Policy Evaluation

Monte Carlo Evaluation: Bias and Variance for MC

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First-Visit Monte Carlo (MC) On Policy Evaluation

Initialize $N(s) = 0$, $G(s) = 0 \ \forall s \in S$

Loop

- Sample episode $i = s_{i,1}, a_{i,1}, r_{i,1}, s_{i,2}, a_{i,2}, r_{i,2}, \dots$
- Define $G_{i,t} = r_{i,t} + \gamma r_{i,t+1} + \gamma^2 r_{i,t+2} + \dots$
- For each state s visited in episode i
  - for first time t that state s is visited in episode i
    - Increment counter of total first visits: $N(s) = N(s) + 1$
    - Increment total return $G(s) = G(s) + G_{i,t}$
    - Update estimate $V^\pi(s) = G(s)/N(s)$

Recap: Bias, Variance and MSE

- Consider a statistical model that is parameterized by θ and that determines a probability distribution over observed data $P(x|\theta)$
- Consider a statistic $\hat{\theta}$ that provides an estimate of θ and is a function of observed data x
  - E.g. for a Gaussian distribution with known variance, the average of a set of i.i.d data points is an estimate of the mean of the Gaussian
- Definition: the bias of an estimator $\hat{\theta}$ is:

$$Bias_\theta(\hat{\theta}) = \mathbb{E}_{x|\theta}[\hat{\theta}] - \theta$$

- Definition: the variance of an estimator $\hat{\theta}$ is:

$$Var(\hat{\theta}) = \mathbb{E}_{x|\theta}[(\hat{\theta} - \mathbb{E}[\hat{\theta}])^2]$$

- Definition: mean squared error (MSE) of an estimator $\hat{\theta}$ is

$$MSE(\hat{\theta}) = Var(\hat{\theta}) + Bias_\theta(\hat{\theta})$$

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Properties:

- V^π estimator is an unbiased estimator of true $\mathbb{E}_\pi[G_t \mid s_t = s]$
- By law of large numbers, as $N(s) \to \inf, V^\pi(s) \to \mathbb{E}_\pi[G_t \mid s_t = s]$
- every-visit MC estimator:
  - is biased estimator of V^π (observations are correlated $\leadsto$ not i.i.d)
  - often better RMSE, because more data per state

Monte Carlo (MC) Policy Evaluation Key Limitations

- Generally high variance estimator
  - Reducing variance can require a lot of data
  - In cases where data is very hard or expensive to acquire, or the stakes are high, MC may be impractical

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- Requires episodic settings
  - Episode must end before data from episode can be used to update V

Monte Carlo (MC) Policy Evaluation Summary

- Aim: estimate $V^\pi(s)$ given episodes generated under policy π
  - $s_{i,1}, a_{i,1}, r_{i,1}, s_{i,2}, a_{i,2}, r_{i,2}, \ldots$ where the actions are sampled from π
  - $G_{i,t} = r_{i,t} + \gamma r_{i,t+1} + \gamma^2 r_{i,t+2} + \ldots$ under policy π
  - $V^\pi(s) = \mathbb{E}[G_t \mid s_t = s]$
- Simple: Estimates expectation by empirical average (given episodes sampled from policy of interest)
- Updates V estimate using sample of return to approximate the expectation
- No bootstrapping
- Does not assume Markov process