生醫影像導論

Introduction to Biomedical Imaging

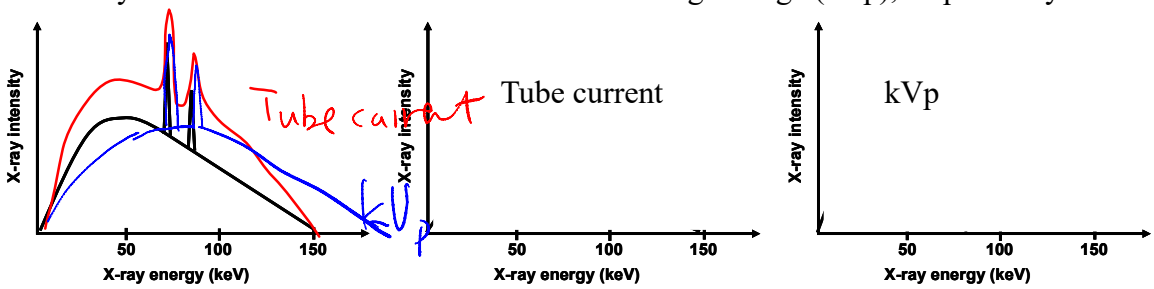
(11020EE 441000)

Homework #3: X-Ray Imaging

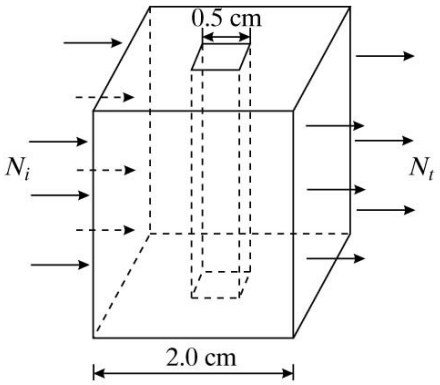
Due: 24:00, 06/05/2010

評分方式： 滿分 100 分，每小題 20 分

1. The left most figure below shows the intensity of X-rays produced from a source as a function of their energy (i.e., X-ray energy spectrum). With respect to the reference graph shown on the left, plot the corresponding X-ray energy spectrum once you increase the tube current and accelerating voltage (kVp), respectively.



2. A slab of soft tissue with one blood vessel running in the middle is imaged under an X-ray imaging system, as shown in the following figure. For ease of computation, assume the tissue and the vessel both have squared-shaped cross-sections; the dimensions are shown in the figure. Assume that the X-ray source produces $N_i = 4 \times 10^6$ photons at either 15 keV or 40 keV, and the photons are uniformly shed upon the side of the tissue. The linear attenuation coefficients μ of the soft tissue, blood, and a radiographic contrast agent at two energy levels are given in the table below.



	Linear attenuation coefficient (cm^{-1})		
Energy (keV)	Soft tissue	Blood	Contrast agent
15	4.0	3.0	5.0

40	0.4	0.2	20

Ignoring photon noise, detector efficiency, beam divergence, Compton scattering, path length, and magnification effects,

(a) Determine the total number of exiting photons at the two energy levels, respectively. At which energy level are more photons absorbed.

(b) Calculate the local contrast [$(I_t\text{-}I_b)/I_b$, t: target, b: background] of the blood vessel (target) at each energy level. At which energy level is the local contrast of the blood vessel better?

(c) Suppose the contrast agent whose linear attenuation coefficient is given in the table were injected into the blood vessel. Would you expect there to be much difference in the local contrast at 15 keV after injection? How about at 40 keV? (No calculations, please, just mathematical reasoning)

(d) Explain why it is expected that the linear attenuation coefficient of the contrast agent is much larger at 40 keV than at 15 keV?

