Policy Evaluation

Summary: Policy Evaluation

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Bias/Variance of Model-free Policy Evaluation Algorithms

- Return G_t is an unbiased estimate of $V^\pi(s_t)$
- TD target $[r_t + \gamma V^\pi(s_{t+1})]$ is biased estimate of $V^\pi(s)$
- But often TD much lower variance than a single return G_t
  - MC: Return function of multi-step sequence of random actions, states & rewards
  - TD target only has one random action, reward and next state

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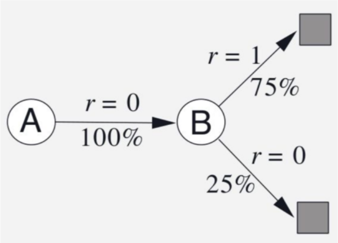
  - Consistent (converges to true) even with function approximation
- TD
  - Some bias
  - Lower variance
  - TD(0) converges to true value with tabular representation
  - TD(0) does not always converge with function approximation

AB Example [Sutton & Barto, 2018]



- Two states A, B with $\gamma = 1$
- Given 8 episodes of experience:
  - $A, 0, B, 0$
  - $B, 1$ (observed 6 times)
  - $B, 0$
- Under batch (offline) solution for this finite set of observations, what do MC and TD(0) converge to?
- Imagine run TD updates over data infinite number of times?

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- For B:
  - MC: $V(B) = \frac{6}{8} = 0.75$
  - TD: $V(B) = \frac{6}{8} = 0.75$

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- For B:
  - MC: $V(B) = \frac{6}{8} = 0.75$
  - TD: $V(B) = \frac{6}{8} = 0.75$
- For A:
  - MC: only one episode with $A \rightsquigarrow V(A) = 0$
  - TD: bootstraps from $V(B) \rightsquigarrow V(A) = 0.75$

$$V^\pi(s) = V^\pi(s) + \alpha(\underbrace{[r_t + \gamma V^\pi(s_{t+1})]}_{\text{TD target}} - V^\pi(s))$$

$\rightsquigarrow$ Monte Carlo in batch setting converges to minimal MSE (mean squared error)

$\rightsquigarrow$ TD(0) converges to DP policy V^π for the MDP with the maximum likelihood model estimates

Efficiency

- Data efficiency & Computational efficiency
- In simplest TD, use (s, a, r, s') once to update $V(s)$
  - $O(1)$ operation per update
  - In an episode of length L, $O(L)$
- In MC have to wait till episode finishes, then also $O(L)$
- MC can be more data efficient than simple TD in non-Markov domains
- TD can exploit Markov structure $\leadsto$ leveraging this is helpful

Summary: Policy Evaluation

Estimating the expected return of a particular policy if don't have access to true MDP models.
Example: evaluating average purchases per session of new product recommendation system

- Dynamic Programming
- Monte Carlo policy evaluation
  - Policy evaluation when we don't have a model of how the world works
- Temporal Difference (TD)
- Metrics to evaluate and compare algorithms
  - Robustness to Markov assumption
  - Bias/variance characteristics
  - Data efficiency
  - Computational efficiency