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Laboratory 5

Problem : Public-key cryptography (ElGamal)

In cryptography, the ElGamal encryption system is an asymmetric key encryption algorithm for public-key cryptography which is based on the Diffie–Hellman key exchange. It was described by Taher Elgamal in 1984.[1] ElGamal encryption is used in the free GNU Privacy Guard software, recent versions of PGP, and other cryptosystems. The Digital Signature Algorithm is a variant of the ElGamal signature scheme, which should not be confused with ElGamal encryption.

ElGamal encryption can be defined over any cyclic group G. Its security depends upon the difficulty of a certain problem in G related to computing discrete logarithms.

ElGamal encryption consists of three components: the key generator, the encryption algorithm, and the decryption algorithm.

Key generation

The key generator works as follows:

- Alice generates an efficient description of a multiplicative cyclic group G of order q with generator g. See below for a discussion on the required properties of this group.
- Alice chooses a random x from $\{0, \dots, q-1\}$.
- Alice computes $h = g^x$.
- Alice publishes h, along with the description of G, q, g, as her **public key**. Alice retains x as her **private key** which must be kept secret.

Encryption

The encryption algorithm works as follows: to encrypt a message m to Alice under her public key (G, q, g, h),

- Bob chooses a random y from $\{0, \dots, q-1\}$, then calculates $c_1 = g^y$.
- Bob calculates the shared secret $s = h^y$. Since a new y is generated for every message[*why?*], y is also called an ephemeral key.

The steps above can be computed ahead of time.

- Bob converts his secret message m into an element m' of G.
- Bob calculates $c_2 = m' \cdot s$.
- Bob sends the ciphertext $(c_1, c_2) = (g^y, m' \cdot h^y) = (g^y, m' \cdot (g^x)^y)$ to Alice.

Decryption

The decryption algorithm works as follows: to decrypt a ciphertext (c_1, c_2) with her private key x,

- Alice calculates the shared secret $s = c_1^x$
- and then computes $m' = c_2 \cdot s^{-1}$ which she then converts back into the plaintext message m.

The decryption algorithm produces the intended message, since

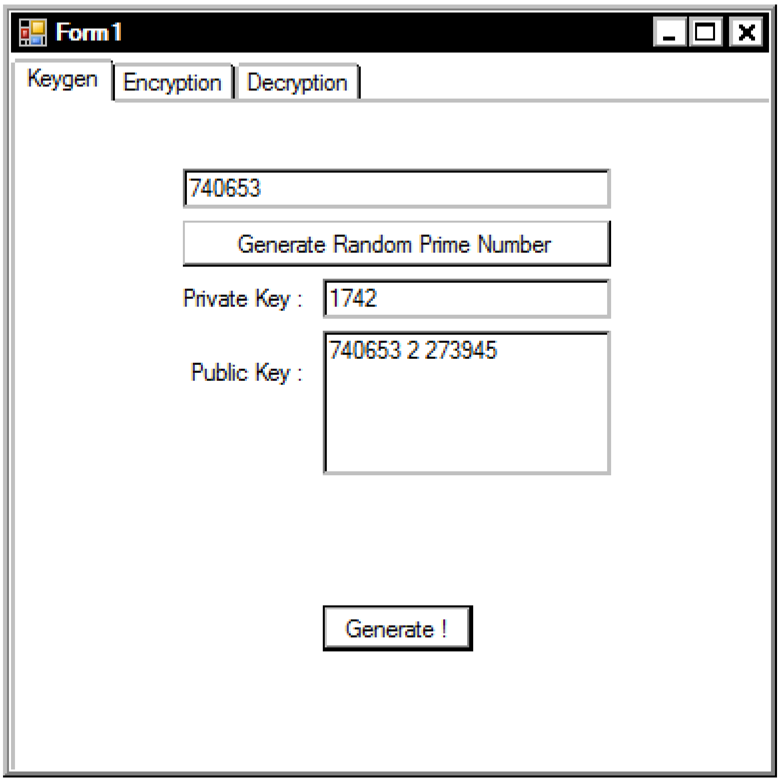
$$c_2 \cdot s^{-1} = m' \cdot h^y \cdot (g^{xy})^{-1} = m' \cdot g^{xy} \cdot g^{-xy} = m'.$$

The ElGamal cryptosystem is usually used in a hybrid cryptosystem. I.e., the message itself is encrypted using a symmetric cryptosystem and ElGamal is then used to encrypt the key used for the symmetric cryptosystem. This allows encryption of messages that are longer than the size of the group G.

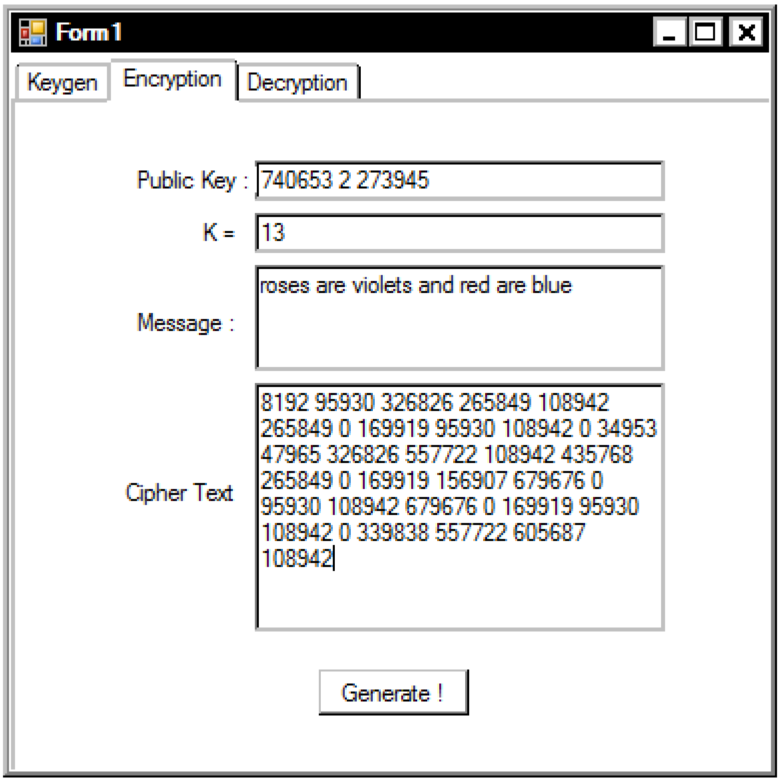
In our case, we have applied the algorithm as follows: each letter from the input message is converted to a numeric form, representing the letter's position in the alphabet, and then it is encrypted. When decrypting, the algorithm is applied in the same manner.

Later on, we present a running example of our application.

Firstly, a key is generated (we only give the private key manually).



Then, the message is encrypted.



Using the same key, the message is decrypted.