(a) Since the number of photons are uniformly shed upon the side of the whole tissue, it is clear that $1/4(= 0.5/2.0)$ of the total incident photons will go through the blood vessel, and the remaining $3/4N_i$ photons only pass through the soft tissue. Hence the total number of photons can be computed as follows

$$\begin{aligned} N_t &= \frac{N_i}{4} e^{-\mu_{\text{vessel}} L_{\text{vessel}} - \mu_{\text{tissue}} (L_{\text{tissue}} - L_{\text{vessel}})} + \frac{3N_i}{4} N_i e^{-\mu_{\text{tissue}} L_{\text{tissue}}} \\ &= \frac{N_i}{4} e^{-0.5\mu_{\text{vessel}}} e^{-1.5\mu_{\text{tissue}}} + \frac{3N_i}{4} e^{-2\mu_{\text{tissue}}}. \end{aligned}$$

At 15 keV,

$$\begin{aligned} N_t &= \frac{4 \times 10^6}{4} e^{-0.5\times 3.0} e^{-1.5\times 4.0} + \frac{3 \times 4 \times 10^6}{4} e^{-2\times 4.0} \\ &\approx 1006 + 553 \\ &= 1559. \end{aligned}$$

At 40 keV,

$$\begin{aligned} N_t &= \frac{4 \times 10^6}{4} e^{-0.5\times 0.2} e^{-1.5\times 0.4} + \frac{3 \times 4 \times 10^6}{4} e^{-2\times 0.4} \\ &\approx 4.96 \times 10^5 + 1.35 \times 10^6 \\ &\approx 1.84 \times 10^6. \end{aligned}$$

We see from this analysis that, at the lower energy level 15 keV, more photons are absorbed, because the linear attenuation coefficients of the tissue and the blood vessel are both high at 15 keV than at 40 keV.

(b) Since the incident photons are uniformly shed upon the tissue, the photon density p, that is, the number of photons per unit area, is a constant and can be computed as $p = N_i/A$, where A is the area of the side of the tissue. Notice that the value of p does not affect the local contrast computation, which can be shown as follows. The background intensity N_b is simply

$$N_b = pe^{-\mu_{\text{tissue}} L_{\text{tissue}}} = pe^{-2.0\mu_{\text{tissue}}}.$$

The object intensity N_o is given by

$$\begin{aligned} N_o &= pe^{-\mu_{\text{vessel}} L_{\text{vessel}} - \mu_{\text{tissue}} (L_{\text{tissue}} - L_{\text{vessel}})} \\ &= pe^{-0.5\mu_{\text{vessel}} - 1.5\mu_{\text{tissue}}}. \end{aligned}$$

Hence, the local contrast can be computed as

$$\begin{aligned} C &= \frac{N_o - N_b}{N_b} \\ &= \frac{pe^{-0.5\mu_{\text{vessel}} - 1.5\mu_{\text{tissue}}} - pe^{-2.0\mu_{\text{tissue}}}}{pe^{-2.0\mu_{\text{tissue}}}} \\ &= e^{-0.5(\mu_{\text{vessel}} - \mu_{\text{tissue}})} - 1. \end{aligned}$$

At 15 keV, the local contrast is

$$C_{15} = e^{-0.5\times(3.0-4.0)} - 1 \approx 0.649.$$

At 40 keV, the local contrast is

$$C_{40} = e^{-0.5\times(0.2-0.4)} - 1 \approx 0.105.$$

Hence, the local contrast is higher (better) at 15 keV.

(c) As we derived in part (b), the local contrast is totally determined by the difference between the linear attenuation coefficients of the soft tissue and the blood vessel. As can be seen from the table, at 15 keV this difference does not change much after the contrast agent is injected into the blood vessel. Hence, it would be expected that the local contrast (in its absolute value) does not change much. (The new contrast is actually 0.393 in absolute value.)

At 40 keV, the linear attenuation coefficient of the contrast agent is hugely different from that of the soft tissue and the original blood vessel. Thus, it can be expected that the local contrast of the blood vessel will be largely changed (improved) after the contrast agent is injected in. (The new contrast is actually 0.999 in absolute value.)

