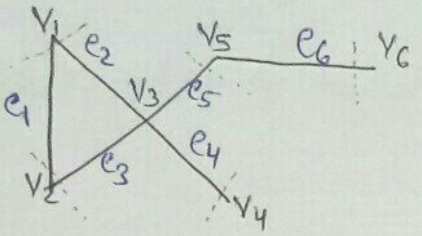




(d,e) → bridge

Cutset → A set of edges of a graph is called cutset of G if removal of set of edges from G convert it connected to disconnected graph.

→ But not any subset of set of edges can do this.



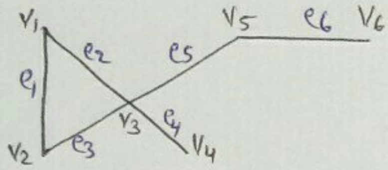
$E_1 = \{e_1, e_2, e_3\}$ ← No

$E_2 = \{e_1, e_2\}$ ← cut set ✓

$E_2 \subset E_1$

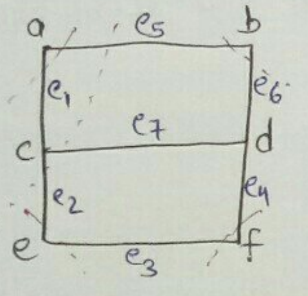
✓ ✗

Edge Connectivity → Let G(V,E) be a connected graph & there may be so many cutset of G which disconnected the graph. The no. of edges in smallest cutset is called the edge connectivity.



$E_1 = \{e_1, e_2\}$
$E_2 = \{e_1, e_3\}$
$E_3 = \{e_4\}$
$E_4 = \{e_5\}$
$E_5 = \{e_6\}$

edge connectivity = 1

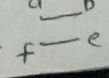


$\{e_1, e_5\}$

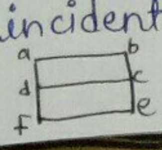
edge connectivity = 2

Cut Vertex → The vertex of connected graph is called cut vertex if after deleting this vertex graph G is disconnected.

Vertex Connectivity → The minm no. of vertex after removal of which a connected graph converted into a disconnected graph with 2 or more component. This minm no. of vertex is called vertex connectivity. when vertex is removed the edge incident to that vertex automatically removed.



c&d remove →

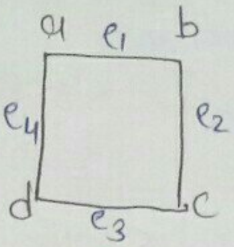


vertex connectivity = 2

* आयलर Euler Graph → (E → edge related)

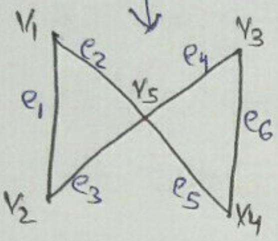
Euler circuit → A circuit of connected graph is called euler circuit if it contain all the edges of G w/o any edge repeatation.

1. closed walk
2. all edges
3. No edge repetation

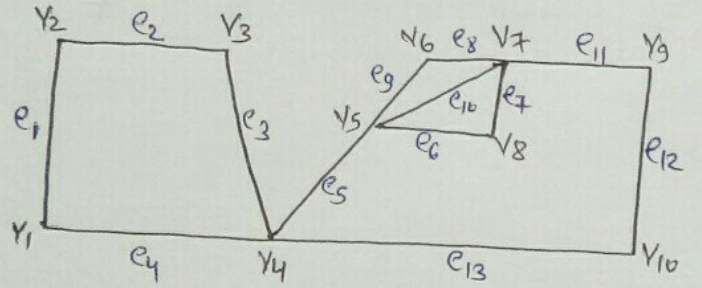


Euler circuit → $\underline{a}e_1be_2ce_3de_4\underline{a}$

Euler Graph → A graph is called a euler graph if there exist an euler circuit.



euler circuit → $\underline{v_1}e_2v_5e_5v_4e_6v_3v_4v_5e_3v_2e_1\underline{v_1}$



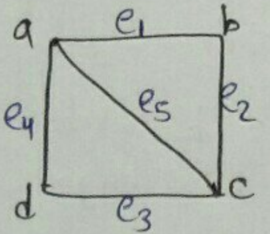
euler circuit →

$\underline{v_1}e_1v_2e_2v_3e_3v_4e_5v_5e_9v_6e_8$
$v_7e_{10}v_5v_6v_8e_7v_7e_{11}v_9e_{12}v_{10}e_{13}$
$v_4e_4\underline{v_1}$

Euler Trail → open walk, All edge
→ no edge repeat

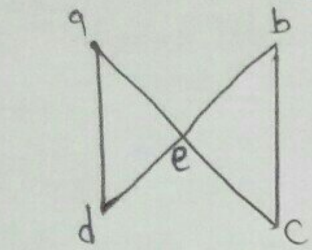
* Hamiltonian Graph → Hamiltonian cycle exist [H → Vertex related]

- → closed walk
- → No edge rep
- → No vertex rep
- * → All Vertex exist



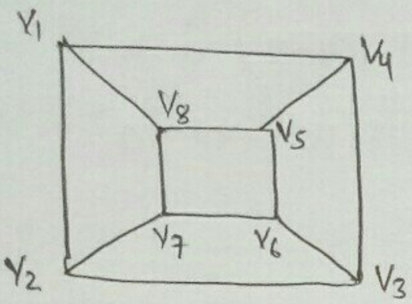
$\underline{a}e_1be_2ce_3de_4\underline{a}$ → Hamiltonian Cycle ∴ H.G.
→ No Euler circuit

$ae_1be_2c(e_5)ae_4de_3c(e_5)a$ ← No E.G.



