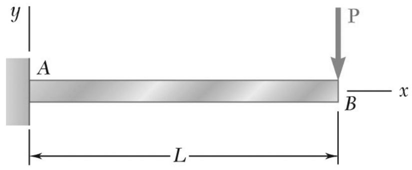
CHAPTER 9



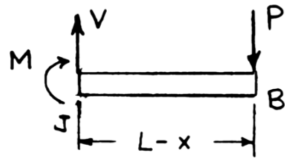
PROBLEM 9.1

For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

SOLUTION

$$+\circlearrowleft \Sigma M_J = 0: \quad -M - P(L-x) = 0$$

$$M = -P(L-x)$$



$$EI\frac{d^2y}{dx^2} = -P(L-x) = -PL + Px$$

$$EI\frac{dy}{dx} = -PLx + \frac{1}{2}Px^2 + C_1$$

$$\left[x = 0, \ \frac{dy}{dx} = 0\right]: \qquad 0 = -0 + 0 + C_1 \qquad C_1 = 0$$

$$EIy = -\frac{1}{2}PLx^2 + \frac{1}{6}Px^3 + C_1x + C_2$$

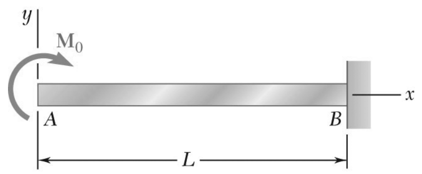
$$[x = 0, \ y = 0]: \qquad 0 = -0 + 0 + 0 + C_2 \qquad C_2 = 0$$

(*a*) Elastic curve: $y = -\dfrac{Px^2}{6EI}(3L - x)$ ◀

$$\frac{dy}{dx} = -\frac{Px}{2EI}(2L - x)$$

(*b*) y at $x = L$: $\quad y_B = -\dfrac{PL^2}{6EI}(3L - L) = -\dfrac{PL^3}{3EI}$ $\qquad y_B = \dfrac{PL^3}{3EI} \downarrow$ ◀

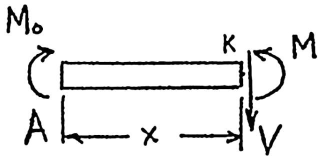
(*c*) $\dfrac{dy}{dx}$ at $x = L$: $\quad \left.\dfrac{dy}{dx}\right|_B = -\dfrac{PL}{2EI}(2L - L) = -\dfrac{PL^2}{2EI}$ $\qquad \theta_B = \dfrac{PL^2}{2EI}$ ◀



PROBLEM 9.2

For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

SOLUTION



$$+\circlearrowleft \sum M_K = 0: \; -M_0 + M = 0$$

$$M = M_0$$

$$EI\frac{d^2y}{dx} = M = M_0$$

$$EI\frac{dy}{dx} = M_0x + C_1$$

$$\left[x = L, \; \frac{dy}{dx} = 0\right]: \qquad 0 = M_0L + C_1 \qquad C_1 = -M_0L$$

$$EIy = \frac{1}{2}M_0x^2 + C_1x + C_2$$

$$[x = L, \; y = 0] \qquad 0 = \frac{1}{2}M_0L^2 - M_0L^2 + C_2 \qquad C_2 = \frac{1}{2}M_0L^2$$

(*a*) Elastic curve:

$$y = \frac{M_0}{2EI}(x^2 - 2Lx + L^2) \blacktriangleleft$$

$$y = \frac{M_0}{2EI}(L - x)^2 \blacktriangleleft$$

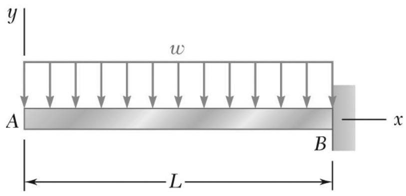
(*b*) y at $x = 0$:

$$y_A = \frac{M_0}{2EI}(L - 0)^2 \qquad y_A = \frac{M_0L^2}{2EI} \uparrow \blacktriangleleft$$

(*c*) $\frac{dy}{dx}$ at $x = 0$:

$$\frac{dy}{dx} = -\frac{M_0}{EI}(L - x) = -\frac{M_0}{EI}(L - 0) = -\frac{M_0L}{EI}$$

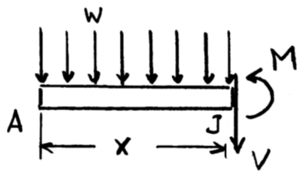
$$\theta_A = \frac{w_0L}{EI} \blacktriangleleft$$



PROBLEM 9.3

For the loading shown, determine (a) the equation of the elastic curve for the cantilever beam AB, (b) the deflection at the free end, (c) the slope at the free end.

SOLUTION



$$+\circlearrowleft \Sigma M_J = 0: \quad (wx)\frac{x}{2} + M = 0$$

$$M = -\frac{1}{2}wx^2$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + C_1$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right]: \quad 0 = -\frac{1}{6}wL^3 + C_1 \qquad C_1 = \frac{1}{6}wL^3$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}wL^3$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{6}wL^3x + C_2$$

$$[x = L,\ y = 0] \quad 0 = -\frac{1}{24}wL^4 + \frac{1}{6}wL^4 + C_2 = 0$$

$$C_2 = \left(\frac{1}{24} - \frac{1}{6}\right)wL^4 = -\frac{3}{24}wL^4$$

(a) Elastic curve:

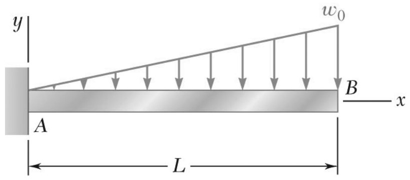
$$y = -\frac{w}{24EI}(x^4 - 4L^3x + 3L^4) \blacktriangleleft$$

(b) y at $x = 0$:

$$y_A = -\frac{3wL^4}{24EI} = -\frac{wL^4}{8EI} \qquad y_A = \frac{wL^4}{8EI} \downarrow \blacktriangleleft$$

(c) $\frac{dy}{dx}$ at $x = 0$:

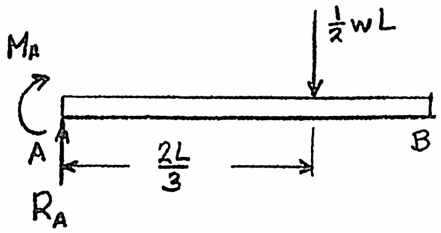
$$\left.\frac{dy}{dx}\right|_A = \frac{wL^3}{6EI} \qquad \theta_A = \frac{wL^3}{6EI} \blacktriangleleft$$



PROBLEM 9.4

For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

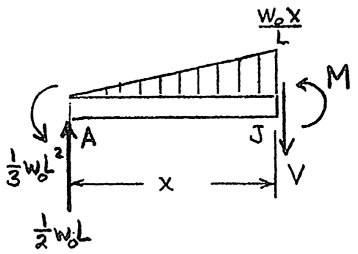
SOLUTION

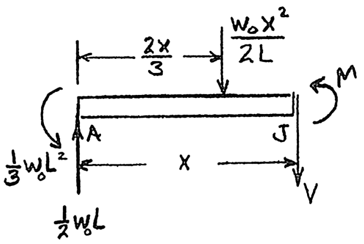


$+\uparrow \sum F_y = 0:$

$$R_A = \frac{1}{2}wL = 0$$

$$R_A = \frac{1}{2}w_0 L$$





$+\circlearrowleft \sum M_A = 0:$

$$-M_A = \frac{2L}{3}\cdot\frac{wL}{2} = 0$$

$$M_A = -\frac{1}{3}w_0L^2$$

$$+\circlearrowleft \Sigma M_J = 0: \quad \frac{1}{3}w_0L^2 - \frac{1}{2}w_0Lx + \frac{w_0x^2}{2L}\cdot\frac{x}{3} + M = 0$$

$$M = -\frac{1}{3}w_0L^2 + \frac{1}{2}w_0Lx - \frac{w_0x^3}{6L}$$

$$EI\frac{d^2y}{dx^2} = -\frac{1}{3}w_0L^2 + \frac{1}{2}w_0Lx - \frac{w_0x^3}{6L}$$

$$EI\frac{dy}{dx} = -\frac{1}{3}w_0L^2x + \frac{1}{4}w_0Lx^2 - \frac{w_0x^4}{24L} + C_1$$

$$\left[x = 0, \frac{dy}{dx} = 0\right]: \qquad 0 = -0 + 0 - 0 + C_1 \qquad\qquad C_1 = 0$$

$$EIy = -\frac{1}{6}w_0L^2x^2 + \frac{1}{12}w_0Lx^3 - \frac{w_0x^5}{120L} + C_2$$

$$[x = 0, y = 0]: \qquad 0 = -0 + 0 - 0 + 0 + C_2 \qquad\qquad C_2 = 0$$

(*a*) Elastic curve:

$$y = -\frac{w_0}{EIL}\left(\frac{1}{6}L^3x^2 - \frac{1}{12}Lx^4 + \frac{1}{120}x^5\right) \blacktriangleleft$$

PROBLEM 9.4 *(Continued)*

(*b*) $\underline{y \text{ at } x = L}$:

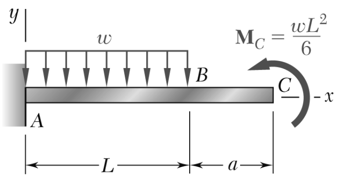
$$y_B = -\frac{w_0 L^4}{EI}\left(\frac{1}{6} - \frac{1}{12} + \frac{1}{120}\right) = -\frac{11}{120}\frac{w_0 L^4}{EI}$$

$$y_B = \frac{11}{120}\frac{w_0 L^4}{EI} \downarrow \blacktriangleleft$$

(*c*) $\underline{\frac{dy}{dx} \text{ at } x = L}$:

$$\left.\frac{dy}{dx}\right|_B = -\frac{w_0 L^3}{EI}\left(\frac{1}{3} - \frac{1}{4} + \frac{1}{24}\right) = -\frac{1}{8}\frac{w_0 L^4}{EI}$$

$$\theta_B = \frac{1}{8}\frac{w_0 L^3}{EI} \blacktriangleleft$$



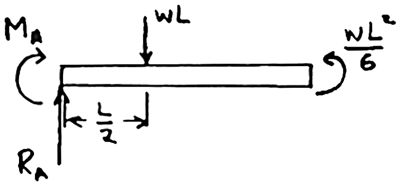
$[x = 0,\ y = 0]$

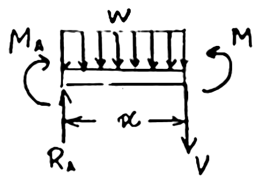
$\left[x = 0, \dfrac{dy}{dx} = 0\right]$

PROBLEM 9.5

For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.

SOLUTION





Using *ABC* as a free body,

$$+\uparrow \Sigma F_y = 0: \quad R_A - wL = 0 \quad R_A = wL$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - (wL)\left(\frac{L}{2}\right) + \frac{wL^2}{6} = 0$$

$$M_A = -\frac{1}{3}wL^2$$

Using *AJ* as a free body, (Portion *AB* only)

$$+\circlearrowleft \Sigma M_J = 0: \; M + (wx)\left(\frac{x}{2}\right) - R_A x - M_A = 0$$

$$M = -\frac{1}{2}wx^2 + R_A x + M_A$$

$$= -\frac{1}{2}wx^2 + wLx - \frac{1}{3}wL^2$$

$$EI\frac{d^2y}{dx^2} = -\frac{1}{2}wx^2 + wLx - \frac{1}{3}wL^2$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{2}wLx^2 - \frac{1}{3}wLx + C_1$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right]: \quad -0 + 0 - 0 + C_1 = 0 \quad \therefore \quad C_1 = 0$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{6}wLx^3 - \frac{1}{6}wLx^2 + C_2$$

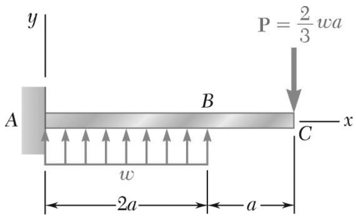
$$[x = 0,\ y = 0]: \quad -0 + 0 - 0 + C_2 = 0 \quad \therefore \quad C_2 = 0$$

(*a*) Elastic curve over *AB*: $\quad y = \dfrac{w}{24EI}(-x^4 + 4Lx^3 - 4L^2x^2)$ ◀

$$\frac{dy}{dx} = \frac{w}{6EI}(-x^3 + 2Lx^2 - L^2x)$$

(*b*) y at $x = L$: $\quad y_B = -\dfrac{wL^4}{24EI} \qquad y_B = \dfrac{wL^4}{24EI}\downarrow$ ◀

(*c*) $\dfrac{dy}{dx}$ at $x = L$: $\quad \left.\dfrac{dy}{dx}\right|_B = 0 \qquad \theta_B = 0$ ◀

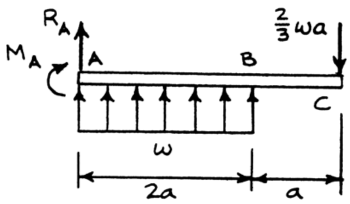


PROBLEM 9.6

For the cantilever beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at *B*, (*c*) the slope at *B*.

SOLUTION

FBD *ABC*:



Using *ABC* as a free body,

$$+\uparrow \Sigma F_y = 0: \quad R_A + 2wa - \frac{2}{3}wa = 0$$

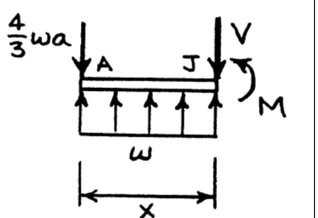
$$R_A = -\frac{4}{3}wa = \frac{4}{3}wa \downarrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A + (2wa)(a) - \left(\frac{2}{3}wa\right)(3a) = 0$$

$$M_A = 0$$

Using *AJ* as a free body,

FBD *AJ*:



$$+\circlearrowleft \Sigma M_J = 0: \quad M + \left(\frac{4}{3}wa\right)(x) - (wx)\left(\frac{x}{2}\right) = 0$$

$$M = \frac{1}{2}wx^2 - \frac{4}{3}wax$$

$$EI\frac{d^2y}{dx^2} = \frac{1}{2}wx^2 - \frac{4}{3}wax$$

$$EI\frac{dy}{dx} = \frac{1}{6}wx^3 - \frac{2}{3}wax^2 + C_1$$

$$\left[x = 0, \ \frac{dy}{dx} = 0\right]: \quad 0 = 0 - 0 + C_1 \quad \therefore \quad C_1 = 0$$

$$EIy = \frac{1}{24}wx^4 - \frac{2}{9}wax^3 + C_2$$

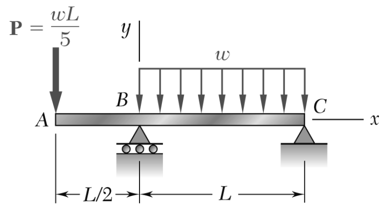
$$[x = 0, \ y = 0]: \quad 0 = 0 - 0 + C_2 \quad \therefore \quad C_2 = 0$$

(*a*) Elastic curve over *AB*: $\quad y = \dfrac{w}{72EI}(3x^4 - 16ax^3)$ ◀

$$\frac{dy}{dx} = \frac{w}{6EI}(x^3 - 4ax^2)$$

(*b*) y at $x = 2a$: $\quad y_B = -\dfrac{10wa^4}{9EI}$ $\qquad y_B = \dfrac{10wa^4}{9EI} \downarrow$ ◀

(*c*) $\dfrac{dy}{dx}$ at $x = 2a$: $\quad \left(\dfrac{dy}{dx}\right)_B = -\dfrac{4wa^3}{3EI}$ $\qquad \theta_B = \dfrac{4wa^3}{3EI}$ ◀

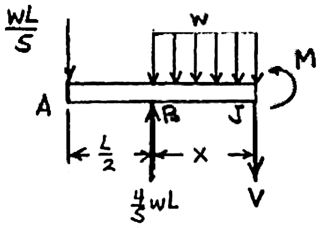


$$x = -\frac{L}{2} \quad \begin{bmatrix} x=0 \\ y=0 \end{bmatrix} \quad \begin{bmatrix} x=L \\ y=0 \end{bmatrix}$$

PROBLEM 9.7

For the beam and loading shown, determine (*a*) the equation of the elastic curve for portion *BC* of the beam, (*b*) the deflection at midspan, (*c*) the slope at *B*.

SOLUTION



Using *ABC* as a free body,

$$+\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{wL}{5}\right)\left(\frac{3L}{2}\right) - R_B L + (wL)\left(\frac{L}{2}\right) = 0$$

$$R_B = \frac{4}{5}wL$$

For portion *BC* only, $(0 < x < L)$

$$+\circlearrowleft \Sigma M_J = 0: \quad \frac{wL}{5}\left(\frac{L}{2} + x\right) - \frac{4}{5}wLx + (wx)\frac{x}{2} + M = 0$$

$$M = \frac{3}{5}wLx - \frac{1}{2}wx^2 - \frac{1}{10}wL^2$$

$$EI\frac{d^2y}{dx^2} = \frac{3}{5}wLx - \frac{1}{2}wx^2 - \frac{1}{10}wL^2$$

$$EI\frac{dy}{dx} = \frac{3}{10}wLx^2 - \frac{1}{6}wx^3 - \frac{1}{10}wL^2x + C_1$$

$$EIy = \frac{1}{10}wLx^3 - \frac{1}{24}wx^4 - \frac{1}{20}wL^2x^2 + C_1x + C_2$$

$$[x=0,\ y=0]: \quad 0 = 0 - 0 - 0 + 0 + C_2 \qquad C_2 = 0$$

$$[x=L,\ y=0]: \quad 0 = \left(\frac{1}{10} - \frac{1}{24} - \frac{1}{20}\right)wL^4 + C_1L + 0 \qquad C_1 = -\frac{1}{120}wL^3$$

(*a*) Elastic curve:

$$y = \frac{w}{EI}\left(\frac{1}{10}Lx^3 - \frac{1}{24}x^4 - \frac{1}{20}L^2x^2 - \frac{1}{120}L^3x\right) \blacktriangleleft$$

$$\frac{dy}{dx} = \frac{w}{EI}\left(\frac{3}{10}Lx^2 - \frac{1}{6}x^3 - \frac{1}{10}L^2x - \frac{1}{120}L^3\right)$$

PROBLEM 9.7 *(Continued)*

(*b*) $\underline{y @ x = \dfrac{L}{2}}$:

$$y_M = \frac{w}{EI}\left\{\frac{1}{10}L\left(\frac{L}{2}\right)^3 - \frac{1}{24}\left(\frac{L}{2}\right)^4 - \frac{1}{20}L^2\left(\frac{L}{2}\right)^2 - \frac{1}{120}L^3\left(\frac{L}{2}\right)\right\}$$

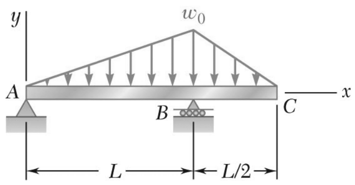
$$= \frac{wL^4}{EI}\left\{\frac{1}{80} - \frac{1}{384} - \frac{1}{80} - \frac{1}{240}\right\} = -\frac{13wL^4}{1920EI}$$

$$y_M = \frac{13wL^4}{1920EI}\downarrow \blacktriangleleft$$

(*c*) $\underline{\dfrac{dy}{dx} @ x = 0}$:

$$\left.\frac{dy}{dx}\right|_B = \frac{w}{EI}\left(0 - 0 - 0 - \frac{1}{120}L^3\right) = -\frac{wL^3}{120EI}$$

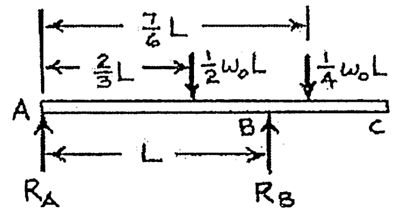
$$\theta_B = \frac{wL^3}{120EI} \blacktriangleleft$$



PROBLEM 9.8

For the beam and loading shown, determine (*a*) the equation of the elastic curve for portion *AB* of the beam, (*b*) the deflection at midspan, (*c*) the slope at *B*.

SOLUTION



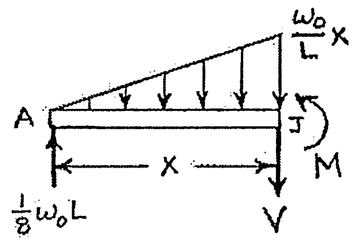
Reactions:

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_AL + \left(\frac{1}{2}w_0L\right)\left(\frac{1}{3}L\right) - \left(\frac{1}{4}w_0L\right)\left(\frac{1}{6}L\right) = 0$$

$$R_A = \frac{1}{8}w_0L$$

Boundary conditions: $[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

For portion *AB* only, $(0 \le x < L)$



$$+\circlearrowleft \Sigma M_J = 0: -\frac{1}{8}w_0Lx + \frac{1}{2}\left(\frac{w_0}{L}x\right)(x)\left(\frac{x}{3}\right) + M = 0$$

$$M = \frac{1}{8}w_0Lx - \frac{1}{6}\frac{w_0}{L}x^3$$

$$EI\frac{d^2y}{dx^2} = \frac{1}{8}w_0Lx - \frac{1}{6}\frac{w_0}{L}x^3$$

$$EI\frac{dy}{dx} = \frac{1}{16}w_0Lx^2 - \frac{1}{24}\frac{w_0}{L}x^4 + C_1$$

$$EIy = \frac{1}{48}w_0Lx^3 - \frac{1}{120}\frac{w_0}{L}x^5 + C_1x + C_2$$

$$[x = 0,\ y = 0]: \quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$$

$$[x = L,\ y = 0]: \quad 0 = \frac{1}{48}w_0L^4 - \frac{1}{120}w_0L^4 + C_1L \quad C_1 = -\frac{1}{80}w_0L^3$$

(*a*) Elastic curve:

$$y = \frac{w_0}{EIL}\left(\frac{1}{48}L^2x^3 - \frac{1}{120}x^5 - \frac{1}{80}L^4x\right) \blacktriangleleft$$

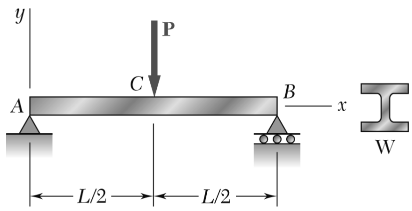
$$\frac{dy}{dx} = \frac{w_0}{EIL}\left(\frac{1}{16}L^2x^2 - \frac{1}{24}x^4 - \frac{1}{80}L^4\right)$$

(*b*) y at $x = \frac{L}{2}$:

$$y_{L/2} = \frac{w_0}{EIL}\left(\frac{L^5}{384} - \frac{L^5}{3840} - \frac{L^5}{160}\right) = -\frac{15w_0L^4}{3840EI} \qquad y_{L/2} = \frac{w_0L^4}{256EI} \downarrow \blacktriangleleft$$

(*c*) $\frac{dy}{dx}$ at $x = L$:

$$\left.\frac{dy}{dx}\right|_B = \frac{w_0}{EIL}\left(\frac{L^4}{16} - \frac{L^4}{24} - \frac{L^4}{80}\right) = +\frac{2w_0L^3}{240EI} \qquad \theta_B = \frac{w_0L^3}{120EI} \blacktriangleleft$$



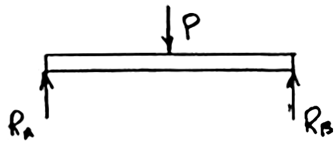
$[x=0,\ y=0]\qquad [x=L,\ y=0]$

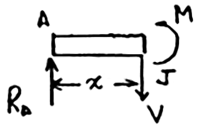
$\left[x=\frac{L}{2},\ \frac{dy}{dx}=0\right]$

PROBLEM 9.9

Knowing that beam AB is a W130×23.8 rolled shape and that $P = 50$ kN, $L = 1.25$ m, and $E = 200$ GPa, determine (*a*) the slope at A, (*b*) the deflection at C.

SOLUTION





Use symmetry boundary condition at C.

By symmetry, $\quad R_A = R_B = \frac{1}{2}P$

Using free body AJ, $\quad 0 \le x \le \frac{L}{2}$

$$+\circlearrowleft \Sigma M_J = 0:\quad M - R_A x = 0$$

$$M = R_A x = \frac{1}{2}Px$$

$$EI\frac{d^2y}{dx^2} = \frac{1}{2}Px$$

$$EI\frac{dy}{dx} = \frac{1}{4}Px^2 + C_1$$

$$EIy = \frac{1}{12}Px^3 + C_1x + C_2$$

$$[x=0,\ y=0]:\quad 0 = 0+0+C_2 \quad \therefore \quad C_2 = 0$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right]:\quad 0 = \frac{1}{4}P\left(\frac{L}{2}\right)^2 + C_1 \quad C_1 = -\frac{1}{16}PL^2$$

<u>Elastic curve:</u> $\quad y = \frac{PL}{48EI}(4x^3 - 3L^2x)$

$$\frac{dy}{dx} = \frac{PL}{16EI}(4x^2 - L^2)$$

<u>Slope at $x = 0$:</u> $\quad \left.\frac{dy}{dx}\right|_A = -\frac{PL^3}{16EI} \qquad \theta_A = \frac{PL^2}{16EI}$ ↘

<u>Deflection at $x = \frac{L}{2}$:</u> $\quad y_C = -\frac{PL^3}{48EI} \qquad y_C = \frac{PL^3}{48EI}\downarrow$

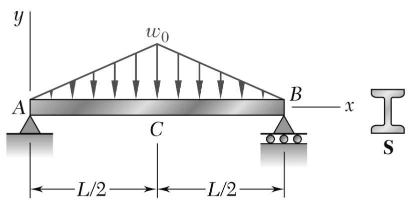
PROBLEM 9.9 *(Continued)*

Data: $P = 50\times 10^3$ N, $I = 8.80\times 10^6\ \text{mm}^4 = 8.80\times 10^{-6}\ \text{m}^4$

$E = 200\times 10^9$ Pa $\quad EI = 1.76\times 10^6\ \text{N}\cdot\text{m}^2 \quad L = 1.25$ m

(*a*) $$\theta_A = \frac{(50\times 10^3)(1.25)^2}{(16)(1.76\times 10^6)}$$ $\theta_A = 2.77\times 10^{-3}$ rad ◀

(*b*) $$y_C = \frac{(50\times 10^3)(1.25)^3}{(48)(1.76\times 10^6)} = 1.156\times 10^{-3}\ \text{m}$$ $y_C = 1.156\ \text{mm}\downarrow$ ◀



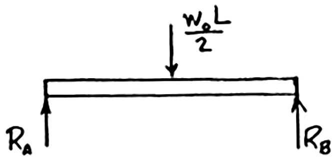
$[x = 0, \quad y = 0] \quad [x = L, \quad y = 0]$

$$\left[x = \frac{L}{2}, \ \frac{dy}{dx} = 0\right]$$

PROBLEM 9.10

Knowing that beam AB is an S200×27.4 rolled shape and that $w_0 = 60$ kN/m, $L = 2.7$ m, and $E = 200$ GPa, determine (*a*) the slope at A, (*b*) the deflection at C.

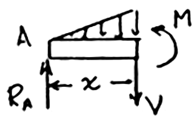
SOLUTION



Use symmetry boundary conditions at C.

Using free body ACB and symmetry,

$$R_A = R_B = \frac{1}{4}w_0L$$



For $\quad 0 < x < \dfrac{L}{2} \qquad w = \dfrac{2w_0x}{L}$

$$\frac{dV}{dx} = -w = -\frac{2w_0x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_0x^2}{L} + R_A = \frac{w_0}{L}\left(\frac{1}{4}L^2 - x^2\right)$$

$$M = \frac{w_0}{L}\left(\frac{1}{4}L^2x - \frac{1}{3}x^3\right) + C_M$$

But $\quad M = 0 \quad$ at $\quad x = 0; \qquad$ hence $C_M = 0$

$$EI\frac{d^2y}{dx^2} = \frac{w_0}{L}\left(\frac{1}{4}L^2x - \frac{1}{3}x^3\right)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L}\left(\frac{1}{8}L^2x^2 - \frac{1}{12}x^4\right) + C_1$$

$\left[x = \dfrac{L}{2}, \dfrac{dy}{dx} = 0\right]$: $\qquad 0 = \dfrac{w_0}{L}\left(\dfrac{1}{32}L^4 - \dfrac{1}{192}L^4\right) + C_1 = 0 \qquad C_1 = -\dfrac{5}{192}w_0L^3$

$$EIy = \frac{w_0}{L}\left(\frac{1}{24}L^2x^3 - \frac{1}{120}x^5\right) - \frac{5}{192}w_0L^3x + C_2$$

$[x = 0, \ y = 0]$: $\qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$

PROBLEM 9.10 *(Continued)*

Elastic curve:
$$y = \frac{w_0}{EIL}\left(\frac{1}{24}L^2x^3 - \frac{1}{60}x^5 - \frac{5}{192}L^4x\right)$$
$$\frac{dy}{dx} = \frac{w_0}{EIL}\left(\frac{1}{8}L^2x^2 - \frac{1}{12}x^4 - \frac{5}{192}L^4\right)$$

Data: $w_0 = 60$ kN/m, $E = 200$ GPa, $I = 23.9\times10^6$ mm^4

$$EI = (200\times10^9)(23.9\times10^{-6}) = 4.78\times10^6 \text{ Nm}^2$$
$$L = 2.7 \text{ m}$$

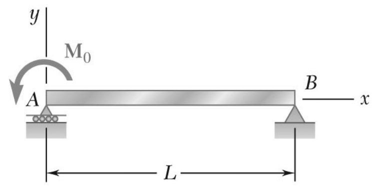
(*a*) Slope at $x = 0$: $\frac{dy}{dx} = \frac{60000}{(4.78\times10^6)(2.7)}\left[-\left(\frac{5}{192}\right)(2.7)^4\right] = -6.434\times10^3$

$$\theta_A = 6.43\times10^{-3} \text{ rad} \blacktriangleleft$$

(*b*) Deflection at $x = 1.35$ m:

$$y = \frac{60000}{(4.78\times10^6)(2.7)}\left[\left(\frac{1}{24}\right)(2.7)^2(1.35)^3 - \frac{1}{60}(1.35)^5 - \frac{5}{192}(2.7)^4(1.35)\right] = -0.0056\times10^{-3} \text{ m}$$

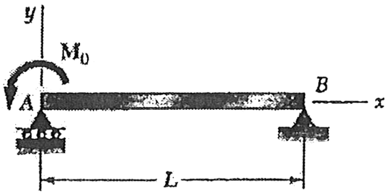
$$= 5.6 \text{ mm}\downarrow \blacktriangleleft$$



PROBLEM 9.11

(*a*) Determine the location and magnitude of the maximum deflection of beam *AB*. (*b*) Assuming that beam *AB* is a W360 × 64, $L = 3.5$ m, and $E = 200$ GPa, calculate the maximum allowable value of the applied moment $\mathbf{M}_0$ if the maximum deflection is not to exceed 1 mm.

SOLUTION



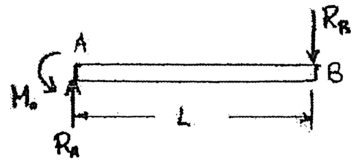
Using entire beam as a free body,

$$+\circlearrowleft \Sigma M_B = 0: \quad M_0 - R_A L = 0 \quad R_A = \frac{M_0}{L}$$

Using portion AJ,

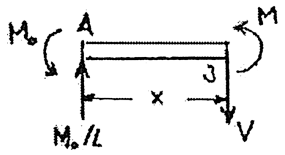
$[x = 0, \ y = 0]$ $[x = L, \ y = 0]$

$$+\circlearrowleft \Sigma M_J = 0: \quad M_0 - \frac{M_0}{L}x + M = 0$$



$$M = \frac{M_0}{L}(x - L)$$

$$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}(x - L)$$



$$EI\frac{dy}{dx} = \frac{M_0}{L}\left(\frac{1}{2}x^2 - Lx\right) + C_1$$

$$EIy = \frac{M_0}{L}\left(\frac{1}{6}x^3 - \frac{1}{2}Lx^2\right) + C_1x + C_2$$

$[x = 0, \ y = 0]$ $\quad 0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$

$[x = L, \ y = 0]$ $\quad 0 = \dfrac{M_0}{L}\left(\dfrac{1}{6}L^3 - \dfrac{1}{2}L^3\right) + C_1L + 0 \quad C_1 = \dfrac{1}{3}M_0L$

$$y = \frac{M_0}{EIL}\left(\frac{1}{6}x^3 - \frac{1}{2}Lx^2 + \frac{1}{3}L^2x\right) \qquad \frac{dy}{dx} = \frac{M_0}{EIL}\left(\frac{1}{2}x^2 - Lx + \frac{1}{3}L^2\right)$$

(*a*) To find location maximum deflection, set $\dfrac{dy}{dx} = 0.$

$$\frac{1}{2}x_m^2 - Lx_m + \frac{1}{3}L^2 = 0 \qquad x_m = L - \sqrt{L^2 - (4)\left(\frac{1}{2}\right)\left(\frac{1}{3}L^2\right)} = \left(1 - \sqrt{\frac{1}{3}}\right)L$$

$$= 0.42265L \qquad x_m = 0.423L \blacktriangleleft$$

$$y_m = \frac{M_0L^2}{EI}\left\{\left(\frac{1}{6}\right)(0.42265)^3 - \left(\frac{1}{2}\right)(0.42265)^2 + \left(\frac{1}{3}\right)(0.42265)\right\} \qquad y_m = 0.06415\frac{M_0L^2}{EI} \blacktriangleleft$$

PROBLEM 9.11 ***(Continued)***

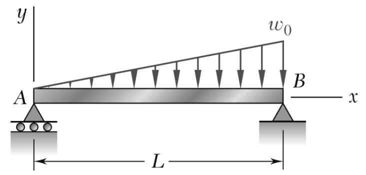
Solving for M_0, $$M_0 = \frac{EIy_m}{0.06415L^2}$$

(b) Data: $$E = 200 \times 10^9 \text{ Pa}, \quad I = 178 \times 10^6 \text{ mm}^4 = 178 \times 10^{-6} \text{ m}^4$$

$$L = 3.5 \text{ m} \quad y_m = 1 \text{ mm} = 10^{-3} \text{ m}$$

$$M_0 = \frac{(200 \times 10^9)(178 \times 10^{-6})(10^{-3})}{(0.06415)(3.5)^2} = 45.3 \times 10^3 \text{ N} \cdot \text{m}$$

$$M_0 = 45.3 \text{ kN} \cdot \text{m} \blacktriangleleft$$

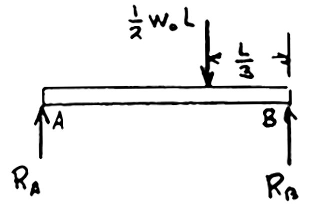


$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

PROBLEM 9.12

For the beam and loading shown, (*a*) express the magnitude and location of the maximum deflection in terms of w_0, L, E, and I. (*b*) Calculate the value of the maximum deflection, assuming that beam AB is a W460×74 rolled shape and that $w_0 = 60$ kN/m, $L = 6$ m, and $E = 200$ GPa.

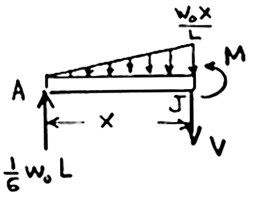
SOLUTION



Using entire beam as a free body,

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \left(\frac{1}{2}w_0 L\right)\left(\frac{L}{3}\right) = 0$$

$$R_A = \frac{1}{6}w_0 L$$



Using AJ as a free body,

$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{1}{6}w_0 L x + \left(\frac{1}{2}\frac{w_0 x^2}{L}\right)\left(\frac{x}{3}\right) + M = 0$$

$$M = \frac{1}{6}w_0 L x - \frac{1}{6}\frac{w_0}{L}x^3$$

$$EI\frac{d^2y}{dx^2} = \frac{1}{6}w_0 L x - \frac{1}{6}\frac{w_0}{L}x^3$$

$$EI\frac{dy}{dx} = \frac{1}{12}w_0 L x^2 - \frac{1}{24}\frac{w_0}{L}x^4 + C_1$$

$$EIy = \frac{1}{36}w_0 L x^3 - \frac{1}{120}\frac{w_0}{L}x^5 + C_1 x + C_2$$

$[x = 0,\ y = 0]$: $\quad 0 = 0 - 0 + 0 + C_2 \qquad \therefore \quad C_2 = 0$

$[x = L,\ y = 0]$: $\quad 0 = \frac{1}{36}w_0 L^4 - \frac{1}{120}w_0 L^4 + C_1 L + 0 \quad \therefore \quad C_1 = -\frac{7 w_0 L^3}{360}$

<u>Elastic curve</u>:

$$y = \frac{w_0}{EI}\left\{\frac{1}{36}Lx^3 - \frac{1}{120}\frac{x^5}{L} - \frac{7}{360}L^3 x\right\}$$

$$\frac{dy}{dx} = \frac{w_0}{EI}\left\{\frac{1}{12}Lx^2 - \frac{1}{24}\frac{x^4}{L} - \frac{7}{360}L^3\right\}$$

PROBLEM 9.12 *(Continued)*

To find location of maximum deflection, set $\frac{dy}{dx} = 0.$

$$15x_m^4 - 30L^2x_m^2 + 7L^4 = 0 \qquad x_m^2 = \frac{30L^2 - \sqrt{900L^4 - 420L^4}}{30}$$

$$x_m^2 = \left(1 - \sqrt{\frac{8}{15}}\right)L^2 = 0.2697L^2 \qquad x_m = 0.5193L \blacktriangleleft$$

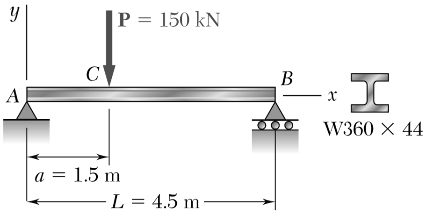
$$y_m = \frac{w_0}{EI}\left\{\frac{1}{36}L(0.5193L)^3 - \frac{1}{120}\frac{(0.5193L^5)}{L} - \frac{7}{360}L^3(0.5193L)\right\}$$

$$= -0.00652\frac{w_0L^4}{EI} \quad \text{or} \quad 0.00652\frac{w_0L^4}{EI}\downarrow \blacktriangleleft$$

<u>Data</u>: $w_0 = 60 \text{ kN/m} = 60\times10^3 \text{ N/m} \quad L = 6 \text{ m}$

For W460×74, $I = 333\times10^6 \text{ mm}^4 = 333\times10^{-6} \text{ m}^4$

$$y_m = \frac{(0.00652)(60\times10^3)(6)^4}{(200\times10^9)(333\times10^{-6})} = 7.61\times10^{-3} \text{ m} \qquad y_m = 7.61 \text{ mm}\downarrow \blacktriangleleft$$



PROBLEM 9.13

For the beam and loading shown, determine the deflection at point C. Use $E = 200$ GPa.

$[x = 0,\ y = 0]$ $\qquad$ $[x = L,\ y = 0]$

$[x = a,\ y = y]$

$$\left[x = a,\ \frac{dy}{dx} = \frac{dy}{dx}\right]$$

SOLUTION

Let $\qquad b = L - a$

Reactions: $\quad R_A = \dfrac{Pb}{L}\uparrow, \quad R_B = \dfrac{Pa}{L}\uparrow$

Bending moments:

$$0 < x < a: \quad M = \frac{Pb}{L}x$$

$$a < x < L: \quad M = \frac{P}{L}[bx - L(x-a)]$$

$0 < x < a$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}(bx)$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left(\frac{1}{2}bx^2\right) + C_1 \qquad (1)$$

$$EIy = \frac{P}{L}\left(\frac{1}{6}bx^3\right) + C_1x + C_2 \qquad (2)$$

$a < x < L$

$$EI\frac{d^2y}{dx^2} = \frac{P}{L}[bx - L(x-a)]$$

$$EI\frac{dy}{dx} = \frac{P}{L}\left[\frac{1}{2}bx^2 - \frac{1}{2}L(x-a)^2\right] + C_3 \qquad (3)$$

$$EIy = \frac{P}{L}\left[\frac{1}{6}bx^3 - \frac{1}{6}L(x-a)^3\right] + C_3x + C_4 \qquad (4)$$

$[x = 0,\ y = 0]$ Eq. (2): $\quad 0 = 0 + 0 + C_2 \qquad \therefore\ C_2 = 0$

$\left[x = a,\ \dfrac{dy}{dx} = \dfrac{dy}{dx}\right]$ Eqs. (1) and (3): $\quad \dfrac{P}{L}\left(\dfrac{1}{\cancel{2}}\cancel{b}a^2\right) + C_1 = \dfrac{P}{L}\left[\dfrac{1}{\cancel{2}}\cancel{b}a^2 + 0\right] + C_3 \quad \therefore\ C_3 = C_1$

$[x = a,\ y = y]$ Eqs. (2) and (4): $\quad \dfrac{P}{L}\left(\dfrac{1}{\cancel{6}}\cancel{b}a^3\right) + C_1a + C_2$

$$= \frac{P}{L}\left[\frac{1}{\cancel{6}}\cancel{b}a^3 + 0\right] + \cancel{C_1}a + C_4 \quad \therefore\ C_4 = C_2 = 0$$

PROBLEM 9.13 *(Continued)*

$$[x = L,\ y = 0] \quad \text{Eq. (4):} \qquad \frac{P}{L}\left[\frac{1}{6}bL^3 - \frac{1}{6}L(L-a)^3\right] + C_3 L = 0$$

$$C_1 = C_3 = \frac{P}{L}\left[\frac{1}{6}(L-a)^3 - \frac{1}{6}bL^2\right] = \frac{P}{L}\left(\frac{1}{6}b^3 - \frac{1}{6}bL^2\right)$$

Make $x = a$ in Eq. (2).

$$y_C = \frac{P}{EIL}\left[\frac{1}{6}ba^3 + \frac{1}{6}b^3a - \frac{1}{6}bL^2a\right] = \frac{P(ba^3 + b^3a - L^2ab)}{6EIL}$$

Data: $P = 150$ kN, $E = 200$ GPa

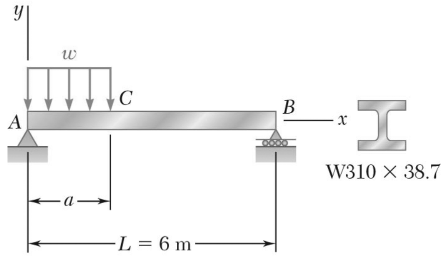
$L = 4.5$ m, $a = 1.5$ m, $b = 3$ m

$I = 122\times 10^{-6}\ \text{m}^4$, $EI = 24.4\times 10^6\ \text{Nm}^2$

$$y_C = \frac{150\times 10^3}{(24.4\times 10^6)(4.5)}[(3)(1.5)^3 + (3)^3(1.5) - (4.5)^2(1.5)(3)]$$

$$= -9.22\times 10^{-3}\ \text{m}$$

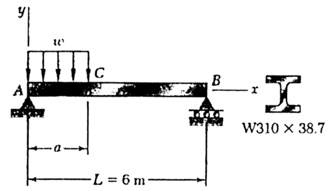
$y_C = 9.2$ mm $\downarrow$ ◀



PROBLEM 9.14

For the beam and loading shown, knowing that $a = 2$ m, $w = 50$ kN/m, and $E = 200$ GPa, determine (*a*) the slope at support *A*, (*b*) the deflection at point *C*.

SOLUTION



Using ACB as a free body and noting that $L = 3a$,

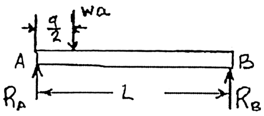
$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - (wa)\left(\frac{a}{2}\right) = 0$$

$[x = 0,\ y = 0]$ $\qquad$ $[x = L,\ y = 0]$

$$R_B = (wa)\frac{a}{2L} = \frac{1}{6}wa$$

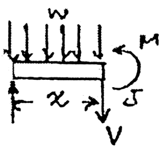
$[x = a,\ y = y]$

$$\left[x = a,\ \frac{dy}{dx} = \frac{dy}{dx}\right]$$



$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - wa = 0 \qquad R_A = \frac{5}{6}wa$$

$0 \le x \le a$



$$+\circlearrowleft \Sigma M_J = 0:$$

$$M - R_A x + (wx)\left(\frac{x}{2}\right) = 0$$

$$M = R_A x - \frac{1}{2}wx^2$$

$$EI\frac{d^2y}{dx^2} = R_A x - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$$

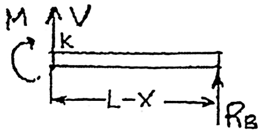
$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x + C_2$$

$$[x = 0,\ y = 0] \quad 0 = 0 - 0 + 0 + C_2 \quad C_2 = 0$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$$

$a \le x \le L$



$$+\circlearrowleft \Sigma M_K = 0:$$

$$-M + R_B(L - x) = 0$$

$$M = R_B(L - x)$$

$$EI\frac{d^2y}{dx^2} = R_B(L - x)$$

$$EI\frac{dy}{dx} = -\frac{1}{2}R_B(L - x)^2 + C_3$$

$$EIy = \frac{1}{6}R_B(L - x)^3 + C_3 x + C_4$$

$$[x = L,\ y = 0] \quad 0 = 0 + C_3 L + C_4 \qquad C_4 = -C_3 L$$

$$EIy = \frac{1}{6}R_B(L - x)^2 - C_3(L - x)$$

$$EI\frac{dy}{dx} = -\frac{1}{2}R_B(L - x)^2 + C_3$$

PROBLEM 9.14 *(Continued)*

$$\left[x=a,\ \frac{dy}{dx}=\frac{dy}{dx}\right] \qquad \frac{1}{2}R_A a^2 - \frac{1}{6}wa^3 + C_1 = -\frac{1}{2}R_B(2a)^2 + C_3$$

$$C_3 = C_1 + \frac{1}{2}R_A a^2 - \frac{1}{6}wa^3 + \frac{1}{2}R_B(2a)^2 = C_1 + \frac{7}{12}wa^3$$

$$[x=a,\ y = y] \qquad \frac{1}{6}R_A a^3 - \frac{1}{24}wa^4 + C_1 a = \frac{1}{6}R_B(2a)^3 - \left(C_1 + \frac{7}{12}wa^3\right)(2a)$$

$$3C_1 a = -\frac{1}{6}R_A a^3 + \frac{1}{24}wa^4 + \frac{1}{6}R_B(2a)^3 - \frac{7}{12}wa^2(2a) = -\frac{25}{24}wa^4$$

$$C_1 = -\frac{25}{72}wa^3$$

For $0 \le x \le a$, $EIy = \dfrac{5}{36}wax^3 - \dfrac{1}{24}wx^4 - \dfrac{25}{72}wa^3x$

$$EI\frac{dy}{dx} = \frac{5}{12}wax^2 - \frac{1}{6}wx^3 - \frac{25}{72}wa^3$$

<u>Data:</u> $w = 50\times 10^3$ N/m, $a = 2$ m, $E = 200\times 10^9$ Pa

$I = 84.9\times 10^6\ \text{mm}^4 = 84.9\times 10^{-6}\ \text{m}^4$, $EI = 16.98\times 10^6\ \text{N}\cdot\text{m}^2$

(*a*) <u>Slope at $x = 0$</u>:

$$16.98\times 10^6 \left.\frac{dy}{dx}\right|_A = 0 - 0 - \frac{25}{72}(50\times 10^3)(2)^3$$

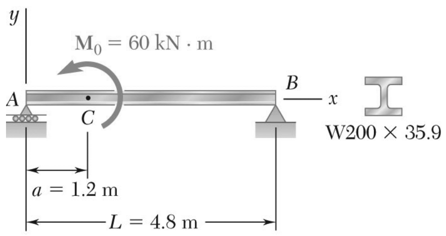
$$\left.\frac{dy}{dx}\right|_A = \theta_A = -8.18\times 10^{-3} \qquad \theta_A = 8.18\times 10^{-3}\ \text{rad} \searrow \blacktriangleleft$$

(*b*) <u>Deflection at $x = 2$ m</u>:

$$EIy_C = \frac{5}{36}wa^4 - \frac{1}{24}wa^4 - \frac{25}{72}wa^4 = -\frac{1}{4}wa^4$$

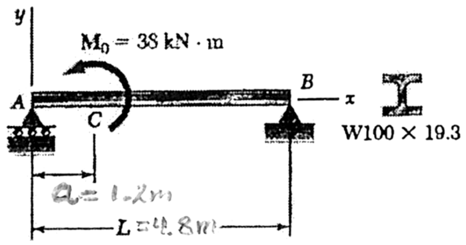
$$16.98\times 10^6\, y_C = -\frac{1}{4}(50\times 10^3)(2)^4 \quad y_C = -11.78\times 10^{-3}\ \text{m} \qquad y_C = 11.78\ \text{mm} \downarrow \blacktriangleleft$$

PROBLEM 9.15



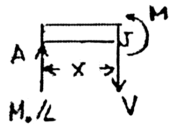
For the beam and loading shown, determine the deflection at point C. Use $E = 200$ GPa.

SOLUTION



<u>Reactions:</u> $R_A = M_0/L \uparrow, \quad R_B = M_0/L \downarrow$

$0 < x < a$: $\quad +\circlearrowleft \sum M_J = 0$:



$$-\frac{M_0}{L}x + M = 0$$

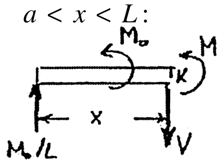
$$M = \frac{M_0}{L}x$$

$[x = 0, y = 0] \qquad [x = L, y = 0]$

$[x = a, y = y]$

$$\left[x = a, \frac{dy}{dx} = \frac{dy}{dx}\right]$$

$a < x < L$: $\quad +\circlearrowleft M_K = 0$:



$$-\frac{M_0}{L}x + M_0 + M = 0$$

$$M = \frac{M_0}{L}(x - L)$$

<u>$0 < x < a$</u>

$$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}x$$

$$EI\frac{dy}{dx} = \frac{M_0}{L}\left(\frac{1}{2}x^2\right) + C_1 \qquad (1)$$

$$EIy = \frac{M_0}{L}\left(\frac{1}{6}x^3\right) + C_1x + C_2 \qquad (2)$$

<u>$a < x < L$</u>

$$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}(x - L)$$

$$EI\frac{dy}{dx} = \frac{M_0}{L}\left(\frac{1}{2}x^2 - Lx\right) + C_3 \qquad (3)$$

$$EIy = \frac{M_0}{L}\left(\frac{1}{6}x^3 - \frac{1}{2}Lx^2\right) + C_3x + C_4 \qquad (4)$$

$[x = 0, \ y = 0]$ Eq. (2): $\quad 0 = 0 + 0 + C_2 \qquad C_2 = 0$

$\left[x = a, \frac{dy}{dx} = \frac{dy}{dx}\right]$ Eqs. (1) and (3): $\quad \frac{M_0}{L}\left(\frac{1}{2}a^2\right) + C_1 = \frac{M_0}{L}\left(\frac{1}{2}a^2 - La\right) + C_3$

$$C_3 = C_1 + M_0a$$

PROBLEM 9.15 *(Continued)*

$[x = a,\ y = y]$ Eqs. (2) and (4): $\frac{M_0}{L}\left(\frac{1}{\cancel{6}}\cancel{a^3}\right) + \cancel{C_1 a} = \frac{M_0}{L}\left(\frac{1}{\cancel{6}}\cancel{a^3} - \frac{1}{2}La^2\right) + (\cancel{C_1} + M_0 a)a + C_4$

$$C_4 = -\frac{1}{2}M_0 a^2$$

$[x = L,\ y = 0]$ Eq. (4): $\frac{M_0}{L}\left(\frac{1}{6}L^3 - \frac{1}{2}L^3\right) + (C_1 + M_0 a)L - \frac{1}{2}M_0 a^2 = 0$

$$C_1 = \frac{M_0}{L}\left(\frac{1}{3}L^2 + \frac{1}{2}a^2 - aL\right)$$

<u>Elastic curve for $0 < x < a$</u>: $y = \frac{M_0}{EIL}\left[\frac{1}{6}x^3 + \left(\frac{1}{3}L^2 + \frac{1}{2}a^2 - aL\right)x\right]$

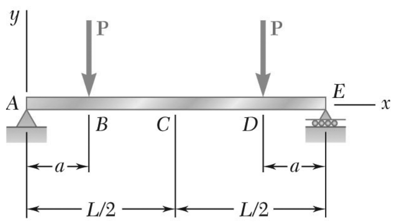
Make $x = a$. $y_C = \frac{M_0}{EIL}\left[\frac{1}{6}a^3 + \frac{1}{3}L^2 a + \frac{1}{2}a^3 - a^2 L\right] = \frac{M_0}{EIL}\left[\frac{2}{3}a^3 + \frac{1}{3}L^2 a - La^2\right]$

<u>Data:</u> $E = 200 \times 10^9\ \text{Pa},\ I = 34.4 \times 10^6\ \text{mm}^4 = 34.4 \times 10^{-6}\ \text{m}^4,\ M_0 = 60 \times 10^3\ \text{N} \cdot \text{m}$

$a = 1.2\ \text{m},\ L = 4.8\ \text{m}$

$$y_C = \frac{(60 \times 10^3)\left[(2)(1.2)^3/3 + (4.8)^2(1.2)/3 - (4.8)(1.2)^2\right]}{(200 \times 10^9)(34.4 \times 10^{-6})(4.8)} = 6.28 \times 10^{-3}\ \text{m}$$

$y_C = 6.28\ \text{mm} \uparrow$ ◀



PROBLEM 9.16

Knowing that beam AE is an $S200 \times 27.4$ rolled shape and that $P = 17.5$ kN, $L = 2.5$ m, $a = 0.8$ m and $E = 200$ GPa, determine (a) the equation of the elastic curve for portion BD, (b) the deflection at the center C of the beam.

SOLUTION

Consider portion ABC only. Apply symmetry about C.

Reactions: $\quad R_A = R_E = P$

Boundary conditions: $[x = 0,\ y = 0]$, $\quad [x = a,\ y = y]$, $\quad \left[x = a,\ \frac{dy}{dx} = \frac{dy}{dx}\right]$, $\quad \left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right]$

$0 < x < a$

$$EI\frac{d^2y}{dx^2} = M = Px$$

$$EI\frac{dy}{dx} = \frac{1}{2}Px^2 + C_1 \qquad (1)$$

$$EIy = \frac{1}{6}Px^3 + C_1x + C_2 \qquad (2)$$

$$[x = 0,\ y = 0] \rightarrow C_2 = 0$$

$a < x < L - a$

$$EI\frac{d^2y}{dx^2} = M = Pa$$

$$EI\frac{dy}{dx} = Pax + C_3$$

$$EIy = \frac{1}{2}Pax^2 + C_3x + C_4$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right] \rightarrow C_3 = -\frac{1}{2}PaL$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right] \qquad \frac{1}{2}Pa^2 + C_1 = Pa^2 - \frac{1}{2}PaL \qquad C_1 = \frac{1}{2}Pa^2 - \frac{1}{2}PaL$$

$$\left[x = \frac{L}{2},\ y = y\right] \qquad \frac{1}{6}Pa^3 + \left(\cancel{\frac{1}{2}Pa^2} - \cancel{\frac{1}{2}PaL}\right)a = \cancel{\frac{1}{2}Pa^3} - \cancel{\frac{1}{2}Pa^2}L + C_4$$

$$C_4 = \frac{1}{6}Pa^3$$

(a) Elastic curve for portion BD:

$$y = \frac{1}{EI}\left(\frac{1}{2}Pax^2 + C_3x + C_4\right)$$

$$y = \frac{P}{EI}\left(\frac{1}{2}ax^2 - \frac{1}{2}aLx + \frac{1}{6}a^3\right) \blacktriangleleft$$

PROBLEM 9.16 *(Continued)*

For <u>deflection at *C*</u>,

set $$x = \frac{L}{2}.$$

$$y_C = \frac{P}{EI}\left(\frac{1}{8}aL^2 - \frac{1}{4}aL^2 + \frac{1}{6}a^3\right)$$

$$= -\frac{Pa}{EI}\left(\frac{1}{8}L^2 - \frac{1}{6}a^2\right)$$

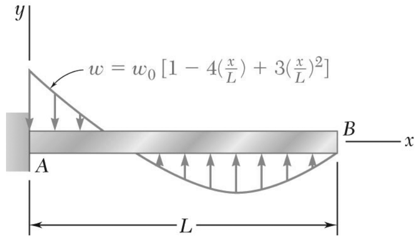
<u>Data</u>: $I = 23.9 \times 10^6 \text{mm}^4 = 23.9 \times 10^{-6} \text{m}^4,$

$E = 200 \times 10^9 \text{Pa}$

$P = 17.5 \times 10^3 \text{N},$

$L = 2.5 \text{ m}, \quad a = 0.8 \text{ m}$

(*b*) $$y_C = -\frac{(17.5 \times 10^3)(0.8)}{(200 \times 10^9)(23.9 \times 10^6)}\left\{\frac{2.5^2}{8} - \frac{0.8^2}{6}\right\} = -1.976 \times 10^{-3} \text{m}$$ $y_C = 1.976 \text{ mm} \downarrow$ ◀



PROBLEM 9.17

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the free end.

SOLUTION

Boundary conditions are shown at right.

$$[x=0,\ y=0] \qquad [x=L,\ V=0]$$
$$\left[x=0,\ \frac{dy}{dx}=0\right] \qquad [x=L,\ M=0]$$

$$\frac{dV}{dx}=-w=-w_0\left[1-4\left(\frac{x}{L}\right)+3\left(\frac{x}{L}\right)^2\right]$$

$$V=-w_0\left[x-\frac{2x^2}{L}+\frac{x^3}{L^2}\right]+C_V$$

$$[x=L,\ V=0]:\ 0=-w_0[L-2L+L]+C_V=0 \qquad C_V=0$$

$$\frac{dM}{dx}=V=-w_0\left[x-\frac{2x^2}{L}+\frac{x^3}{L^2}\right]$$

$$M=-w_0\left[\frac{x^2}{2}-\frac{2x^3}{3L}+\frac{x^4}{4L^2}\right]+C_M$$

$$[x=L,\ M=0]:\ \ 0=-w_0\left[\frac{1}{2}L^2-\frac{2}{3}L^2+\frac{1}{4}L^2\right]+C_M \qquad C_M=\frac{1}{12}w_0L^2$$

$$EI\frac{d^2y}{dx}=M=-w_0\left[\frac{1}{2}x^2-\frac{2}{3}\frac{x^3}{L}+\frac{1}{4}\frac{x^4}{L^2}-\frac{1}{12}L^2\right]$$

$$EI\frac{dy}{dx}=-w_0\left[\frac{1}{6}x^3-\frac{1}{6}\frac{x^4}{L}+\frac{1}{20}\frac{x^5}{L^2}-\frac{1}{12}L^2x\right]+C_1$$

$$\left[x=0,\ \frac{dy}{dx}=0\right] \qquad C_1=0$$

$$EIy=-w_0\left[\frac{1}{24}x^4-\frac{1}{30}\frac{x^5}{L}+\frac{1}{120}\frac{x^6}{L^2}-\frac{1}{24}L^2x^2\right]+C_2$$

$$[x=0,\ y=0] \qquad C_2=0$$

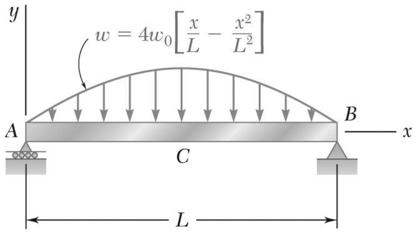
PROBLEM 9.17 *(Continued)*

(*a*) Elastic curve:

$$y = -\frac{w_0}{EIL^2}\left(\frac{1}{24}L^2x^4 - \frac{1}{30}Lx^5 + \frac{1}{120}x^6 - \frac{1}{24}L^4x^2\right) \blacktriangleleft$$

(*b*) Deflection at $x = L$:

$$y_B = -\frac{w_0}{EIL^2}\left(\frac{1}{24}L^6 - \frac{1}{30}L^6 + \frac{1}{120}L^6 - \frac{1}{24}L^6\right) = -\frac{w_0L^4}{40EI} \qquad y_B = \frac{w_0L^4}{40EI}\downarrow \blacktriangleleft$$



$[x = 0,\ M = 0]$ $[x = L,\ M = 0]$

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

PROBLEM 9.18

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection at the midpoint of the span.