(d) The explanation is that the contrast agent material has K-shell electrons whose binding energy is slightly lower than 40 keV but higher than 15 keV. When x-ray photons with an energy of 40 keV enter the material, photoelectric interaction will cause electrons from the K-shell to be ejected and the x-ray photons will be completely absorbed. This effect, called K-edge absorption, significantly increases the attenuation coefficient of the contrast agent.

3. Interesting properties of tissues can sometimes be revealed by imaging them at two x-ray energies. Suppose you are going to use your conventional chest x-ray setup to make two films. One where the highest x-ray energies are at 30 keV and the other where the highest energies are at 100 keV.

(a) What physical parameter will you change, and to what values will you set it, in order to make these two images?

(b) Assuming nothing else changes, which film would you expect to be more exposed than the other? Explain.

(c) At which energy is Compton scattering more of a problem?

(d) Suppose you took the two exposed films and subtracted their optical densities, creating a third image: $D(x,y) = D(x,y; E_h) - D(x,y; E_l)$ [h: high energy, l: low

energy]. Describe in mathematical terms what D(x,y) measures.

(a) You would change the peak tube voltage, or kVp. To generate the first film, you would use a tube voltage = 30 kVp, and to generate the second film, you would use a tube voltage = 100kVp.

(b) If you did not change anything else, the second film, taken at 100 kVp, would be more exposed. That is because the body is more transparent at higher x-ray energies, so more x-rays would get through to expose the film. The high energy x-rays, when stopped by the intensifying screen, will generate more light output as well adding to the exposure. It is true that the intensifying screen is also more transparent at higher energies, but the x-ray spectrum at 100 kVp also contains lower energy x-rays that would contribute to the overall exposure.

(c) we know that Compton scattering events become an increasingly larger fraction of the events as the x-ray energy increases. Therefore, Compton scattering will be more of "a problem" at 100 keV versus 30 keV, yielding lower contrast images.

(d)

$$\begin{aligned} D(x,y) &= D(x,y;E_h) - D(x,y;E_l) \\ &= \Gamma \log_{10}(X_h/X_0) - \Gamma \log_{10}(X_l/X_0) \\ &= \Gamma \log_{10}\left(\frac{X_h}{X_l}\right) \end{aligned}$$

Since $X_h \geq X_l$ (in general), $D(x,y)$ will be a non-negative image revealing the relative additional "transparency" of tissues at the higher x-ray energies. (Note: If the two energies were used on "opposite sides" of the k-edge of a contrast agent, then the difference $D(x,y;E_l) - D(x,y;E_h)$ would be used instead, since the attenuation at the higher energy would be larger.)

4. Describe "Dual Energy Imaging" at least including the purpose and principles.
Dual Energy Imaging 成像原理是由兩張 X-ray 影像所組成，測定方式由 Dual-energy X-ray absorptiometry(DXA)，兩張影像分別在不同 photon energy 下成像，即兩張各有不同的衰減係數，再依權重將一張影像減去另一張，即可選擇僅保留骨頭或僅保留軟組織，通常是用來檢測骨質疏鬆、骨頭密度，舉文獻中的例子來說，如 fig.1，從不同能量下擷取 X-ray image，可看到 low energy 中骨頭約 8 個 unit 軟組織約 3 個 unit，high energy 中骨頭約 4 個 unit 軟組織約 2 個 unit，再選擇所要保留的東西(骨頭或組織)，依權種下去計算，如 Fig.2 權重選擇，就可以去除掉不想保留的部分。

