Machine Learning Assignment - 1

(a)

$f(z) = \log_e(1+z)$, where $z = x^T x$, $x \in \mathbb{R}^d$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} \qquad x^T = [x_1 \ x_2 \ \ldots \ x_d]$$

$$x^T x = [x_1^2 + x_2^2 + \ldots + x_d^2]$$

using chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dx}$$

$$= \frac{d}{dz}\left(\log_e(1+z)\right) \cdot \frac{d}{dx}\left(x^T x\right)$$

$$= \frac{d}{dz}\left(\ln(1+z)\right) \cdot \frac{d}{dx}\left(x_1^2 + x_2^2 + \ldots + x_d^2\right)$$

$$= \frac{1}{1+z} \frac{d}{dz}(z) \cdot \left(2x_1 + 2x_2 + \cdots + 2x_d\right)$$

$$= \frac{1}{1+z} \cdot 2 \cdot \left(x_1 + x_2 + \cdots + x_d\right)$$

$$= \frac{1}{1+z} \, 2 \sum_{i=1}^{d} x_i$$

$$= \frac{2}{1+x^T x} \sum_{i=1}^{d} x_i \quad \text{Ans}$$

(b) $f(z) = e^{-\frac{z}{2}}$

$z = g(y) = y^T S^{-1} y$

$y = h(x) = x - \mu$

Where $x, \mu \in \mathbb{R}^d$ and $S \in \mathbb{R}^{d \times d}$

using the chain rule method

$$\frac{df}{dx} = \frac{df}{dz} \times \frac{dz}{dy} \times \frac{dy}{dx}$$

$$\frac{df}{dz} = \frac{d}{dz}\left(e^{-1/2 z}\right)$$

$$= -\frac{e^{-z/2}}{2}$$

$$\frac{dz}{dy} = \frac{d}{dy}\left(y^T s^{-1} y\right)$$

$$= \lim_{h \to 0} \frac{g(y+h) - g(y)}{h}$$

$$= \lim_{h \to 0} \frac{(y^T + h) s^{-1} (y+h) - y^T s^{-1} y}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{y^T s^{-1} h} + y^T s^{-1} h + h s^{-1} y + s^{-1} h^2 - \cancel{y^T s^{-1} y}}{h}$$

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$$= \lim_{h \to 0} \frac{y^T S^{-1} h + h S^{-1} y + S^{-1} h^2}{h}$$

$$= \lim_{h \to 0} \frac{h\left(y^T S^{-1} + S^{-1} y + S^{-1} h\right)}{h}$$

$$\cancel{= \lim_{h \to 0} \left(y^T S^{-1} + S^{-1} y\right)}$$

$$= \lim_{h \to 0} \left(y^T S^{-1} + S^{-1} y + S^{-1} h\right)$$

$$= y^T S^{-1} + S^{-1} y + \lim_{h \to 0} S^{-1} h$$

$$= y^T S^{-1} + S^{-1} y$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x - \mu\right)$$

$$= 1$$

$$\therefore \frac{df}{dx} = -\frac{e^{-7/2}}{2} \cdot \left(y^T s^{-1} + s^{-1} y\right) \cdot 1$$

$$= -\frac{e^{-7/2}}{2} \frac{1}{s} \left(y^T + y\right)$$ Ans

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