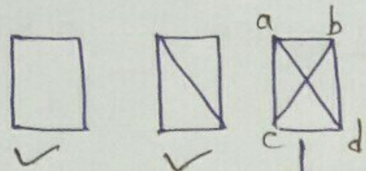
aebceda → Euler G. ✓

→ No H.G.

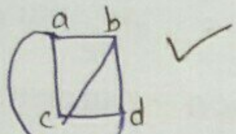


$V_1 V_8 V_5 V_4 V_3 V_6 V_7 V_2 V_1$ → H.G. ✓

* Planner / Planer Graph → G(V, E), in plan, graph have no intersect edge.



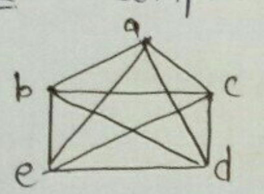
in plan 2D

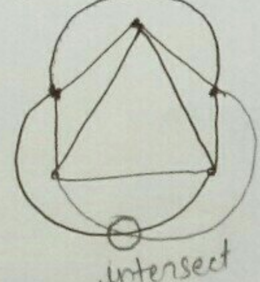


→ A graph is called planner G if it can be represent in a plan, in which no two of its edge intersect.

or A graph is a planer graph if its edges are not intersect.

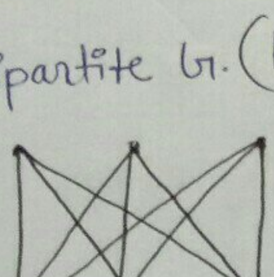
eg → Complete graph → (K_5)

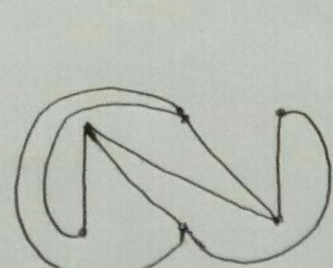




non-planer G.

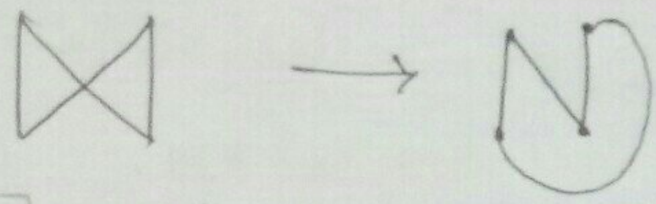
Bipartite G. ($K_{3,3}$)



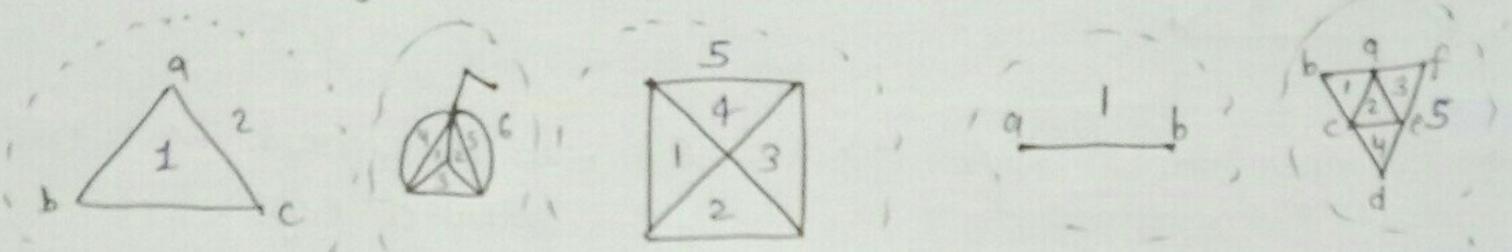


non-planer G.

$V_1 \cap V_2 = \phi$

$K_{2,2}$  (Planer Graph)

* **Region** → A planer G. divided plan into d/t regions, which we get by cutting the G. (paper) into pieces along its edges. The pieces which we obtain are called regions of that Graph.

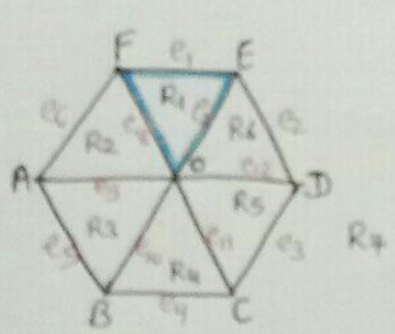


2 region   6 region   5 region   1 region   5 region

→ outer region of closed graph : infinite region (1)

→ region can't be defined for non-planner graph.

