Toy unification in a cellular automaton

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Abstract

Unification of the four forces is arguably achieved in a toy universe based on a fully deterministic, Euclidean, 4-torus cellular automaton using a constructive approach. Each cell contains one integer number forming bubble-like patterns propagating at the speed of light, interacting and being reemitted constantly. The collective behavior of these integers looks like patterns of classical and quantum physics. Although essentially nonlocal, it preserves the no-signalling principle. This flexible model predicts that gravity is not quantized.

Keywords: cellular automaton, graviton, beyond Standard Model, unification, nonlocality

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1 Introduction

Cellular automata are mathematical idealizations of physical systems in which space and time are discrete. The idea of modeling our universe using cellular automata is not new, discreteness is seen by many authors (Refs. [1–8] form a small list) as a solution for the divergences of the Standard Model (SM), and is supported by the existence of a fundamental Planck volume, suggesting that structures smaller than this tiny volume should not be relevant to the theory. This cellular automaton can be regarded as a model beyond the SM.

Quantum mechanics (QM), despite its resounding success, gives us a somewhat blurred image of the universe because of its base on the uncertainty principle, the point particles, and its most accepted interpretation is based on probabilities. Recent results from experimental physics, which far surpass the precision achieved by QM predictions, require a new model of the universe in which QM is only a limiting case.

Can nature be modeled as a cellular automaton? The model described here is designed to investigate this possibility. The emergence of a unified theory of physics is the ultimate goal of a final version based on this approach. Here the automaton is a couple of simple cubic grids closed on themselves as a 4-torus where one *tile* (formatted integer number) is attached to each cell. The cell has a processor, or logical circuit, and interacts with its eight nearest neighbors only (von Neumann convention). Preons are modified under the tick of a central clock. The Planck length is the natural candidate to be used as the distance between the automaton cells.

The approach adopted in this work is a constructive one [9, 10]. Whenever possible, I try to emulate directly the laws of physics, probing the most adequate heuristics.

On the other hand, I'm not saying that the Universe is a vast computer, in fact, I'm attempting to model Planck scale physics using a cellular automaton. Except for developing the basic principles, the construction of an automaton for directly solving cosmological problems, or even complex molecules, is inconceivable. Its complete usefulness will mainly come through mathematical analysis in the approximation of large numbers, which is beyond the scope of the present work. Furthermore, this automaton can not be considered either quantum or classical. Actually, the regularities or patterns generated by the system is that might be considered quantum-like or classical-like—they have no *a priori* meaning.

2 Basic concepts

In this section, some concepts are loosely explored before the rigorous sistematization of the model in Sec. 3.

2.1 Tile properties

A *tile* is a long integer formatted in many smaller integer fields representing signed or unsigned values of variable sizes or as vectors in 3d space. Normalization of vector components, if necessary, is done in a $D/2$ length. Detailed description of all properties can be accessed in Table 1 (in practice, additional fields are used for wavefront propagation, burst propagation, sinewave generation etc.). Since a w dimension value is unique to each preon, the product of their values is used to mark a pair of preons as entangled.

2.2 The cellular automaton

The cellular automaton is a dual Euclidean lattice 4-torus of dimension SIDE, where a single tile is attached to each lattice cell. The lattice is an abstract entity used to organize the cells. The distance between cells is L and the clock period (p_1) is T. Each lattice is alternatively principal (read-only) or dual (draft). D is the main diagonal of the lattice. Three dimensions are spatial and the fourth corresponds to internal degrees of freedom. When propagating as a spherical wavefront at the speed of light, tiles are called a *preon*. When diffusing as a superluminal wavefront at the maximum speed allowed in the automaton, they form a *burst*. Finally, when propagating in isolation at the speed of light, the tile is a *graviton* (GRAV).

