
Autonomous Agents

Report Assignment 1: Single Agent Planning

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1 Introduction

This report has been written for the Master Artificial Intelligence course Autonomous Agents. This report will contain the answers, motivation and explanation for our implementations of the tasks we had to accomplish in our first assignment for this course. These tasks were centered around the topic of “Single Agent Planning”.

1.1 The Environment

In all tasks there is assumed to be a grid world (of 11×11) with a predator and a prey in it. The agents can both move one tile forward each iteration. The direction they take (or if they move at all) is affected by probabilities (their policies). If they move over the edge of the grid they end up at the opposing side of the grid. The prey will never step into the predator. We are focused on improving the decisions of one agent, the predator.

2 Simulating the environment

A first task was to write a simulator for the environment as defined in Section 1.1.

The choice has been made to not encode the positions of the agents as part of a grid, i.e. matrix. Instead the agents both know their own position and on each iteration the position of the other agent is given as input by the environment, as we have stated in the Agent interface. This is needed for prey to see if the predator is next to it, to prevent it moving towards the prey. In the predator’s case it might be necessary later on to know the position of the prey although it is not necessary for this particular sub-assignment.

As part of the assignment a mean and a standard deviation was asked for 100 runs with the use of the random policy for the predator’s behaviour. For the exact output, one can consult Appendix A. The lowest amount of time steps observed was 19 time steps. The optimal amount of time steps given that the prey would remain still throughout the trial run would be 10. The highest amount observed was 1194 steps. The average amount of time steps was 296.93 time steps and the standard deviation was 244.55 time steps (rounded up). This is a clear indication of the inefficiency of the random policy in this particular setting.

3 Planning for the Environment

It is possible for the predator to develop a more sophisticated policy by planning. Given that the whole environment is known to the agent, we can make use of several dynamic programming planning algorithms, and we will discuss our implementation of these in the subsections below.

3.1 Policy Evaluation

The algorithm for Policy Evaluation follows from turning the Bellman equation into an update rule, such that each iteration of Policy Evaluation, Equation 1 is used for every $s \in \mathcal{S}$ (where \mathcal{S} denotes the set of states).

$$V_{k+1}(s) \leftarrow \sum_a \pi(s, a) \sum_{s'} \mathcal{P}_{ss'}^a (\mathcal{R}_{ss'}^a + \gamma V_k(s')) \quad (1)$$

Because we used the random policy for the predator, and there were five actions possible for the predator, Equation 1 could be simplified to Equation 2.

$$V_{k+1}(s) \leftarrow \frac{1}{5} \sum_a \sum_{s'} \mathcal{P}_{ss'}^a (\mathcal{R}_{ss'}^a + \gamma V_k(s')) \quad (2)$$

Where $\mathcal{P}_{ss'}^a$ has to take into account the possible movements of the prey. Note that $\mathcal{R}_{ss'}^a$ was always zero, except for the single goal state.

We implemented Policy Evaluation using a 121×121 matrix to hold the V -values for the states. It took 111 iterations to converge using $\theta = 0$ and $\gamma = 0.8$.

In the following list we show some values that Policy Evaluation found for specific states:

- **Predator(0,0), Prey(5,5):** 0.0060
- **Predator(2,3), Prey(5,4):** 0.1820
- **Predator(2,10), Prey(10,0):** 0.1820
- **Predator(10,10), Prey(0,0):** 1.1950

3.2 (SC) Policy Iteration

Bla bla Uitleg algoritme

The values we found by running Policy Iteration, letting $\theta = 0$ and varying γ to be 0.1, 0.5, 0.7 and 0.9, are exactly the same as those we found with our implementation of Value Iteration, and can thus be found in Tables 1, 2, 3 and 4.

The convergence speed in numbers of iterations is different, however. We give an overview in Table 5.

	Iterations V.I.	Iterations P.I.
$\gamma = 0.1$	20	8
$\gamma = 0.5$	28	7
$\gamma = 0.7$	31	7
$\gamma = 0.9$	34	8

Table 1: Comparison of iterations of Value Iteration and Policy Iteration

However, note that each iteration of Policy Iteration involves a call to Policy Evaluation and Policy Improvement, and both Policy Evaluation and Policy Improvement will sweep through the whole state space. Policy Improvement will do this one time, but Policy Evaluation will often do this multiple times (until the largest change of value is below the threshold θ). Therefore one would be mistaken to compare these two algorithms on the number of their largest-level iterations used as an indication of computational complexity.