SOLUTION

Boundary conditions at *A* and *B* are noted.

$$w = \frac{w_0}{L^2}(4Lx - 4x^2)$$

$$\frac{dV}{dx} = -w = \frac{w_0}{L^2}(4x^2 - 4Lx)$$

$$\frac{dM}{dx} = V = \frac{w_0}{L^2}\left(\frac{4}{3}x^3 - 2Lx^2\right) + C_1$$

$$M = \frac{w_0}{L^2}\left(\frac{1}{3}x^4 - \frac{2}{3}Lx^3\right) + C_1x + C_2$$

$[x = 0,\ M = 0]$ $\quad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$[x = L,\ M = 0]$ $\quad 0 = \frac{w_0}{L^2}\left(\frac{1}{3}L^4 - \frac{2}{3}L^4\right) + C_1L + 0 \qquad C_1 = \frac{1}{3}w_0L$

$$EI\frac{d^2y}{dx^2} = M = \frac{w_0}{L^2}\left(\frac{1}{3}x^4 - \frac{2}{3}Lx^3 + \frac{1}{3}L^3x\right)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L^2}\left(\frac{1}{15}x^5 - \frac{1}{6}Lx^4 + \frac{1}{6}L^3x^2\right) + C_3$$

$$EIy = \frac{w_0}{L^2}\left(\frac{1}{90}x^6 - \frac{1}{30}Lx^5 + \frac{1}{18}L^3x^3\right) + C_3x + C_4$$

$[x = 0,\ y = 0]$ $\quad 0 = 0 + 0 + 0 + 0 + C_4 \qquad C_4 = 0$

$[x = L,\ y = 0]$ $\quad 0 = \frac{w_0}{L^2}\left(\frac{1}{90}L^6 - \frac{1}{30}L^6 + \frac{1}{18}L^6\right) + C_3L + 0 \qquad C_3 = -\frac{1}{30}w_0L^3$

(*a*) Elastic curve:

$$y = \frac{w_0}{EIL^2}\left(\frac{1}{90}x^6 - \frac{1}{30}Lx^5 + \frac{1}{18}L^3x^3 - \frac{1}{30}L^5x\right) \blacktriangleleft$$

$$\frac{dy}{dx} = \frac{w_0}{EIL^2}\left(\frac{1}{15}x^5 - \frac{1}{6}Lx^4 + \frac{1}{6}L^3x^2 - \frac{1}{30}L^5\right)$$

PROBLEM 9.18 *(Continued)*

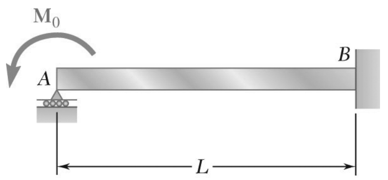
(*b*) Slope at end *A*: Set $x = 0$ in $\frac{dy}{dx}$. $\left.\frac{dy}{dx}\right|_A = -\frac{1}{30}\frac{w_0 L^3}{EI}$

$$\theta_A = \frac{1}{30}\frac{w_0 L^3}{EI} \blacktriangleleft$$

(*c*) Deflection at midpoint: Set $x = \frac{L}{2}$ in y.

$$y_C = \frac{w_0 L^4}{EI}\left\{\left(\frac{1}{90}\right)\left(\frac{1}{2}\right)^6 - \left(\frac{1}{30}\right)\left(\frac{1}{2}\right)^5 + \frac{1}{18}\left(\frac{1}{2}\right)^3 - \frac{1}{30}\left(\frac{1}{2}\right)\right\}$$

$$= \frac{w_0 L^4}{EI}\left\{\frac{1}{5760} - \frac{1}{960} + \frac{1}{144} - \frac{1}{60}\right\} = -\frac{61}{5760}\frac{w_0 L^4}{EI} \qquad y_C = \frac{61}{5760}\frac{w_0 L^4}{EI} \downarrow \blacktriangleleft$$



$[x = 0,\ \ y = 0]$ $\qquad$ $[x = L,\ \ y = 0]$

$$\left[\ x = L,\ \frac{dy}{dx} = 0\right]$$

PROBLEM 9.19

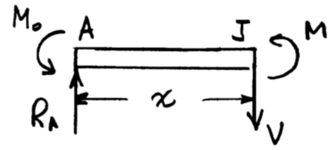
For the beam and loading shown, determine the reaction at the roller support.

SOLUTION

Reactions are statically indeterminate.

Boundary conditions are shown above.

Using free body *AJ*,



$$+\circlearrowleft \Sigma M_J = 0: \quad M_0 - R_A x + M = 0$$

$$M = R_A x - M_0$$

$$EI\frac{d^2y}{dx^2} = R_A x - M_0$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0 x + C_1$$

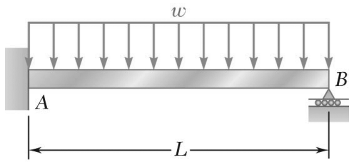
$$\left[x = L,\ \frac{dy}{dx} = 0\right] \quad 0 = \frac{1}{2}R_A L^2 - M_0 L + C_1$$

$$C_1 = M_0 L - \frac{1}{2}R_A L^2$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0 x^3 + C_1 x + C_2$$

$$[x = 0,\ \ y = 0] \qquad C_2 = 0$$

$$[x = L,\ \ y = 0] \quad 0 = \frac{1}{6}R_A L^3 - \frac{1}{2}M_0 L^2 + \left(M_0 L - \frac{1}{2}R_A L^2\right)L + 0 \qquad R_A = \frac{3}{2}\frac{M_0}{L}\uparrow \blacktriangleleft$$



$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

PROBLEM 9.20

For the beam and loading shown, determine the reaction at the roller support.

SOLUTION

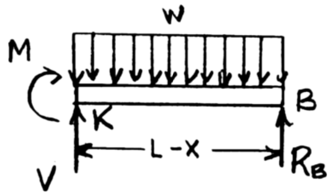
Reactions are statically indeterminate.

Boundary conditions are shown above.

Using free body *KB*,

$$+\circlearrowleft M_K = 0:\quad R_B(L - x) - w(L - x)\left(\frac{L - x}{2}\right) - M = 0$$

$$M = R_B(L - x) - \frac{1}{2}w(L - x)^2$$



$$EI\frac{d^2y}{dx^2} = R_B(L - x) - \frac{1}{2}w(L - x)^2$$

$$EI\frac{dy}{dx} = -\frac{1}{2}R_B(L - x)^2 + \frac{1}{6}w(L - x)^3 + C_1$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right]:\quad 0 = -\frac{1}{2}R_BL^2 + \frac{1}{6}wL^3 + C_1$$

$$C_1 = \frac{1}{2}R_BL^2 - \frac{1}{6}wL^3$$

$$EI_y = \frac{1}{6}R_B(L - x)^3 - \frac{1}{24}w(L - x)^4 + C_1x + C_2$$

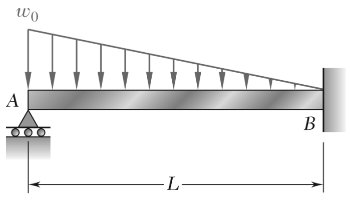
$$[x = 0,\ y = 0]:\quad 0 = \frac{1}{6}R_BL^3 - \frac{1}{24}wL^3 + C_2$$

$$C_2 = -\frac{1}{6}R_BL^3 + \frac{1}{24}wL^4$$

$$[x = L,\ y = 0]:\quad 0 = 0 - 0 + C_1L + C_2$$

$$\frac{1}{2}R_BL^3 - \frac{1}{6}wL^4 - \frac{1}{6}R_BL^3 + \frac{1}{24}wL^4 = 0$$

$$R_B = \frac{3}{8}wL \uparrow \blacktriangleleft$$



PROBLEM 9.21

For the beam and loading shown, determine the reaction at the roller support.

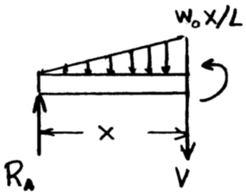
$$[x = 0,\ y = 0] \qquad [x = L,\ y = 0]$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right]$$

SOLUTION

Reactions are statically indeterminate.

Boundary conditions are shown above.



$$w = \frac{w_0}{L}(L - x)$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L}(L - x)$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L}\left(Lx - \frac{1}{2}x^2\right) + R_A$$

$$M = -\frac{w_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + R_A x$$

$$EI\frac{d^2y}{dx^2} = -\frac{w_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + R_A x$$

$$EI\frac{dy}{dx} = -\frac{w_0}{L}\left(\frac{1}{6}Lx^3 - \frac{1}{24}x^4\right) + \frac{1}{2}R_A x^2 + C_1$$

$$EIy = -\frac{w_0}{L}\left(\frac{1}{24}Lx^4 - \frac{1}{120}x^5\right) + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$$[x = 0,\ y = 0] \qquad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$$

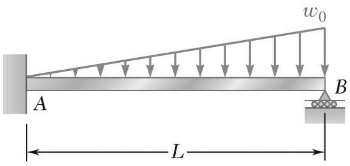
$$\left[x = L,\ \frac{dy}{dx} = 0\right] \qquad -\frac{w_0}{L}\left(\frac{1}{6}L^4 - \frac{1}{24}L^4\right) + \frac{1}{2}R_A L^2 + C_1 = 0$$

$$C_1 = \frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2$$

$$[x = L,\ y = 0] \qquad -\frac{w_0}{L}\left(\frac{1}{24}L^5 - \frac{1}{120}L^5\right) + \frac{1}{6}R_A L^3 + \left(\frac{1}{8}w_0 L^3 - \frac{1}{2}R_A L^2\right)L = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_A = \left(\frac{1}{8} - \frac{1}{24} + \frac{1}{120}\right)w_0 L$$

$$\frac{1}{3}R_A = \frac{11}{120}w_0 L \qquad R_A = \frac{11}{40}w_0 L \uparrow \blacktriangleleft$$



$[x = 0,\ y = 0]$ $\qquad$ $[x = L,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

PROBLEM 9.22

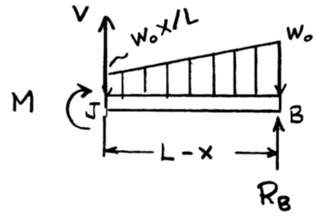
For the beam and loading shown, determine the reaction at the roller support.

SOLUTION

Reactions are statically indeterminate.

Boundary conditions are shown above.

Using free body JB,



$$+\circlearrowleft \Sigma M_J = 0:\quad -M + R_B(L-x) + \frac{1}{2}w_0(L-x)\frac{2}{3}(L-x) + \frac{1}{2}\frac{w_0x}{L}(L-x)\frac{1}{3}(L-x) = 0$$

$$M = R_B(L-x) - \frac{w_0}{6L}[2L(L-x)^2 + x(L-x)^2]$$

$$= R_B(L-x) - \frac{w_0}{6L}[2L^3 - 4L^2x + 2Lx^2 + xL^2 - 2Lx^2 + x^3]$$

$$= R_B(L-x) - \frac{w_0}{6L}(x^3 - 3L^2x + 2L^3)$$

$$EI\frac{d^2y}{dx^2} = R_B(L-x) - \frac{w_0}{6L}(x^3 - 3L^2x + 2L^3)$$

$$EI\frac{dy}{dx} = R_B\left(Lx - \frac{1}{2}x^2\right) - \frac{w_0}{6L}\left(\frac{1}{4}x^4 - \frac{3}{2}L^2x^2 + 2L^3x\right) + C_1$$

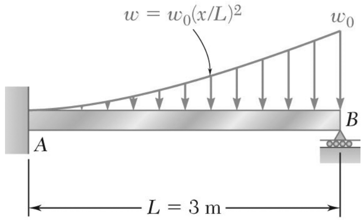
$$EIy = R_B\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) - \frac{w_0}{6L}\left(\frac{1}{20}x^5 - \frac{1}{2}L^2x^3 + L^3x^2\right) + C_1x + C_2$$

$$[x = 0,\ y = 0] \quad \to C_2 = 0$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \to C_1 = 0$$

$$[x = L,\ y = 0] \quad 0 = R_BL^3\left(\frac{1}{2} - \frac{1}{6}\right) - \frac{w_0L^4}{6}\left(\frac{1}{20} - \frac{1}{2} + 1\right)$$

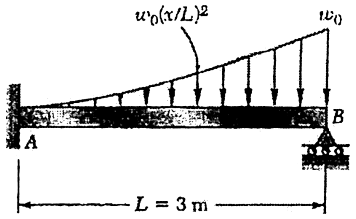
$$\frac{1}{3}R_B = \left(\frac{1}{6}\right)\left(\frac{11}{20}\right)w_0L \qquad\qquad R_B = \frac{11}{40}w_0L \uparrow \blacktriangleleft$$



PROBLEM 9.23

For the beam shown, determine the reaction at the roller support when $w_0 = 15$ kN/m.

SOLUTION



Reactions are statically indeterminate.

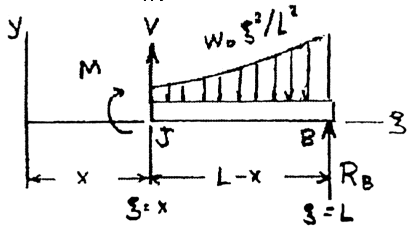
Boundary conditions are shown at left.

Using free body JB,

$+\circlearrowleft \Sigma M_J = 0:$

$[x = 0,\ y = 0]$ $\qquad [x = L,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$



$$-M + \int_x^L \frac{w_0}{L^2}\xi^2(\xi - x)d\xi + R_B(L - x) = 0$$

$$M = \frac{w_0}{L^2}\int_x^L \xi^2(\xi - x)d\xi - R_B(L - x)$$

$$= \frac{w_0}{L^2}\left(\frac{1}{4}\xi^4 - \frac{1}{3}x\xi^3\right)\bigg|_x^L - R_B(L - x)$$

$$= \frac{w_0}{L^2}\left(\frac{1}{4}L^4 - \frac{1}{3}L^3x + \frac{1}{12}x^4\right) - R_B(L - x)$$

$$EI\frac{d^2y}{dx^2} = \frac{w_0}{L^2}\left(\frac{1}{4}L^4 - \frac{1}{3}L^3x + \frac{1}{12}x^4\right) - R_B(L - x)$$

$$EI\frac{dy}{dx} = \frac{w_0}{L^2}\left(\frac{1}{4}L^4x - \frac{1}{6}L^3x^2 + \frac{1}{60}x^5\right) - R_B\left(Lx - \frac{1}{2}x^2\right) + C_1$$

$$EIy = \frac{w_0}{L^2}\left(\frac{1}{8}L^4x^2 - \frac{1}{18}L^3x^3 + \frac{1}{360}x^6\right) - R_B\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3\right) + C_1x + C_2$$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$ $\qquad 0 = 0 + 0 + C_1 \qquad C_1 = 0$

$[x = 0,\ y = 0]$ $\qquad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

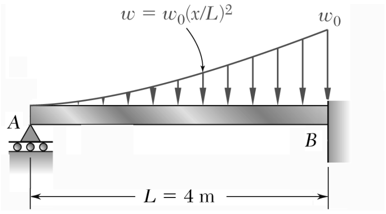
$[x = L,\ y = 0]$ $\qquad \left(\dfrac{1}{8} - \dfrac{1}{18} + \dfrac{1}{360}\right)w_0L^4 - \left(\dfrac{1}{2} - \dfrac{1}{6}\right)R_BL^3 = 0$

$$\frac{13}{180}w_0L^4 - \frac{1}{3}R_BL^3 = 0 \qquad R_B = \frac{13}{60}w_0L$$

<u>Data</u>: $\qquad w_0 = 15$ kN/m $\qquad L = 3$ m

$$R_B = \frac{13}{60}(15)(3) = 9.75 \text{ kN}$$

$R_B = 9.75 \text{ kN} \uparrow$ ◀



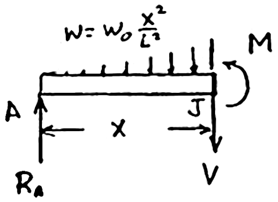
$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$\left[x = L,\ \dfrac{dy}{dx} = 0\right]$

PROBLEM 9.24

For the beam shown, determine the reaction at the roller support when $w_0 = 65$ kN/m.

SOLUTION



Reactions are statically indeterminate.

Boundary conditions are shown at left.

$$w = w_0 \frac{x^2}{L^2}$$

$$\frac{dV}{dx} = -w = -\frac{w_0}{L^2} x^2$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L^2}\frac{x^3}{3} + R_A$$

$$M = -\frac{w_0}{L^2}\frac{x^4}{12} + R_A x$$

$$EI\frac{d^2y}{dx^2} = -\frac{w_0}{L^2}\frac{x^4}{12} + R_A x$$

$$EI\frac{dy}{dx} = -\frac{w_0}{L^2}\frac{x^5}{60} + \frac{1}{2}R_A x^2 + C_1$$

$$EIy = -\frac{w_0}{L^2}\frac{x^6}{360} + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$[x = 0,\ y = 0]$: $\quad 0 = 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$\left[x = L,\ \dfrac{dy}{dx} = 0\right]$: $\quad -\dfrac{1}{60}w_0 L^3 + \dfrac{1}{2}R_A L^2 + C_1 = 0 \qquad C_1 = \dfrac{1}{60}w_0 L^3 - \dfrac{1}{2}R_A L^2$

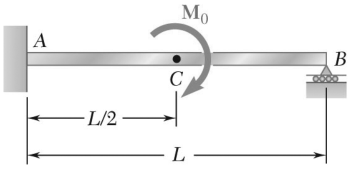
$[x = L,\ y = 0]$: $\quad -\dfrac{1}{360}w_0 L^4 + \dfrac{1}{6}R_A L^3 + \left(\dfrac{1}{60}w_0 L^3 - \dfrac{1}{2}R_A L^3\right)L = 0$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_A = \left(\frac{1}{60} - \frac{1}{360}\right)w_0 L$$

$$\frac{1}{3}R_A = \frac{1}{72}w_0 L \qquad R_A = \frac{1}{18}w_0 L$$

Data: $\quad w_0 = 65$ kN/m, $\quad L = 4$ m

$R_A = \dfrac{1}{18}(65)(4) = 14.44$ kN $\uparrow$ ◀



PROBLEM 9.25

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

$[x = 0,\ y = 0]$ $\qquad$ $[x =$

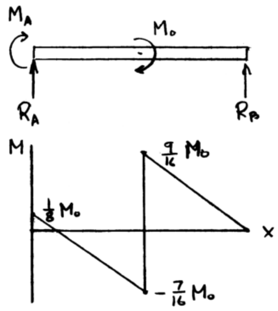
$\left[x = 0,\ \dfrac{dy}{dx} = 0 \right]$

SOLUTION

Reactions are statically indeterminate.

$$+\uparrow \Sigma F_y = 0: \qquad R_A + R_B = 0 \qquad R_A = -R_B$$

$$+\circlearrowleft \Sigma M_A = 0: \qquad -M_A - M_0 + R_B L = 0 \qquad M_A = R_B L - M_0$$



$$\underline{0 < x < \frac{L}{2}}$$

$$M = R_B x + M_A = -M_0 + R_B L - R_B x$$

$$EI \frac{d^2 y}{dx^2} = -M_0 + R_B (L - x)$$

$$EI \frac{dy}{dx} = -M_0 x + R_B \left(Lx - \frac{1}{2} x^2 \right) + C_1$$

$$EIy = -\frac{1}{2} M_0 x^2 + R_B \left(\frac{1}{2} L x^2 - \frac{1}{6} x^3 \right) + C_1 x + C_2$$

$$\underline{\frac{L}{2} < x < L}$$

$$M = R_B (L - x)$$

$$EI \frac{d^2 y}{dx^2} = R_B (L - x)$$

$$EI \frac{dy}{dx} = R_B \left(Lx - \frac{1}{2} x^2 \right) + C_3$$

$$EIy = R_B \left(\frac{1}{2} L x^2 - \frac{1}{6} x^3 \right) + C_3 x + C_4$$

PROBLEM 9.25 *(Continued)*

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \quad 0 + 0 + C_1 = 0 \qquad C_1 = 0$$

$$[x = 0,\ y = 0] \quad 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]$$

$$-M_0\frac{L}{2} + \cancel{R_B\left(\frac{1}{2}L^2 - \frac{1}{6}L^2\right)} = \cancel{R_B\left(\frac{1}{2}L^2 - \frac{1}{6}L^2\right)} + C_3 \qquad C_3 = -\frac{M_0L}{2}$$

$$\left[x = \frac{L}{2},\ y = y\right]$$

$$-\frac{1}{2}M_0\left(\frac{L}{2}\right)^2 + \cancel{R_B\left(\frac{1}{8}L^3 - \frac{1}{48}L^3\right)} = \cancel{R_B\left(\frac{1}{8}L^3 - \frac{1}{48}L^3\right)} + C_3\frac{L}{2} + C_4$$

$$C_4 = -\frac{1}{8}M_0L^2 - \frac{1}{2}C_3L$$

$$= \left(-\frac{1}{8} + \frac{1}{4}\right)M_0L^2 = \frac{1}{8}M_0L^2$$

$$[x = L,\ y = 0]$$

$$R_B\left(\frac{1}{2}L^3 - \frac{1}{6}L^3\right) + \frac{M_0L}{2}L + \frac{1}{8}M_0L^2 = 0$$

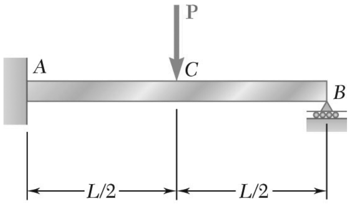
$$\left(\frac{1}{2} - \frac{1}{6}\right)R_BL^3 = \left(\frac{1}{2} - \frac{1}{8}\right)M_0L^2 \quad \frac{1}{3}R_B = \frac{3}{8}\frac{M_0}{L}$$

$$R_B = \frac{9}{8}\frac{M_0}{L}\uparrow \blacktriangleleft$$

$$M_A = \frac{9}{8}M_0 - M_0 = \frac{1}{8}M_0 \qquad M_A = \frac{1}{8}M_0 \blacktriangleleft$$

$$M_{C^-} = -M_0 + \frac{9}{8}\frac{M_0}{L} = -\frac{7}{16}M_0 \qquad M_{C^-} = -\frac{7}{16}M_0 \blacktriangleleft$$

$$M_{C^+} = R_B\left(L - \frac{L}{2}\right) = \frac{9}{8}\frac{M_0}{L}\left(\frac{L}{2}\right) = \frac{9}{16}M_0 \qquad M_{C^+} = \frac{9}{16}M_0 \blacktriangleleft$$



PROBLEM 9.26

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

$\left[x = 0,\ \frac{dy}{dx} = 0\right]$

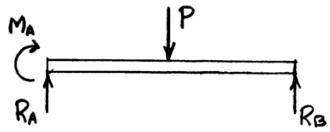
SOLUTION

Reactions are statically indeterminate.

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - P = 0 \quad R_A = P - R_B$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A + \frac{1}{2}PL - R_B L = 0$$

$$M_A = R_B L - \frac{1}{2}PL$$



$$\underline{0 < x < \frac{1}{2}L:}$$

$$M = M_A + R_A x$$

$$EI\frac{d^2y}{dx^2} = M_A + R_A x$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$$\underline{\frac{1}{2}L < x < L:}$$

$$M = M_A + R_A x - P\left(x - \frac{1}{2}L\right)$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\left(x - \frac{1}{2}L\right)$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P\left(x - \frac{1}{2}L\right)^2 + C_3$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\left(x - \frac{L}{2}\right)^3 + C_3 x + C_4$$

PROBLEM 9.26 *(Continued)*

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \quad 0 + 0 + C_1 = 0 \qquad C_1 = 0$$

$$[x = 0,\ y = 0] \quad 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]$$

$$\frac{1}{2}M_A L + \frac{1}{8}R_A L^2 + 0 = \frac{1}{2}M_A L + \frac{1}{8}R_A L^2 - 0 + C_3 \qquad C_3 = 0$$

$$\left[x = \frac{L}{2},\ y = y\right]$$

$$\frac{1}{8}M_A L^2 + \frac{1}{48}R_A L^3 + 0 + 0 = \frac{1}{8}M_A L^2 + \frac{1}{48}R_A L^3 - 0 + 0 + C_4 \quad C_4 = 0$$

$$[x = L,\ y = 0]$$

$$\frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{48}PL^3 + 0 + 0 = 0$$

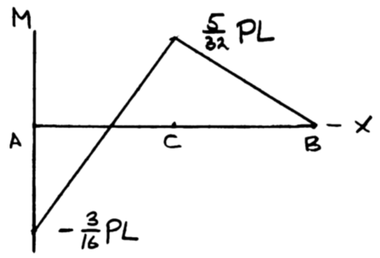
$$\frac{1}{2}\left(R_B L - \frac{1}{2}P\right)L^3 + \frac{1}{6}(P - R_B)L^3 - \frac{1}{48}PL^3 = 0 \qquad R_B = \frac{5}{16}P \uparrow \blacktriangleleft$$

$$R_A = P - \frac{5}{16}P \qquad R_A = \frac{7}{16}P \uparrow$$

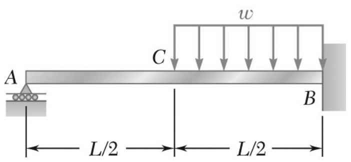
$$M_A = \frac{5}{16}PL - \frac{1}{2}PL \qquad M_A = -\frac{3}{16}PL \blacktriangleleft$$

$$M_C = R_B\left(\frac{L}{2}\right) = \left(\frac{5}{16}P\right)\left(\frac{L}{2}\right) \qquad M_C = \frac{5}{32}PL \blacktriangleleft$$

$$M_B = 0 \blacktriangleleft$$



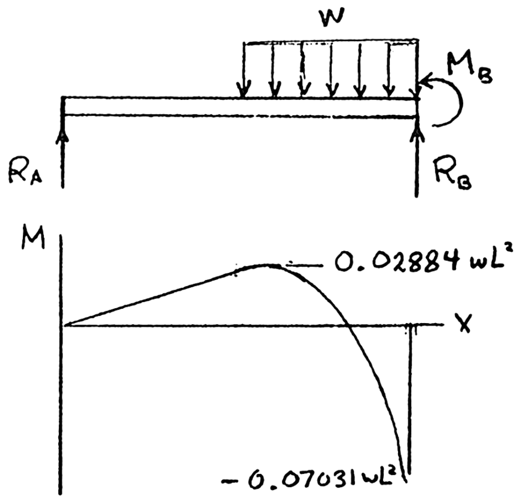
Bending moment diagram



PROBLEM 9.27

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION



Reactions are statically indeterminate.

$$0 < x < \frac{L}{2}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x \tag{1}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1 \tag{2}$$

$$EIy = \frac{1}{6}R_A x^3 + C_1 x + C_2 \tag{3}$$

$$\frac{L}{2} < x < L$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2}w\left(x - \frac{L}{2}\right)^2 \tag{4}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}w\left(x - \frac{L}{2}\right)^3 + C_3 \tag{5}$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{24}w\left(x - \frac{L}{2}\right)^4 + C_3 x + C_4 \tag{6}$$

$[x = 0,\ y = 0]$ $\quad 0 = 0 + 0 + C_2 \quad\quad C_2 = 0$

$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]$ $\quad \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} + C_1 = \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} + 0 + C_3 \quad\quad C_1 = C_3$

$\left[x = \frac{L}{2},\ y = y\right]$ $\quad \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} + \cancel{C_1\frac{L}{2}} + C_2 = \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} - 0 + \cancel{C_3\frac{L}{2}} + C_4 \quad\quad C_2 = C_4 = 0$

$\left[x = L,\ \frac{dy}{dx} = 0\right]$ $\quad \frac{1}{2}R_x L^2 - \frac{1}{6}w\left(\frac{L}{2}\right)^3 + C_3 = 0 \quad\quad C_3 = \frac{1}{48}wL^3 - \frac{1}{2}R_A L^2$

$[x = L,\ y = 0]$ $\quad \frac{1}{6}R_A L^2 - \frac{1}{24}w\left(\frac{L}{2}\right)^4 + \left(\frac{1}{48}wL^3 - \frac{1}{2}R_A L^2\right)L + 0 = 0$

PROBLEM 9.27 *(Continued)*

$$\left(\frac{1}{2}-\frac{1}{6}\right)R_A L^3=\left(\frac{1}{48}-\frac{1}{384}\right)wL^4 \qquad \frac{1}{3}R_A=\frac{7}{384}wL \qquad R_A=\frac{7}{128}wL\uparrow \blacktriangleleft$$

From (1), with $x=\dfrac{L}{2}$, $\quad M_C=R_A\left(\dfrac{L}{2}\right)=\dfrac{7}{256}wL^2 \qquad M_C=0.02734\,wL^2 \blacktriangleleft$

From (4), with $x=L$, $\quad M_B=R_A L-\dfrac{1}{2}w\left(\dfrac{L}{2}\right)^2=\left(\dfrac{7}{128}-\dfrac{1}{8}\right)wL-\dfrac{9}{128}wL^2$

$$M_B=-0.07031wL \blacktriangleleft$$

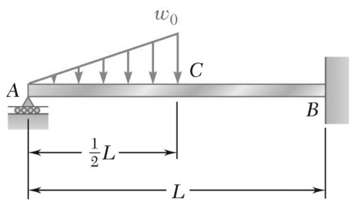
Location of maximum positive M:

$$\frac{L}{2}<x<L \qquad V_m=R_A-w\left(x_m-\frac{L}{2}\right)=0 \qquad x_m-\frac{L}{2}=\frac{R_A}{w}=\frac{7}{128}L$$

$$x_m=\frac{L}{2}+\frac{7}{128}L=\frac{71}{128}L$$

From (4), with $x=x_m$, $\quad M_m=R_A x_m-\dfrac{1}{2}w\left(x_m-\dfrac{L}{2}\right)^2$

$$=\left(\frac{7}{128}wL\right)\left(\frac{71}{128}L\right)-\frac{1}{2}w\left(\frac{7}{128}L\right)^2 \qquad M_m=0.02884\,wL^2 \blacktriangleleft$$

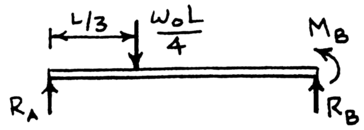


PROBLEM 9.28

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION

Reactions are statically indeterminate.



$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - \frac{w_0 L}{4} = 0 \qquad R_B = \frac{w_0 L}{4} - R_A$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \left(\frac{w_0 L}{4}\right)\left(\frac{2L}{3}\right) + M_B = 0 \qquad M_B = R_A L - \frac{w_0 L^2}{6}$$

$\underline{0 \le x \le \dfrac{L}{2}:}$

$$w = \frac{2w_0}{L}x$$

$$V = R_A - \frac{w_0}{L}x^2$$

$$M = R_A x - \frac{w_0}{3L}x^3$$

$$EI\frac{d^2y}{dx^2} = R_A x - \frac{w_0}{3L}x^3$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{12}\frac{w_0}{L}x^4 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{60}\frac{w_0}{L}x^5 + C_1 x + C_2$$

$\underline{\dfrac{L}{2} \le x \le L:}$

$$M = R_A x - \frac{w_0 L}{4}\left(x - \frac{L}{3}\right)$$

$$EI\frac{d^2y}{dx^2} = R_A x - w_0 L\left(\frac{1}{4}x - \frac{1}{12}L\right)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - w_0 L\left(\frac{1}{8}x^2 - \frac{1}{12}Lx\right) + C_3$$

$$EIy = \frac{1}{6}R_A x^3 - w_0 L\left(\frac{1}{24}x^3 - \frac{1}{24}Lx^2\right) + C_3 x + C_4$$

$$[x = 0,\ y = 0]: \quad 0 = 0 - 0 + 0 + C_2 \qquad \therefore \quad C_2 = 0$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]: \ \cancel{\frac{1}{8}R_A L^2} - \frac{1}{192}w_0 L^3 + C_1 = \cancel{\frac{1}{8}R_A L^2} + \frac{1}{96}w_0 L^3 + C_3 \ \therefore \ C_3 = C_1 - \frac{1}{64}w_0 L$$

$$\left[x = \frac{L}{2},\ y = y\right]: \ \cancel{\frac{1}{48}R_A L^3} - \frac{1}{1920}w_0 L^4 + \cancel{\frac{1}{2}C_1 L} + 0$$

$$= \cancel{\frac{1}{48}R_A L^3} - w_0 L\left(\frac{1}{192}L^3 - \frac{1}{96}L^3\right)$$

$$+\left(\cancel{C_1} - \frac{1}{64}w_0 L^3\right)\left(\frac{L}{2}\right) + C_4 \qquad \therefore \quad C_4 = \frac{1}{480}w_0 L^4$$

PROBLEM 9.28 *(Continued)*

$$\left[x = L, \frac{dy}{dx} = 0\right]: \frac{1}{2}R_A L^2 - w_0 L\left(\frac{1}{8}L^2 - \frac{1}{12}L^2\right) + C_3 = 0 \quad \therefore \quad C_3 = -\frac{1}{2}R_A L^2 + \frac{1}{24}w_0 L^3$$

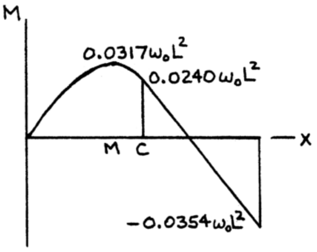
$$[x = L, \ y = 0]: \frac{1}{6}R_A L^3 - w_0 L\left(\frac{1}{24}L^3 - \frac{1}{24}L^3\right) + \left(-\frac{1}{2}R_A L^2 + \frac{1}{24}w_0 L^3\right)(L) + \frac{1}{480}w_0 L^4 = 0$$

$$R_A = \frac{21}{160}w_0 L \uparrow \blacktriangleleft$$

$$R_B = \frac{w_0 L}{4} - \frac{21}{160}w_0 L \qquad R_B = \frac{19}{160}w_0 L \uparrow$$

$$M_B = \frac{21}{160}w_0 L^2 - \frac{w_0 L^2}{6} \qquad M_B = -\frac{17}{480}w_0 L^2 = -0.0354 w_0 L^2 \blacktriangleleft$$

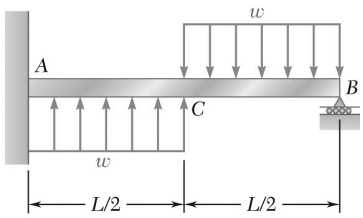
Over $0 < x < \dfrac{L}{2}, \quad V = R_A - \dfrac{w_0}{L}x^2 = \dfrac{21}{160}w_0 L - \dfrac{w_0}{L}x^2$



$$V = 0 \quad \text{at} \quad x = x_m = 0.36228L$$

$$M = \frac{21}{160}w_0 L x - \frac{w_0}{3L}x^3 \quad M_A = M(x = 0) = 0$$

$$M_C = M\left(x = \frac{L}{2}\right) = 0.0240 w_0 L^2 \quad M_m = M(x_m = 0.36228L) = 0.0317 w_0 L^2 \blacktriangleleft$$



PROBLEM 9.29

Determine the reaction at the roller support and the deflection at point C.

$[x = 0,\ y = 0]$ $\qquad$ $[x = L,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

$\left[x = \dfrac{L}{2},\ y = y\right]$

$\left[x = \dfrac{L}{2},\ \dfrac{dy}{dx} = \dfrac{dy}{dx}\right]$

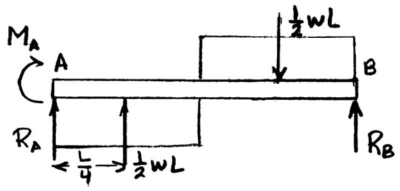
SOLUTION

Reactions are statically indeterminate.

$$+\uparrow \Sigma F_y = 0: \quad R_A + \frac{1}{2}wL - \frac{1}{2}wL + R_B = 0 \quad R_A = -R_B$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - \left(\frac{1}{2}wL\right)\frac{L}{2} + R_B L = 0$$

$$M_A = R_B L - \frac{1}{4}wL^2$$



From A to C: $\qquad 0 < x \le \dfrac{L}{2}$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x + \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{6}wx^3 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{24}wx^4 + C_1 x + C_2$$

PROBLEM 9.29 *(Continued)*

<u>From C to B:</u> $\qquad \dfrac{L}{2} \le x < L$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x + \frac{1}{2}wL\left(x - \frac{L}{4}\right) - \frac{1}{2}w\left(x - \frac{L}{2}\right)^2$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + \frac{1}{4}wL\left(x - \frac{L}{4}\right)^2 - \frac{1}{6}w\left(x - \frac{L}{2}\right)^3 + C_3$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + \frac{1}{12}wL\left(x - \frac{L}{4}\right)^3 - \frac{1}{24}w\left(x - \frac{L}{2}\right)^4 + C_3 x + C_4$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \quad 0 + 0 + 0 + C_1 = 0 \qquad\qquad C_1 = 0$$

$$[x = 0,\ y = 0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \qquad\qquad C_2 = 0$$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]$$

$$\cancel{M_A\frac{L}{2}} - \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} + \frac{1}{6}w\left(\frac{L}{2}\right)^3 = \cancel{M_A\frac{L}{2}} + \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2}$$

$$+ \frac{1}{4}wL\left(\frac{L}{4}\right)^2 - 0 + C_3$$

$$C_3 = \left(\frac{1}{48} - \frac{1}{64}\right)wL^3 = \frac{1}{192}wL^3$$

$$\left[x = \frac{L}{2},\ y = y\right]$$

$$\cancel{\frac{1}{2}M_A\left(\frac{L}{2}\right)^2} + \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} + \frac{1}{24}w\left(\frac{L}{2}\right)^4$$

$$= \cancel{\frac{1}{2}M_A\left(\frac{L}{2}\right)^2} + \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} + \frac{1}{12}wL\left(\frac{L}{4}\right)^3$$

$$- 0 + \frac{1}{192}wL^3\left(\frac{L}{2}\right) + C_4$$

PROBLEM 9.29 *(Continued)*

$$C_4 = \left(\frac{1}{384} - \frac{1}{768} - \frac{1}{384}\right)wL^4 = -\frac{1}{768}wL^4$$

$[x = L, \; y = 0]$

$$\frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 + \frac{1}{12}wL\left(\frac{3L}{4}\right)^3 - \frac{1}{24}w\left(\frac{L}{2}\right)^4$$

$$+\frac{1}{192}wL^3(L) - \frac{1}{768}wL^4 = 0$$

$$\frac{1}{2}\left(R_B L - \frac{1}{4}wL^2\right)L^2 + \frac{1}{6}(-R_B)L^3 + \left(\frac{27}{768} - \frac{1}{384} + \frac{1}{192} - \frac{1}{768}\right)wL^4 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_B L^3 = \left(\frac{1}{8} - \frac{7}{192}\right)wL^4 \quad \frac{1}{3}R_B = \frac{17}{192}wL \qquad R_B = \frac{17}{64}wL \uparrow \blacktriangleleft$$

$$R_A = -R_B = -\frac{17}{64}wL$$

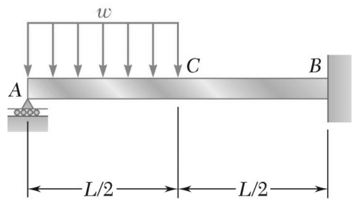
$$M_A = R_B L - \frac{1}{4}wL^2 = \left(\frac{17}{64} - \frac{1}{4}\right)wL^2 = \frac{1}{64}wL^2$$

<u>Deflection at *C*</u>: $\left(y \text{ at } x = \frac{L}{2}\right)$

$$EIy_C = \frac{1}{2}M_A\left(\frac{L}{2}\right)^2 + \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 + \frac{1}{24}w\left(\frac{L}{2}\right)^4$$

$$= \frac{1}{2}\left(\frac{1}{64}wL^2\right)\left(\frac{L}{2}\right)^2 + \frac{1}{6}\left(-\frac{17}{64}wL\right)\left(\frac{L}{2}\right)^3 + \frac{1}{24}w\left(\frac{L}{2}\right)^4$$

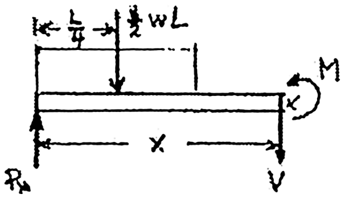
$$= \left(\frac{1}{512} - \frac{17}{3072} + \frac{1}{384}\right)wL^4 = -\frac{1}{1024}wL^4 \qquad y_C = \frac{1}{1024}\frac{wL^4}{EI} \downarrow \blacktriangleleft$$



PROBLEM 9.30

Determine the reaction at the roller support and the deflection at point C.

SOLUTION



Reactions are statically indeterminate.

$$0 < x < \frac{L}{2}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}wx^3 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{24}wx^4 + C_1 x + C_2$$

$\frac{L}{2} < x < L$ (See free body diagram.)

$$+\circlearrowleft \sum M_K = 0: -R_A x + \frac{1}{2}wL\left(x - \frac{L}{4}\right) + M = 0$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{1}{2}wL\left(x - \frac{1}{4}L\right)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{4}wL\left(x - \frac{L}{4}\right)^2 + C_3$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{12}wL\left(x - \frac{L}{4}\right)^3 + C_3 x + C_4$$

$[x = 0,\ y = 0]$: $\quad 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$$\left[x = \frac{L}{2},\ \frac{dy}{dx} = \frac{dy}{dx}\right]: \quad \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} - \frac{1}{6}w\left(\frac{L}{2}\right)^3 + C_1 = \cancel{\frac{1}{2}R_A\left(\frac{L}{2}\right)^2} - \frac{1}{4}wL\left(\frac{L}{4}\right)^2 + C_3$$

$$C_1 = C_3 + \frac{1}{48}wL^3 - \frac{1}{64}wL^3 = C_3 + \frac{1}{192}wL^3$$

$$\left[x = \frac{L}{2},\ y = y\right]: \quad \cancel{\frac{1}{6}R_A\left(\frac{L}{2}\right)^3} - \frac{1}{24}w\left(\frac{L}{2}\right)^4 + \left(\cancel{C_3} + \frac{1}{192}wL^3\right)\frac{L}{2} = \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 - \frac{1}{12}wL\left(\frac{L}{4}\right)^3 + \cancel{C_3\frac{L}{2}} + C_4$$

$$C_4 = -\frac{1}{384}wL^4 + \frac{1}{384}wL^4 + \frac{1}{768}wL^4 = \frac{1}{768}wL^4$$

PROBLEM 9.30 *(Continued)*

$$\left[x = L,\ \frac{dy}{dx} = 0\right]: \qquad \frac{1}{2}R_A L^2 - \frac{1}{4}wL\left(\frac{3L}{4}\right)^2 + C_3 = 0 \qquad C_3 = \frac{9}{64}wL^3 - \frac{1}{2}R_A L^2$$

$$[x = L,\ y = 0]: \qquad \frac{1}{6}R_A L^3 - \frac{1}{12}wL\left(\frac{3L}{4}\right)^3 + \left(\frac{9}{64}wL^3 - \frac{1}{2}R_A L^2\right)L + \frac{1}{768}wL^4 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_A L^3 = \left(\frac{9}{64} - \frac{27}{768} + \frac{1}{768}\right)wL^4 \qquad \frac{1}{3}R_A = \frac{41}{384}wL \qquad R_A = \frac{41}{128}wL \uparrow \blacktriangleleft$$

$$C_3 = \frac{9}{64}wL^3 - \frac{1}{2}\frac{41}{128}wL^3 = -\frac{5}{256}wL^3$$

$$C_1 = -\frac{5}{256}wL^3 + \frac{1}{192}wL^3 = -\frac{11}{768}wL^3$$

<u>Deflection at C</u>: $\left(y \text{ at } x = \frac{L}{2}\right)$

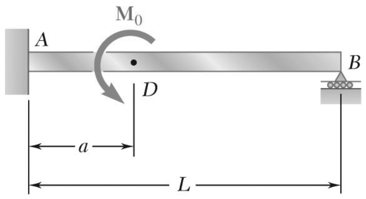
$$y_C = \frac{wL^4}{EI}\left\{\frac{1}{6}\cdot\frac{41}{128}\cdot\left(\frac{1}{2}\right)^3 - \frac{1}{24}\cdot\left(\frac{1}{2}\right)^4 - \frac{11}{768}\cdot\frac{1}{2} + 0\right\}$$

$$= \left(\frac{41}{6144} - \frac{1}{384} - \frac{11}{1536}\right)\frac{wL^4}{EI} = -\frac{19}{6144}\frac{wL^4}{EI} \qquad y_C = \frac{19}{6144}\frac{wL^4}{EI} \downarrow \blacktriangleleft$$

or

$$y_C = \frac{wL^4}{EI}\left\{\frac{1}{6}\cdot\frac{41}{128}\left(\frac{1}{2}\right)^3 - \frac{1}{12}\cdot\left(\frac{1}{4}\right)^3 + \frac{5}{256}\cdot\frac{1}{2} + \frac{1}{768}\right\}$$

$$= \left(\frac{41}{6144} - \cancel{\frac{1}{768}} - \frac{5}{512} + \cancel{\frac{1}{768}}\right)\frac{wL^4}{EI} = -\frac{19}{6144}\frac{wL^4}{EI}$$



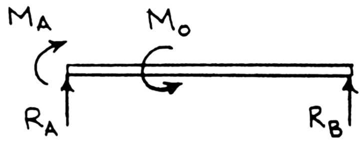
PROBLEM 9.31

Determine the reaction at the roller support and the deflection at point D if a is equal to $L/3$.

SOLUTION

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B = 0 \quad R_A = -R_B$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_0 - M_A + R_B L = 0 \quad M_A = R_B L + M_0$$



$\underline{0 \le x \le a:}$

$$M = M_A + R_A x$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$\underline{a \le x \le L:}$

$$M = M_A + R_A x - M_0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - M_0$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - M_0 x + C_3$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{2}M_0 x^2 + C_3 x + C_4$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right]: \quad 0 + 0 + C_1 = 0 \qquad \therefore \quad C_1 = 0$$

$$[x = 0,\ y = 0]: \quad 0 + 0 + 0 + C_2 = 0 \qquad \therefore \quad C_2 = 0$$

$$\left[x = a,\ \frac{dy}{dx} = \frac{dy}{dx}\right]:$$

$$M_A a + \frac{1}{2}R_A a^2 = M_A a + \frac{1}{2}R_A a^2 - M_0 a + C_3 \qquad \therefore \quad C_3 = M_0 a$$

$$[x = a,\ y = y]:$$

$$\frac{1}{2}M_A a^2 + \frac{1}{6}R_A a^3 = \frac{1}{2}M_A a^2 + \frac{1}{6}R_A a^3 - \frac{1}{2}M_0 a^2 + (M_0 a)(a) + C_4 \quad \therefore \quad C_4 = -\frac{1}{2}M_0 a^2$$

PROBLEM 9.31 *(Continued)*

$[x = L, \ y = 0]:$

$$\frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{2}M_0 L^2 + (M_0 a)(L) - \frac{1}{2}M_0 a^2 = 0$$

$$\frac{1}{2}(R_B L + M_0)L^2 + \frac{1}{6}(-R_B)L^3 - \frac{1}{2}M_0 L^2 + M_0 aL - \frac{1}{2}M_0 a^2 = 0$$

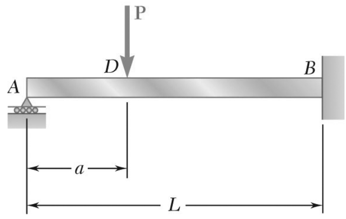
$$R_B = \frac{3M_0 a}{2L^3}(a - 2L) = \frac{3M_0}{2L^3}\left(\frac{L}{3}\right)\left(\frac{L}{3} - 2L\right) = -\frac{5M_0}{6L} \qquad R_B = \frac{5M_0}{6L} \downarrow \blacktriangleleft$$

<u>Deflection at *D*</u>: $\left(y \text{ at } x = a = \frac{L}{3}\right)$

$$y_D = \frac{1}{EI}\left\{\frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3\right\}$$

$$= \frac{1}{EI}\left\{\frac{1}{2}\left(-\frac{5M_0}{6L}L + M_0\right)\left(\frac{L}{3}\right)^2 + \frac{1}{6}\left(+\frac{5M_0}{6L}\right)\left(\frac{L}{3}\right)^3\right\}$$

$$= -\frac{7M_0 L^2}{486EI} \qquad y_D = \frac{7M_0 L^2}{486EI} \uparrow \blacktriangleleft$$



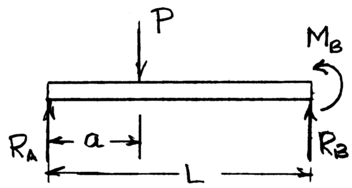
PROBLEM 9.32

Determine the reaction at the roller support and the deflection at point D if a is equal to $L/3$.

SOLUTION

$\underline{0 \le x \le a:}$

$$M = R_A x$$



$$EI\frac{d^2y}{dx^2} = M = R_A x$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 + C_1 x + C_2$$

$\underline{a \le x \le L:}$

$$M = R_A x - P(x-a)$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - P(x-a)$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{2}P(x-a)^2 + C_3$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{6}P(x-a)^3 + C_3 x + C_4$$

$$[x = 0,\ y = 0]:\quad 0 + 0 + C_2 = 0 \qquad \therefore\ C_2 = 0$$

$$\left[x = a,\ \frac{dy}{dx} = \frac{dy}{dx}\right]:$$

$$\frac{1}{2}R_A a^2 + C_1 = \frac{1}{2}R_A a^2 - 0 + C_3 \qquad \therefore\ C_1 = C_3$$

$$[x = a,\ y = y]:$$

$$\frac{1}{6}R_A a^3 + C_1 a + 0 = \frac{1}{6}R_A a^3 - 0 + C_1 a + C_4 \qquad \therefore\ C_4 = 0$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right]:$$

$$\frac{1}{2}R_A L^2 - \frac{1}{2}P(L-a)^2 + C_3 = 0 \qquad \therefore\ C_3 = \frac{1}{2}P(L-a)^2 - \frac{1}{2}R_A L^2$$

PROBLEM 9.32 *(Continued)*

$[x = L, \ y = 0]:$

$$\frac{1}{6}R_A L^3 - \frac{1}{6}P(L-a)^3 + \left[\frac{1}{2}P(L-a)^2 - \frac{1}{2}R_A L^2\right](L) + 0 = 0$$

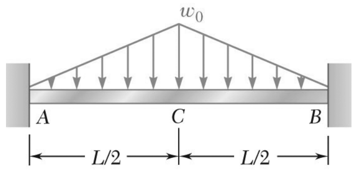
$$R_A = \frac{P}{2L^3}(2L^3 - 3aL^2 + a^3) = \frac{P}{2L^3}\left(2L^3 - L^3 + \frac{L^3}{9}\right) \qquad R_A = \frac{14}{27}P \uparrow \blacktriangleleft$$

Deflection at D: $\left(y \text{ at } x = a = \frac{L}{3}\right)$

$$y_D = \frac{1}{EI}\left\{\frac{1}{6}R_A\left(\frac{L}{3}\right)^3 + C_1\left(\frac{L}{3}\right)\right\}$$

$$= \frac{1}{EI}\left\{\frac{1}{6}\left(\frac{14}{27}P\right)\left(\frac{L}{3}\right)^3 + \left[\frac{1}{2}P\left(L - \frac{L}{3}\right)^2 - \frac{1}{2}\left(\frac{14}{27}P\right)L^2\right]\left(\frac{L}{3}\right)\right\}$$

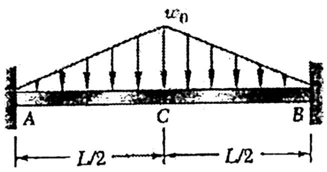
$$= -\frac{20}{2187}\frac{PL^3}{EI} \qquad y_D = \frac{20}{2187}\frac{PL^3}{EI} \downarrow \blacktriangleleft$$



PROBLEM 9.33

Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.

SOLUTION



Reactions are statically indeterminate.

Because of symmetry, $\frac{dy}{dx} = 0$ and $V = 0$ at $x = \frac{L}{2}$.

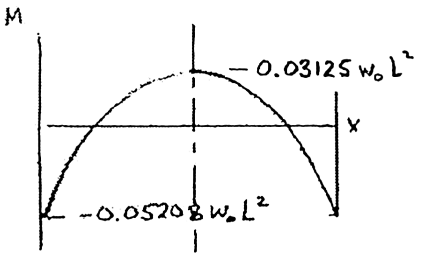
$[x = 0,\ y = 0]$ | $[x = L,\ y = 0]$

$\left[x = 0,\ \frac{dy}{dx} = 0\right]$ | $\left[x = L,\ \frac{dy}{dx} = 0\right]$

$\left[x = \frac{L}{2},\ V = 0\right]$

$\left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right]$

Use portion AC of beam. $\left(0 < x \leq \frac{L}{2}\right)$



$$\frac{dV}{dx} = -w = -2\frac{w_0}{L}x$$

$$\frac{dM}{dx} = V = -\frac{w_0}{L}x^2 + R_A \tag{1}$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{3}\frac{w_0}{L}x^3 + R_A x + M_A \tag{2}$$

$$EI\frac{dy}{dx} = -\frac{1}{12}\frac{w_0}{L}x^4 + \frac{1}{2}R_A x^2 + M_A x + C_1 \tag{3}$$

$$EIy = -\frac{1}{60}\frac{w_0}{L}x^2 + \frac{1}{6}R_A x^3 + \frac{1}{2}M_A x^2 + C_1 x + C_2 \tag{4}$$

$\left[x = 0,\ \frac{dy}{dx} = 0\right]$: $\quad 0 = 0 + 0 + 0 + C_1 \qquad C_1 = 0$

$[x = 0,\ y = 0]$: $\quad 0 = 0 + 0 + 0 + 0 + C_2 \qquad C_2 = 0$

$\left[x = \frac{L}{2},\ V = 0\right]$: $\quad -\frac{w_0}{L}\left(\frac{L}{2}\right)^2 + R_A = 0 \qquad R_A = \frac{wL}{4}$ ◀

$\left[x = \frac{L}{2},\ \frac{dy}{dx} = 0\right]$: $\quad -\frac{1}{12}\frac{w_0}{L}\left(\frac{L}{2}\right)^4 + \frac{1}{2}\left(\frac{1}{4}w_0 L\right)\left(\frac{L}{2}\right)^2 + M_A\frac{L}{2} + 0 = 0$

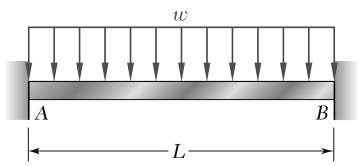
PROBLEM 9.33 *(Continued)*

$$M_A = -2\left(\frac{1}{32} - \frac{1}{192}\right)w_0L^2 = -\frac{5}{96}w_0L^2 \qquad M_A = -0.05208w_0L^2 \blacktriangleleft$$

From (2), with $x = \dfrac{L}{2}$,

$$M_C = -\frac{1}{3}\frac{w_0}{L}\left(\frac{L}{2}\right)^3 + \left(\frac{1}{4}w_0L\right)\left(\frac{L}{2}\right) - \frac{5}{96}w_0L^{12}$$

$$= \left(-\frac{1}{24} + \frac{1}{8} - \frac{5}{96}\right)w_0L^2 = \frac{1}{32}w_0L^2 \qquad M_C = 0.03125w_0L^2 \blacktriangleleft$$



$$[x=0,\ y=0] \qquad [x=L,\ y=0]$$

$$\left[x=0,\ \frac{dy}{dx}=0\right] \qquad \left[x=L,\ \frac{dy}{dx}=0\right]$$

PROBLEM 9.34

Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.

SOLUTION

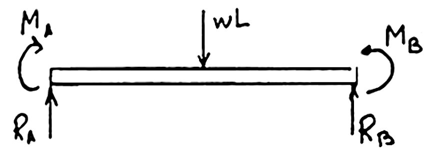
Reactions are statically indeterminate.

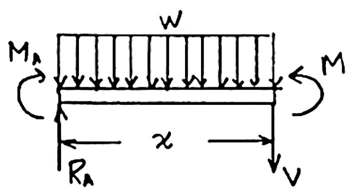
By symmetry, $\quad R_B = R_A; \quad M_B = M_A$

$$\frac{dy}{dx} = 0 \text{ at } x = \frac{L}{2}$$

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - wL = 0 \qquad R_B = R_A = \frac{1}{2}wL\uparrow \blacktriangleleft$$

Over entire beam, $\quad M = M_A + R_A x - \frac{1}{2}wx^2$





$$EI\frac{d^2y}{dx^2} = M_A + \frac{1}{2}wLx - \frac{1}{2}wx^2$$

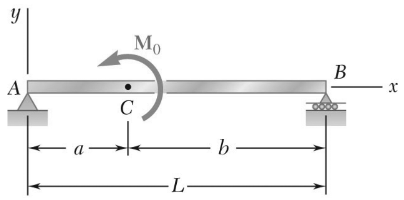
$$EI\frac{dy}{dx} = M_A x + \frac{1}{4}wLx^2 - \frac{1}{6}wx^3 + C_1$$

$$\left[x=0,\ \frac{dy}{dx}=0\right]: \quad 0+0-0+C_1=0 \quad \therefore \quad C_1 = 0$$

$$\left[x=\frac{L}{2},\ \frac{dy}{dx}=0\right]: \quad \frac{1}{2}M_A L + \frac{1}{16}wL^3 - \frac{1}{48}wL^3 + 0 = 0$$

$$M_A = -\frac{1}{12}wL^2 \blacktriangleleft$$

$$M = -\frac{1}{12}wL^2 + \frac{1}{2}wLx - \frac{1}{2}wx^2 \qquad M = w[6x(L-x) - L^2]/12 \blacktriangleleft$$



$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

PROBLEM 9.35

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection of point *C*.

SOLUTION

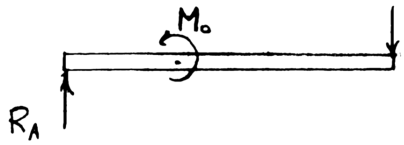
<u>Reactions:</u>

$$R_A = \frac{M_0}{L}\uparrow, \quad R_B = \frac{M_0}{L}\downarrow$$

$$0 < x < a \qquad M = R_A x$$

$$a < x < L \qquad M = R_A x - M_0$$

Using singularity functions,



$$EI\frac{d^2y}{dx^2} = M = R_A x - M_0\langle x-a\rangle^0$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0\langle x-a\rangle^1 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0\langle x-a\rangle^2 + C_1 x + C_2$$

$$[x = 0, y = 0] \qquad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$$

$$[x = L, y = 0] \qquad \frac{1}{6}R_A L^3 - \frac{1}{2}M_0(L-a)^2 + C_1 L + 0 = 0$$

$$C_1 L = -\frac{1}{6}\frac{M_0}{L}L^3 + \frac{1}{2}M_0 b^2$$

$$C_1 = \frac{M_0}{6L}(3b^2 - L^2)$$

(*a*) <u>Elastic curve</u>: $$y = \frac{1}{EI}\left\{\frac{1}{6}\frac{M_0}{L}x^3 - \frac{1}{2}M_0\langle x-a\rangle^2 + \frac{M_0}{6L}(3b^2 - L^2)x\right\}$$

$$y = \frac{M_0}{6EIL}\left\{x^3 - 3L\langle x-a\rangle^2 + (3b^2 - L^2)x\right\} \blacktriangleleft$$

$$\frac{dy}{dx} = \frac{M_0}{6EIL}\left\{3x^2 - 6L\langle x-a\rangle^1 + (3b^2 - L^2)\right\}$$

PROBLEM 9.35 *(Continued)*

(*b*) Slope at *A*: $\left(\dfrac{dy}{dx} \text{ at } x = 0\right)$

$$\theta_A = \frac{M_0}{6EIL}\{0 - 0 + 3Lb^2 - L^3\} \qquad \theta_A = \frac{M_0}{6EIL}(3b^2 - L^2) \blacktriangleleft$$

(*c*) Deflection at *C*: (y at $x = a$)

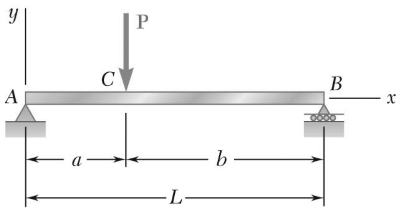
$$y_C = \frac{M_0}{6EIL}\{a^3 - 0 + (3b^2 - L^2)a\}$$

$$= \frac{M_0 a}{6EIL}\{a^2 + 3b^2 - (a+b)^2\}$$

$$= \frac{M_0 a}{6EIL}\{\cancel{a^2} + 3b^2 - \cancel{a^2} - 2ab - b^2\}$$

$$= \frac{M_0 a}{6EIL}\{2b^2 - 2ab\}$$

$$y_C = \frac{M_0 ab}{3EIL}(b - a) \uparrow \blacktriangleleft$$



$[x = 0,\ M = 0]$ $[x = L,\ M = 0]$

$[x = 0,\ y = 0]$ $[x = L,\ y = 0]$

PROBLEM 9.36

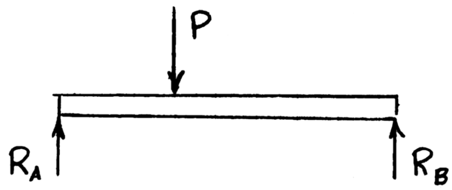
For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection of point *C*.