* **Degree of a Region** → The length of the closed circuit which cover the region is called the degree of the region & it's represented by deg (R).



planner Graph

$\deg(R_1) = F\underline{e_1}E\underline{e_7}O\underline{e_8}F = 3$

$\deg(R_2) = 3$

$\deg(R_3) = 3$

$\deg(R_4) = 3$

$\deg(R_5) = 3$

$\deg(R_6) = 3$

$\deg(R_7) = = 6$

(length of circuit) no. of edges

Euler Formula → Connected Planer Graph $G(V,E)$

v, e, r

For any connected planner $G(V,E)$ with vertices V, edges e & regions r then $\boxed{v - e + r = 2}$

→ Principle of mathematical Induction

→ 3 case
- $n = 1$
- $n = m$
- $n = m+1$

① $n = 1$

$\cdot v_1$

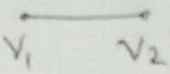
$v - e + r = 2$

$v = 1, \; e = 0, \; r = 1$

$1 - 0 + 1 = 2$

$2 = 2$

$n = 2$



$v = 2 \quad e = 1 \quad r = 1$

$2 - 1 + 1 = 2$

$2 = 2$

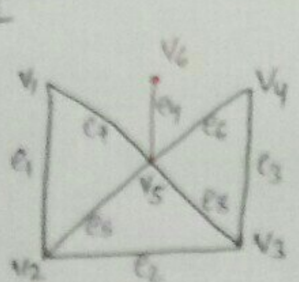
② $n = m$

Les us consider $v - e + r = 2$

③ $n = m+1$

2 cases
- $v = v+1$ (add one vertex)
- $e = e+1$ (add one edge)

I



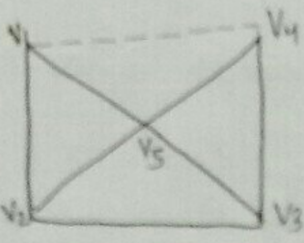
$v - e + r = 2$

$(v+1) - (e+1) + r = 2$

$v + 1 - e - 1 + r = 2$

$\boxed{v - e + r = 2}$

II



$v - (e+1) + (r+1) = 2$

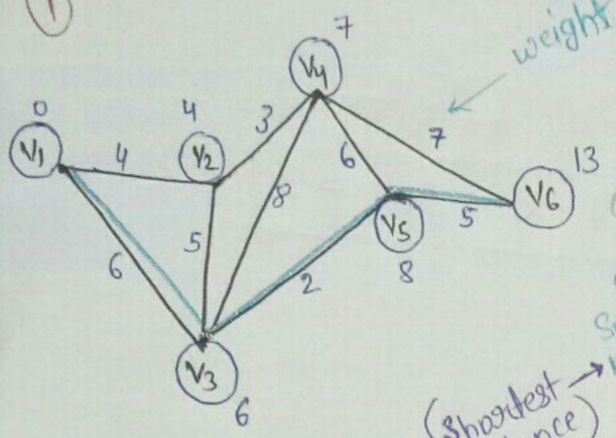
$v - e - 1 + r + 1 = 2$

$\boxed{v - e + r = 2}$

* Weighted Graph → Dijkstra method → Find ① Shortest Distance
② Shortest Path

(Series of iterations → labeling procedure)
$L_0(V_1) = 0,\ L_0(V_i) = \infty$

①



	V_1	V_2	V_3	V_4	V_5	V_6
0th iteration L_0	$\boxed{0}$ ✓	∞	∞	∞	∞	∞
L_1	$\boxed{0}$	$\boxed{4}$ ✓	6	∞	∞	∞
L_2	$\boxed{0}$	$\boxed{4}$	6/9 $\boxed{6}$ ✓	7	∞	∞
L_3	$\boxed{0}$	$\boxed{4}$	$\boxed{6}$	14/7 $\boxed{7}$ ✓	$\boxed{8}$	∞
L_4	$\boxed{0}$	$\boxed{4}$	$\boxed{6}$	$\boxed{7}$	13/8 $\boxed{8}$ ✓	14
L_5	$\boxed{0}$	$\boxed{4}$	$\boxed{6}$	$\boxed{7}$	$\boxed{8}$	14/13 $\boxed{13}$ ✓

assign smallest number (Shortest Distance)

Find :
$V_1 \longrightarrow V_6$

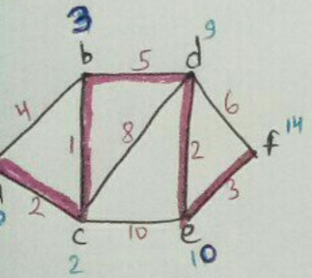
Shortest Distance → $\boxed{13}$ ✓

Shortest Path → $V_1 \to V_2 \to V_4 \to V_6 \Rightarrow 4+3+7 = 14$

$\boxed{V_1 \to V_3 \to V_5 \to V_6 \Rightarrow 6+2+5 = 13}$ ✓

weightadd

②



$a \longrightarrow f$

a	b	c	d	e	f
$\boxed{0}$ ✓	∞	∞	∞	∞	∞
$\boxed{0}$	$\boxed{3}$ ✓	2	∞	∞	∞
$\boxed{0}$	$\boxed{3}$	$\boxed{2}$ ✓	9	∞	∞
$\boxed{0}$	$\boxed{3}$	$\boxed{2}$	$\boxed{8}$ ✓	12	∞
$\boxed{0}$	$\boxed{3}$	$\boxed{2}$	$\boxed{8}$	$\boxed{10}$ ✓	15
$\boxed{0}$	$\boxed{3}$	$\boxed{2}$	$\boxed{8}$	$\boxed{10}$	$\boxed{13}$ ✓

