Neural Networks and the XOR problem

$X = [x_1, x_2]$ input

$W = [w_1, w_2]$

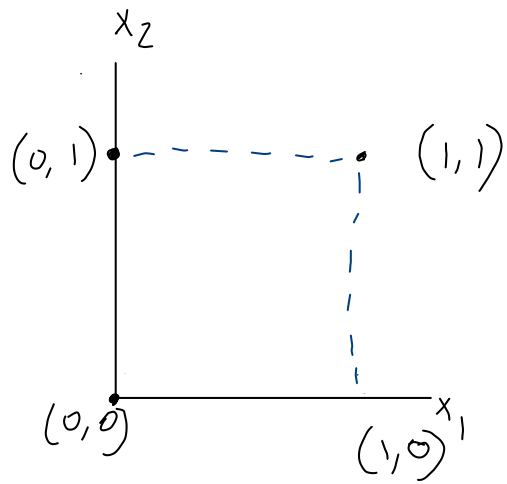
T = True = 1

F = False = 0

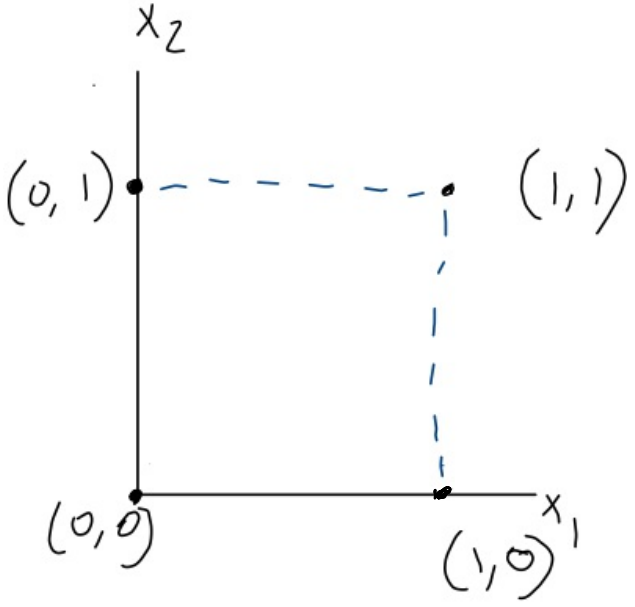
Logical AND operator

x_1	x_2	$x_1 \wedge x_2$
T	T	T
T	F	F
F	T	F
F	F	F

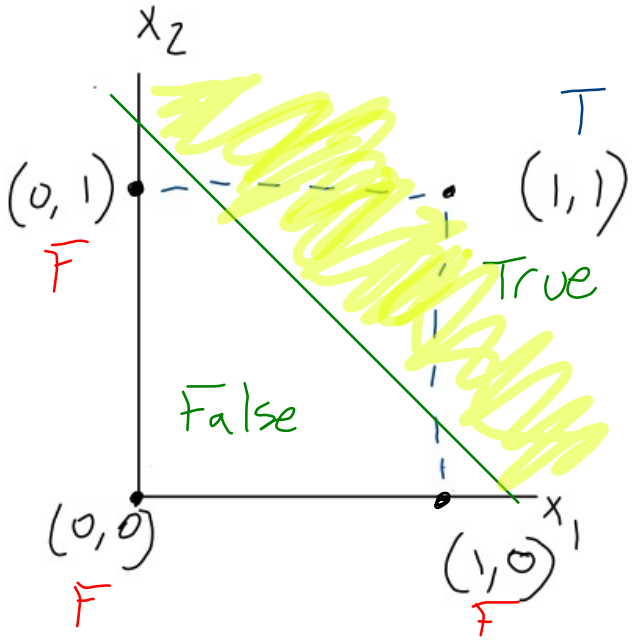
Graph



AND



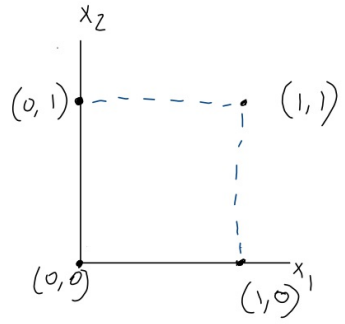
Can we draw a line to separate (x1 AND x2) so it returns true and false properly?