Table 1: Tile fields

Field	Name	Type	Values
p_1	Clock	UI	Incremented in unison after T seconds
$\overrightarrow{p}_2$	Origin	SV	*null* or N_D possible directions. $\lvert\overrightarrow{p}_2\rvert$ =preon radius
p_3	Charge	SI2	0, ± 1
p_4	Chirality	SI2	0, ± 1 (RMLAM, RM, LM)
p_5	Color and conjugation	UI6	R G B R' G' B' (LEPT: 111000, ANTILEPT: 000111)
$\overrightarrow{p}_6$	Spin	SV	*null* or N_D possible directions
p_7	Gravity	UI1	ON, OFF
p_8	Entanglement	2 UI	$0..SIDE^2$
p_9	Sinusoidal phase	SI	$-SIDE/2..+SIDE/2$
p_{10}	Frequency	UI	$0..SIDE$
p_{11}	Helicity	UI1	0, right; 1, left
p_{12}	Interaction	UI4	UNDEF, UXP, UXU, PXP, UXG
p_{13}	Interference	SI	$-SIDE/2..+SIDE/2$
$\overrightarrow{p}_{14}$	Return path	SV	Three integers between $0..SIDE$ (Used to find OP)
p_{15}	Cohesion	UI1	ON, OFF
$\overrightarrow{p}_{16}$	LM direction	SV	*null* or N_D possible directions

Bursts are just low level messengers, so support the no-signalling principle. Typical information driven by a burst is information for the reissue of a preon. While the wavefront during the spread of the preon is synchronous in order to guarantee a perfectly spherical shape, the wavefront in a burst is asynchronous and therefore much faster, or superluminal, fitting entirely between two consecutive light steps (one light step is $LIGHT = 2D$ clock ticks).

2.3 Combinations of preons

Isolated or unpaired preons act as fragments of charge (Us). Two overlapping preons can form a preon pair (P). Us typically interact with other Us and Ps. Ps can sometimes interact with other Ps. Elementary particles are composite systems of Us and Ps, carrying HBAR/2 quantity of intrinsic AM, in the case of fermions, or HBAR, in the case of bosons.

2.4 Isotropy

Isotropy is a consequence of the fact that preons propagate as a nearly perfect wavefront. With the solution above, I fully solved the isotropy problem. Clearly, isotropy granularity depends on the size SIDE of the universal cube. Considering all combinations of normalized 3d vectors that can be formed with that value, it can be stated that the number of possible directions N_D can be calculated as

$$N_D = 2\pi\,(SIDE/2)\,.$$

For $SIDE = 128$, we have approximately $N_D = 102,943$ Ps. This number expresses the best isotropy possible in such a small automaton.

A key ingredient to achieve an isotropic behavior on an automaton is the generation of an isotropic wavefront. One difference between mine and most cited automata is that light speed is not one lattice spacing per clock tick, but is a larger count. Isotropic propagation of a wavefront is achieved in the limit when the number of cells tends to infinity by using the approach developed by Case, Rajan and Shende [11]. The novel feature of that work is that, to obtain the isotropy, is required for each expansion step, executing n steps of the basic algorithm of the automaton, where n is two times the diameter of the universe D (space diagonal). Henceforth we will refer either to lattice speed s or to light speed c. Then we have the relation

$$s = 2\,D\,c.$$

In order to synchronize the preons forming a wavefront, it receives the value

$$t = \lceil 2D\,|p_2|\ + 0.5 \rceil.$$

Actually, to avoid undesired superposition of a preon wavefront with a burst or gravitons on a common layer (w address), the time frame is segmented in two steps: one, when the bursts are active, has a duration of BURST time units. The other, when preons and gravitons are active, has a duration of $2\,D$ time units. The entire frame is termed SYNCH.

2.5 PWM encoding

Discretized properties sine wave and polarization cannot be used directly, but must first be converted to a PWM sequence, ruling out the need of an interaction detection mechanism based on a pseudorandom number generator.

3 Theory

3.1 Ontology

Definition 1. Property *formats*: SI, signed integer; SI3, 3-bit SI; UI, unsigned integer; SV, signed 3d-vector, with $N_D = \pi\,({}^{SIDE}/_{2})^2$ possible directions. The default length is SIDE.

Definition 2. *Tile* is a formatted $(p_1,\, p_2,\, ...)$ N-integer (see Table 1).

Definition 3. The *cellular automaton* is a dual Euclidean lattice 4-torus of dimension SIDE, where a single tile is attached to each cell. The distance between cells is L and the clock period (p_1) is T. Each lattice is alternatively principal (read-only) or dual (draft). D is the main diagonal of the lattice.