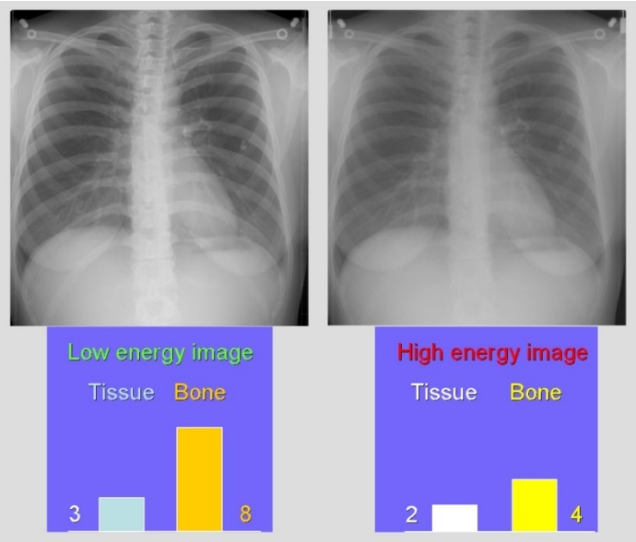
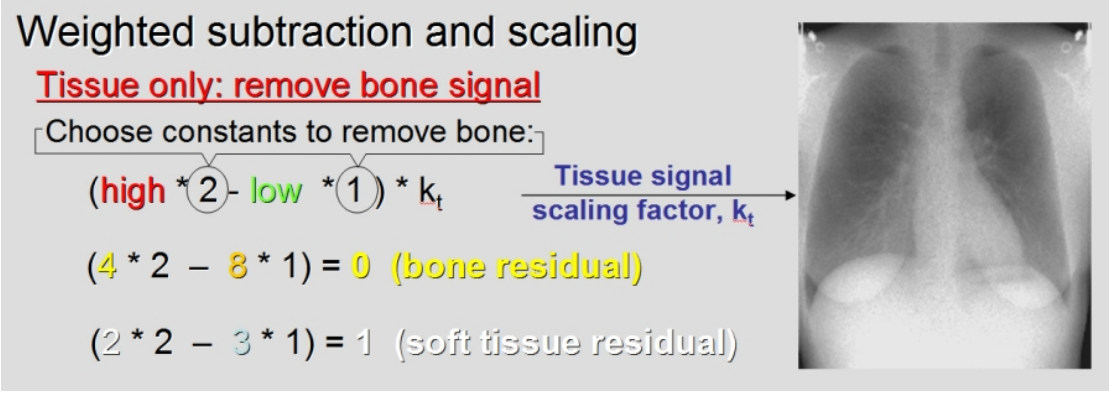


Fig 1. two images acquired at low energy and high energy



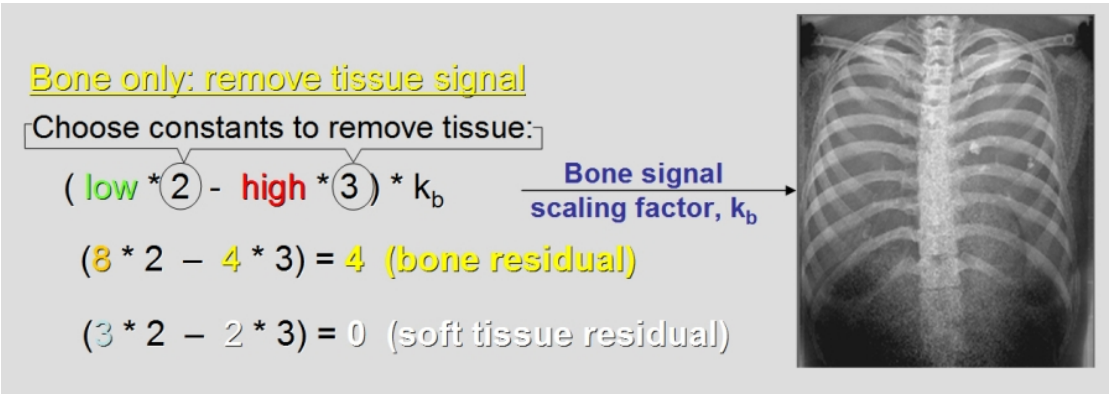
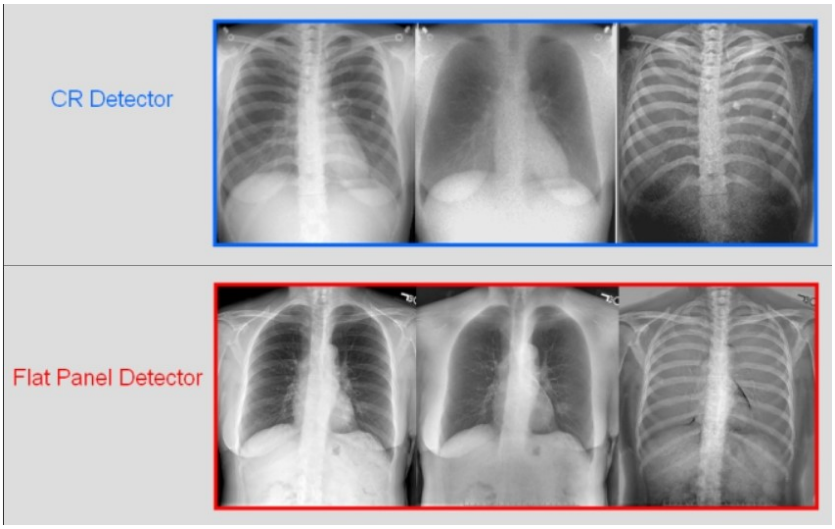