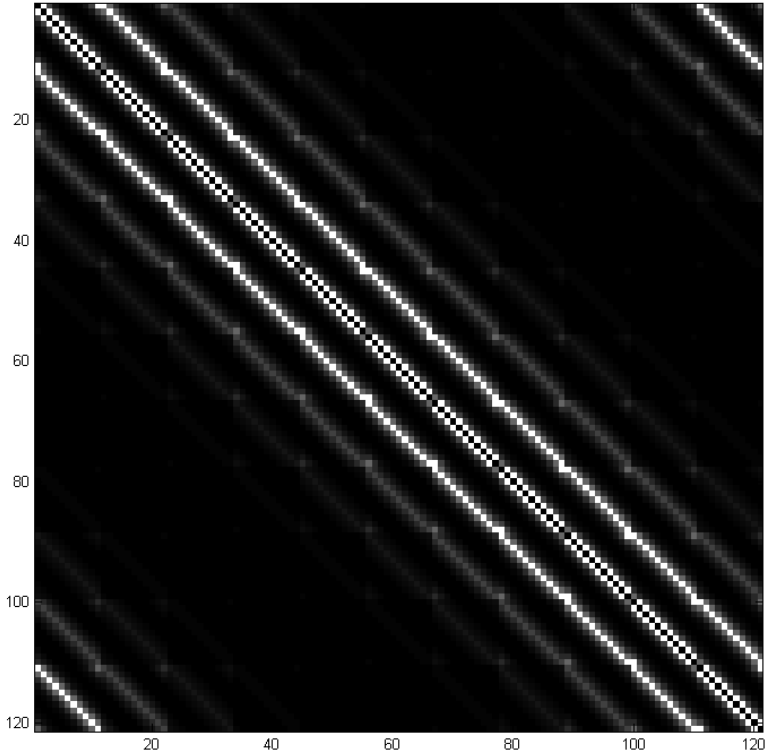


Figure 1: Colormap of the V -values resulting from Policy Evaluation for $\theta = 0$ and $\gamma = 0.8$. Each axis represents all tiles of the grid world. The brighter the color the higher the corresponding V -value.

Note, too, that it would be a mistake to compare between the number of iterations for differently parameterized Policy Iteration algorithms. For example, using $\gamma = 0.1$ we need the same number of iterations as when using $\gamma = 0.9$ for convergence, however, comparing the number of times that Policy Evaluation is invoked under these settings, we saw that using the setting of $\gamma = 0.9$ Policy Iteration had to invoke it approximately 2.45 times as much as when using the setting of $\gamma = 0.1$.

3.3 Value Iteration

Value Iteration is another planning algorithm. This algorithm combines the steps of policy evaluation and policy improvement. The algorithm does so by replacing the value of the $(k+1)^{th}$ iteration with the expected reward of the action that maximizes this expectation (based on the V -value of s' of the k^{th} iteration and the immediate reward of s'), instead of a weighted sum of the expected rewards of all actions. This algorithm uses two parameters, θ , which specifies for which amount of change in the v -values the algorithm terminates, and γ , which is the learning rate.

The Value Iteration algorithm has been used on the MDP of this assignment using different settings of γ . The used values are $\gamma = 0.1$, $\gamma = 0.5$, $\gamma = 0.7$ and $\gamma = 0.9$. For each of these runs θ is set to 0. Since the used state representation results in 11^4 states, we only provide the V -values for the states in which the prey is located at (5,5). The results can be found in Tables 1, 2, 3 and 4. A representation of the V -values which nicely shows the decrease of the V -values further from the goal state can be found in figure 2. Here, the symmetry of the state space becomes apparent.

As expected the V -values are higher near the goal state, which is (5,5) in this specific case. This will cause the predator to move towards the prey in all cases. Also the values further from the goal state decrease

faster in value for lower learning rates. The maximal V -value is 10, which is to be expected with a maximal reward of 10.

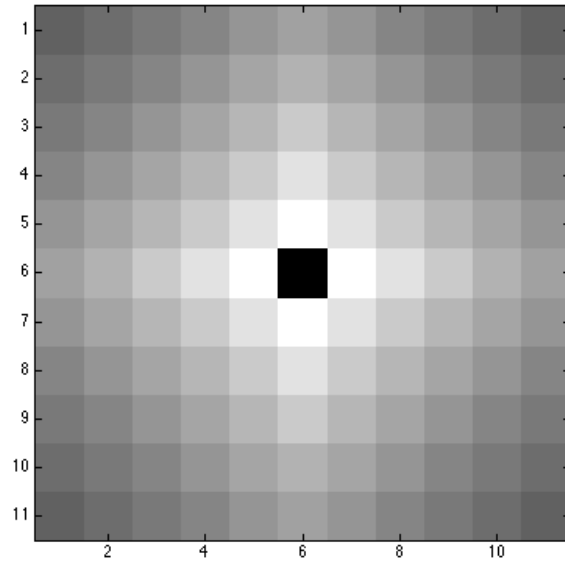


Figure 2: Colormap of the V -values resulting from Value Iteration for $\theta = 0$ and $\gamma = 0.9$.
The brighter the color the higher the corresponding V -value.