Definition 4. A *preon* is a spherical wavefront of tiles occupying the same w address, expanding at the speed of light $c = {}^{L}/_{LIGHT}$. If its p_7 field is ON, it is considered *real*, otherwise, *virtual*.

Definition 5. *Graviton* (G) is a tile that propagates in a straight line at the speed of light. It vanishes after traveling the distance of $SIDE/2$ units in the direction of its spin.

Definition 6. A *burst* is a cubic wavefront occupying the same w address, expanding at the maximum speed $s = {}^{L}/_{T}$—a superluminal messenger.

Definition 7. *Unpaired* (U) is a non-overlapping preon. It works like a charge fragment.

Definition 8. *Pair* (P) are two overlapping preons. A trivial P is a P with trivial properties.

Definition 9. The PWM function is

$$pwm(n) = n \,\mathbf{mod}\, STEP < (n/NSTEPS)\,.$$

where $STEP = log_2\,(SIDE)$ *and* $NSTEPS = {}^{SIDE}/_{STEP}$.

Definition 10. The *input parameters* are $SIDE = 1 \cdot 10^{62}$, $L = one\ Planck\ length$, $T = Planck\ time/3 \cdot SIDE$, $RAMP = 1 \cdot 10^{10}\,Planck\ lengths$, $LOST = log_2\,(SIDE)$ and $HBAR$. They are used for mapping to the real world.[1]

3.2 Dynamics

Axiom 1. *Isotropy and spherical wavefront generation are achieved applying the method described in Ref. [11].*

Axiom 2. *A visit-once-tree is used during preon and burst expansion to avoid cell access conflicts (see Appendix).*

[1] HBAR must be inferred from experimental data. Since it varies extremely slowly and reflects the present cosmological era of the universe, it is represented by a constant.

Axiom 3. *The phase of preons is given by a sinusoidal field calculated by means of a* Direct Form Oscillator cf. *[12]. When preons are superposed, the generator is bumped multiple times.*

Define the constants

$$\begin{aligned} k &= 2\cos(\omega T), \\ U_1 &= SIDE\sin(-2\omega T), \\ U_2 &= SIDE\sin(-\omega T). \end{aligned}$$

At the beginning of each wave do

$$u_0 = 1;\ u_1 = U_1;\ u_2 = U_2.$$

The evolution law is

$$\begin{aligned} u_3 &= k\,u_2 - u_1, \\ u_1 &= u_2, \\ u_2 &= u_3. \end{aligned}$$

Axiom 4. *The conflict of bursts in the same layer must be solved taking into account the octets involved.*

Axiom 5. *Interference derives from a track left by the preons on the visited cells (p_{13} field), inspired by work of Sciarretta [7]. The value algebraically added by the sinusoidal phase on the cell decays absolutely and exponentially after each light step [13].*

Axiom 6. *Polarization ($pol_E(), pol_M()$) is the PWM result of the squared cosine of the propagating wave if*

$$\begin{cases} electric & light \,\mathbf{mod}\, cycle < cycle/8 \\ \\ magnetic & light \,\mathbf{mod}\, cycle < cycle/4, \end{cases}$$

where light is the light step counting and cycle is the sinusoidal wavelength in light step units.

Axiom 7. *Preon interaction is detected by mutual comparisons in the* w *dimension within a single clock tick. At the end, the p_{13} field contains the calculated interaction type (UxG, UxU, UxP or PxP) and the p_{28} field contains the w address of the other interacting preon.*

Axiom 8. *A preon launches a burst every time it is reemitted.*

Axiom 9. *Both Us are reemitted at the contact point in a UxU interaction.*

Axiom 10. *If field $p_{15}^P = ON$ in a UxP interaction, the P is reissued at the contact point while the U is reissued at the point calculated by parallel transport of the P's $\overrightarrow{p}_{16}$ vector to the origin of U, else both are reemitted at the contact point.*

Axiom 11. *The LM alignment with neighbors ($\overrightarrow{p}_{29}^1 \bullet \overrightarrow{p}_{29}^2 \sim +1$) is checked in a UxP interaction using PWM. If resonance occurs, $p_{15}^P = p_{15}^P = OFF$. These Ps merge into one photon and escape the influence of the charges, propagating away.*