SOLUTION

$$+\circlearrowleft \Sigma M_B = 0: \ -R_A L + Pb = 0 \qquad R_A = \frac{Pb}{L}$$

$$\frac{dM}{dx} = V = R_A - P\langle x - a\rangle^0 = \frac{Pb}{L} - P\langle x - a\rangle^0$$

$$M = \frac{Pb}{L}x - P\langle x - a\rangle^1 + \cancel{M_A}$$



$$EI\frac{d^2y}{dx^2} = \frac{Pb}{L}x - P\langle x - a\rangle$$

$$EI\frac{dy}{dx} = \frac{Pb}{2L}x^2 - \frac{1}{2}P\langle x - a\rangle^2 + C_1$$

$$EIy = \frac{Pb}{6L}x^3 - \frac{1}{6}P\langle x - a\rangle^3 + C_1 x + C_2$$

$$[x = 0,\ y = 0] \qquad C_2 = 0$$

$$[x = L,\ y = 0] \quad \frac{Pb}{6L}L^3 - \frac{1}{6}P(L - a)^3 + C_1 L = 0$$

$$C_1 = -\frac{1}{6}\frac{R}{L}(bL^2 - b^3) = -\frac{1}{6}\frac{Pb}{L}(L^2 - b^2)$$

(*a*) Elastic curve:

$$y = \frac{P}{EI}\left\{\frac{b}{6L}x^3 - \frac{1}{6}\langle x - a\rangle^3 - \frac{1}{6}\frac{b}{L}(L^2 - b^2)x\right\}$$

$$y = \frac{P}{6EIL}\{bx^3 - L\langle x - a\rangle^3 - b(L^2 - b^2)x\} \ \blacktriangleleft$$

(*b*) Slope at end *A*:

$$EI\left.\frac{dy}{dx}\right|_{x=0} = C_1 = -\frac{Pb}{6L}(L^2 - b^2)$$

$$\theta_A = -\frac{Pb}{6EIL}(L^2 - b^2) \qquad \theta_A = \frac{Pb}{6EIL}(L^2 - b^2) \ \blacktriangleleft$$

PROBLEM 9.36 *(Continued)*

(c) Deflection at C:

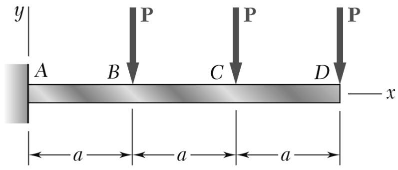
$$EIy_C = \frac{Pb}{6L}a^3 + C_1 a = \frac{Pba^3}{6L} - \frac{Pb}{6L}(L^2 - b^2)a$$

$$= \frac{Pba}{6L}(a^2 - L^2 - b^2)$$

$$y_C = -\frac{Pab}{6EIL}(L^2 - a^2 - b^2)$$

$$= -\frac{Pab}{6EIL}\left\{\cancel{a^2} + 2ab + \cancel{b^2} - \cancel{a^2} - \cancel{b^2}\right\}$$

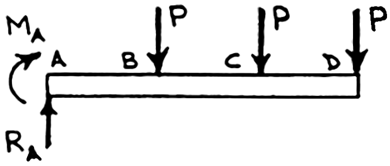
$$= -\frac{Pa^2b^2}{3EIL} \qquad\qquad y_C = \frac{Pa^2b^2}{3EIL} \downarrow ◀$$



PROBLEM 9.37

For the beam and loading shown, determine the deflection at (a) point B, (b) point C, (c) point D.

SOLUTION



$$+\uparrow \Sigma F_y = 0: \quad R_A - P - P - P = 0 \qquad R_A = 3P$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - Pa - P(2a) - P(3a) = 0 \qquad M_A = -6Pa$$

$$\frac{dM}{dx} = V = 3P - P\langle x-a\rangle^0 - P\langle x-2a\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = 3Px - P\langle x-a\rangle^1 - P\langle x-2a\rangle^1 - 6Pa$$

$$EI\frac{dy}{dx} = \frac{3}{2}Px^2 - \frac{1}{2}P\langle x-a\rangle^2 - \frac{1}{2}P\langle x-2a\rangle^2 - 6Pax + C_1$$

$$\left[x=0,\ \frac{dy}{dx}=0\right]: \quad 0-0-0-0+C_1=0 \qquad \therefore C_1 = 0$$

$$EIy = \frac{1}{2}Px^3 - \frac{1}{6}P\langle x-a\rangle^3 - \frac{1}{6}P\langle x-2a\rangle^3 - 3Pax^2 + C_2$$

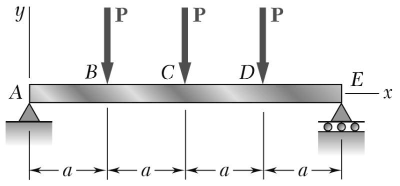
$$[x=0,\ y=0]: \quad 0-0-0-0+C_2 = 0 \qquad \therefore C_2 = 0$$

Elastic curve: $$y = \frac{P}{6EI}\left[3x^3 - \langle x-a\rangle^3 - \langle x-2a\rangle^3 - 18ax^2\right]$$

(a) $x = a$: $\quad y_B = \dfrac{Pa^3}{6EI}[3-0-0-18]$ $\qquad y_B = \dfrac{5Pa^3}{2EI}\downarrow$ ◀

(b) $x = 2a$: $\quad y_C = \dfrac{Pa^3}{6EI}[24-1-0-72]$ $\qquad y_C = \dfrac{49Pa^3}{6EI}\downarrow$ ◀

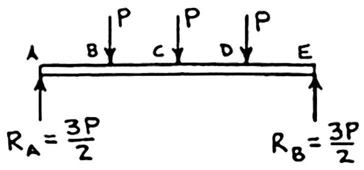
(c) $x = 3a$: $\quad y_D = \dfrac{Pa^3}{6EI}[81-8-1-162]$ $\qquad y_D = \dfrac{15Pa^3}{EI}\downarrow$ ◀



PROBLEM 9.38

For the beam and loading shown, determine the deflection at (*a*) point *B*, (*b*) point *C*, (*c*) point *D*.

SOLUTION



$$\frac{dM}{dx} = V = \frac{3P}{2} - P\langle x-a\rangle^0 - P\langle x-2a\rangle^0 - P\langle x-3a\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = \frac{3P}{2}x - P\langle x-a\rangle^1 - P\langle x-2a\rangle^1 - P\langle x-3a\rangle^1$$

$$EI\frac{dy}{dx} = \frac{3P}{4}x^2 - \frac{1}{2}P\langle x-a\rangle^2 - \frac{1}{2}P\langle x-2a\rangle^2 - \frac{1}{2}P\langle x-3a\rangle^2 + C_1$$

$$EIy = \frac{P}{4}x^3 - \frac{1}{6}P\langle x-a\rangle^3 - \frac{1}{6}P\langle x-2a\rangle^3 - \frac{1}{6}P\langle x-3a\rangle^3 + C_1x + C_2$$

$$[x=0,\ y=0]:\ 0-0-0-0+0+C_2 = 0 \qquad \therefore C_2 = 0$$

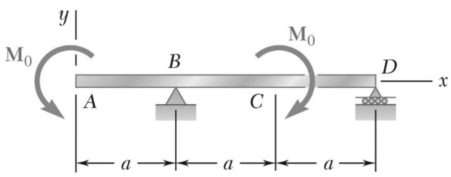
$$[x=4a,\ y=0]:\ 16Pa^3 - \frac{9}{2}Pa^3 - \frac{4}{3}Pa^3 - \frac{1}{6}Pa^3 + 4aC_1 = 0 \qquad \therefore C_1 = -\frac{5}{2}Pa^2$$

<u>Elastic curve</u>: $$y = \frac{P}{12EI}\left[3x^3 - 2\langle x-a\rangle^3 - 2\langle x-2a\rangle^3 - 2\langle x-3a\rangle^3 - 30a^2x\right]$$

(*a*) $x = a$: $y_B = \dfrac{Pa^3}{12EI}[3-0-0-0-30]$ $\qquad y_B = \dfrac{9Pa^3}{4EI}\downarrow$ ◀

(*b*) $x = 2a$: $y_C = \dfrac{Pa^3}{12EI}[24-2-0-0-60]$ $\qquad y_C = \dfrac{19Pa^3}{6EI}\downarrow$ ◀

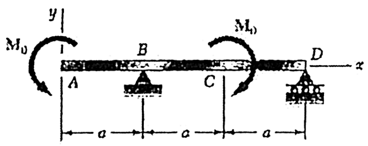
(*c*) $x = 3a$: $y_D = \dfrac{Pa^3}{12EI}[81-16-2-0-90]$ $\qquad y_D = \dfrac{9Pa^3}{4EI}\downarrow$ ◀



PROBLEM 9.39

For the beam and loading shown, determine (*a*) the deflection end *A*, (*b*) the deflection at point *C*, (*c*) the slope at end *D*.

SOLUTION



Since loads self-equilibrate,

$$R_B = 0, \quad R_D = 0$$

$$(0 < x < 2a): \; M = -M_0$$

$[x = a, \; y = 0]$ $[x = 3a, \; y = 0]$ $\qquad (2a < x < 3a): \; M = -M_0 + M_0 = 0$

Using singularity functions,

$$EI\frac{d^2y}{dx^2} = M = -M_0 + M_0\langle x - 2a\rangle^0$$

$$EI\frac{dy}{dx} = -M_0x + M_0\langle x - 2a\rangle^1 + C_1$$

$$EIy = -\frac{1}{2}M_0x^2 + \frac{1}{2}M_0\langle x - 2a\rangle^2 + C_1x + C_2$$

$[x = 3a, \; y = 0]$ $\qquad -\frac{1}{2}M_0(3a)^2 + \frac{1}{2}M_0a^2 + C_1(3a) + C_2 = 0 \qquad 3aC_1 + C_2 = 4M_0a^2$

$[x = a, \; y = 0]$ $\qquad -\frac{1}{2}M_0a^2 + 0 + C_1a + C_2 = 0 \qquad aC_1 + C_2 = \frac{1}{2}M_0a^2$

Subtracting, $\qquad 2aC_1 = \frac{7}{2}M_0a^2 \qquad C_1 = \frac{7}{4}M_0a$

$$C_2 = \frac{1}{2}M_0a^2 - aC_1 = -\frac{5}{4}M_0a^2$$

$$y = \frac{M_0}{EI}\left\{-\frac{1}{2}x^2 + \frac{1}{2}\langle x - 2a\rangle^2 + \frac{7}{4}ax - \frac{5}{4}a^2\right\}$$

$$\frac{dy}{dx} = \frac{M_0}{EI}\left\{-x + \langle x - a\rangle^1 + \frac{7}{4}a\right\}$$

(*a*) <u>Deflection at *A*</u>: $\qquad (y \text{ at } x = 0)$

$$y_A = \frac{M_0a^2}{EI}\left\{-0 + 0 + 0 - \frac{5}{4}\right\} = -\frac{5}{4}\frac{M_0a^2}{EI}, \qquad y_A = \frac{5}{4}\frac{M_0a^2}{EI} \downarrow \blacktriangleleft$$

PROBLEM 9.39 *(Continued)*

(*b*) Deflection at *C*: (y at $x = 2a$)

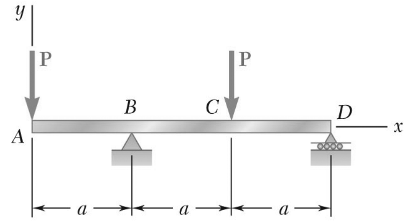
$$y_C = \frac{M_0 a^2}{EI}\left\{-\frac{1}{2}(2)^2 + 0 + \frac{7}{4}(2) - \frac{5}{4}\right\} = \frac{1}{4}\frac{M_0 a^2}{EI}$$

$$y_C = \frac{1}{4}\frac{M_0 a^2}{EI} \uparrow \blacktriangleleft$$

(*c*) Slope at *D*: $\left(\frac{dy}{dx} \text{ at } x = 3a\right)$

$$\theta_D = \frac{M_0 a}{EI}\left\{-3 + 1 + \frac{7}{4}\right\} = -\frac{1}{4}\frac{M_0 a}{EI},$$

$$\theta_D = \frac{1}{4}\frac{M_0 a}{EI} \blacktriangleleft$$

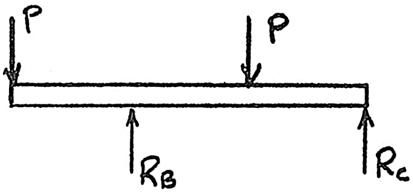


$[x = a,\ y = 0]$ $[x = 3a,\ y = 0]$

PROBLEM 9.40

For the beam and loading shown, determine (*a*) the deflection at end *A*, (*b*) the deflection at point *C*, (*c*) the slope at end *D*.

SOLUTION



Reactions: $R_B = 2P \uparrow, \quad R_D = 0$

$(0 < x < a)$: $V = -P$

$(a < x < 2a)$: $V = -P + 2P$

$(2a < x < 3a)$: $V = -P + 2P - P$

Using singularity functions,

$$\frac{dM}{dx} = V = -P + 2P\langle x-a\rangle^0 - P\langle x-2a\rangle^0$$

$$M = -Px + 2P\langle x-a\rangle^1 - P\langle x-2a\rangle^1 + M_A$$

But $M = 0$ at $x = 0$ $\quad M_A = 0$

$$EI\frac{d^2y}{dx^2} = M = -Px + 2P\langle x-a\rangle^1 - P\langle x-2a\rangle^1 \tag{1}$$

$$EI\frac{dy}{dx} = -\frac{1}{2}Px^2 + P\langle x-a\rangle^2 - \frac{1}{2}P\langle x-2a\rangle^2 + C_1 \tag{2}$$

$$EIy = -\frac{1}{6}Px^3 + \frac{1}{3}P\langle x-a\rangle^3 - \frac{1}{6}P\langle x-2a\rangle^3 + C_1x + C_2 \tag{3}$$

$[x = a,\ y = 0]$ $\quad -\frac{1}{6}Pa^3 + 0 - 0 + C_1a + C_2 = 0 \qquad aC_1 + C_2 = \frac{1}{6}Pa^3 \quad (4)$

$[x = 3a,\ y = 0]$ $\quad -\frac{1}{6}P(3a)^3 + \frac{1}{3}P(2a)^3 - \frac{1}{6}Pa^3 + C_1(3a) + C_2 = 0 \qquad 3aC_1 + C_2 = 2Pa^2 \quad (5)$

Eq (5) – Eq (4) $\quad 2C_1a + \frac{11}{6}Pa^2 \qquad C_1 + \frac{11}{12}Pa^2$

$$C_2 = \frac{1}{6}Pa^2 - aC_1 = -\frac{3}{4}Pa^3$$

$$y = \frac{P}{EI}\left\{-\frac{1}{6}x^3 + \frac{1}{3}\langle x-a\rangle^3 - \frac{1}{6}\langle x-2a\rangle^3 + \frac{11}{12}a^2x - \frac{3}{4}a^3\right\}$$

$$\frac{dy}{dx} = \frac{P}{EI}\left\{-\frac{1}{2}x^2 + \langle x-a\rangle^2 - \frac{1}{2}\langle x-2a\rangle^2 + \frac{11}{12}a^2\right\}$$

PROBLEM 9.40 *(Continued)*

(*a*) Deflection at *A*: (y at $x = 0$)

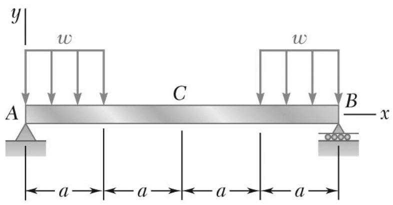
$$y_A = \frac{Pa^3}{EI}\left\{-0 + 0 - 0 + 0 - \frac{3}{4}\right\} = -\frac{3}{4}\frac{Pa^3}{EI} \qquad y_A = \frac{3}{4}\frac{Pa^3}{EI} \downarrow \blacktriangleleft$$

(*b*) Deflection at *C*: (y at $x = 2a$)

$$y_C = \frac{Pa^3}{EI}\left\{-\frac{1}{6}(2)^3 + \frac{1}{3}(1)^3 - 0 + \frac{11}{12}(2) - \frac{3}{4}\right\} \qquad y_C = \frac{1}{12}\frac{Pa^3}{EI} \uparrow \blacktriangleleft$$

(*c*) Slope at D: $\left(\dfrac{dy}{dx} \text{ at } x = 3a\right)$

$$\theta_D = \frac{Pa^2}{EI}\left\{-\frac{1}{2}(3)^2 + (2)^2 - \frac{1}{2}(1)^2 + \frac{11}{12}\right\} = -\frac{1}{12}\frac{Pa^2}{EI} \qquad \theta_D = \frac{1}{12}\frac{Pa^2}{EI} \blacktriangleleft$$



PROBLEM 9.41

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the midpoint *C*.

SOLUTION

By symmetry,

$$R_A = R_B = wa$$

$$w(x) = w - w\langle x-a\rangle^0 + w\langle x-3a\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w + w\langle x-a\rangle^0 - w\langle x-3a\rangle^0$$

$$\frac{dM}{dx} = V = R_A - wx + w\langle x-a\rangle^1 - w\langle x-3a\rangle^1$$

$$M = M_A + R_A x - \frac{1}{2}wx^2 + \frac{1}{2}w\langle x-a\rangle^2 - \frac{1}{2}w\langle x-3a\rangle^2 \quad (\text{with } M_A = 0)$$

$$EI\frac{d^2y}{dx^2} = M = wax - \frac{1}{2}wx^2 + \frac{1}{2}w\langle x-a\rangle^2 - \frac{1}{2}w\langle x-3a\rangle^2$$

$$EI\frac{dy}{dx} = \frac{1}{2}wax^2 - \frac{1}{6}wx^3 + \frac{1}{6}w\langle x-a\rangle^3 - \frac{1}{6}w\langle x-3a\rangle^3 + C_1$$

$$EIy = \frac{1}{6}wax^3 - \frac{1}{24}wx^4 + \frac{1}{24}w\langle x-a\rangle^4 - \frac{1}{24}w\langle x-3a\rangle^4 + C_1x + C_2$$

$$[x = 0,\ y = 0]: \quad 0 - 0 + 0 - 0 + 0 + C_2 = 0 \quad \therefore \quad C_2 = 0$$

$$[x = 4a,\ y = 0]: \quad \frac{1}{6}wa(4a)^3 - \frac{1}{24}w(4a)^4 + \frac{1}{24}w(3a)^4 - \frac{1}{24}w(a)^4 + C_1(4a) = 0$$

$$\therefore \quad C_1 = -\frac{5}{6}wa^3$$

(*a*) Equation of elastic curve:

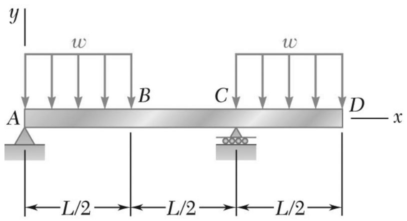
$$y = \frac{w}{EI}\left[\frac{1}{6}ax^3 - \frac{1}{24}x^4 + \frac{1}{24}\langle x-a\rangle^4 - \frac{1}{24}\langle x-3a\rangle^4 - \frac{5}{6}a^3x\right] \blacktriangleleft$$

(*b*) Deflection at *C*: $(y \text{ at } x = 2a)$

$$y_C = \frac{wa^4}{EI}\left[\frac{1}{6}(2)^3 - \frac{1}{24}(2)^4 + \frac{1}{24}(1)^4 - 0 - \frac{5}{6}(2)\right]$$

$$= -\frac{23}{24}\frac{wa^4}{EI}$$

$$y_C = \frac{23}{24}\frac{wa^4}{EI} \downarrow \blacktriangleleft$$



PROBLEM 9.42

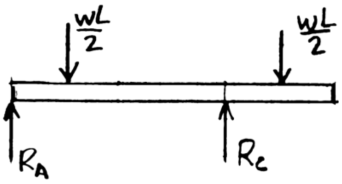
For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at point *B*, (*c*) the deflection at point *D*.

SOLUTION

Use free body *ABCD* with the distributed loads replaced by equivalent concentrated loads.

$$+\circlearrowleft \Sigma M_C = 0: \quad -R_A L + \left(\frac{wL}{2}\right)\left(\frac{3L}{4}\right) - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) = 0$$

$$R_A = \frac{1}{4}wL$$



$$+\circlearrowleft \Sigma M_A = 0: \quad R_C L - \left(\frac{wL}{2}\right)\left(\frac{L}{4}\right) - \left(\frac{wL}{2}\right)\left(\frac{5L}{4}\right) = 0$$

$$R_C = \frac{3}{4}wL$$

$$\frac{dV}{dx} = -w = -w + w\left\langle x - \frac{L}{2}\right\rangle^0 - w\langle x - L\rangle^0$$

Integrating and adding terms to account for the reactions,

$$\frac{dM}{dx} = V = -wx + w\left\langle x - \frac{L}{2}\right\rangle^1 - w\langle x - L\rangle^1 + R_A + R_C\langle x - L\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2 + \frac{1}{2}w\left\langle x - \frac{L}{2}\right\rangle^2 - \frac{1}{2}w\langle x - L\rangle^2 + R_A x + R_C\langle x - L\rangle^1$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}w\left\langle x - \frac{L}{2}\right\rangle^3 - \frac{1}{6}w\langle x - L\rangle^3 + \frac{1}{2}R_A x^2 + \frac{1}{2}R_C\langle x - L\rangle^2 + C_1$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4 - \frac{1}{24}w\langle x - L\rangle^4 + \frac{1}{6}R_A x^3 + \frac{1}{6}R_C\langle x - L\rangle^3 + C_1 x + C_2$$

$[x = 0,\ y = 0]$ $\quad -0 + 0 - 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x = L,\ y = 0]$ $\quad -\cancel{\frac{1}{24}wL^4} + \frac{1}{24}w\left(\frac{L}{2}\right)^4$

$$-0 + \cancel{\frac{1}{6}\left(\frac{wL}{4}\right)}L^3 + 0 + C_1 L + 0 = 0 \qquad C_1 = -\frac{1}{384}wL^3$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4 + \frac{1}{24}w\langle x - L\rangle^4$$

$$+\frac{1}{6}\left(\frac{wL}{4}\right)x^3 + \frac{1}{6}\left(\frac{3wL}{4}\right)\langle x - L\rangle^3 - \frac{1}{384}wL^3 x$$

PROBLEM 9.42 *(Continued)*

(*a*) Elastic curve:

$$y = \frac{w}{24EI}\left\{-x^4 + \left\langle x - \frac{L}{2}\right\rangle^4 - \langle x - L\rangle^4 + Lx^3 + 3L\langle x - L\rangle^3 - \frac{1}{16}L^3x\right\} \blacktriangleleft$$

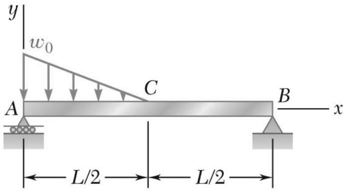
(*b*) Deflection at *B*: $\left(y \text{ at } x = \frac{L}{2}\right)$

$$y_B = \frac{w}{24EI}\left\{-\left(\frac{L}{2}\right)^4 + 0 - 0 + (L)\left(\frac{L}{2}\right)^3 + 0 - \left(\frac{1}{16}L^3\right)\left(\frac{L}{2}\right)\right\} \qquad y_B = \frac{wL^4}{768EI} \uparrow \blacktriangleleft$$

(*c*) Deflection at *D*: $\left(y \text{ at } x = \frac{3L}{2}\right)$

$$y_D = \frac{w}{24EI}\left\{-\left(\frac{3L}{2}\right)^4 + L^4 - \left(\frac{L}{2}\right)^4 + (L)\left(\frac{3L}{2}\right)^3 + (3L)\left(\frac{L}{2}\right)^3 - \left(\frac{1}{16}L\right)\left(\frac{3L}{2}\right)\right\}$$

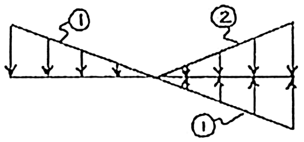
$$= -\frac{5wL^4}{256EI} \qquad y_D = \frac{5wL^4}{256EI} \downarrow \blacktriangleleft$$



PROBLEM 9.43

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the midpoint *C*.

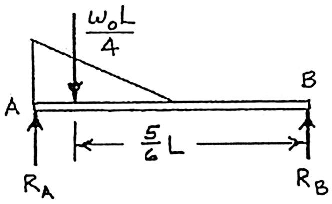
SOLUTION



Distributed loads: $k = \dfrac{2w_0}{L}$

$$(1) \quad w_1(x) = w_0 - kx \qquad (2) \quad w_2(x) = k\left\langle x - \frac{L}{2}\right\rangle^1$$

$$+\circlearrowleft \Sigma M_B = 0: \quad \left(\frac{w_0 L}{4}\right)\left(\frac{5}{6}L\right) - R_A L = 0 \qquad R_A = \frac{5}{24}w_0 L \uparrow$$



$$w(x) = w_0 - kx + k\left\langle x - \frac{L}{2}\right\rangle^1 = w_0 - \frac{2w_0}{L}x + \frac{2w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^1$$

$$\frac{dV}{dx} = -w = -w_0 + \frac{2w_0}{L}x - \frac{2w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^1$$

$$\frac{dM}{dx} = V = \frac{5}{24}w_0 L - w_0 x + \frac{w_0}{L}x^2 - \frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^2$$

$$EI\frac{d^2y}{dx^2} = M = \frac{5}{24}w_0 Lx - \frac{1}{2}w_0 x^2 + \frac{1}{3}\frac{w_0}{L}x^3 - \frac{1}{3}\frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^3$$

$$EI\frac{dy}{dx} = \frac{5}{48}w_0 Lx^2 - \frac{1}{6}w_0 x^3 + \frac{1}{12}\frac{w_0}{L}x^4 - \frac{1}{12}\frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^4 + C_1$$

$$EIy = \frac{5}{144}w_0 Lx^3 - \frac{1}{24}w_0 x^4 + \frac{1}{60}\frac{w_0}{L}x^5 - \frac{1}{60}\frac{w_0}{L}\left\langle x - \frac{L}{2}\right\rangle^5 + C_1 x + C_2$$

$$[x = 0,\ y = 0]: \qquad C_2 = 0$$

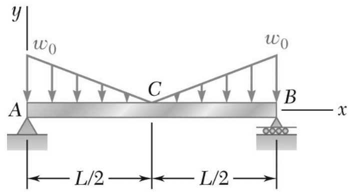
$$[x = L,\ y = 0]: \quad \frac{5}{144}w_0 L^4 - \frac{1}{24}w_0 L^4 + \frac{1}{60}w_0 L^4 - \frac{1}{60}\frac{w_0}{L}\left(\frac{L}{2}\right)^5 + C_1 L = 0 \qquad C_1 = -\frac{53}{5760}w_0 L^3$$

(*a*) Equation of elastic curve:

$$y = w_0\left[96x^5 - 96\left\langle x - \frac{L}{2}\right\rangle^5 - 240Lx^4 + 200L^2x^3 - 53L^4x\right]/5760\,EIL \blacktriangleleft$$

(*b*) Deflection at *C*: $\left(y \quad \text{at} \quad x = \dfrac{L}{2}\right)$

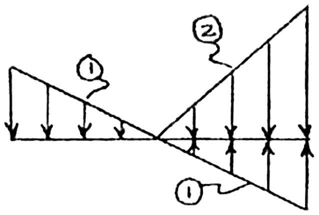
$$y_C = \frac{w_0 L^4}{5760EI}\left(\frac{96}{32} - 0 - \frac{240}{16} + \frac{200}{8} - \frac{53}{2}\right) = -\frac{3w_0 L^4}{1280EI} \qquad y_C = \frac{3w_0 L^4}{1280EI} \downarrow \blacktriangleleft$$

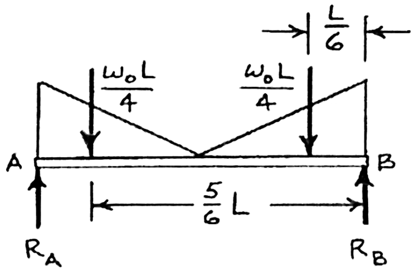


PROBLEM 9.44

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the deflection at the midpoint *C*.

SOLUTION





Distributed loads: $k_1 = \dfrac{2w_0}{L} \qquad k_2 = \dfrac{4w_0}{L}$

$$(1) \qquad w_1(x) = w_0 - k_1 x$$

$$(2) \qquad w_2(x) = k_2 \left\langle x - \frac{L}{2} \right\rangle^1$$

$$+\circlearrowleft \Sigma M_B = 0: \quad \left(\frac{w_0 L}{4}\right)\left(\frac{5}{6}L\right) + \left(\frac{w_0 L}{4}\right)\left(\frac{L}{6}\right) + R_A L = 0$$

$$R_A = \frac{w_0 L}{4} \uparrow$$

$$w(x) = w_0 - k_1 x + k_2 \left\langle x - \frac{L}{2} \right\rangle^1$$

$$= w_0 - \frac{2w_0}{L}x + \frac{4w_0}{L}\left\langle x - \frac{L}{2} \right\rangle^1$$

$$\frac{dV}{dx} = -w = -w_0 + \frac{2w_0}{L}x - \frac{4w_0}{L}\left\langle x - \frac{L}{2} \right\rangle^1$$

$$\frac{dM}{dx} = V = \frac{w_0 L}{4} - w_0 x + \frac{w_0}{L}x^2 - \frac{2w_0}{L}\left\langle x - \frac{L}{2} \right\rangle^2$$

$$EI\frac{d^2y}{dx^2} = M = \frac{1}{4}w_0 L x - \frac{1}{2}w_0 x^2 + \frac{1}{3}\frac{w_0}{L}x^3 - \frac{2}{3}\frac{w_0}{L}\left\langle x - \frac{L}{2} \right\rangle^3$$

$$EI\frac{dy}{dx} = \frac{1}{8}w_0 L x^2 - \frac{1}{6}w_0 x^3 + \frac{1}{12}\frac{w_0}{L}x^4 - \frac{1}{6}\frac{w_0}{L}\left\langle x - \frac{L}{2} \right\rangle^4 + C_1$$

$$EIy = \frac{1}{24}w_0 L x^3 - \frac{1}{24}w_0 x^4 + \frac{1}{60}\frac{w_0}{L}x^5 - \frac{1}{30}\frac{w_0}{L}\left\langle x - \frac{L}{2} \right\rangle^5 + C_1 x + C_2$$

$$[x = 0,\ y = 0]: \qquad C_2 = 0$$

$$[x = L,\ y = 0]: \quad \frac{1}{24}w_0 L^4 - \frac{1}{24}w_0 L^4 + \frac{1}{60}w_0 L^4 - \frac{1}{30}\frac{w_0}{L}\left(\frac{L}{2}\right)^5 + C_1 L = 0$$

$$C_1 = -\frac{1}{64}w_0 L^3$$

PROBLEM 9.44 *(Continued)*

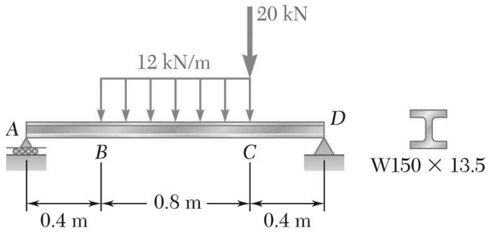
(*a*) Equation of elastic curve:

$$y = w_0\left[16x^5 - 32\left\langle x - \frac{L}{2}\right\rangle^5 - 40Lx^4 + 40L^2x^3 - 15L^4x\right]\Big/ 960\,EIL \blacktriangleleft$$

(*b*) Deflection at C: $\left(y \quad \text{at} \quad x = \frac{L}{2}\right)$

$$y_C = \frac{w_0L^4}{960EI}\left(\frac{1}{2} - 0 - \frac{5}{2} + 5 - \frac{15}{2}\right) = -\frac{3w_0L^4}{640EI}$$

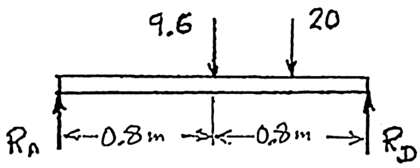
$$y_C = \frac{3w_0L^4}{640EI} \downarrow \blacktriangleleft$$



PROBLEM 9.45

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION



Units: Forces in kN, lengths in m

$+\circlearrowleft M_D = 0:$

$-1.6\,R_A + (9.6)(0.8) + (20)(0.4) = 0$

$R_A = 9.8$ kN

$$w(x) = 12\langle x - 0.4\rangle^0 - 12\langle x - 1.2\rangle^0 \text{ kN/m}$$

$$\frac{dV}{dx} = -w(x) = -12\langle x - 0.4\rangle^0 + 12\langle x - 1.2\rangle^0 \text{ kN/m}$$

$$\frac{dM}{dx} = V = 9.8 - 12\langle x - 0.4\rangle^1 + 12\langle x - 1.2\rangle^1 - 20\langle x - 1.2\rangle^0 \qquad \text{kN}$$

$$EI\frac{d^2y}{dx^2} = M = 9.8x - 6\langle x - 0.4\rangle^2 + 6\langle x - 1.2\rangle^2 - 20\langle x - 1.2\rangle^1 \qquad \text{kN}\cdot\text{m}$$

$$EI\frac{dy}{dx} = 4.9x^2 - 2\langle x - 0.4\rangle^3 + 2\langle x - 1.2\rangle^3 - 10\langle x - 1.2\rangle^2 + C_1 \qquad \text{kN}\cdot\text{m}^2$$

$$EIy = 1.63333x^3 - \frac{1}{2}\langle x - 0.4\rangle^4 + \frac{1}{2}\langle x - 1.2\rangle^4 - \frac{10}{3}\langle x - 1.2\rangle^3 + C_1x + C_2 \quad \text{kN}\cdot\text{m}^3$$

$$[x = 0,\ y = 0]:\ 0 - 0 + 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = 1.6,\ y = 0]:\ (1.63333)(1.6)^3 - \frac{1}{2}(1.2)^4 + \frac{1}{2}(0.4)^4 - \frac{10}{3}(0.4)^3 + C_1(1.6) + 0 = 0$$

$$C_1 = -3.4080 \text{ kN}\cdot\text{m}^2$$

Data: $E = 200\times 10^9$ Pa, $I = 6.83\times 10^6 \text{ mm}^4 = 6.83\times 10^{-6} \text{ mm}^4$

$$EI = (200\times 10^4)(6.83\times 10^{-6}) = 1.366\times 10^6 \text{ N}\cdot\text{m}^2 = 1366 \text{ kN}\cdot\text{m}^2$$

(*a*) Slope at *A*: $\left(\dfrac{dy}{dx} \quad \text{at} \quad x = 0\right)$

$$EI\frac{dy}{dx} = 0 - 0 + 0 - 0 - 3.4080 \quad \text{kN}\cdot\text{m}^2$$

$$\theta_A = -\frac{3.4080}{1366} = -2.49\times 10^{-3} \text{ rad} \qquad \theta_A = 2.49\times 10^{-3} \text{ rad} \blacktriangleleft$$

PROBLEM 9.45 *(Continued)*

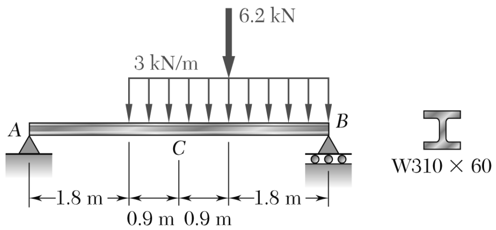
(b) <u>Deflection at C</u>: (y at $x = 1.2$ m)

$$EIy_C = (1.63333)(1.2)^3 - \frac{1}{2}(0.8)^4 + 0 - 0 - (3.4080)(1.2) + 0$$

$$= -1.4720 \text{ kN} \cdot \text{m}^3$$

$$y_C = -\frac{1.4720}{1366} = -1.078 \times 10^{-3} \text{m}$$

$y_C = 1.078$ mm ↓ ◀



PROBLEM 9.46

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN, lengths in meters.

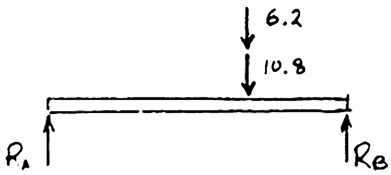
$$+\circlearrowleft \Sigma M_B = 0: \quad -5.4R_A - (1.8)(6.2 + 10.8) = 0$$

$$R_A = 5.6667 \text{ kN}$$

$$w(x) = 3\langle x - 1.8\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -3\langle x - 1.8\rangle^0$$

$$\frac{dM}{dx} = V = 5.6667 - 3\langle x - 1.8\rangle^1 - 6.2\langle x - 3.6\rangle^0$$



$$EI\frac{d^2y}{dx^2} = M = 5.6667x - \frac{3}{2}\langle x - 1.8\rangle^2 - 6.2\langle x - 3.6\rangle^1 \qquad \text{kN} \cdot \text{m}$$

$$EI\frac{dy}{dx} = 2.8333x^2 - \frac{1}{2}\langle x - 1.8\rangle^3 - 3.1\langle x - 3.6\rangle^2 + C_1 \qquad \text{kN} \cdot \text{m}^2$$

$$EIy = 0.9444x^3 - \frac{1}{8}\langle x - 1.8\rangle^4 - 1.0333\langle x - 3.6\rangle^3 + C_1x + C_2 \qquad \text{kN} \cdot \text{m}^3$$

$$[x = 0,\ y = 0]: \quad 0 - 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = 5.4,\ y = 0]: \quad (0.9444)(5.4)^3 - \frac{1}{8}(3.6)^4 - 1.0333(1.8)^3 + C_1(5.4) + 0 = 0$$

$$C_1 = -22.535 \text{ kN} \cdot \text{m}^2$$

Data: $E = 200 \times 10^9$ Pa, $I = 129 \times 10^6 \text{ mm}^4 = 129 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^9)(129 \times 10^{-6}) = 25.8 \times 10^6 \text{ N} \cdot \text{m}^2 = 25.8 \times 10^3 \text{ kN} \cdot \text{m}^2$$

(*a*) Slope at *A*: $\left(\dfrac{dy}{dx} \text{ at } x = 0\right)$

$$EI\frac{dy}{dx} = 0 - 0 - 0 - 22.535 \text{ kN} \cdot \text{m}^2$$

$$\theta_A = -\frac{22.535}{25.8 \times 10^3} = -873 \times 10^{-6}$$

$$\theta_A = 0.873 \times 10^{-3} \text{ rad} \ ◀$$

PROBLEM 9.46 *(Continued)*

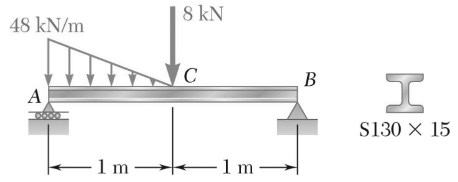
(*b*) Deflection at *C*: (y at $x = 2.7$ m)

$$EIy_C = (0.9444)(2.7)^3 - \frac{1}{8}(0.9)^4 - 0 - (22.585)(2.7) + 0$$

$$= -42.337 \text{ kN} \cdot \text{m}^3$$

$$y_C = -\frac{42.337}{25.8 \times 10^3} = -1.641 \times 10^{-3} \text{ m}$$

$y_C = 1.641 \text{ mm} \downarrow$ ◀

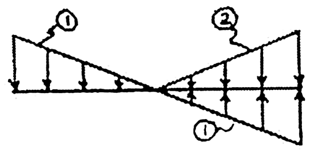


PROBLEM 9.47

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 200$ GPa.

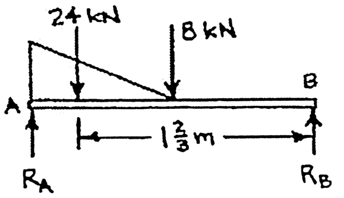
SOLUTION

Distributed loads: (1) $w_1(x) = w_0 - kx$ (2) $w_2 = k\langle x-1\rangle^1$



$$w_0 = 48 \text{ kN/m}, \quad k = 48 \text{ kN/m}^2$$

$$+\circlearrowleft \Sigma M_B = 0: -2R_A + (24)\left(1\frac{2}{3}\right) + (8)(1) = 0 \qquad R_A = 24 \text{ kN} \uparrow$$



$$w(x) = w_0 - kx + k\langle x-1\rangle^1 = 48 - 48x + 48\langle x-1\rangle^1$$

$$\frac{dV}{dx} = -w = -48 + 48x - 48\langle x-1\rangle^1 \quad \text{kN/m}$$

$$\frac{dM}{dx} = V = 24 - 48x + 24x^2 - 24\langle x-1\rangle^2 - 8\langle x-1\rangle^0 \quad \text{kN}$$

$$EI\frac{d^2y}{dx^2} = M = 24x - 24x^2 + 8x^3 - 8\langle x-1\rangle^3 - 8\langle x-1\rangle^1 \quad \text{kN}\cdot\text{m}$$

$$EI\frac{dy}{dx} = 12x^2 - 8x^3 + 2x^4 - 2\langle x-1\rangle^4 - 4\langle x-1\rangle^2 + C_1 \quad \text{kN}\cdot\text{m}^2$$

$$EIy = 4x^3 - 2x^4 + \frac{2}{5}x^5 - \frac{2}{5}\langle x-1\rangle^5 - \frac{4}{3}\langle x-1\rangle^3 + C_1x + C_2 \quad \text{kN}\cdot\text{m}^3$$

$$[x = 0,\ y = 0]:\ 0 - 0 + 0 - 0 - 0 + 0 + C_2 = 0 \qquad \therefore\ C_2 = 0$$

$$[x = 2,\ y = 0]:\ 4(2)^3 - 2(2)^4 + \frac{2}{5}(2)^5 - \frac{2}{5}(1)^5 - \frac{4}{3}(1)^3 + C_1(2) = 0 \quad \therefore\ C_1 = -\frac{83}{15} \text{ kN}\cdot\text{m}^2$$

Data: $E = 200(10^6) \text{ kN/m}^2 \quad I = 5.12(10^6) \text{ mm}^4 = 5.12(10^{-6}) \text{ m}^4$

$$EI = (200\times 10^6)(5.12\times 10^{-6}) = 1024 \text{ kN}\cdot\text{m}^2$$

(*a*) Slope at *A*: $\left(\dfrac{dy}{dx} \quad \text{at} \quad x=0\right)$

$$EI\theta_A = 0 - 0 + 0 - 0 - 0 - \frac{83}{15} \text{ kN}\cdot\text{m}^2$$

$$\theta_A = -\frac{83/15}{1024} = -5.4036\times 10^{-3} \text{ rad} \qquad \theta_A = -5.40\times 10^{-3} \text{ rad} \ ◀$$

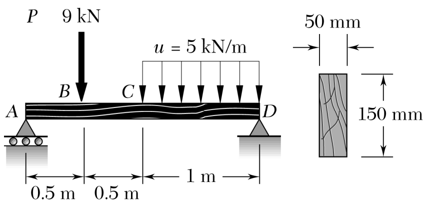
PROBLEM 9.47 *(Continued)*

(*b*) Deflection at *C*: (y at $x = 1\text{ m}$)

$$EIy_C = 4(1)^3 - 2(1)^4 + \frac{2}{5}(1)^4 - 0 - 0 - \frac{83}{15}(1) = -3.1333\text{ kN}\cdot\text{m}^3$$

$$y_C = -\frac{3.1333}{1024} = -3.0599\times 10^{-3}\text{ m}$$

$y_C = 3.06\text{ mm}\downarrow$ ◀



PROBLEM 9.48

For the timber beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $F = 12$ GPa.

SOLUTION

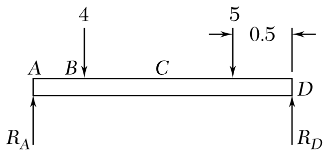
<u>Units</u>: Forces in kN, lengths in meters.

$$I = \frac{1}{12}(50)(150)^3 = 14.0625\times 10^6\ \text{mm}^4$$

$$= 14.0625\times 10^{-6}\ \text{m}^4$$

$$EI = (12\times 10^9)(14.0625\times 10^{-6})$$

$$= 168.75\times 10^3\ \text{N}\cdot\text{m}^2 = 168.75\ \text{kN}\cdot\text{m}^2$$



$$+\circlearrowleft M_D = 0:\quad -2R_A + (1.5)(4) + (0.5)(5) = 0$$

$$R_A = 4.25\ \text{kN}$$

$$w(x) = 5\langle x-1\rangle^0 \qquad \text{kN}\cdot\text{m}$$

$$\frac{dV}{dx} = -w = -5\langle x-1\rangle^0\ \text{kN/m}$$

$$\frac{dM}{dx} = V = -5\langle x-1\rangle^1 + 4.25 - 4\langle x-0.5\rangle^0 \qquad \text{kN}$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{5}{2}\langle x-1\rangle^2 + 4.25x - 4\langle x-0.5\rangle^1 \qquad \text{kN}\cdot\text{m}$$

$$EI\frac{dy}{dx} = -\frac{5}{6}\langle x-1\rangle^3 + 2.125x^2 - 2\langle x-0.5\rangle^2 + C_1 \qquad \text{kN}\cdot\text{m}^2$$

$$EIy = -\frac{5}{24}\langle x-1\rangle^4 + \frac{2.125}{3}x^3 - \frac{2}{3}\langle x-0.5\rangle^3 + C_1x + C_2 \qquad \text{kN}\cdot\text{m}^3$$

$[x = 0,\ y = 0]$ $\quad -0 + 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x = 2m,\ y = 0]$ $\quad -\left(\frac{5}{24}\right)(1)^4 + \left(\frac{2.125}{3}\right)(2)^3 - \left(\frac{2}{3}\right)(1.5)^3 + 2C_1 = 0$

$$C_1 = -1.60417\ \text{kN}\cdot\text{m}^2$$

(*a*) <u>Slope at end *A*</u>: $\left(\dfrac{dy}{dx}\ \text{at}\ x = 0\right)$

$$EI\left(\frac{dy}{dx}\right)_A = -0 + 0 - 0 + C_1$$

$$\left(\frac{dy}{dx}\right)_A = \frac{C_1}{EI} = \frac{-1.60417}{168.75} = -9.51\times 10^{-3} \qquad \theta_A = 9.51\times 10^{-3}\ \text{rad} \blacktriangleleft$$

PROBLEM 9.48 *(Continued)*

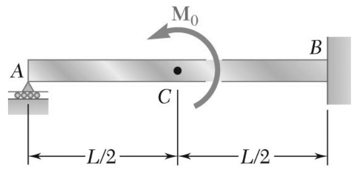
(*b*) Deflection at midpoint *C*: (y at $x = 1\,\text{m}$)

$$EIy_C = -0 + \left(\frac{2.125}{3}\right)(1)^3 - \left(\frac{2}{3}\right)(0.5)^3 + (-1.60417)(1)$$

$$= -979.17 \times 10^{-3}\,\text{kN} \cdot \text{m}^3$$

$$y_C = \frac{-979.17 \times 10^{-3}}{168.75} = -5.80 \times 10^{-3}\,\text{m}$$

$y_C = 5.80\,\text{mm}\downarrow$ ◀



$[x = 0,\ y = 0]$ $\quad \left[x = L,\ \dfrac{dy}{dx} = 0\right]$

$[x = L,\ y = 0]$

PROBLEM 9.49

For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *C*.

SOLUTION

For $\quad 0 \le x \le \dfrac{1}{2}, \qquad M = R_A x$

For $\quad \dfrac{L}{2} \le x \le L, \qquad M = R_A x - M_0$

Then

$$EI\frac{d^2y}{dx^2} = M = R_A x - M_0\left\langle x - \frac{L}{2}\right\rangle^0$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - M_0\left\langle x - \frac{L}{2}\right\rangle^1 + C_1$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{2}M_0\left\langle x - \frac{L}{2}\right\rangle^2 + C_1 x + C_2$$

$$[x = 0,\ y = 0] \quad 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right] \quad \frac{1}{2}R_A L^2 - M_0\left(\frac{L}{2}\right) + C_1 = 0 \qquad C_1 = \frac{1}{2}(M_0 L - R_A L^2)$$

$$[x = L,\ y = 0] \quad \frac{1}{6}R_A L^3 - \frac{1}{2}M_0\left(\frac{L}{2}\right)^2 + \frac{1}{2}(M_0 L - R_A L^2)L + 0 = 0$$

$$-\frac{1}{3}R_A L^3 + \frac{3}{8}M_0 L^2 = 0$$

(*a*) Reaction at A: $\qquad M_A = 0, \qquad R_A = \dfrac{9M_0}{8L} \uparrow$ ◀

$$C_1 = \frac{1}{2}\left[M_0 L - \left(\frac{9M_0}{8L}\right)(L^2)\right] = -\frac{1}{16}M_0 L$$

$$EIy = \frac{1}{6}\left(\frac{9M_0}{8L}\right)x^3 - \frac{1}{2}M_0\left\langle x - \frac{L}{2}\right\rangle^2 - \frac{1}{16}M_0 L x + 0$$

PROBLEM 9.49 *(Continued)*

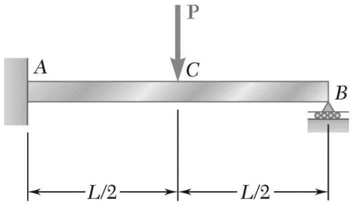
Elastic curve:
$$y = \frac{M_0}{EIL}\left\{\frac{9}{8}x^3 - \frac{1}{2}L\left\langle x - \frac{L}{2}\right\rangle^2 - \frac{1}{16}L^2 x\right\}$$

(*b*) Deflection at point *C*:
$$\left(y \text{ at } x = \frac{L}{2}\right)$$

$$y_C = \frac{M_0}{EIL}\left\{\left(\frac{1}{6}\right)\left(\frac{9}{8}\right)\left(\frac{L}{2}\right)^3 - 0 - \left(\frac{1}{16}L^2\right)\left(\frac{L}{2}\right)\right\}$$

$$= -\frac{M_0L^2}{128EI}$$

$$y_C = \frac{M_0L^2}{128\,EI} \downarrow \blacktriangleleft$$



$[x = 0,\ y = 0]$ $\quad$ $[x = L,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

PROBLEM 9.50

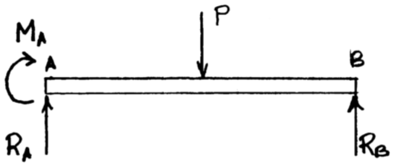
For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *C*.

SOLUTION

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - P = 0 \qquad R_A = P - R_B$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A - P\frac{L}{2} + R_B L = 0 \qquad M_A = R_B L - \frac{1}{2}PL$$

Reactions are statically indeterminate.



$$\frac{dM}{dx} = V = R_A - P\left\langle x - \frac{L}{2}\right\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\left\langle x - \frac{L}{2}\right\rangle^1$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P\left\langle x - \frac{L}{2}\right\rangle^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\left\langle x - \frac{L}{2}\right\rangle^3 + C_1 x + C_2$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \quad 0 + 0 + 0 + C_1 = 0 \qquad C_1 = 0$$

$$[x = 0,\ y = 0] \quad 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = L,\ y = 0] \quad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{6}P\left(\frac{L}{2}\right)^3 + 0 + 0 = 0$$

$$\frac{1}{2}\left(R_B L - \frac{1}{2}PL\right)L^2 + \frac{1}{6}(P - R_B)L^3 - \frac{1}{48}PL^3 = 0$$

$$\left(\frac{1}{2} - \frac{1}{6}\right)R_B L^3 = \left(\frac{1}{4} - \frac{1}{6} + \frac{1}{48}\right)PL^3 \qquad \frac{1}{3}R_B = \frac{5}{48}P \qquad (a)\ R_B = \frac{5}{16}P\uparrow \blacktriangleleft$$

$$R_A = P - \frac{5}{16}P = \frac{11}{16}P$$

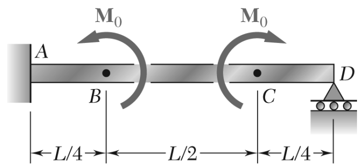
$$M_A = \frac{5}{16}PL - \frac{1}{2}PL = -\frac{3}{16}PL$$

PROBLEM 9.50 *(Continued)*

(*b*) Deflection at C: $\left(y \text{ at } x = \dfrac{L}{2}\right)$

$$y_C = \frac{1}{EI}\left\{\frac{1}{2}M_A\left(\frac{L}{2}\right)^2 + \frac{1}{6}R_A\left(\frac{L}{2}\right)^3 + 0 + 0 + 0\right\}$$

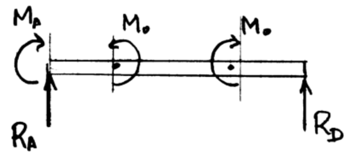
$$= \frac{PL^3}{EI}\left\{\left(\frac{1}{2}\right)\left(-\frac{3}{16}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{6}\right)\left(\frac{11}{16}\right)\left(\frac{1}{8}\right)\right\} = -\frac{7}{168}\frac{PL^3}{EI} \qquad y_C = \frac{7}{168}\frac{PL^3}{EI} \downarrow \blacktriangleleft$$



PROBLEM 9.51

For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *B*.

SOLUTION



$$+\uparrow \Sigma F_y = 0: \quad R_A + R_D = 0 \qquad R_A = -R_D$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -M_A + M_0 - M_0 + RL = 0$$

$$M_A = R_D L$$

$$M(x) = M_A + R_A x - M_0 \left\langle x - \frac{L}{4} \right\rangle^0 + M_0 \left\langle x - \frac{3L}{4} \right\rangle^0$$

$$EI\frac{d^2y}{dx^2} = R_D L - R_D x - M_0 \left\langle x - \frac{L}{4} \right\rangle^0 + M_0 \left\langle x - \frac{3L}{4} \right\rangle^0$$

$$EI\frac{dy}{dx} = R_D L x - \frac{1}{2} R_D x^2 - M_0 \left\langle x - \frac{L}{4} \right\rangle^1 + M_0 \left\langle x - \frac{3L}{4} \right\rangle^1 + C_1$$

$$\left[x = 0, \frac{dy}{dx} = 0 \right] \quad 0 - 0 - 0 + 0 + C_1 = 0$$

$$C_1 = 0$$

$$EIy = \frac{1}{2} R_D L x^2 - \frac{1}{6} R_D x^3 - \frac{1}{2} M_0 \left\langle x - \frac{L}{4} \right\rangle^1 + \frac{1}{2} M_0 \left\langle x - \frac{3L}{4} \right\rangle^1 + C_2$$

$$[x = 0,\ y = 0] \quad 0 - 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = L,\ y = 0] \quad \frac{1}{2} R_D L^3 - \frac{1}{6} R_D L^3 - \frac{1}{2} M_0 \left(\frac{3L}{4} \right)^2 + \frac{1}{2} M_0 \left(\frac{L}{4} \right)^2 = 0$$

(*a*) Reaction at *D*:

$$R_D = \frac{3M_0}{4L} \uparrow \blacktriangleleft$$

$$EIy = \frac{1}{2}\left(\frac{3M_0}{4L} \right) L x^2 - \frac{1}{6}\left(\frac{3M_0}{4L} \right) x^3 - \frac{1}{2} M_0 \left\langle x - \frac{L}{4} \right\rangle^2 + \frac{1}{2} M_0 \left\langle x - \frac{3L}{4} \right\rangle^2$$

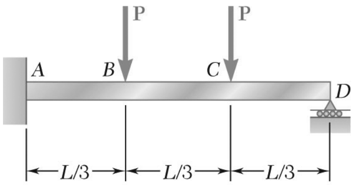
Elastic curve:

$$y = \frac{M_0}{EIL} \left\{ \frac{3}{8} L x^2 - \frac{1}{8} x^3 - \frac{1}{2} L \left\langle x - \frac{L}{4} \right\rangle^2 + \frac{1}{2} L \left\langle x - \frac{3L}{8} \right\rangle^2 \right\}$$

(*b*) Deflection at point *B*: $\left(y \text{ at } x = \frac{L}{4} \right)$

$$y_B = \frac{M_0}{EIL} \left\{ \left(\frac{3}{8} L \right) \left(\frac{L}{4} \right)^2 - \left(\frac{1}{8} \right) \left(\frac{L}{4} \right)^3 - 0 + 0 \right\}$$

$$y_B = \frac{11 M_0 L^2}{512 EI} \uparrow \blacktriangleleft$$

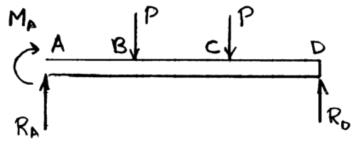


PROBLEM 9.52

For the beam and loading shown, determine (*a*) the reaction at the roller support, (*b*) the deflection at point *B*.

SOLUTION

$$+\uparrow \Sigma F_y = 0: \qquad R_A - P - P + R_D = 0 \qquad R_A = 2P - R_D$$



$$+\circlearrowleft \Sigma M_A = 0: \qquad -M_A - \frac{PL}{3} - \frac{2PL}{3} + R_D L = 0$$

$$M_A = R_D L - PL$$

$$\frac{dM}{dx} = V = R_A - P\left\langle x - \frac{L}{3}\right\rangle^0 - P\left\langle x - \frac{2L}{3}\right\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\left\langle x - \frac{L}{3}\right\rangle^1 - P\left\langle x - \frac{2L}{3}\right\rangle^1$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P\left\langle x - \frac{L}{3}\right\rangle^2 - \frac{1}{2}P\left\langle x - \frac{2L}{3}\right\rangle^2 + C_1$$

$$\left[x = 0, \frac{dy}{dx} = 0\right] \quad 0 + 0 - 0 - 0 + C_1 = 0 \qquad C_1 = 0$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\left\langle x - \frac{L}{3}\right\rangle^3 - \frac{1}{6}P\left\langle x - \frac{2L}{3}\right\rangle^3 + C_2$$

$$[x = 0, \; y = 0] \; 0 + 0 - 0 - 0 + C_2 = 0 \qquad C_2 = 0$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\left\langle x - \frac{L}{3}\right\rangle^3 - \frac{1}{6}P\left\langle x - \frac{2L}{3}\right\rangle^3$$

$$[x = L, \; y = 0] \quad \frac{1}{2}(R_D L - PL)L^2 + \frac{1}{6}(2P - R_D)L^3 - \frac{1}{6}P\left(\frac{2L}{3}\right)^3 - \frac{1}{6}P\left(\frac{L}{3}\right)^3 = 0$$

$$\frac{1}{3}R_D L^3 - \frac{2}{9}PL^3 = 0$$

(*a*) Reaction at *D*: $R_D = \frac{2}{3}P\uparrow$ ◀

$$M_A = \frac{2}{3}PL - PL = -\frac{1}{3}PL \qquad R_A = 2P - \frac{2}{3}P = \frac{4}{3}P$$

$$EIy = \frac{1}{2}\left(-\frac{1}{3}PL\right)x^2 + \frac{1}{6}\left(\frac{4}{3}P\right)x^2 - \frac{1}{6}P\left\langle x - \frac{L}{3}\right\rangle^3 - \frac{1}{6}\left\langle x - \frac{2L}{3}\right\rangle^3$$

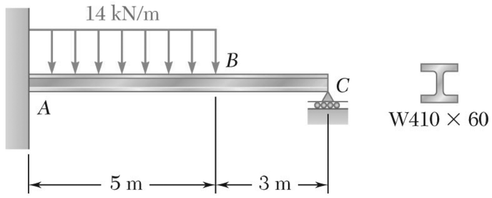
PROBLEM 9.52 *(Continued)*

Elastic curve:
$$y = \frac{P}{EI}\left\{-\frac{1}{6}Lx^2 + \frac{2}{9}x^3 - \frac{1}{6}\left\langle x - \frac{L}{3}\right\rangle^3 - \frac{1}{6}\left\langle x - \frac{2L}{3}\right\rangle^3\right\}$$

(*b*) Deflection at *B*: $\left(y \text{ at } x = \frac{L}{3}\right)$

$$y_B = \frac{P}{EI}\left\{-\left(\frac{1}{6}L\right)\left(\frac{L}{3}\right)^2 + \frac{2}{9}\left(\frac{L}{3}\right)^3 - 0 - 0\right\}$$
$$= -\frac{5PL^3}{486EI}$$

$$y_B = \frac{5PL^3}{486EI} \downarrow \blacktriangleleft$$



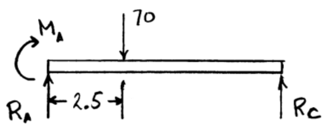
PROBLEM 9.53

For the beam and loading shown, determine (*a*) the reaction at point *C*, (*b*) the deflection at point *B*. Use $E = 200$ GPa.