0	1	2	3	4	5	6	7	8	9	10
0	0.000000	0.000002	0.000011	0.000074	0.000438	0.001730	0.000438	0.000074	0.000011	0.000002
1	0.000002	0.000011	0.000075	0.000498	0.003195	0.013773	0.003195	0.000498	0.000075	0.000011
2	0.000011	0.000075	0.000498	0.003443	0.021739	0.117564	0.021739	0.003443	0.000498	0.000075
3	0.000075	0.000498	0.003443	0.021739	0.166976	0.816327	0.166976	0.021739	0.003443	0.000498
4	0.000498	0.003195	0.021739	0.166976	0.816327	10.000000	0.816327	0.166976	0.021739	0.003195
5	0.003195	0.021739	0.166976	0.816327	10.000000	0.000000	0.816327	0.166976	0.021739	0.003195
6	0.021739	0.166976	0.816327	10.000000	0.000000	0.000000	0.816327	0.166976	0.021739	0.003195
7	0.000498	0.003443	0.021739	0.166976	0.816327	0.166976	0.021739	0.003443	0.000498	0.000075
8	0.000075	0.000498	0.003443	0.021739	0.117564	0.021739	0.003443	0.000498	0.000075	0.000011
9	0.000002	0.000011	0.000075	0.000498	0.003195	0.013773	0.003195	0.000498	0.000075	0.000011
10	0.000000	0.000002	0.000011	0.000074	0.000438	0.001730	0.000438	0.000074	0.000011	0.000002

Table 2: The V -values for Value Iteration, with $\gamma = 0.1$ and the prey at position (5,5). The convergence speed is 20 iterations.

0	1	2	3	4	5	6	7	8	9	10
0	0.0267	0.0502	0.0943	0.1768	0.3328	0.3328	0.1768	0.0943	0.0502	0.0267
1	0.0502	0.0924	0.1769	0.3390	0.6448	0.6448	0.3390	0.1769	0.0924	0.0502
2	0.0943	0.1769	0.3390	0.6498	1.2435	2.2027	0.6498	0.3390	0.1769	0.0943
3	0.1768	0.3390	0.6498	1.2435	2.3977	4.4444	2.3977	1.2435	0.6498	0.3390
4	0.3328	0.6448	1.2435	2.3977	4.4444	10.0000	4.4444	2.3977	1.2435	0.6448
5	0.5332	1.0814	2.2027	4.4444	10.0000	0.0000	10.0000	4.4444	2.2027	1.0814
6	0.3328	0.6448	1.2435	2.3977	4.4444	10.0000	4.4444	2.3977	1.2435	0.6448
7	0.1768	0.3390	0.6498	1.2435	2.3977	4.4444	2.3977	1.2435	0.6498	0.3390
8	0.0943	0.1769	0.3390	0.6498	1.2435	2.2027	0.6498	0.3390	0.1769	0.0943
9	0.0502	0.0924	0.1769	0.3390	0.6448	1.0814	0.6448	0.3390	0.0924	0.0502
10	0.0267	0.0502	0.0943	0.1768	0.3328	0.3328	0.1768	0.0943	0.0502	0.0267

Table 3: The V -values for Value Iteration, with $\gamma = 0.5$ and the prey at position (5,5). The convergence speed is 28 iterations.