Axiom 12. *When a property of a particle is changed by interacting with another particle, before they merge their entanglement fields ($p_8^1 = p_8^2 = w^1 w^2$), the fact is communicated by the burst to all its entangled components, even if space-like separated, which in turn assume opposite spin direction.*

Axiom 13. *Conceptually, when a free P ($p_{15}^P = OFF$) propagates, its spin direction rotates about the propagation direction. Since this direction is undefined, except at the moment of detection, then the direction of vector $\overrightarrow{p}_6$ is only known (calculated) at that precise moment. Vector rotation uses the CORDIC method* cf. *[14].*

Axiom 14. *The wavefront of a real preon continues propagating as a G after reemission.*

Axiom 15. *The graviton detection interaction is a UxG interaction triggered by*

$$p_7^U = ON.$$

As a result, $p_{12}^U = UXG$ *(this kind of operation continues until the G vanishes by wrapping).*

Axiom 16. *The gravitational acceleration interaction is a UxP interaction triggered by*

$$p_{12}^U = UXG \wedge virtual\,(P)\,.$$

As a result, $p_{15}^P = ON$, $p_{12}^U = UNDEF$ *and P is aligned with the* $\overrightarrow{p}_6$ *direction of the origin G.*

Axiom 17. *The static EM interaction is a UxP interaction triggered by*

$$pwm\,(p_{13}) \wedge trivial\,(P)\,.$$

As a result, $p_3^1 = p_3^2 = p_3^U$ *and* $p_6^1 = p_6^2 = p_6^U$.

Axiom 18. *The Coulomb interaction is a UxP interaction where* $p_3^U = \pm 1$ *and* $p_3^1 = p_3^2 = \pm 1$. *The test for repulsion/attraction is given by*

$$\begin{cases} I_R = p_3^A \neq 0 \wedge p_3^B \neq 0 \wedge p_3^A = p_3^B \wedge pwm(p_{13}) \wedge pol_E\,() \\ I_A = p_3^A \neq 0 \wedge p_3^B \neq 0 \wedge p_3^A \neq p_3^B \wedge pwm(p_{13}) \wedge pol_E\,()\,. \end{cases}$$

As a result, the P's $\overrightarrow{p}_{29}$ *field is aligned with the U and P centers, either attracting or repulsing depending on the signs involved.*

Axiom 19. *The magnetic interaction is a UxP interaction triggered by*

$$I_M = \overrightarrow{p}_6^1 = \overrightarrow{p}_6^2 \neq \overrightarrow{0} \wedge pwm(p_{13}) \wedge pol_M\,()\,.$$

As a result, the P has its LM direction perpendicular to its spin and the radial direction (normalized cross product).

Axiom 20. *The absorption interaction is a UxP interaction triggered by*

$$p_3^1 = -p_3^2 \wedge p_3^U \neq 0 \wedge p_{15}^1 = p_{15}^2 = OFF,$$

As a result, it happens the reemission of all the photon's entangled superposed components (local components) from the contact point and flipping of the spin of the remaining entangled components (remote components).

Axiom 21. *The weak interaction is a UxP interaction triggered by*

$$I_W = \left(p_4^A \,\&\, p_4^B\right) \neq 0 \wedge pwm\left(p_{13}^A\right) \wedge pwm\left(p_{13}^B\right).$$

Axiom 22. *The strong interaction is a UxP interaction triggered by*

$$I_S = (p_5^A \,\&\, p_5^B) \neq 0.$$

As a result, the P is aligned with the radial direction in an attractive configuration.

Axiom 23. *Opposite Ps can become trivial in a PxP interaction, thereby avoiding unbounded accumulation. The filter is a PWM test on the cosine of the angle between their LM directions.*

$$\overrightarrow{p}_{29}^1 \bullet \overrightarrow{p}_{29}^2 \sim -1.$$

Axiom 24. *Interactions are processed in the following order: inertia, gravity force, strong force, Coulomb force, magnetic force, weak force. The first one that occurs inhibits all others.*

3.3 Theorems

Theorem 1. *A burst fits within a light step*

Proof. One light step is $LIGHT = 2D$. For the burst step, the total raw time necessary for the operation is given by the recurrence relation

$$\begin{aligned} a_0 &= 7, \\ a_n &= 2a_{n-1} - 1, \end{aligned}$$

which can be recast as the function

$$f(n) = 12 \times 2^{n-1} + 1,$$

where $n = ORDER - 2$ and $ORDER = log_2 SIZE$, assumed a big positive integer. Then

$$2D > 12 \times 2^{ORDER-3},$$

$$2^{ORDER} > 6 \times 2^{ORDER-3},$$

$$8 > 6.$$

□

Theorem 2. *Graviton number is not a conserved quantity, implying an arrow of time.*

Proof. It is a direct consequence of Axiom **??**. □

4 Discussion

Based on the axiomatic body presented above, I state now some conjectures related to expected behavior of the automaton.