$[x = 0,\ y = 0]$ $\qquad$ $[x = 8,\ y = 0]$

$\left[x = 0,\ \dfrac{dy}{dx} = 0\right]$

SOLUTION



Units: Forces in kN; lengths in m.

$$+\uparrow \Sigma F_y = 0: \quad R_A - 70 + R_C = 0$$

$$R_A = 70 - R_C \quad \text{kN}$$

$$+\circlearrowleft M_A = 0: \quad -M_A - (70)(2.5) + 8R_C = 0$$

$$M_A = 8R_C - 175 \quad \text{kN} \cdot \text{m}$$

Reactions are statically indeterminate.

$$w(x) = 14 - 14\langle x - 5\rangle^0 \qquad \text{kN/m}$$

$$\frac{dV}{dx} = -w = -14 + 14\langle x - 5\rangle^0 \qquad \text{kN/m}$$

$$\frac{dM}{dx} = V = R_A - 14x + 14\langle x - 5\rangle^1 \qquad \text{kN}$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - 7x^2 + 7\langle x - 5\rangle^2 \qquad \text{kN} \cdot \text{m}$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{7}{3}x^3 + \frac{7}{3}\langle x - 5\rangle^3 + C_1 \qquad \text{kN} \cdot \text{m}^2$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{7}{12}x^4 + \frac{7}{12}\langle x - 5\rangle^4 + C_1 x + C_2 \qquad \text{kN} \cdot \text{m}^3$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \qquad 0 + 0 + 0 + 0 + C_1 = 0 \qquad C_1 = 0$$

$$[x = 0,\ y = 0] \qquad 0 + 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = 8,\ y = 0] \qquad \frac{1}{2}M_A(8)^2 + \frac{1}{6}R_A(8)^3 - \frac{7}{12}(8)^4 + \frac{7}{12}(3)^4 + 0 + 0 = 0$$

PROBLEM 9.53 *(Continued)*

$$32(8R_C - 175) + \frac{512}{6}(70 - R_C) - \frac{28105}{12} = 0$$

$$170.667R_C = 5600 - \frac{35840}{6} + \frac{28105}{12} = 1968.75 \qquad R_C = 11.536 \text{ kN}\uparrow$$

(*a*) Reduction at *C*: $R_C = 11.546 \text{ kN}\uparrow$ ◀

$$M_A = (8)(11.536) - 175 = -82.715 \text{ kN} \cdot \text{m}$$

$$R_A = 70 - 11.536 = 58.464 \text{ kN}$$

Data: $E = 200 \times 10^9 \text{Pa} \qquad I = 216 \times 10^6 \text{mm}^4 = 216 \times 10^{-6} \text{m}^4$

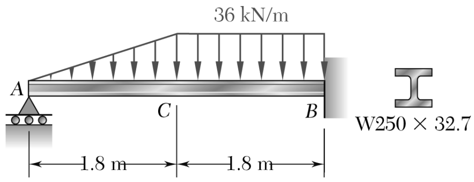
$$EI = (200 \times 10^9)(216 \times 10^{-6}) = 43.2 \times 10^6 \text{N} \cdot \text{m}^2 = 43200 \text{ kN} \cdot \text{m}^2$$

(*b*) Deflection at *B*: $(y \text{ at } x = 5 \text{ m})$

$$EIy_B = \frac{1}{2}(-82.715)(5)^2 + \frac{1}{6}(58.464)(5)^3 - \frac{7}{12}(5)^4 = -180.52 \text{ kN} \cdot \text{m}^3$$

$$y_B = -\frac{180.52}{43200} = -4.18 \times 10^{-3} \text{m}$$

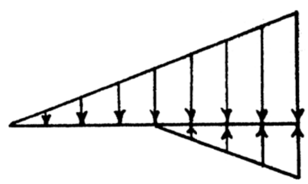
$y_B = 4.18 \text{ mm} \downarrow$ ◀



PROBLEM 9.54

For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION



$$k = \frac{36 \text{ kN/m}}{1.8 \text{ m}} = 20 \text{ kN/m}^2$$

$$w(x) = 20x - 20\langle x-1.8\rangle^1 \text{ kN/m}$$

$$\frac{dV}{dx} = -w(x) = -20x + 20\langle x-1.8\rangle^1 \text{ kN/m}$$

$$\frac{dM}{dx} = V = R_A - 10x^2 + 10\langle x-1.8\rangle^2 \text{ kN}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - \frac{10}{3}x^3 + \frac{10}{3}\langle x-1.8\rangle^3 \text{ kN}\cdot\text{m}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{5}{6}x^4 + \frac{5}{6}\langle x-1.8\rangle^4 + C_1 \text{ kN}\cdot\text{m}^3$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{1}{6}x^5 + \frac{1}{6}\langle x-1.8\rangle^5 + C_1 x + C_2 \text{ kN}\cdot\text{m}^3$$

$$[x=0, y=0]: \quad 0-0+0+0+C_2 = 0 \qquad \therefore \quad C_2 = 0$$

$$\left[x = 3.6 \text{ m}, \frac{dy}{dx} = 0\right]: \quad \frac{1}{2}R_A(3.6)^2 - \frac{5}{6}(3.6)^4 + \frac{5}{6}(1.8)^4 + C_1 = 0$$

$$\therefore \quad C_1 = 131.22 - 6.48R_A \quad \text{kN}\cdot\text{m}^2$$

$$[x = 3.6 \text{ m}, y = 0]: \quad \frac{1}{6}R_A(3.6)^3 - \frac{1}{6}(3.6)^5 + \frac{1}{6}(1.8)^5 + (131.22 - 6.48R_A)(3.6) = 0$$

$$(7.776 - 23.328)R_A = -374.76 \qquad R_A = 24.097 \text{ kN}$$

(*a*) Reaction at *A*: $\quad R_A = 24.1 \text{ kN}\uparrow$ ◀

$$C_1 = 131.22 - 6.48(24.097) = -24.929 \text{ kN}\cdot\text{m}^2$$

Data: $\quad E = 200\times10^6$ kPa $\quad I = 48.9\times10^{-6} \text{ m}^4$

$$EI = (200\times10^6)(48.9\times10^{-6}) = 9780 \text{ kN}\cdot\text{m}^2$$

(*b*) Deflection at *C*: $\quad$ (y at $x = 1.8$ m)

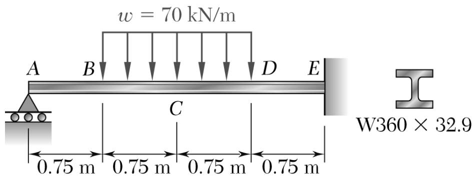
$$EIy_C = \frac{1}{6}(24.097)(1.8)^3 - \frac{1}{6}(1.8)^5 + 0 - 24.929(1.8)$$

$$= -24.599 \text{ kN}\cdot\text{m}^3$$

$$y_C = -\frac{24.599}{9780} = -2.515\times10^{-3} \text{ m}$$

$y_C = 2.5 \text{ mm}\downarrow$ ◀

PROBLEM 9.55



$[x = 0,\ y = 0]$ $\qquad$ $[x = 3,\ y = 0]$

$[x = 0,\ M = 0]$ $\qquad$ $\left[x = 3,\ \dfrac{dy}{dx} = 0\right]$

For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN, lengths in m

$$w(x) = 70\langle x - 0.75\rangle^0 - 70\langle x - 2.25\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -70\langle x - 0.75\rangle^0 + 70\langle x - 2.25\rangle^0 \text{ kN}^2/\text{m}$$

$$\frac{dM}{dx} = V = R_A - 70\langle x - 0.75\rangle^1 + 70\langle x - 2.25\rangle^1 \text{ kN}$$

$$EI\frac{d^2y}{dx^2} = M = R_A x - 35\langle x - 0.75\rangle^2 + 35\langle x - 2.25\rangle^2 \text{ kN}\cdot\text{m}$$

$$EI\frac{dy}{dx} = \frac{1}{2}R_A x^2 - \frac{35}{3}\langle x - 0.75\rangle^3 + \frac{35}{3}\langle x - 2.25\rangle^3 + C_1 \text{ kN}\cdot\text{m}^2$$

$$EIy = \frac{1}{6}R_A x^3 - \frac{35}{12}\langle x - 0.75\rangle^4 + \frac{35}{12}\langle x - 2.25\rangle^4 + C_1 x + C_2 \text{ kN}\cdot\text{m}^3$$

$$[x = 0,\ y = 0]:\ 0 + 0 + 0 + 0 + C_2 = 0 \quad \therefore \quad C_2 = 0$$

$$\left[x = 3, \frac{dy}{dx} = 0\right]:\ \frac{1}{2}R_A(3)^2 - \frac{35}{3}(2.25)^3 + \frac{35}{3}\langle 0.75\rangle^3 + C_1 = 0$$

$$C_1 = 128 - 4.5R_A \text{ kN}\cdot\text{m}^2$$

$$[x = 3,\ y = 0]:\ \frac{1}{6}R_A(3)^3 - \frac{35}{12}(2.25)^4 + \frac{35}{12}(0.75)^9 + (128 - 4.5R_A)(3) + 0 = 0$$

$$9R_A = 310.2$$

$R_A = 34.5$ kN ↑ ◀

$$C_1 = 128 - 4.5(34.5) = -27.25 \text{ kN}\cdot\text{m}^2$$

Data: $E = 200$ GPa, $I = 82.7\times10^{-6}$ m^4

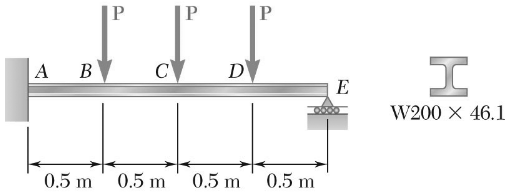
$$EI = (200\times10^6)(82.7\times10^{-6}) = 16540 \text{ kN}\cdot\text{m}^2$$

(*b*) Deflection at *C*: $\qquad$ (y at $x = 1.5$ m)

$$EIy_C = \frac{1}{6}(34.5)(1.5)^3 - \frac{35}{12}(0.75)^4 + 0 - (27.25)(1.5) = -22.39$$

$$y_C = -\frac{22.39}{16540} = -1.35\times10^{-3} \text{ m}$$

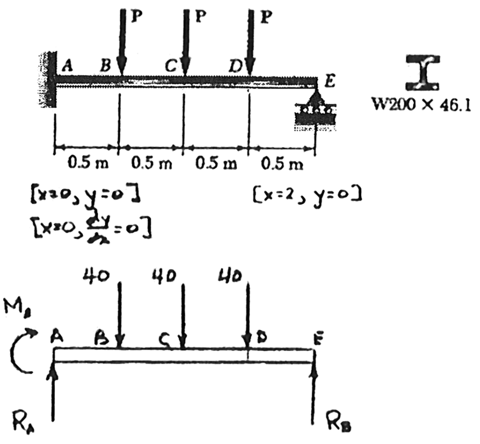
$y_C = 1.35$ mm ↓ ◀



PROBLEM 9.56

For the beam shown and knowing that $P = 40$ kN, determine (*a*) the reaction at point *E*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION



Units: Forces in kN; lengths in m.

$$+\uparrow \sum F_y = 0: \quad R_A - 40 - 40 - 40 + R_E = 0$$

$$R_A = 120 - R_E \quad \text{kN}$$

$$+\circlearrowleft M_A = 0: \quad -M_A - 20 - 40 - 60 + 2R_B = 0$$

$$M_A = 2R_E - 120 \qquad \text{kN} \cdot \text{m}$$

Reactions are statically indeterminate.

$$\frac{dM}{dx} = V = R_A - 40\langle x - 0.5\rangle^0 - 40\langle x - 1\rangle^0 - 40\langle x - 1.5\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - 40\langle x - 0.5\rangle^1 - 40\langle x - 1\rangle^1 - 40\langle x - 1.5\rangle^1$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - 20\langle x - 0.5\rangle^2 - 20\langle x - 1\rangle^2 - 20\langle x - 1.5\rangle^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{20}{3}\langle x - 0.5\rangle^3 - \frac{20}{3}\langle x - 1\rangle^3 - \frac{20}{3}\langle x - 1.5\rangle^3 + C_1 x + C_2$$

$$\left[x = 0, \ \frac{dy}{dx} = 0\right] \qquad 0 + 0 + 0 + 0 + 0 + C_1 = 0 \qquad\qquad C_1 = 0$$

$$[x = 0, \ y = 0] \qquad 0 + 0 + 0 + 0 + 0 + 0 + C_2 = 0 \qquad\qquad C_2 = 0$$

$$[x = 2, \ y = 0] \qquad \frac{1}{2}M_A(2)^2 + \frac{1}{6}R_A(2)^3 - \frac{20}{3}(1.5)^3 - \frac{20}{3}(1)^3 - \frac{20}{3}(0.5)^3 + 0 + 0 = 0$$

(*a*) Reaction at *E*:

$$\frac{1}{2}(2R_E - 120)(2)^2 + \frac{1}{6}(120 - R_E)(2)^3 = 30$$

$$2.66667\, R_E = 30 + 240 - 160 = 110 \qquad\qquad R_E = 41.25 \text{ kN} \uparrow \blacktriangleleft$$

$$M_A = (2)(41.25) - 120 = -37.5 \text{ kN} \cdot \text{m}$$

$$R_A = 120 - 41.25 = 78.25 \text{ kN}$$

PROBLEM 9.56 *(Continued)*

Data: $E = 200 \times 10^9$ Pa, $I = 45.8 \times 10^6\ \text{mm}^4 = 45.8 \times 10^{-6}\ \text{m}^4$

$$EI = (200 \times 10^9)(45.8 \times 10^{-6}) = 9.16 \times 10^6\ \text{N} \cdot \text{m}^2 = 9160\ \text{kN} \cdot \text{m}^2$$

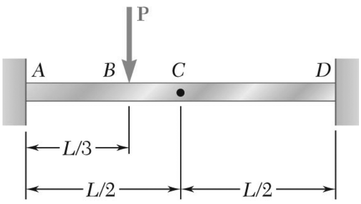
(*b*) Deflection at *C*: (y at $x = 1$ m)

$$EIy_C = \frac{1}{2}(-37.5)(1)^2 + \frac{1}{6}(78.75)(1)^3 - \frac{20}{3}(0.5)^3 - 0 - 0 + 0 + 0$$

$$= -6.4583\ \text{kN} \cdot \text{m}^3$$

$$y_C = -\frac{6.4583}{9160} = -0.705 \times 10^{-3}\ \text{m}$$

$y_C = 0.705$ mm $\downarrow$ ◀



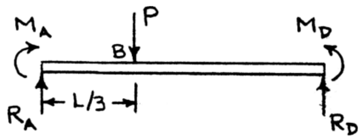
PROBLEM 9.57

For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at midpoint *C*.

SOLUTION

$$\frac{dM}{dx} = V = R_A - P\left\langle x - \frac{L}{3}\right\rangle^0$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - P\left\langle x - \frac{L}{3}\right\rangle^1$$



$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{2}P\left\langle x - \frac{L}{3}\right\rangle^2 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{6}P\left\langle x - \frac{L}{3}\right\rangle^3 + C_1 x + C_2$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \quad 0 + 0 - 0 + C_1 = 0 \qquad \therefore\ C_1 = 0$$

$$[x = 0,\ y = 0] \quad 0 + 0 - 0 + C_2 = 0 \qquad \therefore\ C_2 = 0$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right] \quad M_A L + \frac{1}{2}R_A L^2 - \frac{1}{2}P\left(\frac{2L}{3}\right)^2 = 0 \qquad (1)$$

$$[x = L,\ y = 0] \quad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^3 - \frac{1}{6}P\left(\frac{2L}{3}\right)^3 = 0 \qquad (2)$$

(*a*) Solving Eqs. (1) and (2) simultaneously,

$$R_A = \frac{20}{27}P \qquad M_A = -\frac{4}{27}PL$$

$$R_A = \frac{20}{27}P \uparrow \blacktriangleleft$$

$$M_A = \frac{4}{27}PL \circlearrowright \blacktriangleleft$$

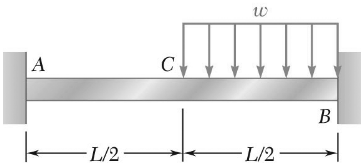
Elastic curve:

$$y = \frac{P}{EI}\left[-\frac{2}{27}Lx^2 + \frac{10}{81}x^3 - \frac{1}{6}\left\langle x - \frac{L}{3}\right\rangle^3\right]$$

(*b*) Deflection at midpoint *C*: $\left(y \text{ at } x = \frac{L}{2}\right)$

$$y_C = \frac{P}{EI}\left[-\frac{2}{27}L\left(\frac{L}{2}\right)^2 + \frac{10}{81}\left(\frac{L}{2}\right)^3 - \frac{1}{6}\left(\frac{L}{6}\right)^3\right] = -\frac{5PL^3}{1296EI}$$

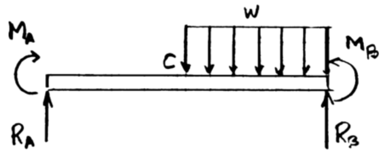
$$y_C = \frac{5PL^3}{1296EI} \downarrow \blacktriangleleft$$



PROBLEM 9.58

For the beam and loading shown, determine (*a*) the reaction at point *A*, (*b*) the deflection at midpoint *C*.

SOLUTION



$$w(x) = w\left\langle x - \frac{L}{2}\right\rangle^0$$

$$\frac{dV}{dx} = -w(x) = -w\left\langle x - \frac{L}{2}\right\rangle^0$$

$$\frac{dM}{dx} = V = R_A - w\left\langle x - \frac{L}{2}\right\rangle^1$$

$$EI\frac{d^2y}{dx^2} = M = M_A + R_A x - \frac{1}{2}w\left\langle x - \frac{L}{2}\right\rangle^2$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{2}R_A x^2 - \frac{1}{6}w\left\langle x - \frac{L}{2}\right\rangle^3 + C_1$$

$$EIy = \frac{1}{2}M_A x^2 + \frac{1}{6}R_A x^3 - \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4 + C_1 x + C_2$$

$$\left[x = 0,\ \frac{dy}{dx} = 0\right] \qquad 0 + 0 - 0 + C_1 = 0 \qquad C_1 = 0$$

$$[x = 0,\ y = 0] \qquad 0 + 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$\left[x = L,\ \frac{dy}{dx} = 0\right] \qquad M_A L + \frac{1}{2}R_A L^2 - \frac{1}{6}w\left(\frac{L}{2}\right)^3 = 0 \qquad (1)$$

$$[x = L,\ y = 0] \qquad \frac{1}{2}M_A L^2 + \frac{1}{6}R_A L^2 - \frac{1}{24}w\left(\frac{L}{2}\right)^4 = 0 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

(*a*) $$R_A = \frac{3wL}{32} \qquad M_A = -\frac{5wL^2}{192} \qquad R_A = \frac{3wL}{32}\uparrow \blacktriangleleft$$

$$M_A = \frac{5wL^2}{192} \circlearrowright \blacktriangleleft$$

$$EIy = -\frac{5}{384}wL^2x^2 + \frac{3}{192}wLx^3 - \frac{1}{24}w\left\langle x - \frac{L}{2}\right\rangle^4$$

PROBLEM 9.58 *(Continued)*

Elastic curve:

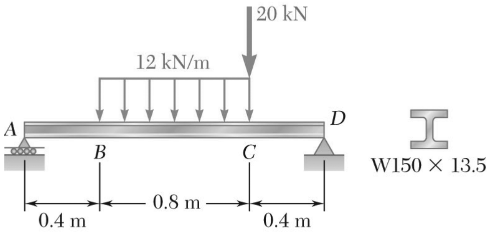
$$y = \frac{w}{EI}\left\{-\frac{5}{384}L^2x^2 + \frac{3}{192}Lx^3 - \frac{1}{24}\left\langle x - \frac{L}{2}\right\rangle^4\right\}$$

(*b*) Deflection at midpoint *C*: $\left(y \text{ at } x = \frac{L}{2}\right)$

$$y_C = \frac{w}{EI}\left\{\left(-\frac{5}{384}L^2\right)\left(\frac{L}{2}\right)^2 + \left(\frac{3}{192}L\right)\left(\frac{L}{2}\right)^3 - 0\right\}$$

$$= -\frac{wL^4}{768EI}$$

$$y_B = \frac{wL^4}{768EI} \downarrow \blacktriangleleft$$



PROBLEM 9.59

For the beam and loading of Prob. 9.45, determine the magnitude and location of the largest downward deflection.

SOLUTION

See solution to Prob. 9.45 for the derivation of the equations used in the following:

$$EI = EI = 1366 \text{ kN} \cdot \text{m}^2$$

$$EI\frac{dy}{dx} = 4.9x^2 - 2\langle x - 0.4\rangle^3 + 2\langle x - 1.2\rangle^3 - 10\langle x - 1.2\rangle^2 - 3.4080 \text{ kN} \cdot \text{m}^2$$

$$EIy = 1.63333x^3 - \frac{1}{2}\langle x - 0.4\rangle^4 + \frac{1}{2}\langle x - 1.2\rangle^4 - \frac{10}{3}\langle x - 1.2\rangle^3 - 3.4080x \text{ kN} \cdot \text{m}^3$$

To find the location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $0.4 < x < 1.2$.

$$4.9x^2 - 2(x - 0.4)^3 - 3.4080 = f(x) = 0$$

Solve by iteration: $x = 0.8 \quad 0.858 \quad 0.857 \quad 0.8570$ $\qquad x_m = 0.8570 \text{ m}$ ◀

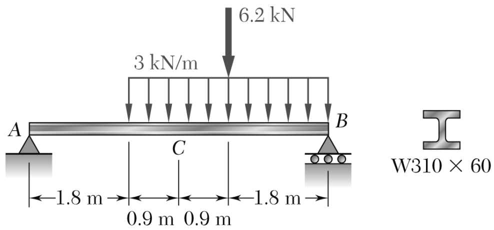
$df/dx = 6.88 \quad 7.123 \quad 7.145$

$$EIy_m = (1.63333)(0.8570)^3 - \frac{1}{2}(0.8570 - 0.4)^4 - (3.4080)(0.8570)$$

$$= -1.9144 \text{ kN} \cdot \text{m}$$

$$y_m = -\frac{1.9144}{1366} = -1.401 \times 10^{-3} \text{m}$$

$y_m = 1.401 \text{ mm} \downarrow$ ◀



PROBLEM 9.60

For the beam and loading indicated, determine the magnitude and location of the largest downward deflection.

SOLUTION

See solution to Prob. 9.47 for the derivation of the equations used in the following:

$$EI = 25.8\times 10^3 \text{ kN}\cdot\text{m}^2$$

$$EI\frac{dy}{dx} = 2.8333\,x^2 - \frac{1}{2}\langle x-1.8\rangle^2 - 3.1\langle x-3.6\rangle^2 - 22.535$$

$$EIy = 0.9444x^3 - \frac{1}{8}\langle x-1.8\rangle^3 - 1.03333\langle x-3.6\rangle^3 - 22.535x$$

To find the location of maximum $|y|$, set $\dfrac{dy}{dx} = 0.$ Assume $1.8 \le x_m < 3.6$

$$EI\frac{dy}{dx} = 2.8333x_m^2 - \frac{1}{2}(x_m - 1.8)^2 - 22.535 = 0$$

Solving by iteration: $x_m = 3,\ 2.86,\ 2.855$ $\qquad x_m = 2.855 \text{ m}$ ◀

$$df/dx = 15.8,\ 15.15$$

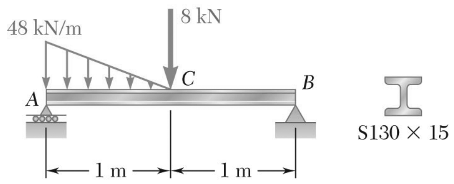
$$EIy_m = 0.9444x_m^3 - \frac{1}{8}(x_m - 1.8)^3 - 22.535x_m$$

$$= (0.9444)(2.855)^3 - \frac{1}{8}(2.855-1.8)^3 - (22.535)(2.855) = -42.507 \text{ kN}\cdot\text{m}^3$$

$$y_m = -\frac{42.507}{25.8\times 10^3} = -1.648\times 10^{-3} \text{ m}$$

$y_m = 1.648 \text{ mm} \downarrow$ ◀

PROBLEM 9.61



For the beam and loading of Prob.9.47, determine the magnitude and location of the largest downward deflection.

SOLUTION

See solution to Prob. 9.47 for the derivation of the equations used in the following:

$$EI = 1024\ \text{kN}\cdot\text{m}^2$$

$$EI\frac{dy}{dx} = 12x^2 - 8x^3 + 2x^4 - 2\langle x-1\rangle^4 - 4\langle x-1\rangle^2 - \frac{83}{15} \quad \text{kN}\cdot\text{m}^2$$

$$EIy = 4x^3 - 2x^4 + \frac{2}{5}x^5 - \frac{2}{5}\langle x-1\rangle^5 - \frac{4}{3}\langle x-1\rangle^3 - \frac{83}{15}x \quad \text{kN}\cdot\text{m}^3$$

To find location of maximum $|y|$, set $\frac{dy}{dx} = 0$. Assume $0 < x < 1$ m.

$$EI\frac{dy}{dx} = 12x^2 - 8x^3 + 2x^4 - \frac{83}{15} = 0$$

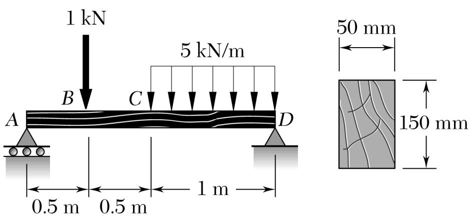
Solving: $x = 0.94166$ m $\qquad x_m = 0.942\ \text{m}$ ◀

$$EIy_m = 4(0.94166)^3 - 2(0.94166)^4 + \frac{2}{5}(0.94166)^5 - \frac{83}{15}(0.94166)$$

$$= -3.1469\ \text{kN}\cdot\text{m}^3$$

$$y_m = -\frac{3.1469}{1024} = -3.0731\times10^{-3}\ \text{m}$$

$y_m = 3.07\ \text{mm} \downarrow$ ◀



PROBLEM 9.62

For the beam and loading of Prob. 9.48, determine the magnitude and location of the largest downward deflection.

SOLUTION

See solution to Prob. 9.48 for the derivation of equations used in the following:

$$EI = 168.75 \text{ kN}\cdot\text{m}^2$$

$$C_1 = -1.60417 \text{ kN}\cdot\text{m}^2, \quad C_2 = 0$$

$$EI\frac{dy}{dx} = -\frac{5}{6}\langle x-1\rangle^3 + 2.125x^2 - 2\langle x-0.5\rangle^2 + C_1 \quad \text{kN}\cdot\text{m}^2$$

$$EIy = -\frac{5}{24}\langle x-1\rangle^4 + \frac{2.125}{3}x^3 - \frac{2}{3}\langle x-0.5\rangle^2 + C_1x + C_2 \quad \text{kN}\cdot\text{m}^3$$

Compute slope at C. $\left(\frac{dy}{dx} \text{ at } x = 1 \text{ m}\right)$

$$EI\left(\frac{dy}{dx}\right)_C = 0 + (2.125)(1)^2 - 2(0.5)^2 - 1.60417 = 20.83\times10^{-3} \text{ kN}\cdot\text{m}^2$$

Since the slope at C is positive, the largest deflection occurs in portion BC, where

$$EI\frac{dy}{dx} = 2.125x^2 - 2(x-0.5)^2 - 1.60417$$

$$EIy = \frac{2.125}{3}x^3 - \frac{2}{3}(x-0.5)^3 - 1.60417x$$

To find the location of the largest downward deflection, set $\frac{dy}{dx} = 0.$

$$2.125x_m^2 - 2(x_m^2 - x_m + 0.25) - 1.60417$$

$$= 0.125x_m^2 + 2x_m - 2.10417 = 0$$

$$x = 1.0521 - 0.0625x^2$$

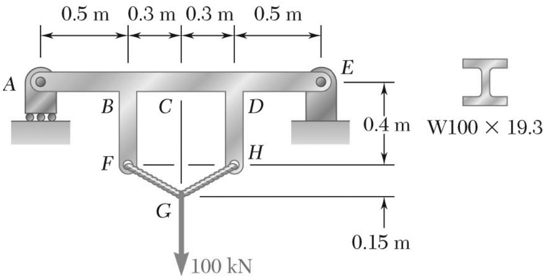
Solve by iteration. $x_m = 1 \quad 0.989 \quad 0.991$ $\qquad x_m = 0.991 \text{ m}$ ◀

$$EIy_m = \left(\frac{2.125}{3}\right)(0.991)^3 - \frac{2}{3}(0.991-0.5)^3 - (1.60417)(0.991)$$

$$= -0.97927 \text{ kN}\cdot\text{m}^3$$

$$y_m = \frac{-0.97927}{168.75} = -5.80\times10^{-3} \text{ m}$$

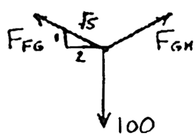
$y_m = 5.80 \text{ mm} \downarrow$ ◀

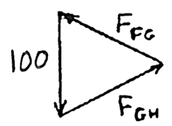


PROBLEM 9.63

The rigid bars *BF* and *DH* are welded to the rolled-steel beam *AE* as shown. Determine for the loading shown (*a*) the deflection at point *B*, (*b*) the deflection at midpoint *C* of the beam. Use $E = 200$ GPa.

SOLUTION



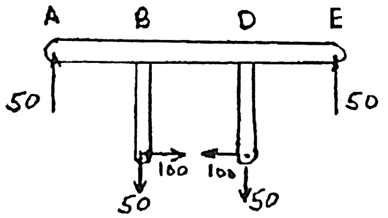


Use joint *G* as a free body,

By symmetry, $F_{GH} = F_{FG}$

$$+\uparrow \sum F_y = 0: \quad 2F_{GHy} - 100 = 0 \qquad F_{GHy} = 50 \text{ kN}$$

$$F_{GHx} = 2\, F_{GHy} = 100 \text{ kN.}$$



Forces in kN; lengths in m.

$$V = 50 - 50\langle x - 0.5\rangle^0 - 50\langle x - 1.1\rangle^0 \quad \text{kN}$$

$$M = 50x - 50\langle x - 0.5\rangle^1 - 50\langle x - 1.1\rangle^0$$

$$+40\langle x - 0.5\rangle^0 - 40\langle x - 1.1\rangle^0 \quad \text{kN} \cdot \text{m}$$

$$EI\frac{dy}{dx} = 25x^2 - 25\langle x - 0.5\rangle^2 - 25\langle x - 1.1\rangle^2 - 40\langle x - 0.5\rangle^1 + 40\langle x - 1.1\rangle^1 + C_1 \qquad \text{kN} \cdot \text{m}^2$$

$$EIy = \frac{25}{3}x^3 - \frac{25}{3}\langle x - 0.5\rangle^3 - \frac{25}{3}\langle x - 1.1\rangle^3 - 20\langle x - 0.5\rangle^2 + 20\langle x - 1.1\rangle^2 + C_1 x + C_2 \quad \text{kN} \cdot \text{m}^3$$

$[x = 0,\ y = 0]$ $\qquad C_2 = 0$

$[x = 1.6,\ y = 0]$

$$\left(\frac{25}{3}\right)(1.6)^3 - \left(\frac{25}{3}\right)(1.1)^3 - \left(\frac{25}{3}\right)(0.5)^3 - (20)(1.1)^2 + (20)(0.5)^2 + C_1(1.6) + 0 = 0$$

$$C_1 = -1.75 \text{ kN} \cdot \text{m}^3$$

For EIy_B, $\qquad x = 0.5\,\text{m}$

$$EIy_B = \left(\frac{25}{3}\right)(0.5)^3 - 0 - 0 + 0 - 0 - (1.75)(0.5) = 0.1667 \text{ kN} \cdot \text{m}^3$$

For EIy_C, $\qquad x = 0.8$ m

$$EIy_C = \left(\frac{25}{3}\right)(0.8)^3 - \left(\frac{25}{3}\right)(0.3)^3 - 0 - (20)(0.3)^2 - 0 - (1.75)(0.8) + 0$$

$$= -0.8417 \text{ kN} \cdot \text{m}^3$$

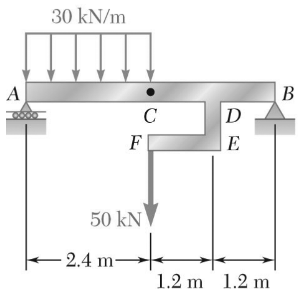
PROBLEM 9.63 *(Continued)*

For $W\,100 \times 19.3$ rolled-steel shape, $I = 4.70 \times 10^6\ \text{mm}^4 = 4.70 \times 10^{-6}\ \text{m}^4$

$$EI = (200 \times 10^9)(4.70 \times 10^{-6}) = 940 \times 10^3\ \text{N} \cdot \text{m}^2 = 940\ \text{kN} \cdot \text{m}^2$$

(*a*) $y_B = \dfrac{0.1667}{940} = 0.177 \times 10^{-3}\ \text{m}$ $\qquad y_B = 0.177\ \text{mm} \uparrow$ ◀

(*b*) $y_C = \dfrac{0.8417}{940} = 0.895 \times 10^{-3}\ \text{m}$ $\qquad y_C = 0.895\ \text{mm} \uparrow$ ◀



PROBLEM 9.64

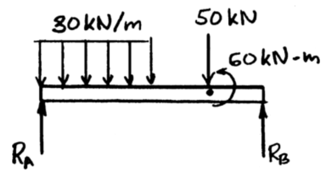
The rigid bar *DEF* is welded at point *D* to the rolled-steel beam *AB*. For the loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at midpoint *C* of the beam. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in meters.

$$+\circlearrowleft M_B = 0: \quad -4.8R_A + (30)(2.4)(3.6) + (50)(2.4) = 0$$

$$R_A = 79 \text{ kN} \uparrow$$



$$I = 212 \times 10^6 \text{mm}^4 = 212 \times 10^{-6} \text{m}^4$$

$$EI = (200 \times 10^9)(212 \times 10^{-6}) = 42.4 \times 10^6 \text{N} \cdot \text{m}^2$$

$$= 42400 \text{ kN} \cdot \text{m}^2$$

$$w(x) = 30 - 30\langle x - 2.4\rangle^0$$

$$\frac{dV}{dx} = -w = -30 + 30\langle x - 2.4\rangle^0 \qquad \text{kN/m}$$

$$\frac{dM}{dx} = V = 79 - 30x + 30\langle x - 2.4\rangle^1 - 50\langle x - 3.6\rangle^0 \qquad \text{kN}$$

$$EI\frac{d^2y}{dx} = M = 79x - 15x^2 + 15\langle x - 2.4\rangle^2 - 50\langle x - 3.6\rangle^1 - 60\langle x - 3.6\rangle^0 \qquad \text{kN} \cdot \text{m}$$

$$EI\frac{dy}{dx} = \frac{79}{2}x^2 - 5x^3 + 5\langle x - 2.4\rangle^3 - 25\langle x - 3.6\rangle^2 - 60\langle x - 3.6\rangle^1 + C_1 \qquad \text{kN} \cdot \text{m}^2$$

$$EIy = \frac{79}{6}x^3 - \frac{5}{4}x^4 + \frac{5}{4}\langle x - 2.4\rangle^4 - \frac{25}{3}\langle x - 3.6\rangle^3 - 30\langle x - 3.6\rangle^2 + C_1x + C_2 \qquad \text{kN} \cdot \text{m}^3$$

$$[x = 0, y = 0] \qquad 0 - 0 + 0 - 0 - 0 + 0 + C_2 = 0 \qquad C_2 = 0$$

$$[x = 4.8, y = 0] \qquad \left(\frac{79}{6}\right)(4.8)^3 - \left(\frac{5}{4}\right)(4.8)^4 + \left(\frac{5}{4}\right)(2.4)^4 - \left(\frac{25}{3}\right)(1.2)^3 - (30)(1.2)^2 + 4.8C_1 = 0$$

$$C_1 = -161.76 \text{ kN} \cdot \text{m}^2$$

PROBLEM 9.64 *(Continued)*

(a) Slope at point A: $\left(\dfrac{dy}{dx} \text{ at } x = 0\right)$

$$EI\left(\frac{dy}{dx}\right)_A = 0 - 0 + 0 - 0 - 0 - 161.76$$

$$= -161.76 \text{ kN} \cdot \text{m}^2$$

$$\left(\frac{dy}{dx}\right)_A = \frac{-161.76}{42400} = -3.82 \times 10^{-3}$$

$\theta_A = 3.82 \times 10^{-3}\text{rad.}$ ◀

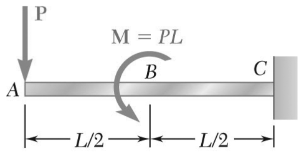
(b) Deflection at midpoint C: (y at $x = 2.4$)

$$EIy_C = \left(\frac{79}{6}\right)(2.4)^3 - \left(\frac{5}{4}\right)(2.4)^4 + 0 - 0 - 0 - (161.76)(2.4) + 0$$

$$= -247.68 \text{ kN} \cdot \text{m}^3$$

$$y_C = \frac{-247.68}{42400} = -5.84 \times 10^{-3}\text{m}$$

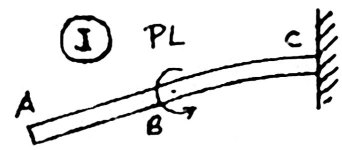
$y_C = 5.84 \text{ mm} \downarrow$ ◀

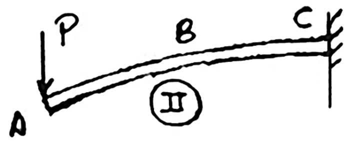


PROBLEM 9.65

For the cantilever beam and loading shown, determine the slope and deflection at the free end.

SOLUTION





Loading I: Counterclockwise couple PL at B.

Case 3 of Appendix D applied to portion BC.

$$\theta'_B = -\frac{(PL)(L/2)}{EI} = \frac{1}{2}\frac{PL^2}{EI}$$

$$y'_B = \frac{(PL)(L/2)^2}{2EI} = \frac{1}{8}\frac{PL^3}{EI}$$

AB remains straight.

$$\theta'_A = \theta'_B = \frac{1}{2}\frac{PL^2}{EI}$$

$$y'_A = y'_B - \left(\frac{L}{2}\right)\theta'_B = -\frac{1}{8}\frac{PL^3}{EI} - \frac{1}{4}\frac{PL^3}{EI} = -\frac{3}{8}\frac{PL^3}{EI}$$

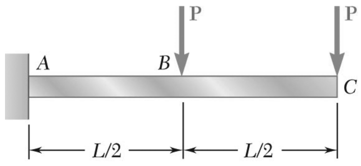
Loading II: Case 1 of Appendix D.

$$\theta''_A = \frac{PL^2}{2EL}, \quad y''_A - \frac{PL^3}{3EI}$$

By superposition,

$$\theta_A = \theta'_A + \theta''_A = \frac{1}{2}\frac{PL^2}{EI} + \frac{1}{2}\frac{PL^2}{EI} = \frac{PL^2}{EI} \qquad \theta_A = \frac{PL^2}{EI} \measuredangle \blacktriangleleft$$

$$y_A = y'_A + y''_A = -\frac{3}{8}\frac{PL^3}{EI} - \frac{1}{3}\frac{PL^3}{EI} = -\frac{17}{24}\frac{PL^3}{EI} \qquad y_A = \frac{17}{24}\frac{PL^3}{EI} \downarrow \blacktriangleleft$$



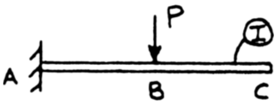
PROBLEM 9.66

For the cantilever beam and loading shown, determine the slope and deflection at the free end.

SOLUTION

<u>Loading I</u>: *P* downward at *B*.

Case 1 of Appendix *D* applied to portion *AB*.



$$\theta'_B = -\frac{P(L/2)^2}{2EI} = -\frac{1}{8}\frac{PL^2}{EI}$$

$$y'_B = -\frac{P(L/2)^3}{3EI} = -\frac{1}{24}\frac{PL^3}{EI}$$

BC remains straight.

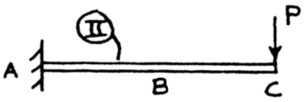
$$\theta'_C = \theta'_B = -\frac{1}{8}\frac{PL^2}{EI}$$

$$y'_C = y'_B - \left(\frac{L}{2}\right)\theta'_B = -\frac{1}{24}\frac{PL^3}{EI} - \frac{1}{16}\frac{PL^3}{EI}$$

$$= -\frac{5}{48}\frac{PL^3}{EI}$$

<u>Loading II</u>: *P* downward at *C*.

Case 1 of Appendix D.

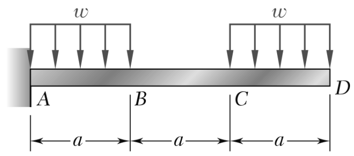


$$\theta''_C = -\frac{PL^2}{2EI} \quad y''_C = -\frac{PL^3}{3EI}$$

By superposition,

$$\theta_C = \theta'_C + \theta''_C = -\frac{1}{8}\frac{PL^2}{EI} - \frac{1}{2}\frac{PL^2}{EI} = -\frac{5}{8}\frac{PL^2}{EI} \qquad \frac{5PL^2}{8EI} \blacktriangleleft$$

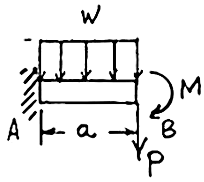
$$y_C = y'_C + y''_C = -\frac{5}{48}\frac{PL^3}{EI} - \frac{1}{3}\frac{PL^3}{EI} = -\frac{21}{48}\frac{PL^3}{EI} \qquad \frac{7PL^3}{16EI}\downarrow \blacktriangleleft$$



PROBLEM 9.67

For the cantilever beam and loading shown, determine the slope and deflection at point B.

SOLUTION



Consider portion AB as a cantilever beam subjected to the three loadings shown.

By statics: $P = wa$

$$M = (wa)\left(a + \frac{a}{2}\right) = \frac{3}{2}wa^2$$

<u>Slope at B</u>. $\theta_B = (\theta_B)_w + (\theta_B)_P + (\theta_B)_M$

Case 2 of App. D. $(\theta_B)_w = -\dfrac{wa^3}{6EI}$

Case 1 of App. D. $(\theta_B)_p = -\dfrac{(wa)a^2}{2EI} = -\dfrac{wa^3}{2EI}$

Case 3 of App. D. $(\theta_B)_M = -\dfrac{(\frac{3}{2}wa^2)^a}{EI} = -\dfrac{3wa^3}{2EI}$

$$\theta_B = -\frac{13wa^3}{6EI}$$

$$\theta_B = \frac{13wa^3}{6EI} \blacktriangleleft$$

<u>Deflection at B</u>: $y_B = (y_B)_w + (y_B)_P + (y_B)_M$

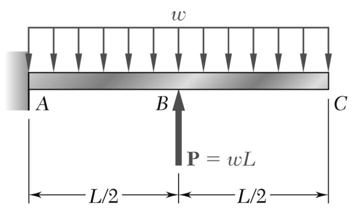
Case 2 of App. D. $(y_B)_w = -\dfrac{wa^4}{8EI}$

Case 1 of App. D. $(y_B)_P = -\dfrac{(wa)a^3}{3EI} = -\dfrac{wa^4}{3EI}$

Case 3 of App. D. $(y_B)_M = -\dfrac{(\frac{3}{2}wa^2)a}{2EI} = -\dfrac{3wa^4}{4EI}$

$$y_B = -\frac{29wa^4}{24EI}$$

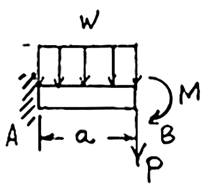
$$y_B = \frac{29wa^4}{24EI}\downarrow \blacktriangleleft$$



PROBLEM 9.68

For the cantilever beam and loading shown, determine the slope and deflection at point B.

SOLUTION



Consider portion AB as a cantilever beam subjected to the three loadings shown.

By statics: $$Q = P - \frac{wL}{2} = \frac{wL}{2}$$

$$M_0 = \left(\frac{wL}{2}\right)\left(\frac{1}{2} - \frac{L}{2}\right) = \frac{wL^2}{8}$$

Slope at B. $$\theta_B = (\theta_B)_w + (\theta_B)_Q + (\theta_B)_M$$

Case 2 of App. D. $$(\theta_B)_w = -\frac{w}{6EI}\left(\frac{L}{2}\right)^3 = -\frac{wL^3}{48EI}$$

Case 1 of App. D. $$(\theta_B)_Q = +\frac{(wL/2)}{2EI}\left(\frac{L}{2}\right)^2 = \frac{wL^3}{16EI}$$

Case 3 of App. D. $$(\theta_B)_M = -\frac{(wL\ 2/8)}{EI}\left(\frac{L}{2}\right) = -\frac{wL^3}{16EI}$$

$$\theta_B = -\frac{wL^3}{48EI} \qquad \theta_B = \frac{wL^3}{48EI} \ \blacktriangleleft$$

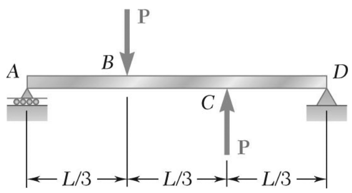
Deflection at B: $$y_B = (y_B)_w + (y_B)_Q + (y_B)_M$$

Case 2 of App. D. $$(y_B)_w = -\frac{W}{8EI}\left(\frac{L}{2}\right)^4 = -\frac{wL^4}{128EI}$$

Case 1 of App. D. $$(y_B)_Q = +\frac{(wL/2)}{3EI}\left(\frac{L}{2}\right)^3 = \frac{wL^4}{48EI}$$

Case 3 of App. D. $$(y_B)_M = -\frac{(wL\ 2/8)}{2EI}\left(\frac{L}{2}\right)^2 = -\frac{wL^4}{64EI}$$

$$y_B = -\frac{wL^4}{384EI} \qquad y_B = \frac{wL^4}{384EI}\downarrow \ \blacktriangleleft$$



PROBLEM 9.69

For the beam and loading shown, determine (*a*) the deflection at *C*, (*b*) the slope at end *A*.

SOLUTION

<u>Loading I</u>: Downward load *P* at *B*.

Use Case 5 of Appendix D with

$$P = P, \quad a = \frac{L}{3}, \quad b = \frac{2L}{3}, \quad L = L, \quad x = \frac{2L}{3}$$

For $x < a$, given elastic curve is $\quad y = \dfrac{Pb}{EIL}[x^3 - (L^2 - b^2)x]$

To obtain elastic curve for $x > a$, replace *x* by $L - x$ and interchange *a* and *b* to get

$$y = \frac{Pa}{6EIL}[(L-x)^3 - (L^2 - a^2)(L-x)] \text{ with } x = \frac{2L}{3} \text{ at point } C.$$

$$y_C = \frac{P(L/3)}{6EIL}\left[\left(\frac{L}{3}\right)^3 - \left(L^2 - \left(\frac{L}{3}\right)^2\right)\left(\frac{L}{3}\right)\right] = -\frac{7}{486}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P(2L/3)[L^2 - (2L/3)^2]}{6EIL} = -\frac{5}{81}\frac{PL^2}{EI}$$

<u>Loading II</u>: Upward load at *C*. Use Case 5 of Appendix *D* with

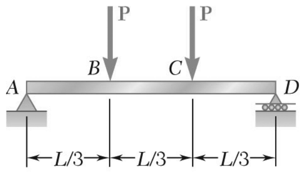
$$P = -P, \quad a = \frac{2L}{3}, \quad b = \frac{L}{3}, \quad L = L, \quad x = a = \frac{2L}{3}$$

$$y_C = -\frac{(-P)(2L/3)^2(L/3)^2}{3EIL} = \frac{4}{243}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{(-P)(L/3)(L^2 - (L/3)^2)}{6EIL} = \frac{4}{81}\frac{PL^2}{EI}$$

(*a*) <u>Deflection at *C*</u>: $\quad y_C = -\dfrac{7}{486}\dfrac{PL^3}{EI} + \dfrac{4}{243}\dfrac{PL^3}{EI}$ $\quad y_C = \dfrac{1}{486}\dfrac{PL^3}{EI} \uparrow$ ◀

(*b*) <u>Slope at *A*</u>: $\quad \theta_A = -\dfrac{5}{81}\dfrac{PL^2}{EI} + \dfrac{4}{81}\dfrac{PL^2}{EI}$ $\quad \theta_A = \dfrac{1}{81}\dfrac{PL^2}{EI}$ ◀



PROBLEM 9.70

For the beam and loading shown, determine (*a*) the deflection at *C*, (*b*) the slope at end *A*.

SOLUTION

Loading I: Load at *B*. Case 5 of Appendix D.

$$a = \frac{L}{3}, \quad b = \frac{2L}{3}, \quad x = \frac{2L}{3}$$

For $x > a$, replace *x* by $L - x$ and interchange *a* and *b*.

$$y = \frac{Pa}{6EIL}[(L-x)^3 - (L^2 - a^2)(L-x)]$$

$$y_C = \frac{P(L/3)}{6EIL}\left[\left(L - \frac{2L}{3}\right)^3 - \left(L^2 - \frac{L^2}{9}\right)\left(L - \frac{2L}{3}\right)\right] = -\frac{7}{486}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P(2L/3)(L^2 - 4L^2/9)}{6EIL} = -\frac{5}{81}\frac{PL^2}{EI}$$

Loading II: Load at *C*. Case 5 of Appendix D.

$$a = \frac{2L}{3}, \quad b = \frac{L}{3}, \quad x = \frac{2L}{3}$$

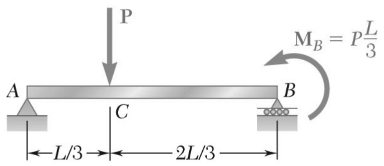
$$y_C = \frac{Pb}{6EIL}[x^3 - (L^2 - b^2)x] = \frac{P(L/3)}{6EIL}\left[\frac{8L^3}{27} - \left(L^2 - \frac{L^2}{9}\right)\left(\frac{2L}{3}\right)\right]$$

$$= -\frac{8}{486}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P(L/3)(L^2 - L^2/9)}{6EIL} = -\frac{4}{81}\frac{PL^2}{EI}$$

(*a*) Deflection at *C*: $y_C = -\frac{7}{486}\frac{PL^3}{EI} - \frac{8}{486}\frac{PL^3}{EI} = -\frac{15}{486}\frac{PL^3}{EI}$ $\qquad y_C = \frac{5}{162}\frac{PL^3}{EI} \downarrow$ ◀

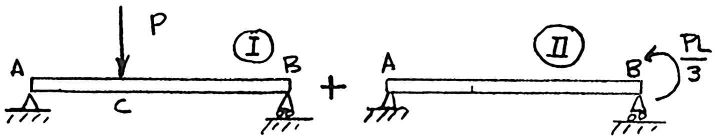
(*b*) Slope at *A*: $\theta_A = -\frac{5}{81}\frac{PL^2}{EI} - \frac{4}{81}\frac{PL^2}{EI} = -\frac{1}{9}\frac{PL^2}{EI}$ $\qquad \theta_A = \frac{1}{9}\frac{PL^2}{EI}$ ◀



PROBLEM 9.71

For the beam and loading shown, determine (*a*) the deflection at *C*, (*b*) the slope at end *A*.

SOLUTION



Loading I: Case 5: $a = \dfrac{L}{3}, \quad b = \dfrac{2L}{3}, \quad P = P, \quad x = a$

$$y_C = -\frac{Pa^2b^2}{6EIL} = -\frac{P}{6EIL}\left(\frac{L}{3}\right)^2\left(\frac{2L}{3}\right)^2 = -\frac{4}{243}\frac{PL^3}{EI}$$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{P}{6EIL}\left(\frac{2L}{3}\right)\left[L^2 - \left(\frac{2L}{3}\right)^2\right] = -\frac{5}{81}\frac{PL^2}{EI}$$

Loading II: Case 7: $M = -\dfrac{PL}{3}, \qquad x = \dfrac{L}{3}$

$$y_C = -\frac{M}{6EIL}(x^3 - L^2x) = +\frac{PL/3}{6EIL}\left\{\left(\frac{L}{3}\right)^3 - L^2\left(\frac{L}{3}\right)\right\} = -\frac{4}{243}\frac{PL^3}{EI}$$

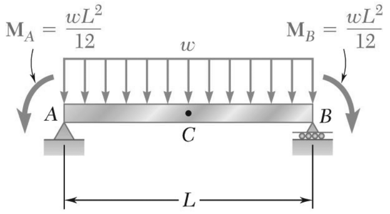
$$\theta_A = +\frac{ML}{6EI} = \frac{-(PL/3)L}{6EI} = -\frac{1}{18}\frac{PL^2}{EI}$$

(*a*) Deflection at *C*: $y_C = -\dfrac{4}{243}\dfrac{PL^3}{EI} - \dfrac{4}{243}\dfrac{PL^3}{EI} = -\dfrac{8}{243}\dfrac{PL^3}{EI}$

$$y_C = \frac{8}{243}\frac{PL^3}{EI} \downarrow \blacktriangleleft$$

(*b*) Slope at *A*: $\theta_A = -\dfrac{5}{81}\dfrac{PL^2}{EI} - \dfrac{1}{18}\dfrac{PL^2}{EI} = -\dfrac{19}{162}\dfrac{PL^2}{EI}$

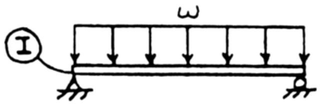
$$\theta_A = \frac{19}{162}\frac{PL^2}{EI} \; \blacktriangleleft$$



PROBLEM 9.72

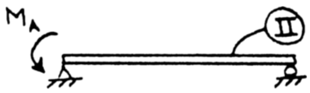
For the beam and loading shown, determine (*a*) the deflection at *C*, (*b*) the slope at end *A*.

SOLUTION



<u>Loading I</u>: Case 6 in Appendix D.

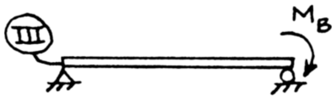
$$y_C = -\frac{5wL^4}{384EI} \qquad \theta_A = -\frac{wL^3}{24EI}$$



<u>Loading II</u>: Case 7 in Appendix D.

$$y_C = -\frac{M_A}{6EIL}\left[\left(\frac{L}{2}\right)^3 - L^2\left(\frac{L}{2}\right)\right] = \frac{1}{16}\frac{M_A L^2}{EI} \quad \theta_A = \frac{M_A L}{3EI}$$

with $\quad M_A = \frac{wL^2}{12} \quad y_C = \frac{1}{192}\frac{wL^4}{EI} \quad \theta_A = \frac{1}{36}\frac{wL^3}{EI}$



<u>Loading III</u>: Case 7 in Appendix D.

$$y_C = \frac{1}{16}\frac{M_B L^3}{EI} \quad \text{(using Loading II result)}$$

$$\theta_A = \frac{M_B L}{6EI}$$

with $\quad M_B = \frac{wL^2}{12} \quad y_C = \frac{1}{192}\frac{wL^4}{EI} \quad \theta_A = \frac{1}{72}\frac{wL^3}{EI}$

(*a*) <u>Deflection at *C*</u>:

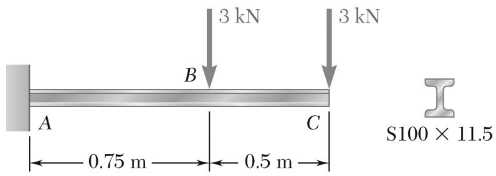
$$y_C = -\frac{5}{384}\frac{wL^4}{EI} + \frac{1}{192}\frac{wL^4}{EI} + \frac{1}{192}\frac{wL^4}{EI} = -\frac{1}{384}\frac{wL^4}{EI}$$

$$y_C = \frac{1}{384}\frac{wL^4}{EI} \downarrow \blacktriangleleft$$

(*b*) <u>Slope at *A*</u>:

$$\theta_A = -\frac{1}{24}\frac{wL^3}{EI} + \frac{1}{36}\frac{wL^3}{EI} + \frac{1}{72}\frac{wL^3}{EI} = 0$$

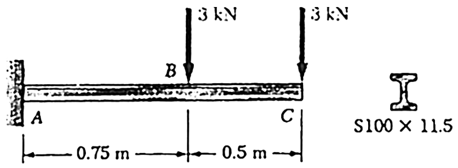
$$\theta_A = 0 \blacktriangleleft$$

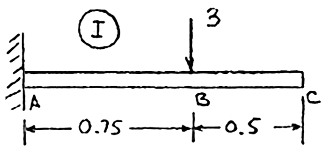


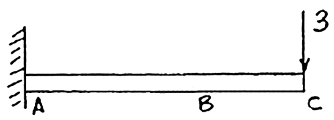
PROBLEM 9.73

For the cantilever beam and loading shown, determine the slope and deflection at end C. Use $E = 200$ GPa.

SOLUTION







Units: Forces in kN; lengths in m.

Loading I: Concentrated load at B

Case 1 of Appendix D applied to portion AB.

$$\theta'_B = -\frac{PL^2}{2EI} = -\frac{(3)(0.75)^2}{2EI} = -\frac{0.84375}{EI}$$

$$y'_B = -\frac{PL^3}{3EI} = -\frac{(3)(0.75)^3}{3EI} = -\frac{0.421875}{EI}$$

Portion BC remains straight.

$$\theta'_C = \theta'_B = -\frac{0.84375}{EI}$$

$$y'_C = y'_B - (0.5)\theta'_B = -\frac{0.84375}{EI}$$

Loading II: Concentrated load at C: Case 1 of Appendix D.

$$\theta''_A = -\frac{PL^2}{2EI} = -\frac{(3)(1.25)^2}{2EI} = -\frac{2.34375}{EI}$$

$$y''_A = -\frac{PL^3}{3EI} = -\frac{(3)(1.25)^3}{3EI} = -\frac{1.953125}{EI}$$

By superposition,

$$\theta_A = \theta'_A + \theta''_A = -\frac{3.1875}{EI}$$

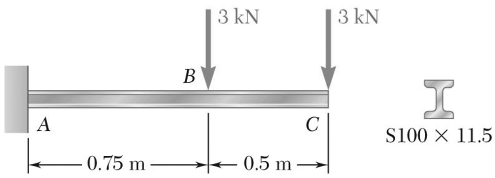
$$y_A = y'_A + y''_A = -\frac{2.796875}{EI}$$

Data: $E = 200 \times 10^9 \text{ Pa}, \quad I = 2.52 \times 10^6 \text{ mm}^4 = 2.52 \times 10^{-6} \text{ m}^4$

$$EI = (200 \times 10^4)(2.52 \times 10^{-6}) = 504 \times 10^3 \text{ N} \cdot \text{m}^2 = 504 \text{ kN} \cdot \text{m}^2$$

Slope at C: $\theta_C = -\frac{3.1875}{504} = -6.32 \times 10^{-3}$ rad $\qquad \theta_C = 6.32 \times 10^{-3}$ rad ◀

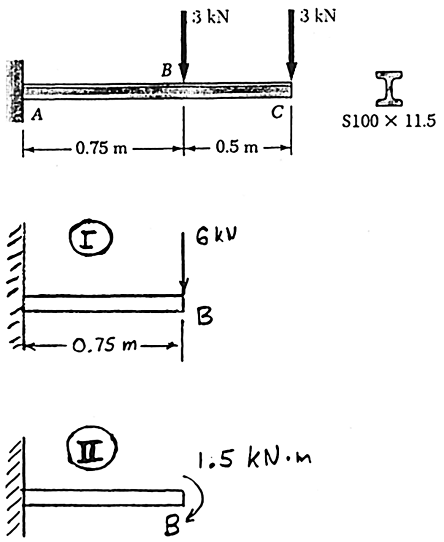
Deflection at C: $y_C = -\frac{2.796875}{504} = -5.55 \times 10^{-3}$ m $\qquad y_C = 5.55$ mm $\downarrow$ ◀



PROBLEM 9.74

For the cantilever beam and loading shown, determine the slope and deflection at point B. Use $E = 200$ GPa.

SOLUTION



Units: Forces in kN; lengths in m.

The slope and deflection at B depend only on the deformation of portion AB.

Reduce the force at C to an equivalent force-couple system at B and add the force already at B to obtain the loadings I and II shown.

Loading I: Case 1 of Appendix D.

$$\theta'_B = -\frac{PL^2}{2EI} = -\frac{(6)(0.75)^2}{2EI} = -\frac{1.6875}{EI}$$

$$y'_B = -\frac{PL^3}{3EI} = -\frac{(6)(0.75)^3}{3EI} = -\frac{0.84375}{EI}$$

Loading II: Case 3 of Appendix D.

$$\theta''_B = -\frac{ML}{EI} = -\frac{(1.5)(0.75)}{EI} = -\frac{1.125}{EI}$$

$$y''_B = -\frac{ML^2}{2EI} = -\frac{(1.5)(0.75)^2}{EI} = -\frac{0.421875}{EI}$$

By superposition,

$$\theta_B = \theta'_B + \theta''_B = -\frac{2.8125}{EI}$$

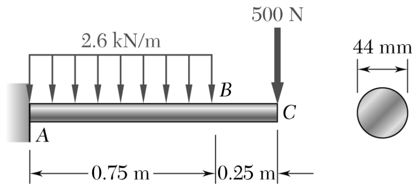
$$y_B = y'_B + y''_B = -\frac{1.265625}{EI}$$

Data: $E = 200 \times 10^9\ \text{Pa}, \quad I = 2.52 \times 10^6\ \text{mm}^4 = 2.52 \times 10^{-6}\ \text{m}^4$

$$EI = (200 \times 10^9)(2.52 \times 10^{-6}) = 504 \times 10^3\ \text{N} \cdot \text{m}^2 = 504\ \text{kN} \cdot \text{m}^2$$

Slope at B: $\theta_B = -\dfrac{2.8125}{504} = -5.58 \times 10^{-3}$ rad $\qquad \theta_B = 5.58 \times 10^{-3}$ rad ◀

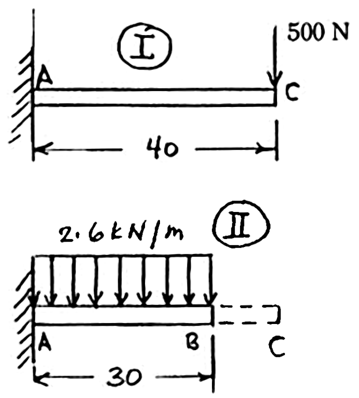
Deflection at B: $y_B = -\dfrac{1.265625}{504} = -2.51 \times 10^{-3}$ m $\qquad y_B = 2.51$ mm ↓ ◀



PROBLEM 9.75

For the cantilever beam and loading shown, determine the slope and deflection at end C. Use $E = 200$ GPa.