shortest distance $= 13$

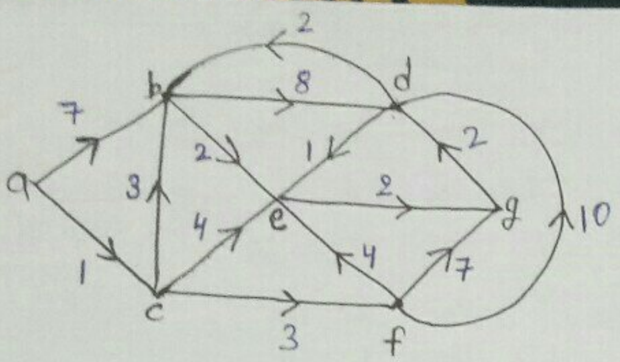
$a \to c \to e \to f \Rightarrow 2 + 10 + 3 = 15$

$a \to b \to d \to f \Rightarrow 4 + 5 + 6 = 15$

$a \to b \to c \to d \to f \Rightarrow 4 + 1 + 8 + 6 = 19$

$a \to b \to d \to e \to f \Rightarrow 4 + 5 + 2 + 3 = 14$

Shortest Path ⇒ $\boxed{a \to c \to b \to d \to e \to f \Rightarrow 2 + 1 + 5 + 2 + 3 = 13}$



Directed graph

a	b	c	d	e	f	g
$\boxed{0}$ ✓	∞	∞	∞	∞	∞	∞
$\boxed{0}$	$\boxed{4}$ ✓	1	∞	∞	∞	∞
$\boxed{0}$	$\boxed{4}$	$\boxed{1}$ ✓	15	∞	∞	∞
$\boxed{0}$	$\boxed{4}$	$\boxed{1}$	$\boxed{9}$ ✓	5	∞	∞
$\boxed{0}$	$\boxed{4}$	$\boxed{1}$	$\boxed{9}$	$\boxed{5}$ ✓	4	∞
$\boxed{0}$	$\boxed{4}$	$\boxed{1}$	$\boxed{9}$	$\boxed{5}$	$\boxed{4}$ ✓	11
$\boxed{0}$	$\boxed{4}$	$\boxed{1}$	$\boxed{9}$	$\boxed{5}$	$\boxed{4}$	$\boxed{7}$ ✓

Shortest Distance ⇒

Shortest path ⇒

→ The $\boxed{\text{length}}$ of a path in a weighted graph be the sum of the weights of the edges of this path.

→ shortest path : a path of least length b/w 2 vertices.

Shortest path Algorithm → discovered by Dutch mathematician Edsger Dijkstra in 1959.
(1930 – 2002)

→ ✱ **Matrix Representation of Undirected Graph** →

① **Adjacent Matrix Representation** →

→ Simple graph

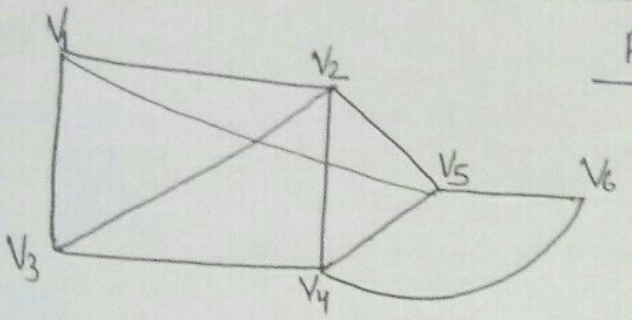
bound Cond^n → No parallel edge

→ No loop

① → Symmetric Matrix ⇒ relative row exchange with relative column then matrix remain same.
(n×n matrix)

② Diagonal elements entry = zero $[V_{11}\ V_{22}\ V_{33}\ V_{44} = 0]$

③ Vertex degree → (row entry sum / column entry sum)