	0	1	2	3	4	5	6	7	8	9	10
0	0.4348	0.6065	0.8453	1.1765	1.6463	2.1169	1.6463	1.1765	0.8453	0.6065	0.4348
1	0.6065	0.8328	1.1750	1.6587	2.3345	3.0759	2.3345	1.6587	1.1750	0.8328	0.6065
2	0.8453	1.1750	1.6587	2.3415	3.3044	4.4805	3.3044	2.3415	1.6587	1.1750	0.8453
3	1.1765	1.6587	2.3415	3.3044	4.6737	6.5116	4.6737	3.3044	2.3415	1.6587	1.1765
4	1.6463	2.3345	3.3044	4.6737	6.5116	10.0000	6.5116	4.6737	3.3044	2.3345	1.6463
5	2.1169	3.0759	4.4805	6.5116	10.0000	0.0000	10.0000	6.5116	4.4805	3.0759	2.1169
6	1.6463	2.3345	3.3044	4.6737	6.5116	10.0000	6.5116	4.6737	3.3044	2.3345	1.6463
7	1.1765	1.6587	2.3415	3.3044	4.6737	6.5116	4.6737	3.3044	2.3415	1.6587	1.1765
8	0.8453	1.1750	1.6587	2.3415	3.3044	4.4805	3.3044	2.3415	1.6587	1.1750	0.8453
9	0.6065	0.8328	1.1750	1.6587	2.3345	3.0759	2.3345	1.6587	1.1750	0.8328	0.6065
10	0.4348	0.6065	0.8453	1.1765	1.6463	2.1169	1.6463	1.1765	0.8453	0.6065	0.4348

Table 4: The V -values for Value Iteration, with $\gamma = 0.7$ and the prey at position (5,5). The convergence speed is 31 iterations.

	0	1	2	3	4	5	6	7	8	9	10
0	3.8831	4.2915	4.7417	5.2374	5.7919	6.2513	5.7919	5.2374	4.7417	4.2915	3.8831
1	4.2915	4.7118	5.2281	5.8024	6.4356	6.9973	6.4356	5.8024	5.2281	4.7118	4.2915
2	4.7417	5.2281	5.8024	6.4401	7.1476	7.8390	7.1476	6.4401	5.8024	5.2281	4.7417
3	5.2374	5.8024	6.4401	7.1476	7.9362	8.7805	7.9362	7.1476	6.4401	5.8024	5.2374
4	5.7919	6.4356	7.1476	7.9362	8.7805	10.0000	8.7805	7.9362	7.1476	6.4356	5.7919
5	6.2513	6.9973	7.8390	8.7805	10.0000	0.0000	10.0000	8.7805	7.8390	6.9973	6.2513
6	5.7919	6.4356	7.1476	7.9362	8.7805	10.0000	8.7805	7.9362	7.1476	6.4356	5.7919
7	5.2374	5.8024	6.4401	7.1476	7.9362	8.7805	7.9362	7.1476	6.4401	5.8024	5.2374
8	4.7417	5.2281	5.8024	6.4401	7.1476	7.8390	7.1476	6.4401	5.8024	5.2281	4.7417
9	4.2915	4.7118	5.2281	5.8024	6.4356	6.9973	6.4356	5.8024	5.2281	4.7118	4.2915
10	3.8831	4.2915	4.7417	5.2374	5.7919	6.2513	5.7919	5.2374	4.7417	4.2915	3.8831

Table 5: The V -values for Value Iteration, with $\gamma = 0.9$ and the prey at position (5,5). The convergence speed is 34 iterations.

4 (SC) State space reduction

In the experiments described in all following sections, unless stated otherwise, we used a state space that ...
(*utleg default state space*)

We will refer to this state space as the “default” state space.

We elaborated on a more efficient representation for the states, and finally contrived one consisting of 30 states, an approximately 488 times smaller one than the default state space. We will refer to this state space as the “efficient” state space. This state space...

(*utleg efficiente state space*)

5 Conclusion

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Appendices

A Simulating the Environment

Our program’s output for the first 100 runs for the random policy predator.

```
Timesteps:421
Timesteps:831
Timesteps:476
Timesteps:74
Timesteps:537
Timesteps:40
Timesteps:468
Timesteps:465
Timesteps:105
Timesteps:123
Timesteps:227
Timesteps:658
Timesteps:696
Timesteps:153
Timesteps:426
Timesteps:431
Timesteps:24
Timesteps:197
Timesteps:517
Timesteps:313
Timesteps:492
Timesteps:213
Timesteps:457
Timesteps:392
Timesteps:47
Timesteps:178
Timesteps:459
Timesteps:624
Timesteps:881
Timesteps:100
Timesteps:244
Timesteps:127
Timesteps:213
Timesteps:145
Timesteps:45
Timesteps:301
Timesteps:628
Timesteps:248
Timesteps:88
Timesteps:123
Timesteps:82
Timesteps:206
Timesteps:181
Timesteps:771
Timesteps:114
Timesteps:238
Timesteps:118
Timesteps:67
Timesteps:41
Timesteps:662
Timesteps:27
Timesteps:73
Timesteps:217
Timesteps:269
Timesteps:382
Timesteps:60
Timesteps:205
Timesteps:64
Timesteps:133
Timesteps:232
Timesteps:148
Timesteps:504
Timesteps:113
Timesteps:316
Timesteps:151
Timesteps:178
Timesteps:53
Timesteps:526
Timesteps:150
Timesteps:690
Timesteps:490
```



```

Timesteps:116
Timesteps:288
Timesteps:79
Timesteps:163
Timesteps:266
Timesteps:566
Timesteps:1194
Timesteps:133
Timesteps:690
Timesteps:136
Timesteps:121
Timesteps:123
Timesteps:492
Timesteps:288
Timesteps:185
Timesteps:19
Timesteps:78
Timesteps:250
Timesteps:42
Timesteps:268
Timesteps:190
Timesteps:231
Timesteps:393
Timesteps:338
Timesteps:100
Timesteps:49
Timesteps:653
Timesteps:1173
Timesteps:421
Average timesteps over 100 trials: 296.93
Standard deviation over 100 trials: 244.54689754728025