SOLUTION



$$C = \frac{1}{2}d = \frac{1}{2}(44) = 22 \text{ mm}$$

$$I = \frac{\pi}{4}C^4 = \frac{\pi}{4}(22)^4 = 183984 \text{ mm}^4$$

$$EI = (200 \times 10^6)(183984 \times 10^{-12}) = 36.8 \text{ kNm}^2$$

Loading I: Case 1 of Appendix D.

$$P = 0.5 \text{ kN}, \quad L = 1 \text{ m}$$

$$(y_C)_1 = -\frac{PL^3}{3EI} = -\frac{(0.5)(1)^3}{(3)(36.8)} = -4.529 \times 10^{-3} \text{ m}$$

$$(\theta_C)_1 = -\frac{PL^2}{2EI} = -\frac{(0.5)(1)^2}{(2)(36.8)} = -6.793 \times 10^{-3}$$

Loading II. Treat portion AB as a cantilever beam. (Case 2)

$$w = 2.6 \text{ kN/m}, \quad L = 0.75 \text{ m}$$

$$(y_B)_2 = -\frac{wL^4}{8EI} = -\frac{(2.6)(0.75)^4}{(8)(36.8)} = -2.794 \times 10^{-3} \text{ m}$$

$$(\theta_B)_2 = -\frac{wL^3}{6EI} = -\frac{(2.6)(0.75)^3}{(6)(36.8)} = -4.968 \times 10^{-3}$$

Portion BC remains straight for loading II.

$$L_{BC} = 0.25 \text{ m}$$

$$(y_C)_2 = (y_B)_2 + L_{BC}(\theta_B)_2 = -4.036 \times 10^{-3} \text{ m}$$

$$(\theta_C)_2 = (\theta_B)_2 = -4.968 \times 10^{-3}$$

Slope at end C: By superposition, $\theta_C = (\theta_C)_1 + (\theta_C)_2$

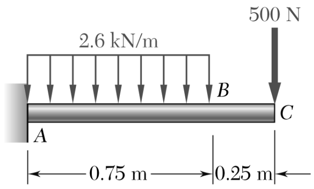
$$\theta_C = -11.761 \times 10^{-3}$$

$\theta_C = 11.76 \times 10^{-3}$ rad ◀

Deflection at end C: By superposition, $y_C = (y_C)_1 + (y_C)_2$

$$y_C = -8.565 \times 10^{-3} \text{ m}$$

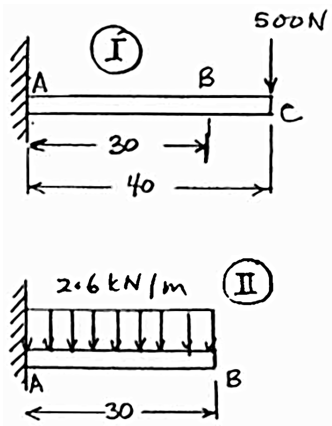
$y_C = 8.57$ mm ↓ ◀



PROBLEM 9.76

For the cantilever beam and loading shown, determine the slope and deflection at point B. Use $E = 200$ GPa.

SOLUTION



$$C = \frac{1}{2}d = \frac{1}{2}(44) = 22 \text{ mm}$$

$$I = \frac{\pi}{4}C^4 = \frac{\pi}{4}(22)^4 = 183984 \text{ mm}^4$$

$$EI = (200\times 10^6)(183984\times 10^{-6}) = 36.8 \text{ kNm}^2$$

Loading I: Case 1 of Appendix D.

$$P = 0.5 \text{ kN} \quad L = 1 \text{ m} \quad x = 0.75 \text{ m}$$

$$y_1 = \frac{P}{6EI}(x^3 - 3Lx^2)$$

$$\theta_1 = \frac{dy}{dx} = \frac{P}{2EI}(x^2 - 2Lx)$$

$$(y_B)_1 = \frac{0.5}{(6)(36.8)}[(0.75)^3 - (3)(1)(0.75)^2]$$

$$= -2.866\times 10^{-3} \text{ mm}$$

$$(\theta_B)_1 = \frac{0.5}{(2)(36.8)}[(0.75)^2 - (2)(1)(0.75)]$$

$$= -6.369\times 10^{-3}$$

Loading II: Case 2 of Appendix D.

$$w = 2.6 \text{ kN/m}, \quad L = 0.75 \text{ m}$$

$$(y_B)_2 = -\frac{wL^4}{8EI} = -\frac{(2.6)(0.75)^4}{(8)(36.8)} = -2.794\times 10^{-3} \text{ m}$$

$$(\theta_B)_2 = -\frac{wL^3}{6EI} = -\frac{(2.6)(0.75)^3}{(6)(36.8)} = -4.968\times 10^{-3}$$

Slope at point B: $\theta_B = (\theta_B)_1 + (\theta_B)_2$

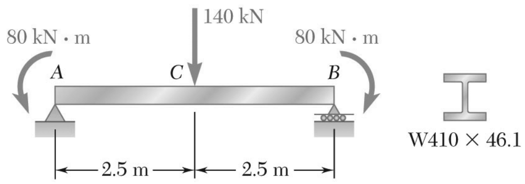
$$\theta_B = -11.337\times 10^{-3}$$

$\theta_B = 11.34\times 10^{-3}$ rad ◀

Deflection at point B: $y_B = (y_B)_1 + (y_B)_2$

$$y_B = -5.66\times 10^{-3} \text{ m}$$

$y_B = 5.66$ mm $\downarrow$ ◀



PROBLEM 9.77

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in m.

Loading I: Moment at *B*.

Case 7 of Appendix D: $M = 80\ \text{kN}\cdot\text{m}, \quad L = 5.0\ \text{m}, \quad x = 2.5\ \text{m}$

$$\theta_A = \frac{ML}{6EI} = \frac{(80)(5.0)}{6EI} = \frac{66.667}{EI}$$

$$y_C = -\frac{M}{6EIL}(x^3 - L^2 x) = -\frac{80}{6EI(5.0)}[2.5^3 - (5.0)^2(2.5)] = \frac{125}{EI}$$

Loading II: Moment at *A*: (Case 7 of Appendix D.)

$$M = 80\ \text{kN}\cdot\text{m}, \quad L = 5.0\ \text{m}, \quad x = 2.5\ \text{m}$$

$$\theta_A = \frac{ML}{3EI} = \frac{(80)(5.0)}{3EI} = \frac{133.333}{EI}$$

$$y_C = \frac{125}{EI} \qquad \text{(Same as loading I.)}$$

Loading III: 140 kN concentrated load at *C*: $P = 140$ kN

$$\theta_A = -\frac{PL^2}{16EI} = -\frac{(140)(5.0)^2}{16EI} = -\frac{218.75}{EI}$$

$$y_C = -\frac{PL^3}{48EI} = -\frac{(140)(5.0)^3}{48EI} = -\frac{364.583}{EI}$$

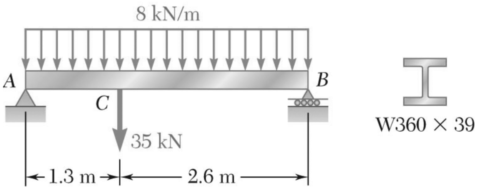
Data: $E = 200\times10^9\ \text{Pa}, \quad I = 156\times10^6\ \text{mm}^4 = 156\times10^{-6}\ \text{m}^4$

$$EI = (200\times10^9)(156\times10^{-6}) = 31.2\times10^6\ \text{N}\cdot\text{m}^2 = 31200\ \text{kN}\cdot\text{m}^2$$

(*a*) Slope at *A*: $\theta_A = \dfrac{67.667 + 133.333 - 218.75}{31200} = -0.601\times10^{-3}\ \text{rad}$

$\theta_A = 0.601\times10^{-3}\ \text{rad}$ ◀

(*b*) Deflection at *C*: $y_C = \dfrac{125 + 125 - 364.583}{31200} = -3.67\times10^{-3}\ \text{m}$ $\qquad y_C = 3.67\ \text{mm} \downarrow$ ◀



PROBLEM 9.78

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION

<u>Units</u>: Forces in kN; lengths in m.

<u>Loading I:</u> 8 kN/m uniformly distributed.

Case 6: $w = 8\text{ kN/m}, \quad L = 3.9\text{ m}, \quad x = 1.3\text{ m}$

$$\theta_A = -\frac{WL^3}{24EI} = -\frac{(8)(3.9)^3}{24EI} = -\frac{19.773}{EI}$$

$$y_C = -\frac{w}{24EI}[x^4 - 2Lx^3 + L^3x] = -\frac{8}{24EI}[(1.3)^4 - (2)(3.9)(1.3)^3 + (3.9)^3(1.3)]$$

$$= -\frac{20.945}{EI}$$

<u>Loading II:</u> 35 kN concentrated load at *C*. Case 5 of Appendix D.

$P = 35\text{ kN}, \quad L = 3.9\text{ m}, \quad a = 1.3\text{ m}, \quad b = 2.6\text{ m}, \quad x = a = 1.3\text{ m}$

$$\theta_A = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(35)(2.6)(3.9^2 - 2.6)^2}{6EI(3.9)} = -\frac{32.861}{EI}$$

$$y_C = -\frac{Pa^2b^2}{3EIL} = -\frac{(35)(1.3)^2(2.6)^2}{3EI(3.9)} = -\frac{34.176}{EI}$$

Data: $E = 200 \times 10^9, \quad I = 102 \times 10^6\text{ mm}^4 = 102 \times 10^{-6}\text{ m}^4$

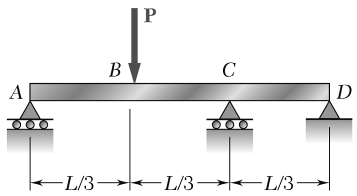
$$EI = (200 \times 10^9)(102 \times 10^{-6}) = 20.4 \times 10^6\text{ N} \cdot \text{m}^2 = 20{,}400\text{ kN} \cdot \text{m}^2$$

(*a*) <u>Slope at *A*</u>:
$$\theta_A = -\frac{19.773 + 32.861}{20{,}400} = -2.58 \times 10^{-3}\text{ rad}$$

$\theta_A = 2.58 \times 10^{-3}$ rad ◀

(*b*) <u>Deflection at *C*</u>:
$$y_C = -\frac{20.934 + 34.176}{20{,}400} = -2.70 \times 10^{-3}\text{ m}$$

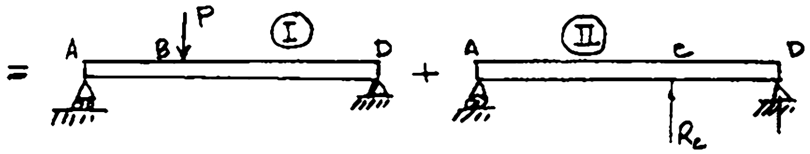
$y_C = 2.70\text{ mm} \downarrow$ ◀



PROBLEM 9.79

For the uniform beam shown, determine the reaction at each of the three supports.

SOLUTION



Consider R_C as redundant and replace loading system by I and II.

Loading I: (Case 5 of Appendix D). $a = \frac{2L}{3}, \quad b = \frac{L}{3}, \quad x = \frac{L}{3}$ at C.

$$(y_C)_1 = \frac{Pb}{6EIL}[x^3 - (L^2 - b^2)x] = \frac{P(L/3)}{6EIL}\left[\left(\frac{L}{3}\right)^3 - \left\{L^2 - \left(\frac{L}{3}\right)^2\right\}\frac{L}{3}\right]$$

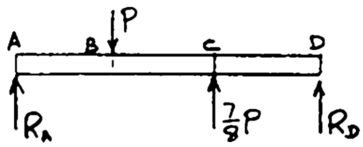
$$= -\frac{7PL^3}{486EI}$$

Loading II: (Case 5 of Appendix D). $a = \frac{2L}{3}, \quad b = \frac{L}{3}$

$$(y_C)_2 = \frac{R_C 2^2 b^2}{3EIL} = \frac{R_C (L/3)^2 (2L/3)^2}{3EIL} = \frac{4R_C L^3}{243EI}$$

Superposition and constraint: $y_C = (y_C)_1 + (y_C)_2 = 0$

$$-\frac{7PL^3}{486EI} + \frac{4R_C}{243EI} = 0 \qquad R_C = \frac{7}{8}P \uparrow \blacktriangleleft$$

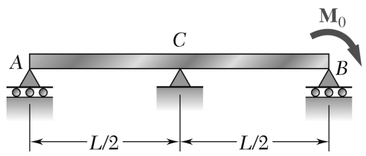


$+\circlearrowleft \Sigma M_D = 0$:

$$-R_A L + P\left(\frac{2L}{3}\right) - \left(\frac{7}{8}P\right)\left(\frac{L}{3}\right) = 0 \qquad R_A = \frac{3}{8}P \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_D - P + \frac{7}{8}P = 0$$

$$R_D = P - \frac{7}{8}P - \frac{3}{8}P = -\frac{1}{4}P \qquad R_D = \frac{4}{4}P \downarrow \blacktriangleleft$$

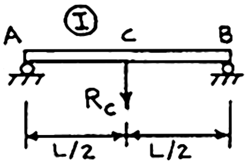


PROBLEM 9.80

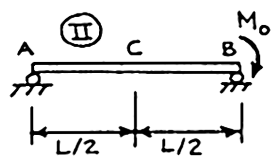
For the uniform beam shown, determine the reaction at each of the three supports.

SOLUTION

Consider R_C as redundant and replace loading system by I and II.



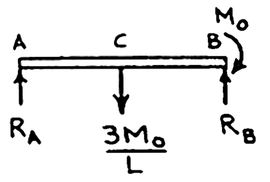
Loading I. (Case 4 of Appendix D): $Y_C' = -\dfrac{R_C L^3}{48EI}$



Loading II. (Case 7 of Appendix D): $Y_C'' = -\dfrac{M_0}{6EIL}\left[\left(\dfrac{L}{2}\right)^3 - L^2\left(\dfrac{L}{2}\right)\right] = \dfrac{M_0 L^2}{16EI}$

Superposition and constraint. $\quad y_C = y_C' + y_C'' = 0$

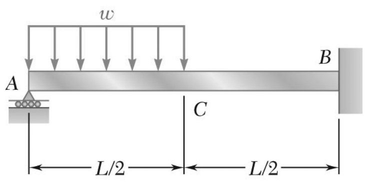
$$-\frac{R_C L^3}{48EI} + \frac{M_0 L^2}{16EI} = 0 \qquad R_C = \frac{3M_0}{L}\downarrow \blacktriangleleft$$



$$+\circlearrowleft \Sigma M_B = 0: \quad -R_A L + \left(\frac{3M_0}{L}\right)\left(\frac{L}{2}\right) - M_0 = 0 \qquad R_A = \frac{M_0}{2L}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{M_0}{2L} - \frac{3M_0}{L} + R_B = 0$$

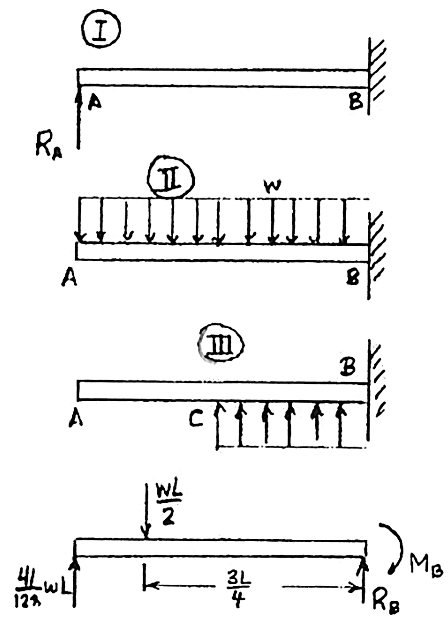
$$R_B = \frac{5M_0}{2L}\uparrow \blacktriangleleft$$



PROBLEM 9.81

For the uniform beam shown, determine (*a*) the reaction at *A,* (*b*) the reaction at *B*.

SOLUTION



Beam is indeterminate to first degree. Consider R_A as redundant and replace the given loading by loadings I, II, and III.

Loading I: (Case 1 of Appendix D.)

$$(y_A)_I = \frac{R_A L^3}{3EI}$$

Loading II: (Case 2 of Appendix D.)

$$(y_A)_{II} = -\frac{wL^4}{8EI}$$

Loading III: (Case 2 of Appendix D (portion *CB*).)

$$(\theta_C)_{III} = -\frac{w(L/2)^3}{6EI} = -\frac{1}{48}\frac{wL^3}{EI}$$

$$(y_C)_{III} = \frac{w(L/2)^4}{8EI} = \frac{1}{128}\frac{wL^4}{EI}$$

Portion AC remains straight.

$$(y_A)_{III} = (y_C)_{III} + \frac{L}{2}(\theta_C)_{III} = \frac{7}{384}\frac{wL^4}{EI}$$

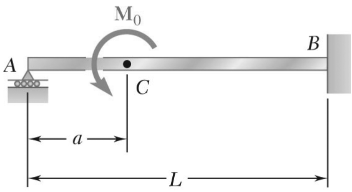
Superposition and constraint: $y_A = (y_A)_I + (y_A)_{II} + (y_A)_{III} = 0$

(*a*) $\frac{1}{3}\frac{R_A L^3}{3EI} - \frac{1}{8}\frac{wL^4}{EI} + \frac{7}{384}\frac{wL^4}{EI} = \frac{1}{3}\frac{R_A L^3}{EI} - \frac{41}{384}\frac{wL^4}{EI} = 0$ $\qquad R_A = \frac{41}{128}wL \uparrow$ ◀

Statics:

(*b*) $+\uparrow \Sigma F_y = 0: \quad \frac{41}{128}wL - \frac{1}{2}wL + R_B = 0$ $\qquad R_B = \frac{23}{128}wL \uparrow$ ◀

$+\circlearrowleft \Sigma M_B = 0: \quad -\left(\frac{41}{128}wL\right)L - \left(\frac{1}{2}wL\right)\left(\frac{3L}{4}\right) - M_B = 0$ $\qquad M_B = \frac{7}{128}wL^2 \circlearrowright$ ◀

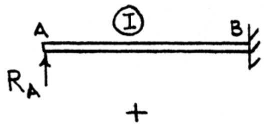


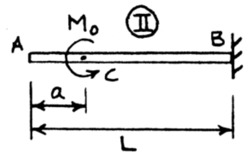
PROBLEM 9.82

For the uniform beam shown, determine (*a*) the reaction at *A*, (*b*) the reaction at *B*.

SOLUTION

Consider R_A as redundant and replace loading system by I and II.





<u>Loading I</u>: (Case 1 of Appendix D.) $\quad y_A' = \dfrac{R_A L^3}{3EI}$

<u>Loading II</u>: Portion BC. (Case 3 of Appendix D.)

$$y_C'' = -\frac{M_0(L-a)^2}{2EI} \qquad \theta_C'' = \frac{M_0(L-a)}{EI}$$

Portion AC is straight.

$$y_A'' = y_C'' - (a)\theta_C$$

$$= -\frac{M_0(L-a)^2}{2EI} - \frac{aM_0(L-a)}{EI}$$

(*a*) Superposition and constraint: $\quad y_A = y_A' + y_A'' = 0$

$$\frac{R_A L^3}{3EI} - \frac{M_0(L-a)^2}{2EI} - \frac{aM_0(L-a)}{EI} = 0$$

$$\frac{2}{3}R_A L^3 - M_0(L-a)(L-a+2a) = 0$$

$$\frac{2}{3}R_A L^3 = M_0(L^2 - a^2)$$

$$R_A = \frac{3M_0}{2L^3}(L^2 - a^2)\uparrow \blacktriangleleft$$

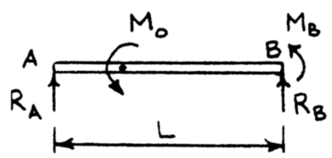
(*b*) $+\uparrow \Sigma F_y = 0: \quad R_A + R_B = 0 \qquad R_B = -R_A$

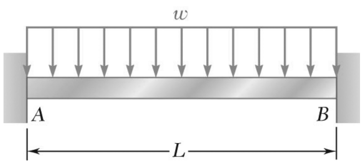
$$R_B = \frac{3M_0}{2L^3}(L^2 - a^2)\downarrow \blacktriangleleft$$

$+\circlearrowleft \Sigma M_B = 0: \quad M_B + M_0 - R_A L = 0 \qquad M_B + M_0 - \dfrac{3}{2}\dfrac{M_0}{L^2}(L^2 - a^2) = 0$

$$M_B = \frac{3M_0}{2L^2}\left(L^2 - a^2 - \frac{2}{3}L^2\right)$$

$$M_B = \frac{M_0}{2L^2}(L^2 - 3a^2) \circlearrowleft \blacktriangleleft$$



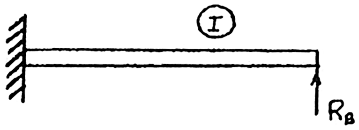


PROBLEM 9.83

For the beam shown, determine the reaction at B.

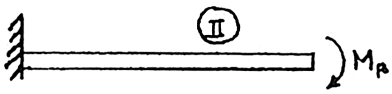
SOLUTION

Beam is second degree indeterminate. Choose R_B and M_B as redundant reactions.



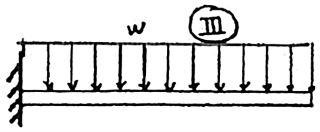
Loading I: Case 1 of Appendix D.

$$(y_B)_I = \frac{R_B L^3}{3EI} \qquad (\theta_B)_I = \frac{R_B L^2}{2EI}$$



Loading II: Case 3 of Appendix D.

$$(y_B)_{II} = -\frac{M_B L^2}{2EI} \qquad (\theta_B)_{II} = -\frac{M_B L}{EI}$$



Loading III: Case 2 of Appendix D.

$$(y_B)_{III} = -\frac{wL^4}{8EI} \qquad (\theta_B)_{III} = -\frac{wL^2}{6EI}$$

Superposition and constraint:

$$y_B = (y_B)_{\mathrm{I}} + (y_B)_{\mathrm{II}} + (y_B)_{\mathrm{III}} = 0$$

$$\frac{L^3}{3EI} R_B - \frac{L^2}{2EI} M_B - \frac{wL^4}{8EI} = 0 \qquad (1)$$

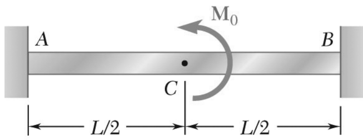
$$\theta_B = (\theta_B)_{\mathrm{I}} + (\theta_B)_{\mathrm{II}} + (\theta_B)_{\mathrm{III}} = 0$$

$$\frac{L^2}{2EI} R_B - \frac{L}{EI} M_B - \frac{wL^3}{6EI} = 0 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$R_B = \frac{1}{2} wL \uparrow \blacktriangleleft$$

$$M_B = \frac{1}{12} wL^2 \circlearrowright \blacktriangleleft$$



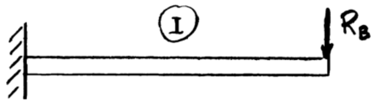
PROBLEM 9.84

For the beam shown, determine the reaction at B.

SOLUTION

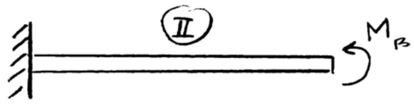
Beam is second degree indeterminate. Choose R_B and M_B as redundant reactions.

<u>Loading I</u>: Case 1 of Appendix D.



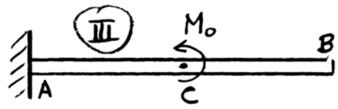
$$(y_B)_{\mathrm{I}} = -\frac{R_B L^3}{3EI}, \quad (\theta_B)_{\mathrm{I}} = -\frac{R_B L^2}{2EI}$$

<u>Loading II</u>: Case 3 of Appendix D.



$$(y_B)_{\mathrm{II}} = \frac{M_B L^2}{2EI}, \quad (\theta_B)_{\mathrm{II}} = \frac{M_B L}{EI}$$

<u>Loading III</u>: Case 3 applied to portion AC.



$$(y_C)_{\mathrm{III}} = \frac{M_0 (L/2)^2}{2EI} = \frac{M_0 L^2}{8EI}$$

$$(\theta_C)_{\mathrm{III}} = \frac{M_0 (L/2)}{EI} = \frac{M_0 L}{2EI}$$

Portion CB remains straight.

$$(y_B)_{\mathrm{III}} = (y_C)_{\mathrm{III}} + \frac{L}{2}(\theta_C)_{\mathrm{III}} = \frac{3}{8}\frac{M_0 L^2}{EI}$$

$$(\theta_B)_{\mathrm{III}} = (\theta_C)_{\mathrm{III}} = \frac{1}{2}\frac{M_0 L^2}{EI}$$

Superposition and constraint:

$$y_B = (y_B)_{\mathrm{I}} + (y_B)_{\mathrm{II}} + (y_B)_{\mathrm{III}} = 0$$

$$-\frac{L^3}{3EI}R_B + \frac{L^2}{2EI}M_B + \frac{3}{8}\frac{M_0 L^2}{EI} = 0 \tag{1}$$

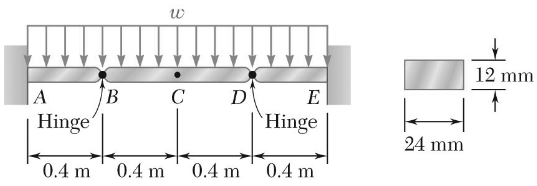
$$\theta_B = (\theta_B)_{\mathrm{I}} + (\theta_B)_{\mathrm{II}} + (\theta_B)_{\mathrm{III}} = 0$$

$$-\frac{L^2}{2EI}R_B + \frac{L}{EI}M_B + \frac{1}{2}\frac{M_0 L}{EI} = 0 \tag{2}$$

Solving Eqs. (1) and (2) simultaneously,

$$R_B = \frac{3}{2}\frac{M_0}{L} \downarrow \blacktriangleleft$$

$$M_B = \frac{1}{4}M_0 \circlearrowright \blacktriangleleft$$



PROBLEM 9.85

A central beam BD is joined at hinges to two cantilever beams AB and DE. All beams have the cross section shown. For the loading shown, determine the largest w so that the deflection at C does not exceed 3 mm. Use $E = 200$ GPa.

SOLUTION

Let $a = 0.4$ m.

Cantilever beams AB and CD.

Cases 1 and 2 of Appendix D. $$y_B = y_D = -\frac{(wa)a^3}{3EI} - \frac{wa^4}{8EI} = -\frac{11}{24}\frac{wa^4}{EI}$$

Beam BCD, with $L = 0.8$ m, assuming that points B and D do not move.

Case 6 of Appendix D.

$$y'_C = -\frac{5wL^4}{384EI}$$

Additional deflection due to movement of points B and D.

$$y''_C = y_B = y_D = -\frac{11}{24}\frac{wa^4}{EI}$$

Total deflection at C: $$y_C = y'_C + y''_C$$

$$y_C = -\frac{w}{EI}\left\{\frac{5L^4}{384} + \frac{11a^4}{24}\right\}$$

Data: $$E = 200\times 10^9\,\text{Pa},$$

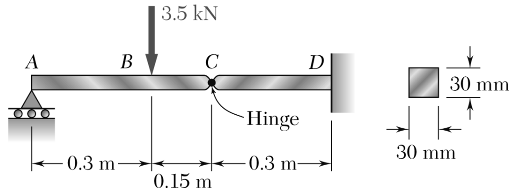
$$I = \frac{1}{12}(24)(12)^3 = 3.456\times 10^{-3}\,\text{mm}^4 = 3.456\times 10^{-9}\,\text{m}^4$$

$$EI = (200\times 10^9)(3.456\times 10^{-9}) = 691.2\ \text{N}\cdot\text{m}^2$$

$$y_C = -3\times 10^{-3}\,\text{m}$$

$$-3\times 10^{-3} = -\frac{w}{691.2}\left\{\frac{(5)(0.8)^4}{384} + \frac{(11)(0.4)^4}{24}\right\} = -24.69^{-6}\,w$$

$w = 121.5$ N/m ◀

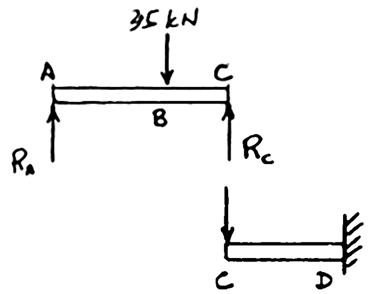


PROBLEM 9.86

The two beams shown have the same cross section and are joined by a hinge at C. For the loading shown, determine (*a*) the slope at point A, (*b*) the deflection at point B. Use $E = 200$ GPa.

SOLUTION

Using free body ABC,



$$+\circlearrowleft \Sigma M_A = 0: \quad 0.45R_C - (0.3)(3.5) = 0 \qquad R_C = 2.33\text{ kN}$$

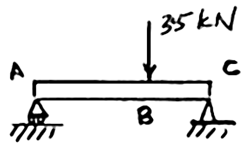
$$E = 200\text{ GPa}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(30)(30)^3 = 67500\text{ mm}^4$$

$$EI = (200\times 10^6)(67500\times 10^{-12}) = 13.5\text{ kNm}^2$$

Using cantilever beam CD with load R_C,

Case 1 of Appendix D:



$$y_C = -\frac{R_C L_{CD}^3}{3EI} = -\frac{(2.33)(0.3)^3}{(3)(13.5)} = 0.001555\text{ mm}$$

Calculation of θ_A' and y_B' assuming that point C does not move.

Case 5 of Appendix D:

$$P = 3.5\text{ kN}, \quad L = 0.45\text{ m}, \quad a = 0.3\text{ m}, \quad b = 0.15\text{ m}$$

$$\theta_A' = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(3.5)(0.15)(0.45^2 - 0.15^2)}{(6)(13.5)(0.45)} = -0.00259\text{ rad}$$

$$y_B' = -\frac{Pb^2a^2}{3EIL} = -\frac{(3.5)(0.15)^2(0.3)^2}{(3)(13.5)(0.45)} = -0.000389\text{ m}$$

Additional slope and deflection due to movement of point C.

$$\theta_A'' = \frac{y_C}{L_{AC}} = -\frac{0.001555}{0.45} = -0.003456\text{ rad}$$

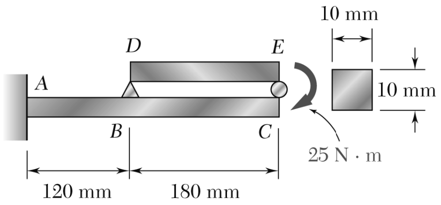
$$y_B'' = \frac{a}{L}y_C = -\frac{(0.3)(0.001555)}{0.45} = -0.0010367\text{ mm}$$

(*a*) Slope at A: $\theta_A = \theta_A' + \theta_A'' = -0.00259 - 0.003456$

$$= -0.006046\text{ rad} = 0.00605\text{ rad} \blacktriangleleft$$

(*b*) Deflection at B: $y_B = y_B' + y_B'' = -0.000389 - 0.0010367$

$$= -1.4257\times 10^{-3}\text{ m} = 1.43\text{ mm} \downarrow \blacktriangleleft$$



PROBLEM 9.87

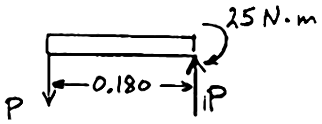
Beam DE rests on the cantilever beam AC as shown. Knowing that a square rod of side 10 mm is used for each beam, determine the deflection at end C if the 25-N · m couple is applied (*a*) to end E of beam DE, (*b*) to end C of beam AC. Use $E = 200$ GPa.

SOLUTION

$$E = 200\times 10^9 \text{ Pa}$$

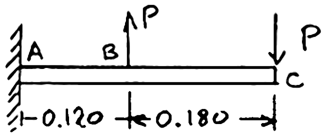
$$I = \frac{1}{12}(10)(10)^3 = 833.33 \text{ mm}^4 = 833.33\times 10^{-12} \text{ m}^4$$

$$EI = 166.667 \text{ N}\cdot\text{m}^2$$



(*a*) Couple applied to beam DE.

Free body DE. $+\circlearrowleft \Sigma M = 0$: $0.180P - 25 = 0 \quad P = 138.889$ N



Loads on cantilever beam ABC are $P\uparrow$ at point B and $P\downarrow$ at point C as shown.

Due to $P\uparrow$ at point B.

Using portion AB and applying Case 1 of Appendix D,

$$(y_B)_1 = \frac{PL^3}{3EI} = \frac{(138.889)(0.120)^3}{(3)(166.667)} = 0.480\times 10^{-3} \text{ m}$$

$$(\theta_B)_1 = \frac{PL^2}{2EI} = \frac{(138.889)(0.120)^2}{(2)(166.667)} = 6.00\times 10^{-3}$$

$$(y_C)_1 = (y_B)_1 + L_{BC}(\theta_B)_1 = 0.480\times 10^{-3} + (0.180)(6.00\times 10^{-3})$$

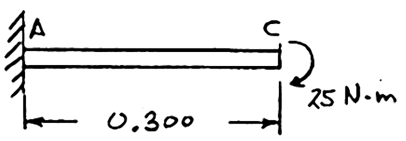
$$= 1.56\times 10^{-3} \text{ m}$$

Due to load $P\downarrow$ at point C: (Case 1 of App. D applied to ABC.)

$$(y_C)_2 = -\frac{PL^3}{3EI} = -\frac{(138.889)(0.120+0.180)^3}{(3)(166.667)} = -7.50\times 10^{-3} \text{ m}$$

Total deflection at point C: $\quad y_C = (y_C)_1 + (y_C)_2 = -5.94\times 10^{-3}$ m

$$y_C = 5.94 \text{ mm}\downarrow \blacktriangleleft$$

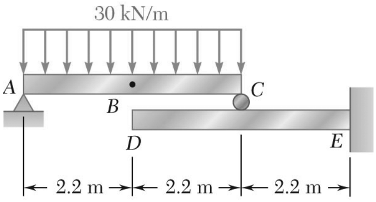


(*b*) Couple applied to beam AC:

Case 3 of Appendix D.

$$y_C = -\frac{ML^2}{2EI} = -\frac{(25)(0.300)^2}{(2)(166.667)} = -6.75\times 10^{-3} \text{ m}$$

$$y_C = 6.75 \text{ mm}\downarrow \blacktriangleleft$$



PROBLEM 9.88

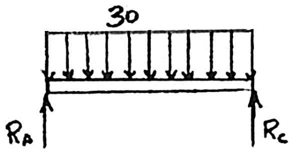
Beam AC rests on the cantilever beam DE as shown. Knowing that a $W410 \times 38.8$ rolled-steel shape is used for each beam, determine for the loading shown (*a*) the deflection at point B, (*b*) the deflection at point D. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in m.

Using free body ABC,

$$+\circlearrowleft \Sigma M_A = 0: \; 4.4 R_C - (4.4)(30)(2.2) = 0$$



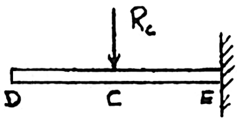
$$R_C = 66.0 \text{ kN}$$

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 125 \times 10^6 \text{ mm}^4 = 125 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(125 \times 10^{-6}) = 25.0 \times 10^{-6} \text{ N} \cdot \text{m}^2$$

$$= 25{,}000 \text{ kN} \cdot \text{m}^2$$



For slope and defection at C, use Case 1 of Appendix D applied to portion CE of beam DCE.

$$\theta_C = \frac{R_C L^2}{2EI} = \frac{(66.0)(2.2)^2}{(2)(25{,}000)} = 6.3888 \times 10^{-3} \text{ rad}$$

$$y_C = -\frac{R_C L^3}{3EI} = \frac{(66.0)(2.2)^3}{(3)(25{,}000)} = -9.3702 \times 10^{-3} \text{ m}$$

Defection at B, assuming that point C does not move.

Use Case 6 of Appendix D. $$(y_B)_1 = -\frac{5WL^4}{384EI} = -\frac{(5)(30)(4.4)^4}{(384)(25{,}000)} = -5.8564 \times 10^{-3}$$

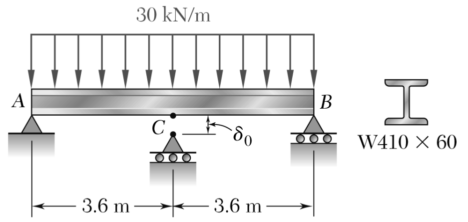
Additional defection at B due to movement of point C: $$(y_B)_2 = \frac{1}{2} y_C = -4.6851 \times 10^{-3} \text{ m}$$

(*a*) Total deflection at B: $$y_B = (y_B)_1 + (y_B)_2 = -10.54 \times 10^{-3} \text{ m} \qquad y_B = 10.54 \text{ mm} \downarrow \blacktriangleleft$$

Portion DC of beam DCB remains straight.

(*b*) Deflection at D: $$y_D = y_C - a\theta_C = -9.3702 \times 10^{-3} - (2.2)(6.3888 \times 10^{-3}) = -23.4 \times 10^{-3} \text{ m}$$

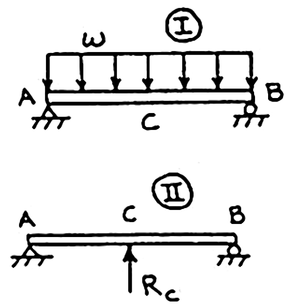
$$y_D = 23.4 \text{ mm} \downarrow \blacktriangleleft$$



PROBLEM 9.89

Before the 30 kN/m load is applied, a gap, $\delta_0 = 20$ mm exists between the W410×60 beam and the support at *C*. Knowing that $E = 200$ GPa, determine the reaction at each support after the uniformly distributed load is applied.

SOLUTION



Data: $\delta_0 = 0.02$ m

$$E = 200\times10^6 \text{ kPa}$$

$$I = 216\times10^{-6} \text{ m}^4$$

$$EI = 43.2\times10^3 \text{ kN}\cdot\text{m}^2$$

Loading I: Case 6 of Appendix D.

$$y'_C = -\frac{5wL^4}{384EI} = -\frac{5(30)(7.2)^4}{384(43.2\times10^3)}$$

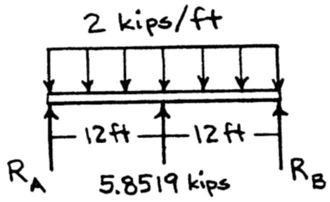
$$= -24.3\times10^{-3} \text{ m}$$

Loading II: Case 4 of Appendix D.

$$y''_C = \frac{R_C L^3}{48EI} = \frac{R_C(7.2)^3}{48(43.2\times10^3)}$$

$$= 0.18\times10^{-3} R_C$$

Deflection at *C*:



$$y_C = y'_C + y''_C = -\delta_0$$

$$-24.3\times10^{-3} + 0.18\times10^{-3} R_C = -20\times10^{-3}$$

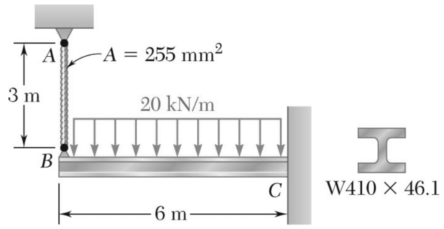
$$R_C = 23.889 \text{ kN} \qquad R_C = 23.9 \text{ kN}\uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (30)(7.2)(3.6) - R_A(7.2) - (23.889)(3.6) = 0$$

$$R_A = 96.055 \qquad R_A = 96.1 \text{ kN}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 96.055 - 30(7.2) + 23.889 + R_B = 0$$

$$R_B = 96.1 \text{ kN}\uparrow \blacktriangleleft$$



PROBLEM 9.90

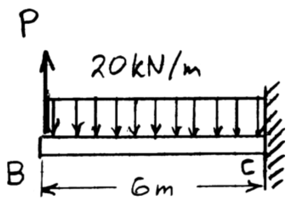
The cantilever beam BC is attached to the steel cable AB as shown. Knowing that the cable is initially taut, determine the tension in the cable caused by the distributed load shown. Use $E = 200$ GPa.

SOLUTION

Let P be the tension developed in member AB and δ_B be the elongation of that member.

Cable AB: $A = 255 \text{ mm}^2 = 255\times10^{-6} \text{ m}^2$

$$\delta_B = \frac{PL}{EA} = \frac{(P)(3)}{(200\times10^9)(255\times10^{-6})} = 58.82\times10^{-9}\,\text{P}$$



Beam BC: $I = 156\times10^6 \text{ mm}^4 = 156\times10^{-6}\,\text{m}^4$

$$EI = (200\times10^9)(156\times10^{-6}) = 31.2\times10^6\,\text{N}\cdot\text{m}^2$$

Loading I: 20 kN/m downward.

Refer to Case 2 of Appendix D.

$$(y_B)_1 = -\frac{wL^4}{8EI} = -\frac{(20\times10^3)(6)^4}{(8)(31.2\times10^6)} = -103.846\times10^{-3}\,\text{m}$$

Loading II: Upward force P at Point B.

Refer to Case 1 of Appendix D.

$$(y_B)_2 = \frac{PL^3}{3EI} = \frac{P(6)^3}{(3)(31.2\times10^6)} = 2.3077\times10^{-6}\,P$$

By superposition, $y_B = (y_B)_1 + (y_B)_2$

Also, matching the deflection at B,

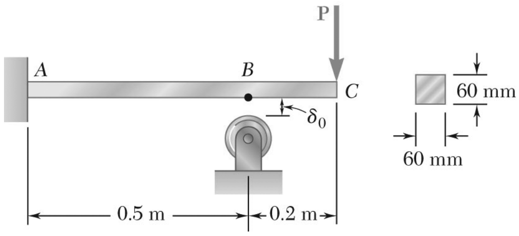
$$y_B = -\delta_B$$

$$-103.846\times10^{-3} + 2.3077\times10^{-6}\,P = -58.82\times10^{-9}\,P$$

$$2.3666\times10^{-6}\,P = 103.846\times10^{-3}$$

$P = 43.9\times10^3$ N

$P = 43.9$ kN ◀



PROBLEM 9.91

Before the load **P** was applied, a gap, $\delta_0 = 0.5\,\text{mm}$, existed between the cantilever beam AC and the support at B. Knowing that $E = 200$ GPa, determine the magnitude of **P** for which the deflection at C is 1 mm.

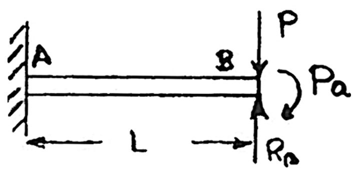
SOLUTION

Let length $AB = L = 0.5$ m

length $BC = a = 0.2$ m

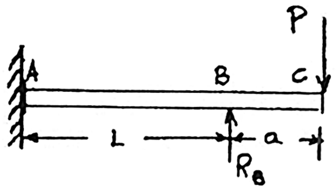
Consider portion AB of beam ABC.

The loading becomes forces P and R_B at B plus the couple Pa. The deflection at B is δ_0. Using Cases 1 and 3 of Appendix D,



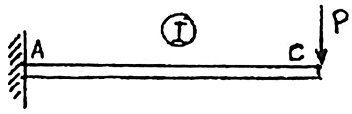
$$\delta_0 = \frac{(P - R_B)L^3}{3EI} + \frac{PaL^2}{2EI}$$

$$\left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)P - \frac{L^3}{3}R_B = EI\delta_0 \qquad (1)$$



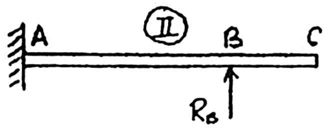
The deflection at C depends on the deformation of beam ABC subjected to loads P and R_B. For loading I, using Case 1 of Appendix D,

$$(\delta_C)_1 = \frac{P(L + a)^3}{3EI}$$



For loading II, using Case 1 of Appendix D,

$$y_B = \frac{R_B L^3}{3EI} \qquad \theta_B = \frac{R_B L^2}{2EI}$$



Portion BC remains straight.

$$y_C = y_B + a\theta_B = \left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)\frac{R_B}{EI}$$

By superposition, the downward deflection at C is

$$\delta_C = \frac{P(L + a)^3}{3EI} - \left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)\frac{R_B}{EI}$$

$$\frac{(L + a)^3}{3}P - \left(\frac{L^3}{3} + \frac{L^2 a}{2}\right)R_B = EI\delta_C \qquad (2)$$

PROBLEM 9.91 *(Continued)*

Data: $E = 200 \times 10^9$ Pa $\qquad I = \frac{1}{12}(60)(60)^3 = 1.08 \times 10^6 \text{ mm}^4 = 1.08 \times 10^{-6} \text{ m}^4$

$$EI = 216 \times 10^3 \text{ N} \cdot \text{m}^2$$

$$\delta_0 = 0.5 \times 10^{-3} \text{ m} \qquad \delta_C = 1.0 \times 10^{-3} \text{ m}$$

Using the data, eqs (1) and (2) become

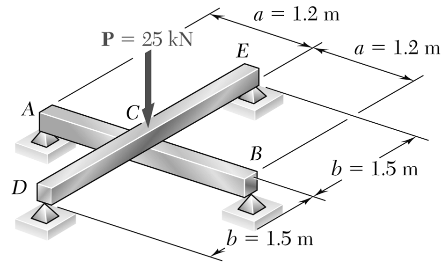
$$0.06667P - 0.04167R_B = 108 \qquad (1)'$$

$$0.11433P - 0.06667R_B = 216 \qquad (2)'$$

Solving simultaneously,

$$P = 5.63 \times 10^3 \text{ N} \qquad P = 5.63 \text{ kN} \downarrow \blacktriangleleft$$

$$R_B = 6.42 \times 10^3 \text{ N}$$



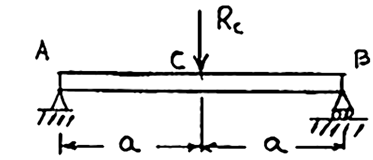
PROBLEM 9.92

For the loading shown, and knowing that beams *AB* and *DE* have the same flexural rigidity, determine the reaction (*a*) at *B*, (*b*) at *E*.

SOLUTION

Units: Forces in kN; lengths in m.

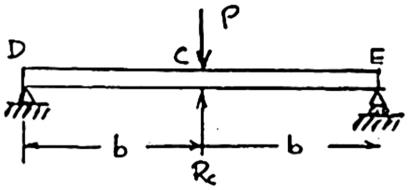
For beam *ACB,* using Case 4 of Appendix D.



$$(y_C)_1 = -\frac{R_C(2a)^3}{48EI}$$

For beam *DCE*, using Case 4 of Appendix D.

$$(y_C)_2 = \frac{(R_C - P)(2b)^3}{48EI}$$



Matching deflections at *C*,

$$-\frac{R_C(2a)^3}{48EI} = \frac{(R_C - P)(2b)^3}{48EI}$$

$$R_C = \frac{Pb^3}{a^3+b^3} = \frac{(25)(1.5)^3}{1.2^3+1.5^3} = 16.53 \text{ kN}$$

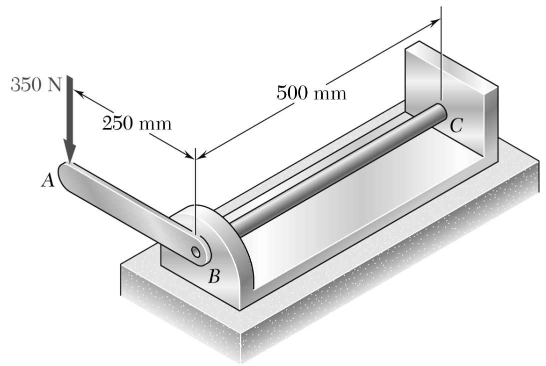
$$P - R_C = 25 - 16.53 = 8.47 \text{ kN}$$

Using free body *ACB*, $+\circlearrowleft M_A = 0$: $2aR_B - aR_C = 0$

$$(a) \quad R_B = \frac{1}{2}R_C = 8.265 \text{ kN}\uparrow \blacktriangleleft$$

Using free body *DCE*, $+\circlearrowleft M_D = 0$: $2bR_E - b(P - R_C) = 0$

$$(b) \quad R_E = \frac{1}{2}(P - R_C) = 4.235 \text{ kN}\uparrow \blacktriangleleft$$

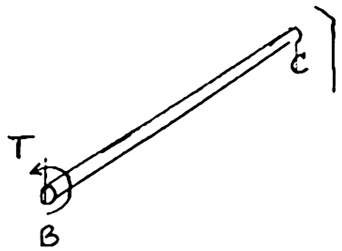


PROBLEM 9.93

A 22-mm-diameter rod BC is attached to the lever AB and to the fixed support at C. Lever AB has a uniform cross section 10 mm thick and 25 mm deep. For the loading shown, determine the deflection of point A. Use $E = 200$ GPa and $G = 77$ GPa

SOLUTION

Deformation of rod BC: (Torsion)



$$C = \frac{1}{2}d = \frac{1}{2}(22) = 11 \text{ mm}$$

$$J = \frac{\pi}{2}C^4 = 22998 \text{ mm}^4$$

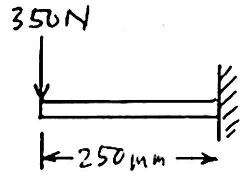
$$T = Pa = (350)(0.25) = 87.5 \text{ N}\cdot\text{m}$$

$$L = 0.5 \text{ m}$$

$$\varphi_B = \frac{TL}{GJ} = \frac{(87.5)(0.5)}{(77\times 10^9)(22998\times 10^{-12})}$$

$$= 0.0247 \text{ rad}$$

Deflection of point A assuming lever AB to be rigid:



$$(y_B)_1 = a\varphi_B = (0.25)(0.0247)$$

$$= 0.006175 \text{ m}\downarrow$$

Additional deflection due to bending of lever AB.

Refer to Case 1 of Appendix D.

$$I = \frac{1}{12}(10)(25)^3 = 13021 \text{ mm}^4$$

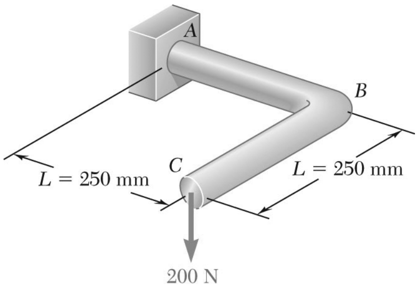
$$(y_A) = \frac{PL^3}{3EI} = \frac{(350)(0.25)^3}{(3)(200\times 10^9)(13021\times 10^{-12})}$$

$$= 2.1\times 10^{-3} \text{ m}\downarrow$$

Total deflection at point A:

$$y_A = (y_A)_1 + (y_A)_2 = 8.28 \text{ mm}\downarrow \blacktriangleleft$$

PROBLEM 9.94



A 16-mm-diameter rod has been bent into the shape shown. Determine the deflection of end C after the 200-N force is applied. Use $E = 200$ GPa and $G = 80$ GPa.

SOLUTION

Let $200\text{ N} = P$.

Consider torsion of rod AB.

$$\phi_B = \frac{TL}{JG} = \frac{(PL)L}{JG} = \frac{PL^2}{JG}$$

$$y'_C = -L\phi_B = -\frac{PL^3}{JG}$$

Consider bending of AB. (Case 1 of Appendix D.)

$$y''_C = y_B = -\frac{PL^3}{3EI}$$

Consider bending of BC. (Case 1 of Appendix D.)

$$y'''_C = -\frac{PL^3}{3EI}$$

Superposition:

$$y_C = y'_C + y''_C + y'''_C$$
$$= -\frac{PL^3}{JG} - \frac{PL^3}{3EI} - \frac{PL^3}{3EI} = -\frac{PL^3}{EI}\left(\frac{EI}{JG} + \frac{2}{3}\right)$$

Data:

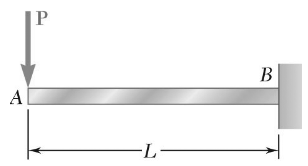
$$G = 80(10^9)\text{ Pa} \qquad J = \frac{1}{2}\pi(0.008)^4 = 6.4340(10^{-9})\text{ m}^4$$

$$E = 200(10^9)\text{ Pa} \qquad I = \frac{1}{2}J = 3.2170(10^{-9})\text{ m}^4$$

$$EI = 643.40\text{ N}\cdot\text{m}^2 \qquad JG = 514.72\text{ N}\cdot\text{m}^2$$

$$y_C = -\frac{(200)(0.25)^3}{643.40}\left(\frac{643.40}{514.72} + \frac{2}{3}\right) = -9.3093(10^{-3})\text{ m}$$

$$y_C = 9.31\text{ mm}\downarrow \blacktriangleleft$$



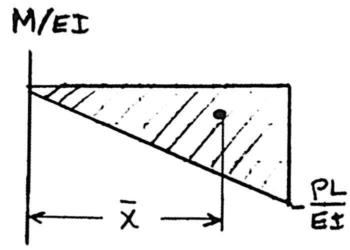
PROBLEM 9.95

For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

SOLUTION

Place reference tangent at *B*.

Draw *M*/*EI* diagram.



$$A = \frac{1}{2}\left(-\frac{PL}{EI}\right)L = -\frac{PL^2}{2EI}$$

$$\bar{x} = \frac{2}{3}L$$

$$\theta_{B/A} = A = -\frac{PL^2}{2EI}$$

$$t_{A/B} = A\bar{x} = \left(-\frac{PL^3}{2EI}\right)\left(\frac{2}{3}L\right) = -\frac{PL^3}{3EI}$$

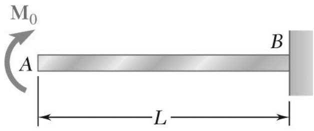
(*a*) Slope at end *A*:

$$\theta_B = \theta_A + A$$

$$0 = \theta_A - \frac{PL^2}{2EI} \qquad \theta_A = \frac{PL^2}{2EI} \blacktriangleleft$$

(*b*) Deflection at *A*:

$$y_A = t_{A/B} = -\frac{PL^3}{3EI} \qquad y_A = \frac{PL^3}{3EI} \downarrow \blacktriangleleft$$



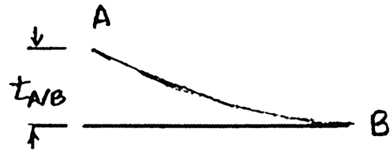
PROBLEM 9.96

For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

SOLUTION

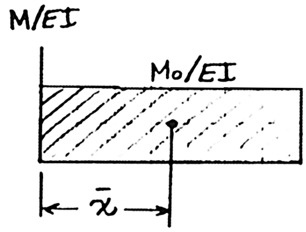
Place reference tangent at *B*.

Draw *M*/*EI* diagram.



$$A = \left(\frac{M_0}{EI}\right)L = \frac{M_0 L}{EI}$$

$$\bar{x} = \frac{1}{2}L$$



$$\theta_{B/A} = A = \frac{M_0 L}{EI}$$

$$t_{B/A} = A\bar{x} = \left(\frac{M_0 L}{EI}\right)\left(\frac{1}{2}L\right)$$

$$= \frac{M_0 L^2}{2EI}$$

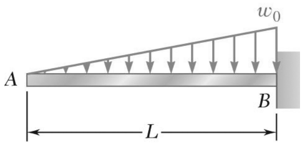
(*a*) Slope at end *A*:

$$\theta_B = \theta_A + A \qquad 0 = \theta_A + \frac{M_0 L}{EI}$$

$$\theta_A = -\frac{M_0 L}{EI} \qquad \theta_A = \frac{M_0 L}{EI} ◀$$

(*b*) Deflection at *A*:

$$y_A = t_{A/B} = \frac{M_0 L^2}{2EI} \qquad y_A = \frac{M_0 L^2}{2EI} \uparrow ◀$$

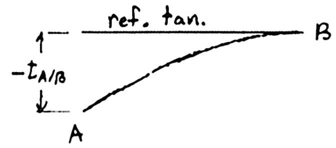


PROBLEM 9.97

For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

SOLUTION

Place reference tangent at *B*.

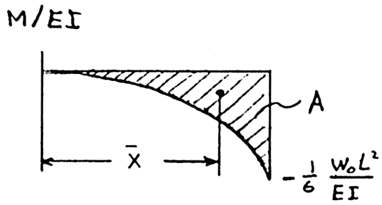


$$\theta_B = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad \left(\frac{1}{2}w_0L\right)\frac{L}{3} + M_B = 0$$

$$M_B = -\frac{1}{6}w_0L^2$$

Draw M/EI curve as cubic parabola.



$$A = -\frac{1}{4}\left(\frac{1}{6}\frac{w_0L^2}{EI}\right)L = -\frac{1}{24}\frac{w_0L^3}{EI}$$

$$\bar{x} = L - \frac{1}{5}L = \frac{4}{5}L$$

By first moment-area theorem,

$$\theta_{B/A} = A = -\frac{1}{24}\frac{w_0L^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \frac{1}{24}\frac{w_0L^3}{EI} = \frac{1}{24}\frac{w_0L^3}{EI}$$

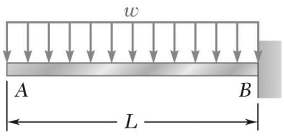
By second moment-area theorem,

$$t_{A/B} = \bar{x}A = \left(\frac{4}{5}L\right)\left(-\frac{1}{24}\frac{w_0L^3}{EI}\right) = -\frac{1}{30}\frac{w_0L^4}{EI}$$

$$y_A = t_{A/B} = -\frac{1}{30}\frac{w_0L^4}{EI}$$

(*a*) $\theta_A = \dfrac{w_0L^3}{24EI}$ ◀

(*b*) $y_A = \dfrac{w_0L^4}{30EI} \downarrow$ ◀



PROBLEM 9.98

For the uniform cantilever beam and loading shown, determine (*a*) the slope at the free end, (*b*) the deflection at the free end.

SOLUTION

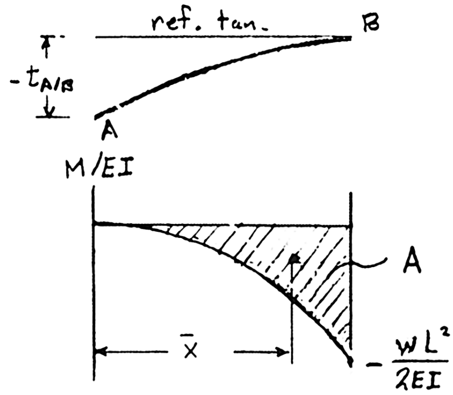
Place reference tangent at *B*.

$$\theta_B = 0$$

Draw *M*/*EI* curve as parabola.

$$A = -\frac{1}{3}\left(\frac{wL^2}{2EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

$$\bar{x} = L - \frac{1}{4}L = \frac{3}{4}L$$



By first moment-area theorem,

$$\theta_{B/A} = A = -\frac{1}{6}\frac{wL^3}{EI}$$

$$\theta_B = \theta_A + \theta_{B/A}$$

$$\theta_A = \theta_B - \theta_{B/A} = 0 + \frac{1}{6}\frac{wL^3}{EI} = \frac{1}{6}\frac{wL^3}{EI}$$

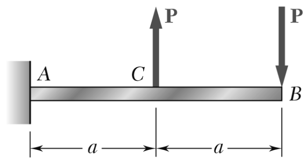
By second moment-area theorem,

$$t_{A/B} = \bar{x}A = \left(\frac{3}{4}L\right)\left(-\frac{1}{6}\frac{wL^3}{EI}\right) = -\frac{1}{8}\frac{wL^4}{EI}$$

$$y_A = t_{A/B} = -\frac{1}{8}\frac{wL^4}{EI}$$

(*a*) $\theta_A = \dfrac{wL^3}{6EI}$ ◀

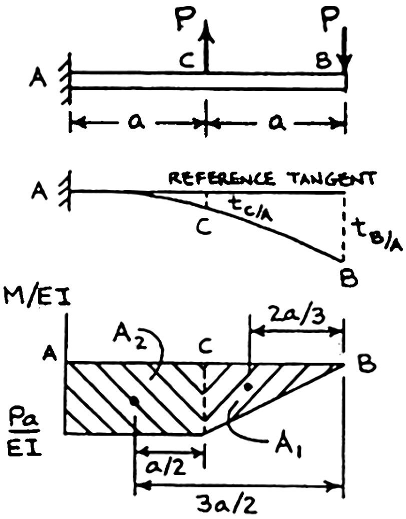
(*b*) $y_A = \dfrac{wL^4}{8EI} \downarrow$ ◀



PROBLEM 9.99

For the uniform cantilever beam and loading shown, determine (*a*) the slope and deflection at (*a*) point *B*, (*b*) point *C*.

SOLUTION



$$A_1 = \frac{1}{2}(a)\left(-\frac{Pa}{EI}\right) = -\frac{Pa^2}{2EI}$$

$$A_2 = (a)\left(-\frac{Pa}{EI}\right) = -\frac{Pa^2}{EI}$$

(*a*) <u>At point *B*:</u>

$$\theta_B = \theta_{B/A} = A_1 + A_2 = -\frac{Pa^2}{2EI} - \frac{Pa^2}{EI} = -\frac{3Pa^2}{2EI}$$

$$\theta_B = \frac{3Pa^2}{2EI} ◄$$

$$y_B = t_{B/A} = A_1\left(\frac{2a}{3}\right) + A_2\left(\frac{3a}{2}\right)$$

$$= \left(-\frac{Pa^2}{2EI}\right)\left(\frac{2a}{3}\right) + \left(-\frac{Pa^2}{EI}\right)\left(\frac{3a}{2}\right) = -\frac{11Pa^3}{6EI}$$

$$y_B = \frac{11Pa^3}{6EI}\downarrow ◄$$

(*b*) <u>At point *C*:</u>

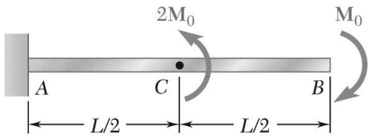
$$\theta_C = \theta_{C/A} = A_2 = -\frac{Pa^2}{EI}$$

$$\theta_C = \frac{Pa^2}{EI} ◄$$

$$y_C = t_{C/A} = A_2\left(\frac{a}{2}\right)$$

$$= \left(-\frac{Pa^2}{EI}\right)\left(\frac{a}{2}\right) = -\frac{Pa^3}{2EI}$$

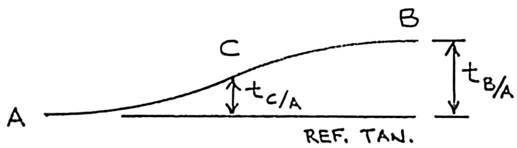
$$y_C = \frac{Pa^3}{2EI}\downarrow ◄$$



PROBLEM 9.100

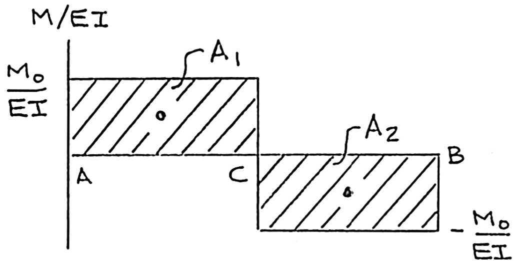
For the uniform cantilever beam and loading shown, determine the slope and deflection at (*a*) point *B*, (*b*) point *C*.