Adj M. R. →

	V_1	V_2	V_3	V_4	V_5	V_6
V_1	0	1	1	0	1	0
V_2	1	0	1	1	1	0
V_3	1	1	0	1	0	0
V_4	0	1	1	0	1	1
V_5	1	1	1	1	0	1
V_6	0	0	0	1	1	0

(2) Incidence Matrix Representation → $G(V, E)$

n vertex , edges

bound Cond^n → No loop

n×e matrix

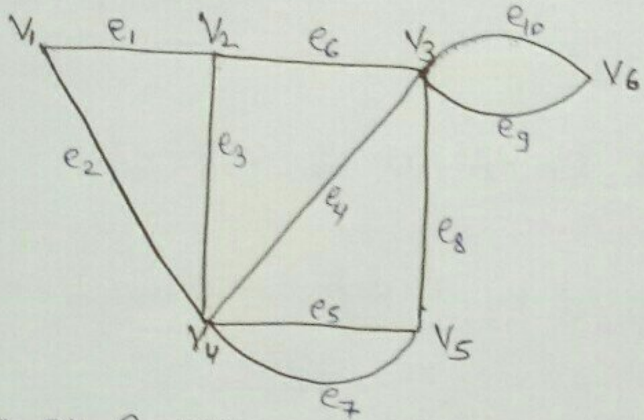
(1) edge column → 2 (entry)

(2) vertex degree → entry add (row, column)

(3) parallel edge → entry same → identical

(4) Isolated Vertex → all entry zero (row)

(5) binary matrix → 0, 1 entry

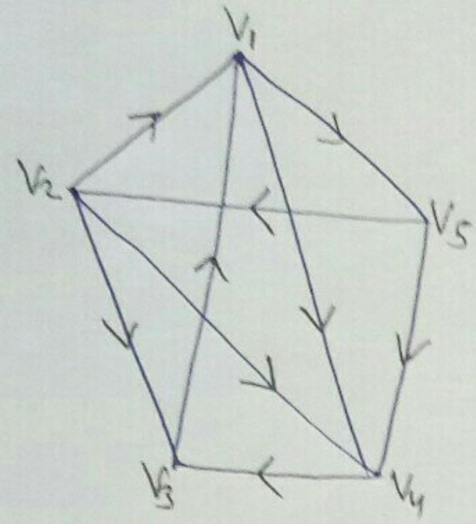


Inci. M. R. ↴

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
V_1	1	1	0	0	0	0	0	0	0	0
V_2	1	0	1	0	0	1	0	0	0	0
V_3	0	0	0	1	0	1	0	1	1	1
V_4	0	1	1	1	1	0	1	0	0	0
V_5	0	0	0	0	1	0	1	1	0	0
V_6	0	0	0	0	0	0	0	0	1	1

* Matrix Representation of Directed Graph →

① Adjacent M.R. → relation b/w V & V
↳ No parallel edge

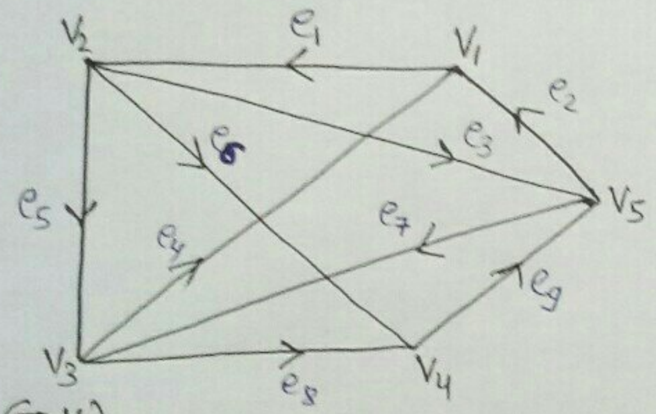


(Initial vertex) / (Terminal vertex)

	V_1	V_2	V_3	V_4	V_5
V_1	0	0	0	1	1
V_2	1	0	1	1	0
V_3	1	0	0	0	0
V_4	0	0	1	0	0
V_5	0	1	0	1	0

Q → Graph → Table
↳ Table → Graph

② Incidence M.R. → V & E, No loop, entries → 0, 1, −1



(T.V)

I.V	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
V_1	1	−1	0	−1	0	0	0	0	0
V_2	−1	0	1	0	1	1	0	0	0
V_3	0	0	0	1	−1	0	−1	1	0
V_4	0	0	0	0	0	−1	0	−1	1
V_5	0	1	−1	0	0	0	1	0	−1

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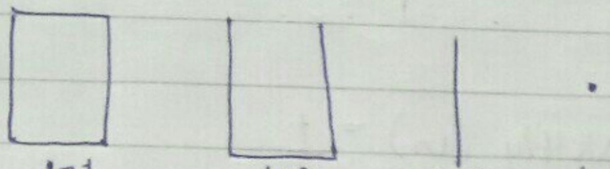
* $\boxed{\text{Tree}}$ → A connected undirected graph w/o any loop, and parallel edges is called a tree.
→ No cycle / circuit
G (V,E)

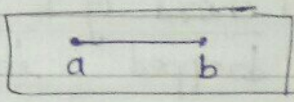


G1 G2 G3 G4
X ✓ ✓ ✓

* $\boxed{\text{Trivial Tree}}$
↓
Single vertex
No edge

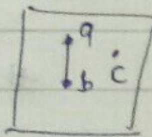
$\boxed{\text{Non - Trivial Tree}}$
↓
more than one vertex