```

B Policy Evaluation

Our program’s output for Policy Evaluation using a discount factor (γ) of 0.8.
The term “maxValueDiff” denotes the threshold θ should be under for the algorithm to terminate.

```

Policy Evaluation, iteration number: 1; maxValueDiff = 2.173444408972901
Policy Evaluation, iteration number: 2; maxValueDiff = 0.594273911906516
Policy Evaluation, iteration number: 3; maxValueDiff = 0.288747386234562
Policy Evaluation, iteration number: 4; maxValueDiff = 0.142434892859105
Policy Evaluation, iteration number: 5; maxValueDiff = 0.077807292807347
Policy Evaluation, iteration number: 6; maxValueDiff = 0.043989076413322
Policy Evaluation, iteration number: 7; maxValueDiff = 0.025523868234365
Policy Evaluation, iteration number: 8; maxValueDiff = 0.015571413160874
Policy Evaluation, iteration number: 9; maxValueDiff = 0.009673932990005
Policy Evaluation, iteration number: 10; maxValueDiff = 0.006050573197382
Policy Evaluation, iteration number: 11; maxValueDiff = 0.003853977807979
Policy Evaluation, iteration number: 12; maxValueDiff = 0.002486803847959
Policy Evaluation, iteration number: 13; maxValueDiff = 0.001609366549420
Policy Evaluation, iteration number: 14; maxValueDiff = 0.001044912566793
Policy Evaluation, iteration number: 15; maxValueDiff = 0.000680717843366
Policy Evaluation, iteration number: 16; maxValueDiff = 0.000452195511078
Policy Evaluation, iteration number: 17; maxValueDiff = 0.000300871717137
Policy Evaluation, iteration number: 18; maxValueDiff = 0.000201864159448
Policy Evaluation, iteration number: 19; maxValueDiff = 0.000136221951843
Policy Evaluation, iteration number: 20; maxValueDiff = 0.000091909267497
Policy Evaluation, iteration number: 21; maxValueDiff = 0.000062025880297
Policy Evaluation, iteration number: 22; maxValueDiff = 0.000042272381132
Policy Evaluation, iteration number: 23; maxValueDiff = 0.000028863910013
Policy Evaluation, iteration number: 24; maxValueDiff = 0.000019849447467
Policy Evaluation, iteration number: 25; maxValueDiff = 0.000013630925504
Policy Evaluation, iteration number: 26; maxValueDiff = 0.000009359323432
Policy Evaluation, iteration number: 27; maxValueDiff = 0.000006460531614
Policy Evaluation, iteration number: 28; maxValueDiff = 0.000004480328471
Policy Evaluation, iteration number: 29; maxValueDiff = 0.000003102386442
Policy Evaluation, iteration number: 30; maxValueDiff = 0.000002145040952
Policy Evaluation, iteration number: 31; maxValueDiff = 0.000001489301996
Policy Evaluation, iteration number: 32; maxValueDiff = 0.000001035209317
Policy Evaluation, iteration number: 33; maxValueDiff = 0.000000718422800
Policy Evaluation, iteration number: 34; maxValueDiff = 0.000000497903405
Policy Evaluation, iteration number: 35; maxValueDiff = 0.000000344679351
Policy Evaluation, iteration number: 36; maxValueDiff = 0.000000238380220
Policy Evaluation, iteration number: 37; maxValueDiff = 0.000000164815624
Policy Evaluation, iteration number: 38; maxValueDiff = 0.000000114177600
Policy Evaluation, iteration number: 39; maxValueDiff = 0.000000079027742
Policy Evaluation, iteration number: 40; maxValueDiff = 0.000000054656798
Policy Evaluation, iteration number: 41; maxValueDiff = 0.00000003776286

```

[illegible]