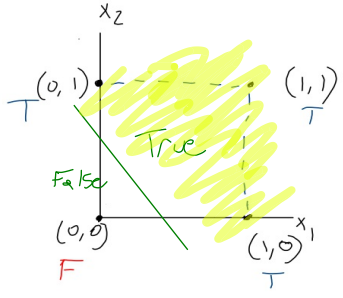
Yes! Anything above the line is true and below the line is false.

Logical OR

x_1	x_2	$x_1 \vee x_2$
T	T	T
T	F	T
F	T	T
F	F	F

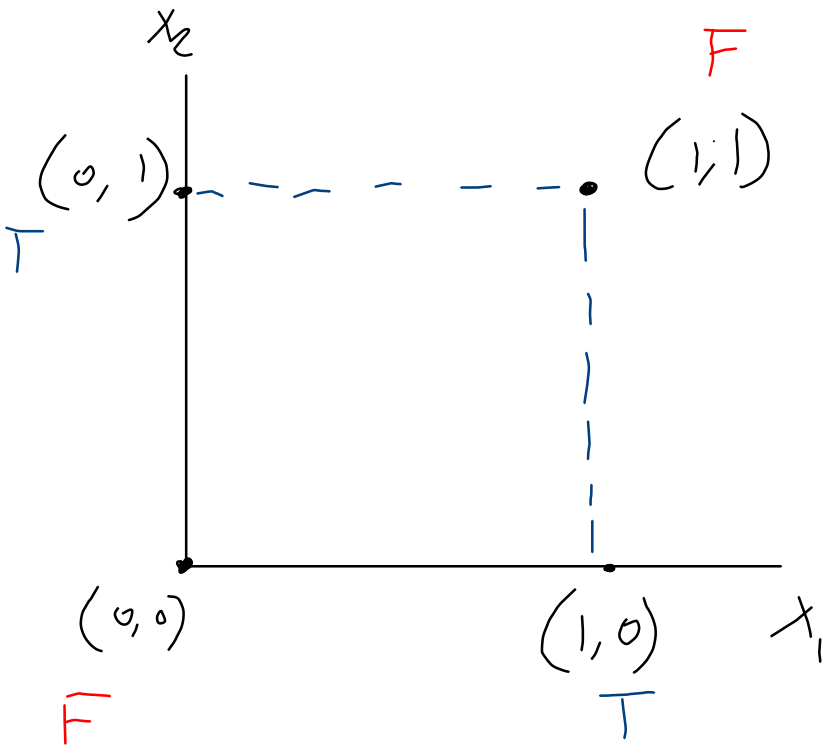


Can we find a line to separate true and false? Yes! Just like the AND



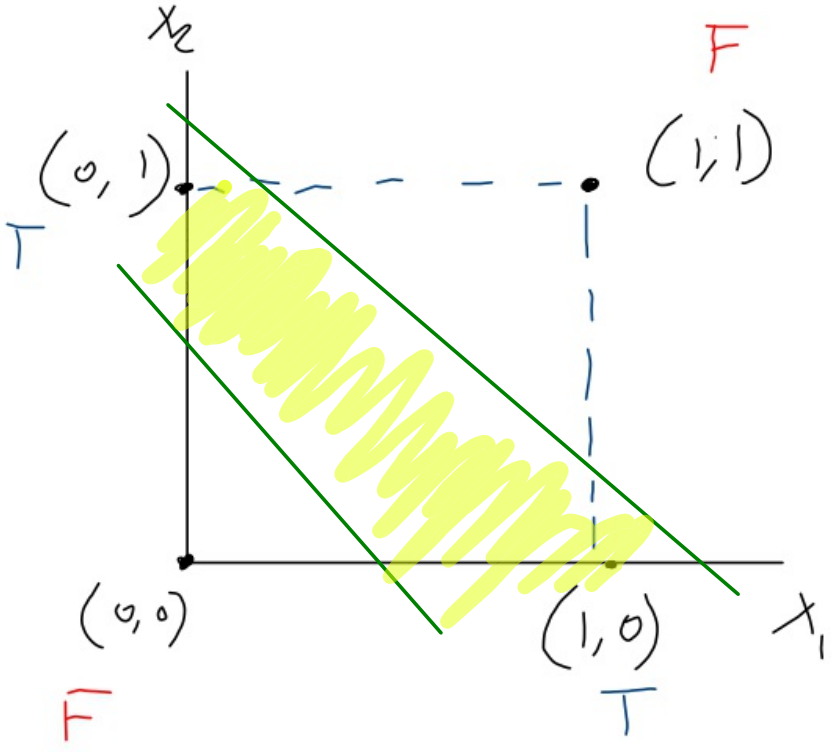