* $\boxed{\text{Pendent Vertex}}$ → degree 1



← Tree ✓

* $\boxed{\text{Isolated vertex}}$ → No connected vertex

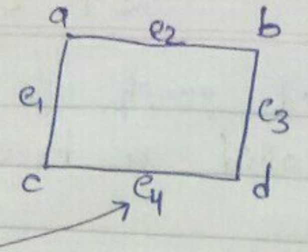


X No tree

* $\boxed{\text{Spanning Tree}}$ → Let G(V, E) be a graph with n vertices & e edges then spanning tree T of G is a subgraph of G having:

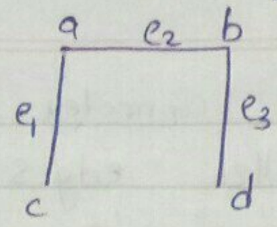
I All the vertex of G
II It is connected subgraph of G
III It does not have any loop / circuit

Branch of tree (e_1, e_2, e_3)



chord of T (e_4)

G



T

$T \subset G$

$R(G) = 3$ (Rank) , Nullity $(G) = 1$

$\boxed{\text{Branch of Tree}}$ → The edges of G that include in spanning tree is called the branch of tree.

$\boxed{\text{chord of T}}$ → The edges of G that are dropped in spanning tree is called chord of T.

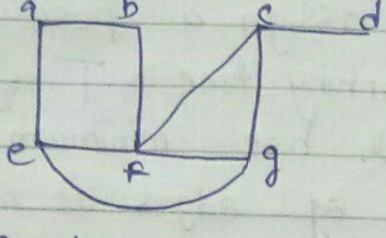
$\boxed{\text{Rank}}$ → The no. of edges of G which are include in spanning tree are called Branch of tree or called the rank of G.

$\boxed{\text{Nullity}}$ → The no. of edges of G which are exclude in spanning tree are called the chord of tree i.e. Nullity of G.

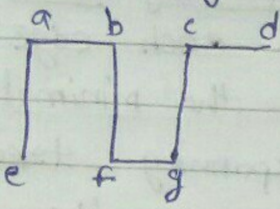
→ G → n edges

→ T → (n-1) edges.

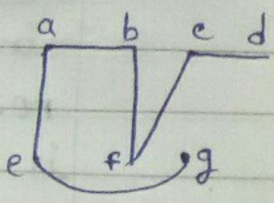
Q. Find all possible spanning tree of a graph.



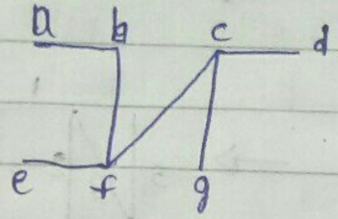
→ (1)



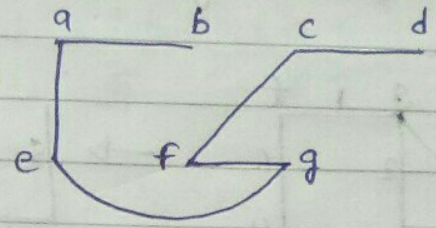
(2)



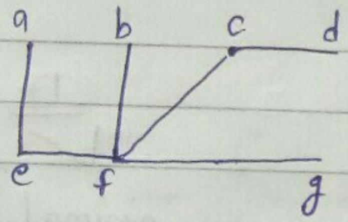
(3)



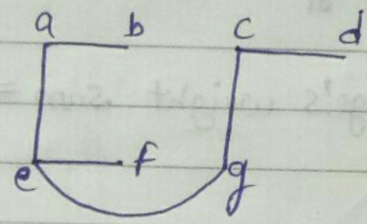
(4)



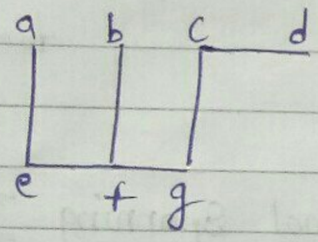
(5)



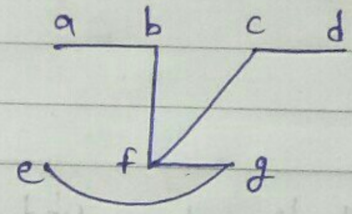
(6)



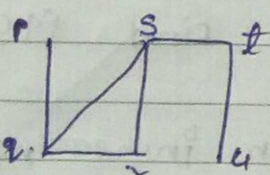
(7)



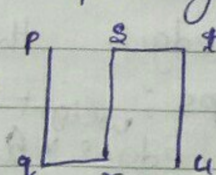
(8)



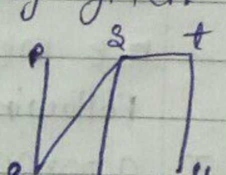
Q. Make all possible spanning tree of given graph.