Conjecture 1. *Clusters of Us and associated Ps tend to produce stable or transient patterns of HBAR/2 Us that I call* fermions*. This quantization effect is supported by the electric Us and the closure of the universe.*

Conjecture 2. *In a fermion, the spins tend to align either outward or inward, forming a spherically symmetric pattern. These states correspond to either spin up or spin down. This conjecture was inspired in the Hofer electron [15]. Coherence inhibts this tendency while decoherence stimulates this dichotomy.*

Conjecture 3. *The magnetic effects of still charge over another still charge cancels out due to spherical symmetry. Ps can break the symmetry of the cloud, which passes into an oval configuration and consequently induces a magnetic dipole.*

Conjecture 4. *A fermion is in a superposition state when one part of the spins of its Us point inward while the other part points outward.*

Conjecture 5. *Gravity is not quantized (adiabatic process).*

Conjecture 6. *Quarks are emergent patterns formed inside hadrons, so are confined. These patterns tend to shrink to a point at higher LM.*

Conjecture 7. *Since leptons and hadrons are composite particles, they can possess radial vibration, like a pulsating sphere [17]. The muon is the first excited state of the radial vibrational state of the electron, the tau is the second, so there is just one stable kind of lepton: the electron. For quarks, the charm is the first excited state of the radial vibrational state of the up. The top is the second radial vibrational state of the up. The strange is the first radial vibrational state of the down. The bottom is the second radial vibrational state of the down. The down is formed when the up captures a charged lepton. We therefore are led to conclude that there is just one kind of stable quark, the up. Therefore, the amount of Ps trapped in these resonance modes gives rise to the* rest mass *of the particles when they emit duo-gravitons in addition to the gravitons emitted by their Us.*

Conjecture 8. *Particle formation, annihilation and decay emerge naturally from the interaction of a huge number of preons.*

5 Conclusion

The construction of a cellular automaton describing the basic laws of nature is a long-term goal, requiring the synergy of many researchers. In this contribution, I presented a tentative solution to the unification problem using a constructive approach.

The results suggest some qualitative resemblance to QM, the SM and experimental data [15, 19–22]. The no-signalling principle is preserved. Since graviton emission is not conditioned to AM transfer, gravity is therefore not quantized.

Note that the term energy was not used anywhere in the text. Far from being heresy, it simply means that it was not necessary to invoke it at this stage of the model's development, even though energy is an ill-defined concept in Physics.

Except for developing the basic principles, the construction of such an automaton for directly solving cosmological problems, or even complex molecules, is inconceivable. Its complete usefulness will mainly come through mathematical analysis in the approximation of large numbers.

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Appendix: Visit-once-tree

To avoid cell access conflict, the path of the expanding preon or burst must be tested using the pseudocode below:

```
▷ Tests whether the direction dir is a valid path in the visit-once-tree.
boolean function isAllowed(dir) begin
    x = p2.x + dirs[dir].x
    y = p2.y + dirs[dir].y
    z = p2.z + dirs[dir].z
    level = abs(x) + abs(y) + abs(z)
    ▷ x-axis
    if x > 0 and y = 0 and z = 0 and dir = 0 then
        return true
    else if x < 0 and y = 0 and z = 0 and dir = 1 then
        return true
    end if
    ▷ y-axis
    else if x = 0 and y > 0 and z = 0 and dir = 2 then
        return true
    else if x = 0 and y < 0 and z = 0 and dir = 3 then
        return true
    end if
    ▷ z-axis
    else if x = 0 and y = 0 and z > 0 and dir = 4 then
        return true
    else if x = 0 and y = 0 and z < 0 and dir = 5 then
        return true
    end if
    ▷ xy plane
    else if x > 0 and y > 0 and z = 0 then
        if level mod 2 = 1 then
            return (dir = 0 and p200 = 2)
        else
            return (dir = 2 and p200 = 0)
        end if
    else if x < 0 and y > 0 and z = 0 then
        if level mod 2 = 1 then
            return (dir = 1 and p200 = 2)
        else
            return (dir = 2 and p200 = 1)
        end if
    else if x > 0 and y < 0 and z = 0 then
        if level mod 2 = 1 then
            return (dir = 0 and p200 = 3)
        else
            return (dir = 3 and p200 = 0)
        end if
    else if x < 0 and y < 0 and z = 0 then
        if level mod 2 = 1then
            return (dir = 1 and p200 = 3)
        else
            return (dir = 3 and p200 = 1)
        end if
    end if
    ▷ yz plane
```