The XOR Problem

X_1	X_2	X_1 XOR X_2
T	T	F
T	F	T
F	T	T
F	F	F

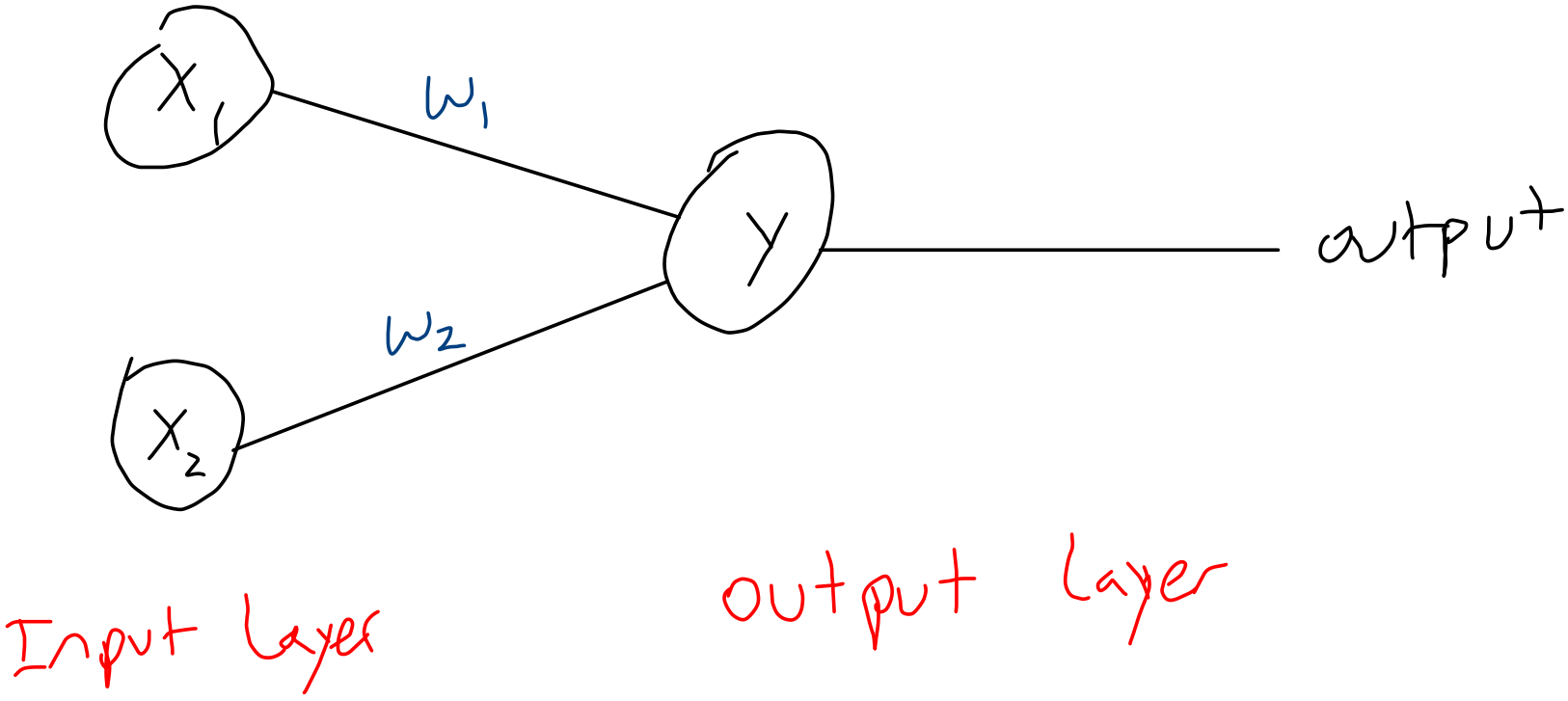


Can we draw a single line to separate the two classes?

Although XOR doesn't seem so different from AND/OR, it's suprisingly more computationally complex. We need two lines, or at least two neurons, to define the decision boundary.