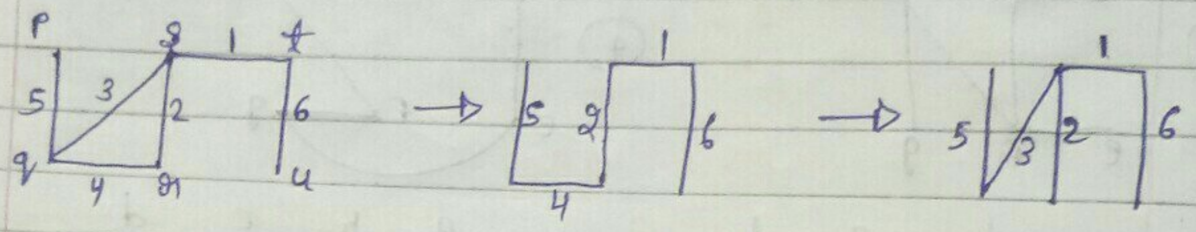
→ (1)



(2)



Minimal Spanning Tree → Let $G(V,E)$ a graph in which each edge are assign a positive real no., then the minimal spanning tree T of G is that spanning tree which has minimum length sum. [sum of the length of edges is minimum.



edge's weight sum ⇒ 18

17 ✓

minimal spanning tree

16-2-22

* Methods to find out Minimal Spanning Tree →

① Kruskal Algorithm →

Let $G(V,E)$ be a graph with n vertices & e weighted edges then we find M.S.T in Following steps:

I arrange the weight edges of G in increasing weight

II then display the n isolated vertices (Point out)

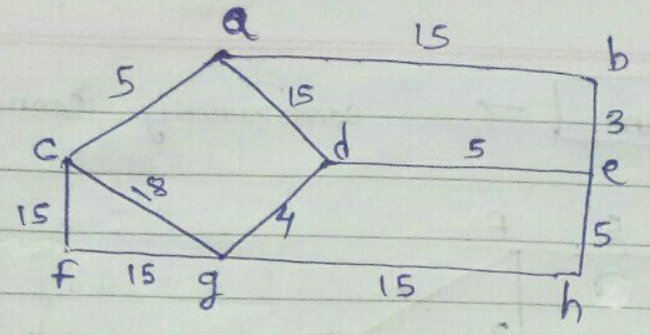
III select the edge of minimum weight & make connection b/w the vertices.

IV then choose the next lowest edges & make connection.

V Continue the process until $(n-1)$ edges cover n vertices, w/o making circuit.

Q Find MST by kruskal algo (write all steps in exam).

Q

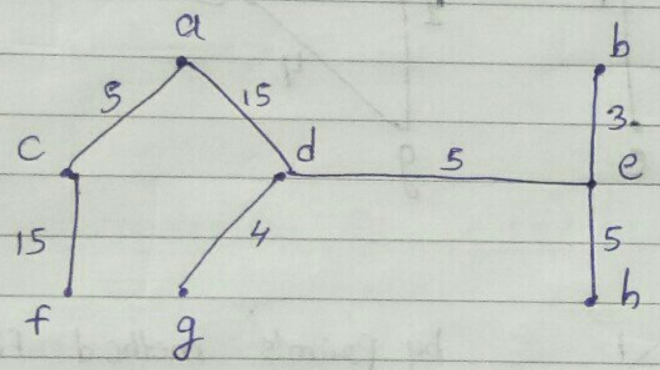


I

(b,e) 3 (g,d) 4 (d,e) 5 (e,h) 5 (a,c) 5 (a,d) 15 (g,h) 15

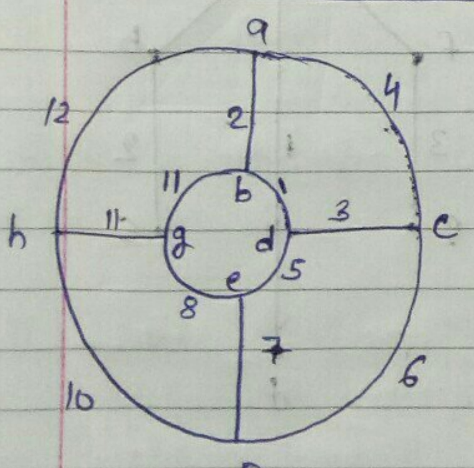
(c,f) 15 (f,g) 15 (a,b) 15 (c,g) 18

II



M.S.T. ⟹ 8 vertex (n), 7 edges (n-1)

Q



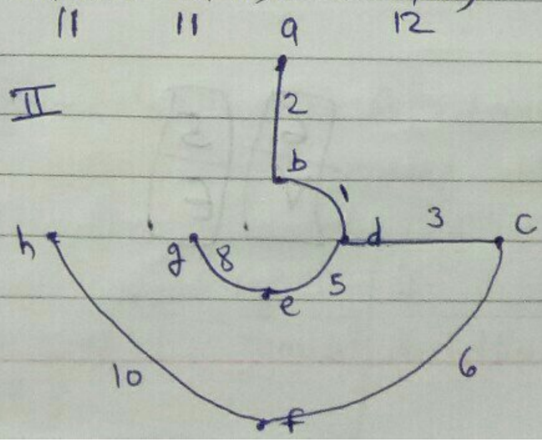
I

1	2	3	4
(b,d)	(b,a)	(c,d)	(a,c)