SOLUTION



Place reference tangent at *A*. $\theta_A = 0$

Draw $\dfrac{M}{EI}$ diagram.



$$A_1 = \left(\frac{M_0}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{2}\frac{M_0L}{EI}$$

$$A_2 = \left(-\frac{M_0}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{2}\frac{M_0L}{EI}$$

(*a*) Slope at *B*:

$$\theta_{B/A} = A_1 + A_2 = \frac{1}{2}\frac{M_0L}{EI} - \frac{1}{2}\frac{M_0L}{EI} = 0$$

$$\theta_B = \theta_A + \theta_{B/A} = 0 \qquad\qquad \theta_B = 0 \blacktriangleleft$$

Deflection at *B*:

$$y_B = t_{B/A} = A_1\left(\frac{L}{2} + \frac{1}{2}\cdot\frac{L}{2}\right) + A_2\left(\frac{1}{2}\cdot\frac{L}{2}\right)$$

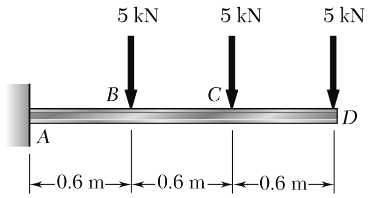
$$= \frac{3}{8}\frac{M_0L^2}{EI} - \frac{1}{8}\frac{M_0L^2}{EI} = \frac{1}{4}\frac{M_0L^2}{EI} \qquad\qquad y_B = \frac{1}{4}\frac{M_0L^2}{EI} \uparrow \blacktriangleleft$$

(*b*) Slope at *C*:

$$\theta_{C/A} = A_1 = \frac{1}{2}\frac{M_0L}{EI} \quad \theta_C = \theta_A + \theta_{C/A} \qquad\qquad \theta_C = \frac{1}{2}\frac{M_0L}{EI} \measuredangle \blacktriangleleft$$

Deflection at *C*:

$$y_C = t_{C/A} = A_1\left(\frac{1}{2}\cdot\frac{L}{2}\right) = \frac{1}{8}\frac{M_0L^2}{EI} \qquad\qquad y_C = \frac{1}{8}\frac{M_0L^2}{EI} \uparrow \blacktriangleleft$$



PROBLEM 9.101

Two C150×12.2 channels are welded back to back and loaded as shown. Knowing that $E = 200$ GPa., determine (*a*) the slope at point *D*, (*b*) the deflection at point *D*.

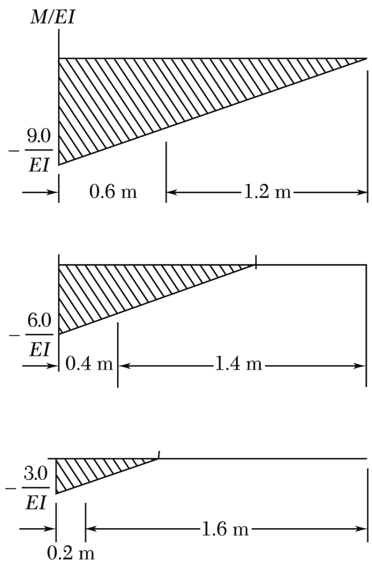
SOLUTION

Units: Forces in kN, lengths in m.

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = (2)(5.35 \times 10^6) = 10.7 \times 10^6 \text{ mm}^4$$

$$EI = (200 \times 10^9)(10.7 \times 10^{-6}) = 2140 \text{ kN} \cdot \text{m}^2$$



Draw $\frac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{(5)(1.8)}{EI} = -\frac{9.0}{EI} \text{ m}^{-1}$$

$$A_1 = \frac{1}{2}\left(\frac{9.0}{EI}\right)(1.8) = -\frac{81}{EI}$$

$$\bar{x}_1 = \frac{1}{3}(1.8) = 0.6 \text{ m}$$

$$\frac{M_2}{EI} = -\frac{(5)(1.2)}{EI} = -\frac{6}{EI} \text{ m}^{-1}$$

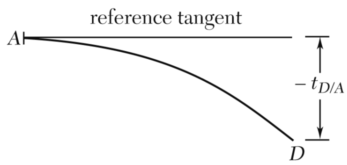
$$A_2 = \frac{1}{2}\left(-\frac{6}{EI}\right)(1.2) = -\frac{3.6}{EI}$$

$$\bar{x}_2 = \frac{1}{3}(1.2) = 0.4 \text{ m}$$

$$\frac{M_3}{EI} = -\frac{(5)(0.6)}{EI} = -\frac{3.0}{EI} \text{ m}^{-1}$$

$$A_3 = \frac{1}{2}\left(-\frac{3.0}{EI}\right)(0.6) = -0.9$$

$$\bar{x}_3 = \frac{1}{3}(0.6) = 0.2 \text{ m}$$



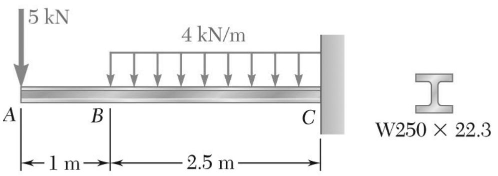
Place reference tangent at *A*. $\qquad \theta_A = 0$

$$\theta_{D/A} = A_1 + A_2 + A_3 = -\frac{12.6}{EI} = -\frac{12.6}{2140} = -5.84 \times 10^{-3} \text{ rad}$$

$$\theta_D = \theta_A + \theta_{D/A} = -5.88 \times 10^{-3} \text{ rad} \blacktriangleleft$$

$$t_{D/A} = \left(-\frac{81}{EI}\right)(1.2) + \left(-\frac{3.6}{EI}\right)\left(\frac{1.4}{3}\right) + \left(-\frac{3.0}{EI}\right)(1.6) = -\frac{19.56}{EI} = -\frac{19.56}{2140} = 9.14 \times 10^{-3} \text{ m}$$

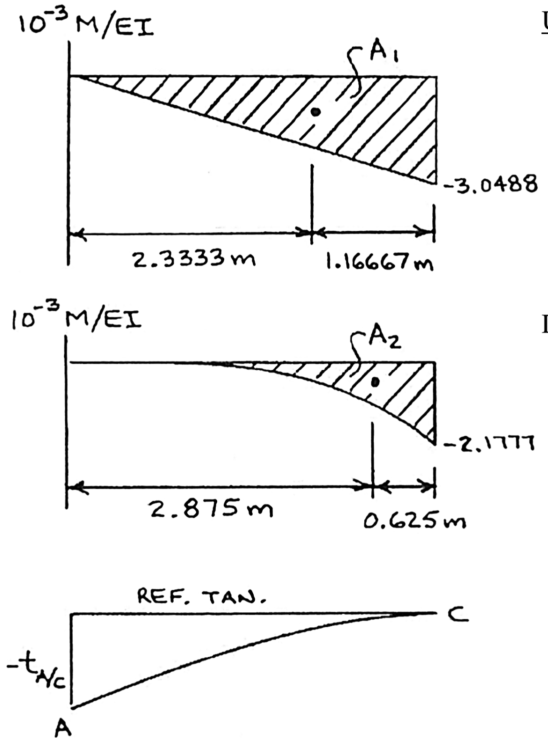
$$y_D = t_{D/A} = 9.14 \times 10^{-3} \text{ m} = 9.14 \text{ mm} \downarrow \blacktriangleleft$$



PROBLEM 9.102

For the cantilever beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*. Use $E = 200$ GPa.

SOLUTION



Units: Forces in kN; lengths in m.

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 28.7 \times 10^6 \text{ mm}^4 = 28.7 \times 10^{-6} \text{m}^4$$

$$EI = (200 \times 10^9)(28.7 \times 10^6)$$

$$= 5.74 \times 10^6 \text{ N} \cdot \text{m}^2$$

$$= 5740 \text{ kN} \cdot \text{m}^2$$

Draw *M*/*EI* diagram by parts:

$$\frac{M_1}{EI} = -\frac{(5)(3.5)}{5740} = -3.0488 \times 10^{-3} \text{m}^{-1}$$

$$A_1 = \frac{1}{2}(-3.0488 \times 10^{-3})(3.5) = -5.3354 \times 10^{-3}$$

$$\bar{x}_1 = \frac{1}{3}(3.5) = 1.16667 \text{ m}$$

$$\frac{M_2}{EI} = -\frac{(4)(2.5)^2}{(2)(5740)} = -2.1777 \times 10^{-3} \text{m}^{-1}$$

$$A_2 = \frac{1}{3}(-2.1777 \times 10^{-3})(2.5) = -1.81475 \times 10^{-3}$$

$$\bar{x}_2 = \frac{1}{4}(2.5) = 0.625 \text{m}$$

Place reference tangent at *C*. $\theta_C = 0$

$$\theta_{C/A} = A_1 + A_2 = -7.1502 \times 10^{-3}$$

(*a*) Slope at *A*: $\theta_A = \theta_C - \theta_{C/A} = 7.1502 \times 10^{-3}$

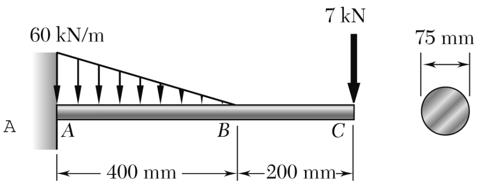
$$\theta_A = 7.15 \times 10^{-3} \text{ rad} \ \blacktriangleleft$$

$$t_{A/C} = (2.3333)(-5.3354 \times 10^{-3}) + (2.875)(-1.81475 \times 10^{-3})$$

$$= -17.6665 \times 10^{-3} \text{ m}$$

(*b*) Deflection at *A*:

$$y_A = t_{AC} = -17.67 \times 10^{-3} \text{ m} \qquad y_A = 17.67 \text{ mm} \downarrow \ \blacktriangleleft$$

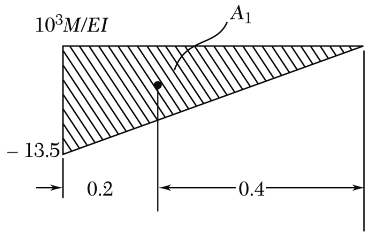


PROBLEM 9.103

For the cantilever beam and loading shown, determine (*a*) the slope at point *C*, (*b*) the deflection at point *C*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN, lengths in m.

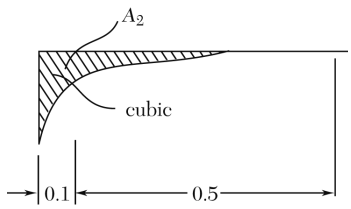


$$E = 200\times10^9\,\text{Pa}$$

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{75}{2}\right)^4 = 1.55\times10^6\ \text{mm}^4$$

$$EI = (200\times10^9)(1.55\times10^{-6}) = 310\ \text{kN}\cdot\text{m}^2$$

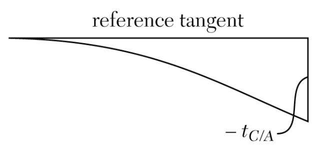
Draw $\frac{M}{EI}$ diagram by parts.



$$\frac{M_1}{EI} = -\frac{(7)(0.6)}{310} = -13.5\times10^{-3}\ \text{m}^{-1}$$

$$A_1 = \frac{1}{2}(-13.5\times10^{-3})(0.6) = -4.05\times10^{-3}$$

$$\bar{x}_1 = \frac{1}{3}(0.6) = 0.2\ \text{m}$$



$$\frac{M_2}{EI} = \frac{\frac{1}{2}(60)(0.4)\left(\frac{1}{3}\cdot0.4\right)}{310} = -5.16\times10^{-3}\ \text{m}^{-1}$$

$$A_2 = \frac{1}{4}(-5.16\times10^{-3})(0.4) = -0.516\times10^{-3}$$

$$\bar{x}_2 = \frac{1}{4}\cdot(0.4) = 0.1\ \text{m}$$

Place reference tangent at *A*. $\theta_A = 0$

$$\theta_{C/A} = A_1 + A_2 = -4.566\times10^{-3}\ \text{rad}$$

$$\theta_C = \theta_A + \theta_{C/A} = -4.566\times10^{-3}\ \text{rad} \quad ◀$$

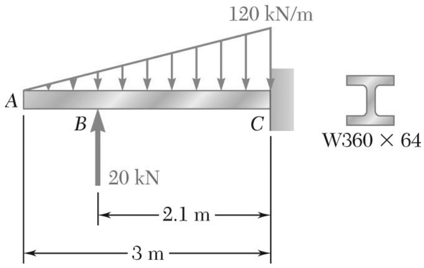
$$t_{C/A} = (0.4)(-4.05\times10^{-3}) + (0.5)(-0.516\times10^{-3})$$

$$= -1.878\times10^{-3}\ \text{m}$$

$$y_C = y_A + (2)(\theta_A) + t_{C/A}$$

$$= 0 + 0 - 1.878\times10^{-3}\ \text{m}\downarrow$$

$$= 1.878\ \text{mm}\downarrow \quad ◀$$



PROBLEM 9.104

For the cantilever beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in meters.

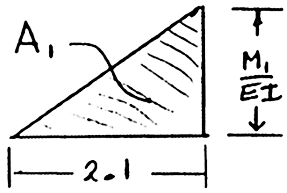
$$I = 178\times10^6 \text{ mm}^4 = 178\times10^{-6}$$

$$EI = (200\times10^9)(178\times10^{-6}) = 35600 \text{ kN}\cdot\text{m}^2$$

Draw $\frac{M}{EI}$ diagram by parts.

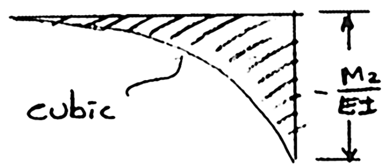
$$\frac{M_1}{EI} = \frac{(20)(2.1)}{35600} = 1.17978\times10^{-3}\,\text{m}^{-1}$$

$$A_1 = \left(\frac{1}{2}\right)(2.1)(1.17978\times10^{-3}) = 1.23876\times10^{-3}$$



$$M_2 = -\frac{\left(\frac{1}{2}\right)(120)(3)(1)}{35600} = -5.0562\times10^{-3}\,\text{m}^{-1}$$

$$A_2 = \left(\frac{1}{4}\right)(3)(-5.0562\times10^{-3}) = -3.7921\times10^{-3}$$



Place reference tangent at *C*.

$$\theta_C = 0$$

(*a*) Slope at *A*:

$$\theta_A = -\theta_{C/A} = -A_1 - A_2$$

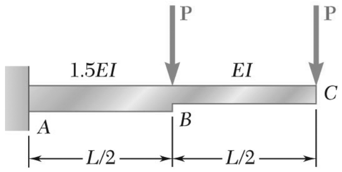
$$\theta_A = 2.55\times10^{-3} \text{ rad} \blacktriangleleft$$

(*b*) Deflection at *A*:

$$y_A = t_{A/C}$$

$$y_C = A_1(3-0.7) + A_2(3-\frac{3}{5}) = -6.25\times10^{-3} \text{ m}$$

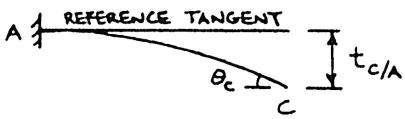
$$y_C = 6.25 \text{ mm}\downarrow \blacktriangleleft$$



PROBLEM 9.105

For the cantilever beam and loading shown, determine (*a*) the slope at point *C*, (*b*) the deflection at point *C*.

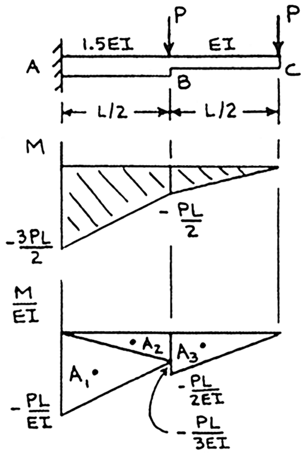
SOLUTION



$$A_1 = \frac{1}{2}\left(-\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = -\frac{PL^2}{4EI}$$

$$A_2 = \frac{1}{2}\left(-\frac{PL}{3EI}\right)\left(\frac{L}{2}\right) = -\frac{PL^2}{12EI}$$

$$A_3 = \frac{1}{2}\left(-\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) = -\frac{PL^2}{8EI}$$



(*a*) Slope at *C*:

$$\theta_C = A_1 + A_2 + A_3$$

$$= -\frac{PL^2}{EI}\left(\frac{1}{4} + \frac{1}{12} + \frac{1}{8}\right) = -\frac{11PL^2}{24EI}$$

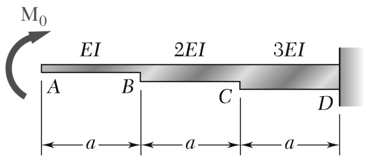
$$\theta_C = \frac{11PL^2}{24EI} \;\blacktriangleleft$$

(*b*) Deflection at *C*:

$$y_C = t_{C/A} = A_1\left(\frac{5}{6}L\right) + A_2\left(\frac{2}{3}L\right) + A_3\left(\frac{1}{3}L\right)$$

$$= \left(-\frac{PL^2}{4EI}\right)\left(\frac{5}{6}L\right) + \left(-\frac{PL^2}{12EI}\right)\left(\frac{2}{3}L\right) + \left(-\frac{PL^2}{8EI}\right)\left(\frac{1}{3}L\right) = -\frac{22PL^3}{72EI}$$

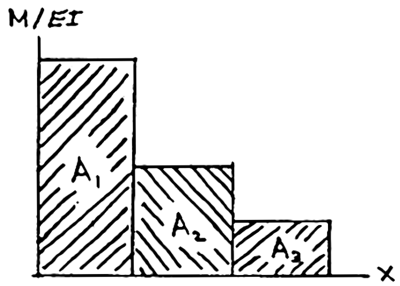
$$y_C = \frac{11PL^3}{36EI} \downarrow \;\blacktriangleleft$$



PROBLEM 9.106

For the cantilever beam and loading shown, determine the deflection and slope at end A caused by the moment $\mathbf{M}_0$.

SOLUTION



Draw $\dfrac{M}{EI}$ diagram.

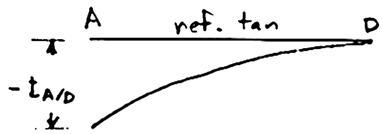
$$A_1 = +\frac{M_0 a}{EI}$$

$$A_2 = +\frac{M_0 a}{2EI}$$

$$A_3 = +\frac{M_0 a}{3EI}$$

$$\theta_0 = 0, \quad y_D = 0$$

Place reference tangent at D.



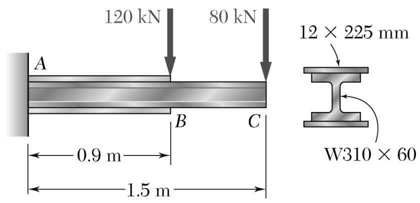
Deflection at A $\quad y_A = t_{A/D}$

$$y_A = A_1\left(\frac{1}{2}a\right) + A_2\left(\frac{3}{2}a\right) + A_3\left(\frac{5}{2}a\right) = \frac{25M_0a^2}{12EI}\uparrow \quad ◀$$

Slope at A. $\quad \theta_A = -\theta_{C/A}$

$$\theta_A = -A_1 - A_2 - A_3 = -\frac{11M_0a}{6EI}$$

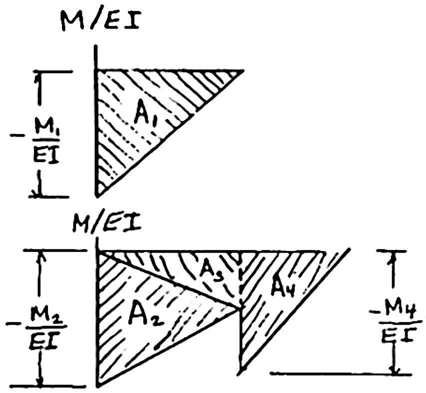
$$\theta_A = \frac{11M_0a}{6EI} \quad ◀$$



PROBLEM 9.107

Two cover plates are welded to the rolled-steel beam as shown. Using $E = 200$ GPa, determine the slope and deflection at end C.

SOLUTION



Use units of kN and m

Over portion BC $\qquad I = 129\times10^{-6}\ \text{m}^4$

$$EI = (200\times10^{6})(129\times10^{-6})$$
$$= 25800\ \text{kN}\cdot\text{m}^2$$

Portion AB:

	$A(\text{mm}^2)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}\,(\text{mm}^4)$
Top plate	2700	145.5	57.16×10^{6}	32400
W310 × 60		0	0	129×10^{6}
Bot. plate	2700	145.5	57.16×10^{6}	32400
Σ			114.32×10^{6}	129.065×10^{6}

$$I = (10^6)(114.32 + 129.065) = 243.385\times10^{6}\ \text{mm}^4$$
$$EI = 48677\ \text{kN}\cdot\text{m}^2$$

Draw $\dfrac{M}{EI}$ diagram.

$$\frac{M_1}{EI} = -\frac{(120)(0.9)}{48677} = -2.219\times10^{-3}\ \text{m}^{-1}$$
$$\frac{M_2}{EI} = -\frac{(80)(1.5)}{48677} = -2.465\times10^{-3}\ \text{m}^{-1}$$
$$\frac{M_4}{EI} = -\frac{(80)(0.6)}{25800} = -1.86\times10^{-3}\ \text{m}^{-1}$$
$$A_1 = \left(\frac{1}{2}\right)(0.9)(-0.002219) = -0.9986\times10^{-3}$$
$$A_2 = \left(\frac{1}{2}\right)(0.9)(0.002465) = -1.10925\times10^{-3}$$
$$A_3 = A_2\left(\frac{0.6}{1.5}\right) = -0.4437\times10^{-3}$$
$$A_4 = \left(\frac{1}{2}\right)(0.6)(-0.00186) = -0.558\times10^{-3}$$

PROBLEM 9.107 *(Continued)*

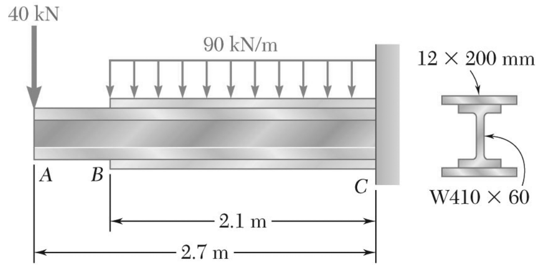
Place reference tangent at A.

Slope at C: $\theta_C = \theta_{C/A} = A_1 + A_2 + A_3 + A_4 = -3.11\times10^{-3}$ ◀

Deflection at C: $y_C = t_{C/A}$

$$y_C = (1.2)(A_1) + (1.2)(A_2) + (0.9)(A_3) + (0.4)(A_4)$$

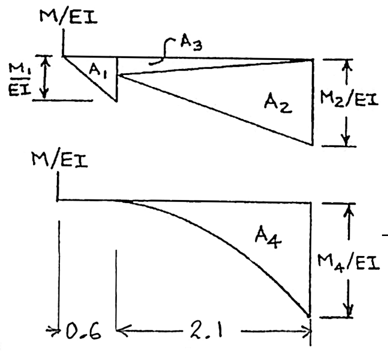
$$= -3.152\times10^{-3}\text{ m} = 3.15\text{ m}\downarrow$$ ◀



PROBLEM 9.108

Two cover plates are welded to the rolled-steel beam as shown. Using $E = 200$ GPa, determine (*a*) the slope at end *A*, (*b*) the deflection at end *A*.

SOLUTION



Portion *AB*: $I = 216 \times 10^6 \text{ mm}^4$

$$EI = (200 \times 10^6 \text{ kPa})(216 \times 10^{-6} \text{ m}^4) = 43{,}200 \text{ kN} \cdot \text{m}^2$$

Portion *BC*:

	$A(\text{mm}^2)$	$d(\text{mm})$	$Ad^2(\text{mm}^4)$	$\bar{I}(\text{mm}^4)$
Top plate	2400	209	104.834×10^6	28,800
W410 × 60				216×10^6
Bot. plate	2400	209	104.834×10^6	28,800
Σ			209.67×10^6	216.06×10^6

$$I = 209.67 \times 10^6 + 216.06 \times 10^6 = 425.73 \times 10^6 \text{ mm}^4$$

$$EI = (200 \times 10^6 \text{ kPa})(425.73 \times 10^{-6} \text{ m}^4) = 85{,}146 \text{ kN} \cdot \text{m}^2$$

Draw $\dfrac{M}{EI}$ diagram:

$$\frac{M_1}{EI} = -\frac{(40)(0.6)}{43200} = -0.55556 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_2}{EI} = -\frac{(40)(2.7)}{85146} = -1.26841 \times 10^{-3} \text{ m}^{-1}$$

$$\frac{M_4}{EI} = -\frac{(90)(2.1)(1.05)}{85146} = -2.3307 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = \frac{1}{2}(0.6)(-0.55556 \times 10^{-3}) = -0.166668 \times 10^{-3}$$

$$A_2 = \frac{1}{2}(2.1)(-1.26841 \times 10^{-3}) = -1.33183 \times 10^{-3}$$

$$A_3 = \frac{0.6}{2.7} A_2 = -0.29596 \times 10^{-3}$$

$$A_4 = \frac{1}{3}(2.1)(-2.3307 \times 10^{-3}) = -1.63149 \times 10^{-3}$$

PROBLEM 9.108 *(Continued)*

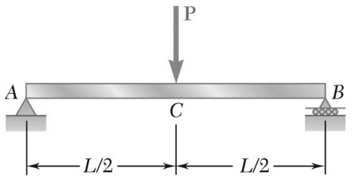
Place reference tangent at C. $\theta_C = 0$

(*a*) Slope at A: $\theta_A = \theta_C - \theta_{A/C} = 0 - (A_1 + A_2 + A_3 + A_4)$ $\theta_A = 3.43 \times 10^{-3}$ rad ◀

(*b*) Deflection at A: $y_A = t_{A/C}$

$$y_A = (0.4)(A_1) + (2)(A_2) + (1.3)(A_3) + (2.175)(A_4) = -6.66 \times 10^{-3}\text{ m}$$

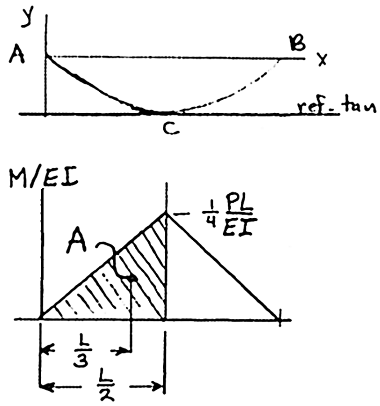
$y_A = 6.66\text{ mm} \downarrow$ ◀



PROBLEM 9.109

For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION



Symmetric beam and loading.

Place reference tangent at *C*.

$$\theta_C = 0, \quad y_C = -t_{A/C}$$

Reactions: $R_A = R_B = \frac{1}{2}P$

Bending moment at *C*: $M_C = \frac{1}{4}PL$

$$A = \frac{1}{2}\left(\frac{1}{4}\frac{PL}{EI}\right)\left(\frac{L}{2}\right) = \frac{1}{16}\frac{PL^2}{EI}$$

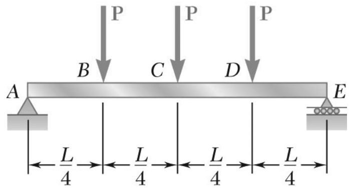
(*a*) Slope at *A*: $\theta_A = \theta_C - \theta_{C/A}$

$$\theta_A = 0 - \frac{1}{16}\frac{PL^2}{EI}$$

$$\theta_A = -\frac{1}{16}\frac{PL^2}{EI} \quad ◀$$

(*b*) Deflection at *C*: $y_C = -t_{A/C} = -A\left(\frac{L}{3}\right) = -\left(\frac{1}{16}\frac{PL^2}{EI}\right)\left(\frac{L}{3}\right)$

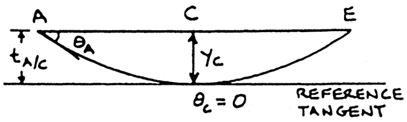
$$y_C = \frac{1}{48}\frac{PL^3}{EI} \downarrow \quad ◀$$

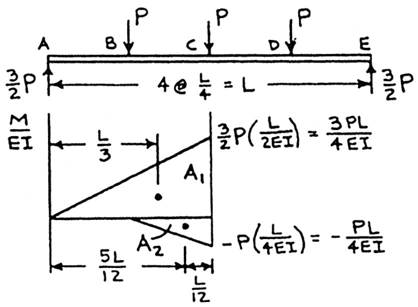


PROBLEM 9.110

For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION





$$A_1 = \frac{1}{2}\left(\frac{3PL}{4EI}\right)\left(\frac{L}{2}\right) = \frac{3PL^2}{16EI}$$

$$A_2 = \frac{1}{2}\left(-\frac{PL}{4EI}\right)\left(\frac{L}{4}\right) = -\frac{PL^2}{32EI}$$

(*a*) <u>Slope at *A*:</u> $\theta_C = \theta_A + \theta_{C/A}; \quad \theta_A = 0 - \theta_{C/A}$

$$\theta_A = -\theta_{C/A} = -(A_1 + A_2)$$

$$= -\left[\frac{3PL^2}{16EI} - \frac{PL^2}{32EI}\right]$$

$$= -\frac{5PL^2}{32EI}$$

$$\theta_A = \frac{5PL^2}{32EI} \ \blacktriangleleft$$

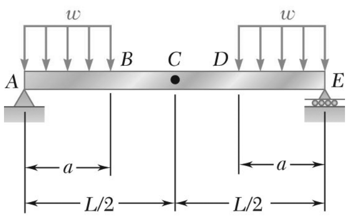
(*b*) <u>Deflection at *C*:</u>

$$t_{A/C} = A_1\left(\frac{L}{3}\right) + A_2\left(\frac{5L}{12}\right)$$

$$= \left(\frac{3PL^2}{16EI}\right)\left(\frac{L}{3}\right) + \left(-\frac{PL^2}{32EI}\right)\left(\frac{5L}{12}\right)$$

$$= \frac{19PL^3}{384EI}$$

$$y_C = -t_{A/C} = -\frac{19PL^3}{384EI} \qquad y_C = \frac{19PL^3}{384EI} \downarrow \ \blacktriangleleft$$



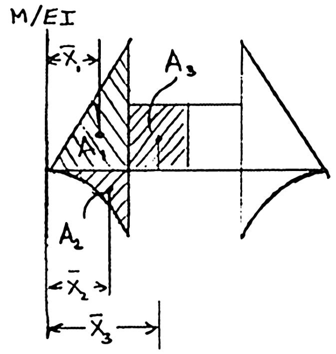
PROBLEM 9.111

For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION

Symmetric beam and loading.

Place reference tangent at *C*. $\theta_C = 0$



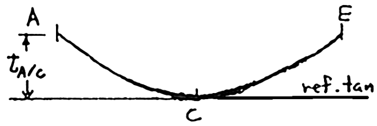
Reactions: $R_A = R_E = wa$

Bending moment:

Over *AB*: $M = wax - \frac{1}{2}wa^2$

Over *BD*: $M = \frac{1}{2}wa^2$

Draw M/EI diagram by parts:



$$\frac{M_1}{EI} = \frac{wa^2}{EI}$$

$$\frac{M_2}{EI} = -\frac{1}{2}\frac{wa^2}{EI}$$

$$\frac{M_3}{EI} = \frac{1}{2}\frac{wa^2}{EI}$$

$$A_1 = \frac{1}{2}\frac{M_1}{EI}a = \frac{1}{2}\frac{wa^3}{EI}$$

$$A_2 = -\frac{1}{3}\frac{M_2}{EI}a = -\frac{1}{6}\frac{wa^3}{EI}$$

$$A_3 = \frac{M_3}{EI}\left(\frac{L}{2} - a\right) = \frac{1}{4}\frac{wa^2}{EI}(L - 2a)$$

(*a*) Slope at *A*:

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{1}{2}\frac{wa^3}{EI} + \frac{1}{6}\frac{wa^3}{EI} - \frac{1}{4}\frac{wa^2}{EI}(L - 2a)$$

$$= -\frac{wa^2}{EI}\left(\frac{1}{4}L - \frac{1}{6}a\right)$$

$$= -\frac{1}{12}\frac{wa^2}{EI}(3L - 2a)$$

$$\theta_A = \frac{wa^2}{12EI}(3L - 2a) \quad ◀$$

PROBLEM 9.111 *(Continued)*

(*b*) Deflection at *C*:

$$y_C = -t_{C/A}$$

$$\bar{x}_1 = \frac{2}{3}a,$$

$$\bar{x}_2 = \frac{3}{4}a,$$

$$\bar{x}_3 = a + \frac{1}{2}\left(\frac{L}{2} - a\right) = \frac{1}{4}(L + 2a)$$

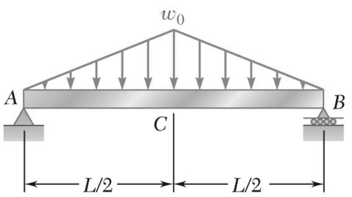
$$y_C = -t_{C/A} = -A_1\bar{x}_1 - A_2\bar{x}_2 - A_3\bar{x}_3$$

$$= -\left(\frac{1}{2}\frac{wa^3}{EI}\right)\left(\frac{2}{3}a\right) + \left(\frac{1}{6}\frac{wa^3}{EI}\right)\left(\frac{3}{4}a\right) - \frac{1}{4}\left(\frac{wa^2}{EI}\right)(L - 2a)\frac{1}{4}(L + 2a)$$

$$= -\frac{1}{3}\frac{wa^3}{EI} + \frac{1}{8}\frac{wa^3}{EI} - \frac{1}{16}\frac{wa^2}{EI}(L^2 - 4a^2)$$

$$= -\frac{wa^2}{EI}\left(\frac{1}{16}L^2 - \frac{1}{24}a^2\right) = -\frac{1}{48}\frac{wa^2}{EI}(3L^2 - 2a^2)$$

$$y_A = \frac{wa^2}{48EI}(3L^2 - 2a^2)\downarrow \blacktriangleleft$$



PROBLEM 9.112

For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

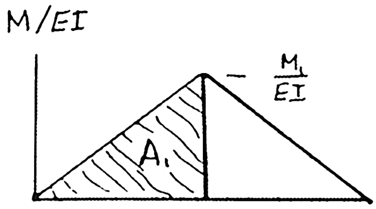
SOLUTION

Symmetric beam and loading.

Place reference tangent at *C*. $\theta_C = 0$

Reactions: $R_A = R_B = \dfrac{w_0 L}{4}$

Draw $\dfrac{M}{EI}$ diagram by parts.

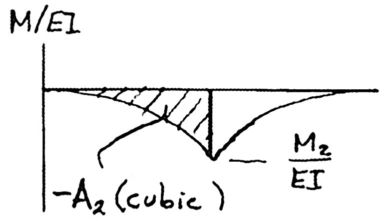


$$\frac{M_1}{EI} = \frac{R_A L}{2} = \frac{w_0 L^2}{8EI}$$

$$A_1 = \frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{M_1}{EI}\right) = \frac{w_0 L^3}{32EI}$$

$$\frac{M_2}{EI} = \frac{1}{EI}\cdot\frac{1}{2}\left(-\frac{w_0 L}{2}\right)\left(\frac{1}{3}\cdot\frac{L}{2}\right) = -\frac{w_0 L^2}{24EI}$$

$$A_2 = \frac{1}{4}\left(\frac{L}{2}\right)\left(-\frac{w_0 L^2}{24EI}\right) = -\frac{w_0 L^3}{192EI}$$



(*a*) Slope at *A*: $\theta_A = -\theta_{C/A}$

$$\theta_A = -A_1 - A_2 = \left(-\frac{1}{32} + \frac{1}{192}\right)\frac{w_0 L^3}{EI}$$

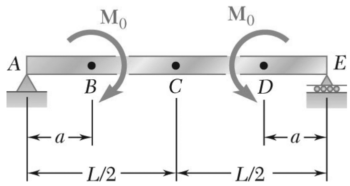
$$\theta_A = \frac{5 w_0 L^3}{192EI} \blacktriangleleft$$

(*b*) Deflection at *C*: $y_C = t_{A/C}$

$$t_{A/C} = A_1\left[\left(\frac{2}{3}\right)\left(\frac{L}{2}\right)\right] + A_2\left[\left(\frac{4}{5}\right)\left(\frac{L}{2}\right)\right] = \left[\left(\frac{1}{3}\right)\left(\frac{1}{32}\right) - \left(\frac{2}{5}\right)\left(\frac{1}{192}\right)\right]\frac{w_0 L^4}{EI}$$

$$= \frac{w_0 L^4}{120EI}$$

$$y_c = \frac{w_0 L^4}{120EI}\downarrow \blacktriangleleft$$



PROBLEM 9.113

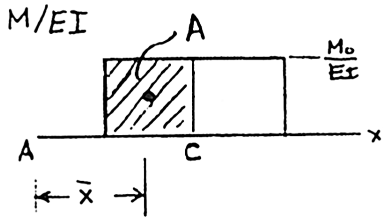
For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION

Symmetric beam and loading.

Place reference tangent at *C*. $\theta_C = 0.$

Draw $\dfrac{M}{EI}$ diagram.



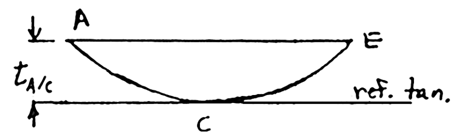
(*a*) Slope at *A*: $\theta_A = 0$

$$A = \frac{M_0}{EI}\left(\frac{L}{2} - a\right) = \frac{1}{2}\frac{M_0}{EI}(L - 2a)$$

$$\theta_A = \theta_C - \theta_{C/A} = 0 - A = -\frac{1}{2}\frac{M}{EI}(L - 2a)$$

$$\theta_A = \frac{1}{2}\frac{M_0}{EI}(L - 2a) \; \blacktriangleleft$$

(*b*) Deflection at *C*:



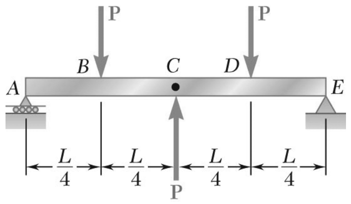
$$\bar{x} = a + \frac{1}{2}\left(\frac{L}{2} - a\right) = \frac{1}{4}(L + 2a)$$

$$y_C = -t_{C/A} = A\bar{x}$$

$$= -\frac{1}{2}\frac{M_0}{EI}(L - 2a)\frac{1}{4}(L + 2a)$$

$$= -\frac{1}{8}\frac{M_0}{EI}(L^2 - 4a^2)$$

$$y_C = \frac{1}{8}\frac{M_0}{EI}(L^2 - 4a^2) \downarrow \; \blacktriangleleft$$



PROBLEM 9.114

For the prismatic beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

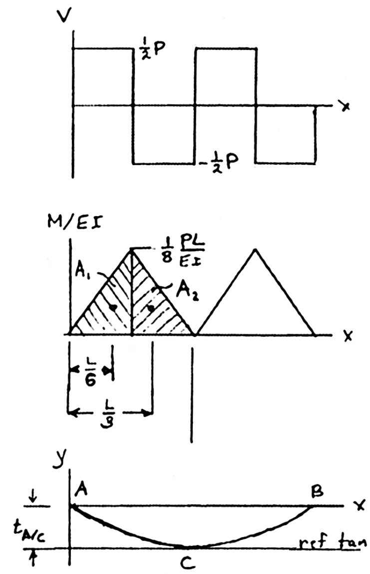
SOLUTION

Symmetric beam and loading.

Place reference tangent at *C*.

$$\theta_C = 0$$

<u>Reactions</u>: $R_A = R_E = \dfrac{1}{2}P$



Draw *V* (shear) and *M/EI* diagrams.

$$A_1 = A_2 = \frac{1}{2}\left(\frac{1}{8}\frac{PL}{EI}\right)\frac{L}{4} = \frac{1}{64}\frac{PL^2}{EI}$$

(*a*) <u>Slope at *A*</u>:

$$\theta_A = \theta_C - \theta_{A/C} = 0 - A_1 - A_2$$

$$= -\frac{1}{32}\frac{PL^2}{EI}$$

$$\theta_A = \frac{PL^2}{32EI} \quad ◀$$

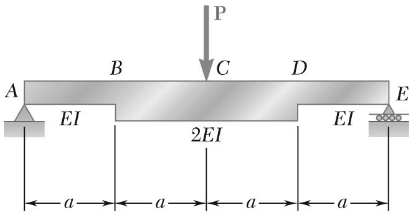
(*b*) <u>Deflection at *C*</u>:

$$y_C = -t_{A/C} = -\left(A_1\frac{L}{6} + A_2\frac{L}{3}\right)$$

$$= -\left(\frac{1}{64}\frac{PL^3}{EI}\cdot\frac{L}{6} + \frac{1}{64}\frac{PL^3}{EI}\cdot\frac{L}{3}\right)$$

$$= -\frac{1}{128}\frac{PL^3}{EI}$$

$$y_C = \frac{PL^3}{128EI}\downarrow \quad ◀$$



PROBLEM 9.115

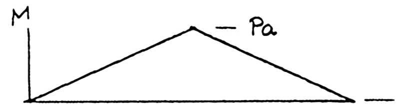
For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the center *C* of the beam.

SOLUTION

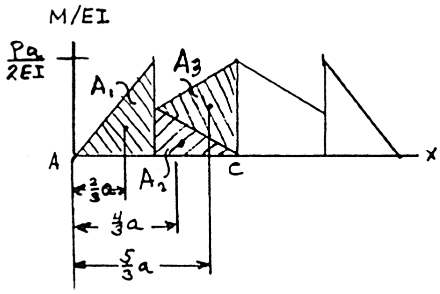
Symmetric beam and loading.

$$R_A = R_E = \frac{1}{2}P$$

$$M_{\max} = \left(\frac{1}{2}P\right)(2a) = Pa$$



Draw M and M/EI diagrams.

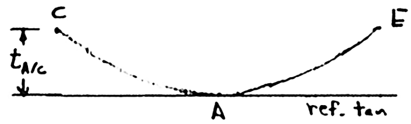


$$A_1 = \frac{1}{2}\left(\frac{Pa}{2EI}\right)a = \frac{1}{4}\frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{Pa}{4EI}\right)a = \frac{1}{8}\frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{Pa}{2EI}\right)a = \frac{1}{4}\frac{Pa^2}{EI}$$

Place reference tangent at C.



$$\theta_C = 0$$

(*a*) Slope at *A*:

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{5}{8}\frac{Pa^2}{EI}$$

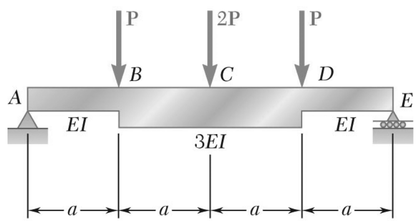
$$\theta_A = \frac{5Pa^2}{8EI} \blacktriangleleft$$

(*b*) Deflection at *C*:

$$|y_C| = t_{A/C} = A_1\left(\frac{2}{3}a\right) + A_2\left(\frac{4}{3}a\right) + A_3\left(\frac{5}{3}a\right)$$

$$= \frac{1}{6}\frac{Pa^3}{EI} + \frac{1}{6}\frac{Pa^3}{EI} + \frac{5}{12}\frac{Pa^3}{EI} = \frac{3}{4}\frac{Pa^3}{EI}$$

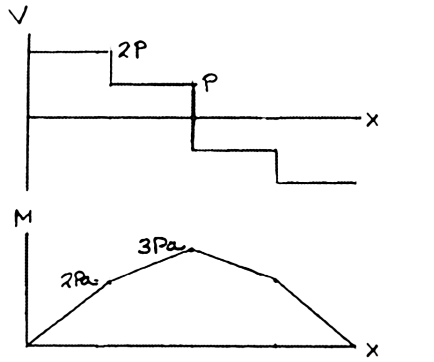
$$y_C = \frac{3Pa^3}{4EI}\downarrow \blacktriangleleft$$



PROBLEM 9.116

For the beam and loading shown, determine (a) the slope at end A, (b) the deflection at the center C of the beam.

SOLUTION



Symmetric beam and loading.

$$R_A = R_E = 2P.$$

Draw V, M, and M/EI diagrams.

$$A_1 = \frac{1}{2}\left(\frac{2Pa}{EI}\right)a = \frac{Pa^2}{EI}$$

$$A_2 = \frac{1}{2}\left(\frac{2}{3}\frac{Pa}{EI}\right)a = \frac{1}{3}\frac{Pa^2}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{Pa}{EI}\right)a = \frac{1}{2}\frac{Pa^2}{EI}$$

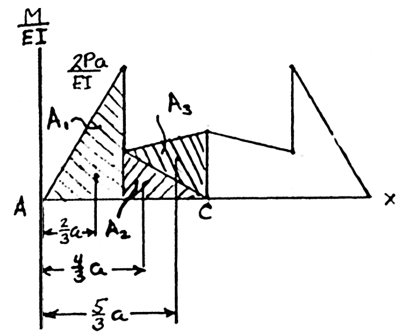
Place reference tangent at C.

$$\theta_C = 0$$

(a) Slope at A:

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$$

$$= -\frac{11}{6}\frac{Pa^2}{EI} \qquad \theta_A = \frac{11Pa^2}{6EI} ◀$$

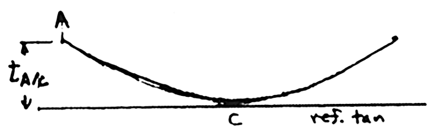


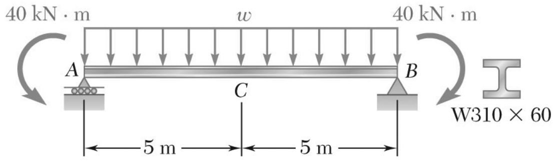
(b) Deflection at C:

$$|y_C| = t_{A/C}$$

$$= A_1\left(\frac{2}{3}a\right) + A_2\left(\frac{4}{3}a\right) + A_3\left(\frac{5}{3}a\right)$$

$$= \frac{35}{18}\frac{Pa^3}{EI} \qquad y_C = \frac{35Pa^3}{18EI} \downarrow ◀$$

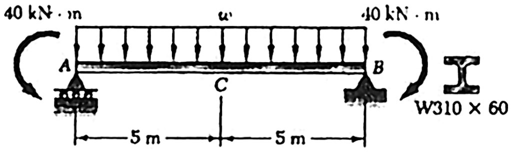




PROBLEM 9.117

For the beam and loading shown and knowing that $w = 8$ kN/m, determine (*a*) the slope at end *A*, (*b*) the deflection at midpoint *C*. Use $E = 200$ GPa.

SOLUTION



$$E = 200\times 10^9\,\text{Pa}$$
$$I = 128\times 10^6\,\text{mm}^4 = 128\times 10^{-6}\,\text{m}^4$$
$$EI = (200\times 10^9)(128\times 10^{-6}) = 25.6\times 10^6\,\text{N}\cdot\text{m}^2$$
$$= 25,600\ \text{kN}\cdot\text{m}^2$$

Symmetrical beam and loading.

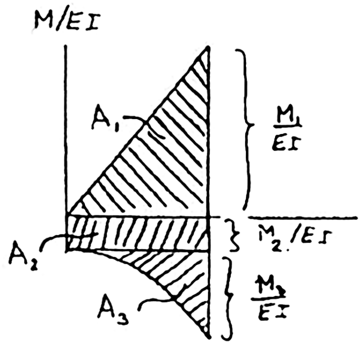
$$R_A = R_B = \frac{1}{2}(8)(10) = 40\ \text{kN}$$

Bending moment:

$$M = 40x - 40 - \frac{1}{2}(8)x^2$$

At $x = 5$,

$$M = 200 - 40 - 100$$



Draw $\dfrac{M}{EI}$ diagram by parts.

$$\frac{M_1}{EI} = \frac{200}{25,600} = 7.8125\times 10^{-3}\,\text{m}^{-1}$$
$$\frac{M_2}{EI} = \frac{-40}{25,600} = -1.5625\times 10^{-3}\,\text{m}^{-1}$$
$$\frac{M_3}{EI} = \frac{-100}{25,600} = -3.9063\times 10^{-3}\,\text{m}^{-1}$$

$$A_1 = \frac{1}{2}(7.8125\times 10^{-3})(5) = 19.5313\times 10^{-3} \qquad \bar{x}_1 = \left(\frac{2}{3}\right)(5) = 3.3333\ \text{m}$$

$$A_2 = -(1.5625)(5) = -7.8125\times 10^{-3} \qquad \bar{x}_2 = \left(\frac{1}{2}\right)(5) = 2.5\ \text{m}$$

$$A_3 = -\frac{1}{3}(3.9063)(5) = -6.5105\times 10^{-3} \qquad \bar{x}_3 = \left(\frac{3}{4}\right)(5) = 3.75\ \text{m}$$

Place reference tangent at *C*. $\qquad \theta_C = 0$

PROBLEM 9.117 *(Continued)*

(*a*) Slope at *A*:

$$\theta_A = \theta_C - \theta_{C/A} \; = 0 - (A_1 + A_2 + A_3)$$

$$\theta_A = -(19.5313\times10^{-3} - 7.8125\times10^{-3} - 6.5105\times10^{-3}) = -5.21\times10^{-3}$$

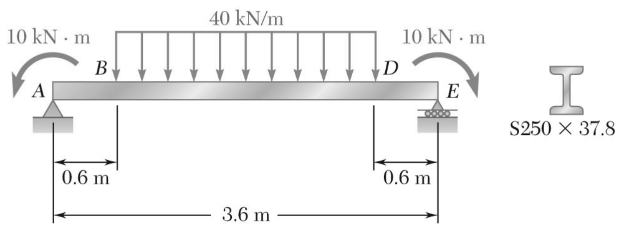
$$\theta_A = 5.21\times10^{-3}\,\text{rad} \blacktriangleleft$$

(*b*) Deflection at *C*:

$$|y_C| = t_{A/C}$$

$$= (19.5313\times10^{-3})(3.3333) - (7.8125\times10^{-3})(2.5) - (6.5105\times10^{-3})(3.75)$$

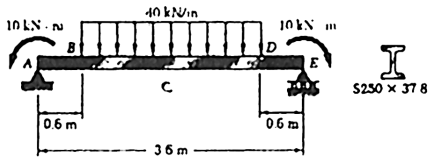
$$= 21.2\times10^{-3}\,\text{m} \qquad y_C = 21.2 \text{ mm} \downarrow \blacktriangleleft$$



PROBLEM 9.118

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint of the beam. Use $E = 200$ GPa.

SOLUTION



Use units of kN and m.

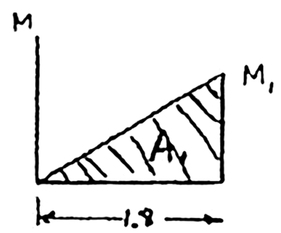
For $S250 \times 37.8$

$$I = 51.2 \times 10^6 \text{mm}^4 = 51.2 \times 10^{-6} \text{m}^4$$

$$EI = (200 \times 10^9)(51.2 \times 10^{-6})$$

$$= 10.24 \times 10^6 \text{ N} \cdot \text{m}^2 = 10{,}240 \text{ kN} \cdot \text{m}^2$$

Place reference tangent at midpoint *C*.

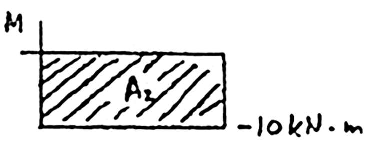


<u>Reactions:</u> $R_A = R_E = \frac{1}{2}(40)(3.6 - 1.2) = 48 \text{ kN} \uparrow$

Draw bending moment diagram of left half of beam by parts.

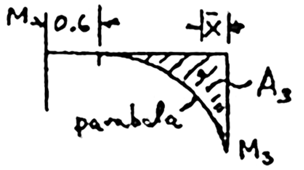
$$M_1 = (48)(1.8) = 86.4 \text{ kN} \cdot \text{m}$$

$$A_1 = \frac{1}{2}(1.8)(86.4) = 77.76 \text{ kN} \cdot \text{m}^2$$



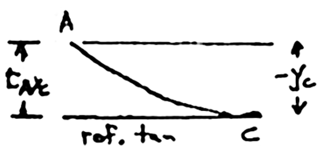
$$A_2 = (1.8)(-10) = -18 \text{ kN} \cdot \text{m}^2$$

$$M_3 = \frac{1}{2}(40)(1.8 - 0.6)^2 = -28.8 \text{ kN} \cdot \text{m}$$



$$A_3 = \frac{1}{3}(1.2)(-28.8) = -11.52 \text{ kN} \cdot \text{m}^2$$

$$\bar{x} = \frac{1}{4}(1.2) = 0.30 \text{ m}$$



(*a*) <u>Slope at end *A*</u>. $\theta_A = -\theta_{A/C}$

$$\theta_A = \frac{1}{EI}\{-A_1 - A_2 - A_3\} = \frac{-77.76 + 18 + 11.52}{10{,}240}$$

$$= -4.71 \times 10^{-3} \text{rad}$$

$$\theta_A = 4.71 \times 10^{-3} \text{rad} \;\; ◀$$

PROBLEM 9.118 *(Continued)*

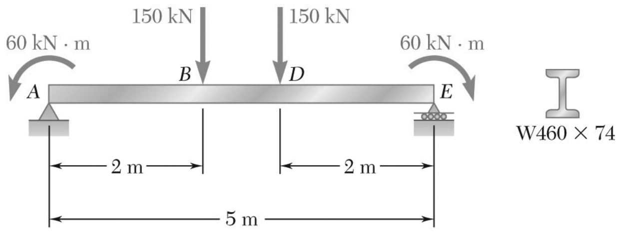
(*b*) Deflection at midpoint *C*: $y_C = -t_{A/C}$

$$t_{A/C} = \frac{1}{EI}\{1.2A_1 + 0.9A_2 + (1.8 - 0.3)A_3\}$$

$$= \frac{(1.2)(77.76) - (0.9)(18) - (1.5)(11.52)}{10,240} = 5.84 \times 10^{-3} \text{ m}$$

$$y_C = -5.84 \times 10^{-3} \text{ m}$$

$y_C = 5.84$ mm ↓ ◀



PROBLEM 9.119

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint of the beam. Use $E = 200$ GPa.

SOLUTION

Use units of kN and m.

For $W460 \times 74$,

$$I = 333 \times 10^6 \text{ mm}^4 = 333 \times 10^{-6} \text{ m}^4$$

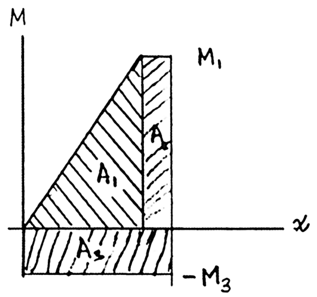
$$EI = (200 \times 10^9)(333 \times 10^{-6})$$

$$= 66.6 \times 10^6 \text{ N} \cdot \text{m}^2 = 66600 \text{ kN} \cdot \text{m}^2$$

Symmetric beam and loading. Place reference tangent at midpoint *C* where $\theta_C = 0$.

Reactions: $R_A = R_E = 150 \text{ kN} \uparrow$

Draw bending moment diagram of left half of beam by parts.



$$M_1 = (2)(150) = 300 \text{ kN} \cdot \text{m}$$

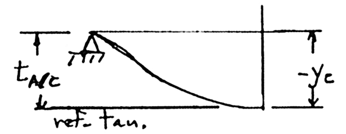
$$A_1 = \left(\frac{1}{2}\right)(2)(300) = 300 \text{ kN} \cdot \text{m}^2$$

$$A_2 = (0.5)(300) = 150 \text{ kN} \cdot \text{m}^2$$

$$M_3 = -60 \text{ kN} \cdot \text{m}$$

$$A_3 = (2.5)(-60) = -150 \text{ kN} \cdot \text{m}^2$$

(*a*) Slope at end *A*: $\theta_A = -\theta_{C/A}$



$$\theta_A = \frac{1}{EI}\{-A_1 - A_2 - A_3\}$$

$$= \frac{-300 - 150 + 150}{66600}$$

$$= -4.50 \times 10^{-3} \text{ rad}$$

$\theta_A = 4.50 \times 10^{-3}$ rad ◀

(*b*) Deflection at midpoint *C*: $y_C = -t_{A/C}$

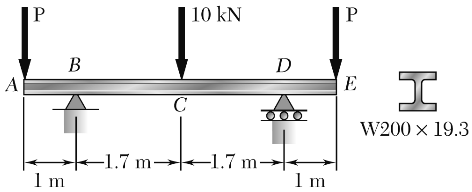
$$t_{A/C} = \frac{1}{EI}\left\{\left(\frac{2}{3} \cdot 2\right)A_1 + \left(2 + \frac{0.5}{2}\right)A_2 + \left(\frac{2.5}{2}\right)A_3\right\}$$

$$= \frac{400 + 337.5 - 187.5}{66600} = 8.26 \times 10^{-3} \text{ m}$$

$$y_C = -8.26 \times 10^{-3} \text{ m}$$

$y_C = 8.26 \text{ mm} \downarrow$ ◀

PROBLEM 9.120



Knowing that $P = 8$ kN, determine (*a*) the slope at end *A*, (*b*) the deflection at midpoint *C*. Use $E = 200$ GPa.

SOLUTION

$$E = 200\times 10^9 \text{ Pa}$$
$$I = 16.6\times 10^6 \text{ mm}^4 = 16.6\times 10^{-6} \text{ m}^4$$
$$EI = (200\times 10^9)(16.6\times 10^{-6}) = 3.32\times 10^6 \text{ N}\cdot\text{m}^2$$
$$= 3320 \text{ kN}\cdot\text{m}^2$$

Symmetric beam and loading:

$$R_A = R_B = P + 5 = 8 + 5 = 13 \text{ kN}$$

Bending moment:

Over *AB*: $M = -Px = -8x$

Over *BC*: $M = -8x + 13(x-1) = 5(x-1) - 8$

Draw $\dfrac{M}{EI}$ diagram by parts.

$$A_1 = \frac{1}{2}\left(\frac{8.5}{EI}\right)(1.7) = \frac{7.225}{EI}$$
$$A_2 = -\frac{1}{2}\left(\frac{8}{EI}\right)(1) = -\frac{4}{EI}$$
$$A_3 = -\left(\frac{8}{EI}\right)(1.7) = -\frac{13.600}{EI}$$

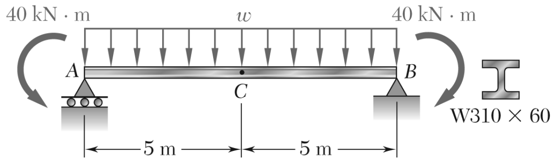
Place reference tangent at *C*. $\theta_C = 0$

(*a*) Slope at *A*: $\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2 + A_3)$

$$\theta_B = -\left(\frac{7.225}{EI} - \frac{4}{EI} - \frac{13.600}{EI}\right) = \frac{10.375}{EI} = \frac{10.375}{3320} = 3.125\times 10^{-3} \text{ rad} \quad ◀$$

(*b*) Deflection at *C*: $y_C = -t_{B/C}$

$$= -(A_1\bar{x}_1 + A_3\bar{x}_3)$$
$$= -\left[\left(\frac{7.225}{EI}\right)\left(\frac{2}{3}(1.7)\right) - \left(\frac{13.600}{EI}\right)\left(\frac{17}{2}\right)\right] = \frac{3.3717}{EI} = \frac{3.3717}{3320}$$
$$= 1.016\times 10^{-3} \text{ m} = 1.016 \text{ mm} \quad ◀$$



PROBLEM 9.121

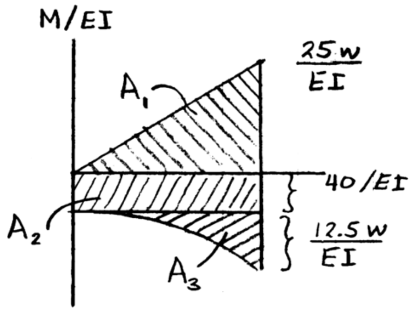
For the beam and loading of Prob. 9.117, determine the value of w for which the deflection is zero at the midpoint C of the beam. Use $E = 200$ GPa.

SOLUTION

Symmetric beam and loading:

$$R_A = R_B = 5w \qquad (w \text{ in kN/m})$$

Bending moment in kN·m:



$$M = 5wx - 40 - \frac{1}{2}wx^2$$

At $x = 5$ m,

$$M = 25w - 40 - 12.5w$$

Draw M/EI diagram by parts.

$$A_1 = \frac{1}{2}\left(\frac{25w}{EI}\right)(5) = \frac{62.5w}{EI}$$

$$A_2 = -\frac{(40)(5)}{EI} = -\frac{200}{EI}$$

$$A_3 = -\frac{1}{3}\left(\frac{12.5w}{EI}\right)(5) = -\frac{20.833w}{EI}$$

$$\bar{x}_1 = \frac{2}{3}(5) = 3.3333 \text{ m}$$

$$\bar{x}_2 = \frac{1}{2}(5) = 2.5 \text{ m}$$

$$\bar{x}_3 = \frac{3}{4}(5) = 3.75 \text{ m}$$

Place reference tangent at C.