else if $x = 0$ **and** $y > 0$ **and** $z > 0$ **then**
    if $level \bmod 2 = 0$ **then**
        return $(dir = 4$ **and** $p_{200} = 2)$
    else
        return $(dir = 2$ **and** $p_{200} = 4)$
    end if
else if $x = 0$ **and** $y < 0$ **and** $z > 0$ **then**
    if $level \bmod 2 = 0$ **then**
        return $(dir = 4$ **and** $p_{200} = 3)$
    else
        return $(dir = 3$ **and** $p_{200} = 4)$
    end if
else if $x = 0$ **and** $y > 0$ **and** $z < 0$ **then**
    if $level \bmod 2 = 0$ **then**
        return $(dir = 5$ **and** $p_{200} = 2)$
    else
        return $(dir = 2$ **and** $p_{200} = 5)$
    end if
else if $x = 0$ **and** $y < 0$ **and** $z < 0$ **then**
    if $level \bmod 2 = 0$ **then**
        return $(dir = 5$ **and** $p_{200} = 3)$
    else
        return $(dir = 3$ **and** $p_{200} = 5)$
    end if
end if
▷ zx plane
else if $x > 0$ **and** $y = 0$ **and** $z > 0$ **then**
    if $level \bmod 2 = 1$ **then**
        return $(dir = 4$ **and** $p_{200} = 0)$
    else
        return $(dir = 0$ **and** $p_{200} = 4)$
    end if
else if $x < 0$ **and** $y = 0$ **and** $z > 0$ **then**
    if $level \bmod 2 = 1$ **then**
        return $(dir = 4$ **and** $p_{200} = 1)$
    else
        return $(dir = 1$ **and** $p_{200} = 4)$
    end if
else if $x > 0$ **and** $y = 0$ **and** $z < 0$ **then**
    if $level \bmod 2 = 1$ **then**
        return $(dir = 5$ **and** $p_{200} = 0)$
    else
        return $(dir = 0$ **and** $p_{200} = 5)$
    end if
else if $x < 0$ **and** $y = 0$ **and** $z < 0$ **then**
    if $level \bmod 2 = 1$**then**
        return $(dir = 5$ **and** $p_{200} = 1)$
    else
        return $(dir = 1$ **and** $p_{200} = 5)$
    end if
else
▷ spirals
    $x_0 = x + SIDE/2$
    $y_0 = y + SIDE/2$
    $z_0 = z + SIDE/2$
    switch level **mod** 3 **do**

case 0
  if $x_0 \neq SIDE/2$ **and** $y_0 \neq SIDE/2$ **then**
    return $(z_0 > SIDE/2$ **and** $dir = 4)$ **or** $(z_0 < SIDE/2$ **and** $dir = 5)$
  end if
  break
case 1
  if $y_0 \neq SIDE/2$ **and** $z_0 \neq SIDE/2$ **then**
    return $(x_0 > SIDE/2$ **and** $dir = 0)$ **or** $(x_0 < SIDE/2$ **and** $dir = 1)$
  end if
  break
case 2
  if $x_0 \neq SIDE/2$ **and** $z_0 \neq SIDE/2$ **then**
    return $(y_0 > SIDE/2$ **and** $dir = 2)$ **or** $(y_0 < SIDE/2$ **and** $dir = 3)$
  end if
  break
end switch
end if
return false
end