(d,e) 5 (e,f) 6 (e,f) 7 (e,g) 8 (h,f) 10

(h,g) 11 (b,g) 11 (a,h) 12

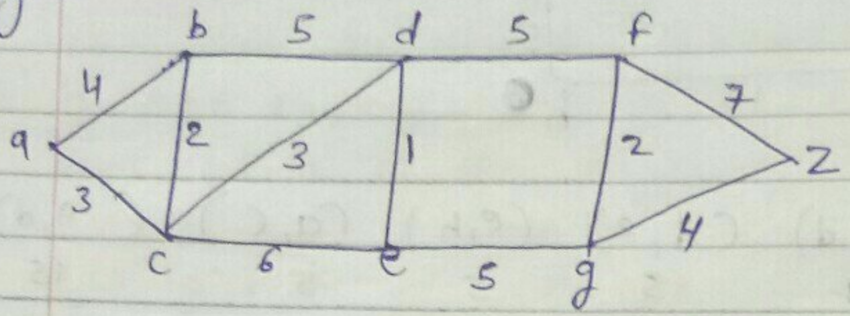
II



8 V 7 E

② # $\boxed{\text{Prim's Algorithm}}$ → continuously connected form

eg →



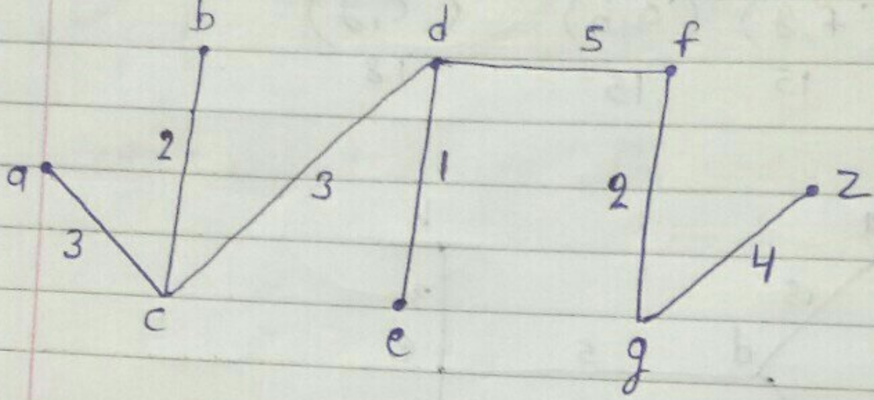
$\boxed{a \to 4}$

c → ~~7~~ ~~3~~ 6

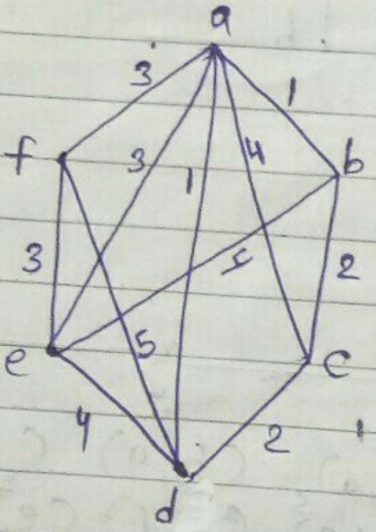
b → ~~4~~, 5

d → 5, ~~1~~, 5

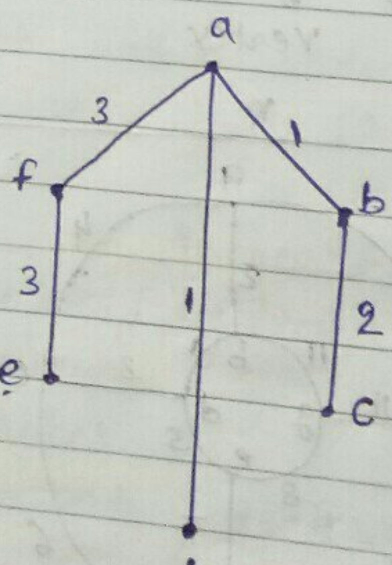
e → 5, ~~6~~



Q



by Prim's method find M.S.T.



$\boxed{\frac{6}{V}}$ $\boxed{\frac{5}{E}}$

Unit - I

Set Theory

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22 2 22

Set → A set is any well-defined collection of objects.

$MCA = \{1, 2, 3, 4, 5, 6, 7, 8\}$
elements

→ The objects are called elements or member of the set.

→ $A = \{a, b, c\}$

capital letter (set)
small letter (elements)

$a \in A$, $b \in A$, $c \in A$, $d \notin A$
belongs to — not belongs to

* Representation of a set →

I **Roster Method** → A set can be represented by actually listing all the elements that belongs to it.

$A = \{a, b, c\}$, $B = \{c, b, a\}$

Equal $A = B$, order matter → No

$A \neq B$ order matter → Yes

distinct elements → in a set → $A = \{a, b, c, a\}$
→ $A = \{a, b, c\}$

II **Set Builder Form** → A set is sometimes define by the property or the rules which characterises all the elements of the set

Set Builder → $A = \{x \mid x \in N, x \leq 7\}$

x → element; | → such that; ∈ → belongs to

Roster → $A = \{1, 2, 3, 4, 5, 6, 7\}$