Fig 2. Weighted subtraction and scaling

現行市面上有兩種醫學成像系統，第一為 "passive" photostimulable storage phosphor imaging plates 可同時獲取兩個圖像，兩個成像板堆疊，其一會先吸收較低能量，再讓第二個成像板吸收較高能量，好處是不會有 motion artifact，壞處是對比不夠強烈，第二種系統是運用 flat-panel detector 搭配兩種不同能量的燈管，好處是對比差異很大，壞處是執行時間較久容易有 artifact。



參考網站:

i. https://www.insideradiology.com.au/dual-energy-ct-scan/
ii. https://www.upstate.edu/radiology/education/rsna/radiography/dual.php

5. Please find out the spatial resolution of common X-ray imaging in μm, and the values of commonly used accelerating voltage (kVp) and tube current (mA or mAs), both of which I never mentioned in my lecture because I have no idea, either but I believe, both of which are body-type dependent.
   - Spatial resolution 查到約為 100μm，由於 CT 也是運用 X-ray 的原理因此我也順便查了一下，傳統 CT 的 spatial resolution 約為 1-2mm，而 high resolution 的 CT 約為 100-200 μm。
   - Common used kVp and mAs: 主要用於醫療目的的旋轉陽極管從 25 kV 至 150 kV，以及檢查行業中使用的固定陽極管從 25 kV 至超過 400 kV。固定式陽極管通常在接近連續的工作狀態下以 1-20 mA 的電流工作，並且一次可以工作數小時，而旋轉陽極管的工作電流超過 1000 mA，但主要用於約 1 毫秒至 10 秒的脈沖模式，另外，約有 20%以下之體型較大的患者須提升 20kVp 的 X-ray beam，可以看出 kVp 會受 body-type 的影響。

參考網站:

i. https://indico.cern.ch/event/418810/contributions/1009332/attachments/869668/1218112/iworid.pdf
ii. http://www.ctlab.geo.utexas.edu/about-ct/resolution-and-size-limitations/
iii. https://www.spellmanhv.com/en/Technical-Resources/Application-Notes-X-Ray-Generators/AN-02
iv. https://pubs.rsna.org/doi/pdf/10.1148/radiol.2521080141#:~:text=Compared%20with%20increases%20in%20distal,increase%20in%20voltage%20results%20

in

Notice:

1. Please hand in your solution files to the LMS elearning system, including your word file of the detailed solutions, the associated Matlab codes, and all the related materials. It would be nice that you can put your codes with comments side by side along with your answer in the word file.
2. Name your solution files "BioMedImg_HW4_StudentID.doc" and "BioMedImg_HW4_StudentID.m", and archive them as a single zip file: BioMedImg_HW4_StudentID.zip.
3. The first line of your word or Matlab file should contain your name and some brief description, e.g., % EE 441000 王小明 u9612345 HW4 06/15/2010
4. Please include "figures" and "reference sources" in your word file