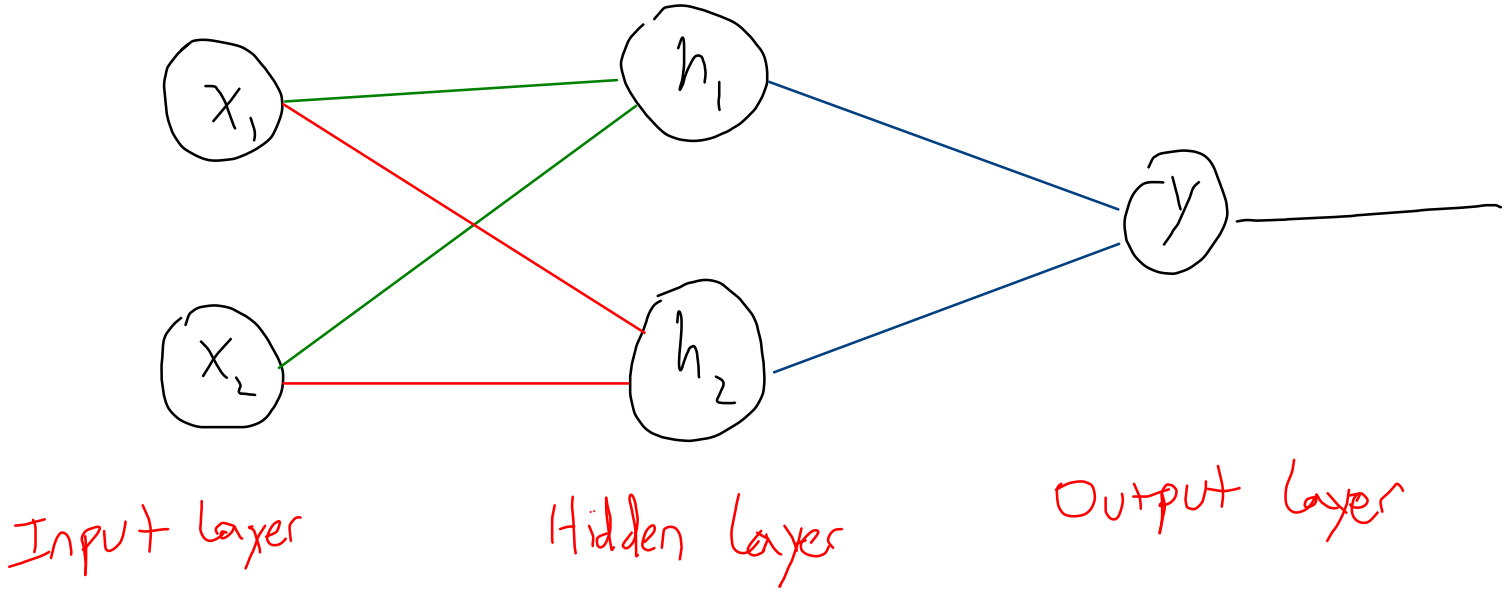


Neural network for AND/OR

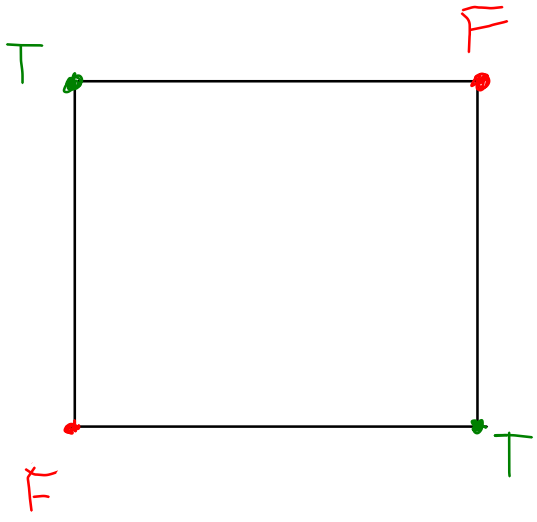


The XOR function can logically be rewritten in terms of AND, OR, and NOT. The NOT function is just what it sounds like, NOT true = false, NOT false = true.

(x1 OR x2) AND (NOT(x1 AND x2)



Imagine the graph for XOR is a square piece of paper. How can we use a single line to separate the two classes?



Fold true corners up.