Deflection at C is zero.

$$t_{A/C} = y_A - y_C = 0$$

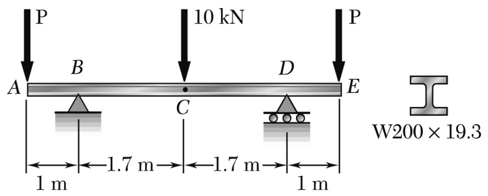
$$A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3 = 0$$

$$\left(\frac{62.5\,w}{EI}\right)(3.3333) - \left(\frac{200}{EI}\right)(2.5) - \left(\frac{20.833\,w}{EI}\right)(3.75) = 0$$

$$\frac{130.21\,w}{EI} - \frac{500}{EI} = 0$$

$$w = \frac{500}{130.21} = 3.84 \text{ kN/m}$$

$w = 3.84$ kN/m ◀



PROBLEM 9.122

For the beam and loading of Prob. 9.120, determine the magnitude of the forces **P** for which the deflection is zero at end A. Use $E = 200$ GPa.

SOLUTION

Symmetric beam and loading:

$$R_A = R_B = P + 5 \qquad (P \text{ in kN})$$

Bending moment:

Over AB: $\quad M = -Px \quad \text{kN}\cdot\text{m}$

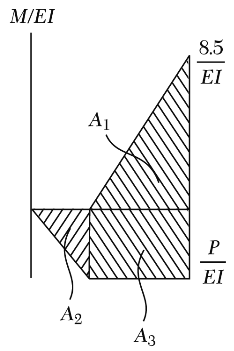
Over BC: $\quad M = -Px + (P+5)(x-1)$

$$= 5(x-1) - P(1)$$

At $\quad x = 2.7 \text{ m}$

$$M = 8.5 - P(1)$$

Draw $\dfrac{M}{EI}$ diagram by parts.



$$A_1 = \frac{1}{2}\left(\frac{8.5}{EI}\right)(1.7) = \frac{7.225}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{P}{EI}\right)(1) = -\frac{0.5P}{EI}$$

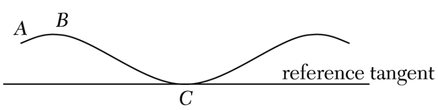
$$A_3 = -\left(\frac{P}{EI}\right)(1.7) = -\frac{1.7P}{EI}$$

Place reference tangent at C.

$$y_A = y_B = 0$$

$$y_A - y_B = 0$$

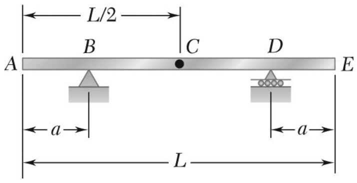
$$t_{A/C} - t_{B/C} = 0$$



$$A_1\left(1 + \frac{2}{3}\cdot 1.7\right) + A_3\left(1 + \frac{1}{2}\cdot 1.7\right) + A_3\left(\frac{2}{3}\right) - A_1\left(\frac{2}{3}\cdot 1.7\right) - A_3\left(\frac{1}{2}\cdot 1.7\right) = 0$$

$$A_1(1) + A_3(1) + A_2\left(\frac{2}{3}\right) = 0$$

$$\frac{7.225}{EI} - \frac{1.7P}{EI} - \frac{0.33333P}{EI} = 0 \quad P = \frac{7.225}{2.0333} = 3.55 \text{ kN} \quad ◀$$



PROBLEM 9.123*

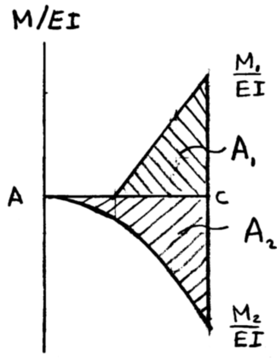
A uniform rod AE is to be supported at two points B and D. Determine the distance a for which the slope at ends A and E is zero.

SOLUTION

Let w = weight per unit length of rod.

Symmetric beam and loading:

$$R_B = R_D = \frac{1}{2}wL$$



Bending moment:

Over AB: $$M = -\frac{1}{2}wx^2$$

Over BCD: $$M = -\frac{1}{2}wx^2 + \frac{1}{2}wL(x-a)$$

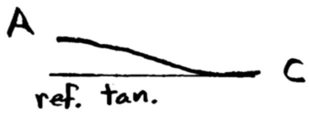
Draw M/EI diagram by parts.

$$\frac{M_1}{EI} = \frac{1}{2}\frac{wL(\frac{L}{2}-a)}{EI} = \frac{1}{4}\frac{wL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = \frac{1}{2}\frac{w(\frac{L}{2})^2}{EI} = -\frac{1}{8}\frac{wL^2}{EI}$$

$$A_1 = \frac{1}{2}\frac{M_1}{EI}\left(\frac{L}{2}-a\right) = \frac{1}{16}\frac{wL(L-2a)^2}{EI}$$

$$A_2 = \frac{1}{3}\left(\frac{M_2}{EI}\right)\frac{L}{2} = -\frac{1}{48}\frac{wL^3}{EI}$$



Place reference tangent at C. $$\theta_C = 0$$

$$\theta_A = \theta_C - \theta_{C/A} = 0 - (A_1 + A_2) = 0$$

$$-\frac{1}{16}\frac{wL(L-2a)^2}{EI} + \frac{1}{48}\frac{wL^3}{EI} = 0$$

Let $u = \frac{a}{L}$ and divide by $\frac{wL^3}{48EI}$.

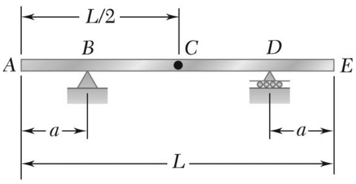
$$1 - 3(1-2u)^2 = 0$$

$$1 - 2u = \frac{\sqrt{3}}{3}$$

$$u = \frac{1}{2}\left(1 - \frac{\sqrt{3}}{3}\right) = 0.21132$$

$$\frac{a}{L} = 0.211$$

$a = 0.211L$ ◀



PROBLEM 9.124*

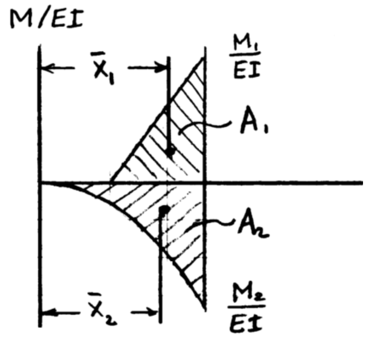
A uniform rod AE is to be supported at two points B and D. Determine the distance a from the ends of the rod to the points of support, if the downward deflections of points A, C, and E are to be equal.

SOLUTION

Let w = weight per unit length of rod.

Symmetric beam and loading:

$$R_B = R_D = \frac{1}{2}wL$$



Bending moment:

Over AB: $$M = -\frac{1}{2}wx^2$$

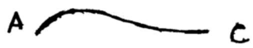
Over BCD: $$M = -\frac{1}{2}wx^2 + \frac{1}{2}wL(x-a)$$

Draw M/EI diagram by parts.

$$\frac{M_1}{EI} = \frac{1}{2}\frac{wL(\frac{L}{2}-a)}{EI} = \frac{1}{4}\frac{wL(L-2a)}{EI}$$

$$\frac{M_2}{EI} = -\frac{1}{2}\frac{w(\frac{L}{2})^2}{EI} = -\frac{1}{8}\frac{wL^2}{EI}$$

$$A_1 = \frac{1}{2}\frac{M_1}{EI}\left(\frac{L}{2}-a\right) = \frac{1}{16}\frac{wL(L-2a)^2}{EI}$$



$$A_2 = \frac{1}{3}\left(\frac{M_2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{48}\frac{wL^3}{EI}$$

$$\bar{x}_1 = a + \frac{2}{3}\left(\frac{L}{2}-a\right) = \frac{1}{3}(L+a)$$

$$\bar{x}_2 = \frac{L}{2} - \frac{1}{4}\left(\frac{L}{2}\right) = \frac{3}{8}L$$

Place reference tangent at C.

$$y_A - y_c = t_{A/C} = 0$$

$$A_1\bar{x}_1 + A_2\bar{x}_2 = 0$$

$$\frac{1}{16}\frac{wL(L-2a)^2}{EI}\frac{1}{3}(L+a) - \frac{1}{48}\frac{wL^3}{EI}\frac{3}{8}L = 0$$

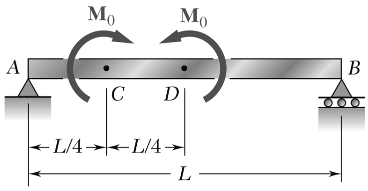
PROBLEM 9.124* *(Continued)*

Let $u = \frac{a}{L}$. Divide by $\frac{wL^4}{48EI}$.

$$(1-2u)^2(1+u) - \frac{3}{8} = 0$$

$$4u^3 - 3u + \frac{5}{8} = 0$$

Solving for u, $\quad u = 0.22315 \quad \frac{a}{L} = 0.223 \quad$ $a = 0.223\,\mathrm{L}$ ◀



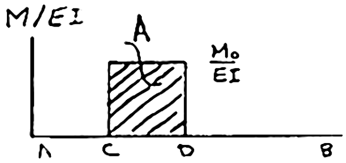
PROBLEM 9.125

For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

SOLUTION

From Statics, $R_A = R_B = 0$

Draw $\dfrac{M}{EI}$ diagram.

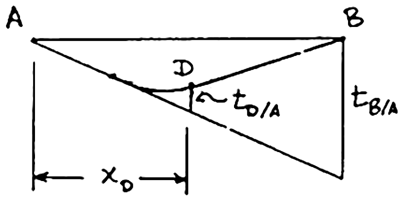


$$A = \left(\frac{M_0}{EI}\right)\left(\frac{L}{4}\right) = \frac{1}{4}\frac{M_0 L}{EI}$$

Place reference tangent at *A*.

$$t_{B/A} = A\left(\frac{L}{2} + \frac{L}{8}\right) = \frac{5}{32}\frac{M_0 L^2}{EI}$$

$$t_{D/A} = A\left(\frac{L}{8}\right) = \frac{1}{32}\frac{M_0 L^2}{EI}$$



(*a*) Deflection at *D*:

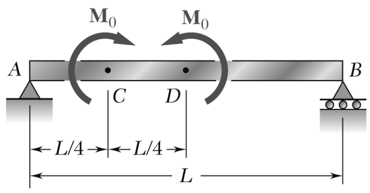
$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A} = t_{D/A} - \frac{1}{2} t_{B/A}$$

$$= \frac{1}{32}\frac{M_0 L^2}{EI} - \frac{5}{64}\frac{M_0 L^2}{EI} = -\frac{3}{64}\frac{M_0 L^2}{EI}$$

$$y_D = \frac{3M_0 L^2}{64EI} \downarrow \blacktriangleleft$$

(*b*) Slope at *A*:

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{5}{32}\frac{M_0 L}{EI} \qquad \theta_A = \frac{5M_0 L}{32EI} \blacktriangleleft$$



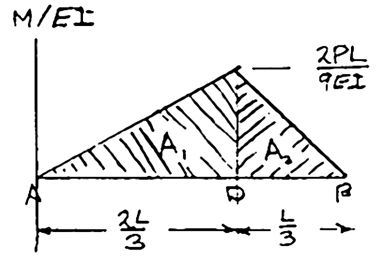
PROBLEM 9.126

For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

SOLUTION

From Statics, $R_A = \frac{1}{3}P \uparrow, \quad R_B = \frac{2}{3}P \uparrow$

$$M_D = R_A\left(\frac{2}{3}L\right) = \frac{2}{9}PL$$

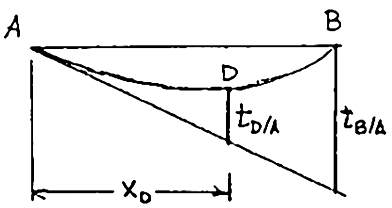


Draw $\frac{M}{EI}$ diagram.

$$A_1 = \frac{1}{2}\left(\frac{2L}{3}\right)\left(\frac{2PL}{9EI}\right) = \frac{2PL^2}{27EI}$$

$$A_2 = \frac{1}{2}\left(\frac{L}{3}\right)\left(\frac{2PL}{9EI}\right) = \frac{PL^2}{27EI}$$

Place reference tangent at *A*.



$$t_{B/A} = A_1\left(\frac{L}{3} + \frac{1}{3}\cdot\frac{2L}{3}\right) + A_2\left(\frac{2}{3}\cdot\frac{L}{3}\right) = \frac{4PL^3}{81EI}$$

$$t_{D/A} = A_1\left(\frac{L}{3}\cdot\frac{2L}{3}\right) = \frac{4PL^3}{243EI}$$

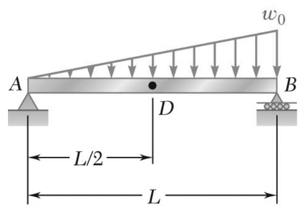
(*a*) Deflection at *D*:

$$y_D = t_{D/A} - \frac{x_D}{L}t_{B/A}$$

$$y_D = \frac{4PL^3}{243EI} - \left(\frac{2}{3}\right)\frac{4PL^3}{81EI}$$

$$= -\frac{4PL^3}{243EI} \qquad y_D = \frac{4PL^3}{243EI}\downarrow ◀$$

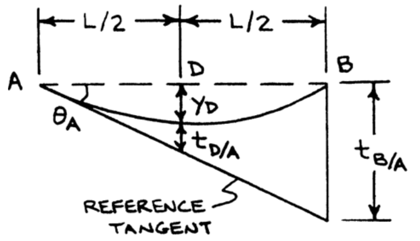
(*b*) Slope at end *A*: $\theta_A = -\frac{t_{B/A}}{L} = -\frac{4}{81}\frac{PL^2}{EI}$ $\qquad \theta_D = \frac{4PL^2}{81EI}$ ◀



PROBLEM 9.127

For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

SOLUTION

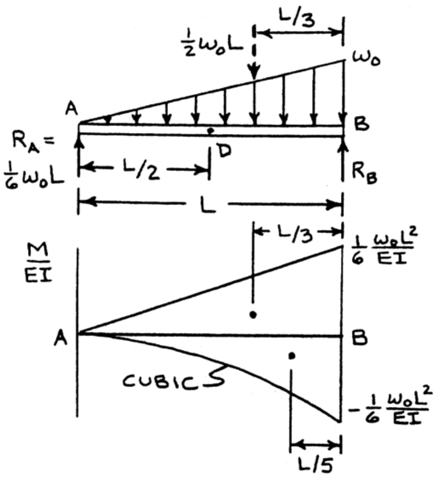


$$t_{B/A} = \frac{1}{2}\left(\frac{1}{6}\frac{w_0L^2}{EI}\right)(L)\left(\frac{L}{3}\right) + \frac{1}{4}\left(-\frac{1}{6}\frac{w_0L^2}{EI}\right)(L)\left(\frac{L}{5}\right)$$

$$= \frac{7w_0L^4}{360EI}$$

$$t_{D/A} = \frac{1}{2}\left(\frac{1}{12}\frac{w_0L^2}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{6}\right) + \frac{1}{4}\left(-\frac{1}{48}\frac{w_0L^2}{EI}\right)\left(\frac{L}{2}\right)\left(\frac{L}{10}\right)$$

$$= \frac{37w_0L^4}{11520EI}$$



(*a*) Deflection at *D*:

$$y_D = \frac{1}{2}t_{B/A} - t_{D/A}$$

$$= \frac{1}{2}\left(\frac{7w_0L^4}{360EI}\right) - \frac{37w_0L^4}{11520EI}$$

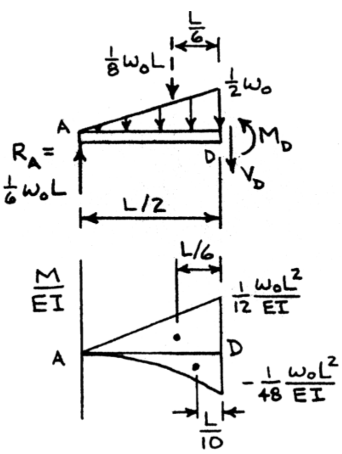
$$= \frac{75w_0L^4}{11520EI}$$

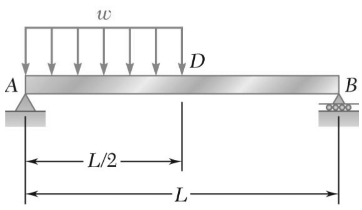
$$y_D = \frac{5w_0L^4}{768EI} \downarrow \blacktriangleleft$$

(*b*) Slope at *A*:

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{7w_0L^3}{360EI}$$

$$\theta_A = \frac{7w_0L^3}{360EI} \blacktriangleleft$$





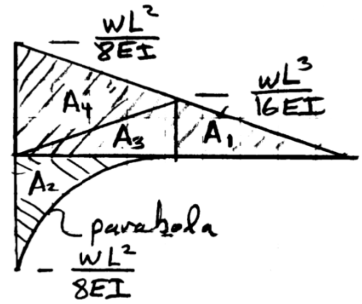
PROBLEM 9.128

For the prismatic beam and loading shown, determine (*a*) the deflection at point *D*, (*b*) the slope at end *A*.

SOLUTION

$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - \frac{wL}{2}\left(\frac{L}{4}\right) = 0$$

$$R_B = \frac{1}{8} wL \uparrow$$



Draw M/EI diagram by parts.

$$\frac{M_1}{EI} = \frac{R_B L}{EI} = \frac{wL^2}{8EI}$$

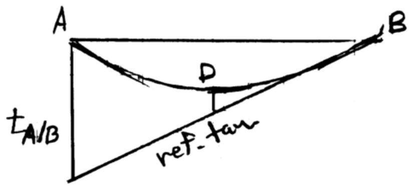
$$\frac{M_2}{EI} = -\frac{wL^3}{8EI}$$

$$A_1 = \left(\frac{1}{2}\right)\left(\frac{L}{2}\right)\frac{wL^2}{16EI} = \frac{wL^3}{64EI}$$

$$A_2 = \left(\frac{1}{3}\right)\left(\frac{L}{2}\right)\frac{wL^2}{8EI} = -\frac{wL^3}{48EI}$$

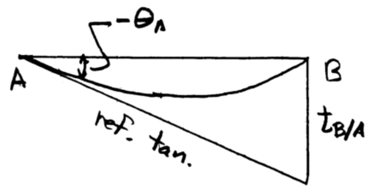
$$A_3 = \left(\frac{1}{2}\right)\left(\frac{L}{2}\right)\frac{wL^2}{16EI} = \frac{wL^3}{64EI}$$

$$A_4 = \left(\frac{1}{2}\right)\left(\frac{L}{2}\right)\frac{wL^2}{8EI} = \frac{wL^3}{32EI}$$



(*a*) Deflection at *D*:

Place reference tangent at *B*.



$$y_D = t_{D/B} - \frac{L}{2} t_{A/B}$$

$$t_{D/A} = \left(\frac{1}{3}\cdot\frac{L}{2}\right)A_1 = \frac{wL^4}{384EI}$$

$$t_{B/A} = \frac{L}{3}(A_1 + A_3 + A_4) + \left(\frac{1}{4}\cdot\frac{L}{2}\right)A_4 = \frac{7wL^4}{384EI}$$

$$y_D = \frac{wL^4}{384EI} - \frac{1}{2}\cdot\frac{7wL^4}{384EI} = -\frac{5wL^4}{768EI}$$

$$y_D = \frac{5wL^4}{768EI} \downarrow \blacktriangleleft$$

PROBLEM 9.128 *(Continued)*

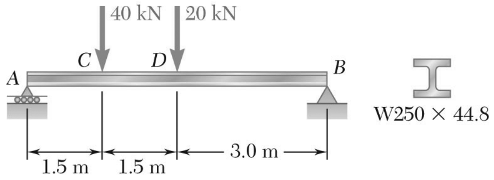
(*b*) Slope at *A*: Place reference tangent at *A*.

$$\theta_A = -\frac{1}{L} t_{B/A}$$

$$= -\left(\frac{1}{L}\right)\left\{\left(\frac{2L}{3}\right)(A_1 + A_3 + A_4) + \left(L - \frac{1}{4}\cdot\frac{L}{2}\right)A_2\right\}$$

$$= -\frac{3wL^3}{128EI}$$

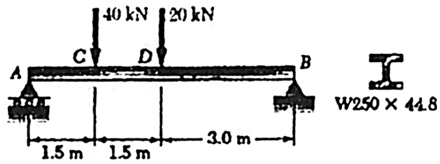
$$\theta_A = \frac{3wL^3}{128EI} \; ◀$$



PROBLEM 9.129

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.

SOLUTION



$$E = 200 \times 10^9 \text{ Pa}$$

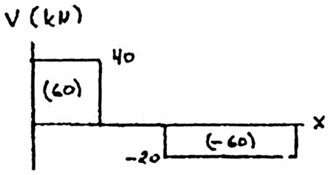
$$I = 70.8 \times 10^6 \text{ mm}^4 = 70.8 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(70.8 \times 10^{-6}) = 19.16 \times 10^6 \text{ N} \cdot \text{m}^2$$

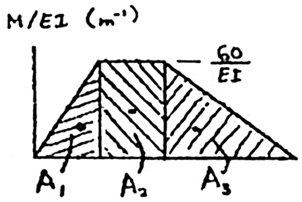
$$= 14{,}160 \text{ kN} \cdot \text{m}^2$$

$$+\circlearrowleft \Sigma M_B = 0: \; -6R_A + (4.5)(40) + (3)(20) = 0$$

$$R_A = 40 \text{ kN}$$



Draw shear and $\dfrac{M}{EI}$ diagrams.



$$A_1 = \frac{1}{2}\left(\frac{60}{EI}\right)(1.5) = \frac{45}{EI}$$

$$A_2 = \left(\frac{60}{EI}\right)(1.5) = \frac{90}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{60}{EI}\right)(3) = \frac{90}{EI}$$

Place reference tangent at *A*.

$$t_{B/A} = A_1(4.5 + 0.5) + A_2(3 + 0.75) + A_3(2.0)$$

$$= \frac{742.5}{EI} \text{ m}$$

$$t_{D/A} = A_1(1.5 + 0.5) + A_2(0.75)$$

$$= \frac{157.5}{EI} \text{ m}$$

(*a*) Slope at *A*:

$$\theta_A = -\frac{t_{B/A}}{L} = -\frac{742.5}{6EI} = -\frac{123.75}{EI} = -\frac{123.75}{14{,}160}$$

$$= -8.74 \times 10^{-3}$$

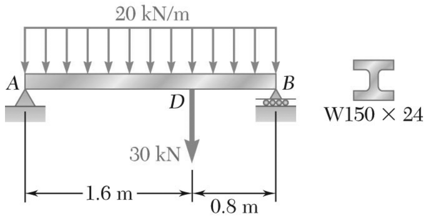
$$\theta_A = 8.74 \times 10^{-3} \text{ rad} \;\blacktriangleleft$$

(*b*) Deflection at *D*:

$$y_D = t_{D/A} - \frac{x_D}{L} t_{B/A} = \frac{157.5}{EI} - \left(\frac{3}{6}\right)\left(\frac{742.5}{EI}\right) = -\frac{213.75}{EI}$$

$$= -\frac{213.75}{14{,}160} = -15.10 \times 10^{-3} \text{ m}$$

$$y_D = 15.10 \text{ mm} \downarrow \;\blacktriangleleft$$



PROBLEM 9.130

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.

SOLUTION

<u>Units</u>: Forces in kN; lengths in meters.

For W150×24,

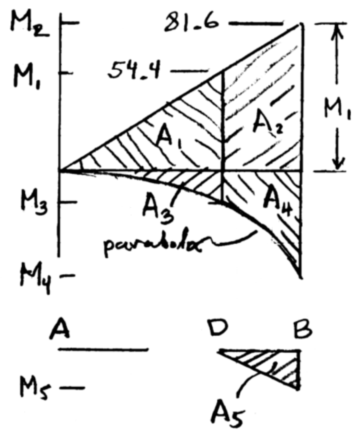
$$I = 13.4\times10^{6}\,\text{mm}^4$$
$$= 13.4\times10^{-6}\,\text{m}^4$$

$$EI = (200\times10^{9})(13.4\times10^{-6}) = 2.68\times10^{6}\,\text{N}\cdot\text{m}^2$$
$$= 2680\ \text{kN}\cdot\text{m}^2$$

$$+\circlearrowleft \Sigma M_B = 0:\quad -2.4R_A + (0.8)(30) + (1.2)(2.4)(20) = 0$$
$$R_A = 34\ \text{kN}\uparrow$$

Draw bending moment diagram by parts.

$$M_1 = (1.6)(34) = 54.4\ \text{kN}\cdot\text{m}$$
$$M_2 = (2.4)(34) = 81.6\ \text{kN}\cdot\text{m}$$
$$M_3 = -\frac{1}{2}(20)(1.6)^2 = -25.6\ \text{kN}\cdot\text{m}$$
$$M_4 = -\frac{1}{2}(20)(2.4)^2 = -57.6\ \text{kN}\cdot\text{m}$$
$$M_5 = -(0.8)(30) = -24\ \text{kN}\cdot\text{m}$$
$$A_1 = \frac{1}{2}(1.6)(54.4) = 43.52\ \text{kN}\cdot\text{m}^2$$
$$A_1 + A_2 = \frac{1}{2}(2.4)(81.6) = 97.92\ \text{kN}\cdot\text{m}^2$$
$$A_3 = \frac{1}{3}(1.6)(-25.6) = -13.6533\ \text{kN}\cdot\text{m}^2$$
$$A_3 + A_4 = \frac{1}{3}(2.4)(-57.6) = -46.08\ \text{kN}\cdot\text{m}^2$$
$$A_5 = \frac{1}{2}(0.8)(-24) = -9.6\ \text{kN}\cdot\text{m}^2$$



PROBLEM 9.130 *(Continued)*

(*a*) Slope at *A*: Place reference tangent at *A*.

$$\theta_A = -\frac{1}{L} t_{B/A}$$

$$t_{B/A} = \frac{1}{EI}\left\{(A_1 + A_2)\left(\frac{1}{3}\right)(2.4) + (A_3 + A_4)\left(\frac{1}{4}\right)(2.4) + A_5\left(\frac{1}{3}\right)(0.8)\right\}$$

$$= \frac{48.128}{2680} = 17.9582 \times 10^{-3}\,\text{m}$$

$$\theta_A = -\frac{17.9582 \times 10^{-3}}{2.4} = -7.48258 \times 10^{-3}$$

$$\theta_A = 7.48 \times 10^{-3}\,\text{rad.} \; ◀$$

(*b*) Deflection at point *D*:

$$y_D = t_{D/A} + \theta_A x_D$$

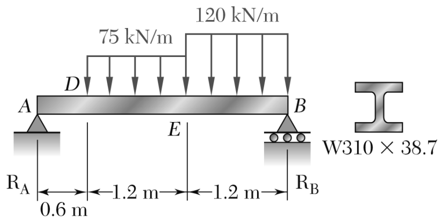
$$t_{D/A} = \frac{1}{EI}\left\{A_1\left(\frac{1}{3}\right)(1.6) + A_2\left(\frac{1}{4}\right)(1.6)\right\}$$

$$= \frac{17.7493}{2680} = 6.62289 \times 10^{-3}\,\text{m}$$

$$y_D = 6.62289 \times 10^{-3} + (-7.48258 \times 10^{-3})(1.6)$$

$$= -5.3492 \times 10^{-3}\,\text{m}$$

$$y_D = 5.35\,\text{mm} \downarrow \; ◀$$



PROBLEM 9.131

For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *E*. Use $E = 200$ GPa.

SOLUTION

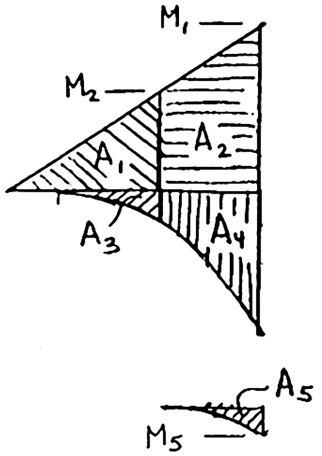
Units: Forces in kN; lengths in m.

For W310×28.7, $I = 85.1\times10^{-6}\ \text{m}^4$

$$EI = (200)(85.1) = 17020\ \text{kN}\cdot\text{m}^2$$

$$+\circlearrowleft \Sigma M_B = 0:\ \ -3R_A + (75)(1.2)(1.8) + (120)(1.2)(0.6) = 0 \qquad R_A = 82.8\ \text{kN}\uparrow$$

Consider loading as 75 kN/m from *D* to *B* plus 45 kN/m from *E* to *B*. Draw bending moment diagram by parts.



$$M_1 = 3R_A = 248.2\ \text{kN}\cdot\text{m}$$

$$M_2 = 1.8R_A = 149\ \text{kN}\cdot\text{m}$$

$$M_3 = -\frac{1}{2}(75)(2.4)^2 = -216\ \text{kN}\cdot\text{m}$$

$$M_4 = -\frac{1}{2}(75)(1.2)^2 = -54\ \text{kN}\cdot\text{m}$$

$$M_5 = -\frac{1}{2}(45)(1.2)^2 = -32.4\ \text{kN}\cdot\text{m}$$

$$A_1 + A_2 = \frac{1}{2}(3)(248.4) = 392.3\ \text{kN}\cdot\text{m}^2$$

$$A_1 = \frac{1}{2}(1.8)(149) = 134\ \text{kN}\cdot\text{m}^2$$

$$A_3 + A_4 = \frac{1}{3}(2.4)(-216) = -172.8\ \text{kN}\cdot\text{m}^2$$

$$A_3 = \frac{1}{3}(1.2)(-54) = -21.6\ \text{kN}\cdot\text{m}^2$$

$$A_5 = \frac{1}{3}(1.2)(-32.4) = -13$$

PROBLEM 9.131 *(Continued)*

(*a*) Slope at *A*: $y_B = y_A + \theta_A L + t_{B/A} \qquad y_A = y_B = 0$

$$\theta_A = -t_{B/A}/L$$

$$t_{B/A} = \frac{1}{EI}\left\{(A_1 + A_2)\left(\frac{1}{3}\right)(3) + (A_3 + A_4)\left(\frac{1}{4}\right)(2.4) + (A_5)\left(\frac{1}{4}\right)(1.2)\right\}$$

$$= \frac{264.72}{17020} = 0.01555 \text{ m}$$

$$\theta_A = -\frac{0.01555}{3} = -5.183\times10^{-3}$$

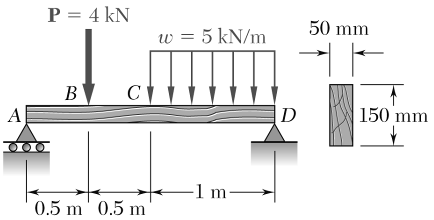
$$\theta_A = 5.18\times10^{-3} \text{ rad} \quad ◀$$

(*b*) Deflection at *E*: $y_E = x_E\theta_A + t_{E/A}$

$$t_{E/A} = \frac{1}{EI}\left\{(A_1)\left(\frac{1}{3}\right)(1.8) + (A_3)\left(\frac{1}{4}\right)(1.2)\right\} = \frac{73.92}{17020} = 4.343\times10^{-3} \text{ m}$$

$$y_E = (1.8)(-0.005183) + 0.004343 = -0.00499 \text{ m}$$

$$= 5 \text{ mm}\downarrow \quad ◀$$



PROBLEM 9.132

For the timber and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at the midpoint *C*. Use $E = 12$ GPa.

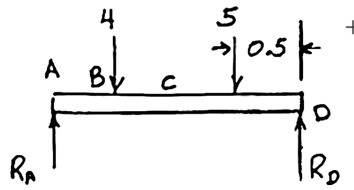
SOLUTION

Units: Forces in kN, lengths in meters.

$$I = \frac{1}{12}(50)(150)^3 = 14.0625\times10^6 \text{ mm}^4$$

$$= 14.0625\times10^{-6} \text{ m}^4$$

$$EI = (12\times10^9)(14.0625\times10^{-6}) = 168.75\times10^3 \text{ N}\cdot\text{m}^2 = 168.75 \text{ kN}\cdot\text{m}^2$$



$$+\circlearrowleft\Sigma M_D = 0: \quad -2R_A + (1.5)(4) + (0.5)(5) = 0 \qquad R_A = 4.25 \text{ kN}$$

$$w(x) = 5\langle x-1\rangle^0 \text{ kN}\cdot\text{m}$$

$$\frac{dV}{dx} = -w = -5\langle x-1\rangle^0 \text{ kN/m}$$

$$\frac{dM}{dx} = V = -5\langle x-1\rangle^1 + 4.25 - 4\langle x-0.5\rangle^0 \text{ kN}$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{5}{2}\langle x-1\rangle^2 + 4.25x - 4\langle x-0.5\rangle^1 \text{ kN}\cdot\text{m}$$

$$EI\frac{dy}{dx} = -\frac{5}{6}\langle x-1\rangle^3 + 2.125x^2 - 2\langle x-0.5\rangle^2 + C_1 \text{ kN}\cdot\text{m}^2$$

$$EIy = -\frac{5}{24}\langle x-1\rangle^4 + \frac{2.125}{3}x^3 - \frac{2}{3}\langle x-0.5\rangle^3 + C_1x + C_2 \text{ kN}\cdot\text{m}^3$$

$$[x=0,\ y=0]: \quad -0+0-0+0+C_2 = 0 \qquad \therefore C_2 = 0$$

$$[x = 2\ m,\ y=0]: -\left(\frac{5}{24}\right)(1)^4 + \left(\frac{2.125}{3}\right)(2)^3 - \left(\frac{2}{3}\right)(1.5)^3 + 2C_1 = 0$$

$$C_1 = -1.60417 \text{ kN}\cdot\text{m}^2$$

(*a*) Slope at end *A*: $\left(\dfrac{dy}{dx} \text{ at } x = 0\right)$

$$EI\left(\frac{dy}{dx}\right)_A = -0+0-0+C_1$$

$$\left(\frac{dy}{dx}\right)_A = \frac{C_1}{EI} = \frac{-1.60417}{168.75} = -9.51\times10^{-3}$$

$$\theta_A = 9.51\times10^{-3} \text{ rad} \ \blacktriangleleft$$

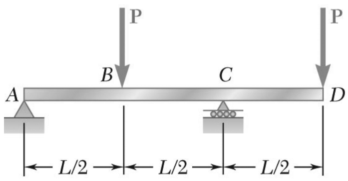
PROBLEM 9.132 *(Continued)*

(*b*) Deflection at midpoint *C*: (y at $x = 1$ m)

$$EIy_C = -0 + \left(\frac{2.125}{3}\right)(1)^3 - \left(\frac{2}{3}\right)(0.5)^3 + (-1.60417)(1) = -979.17 \times 10^{-3}\ \text{kN} \cdot \text{m}^3$$

$$y_C = \frac{-979.17 \times 10^{-3}}{168.75} = -5.80 \times 10^{-3}\ \text{m}$$

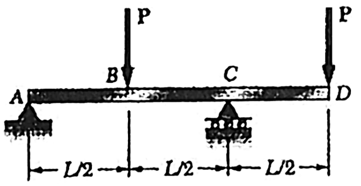
$$y_C = 5.80\ \text{mm}\downarrow \blacktriangleleft$$



PROBLEM 9.133

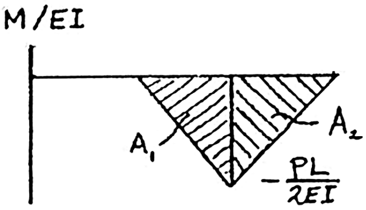
For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *D*.

SOLUTION



$$+\circlearrowleft \Sigma M_C = 0: \quad -R_A L + P\frac{L}{2} - P\frac{L}{2} = 0 \qquad R_A = 0$$

Draw $\dfrac{M}{EI}$ diagram.

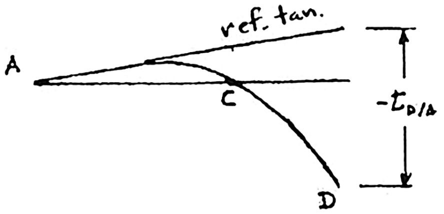


$$A_1 = -\frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{8}\frac{PL^2}{EI}$$

$$A_2 = -\frac{1}{2}\left(\frac{PL}{2EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{8}\frac{PL^2}{EI}$$

Place reference tangent at *A*.

$$t_{C/A} = A_1 \cdot \left(\frac{1}{3}\cdot\frac{L}{2}\right) = -\frac{1}{48}\frac{PL^3}{EI}$$



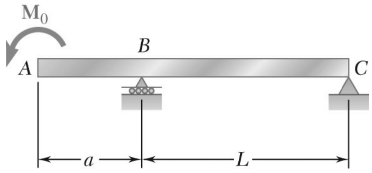
(*a*) Slope at *A*:

$$\theta_A = -\frac{t_{C/A}}{L} \qquad\qquad \theta_A = \frac{1}{48}\frac{PL^2}{EI} \blacktriangleleft$$

(*b*) Deflection at *D*:

$$t_{D/A} = A_1\left(\frac{L}{2}+\frac{L}{6}\right) + A_2\left(\frac{2}{3}\cdot\frac{L}{2}\right) = -\frac{1}{8}\frac{PL^3}{EI}$$

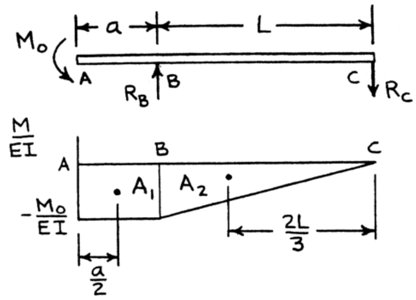
$$y_D = t_{D/A} - \frac{x_D}{L}t_{C/A} = -\frac{1}{8}\frac{PL^3}{EI} - \left(\frac{3}{2}\right)\left(-\frac{1}{48}\frac{PL^3}{EI}\right) \qquad\qquad y_D = -\frac{3}{32}\frac{PL^3}{EI} \blacktriangleleft$$



PROBLEM 9.134

For the beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*.

SOLUTION



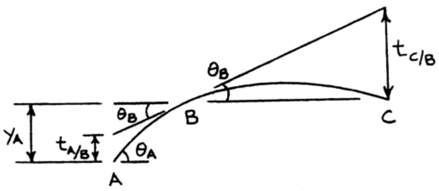
$$A_1 = -\frac{M_0 a}{EI}$$

$$A_2 = -\frac{M_0 L}{2EI}$$

$$t_{C/B} = A_2\left(\frac{2L}{3}\right)$$

$$= \left(-\frac{M_0 L}{2EI}\right)\left(\frac{2L}{3}\right)$$

$$= -\frac{M_0 L^2}{3EI}$$



(*a*) Slope at *A*:

$$\theta_B = \frac{t_{C/B}}{L} = \frac{M_0 L}{3EI}$$

$$\theta_B = \theta_A + \theta_{B/A} = \theta_A + A_1$$

$$\frac{M_0 L}{3EI} = \theta_A - \frac{M_0 a}{EI}$$

$$\theta_A = \frac{M_0}{3EI}(L + 3a) \quad ◀$$

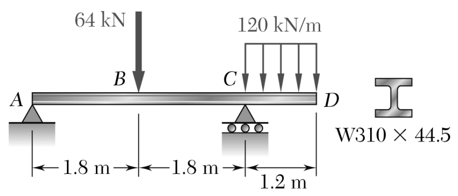
(*b*) Deflection at *A*:

$$t_{A/B} = A_1\left(\frac{a}{2}\right) = -\frac{M_0 a^2}{2EI}$$

$$y_A = \frac{a}{L} t_{C/B} + t_{A/B}$$

$$= \frac{a}{L}\left(-\frac{M_0 L^2}{3EI}\right) - \frac{M_0 a^2}{2EI}$$

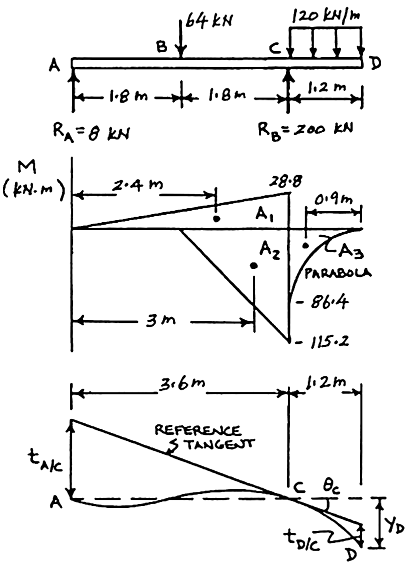
$$y_A = \frac{M_0 a}{6EI}(2L + 3a) \downarrow \quad ◀$$



PROBLEM 9.135

For the beam and loading shown, determine (*a*) the slope at point *C*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.

SOLUTION



Free Body *AD*: $+\circlearrowleft \Sigma M_C = 0: \quad (64)(1.8) - (144)(0.6) - 3.6R_A = 0$

$$R_A = 8 \text{ kN}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 8 - 64 + R_B - 144 = 0$$

$$R_B = 200 \text{ kN}\uparrow$$

For W310×44.5: $I = 99.2 \times 10^{-6} \text{ m}^4$

$$EI = (200\times10^6)(99.2\times10^{-6}) = 19840 \text{ kN}\cdot\text{m}^2$$

(*a*) Slope at *C*:

$$A_1 = \frac{1}{2}(28.8)(3.6) = 51.84 \text{ kN}\cdot\text{m}^2$$

$$A_2 = \frac{1}{2}(-115.2)(1.8) = -103.68 \text{ kN}\cdot\text{m}^2$$

$$EIt_{A/C} = A_1(2.4 \text{ m}) + A_2(3 \text{ m})$$

$$= (51.84)(2.4) + (-103.68)(3) = -186.624 \text{ kN}\cdot\text{m}^3$$

$$t_{A/C} = -\frac{186.624}{19840} = -9.406\times10^{-3} \text{ m}$$

$$\theta_C = \frac{t_{A/C}}{L} = -\frac{9.406\times10^{-3}\text{ m}}{3.6 \text{ m}}$$

$$\theta_C = 2.61\times10^{-3} \text{ rad} \blacktriangleleft$$

(*b*) Deflection at *D*:

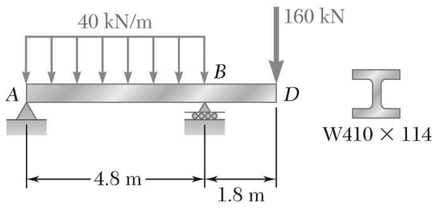
$$EIt_{D/C} = A_1(0.9 \text{ m}) = \frac{1}{3}(-86.4)(1.2)(0.9) = -31.1 \text{ kN}\cdot\text{m}$$

$$t_{D/C} = -\frac{31.1}{19840} = -1.5675\times10^{-3} \text{ m}$$

$$y_D = t_{D/C} + \frac{1.2}{36}t_{A/C} = -1.5675\times10^{-3} + \frac{1}{3}(-9.406\times10^{-3})$$

$$= -4.703\times10^{-3} \text{ m}$$

$$y_C = 4.7 \text{ mm}\downarrow \blacktriangleleft$$



PROBLEM 9.136

For the beam and loading shown, determine (*a*) the slope at point *B*, (*b*) the deflection at point *D*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in meters.

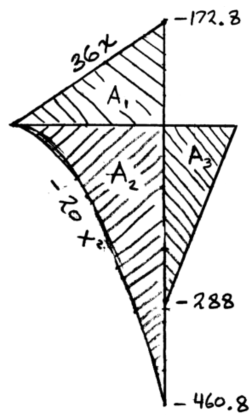
$$I = 462 \times 10^6 \text{ mm}^4 = 462 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(462 \times 10^{-6})$$

$$= 92.4 \times 10^6 \text{ N} \cdot \text{m}^2 = 92400 \text{ kN} \cdot \text{m}^2$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -4.8 R_A + (40)(4.8)(2.4) - (160)(1.8) = 0$$

$$R_A = 36 \text{ kN}$$



Draw bending moment diagram by parts.

$$A_1 = \frac{1}{2}(4.8)(172.8) = 414.72 \text{ kN} \cdot \text{m}^2$$

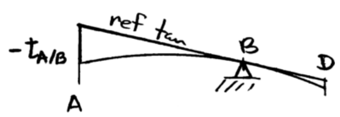
$$A_2 = \frac{1}{3}(4.8)(-460.8) = -737.28 \text{ kN} \cdot \text{m}^2$$

$$A_3 = \frac{1}{2}(1.8)(-288) = -259.2 \text{ kN} \cdot \text{m}^2$$

Place reference tangent at *B*.

(*a*) Slope at *B*.

$$y_A = y_B - L\theta_B + t_{A/B}$$



$$\theta_B = \frac{t_{B/A}}{L} = \frac{1}{EIL}\left\{A_1\left(\frac{2}{3}\right)(4.8) + A_2\left(\frac{3}{4}\right)(4.8)\right\}$$

$$= \frac{-1327.104}{(92400)(4.8)} = -2.9922 \times 10^{-3}$$

$\theta_B = 2.99 \times 10^{-3}$ rad ◄

(*b*) Deflection at *D*.

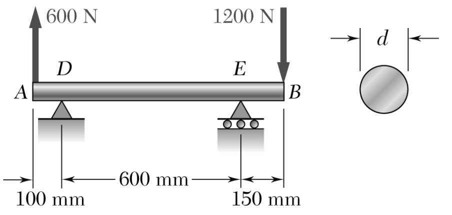
$$y_D = y_B + a\theta_B + t_{D/B}$$

$$= 0 + (1.8)(-2.9922 \times 10^{-3}) - \frac{1}{EI}\left\{A_3\left(\frac{2}{3}\right)(1.8)\right\}$$

$$= -5.3860 \times 10^{-3} - \frac{311.04}{92400}$$

$$= -8.75 \times 10^{-3} \text{ m}$$

$y_D = 8.75 \text{ mm} \downarrow$ ◄



PROBLEM 9.137

Knowing that the beam AB is made of a solid steel rod of diameter $d = 18$ mm, determine for the loading shown (a) the slope at point D, (b) the deflection at point A. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN; lengths in m.

$$C = \frac{1}{2} = \frac{1}{2}(18) = 9 \text{ mm}$$

$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(9)^4 = 5153 \text{ mm}^4$$

$$EI = (200\times10^6)(5153\times10^{-12}) = 1.0306 \text{ kN}\cdot\text{m}^2$$

Draw $\frac{M}{EI}$ diagram by parts by considering the bending moment diagram due to each of the applied loads.

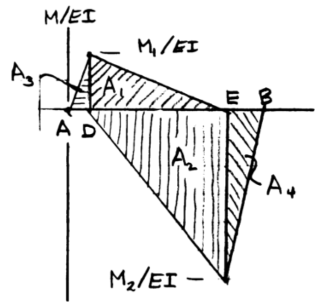
$$\frac{M_1}{EI} = \frac{(0.6)(0.1)}{1.0306} = 0.0582\,\text{m}^{-1}$$

$$\frac{M_2}{EI} = -\frac{(1.2)(0.15)}{1.0306} = -0.1747\,\text{m}^{-1}$$

$$A_1 = \frac{1}{2}(0.6)(0.0582) = 0.01746$$

$$A_2 = \frac{1}{2}(0.6)(-0.1747) = -0.05241$$

$$A_3 = \frac{1}{2}(0.1)(0.0582) = 0.00291$$



Place reference tangent at D.

(a) Slope at point D:

$$y_E = y_D + L\theta_D + t_{E/D} \qquad \theta_D = \frac{-t_{E/D}}{L}$$

$$t_{E/A} = 0.4A_1 + 0.2A_2 = -3.498\times10^{-3}$$

$$\theta_D = \frac{-0.003498}{0.6} = -5.83\times10^{-3}$$

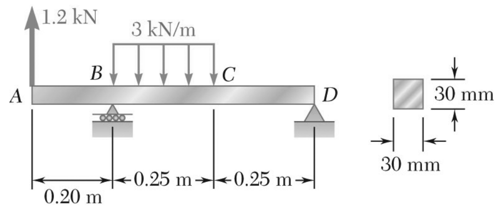
$\theta_D = 5.83\times10^{-3}$ rad ◀

(b) Deflection at A:

$$y_A = y_D - a\theta_D + t_{A/D} = t_{A/D} - a\theta_D$$

$$y_A = A_3\left(\frac{2}{3}\right)(0.1) - (0.1)(0.00583) = -0.389\times10^{-3} \text{ m}$$

$y_A = 0.39$ mm ↓ ◀



PROBLEM 9.138

Knowing that the beam AD is made of a solid steel bar, determine the (a) slope at point B, (b) the deflection at point A. Use $E = 200$ GPa.

SOLUTION

$$E = 200\times10^9\,\text{Pa} \qquad I = \frac{1}{12}(30)(30)^3 = 67.5\times10^3\,\text{mm}^4 = 67.5\times10^{-9}\,\text{m}^4$$

$$EI = (200\times10^9)(67.5\times10^{-9}) = 13500\ \text{N}\cdot\text{m}^2 = 13.5\ \text{kN}\cdot\text{m}^2$$

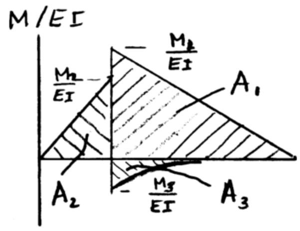
$$+\circlearrowleft \Sigma M_B = 0: \quad -(0.2)(1.2) - (3)(0.25)(0.125) + 5R_D = 0 \qquad R_D = 0.6675\ \text{kN}$$

Draw $\dfrac{M}{EI}$ diagram by parts.

$$M_1 = (0.6675)(0.5) = 0.33375\ \text{kN}\cdot\text{m}$$

$$M_2 = (1.2)(0.2) = 0.240\ \text{kN}\cdot\text{m}$$

$$M_3 = -\frac{1}{2}(3)(0.25)^2 = -0.09375\ \text{kN}\cdot\text{m}$$



$$A_1 = \frac{1}{2}(0.33375)(0.5)/EI = 0.0834375/EI$$

$$A_2 = \frac{1}{2}(0.240)(0.2)/EI = 0.024/EI$$

$$A_3 = \frac{1}{3}(-0.09375)(0.25)/EI = -0.0078125/EI$$

Place reference tangent at B.

$$t_{D/B} = A_1\left(\frac{2}{3}\cdot 0.5\right) + A_3\left(\frac{3}{4}\cdot(0.25) + 0.25\right) = 0.024395/EI$$

(a) Slope at B:

$$\theta_B = -\frac{t_{D/B}}{L} = -\frac{0.024395}{0.5EI} = -\frac{0.048789}{EI}$$

$$= -3.6140\times10^{-3}$$

$\theta_B = 3.61\times10^{-3}$ rad ◀

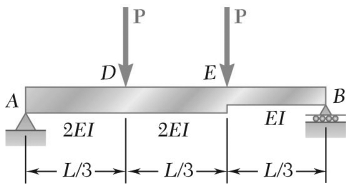
$$t_{A/B} = A_2\left(\frac{2}{3}(0.20)\right) = 0.0032/EI = 0.23704\times10^{-3}\ \text{m}$$

(b) Deflection at A:

$$y_A = t_{A/B} - L_{AB}\theta_B$$

$$= 0.23704\times10^{-3} - (0.2)(-3.6140\times10^{-3}) = 0.960\times10^{-3}\,\text{m}$$

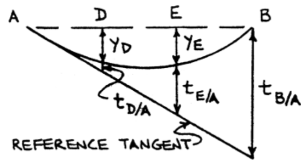
$y_A = 0.960$ mm $\uparrow$ ◀



PROBLEM 9.139

For the beam and loading shown, determine the deflection (*a*) at point *D*, (*b*) at point *E*.

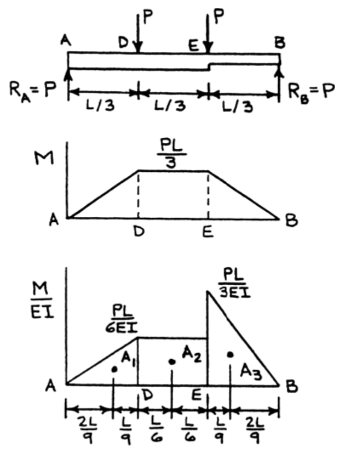
SOLUTION



$$A_1 = \frac{1}{2}\left(\frac{PL}{6EI}\right)\left(\frac{L}{3}\right) = \frac{PL^2}{36EI}$$

$$A_2 = \left(\frac{PL}{6EI}\right)\left(\frac{L}{3}\right) = \frac{PL^2}{18EI}$$

$$A_3 = \frac{1}{2}\left(\frac{PL}{3EI}\right)\left(\frac{L}{3}\right) = \frac{PL^2}{18EI}$$



$$t_{D/A} = A_1\left(\frac{L}{9}\right) = \left(\frac{PL^2}{36EI}\right)\left(\frac{L}{9}\right) = \frac{PL^3}{324EI}$$

$$t_{E/A} = A_1\left(\frac{L}{9}+\frac{L}{3}\right) + A_2\left(\frac{L}{6}\right)$$

$$\left(\frac{PL^3}{36EI}\right)\left(\frac{4L}{9}\right) + \left(\frac{PL^2}{18EI}\right)\left(\frac{L}{6}\right)$$

$$= \frac{7PL^3}{324EI}$$

$$t_{B/A} = A_1\left(\frac{7L}{9}\right) + A_2\left(\frac{L}{2}\right) + A_3\left(\frac{2L}{9}\right)$$

$$= \left(\frac{PL^3}{36EI}\right)\left(\frac{7L}{9}\right) + \left(\frac{PL^2}{18EI}\right)\left(\frac{L}{2}\right) + \left(\frac{PL^2}{18EI}\right)\left(\frac{2L}{9}\right)$$

$$= \frac{5PL^3}{81EI}$$

(*a*) <u>Deflection at *D*</u>:

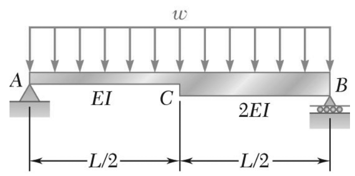
$$y_D = \frac{1}{3}t_{B/A} - t_{D/A} = \frac{1}{3}\left(\frac{5PL^3}{81EI}\right) - \frac{PL^3}{324EI} = \frac{17PL^3}{972EI}$$

$$y_D = \frac{17PL^3}{972EI}\downarrow \blacktriangleleft$$

(*b*) <u>Deflection at *E*</u>:

$$y_E = \frac{2}{3}t_{B/A} - t_{E/A} = \frac{2}{3}\left(\frac{5PL^3}{81EI}\right) - \frac{7PL^3}{324EI} = \frac{19PL^3}{972EI}$$

$$y_E = \frac{19PL^3}{972EI}\downarrow \blacktriangleleft$$



PROBLEM 9.140

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the slope at end *B*, (*c*) the deflection at the midpoint *C*.

SOLUTION

Reactions: $$R_A = R_B = \frac{1}{2}wL$$

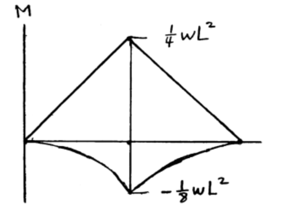
Draw bending moment and *M/EI* diagrams by parts as shown.

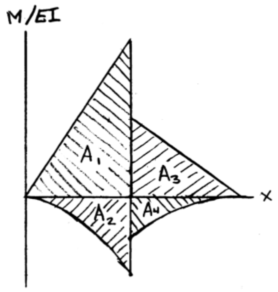
$$A_1 = \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{wL^2}{4EI} = \frac{wL^3}{16EI}$$

$$A_2 = -\frac{1}{3}\cdot\frac{L}{2}\cdot\frac{wL^2}{8EI} = -\frac{wL^3}{48EI}$$

$$A_3 = \frac{1}{2}\cdot\frac{L}{2}\cdot\frac{wL^2}{8EI} = -\frac{wL^3}{32EI}$$

$$A_4 = -\frac{1}{3}\cdot\frac{L}{2}\cdot\frac{wL^2}{16EI} = -\frac{wL^3}{96EI}$$





Place reference tangent at *A*.

(*a*) Slope at end *A*:

$$y_B = y_A + L\theta_A + t_{B/A}$$
$$\theta_A = -t_{B/A}/L$$

$$t_{B/A} = \left(\frac{L}{2}+\frac{L}{6}\right)A_1 + \left(\frac{L}{2}+\frac{L}{8}\right)A_2 + \frac{L}{3}A_3 + \frac{3L}{8}A_4$$
$$= \frac{wL^4}{EI}\left(\frac{1}{24} - \frac{5}{384} + \frac{1}{96} - \frac{1}{256}\right) = \frac{9wL^4}{256EI}$$

$$\theta_A = -\frac{9wL^4}{256EI}\cdot\frac{1}{L} = -\frac{9wL^3}{256EI}$$

$$\theta_A = \frac{9wL^3}{256EI}$$

(*b*) Slope at end *B*:

$$\theta_B = \theta_A + \theta_{B/A} = -\frac{9wL^3}{256EI} + A_1 + A_2 + A_3 + A_4$$

$$\theta_B = \frac{7wL^3}{256EI}$$

$$\theta_B = \frac{7wL^3}{256EI}$$

PROBLEM 9.140 *(Continued)*

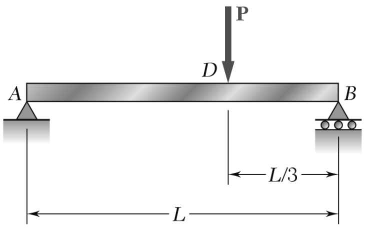
(*c*) Deflection at midpoint *C*:

$$y_A = y_C + \frac{L}{2}\theta_A + t_{C/A}$$

$$t_{C/A} = \left(\frac{L}{6}\right)A_1 + \left(\frac{L}{8}\right)A_2 = \frac{wL^4}{128EI}$$

$$y_C = 0 + \left(\frac{L}{2}\right)\left(-\frac{9wL^3}{256EI}\right) + \frac{wL^4}{128EI} = -\frac{5wL^4}{512EI}$$

$$y_C = \frac{5wL^4}{512EI}\downarrow \blacktriangleleft$$



PROBLEM 9.141

For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

SOLUTION

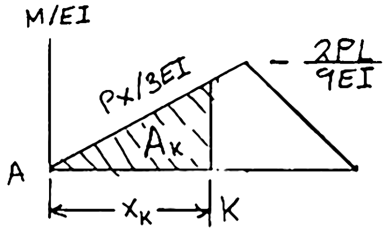
From the solution to Problem 9.126,

$$R_A = \frac{1}{3}P\uparrow, \quad \theta_A = -\frac{4PL^2}{81\,EI}$$

Over portion AD,

$$\frac{M}{EI} = \frac{R_0 x}{EI} = \frac{Px}{3EI}$$

$$A_k = \frac{1}{2}x_k\left(\frac{Px_k}{3EI}\right) = \frac{Fx_k^2}{6EI}$$



$$\theta_K = \theta_A + A_K = 0$$

$$-\frac{4PL^2}{81EI} + \frac{Px_K^2}{6EI} = 0$$

$$x_K^2 = \frac{8}{27}L^2$$

$$x_K = 0.54433L$$

$$A_K = -\theta_A = \frac{4PL^2}{81EI}$$

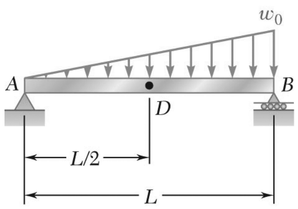
$$t_{A/K} = \left(\frac{2}{3}x_k\right)A_k = 0.01792\frac{PL^3}{EI}$$

$$y_A = y_k + t_{A/K} = 0$$

$$y_K = -t_{A/K} = -0.01792\frac{PL^3}{EI}$$

$$y_K = 0.01792\frac{PL^3}{EI}\downarrow \blacktriangleleft$$

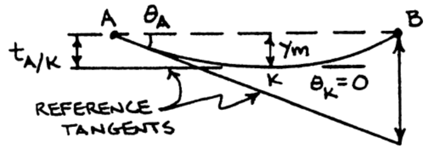
$$x_K = 0.544L \blacktriangleleft$$



PROBLEM 9.142

For the beam and loading of Prob. 9.127, determine the magnitude and location of the largest downward deflection.

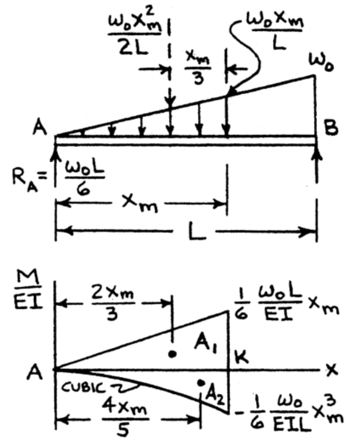
SOLUTION



From Prob. 9.127: $\theta_A = -\dfrac{7w_0L^3}{360EI}$

$$A_1 = \frac{1}{2}\left(\frac{w_0L}{6EI}x_m\right)(x_m) = \frac{w_0Lx_m^2}{12EI}$$

$$A_2 = \frac{1}{4}\left(-\frac{w_0x_m^3}{6EIL}\right)(x_m) = -\frac{w_0x_m^4}{24EIL}$$



Maximum deflection occurs at K, where $\theta_K = 0$.

$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + A_1 + A_2$$

$$0 = -\frac{7w_0L^3}{360EI} + \frac{w_0Lx_m^2}{12EI} - \frac{w_0x_m^4}{24EIL}$$

Rearranging: $$0 = \frac{w_0L^2}{360EI}\left[-7 + 30\left(\frac{x_m}{L}\right)^2 - 15\left(\frac{x_m}{L}\right)^4\right]$$

Solving biquadratic: $\left(\dfrac{x_m}{L}\right)^2 = 0.26970 \qquad x_m = 0.51933L$

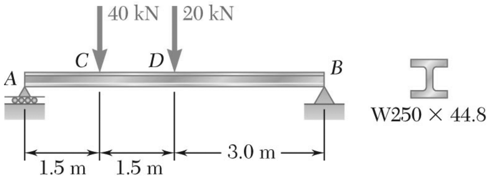
y_m is $0.519L$ from A. ◀

$$t_{A/K} = A_1\frac{2x_m}{3} + A_2\frac{4x_m}{5} = \left(\frac{w_0Lx_m^2}{12EI}\right)\frac{2x_m}{3} + \left(-\frac{w_0x_m^4}{24EIL}\right)\frac{4x_m}{5}$$

$$= \frac{w_0L^4}{90EI}\left[5\left(\frac{x_m}{L}\right)^3 - 3\left(\frac{x_m}{L}\right)^5\right] = \frac{w_0L^4}{90EI}\left[5(0.51933)^3 - 3(0.51933)^5\right]$$

$$= 0.0065222\frac{w_0L^4}{EI} \qquad y_m = |t_{A/K}|$$

$$y_m = 6.52\times10^{-3}\frac{w_0L^4}{EI}$$ ◀



PROBLEM 9.143

For the beam and loading of Prob. 9.129, determine the magnitude and location of the largest downward deflection.

SOLUTION

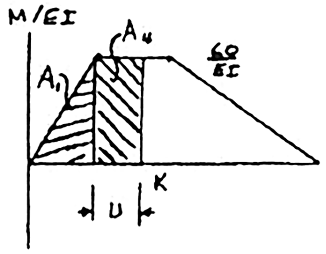
Referring to the solution to Prob. 9.129,

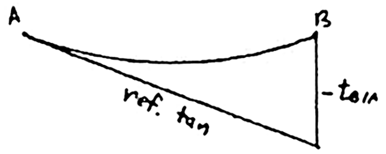
$$EI = 14{,}160\ \text{kN}\cdot\text{m}^2$$

$$R_A = 40\ \text{kN},\quad A_1 = \frac{45}{EI}$$

$$t_{B/A} = \frac{742.5}{EI}\ \text{m}$$

$$\theta_A = -\frac{123.75}{EI}$$





Let K be the location of the maximum deflection. Assume that K lies between C and D.

$$\theta_K = \theta_A + \theta_{K/A}$$

$$= -\frac{123.75}{EI} + A_1 + A_4$$

$$= -\frac{123.75}{EI} + \frac{45}{EI} + \frac{60u}{EI} = 0$$

$$u = \frac{123.75 - 45}{60} = 1.3125\ \text{m}$$

$$x_K = 1.5 + u = 2.8125\ \text{m}$$

$$t_{K/A} = A_1(u + 0.5) + A_4\left(\frac{1}{2}u\right)$$

$$= \frac{45}{EI}(1.8125) + \frac{(60)(1.3125)\left(\frac{1}{2}\right)(1.3125)}{EI} = \frac{133.242}{EI}$$

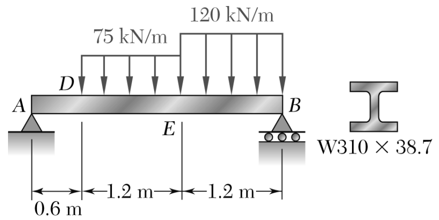
$$y_K = t_{K/A} - \frac{x_K}{L}t_{B/A}$$

$$= \frac{133.242}{EI} - \frac{2.8125}{6}\left(\frac{742.5}{EI}\right) = -\frac{214.80}{EI} = -\frac{214.80}{14{,}160}$$

$$= -15.17\times 10^{-3}\ \text{m}$$

$y_K = 15.17\ \text{mm}\downarrow$ ◀

$x_K = 2.81\ \text{m}$ ◀



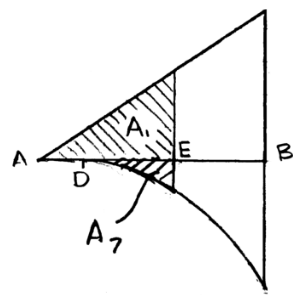
PROBLEM 9.144

For the beam and loading shown, determine the magnitude and location of the largest downward deflection.

PROBLEM 9.144 Beam and loading of Prob. 9.131.

SOLUTION

From the solution to Problem 9.131

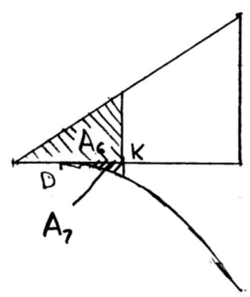


$$EI = 17020 \text{ kNm}^2$$
$$R_A = 82.8 \text{ kN}$$
$$A_1 = 134 \text{ kNm}^2$$
$$A_3 = -21.6 \text{ kNm}^2$$
$$\theta_A = -0.005183$$

Slope at E: $\qquad \theta_E = \theta_A + \theta_{E/A}$

$$\theta_{E/A} = \frac{1}{EI}\{A_1 + A_3\} = \frac{278.767}{41083} = 6.604\times 10^{-3}$$
$$\theta_E = 1.421\times 10^{-3}$$

Since $\theta_E > 0$, the point K of zero slope lies to the left of point E. Let x_K be the coordinate of point K.



$$A_6 = \frac{1}{2}R_A x_K^2 = 41.4\, x_K^2$$
$$A_7 = -\frac{1}{6}(75)(x_K - 0.6)^3$$
$$\theta_K = \theta_A + \theta_{K/A} = \theta_A + \frac{1}{EI}\{A_6 + A_7\} = 0$$
$$A_6 + A_7 + EI\theta_A = 0$$
$$f(x_K) = 41.4x_K^2 - \frac{75}{6}(x_K - 0.6)^3 - 88.2 = 0$$
$$\frac{df}{dx_K} = 82.8\, x_K - 37.5\,(x_K - 0.6)^2$$

PROBLEM 9.144 *(Continued)*

Solve for x_K by iteration.

$$x_K = (x_K)_0 - \frac{f}{df/dx_K}$$

x_K	1.5	1.5442	1.54575	$x_K = 1.545$ m
f	–4.1625	–0.00157	0.145	
df/dx_K	93.825	94.428		

$$A_6 = 98.823 \text{ kN}\cdot\text{m}^2, \quad A_7 = -10.55 \text{ kN}\cdot\text{m}^2$$

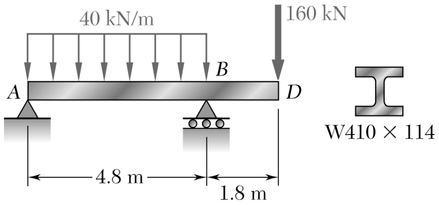
Maximum deflection: $y_A = y_K + t_{A/K} = 0 \qquad y_K = -t_{A/K}$

$$\bar{x}_6 = \frac{2}{3}x_K \quad \bar{x}_7 = 0.6 + \frac{3}{4}(x_K - 0.6) = \frac{3x_K + 0.6}{4}$$

$x_K = 1.545$ m ◀

$$y_7 = -\frac{1}{EI}\{A_6\bar{x}_6 + A_7\bar{x}_7\} = -\frac{87.98}{17020} = -0.005169 \text{ m}$$

$y_K = 5.2$ mm↓ ◀



PROBLEM 9.145

For the beam and loading of Prob. 9.136, determine the largest upward deflection in span *AB*.

SOLUTION

Units: Forces in kN; lengths in meters.

$$I = 462\times10^6\,\text{mm}^4 = 462\times10^{-6}\,\text{m}^4$$

$$EI = (200\times10^9)(462\times10^{-6})$$

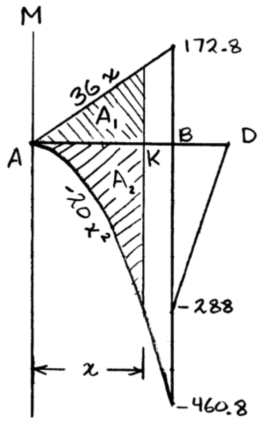
$$= 92.4\times10^6\,\text{N}\cdot\text{m}^2 = 92400\ \text{kN}\cdot\text{m}$$

$$+\circlearrowleft M_B = 0:\quad -4.8R_A + (40)(4.8)(2.4) - (160)(1.8) = 0$$

$$R_A = 36\ \text{kN}$$

$$A_1 = \frac{1}{2}x(36x) = 18x^2$$

$$A_2 = \frac{1}{3}x(-20x^2) = -\frac{20}{3}x^3$$



Place reference tangent at *A*.

$$y_B = y_A + L\theta_A + t_{B/A} = 0$$

$$\theta_A = -\frac{t_{B/A}}{L}$$

$$(A_1)_B = (18)(4.8)^2 = 414.72\ \text{kN}\cdot\text{m}^2$$

$$(A_2)_B = \left(\frac{20}{3}\right)(4.8)^3 = -737.28\ \text{kN}\cdot\text{m}^2$$

$$\theta_A = -\frac{1}{EIL}\left\{(A_1)_B\left(\frac{1}{3}\right)(4.8) + (A_2)_B\left(\frac{1}{4}\right)(4.8)\right\}$$

$$= -\frac{-221.184}{(92400)(4.8)} = 0.49870\times10^{-3}$$

PROBLEM 9.145 *(Continued)*

Locate Point K of maximum deflection.

$$\theta_K = \theta_A + \theta_{K/A} = 0$$
$$EI\theta_A + A_1 + A_2 = 0$$
$$f = 46.08 + 18x_K^2 - \frac{20}{3}x_K^3 = 0 \qquad \frac{df}{dx} = 36x_K - 20x_K^2$$

Solve by iteration.

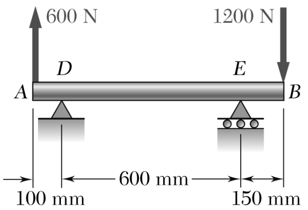
$$x_K = (x_K)_0 - \frac{f}{df/dx}$$

x_K	3	3.39	3.327	3.3251	3.32514 ←
df/dx	–72	–107.8	–101.6	–101.42	
f	28.08	–6.78	–0.188	0.005	

Place reference tangent at K.

$$y_A = y_K + t_{A/K}$$
$$y_A - y_K = -t_{A/K}$$
$$= -\frac{1}{EI}\left\{(A_1)\left(\frac{2}{3}x_K\right) + A_2\left(\frac{3}{4}x_K\right)\right\} = -\frac{1}{EI}\left\{12x_K^3 + 5x_K^4\right\}$$
$$= -\frac{170.064}{92400} = -1.841\times10^{-3}\ \text{m}$$

$y_K = 1.841$ mm ◀



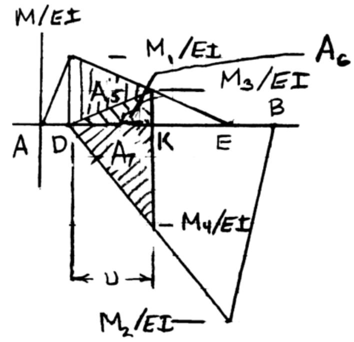
PROBLEM 9.146

For the beam and loading of Prob. 9.137, determine the largest upward deflection in span *DE*.

SOLUTION

Units: Forces in kN; lengths in m.

From the solution to Problem 9.137.



$$EI = 1.0306 \text{ kN}\cdot\text{m}^2$$

$$\frac{M_1}{EI} = 0.0582 \text{ m}^{-1}$$

$$\frac{M_2}{EI} = -0.1747 \text{ m}^{-1}$$

$$\theta_D = 5.83\times10^{-3}$$

Location of maximum deflection:

$$\frac{M_3}{EI} = \frac{M_1}{EI}\left(1-\frac{U}{0.6}\right)$$

$$\frac{M_4}{EI} = \frac{M_2}{EI}\frac{U}{0.6}$$

$$A_5 = \frac{1}{2}\frac{M_1}{EI}\cdot U = 0.0291U$$

$$A_6 = \frac{1}{2}\frac{M_3}{EI}\left(\frac{U}{0.6}\right) = 0.0291\left(1-\frac{U}{0.6}\right)U$$

$$A_7 = \frac{1}{2}\frac{M_4}{EI}\frac{U}{0.6} = -0.0873\left(\frac{U}{0.6}\right)U$$

$$\theta_K = \theta_D + A_5 + A_6 + A_7 = 0$$

Multiply by10^3:

$$0.00583 + 0.0291U + 0.0291\left(1-\frac{U}{0.6}\right)U - (0.0873)\frac{U}{0.6}U = 0$$

$$0.00583 + 0.0582U - 0.097U^2 = 0$$

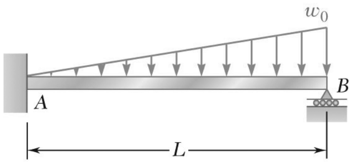
$$U = 0.3807 \text{ m} = 381 \text{ mm}$$

$$A_5 = 0.0111, \quad A_6 = 0.00405, \quad A_7 = -0.0211$$

Maximum deflection in portion *DE*: $y_D = y_K + t_{D/K} = 0$

$$y_K = -t_{D/K} = -\left\{A_5\left(\frac{U}{3}\right) + A_6\left(\frac{2U}{3}\right) + A_7\left(\frac{2U}{3}\right)\right\}$$

$$= -\{-0.00292\} = 2.9 \text{ mm}\uparrow$$ ◀



PROBLEM 9.147

For the beam and loading shown, determine the reaction at the roller support.

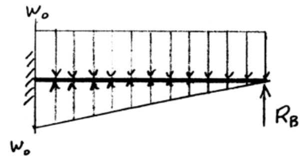
SOLUTION

Remove support B and treat R_B as redundant.

Replace loading by equivalent shown at left.

Draw M/EI diagram for load w_0 and R_B.

Use parts as shown.



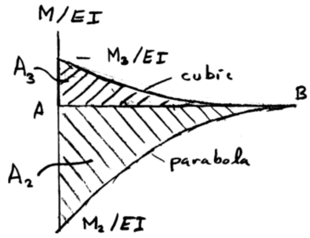
$$A_1 = \frac{1}{2}\left(\frac{R_B L}{EI}\right)(L) = \frac{1}{2}\frac{R_B L^2}{EI}$$

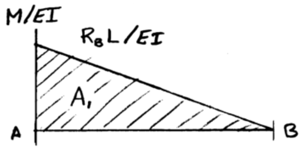
$$M_2 = -\frac{1}{2}w_0 L^2$$

$$A_2 = \frac{1}{3}\left(-\frac{1}{2}\frac{w_0 L^2}{EI}\right)L = -\frac{1}{6}\frac{w_0 L^3}{EI}$$

$$M_3 = \frac{1}{6}\frac{w_0}{L}L^3 = \frac{1}{6}w_0 L^2$$

$$A_3 = \frac{1}{4}\left(\frac{1}{6}\frac{w_0 L^2}{EI}\right)L = \frac{1}{24}\frac{w_0 L^3}{EI}$$





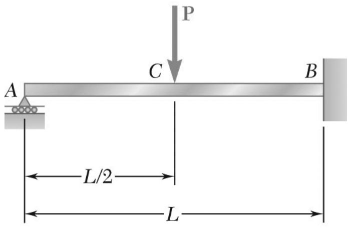
Place reference tangent at A.

$$t_{B/A} = A_1\left(\frac{2}{3}L\right) + A_2\left(\frac{3}{4}L\right) + A_3\left(\frac{4}{5}L\right)$$

$$= \frac{1}{3}\frac{R_B L^3}{EI} - \frac{1}{8}\frac{w_0 L^4}{EI} + \frac{1}{30}\frac{w_0 L^4}{EI} = 0$$

$$R_B = \frac{11}{40}w_0 L \uparrow$$

$R_B = 0.275 w_0 L \uparrow$ ◀



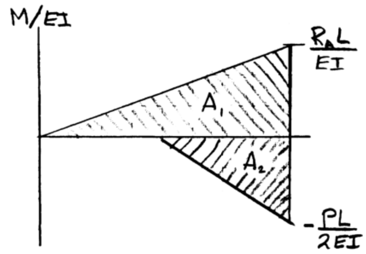
PROBLEM 9.148

For the beam and loading shown, determine the reaction at the roller support.

SOLUTION

Remove support A and treat R_A as redundant.

Draw the M/EI diagram by parts.



$$A_1 = \frac{1}{2} L \frac{R_A L}{EI} = \frac{R_A L^2}{2EI}$$

$$A_2 = -\frac{1}{2}\frac{L}{2}\frac{PL}{2} = -\frac{PL^2}{8EI}$$

Place reference tangent at B.

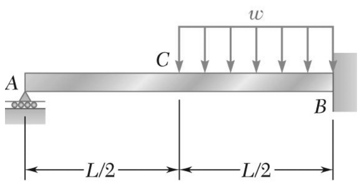
$$y_A = y_B - \theta_B L + t_{A/B} = 0$$

$$t_{A/B} = 0$$

$$A_1\left(\frac{2L}{3}\right) + A_2\left(\frac{L}{2} + \frac{L}{3}\right) = 0$$

$$\frac{R_A L^3}{3EI} - \frac{5PL^3}{48EI} = 0$$

$$R_A = \frac{5}{16} P \uparrow \blacktriangleleft$$



PROBLEM 9.149

For the beam and loading shown, determine the reaction at the roller support.

SOLUTION

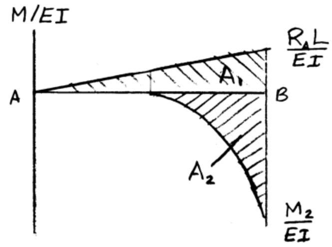
Remove support A and treat R_A as redundant.

Draw M/EI diagram for loads R_A and w.

$$M_2 = -\frac{1}{2}w\left(\frac{L}{2}\right)^2 = -\frac{1}{8}wL^2$$

$$A_1 = \frac{1}{2}\left(\frac{R_A L}{EI}\right)L = \frac{1}{2}\frac{R_A L^2}{EI}$$

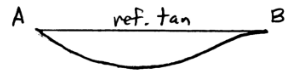
$$A_2 = \frac{1}{3}\left(-\frac{1}{8}\frac{wL^2}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{48}\frac{wL^3}{EI}$$



Place reference tangent at B.

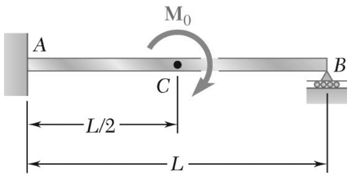
$$t_{A/B} = A_1\left(\frac{2}{3}L\right) + A_2\left(\frac{L}{2} + \frac{3}{4}\frac{L}{2}\right)$$

$$= \frac{1}{3}\frac{R_A L^3}{EI} - \frac{7}{384}\frac{wL^4}{EI} = 0$$



$$R_A = \frac{7}{128}wL$$

$$R_A = \frac{7}{128}wL \uparrow \blacktriangleleft$$



PROBLEM 9.150

For the beam and loading shown, determine the reaction at the roller support.

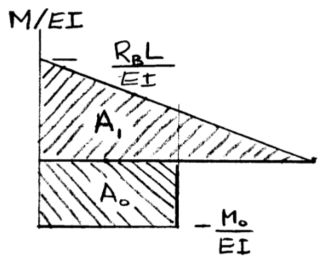
SOLUTION

Remove support B and treat R_B as redundant.

Draw M/EI diagram.

$$A_1 = \frac{1}{2} L \frac{R_A L}{EI} = \frac{R_A L^2}{2EI}$$

$$A_2 = \frac{L}{2} \cdot \frac{M_0 L}{EI} = \frac{M_0 L^2}{2EI}$$



Place reference tangent at A.

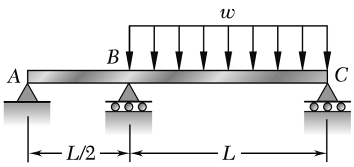
$$y_B = y_A + L\theta_A + t_{B/A} = 0$$

$$t_{B/A} = 0$$

$$A_1\left(\frac{2L}{3}\right) + A_2\left(\frac{L}{2} + \frac{L}{4}\right) = 0$$

$$\frac{R_A L^3}{3EI} - \frac{3M_0 L^2}{8EI} = 0$$

$$R_A = \frac{9}{8}\frac{M_0}{L} \uparrow \blacktriangleleft$$



PROBLEM 9.151

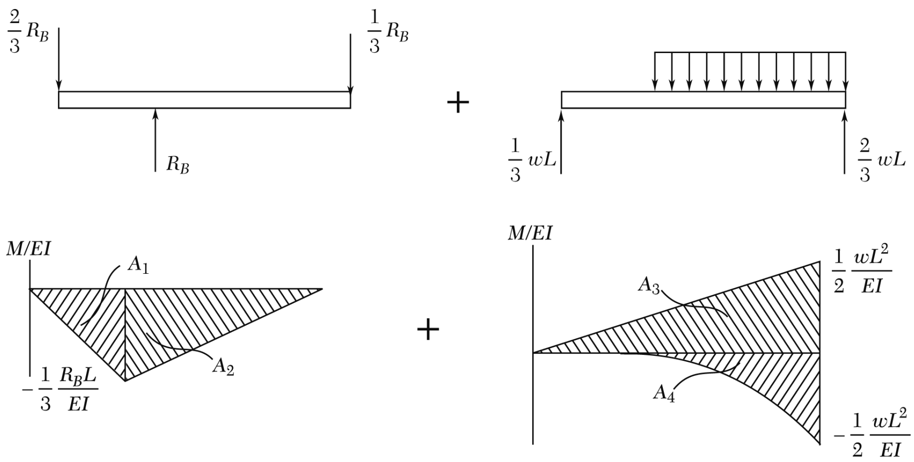
For the beam and loading shown, determine the reaction at each support.

SOLUTION

Remove support *B* and consider R_B as redundant.

Consider loads R_B and *w* separately.

Place reference tangent at *A*.



$$A_1 = \frac{1}{2}\cdot\left(-\frac{1}{3}\frac{R_BL}{EI}\right)\left(\frac{L}{2}\right) = -\frac{1}{12}\frac{R_BL^2}{EI}$$

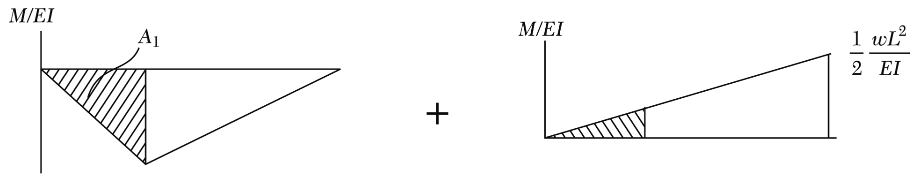
$$A_2 = \frac{1}{2}\left(-\frac{1}{3}\frac{R_BL}{EI}\right)L = -\frac{1}{6}\frac{R_BL^2}{EI}$$

$$A_3 = \frac{1}{2}\left(\frac{1}{2}\frac{wL^2}{EI}\right)\frac{3L}{2} = \frac{3}{8}\frac{wL^2}{EI}$$

$$A_4 = \frac{1}{3}\left(-\frac{1}{2}\frac{wL^2}{EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

$$\begin{aligned} t_{C/A} &= A_1\left(L + \frac{1}{3}\frac{L}{2}\right) + A_2\left(\frac{2}{3}L\right) \\ &\quad + A_3\left(\frac{1}{3}\cdot\frac{3L}{2}\right) + A_4\left(\frac{1}{4}L\right) \\ &= -\frac{7}{72}\frac{R_BL^3}{EI} - \frac{1}{9}\frac{R_BL^3}{EI} \\ &\quad + \frac{3}{16}\frac{wL^4}{EI} - \frac{1}{24}\frac{wL^4}{EI} \\ &= -\frac{5}{24}\frac{R_BL^3}{EI} + \frac{7}{48}\frac{wL^3}{EI} \end{aligned}$$

PROBLEM 9.151 *(Continued)*



$$A_5 = \frac{1}{2}\left(\frac{1}{6}\frac{wL^2}{EI}\right)\frac{L}{2} = \frac{1}{24}\frac{wL^3}{EI}$$

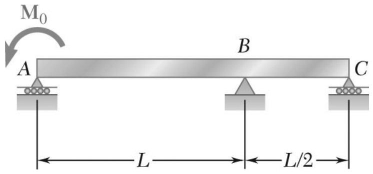
$$t_{B/A} = A_1\left(\frac{1}{3}\frac{L}{2}\right) + A_5\left(\frac{1}{3}\frac{L}{2}\right) = -\frac{1}{72}\frac{R_B L^3}{EI} + \frac{1}{144}\frac{wL^4}{EI}$$

$$y_B = t_{B/A} - \frac{L/2}{3L/2}t_{C/A} = \left(-\frac{1}{72} + \frac{5}{72}\right)\frac{R_B L^3}{EI} + \left(\frac{1}{144} - \frac{7}{144}\frac{wL^4}{EI}\right)$$

$$= \frac{1}{18}\frac{R_B L^3}{EI} - \frac{1}{24}\frac{wL^3}{EI} = 0 \quad R_B = \frac{3}{4}wL\uparrow \quad \blacktriangleleft$$

$$R_A = \frac{1}{3}wL - \frac{2}{3}R_B = \frac{1}{3}wL - \frac{1}{2}wL = -\frac{1}{6}wL = \frac{1}{6}wL\downarrow \quad \blacktriangleleft$$

$$R_C = \frac{2}{3}wL - \frac{1}{3}R_B = \frac{2}{3}wL - \frac{1}{4}wL = \frac{5}{12}wL\uparrow \quad \blacktriangleleft$$



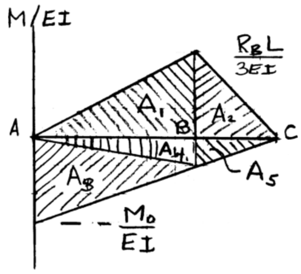
PROBLEM 9.152

For the beam and loading shown, determine the reaction at each support.

SOLUTION

Choose $R_B \downarrow$ as the redundant reaction.

Draw M/EI diagram for the loads R_B and M_0.



$$A_1 = \frac{1}{2}(L)\left(\frac{R_B L}{3EI}\right) = \frac{R_B L^2}{6EI}$$

$$A_2 = \frac{1}{2}\left(\frac{L}{2}\right)\left(\frac{R_B L}{3EI}\right) = \frac{R_B L^2}{12EI}$$

$$A_3 = \frac{1}{2}(L)\left(-\frac{M_0}{EI}\right) = -\frac{M_0 L}{2EI}$$

$$A_4 = \frac{1}{2}(L)\left(\frac{1}{3}\right)\left(-\frac{M_0}{EI}\right) = -\frac{M_0 L}{6EI}$$

$$A_3 + A_4 + A_5 = \frac{1}{2}\left(\frac{3L}{2}\right)\left(-\frac{M_0}{EI}\right) = -\frac{3M_0 L}{4EI}$$

$$y_B = y_A + L\theta_A + t_{B/A} \qquad \theta_A = -t_{B/A}/L$$

$$y_C = y_A + \frac{3L}{2}\theta_A + t_{C/A} = 0 \qquad -\frac{3}{2}t_{B/A} + t_{C/A} = 0$$

$$t_{B/A} = (A_1)\left(\frac{L}{3}\right) + A_3\left(\frac{2L}{3}\right) + A_4\left(\frac{L}{3}\right) = \frac{R_B L^3}{18EI} - \frac{7M_0 L^2}{18EI}$$

$$t_{C/A} = (A_1)\left(\frac{L}{2} + \frac{L}{3}\right) + A_2\left(\frac{L}{3}\right) + (A_3 + A_4 + A_5)(L) = \frac{R_B L^3}{6EI} - \frac{3M_0 L^2}{4EI}$$

$$-\frac{3}{2}t_{B/A} + t_{C/A} = \frac{R_B L^3}{12EI} - \frac{M_0 L^2}{6EI} = 0$$

$$R_B = \frac{2M_0}{L} \downarrow \blacktriangleleft$$

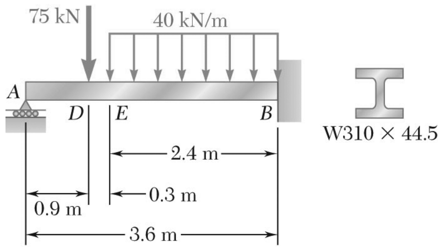
$$+\circlearrowleft \Sigma M_C = 0: \quad M_0 + \frac{L}{2}R_B - \frac{3L}{2}R_A = 0$$

$$R_A = \frac{2}{3L}[M_0 + M_0]$$

$$R_A = \frac{4M_0}{3L} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B + R_C = 0 \quad \frac{4}{3}\frac{M_0}{L} - \frac{2M_0}{L} + R_C = 0$$

$$R_C = \frac{2M_0}{3L} \uparrow \blacktriangleleft$$



PROBLEM 9.153

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

SOLUTION

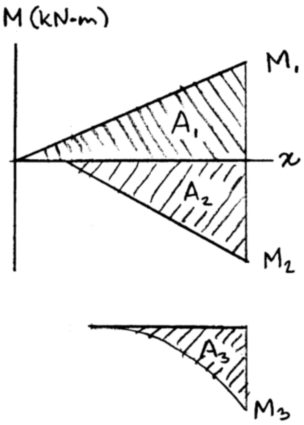
<u>Units</u>: Forces in kN; lengths in meters.

Let R_A be the redundant reaction.

Remove support at A and add reaction $R_A \uparrow$.

Draw bending moment diagram by parts.

$$M_1 = 3.6\, R_A \text{ kN}\cdot\text{m}$$
$$M_2 = -(75)(0.3 + 2.4) = -202.5 \text{ kN}\cdot\text{m}$$
$$M_3 = -\frac{1}{2}(40)(2.4)^2 = -115.2 \text{ kN}\cdot\text{m}$$



$$A_1 = \frac{1}{2}(3.6)(3.6R_A) = 6.48 \text{ kN}\cdot\text{m}^2$$
$$A_2 = \frac{1}{2}(2.7)(-202.5) = -273.375 \text{ kN}\cdot\text{m}^2$$
$$A_3 = \frac{1}{3}(2.4)(-115.2) = -92.16 \text{ kN}\cdot\text{m}^2$$

Place reference tangent at B, where

$$\theta_B = 0 \quad \text{and} \quad y_B = 0.$$

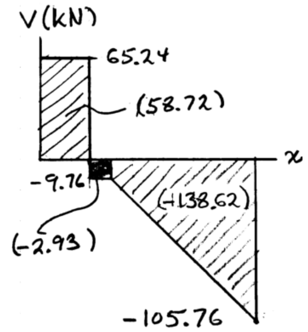
Then

$$y_A = t_{A/B} = 0$$
$$t_{A/B} = \frac{1}{EI}\left\{\left(\frac{2}{3}\cdot 3.6\right)A_1 + \left(0.9 + \frac{2}{3}\cdot 2.7\right)A_2 + \left(0.9 + 0.3 + \frac{3}{4}\cdot 2.4\right)A_3\right\}$$
$$= \frac{1}{EI}\{15.552\, R_A - 1014.5925\} = 0$$

$$R_A = 65.24 \text{ kN}\uparrow \blacktriangleleft$$

PROBLEM 9.153 *(Continued)*

Draw shear diagram.



A to D: $V = R_A = 65.24$ kN

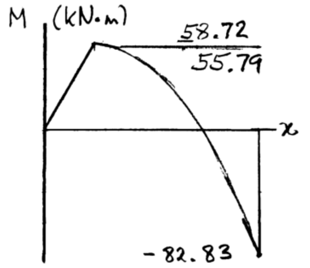
D to E: $V = 65.24 - 75 = -9.76$ kN

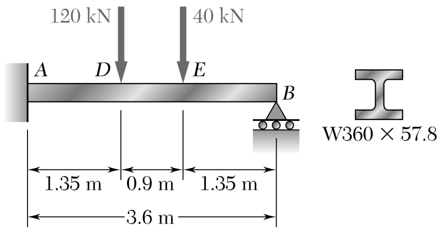
E to B: $V = -9.76 - 40(x - 1.2)$ kN

At B, $V_B = -105.76$ kN

Bending moment diagram: $M_A = 0$

$$M_D = M_A + 58.72 = 58.72 \text{ kN} \cdot \text{m}$$
$$M_E = 58.72 - 2.93 = 55.79 \text{ kN} \cdot \text{m}$$
$$M_B = 55.79 - 138.62 = -82.83 \text{ kN} \cdot \text{m}$$





PROBLEM 9.154

Determine the reaction at the roller support and draw the bending moment diagram for the beam and loading shown.

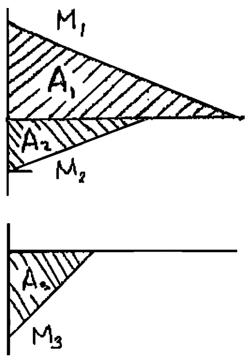
SOLUTION

Units: Forces in kN, lengths in m.

Let R_B be the redundant reaction.

Remove support B and add load R_B.

Draw bending moment diagram by parts.



$$M_1 = 3.6R_B \text{ kN}\cdot\text{m}$$

$$M_2 = -(1.35 + 0.9)(40) = -90 \text{ kN}\cdot\text{m}$$

$$M_3 = -(1.35)(120) = -162 \text{ kN}\cdot\text{m}$$

$$A_1 = \frac{1}{2}(3.6)(3.6R_B) = R_B 6.48 \text{ kN}\cdot\text{m}^2$$

$$A_2 = \frac{1}{2}(2.25)(-90) = -101.25 \text{ kN}\cdot\text{m}^2$$

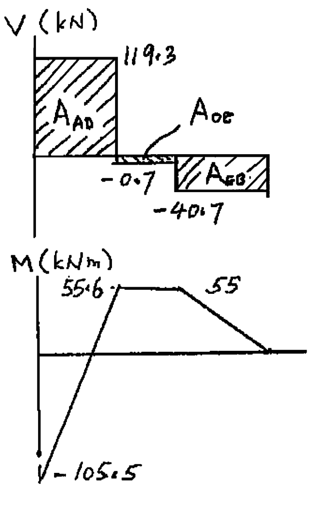
$$A_3 = \frac{1}{2}(1.35)(-162) = -109.35 \text{ kN}\cdot\text{m}^2$$

$$y_B = y_A + 3.6\theta_A + t_{B/A} = 0$$

$$t_{B/A} = 0$$

$$t_{B/A} = \frac{1}{EI}\{(6.48R_B)(2.4) + (-101.25)(2.85) + (-109.35)(3.15)\} = 0$$

$$15.552R_B - 633.015 = 0 \qquad R_B = 40.7 \text{ kN}\uparrow \blacktriangleleft$$



Draw shear diagram working from right to left.

B to E $\quad V = -R_B = -40.7 \text{ kN}$

E to D $\quad V = -40.7 + 40 = -0.7 \text{ kN}$

D to A $\quad V = -0.7 + 120 = 119.3 \text{ kN}$

Areas of shear diagram:

$$A_{AD} = (1.35)(119.3) = 161.055 \text{ kN}\cdot\text{m}$$

$$A_{DE} = (0.9)(-0.7) = -0.63 \text{ kN}\cdot\text{m}$$

$$A_{EB} = (1.35)(-40.7) = 54.945 \text{ kN}\cdot\text{m}$$

PROBLEM 9.154 *(Continued)*

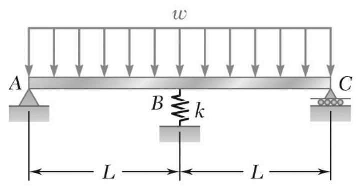
Bending moments:

$$M_A = M_1 + M_2 + M_3 = -105.5\ \text{kN} \cdot \text{m} \quad \blacktriangleleft$$

$$M_D = M_A + M_{AD} = 55.6\ \text{kN} \cdot \text{m} \quad \blacktriangleleft$$

$$M_E = M_D + A_{DE} = 55\ \text{kN} \cdot \text{m} \quad \blacktriangleleft$$

$$M_B = M_E + A_{EB} = 0 \quad \blacktriangleleft$$



PROBLEM 9.155

For the beam and loading shown, determine the spring constant k for which the force in the spring is equal to one-third of the total load on the beam.

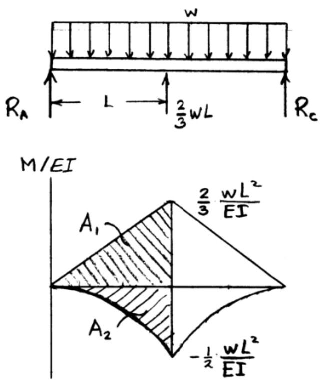
SOLUTION

Symmetric beam and loading: $R_C = R_A$

Spring force: $F = \frac{1}{3}(2wL) = \frac{2}{3}wL$

$$+\uparrow \Sigma F_y = 0: \quad R_A + F - 2wL + R_C = 0$$

$$R_A = R_C = \frac{2}{3}wL$$



Draw M/EI diagram by parts.

$$A_1 = \frac{1}{2}\left(\frac{2}{3}\frac{wL^2}{EI}\right)L = \frac{1}{3}\frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{3}\left(\frac{1}{2}\frac{wL^2}{EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$

Place reference tangent at B. $\theta_B = 0$

$$y_B = -t_{A/B}$$

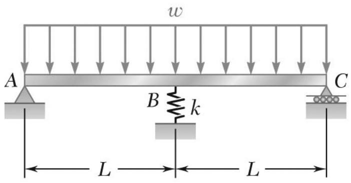
$$= -\left(A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L\right)$$

$$= -\frac{7}{72}\frac{wL^4}{EI}$$

$$F = -ky_B$$

$$k = -\frac{F}{y_B} = \frac{\frac{2}{3}wL}{\frac{7}{72}\frac{wL^4}{EI}}$$

$$k = \frac{48}{7}\frac{EI}{L^3} \blacktriangleleft$$



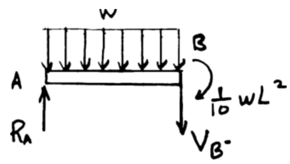
PROBLEM 9.156

For the beam and loading shown, determine the spring constant k for which the bending moment at B is $M_B = -wL^2/10$.

SOLUTION

Using free body AB,

$$+\circlearrowleft M_B = 0: \quad -R_A L + (wL)\left(\frac{L}{2}\right) - \frac{1}{10}wL^2 = 0$$

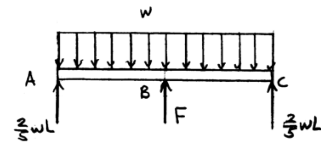


$$R_A = \frac{2}{5}wL \uparrow$$

Symmetric beam and loading: $\quad R_C = R_A$

Using free body ABC, $\quad +\uparrow \Sigma F_y = 0: \quad \frac{2}{5}wL + F + \frac{2}{5}wL - 2wL = 0$

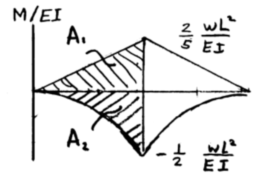
$$F = \frac{6}{5}wL$$



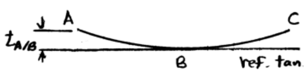
Draw M/EI diagram by parts.

$$A_1 = \frac{1}{2}\left(\frac{2}{5}\frac{wL^2}{EI}\right)L = \frac{1}{5}\frac{wL^3}{EI}$$

$$A_2 = -\frac{1}{3}\left(\frac{1}{2}\frac{wL^2}{EI}\right)L = -\frac{1}{6}\frac{wL^3}{EI}$$



Place reference tangent at B. $\quad \theta_B = 0$



$$y_B = -t_{A/B}$$

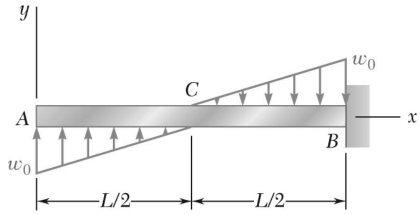
$$= -\left(A_1 \cdot \frac{2}{3}L + A_2 \cdot \frac{3}{4}L\right)$$

$$= -\frac{1}{120}\frac{wL^4}{EI}$$

$$F = -ky_B$$

$$k = -\frac{F}{y_B} = \frac{\frac{6}{5}wL}{\frac{1}{120}\frac{wL^4}{EI}}$$

$$k = 144\frac{EI}{L^3} \blacktriangleleft$$



PROBLEM 9.157

For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

SOLUTION

$$w(x) = \frac{2w_0}{L}x - w_0$$

$$V(x) = -\int w(x)dx = -\int\left(\frac{2w_0}{L}x - w_0\right)dx = -\frac{w_0}{L}x^2 + w_0 x + C_1$$

$$[x=0,\ V=0] \quad 0 = 0+0+C_1 \quad \therefore \quad C_1 = 0$$

$$M(x) = \int V(x)dx = \int\left(-\frac{w_0}{L}x^2 + w_0 x\right)dx = -\frac{w_0}{3L}x^3 + \frac{w_0}{2}x^2 + C_2$$

$$[x=0,\ M=0] \quad 0 = 0+0+C_2 \quad \therefore \quad C_2 = 0$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{w_0}{3L}x^3 + \frac{w_0}{2}x^2$$

$$EI\frac{dy}{dx} = -\frac{w_0}{12L}x^4 + \frac{w_0}{6}x^3 + C_3$$

$$\left[x=L,\ \frac{dy}{dx}=0\right] \quad 0 = -\frac{w_0L^3}{12} + \frac{w_0L^3}{6} + C_3 \quad \therefore \quad C_3 = -\frac{w_0L^3}{12}$$

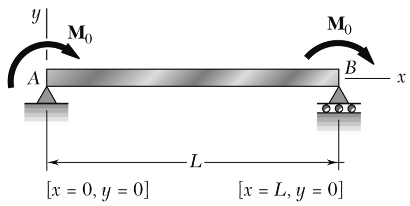
$$EIy = -\frac{w_0}{60L}x^5 + \frac{w_0}{24}x^4 - \frac{w_0L^3}{12}x + C_4$$

$$[x=L,\ y=0] \quad 0 = -\frac{w_0L^4}{60} + \frac{w_0L^4}{24} - \frac{w_0L^4}{12} + C_4 \quad \therefore \quad C_4 = \frac{7w_0L^4}{120}$$

(*a*) Elastic curve: $\quad y = -\frac{w_0}{120EIL}(2x^5 - 5Lx^4 + 10L^4x - 7L^5)$ ◀

(*b*) y at $x = 0$. $\quad y_A = +\frac{7w_0L^4}{120EI} \quad y_A = \frac{7w_0L^4}{120EI} \uparrow$ ◀

(*c*) $\frac{dy}{dx}$ at $x = 0$. $\quad \left.\frac{dy}{dx}\right|_A = -\frac{w_0L^3}{12EI} \quad \theta_A = \frac{w_0L^3}{12EI}$ ◀



PROBLEM 9.158

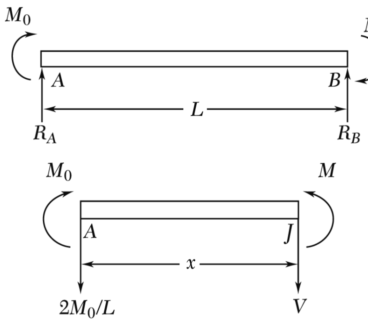
(*a*) Determine the location and magnitude of the maximum absolute deflection in AB between A and the center of the beam. (*b*) Assuming that beam AB is a W460 × 113, $M_0 = 224$ kN · m and $E = 200$ GPa, determine the maximum allowable length L of the beam if the maximum deflection if not to exceed 1.2 mm.

SOLUTION

Using AB as a free body

$$\Sigma M_B = 0 \quad -2M_0 - R_A L = 0$$

$$R_A = \frac{2M_0}{L}$$



Using portion AJ as a free body

$$\Sigma M_J = 0 \quad -M_0 + \frac{2M_0}{L}x + M = 0$$

$$M = \frac{M_0}{L}(L - 2x)$$

$$EI\frac{d^2y}{dx^2} = \frac{M_0}{L}(L - 2x)$$

$$EI\frac{dy}{dx} = \frac{M_0}{L}(Lx - x^2) + C_1$$

$$EIy = \frac{M_0}{L}\left(\frac{1}{2}Lx^2 - \frac{1}{3}x^3\right) + C_1 x + C_2$$

$$[x = 0, \ y = 0] \quad 0 = 0 - 0 + 0 + C_2 \qquad C_2 = 0$$

$$[x = L, \ y = 0] \quad 0 = \frac{M_0}{L}\left(\frac{1}{2}L^3 - \frac{1}{3}L^3\right) + C_1 L + 0 \qquad C_1 = -\frac{1}{6}M_0 L^2$$

$$y = \frac{M_0}{EIL}\left(\frac{1}{2}Lx^2 - \frac{1}{3}x^3 - \frac{1}{6}L^2 x\right) \quad \frac{dy}{dx} = \frac{M_0}{EIL}\left(Lx - x^2 - \frac{1}{6}L^2\right)$$

To find location maximum deflection set $\dfrac{dy}{dx} = 0.$

$$x_m^2 - Lx_m - \frac{1}{6}L^2 = 0 \qquad x_m = \frac{L - \sqrt{L^2 - (4)\left(\frac{1}{6}L^2\right)}}{2} = \frac{1}{2}\left(1 - \frac{\sqrt{3}}{3}\right)L = 0.21132L \quad \blacktriangleleft$$

$$y_m = \frac{M_0 L^2}{EI}\left\{\left(\frac{1}{2}\right)(0.21132)^2 - \left(\frac{1}{3}\right)(0.21132)^2 - \left(\frac{1}{6}\right)(0.21132)\right\} = -0.0160375\frac{M_0 L^2}{EI}$$

$$|y_m| = 0.0160375\frac{M_0 L^2}{EI} \quad \blacktriangleleft$$

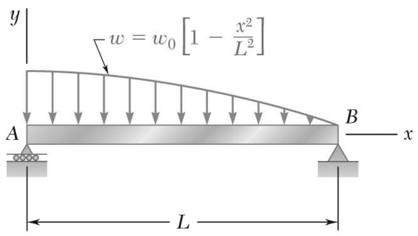
PROBLEM 9.158 *(Continued)*

Solving for L $$L = \left\{ \frac{EI\,|y_m|}{0.0160375 M_0} \right\}^{1/2}$$

Data: $E = 200\times 10^9$ Pa, $I = 556\times 10^6\ \text{mm}^4 = 556\times 10^{-6}\ \text{m}^4$

$|y_m| = 1.2\ \text{mm} = 1.2\times 10^{-3}$ m. $M_0 = 224\times 10^3 = \text{N}\cdot\text{m}$

$$L = \left\{ \frac{(200\times 10^9)(556\times 10^{-6})(1.2\times 10^{-3})}{(0.0160375)(224\times 10^3)} \right\}^{1/2} = 6.09\ \text{m} \quad ◀$$



PROBLEM 9.159

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the deflection at the midpoint of the span.

SOLUTION

$$w = w_0\left[1 - \frac{x^2}{L^2}\right] = \frac{w_0}{L^2}[L^2 - x^2]$$

$$\frac{dV}{dx} = -w = \frac{w_0}{L^2}[x^2 - L^2]$$

$$\frac{dM}{dx} = V = \frac{w_0}{L^2}\left[\frac{x^3}{3} - L^2 x\right] + C_1$$

$$M = \frac{w_0}{L^2}\left[\frac{1}{12}x^4 - \frac{1}{2}L^2x^2\right] + C_1 x + C_2$$

$$[x = 0, M = 0]: \quad 0 = 0 - 0 + 0 + C_2 \qquad \therefore \quad C_2 = 0$$

$$[x = L, M = 0]: \quad 0 = \frac{w_0}{L^2}\left[\frac{1}{12}L^4 - \frac{1}{2}L^4\right] + C_1 L \qquad \therefore \quad C_1 = \frac{5}{12}w_0 L$$

$$EI\frac{d^2y}{dx^2} = M = \frac{w_0}{L^2}\left[\frac{1}{12}x^4 - \frac{1}{2}L^2x^2 + \frac{5}{12}L^3x\right]$$

$$EI\frac{dy}{dx} = \frac{w_0}{L^2}\left[\frac{1}{60}x^5 - \frac{1}{6}L^2x^3 + \frac{5}{24}L^3x^2\right] + C_3$$

$$EIy = \frac{w_0}{L^2}\left[\frac{1}{360}x^6 - \frac{1}{24}L^2x^4 + \frac{5}{72}L^3x^3\right] + C_3 x + C_4$$

$$[x = 0, y = 0]: \quad 0 = 0 - 0 + 0 + 0 + C_4 \qquad \therefore \quad C_4 = 0$$

$$[x = L, y = 0]: \quad 0 = \frac{w_0}{L^2}\left[\frac{1}{360}L^6 - \frac{1}{24}L^6 + \frac{5}{72}L^6\right] + C_3 L \quad \therefore \quad C_3 = -\frac{11}{360}w_0 L^3$$

(*a*) Elastic curve: $y = w_0(x^6 - 15L^2x^4 + 25L^3x^3 - 11L^5x)/360\,EIL^2$ ◀

$$\frac{dy}{dx} = w_0(6x^5 - 60L^2x^3 + 75L^3x^2 - 11L^5)/360\,EIL^2$$

(*b*) Slope at end *A*: Set $x = 0$ in $\frac{dy}{dx}$. $\left.\frac{dy}{dx}\right|_A = -\frac{11}{360}\frac{w_0 L^3}{EI}$ $\qquad \theta_A = \frac{11}{360}\frac{w_0 L^3}{EI}$ ◀

PROBLEM 9.159 *(Continued)*

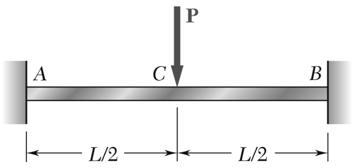
(*c*) Deflection at midpoint (say, point C): Set $x = \dfrac{L}{2}$ in. y.

$$y_C = w_0\left(\frac{1}{64}L^6 - \frac{15}{16}L^6 + \frac{25}{8}L^6 - \frac{11}{2}L^6\right)\Big/360\ EIL^2$$

$$= w_0\left(\frac{1}{64}L^6 - \frac{60}{64}L^6 + \frac{200}{64}L^6 - \frac{352}{64}L^6\right)\Big/360\ EIL^2$$

$$= -\frac{211}{23040}\frac{w_0L^4}{EI}$$

$$y_C = 0.00916\frac{w_0L^4}{EI} \downarrow \blacktriangleleft$$



$[x=0,\ \ y=0]$

$\left[x=0,\ \dfrac{dy}{dx}=0\right] \quad \left[x=\dfrac{L}{2},\ \dfrac{dy}{dx}=0\right]$

PROBLEM 9.160

Determine the reaction at A and draw the bending moment diagram for the beam and loading shown.

SOLUTION

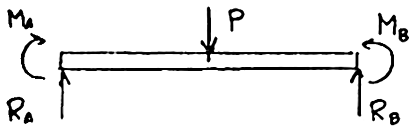
By symmetry, $\quad R_A = R_B \quad$ and $\quad \dfrac{dy}{dx}=0$ at $x=\dfrac{L}{2}$.

$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - P = 0 \qquad\qquad R_A = R_B = \frac{1}{2}P \blacktriangleleft$$

Moment reaction is statically indeterminate.

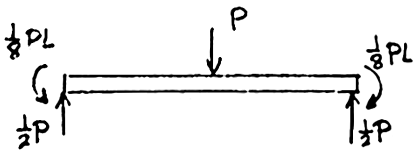
$\underline{0 < x < \dfrac{L}{2}}$

$$M = M_A + R_A x = M_A + \frac{1}{2}Px$$



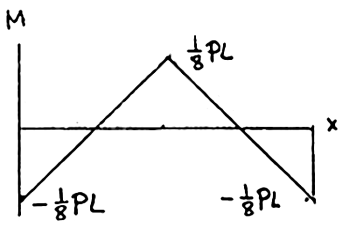
$$EI\frac{d^2y}{dx^2} = M_A + \frac{1}{2}Px$$

$$EI\frac{dy}{dx} = M_A x + \frac{1}{4}Px^2 + C_1$$



$$\left[x=0,\ \ \frac{dy}{dx}=0\right]: \quad 0 - 0 + C_1 = 0 \qquad C_1 = 0$$

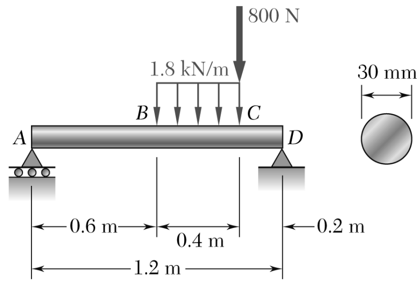
$$\left[x=\frac{L}{2},\ \ \frac{dy}{dx}=0\right]: \quad M_A\frac{L}{2} + \frac{1}{4}P\left(\frac{L}{2}\right)^2 + 0 = 0$$



$$M_A = -\frac{1}{8}PL \qquad\qquad M_A = \frac{1}{8}PL \blacktriangleleft$$

By symmetry, $\quad M_B = M_A = \dfrac{1}{8}PL \quad \blacktriangleleft$

$$M_C = M_A + \frac{1}{2}P\frac{L}{2} = -\frac{1}{8}PL + \frac{1}{4}PL = \frac{1}{8}PL \quad \blacktriangleleft$$



PROBLEM 9.161

For the beam and loading shown, determine (*a*) the slope at end *A*, (*b*) the deflection at point *B*. Use $E = 200$ GPa.

SOLUTION

Units: Forces in kN lengths in meters.

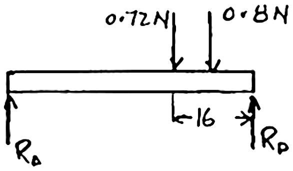
$$C = \frac{1}{2}d = \left(\frac{1}{2}\right)(30) = 15 \text{ mm}$$

$$I = \frac{\pi}{4}C^4 = \frac{\pi}{4}(15)^4 = 39761 \text{ mm}^4$$

$$EI = (200\times 10^6)(39761\times 10^{-12}) = 7.95 \text{ kN}\cdot\text{m}^2$$

Use entire beam *ABCD* as free body.

$$+\circlearrowleft \Sigma M_D = 0: \quad -1.2R_A + (0.72)(0.4) + (0.8)(0.2) = 0 \qquad R_A = 0.373 \text{ kN}\uparrow$$



$$w(x) = 1.8\langle x-0.6\rangle^0 - 1.8\langle x-1\rangle^0 \text{ kN/m}$$

$$\frac{dV}{dx} = -w = -1.8\langle x-0.6\rangle^0 + 1.8\langle x-1\rangle^0 \text{ kN/m}$$

$$\frac{dM}{dx} = V = -1.8\langle x-0.6\rangle^1 + 1.8\langle x-1\rangle^1 + 0.373 - 0.8\langle x-1\rangle^0 \text{ kN}\cdot\text{m}$$

$$EI\frac{d^2y}{dx^2} = M = -0.9\langle x-0.6\rangle^2 + 0.9\langle x-1\rangle^2 + 0.373x - 0.8\langle x-1\rangle^1 \text{ kN}\cdot\text{m}$$

$$EI\frac{dy}{dx} = -0.3\langle x-0.6\rangle^3 + 0.3\langle x-1\rangle^3 + 0.1865x^2 - 0.4\langle x-1\rangle^2 + C_1 \text{ kN}\cdot\text{m}^2$$

$$EIy = -0.075\langle x-0.6\rangle^4 + 0.075\langle x-1\rangle^4 + 0.0622x^3 - 0.133\langle x-1\rangle^3 + C_1x + C_2 \text{ kN}\cdot\text{m}^3$$

$$[x=0,\ y=0]: \quad -0+0+0+0+C_2 = 0 \qquad \therefore C_2 = 0$$

$$[x=1.2,\ y=0]: -0.075(0.6)^4 + 0.075(0.2)^4 + 0.0622(1.2)^3 - 0.133(0.2)^3 + 1.2C_1 = 0$$

$$C_1 = -0.0807 \text{ kN}\cdot\text{m}^2$$

PROBLEM 9.161 *(Continued)*

(*a*) Slope at end *A*: $\left(\dfrac{dy}{dx} \text{ at } x=0\right)$

$$EI\left(\frac{dy}{dx}\right)_A = -0+0+0+C_1$$

$$\left(\frac{dy}{dx}\right)_A = \frac{C_1}{EI} = \frac{-0.0807}{7.95} = -0.01015$$

$\theta_A = 0.01015 \text{ rad}$ ◀

(*b*) Deflection at point *B*: $(y \text{ at } x = 0.6 \text{ m})$

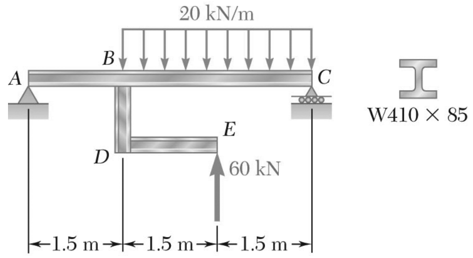
$$EIy_B = -0+0+0.0622(0.6)^3 - 0 + (-0.0807)(0.6)$$

$$= -0.035 \text{ kN}\cdot\text{m}^3$$

$$y_B = -\frac{0.035}{7.95} = 4.402\times10^{-3} \text{ m}$$

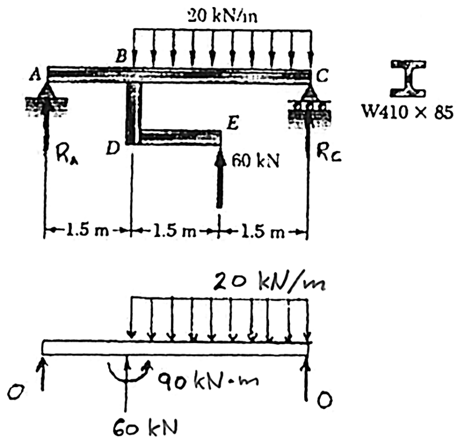
$y_B = 4.4 \text{ mm}\downarrow$ ◀

PROBLEM 9.162



The rigid bar BDE is welded at point B to the rolled-steel beam AC. For the loading shown, determine (a) the slope at point A, (b) the deflection at point B. Use $E = 200$ GPa.

SOLUTION



$+\circlearrowleft M_C = 0:$

$$-4.5R_A + (20)(3)(1.5) - (60)(1.5) = 0 \qquad R_A = 0$$

Units: Forces in kN; lengths in m.

$$EI\frac{d^2y}{dx^2} = M = 60\langle x-1.5\rangle^1 - 90\langle x-1.5\rangle^0 - \frac{1}{2}(20)\langle x-1.5\rangle^2$$

$$EI\frac{dy}{dx} = 30\langle x-1.5\rangle^2 - 90\langle x-1.5\rangle^1 - \left(\frac{1}{6}\right)(20)\langle x-1.5\rangle^3 + C_1$$

$$EIy = 10\langle x-1.5\rangle^3 - 45\langle x-1.5\rangle^2 - \frac{1}{24}(20)\langle x-1.5\rangle^4 + C_1x + C_2$$

Boundary conditions:

$[x = 0,\ y = 0]$ $\quad 0 + 0 + 0 + 0 + C_2 = 0 \qquad C_2 = 0$

$[x = 4.5,\ y = 0]$ $\quad (10)(3)^3 - (45)(3)^2 - \frac{1}{24}(20)(3)^4 + 4.5C_1 + 0 = 0 \qquad C_1 = 45\text{ kN}\cdot\text{m}^2$

Data: $\quad E = 200\times10^9\text{ Pa},\quad I = 316\times10^6\text{ mm}^4 = 316\times10^{-6}\text{ m}^4$

$$EI = (200\times10^9)(316\times10^{-6}) = 63.2\times10^6\text{ N}\cdot\text{m}^2 = 63{,}200\text{ kN}\cdot\text{m}^2$$

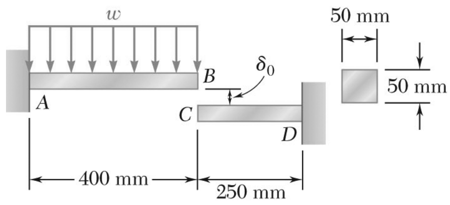
(a) Slope at A: $\quad \left(\frac{dy}{dx}\text{ at } x = 0\right) \qquad EI\theta_A = C_1 = 45\text{ kN}\cdot\text{m}^2$

$$\theta_A = \frac{45}{63{,}200} = 0.712\times10^{-3}\text{ rad} \qquad \theta_A = 0.712\times10^{-3}\text{ rad} \blacktriangleleft$$

(b) Deflection at B: $\quad (y \text{ at } x = 1.5)$

$$EIy_B = (C_1)(1.5) = (45)(1.5) = 67.5\text{ kN}\cdot\text{m}^3$$

$$y_B = \frac{67.5}{63{,}200} = 1.068\times10^{-3}\text{m} \qquad y_B = 1.068\text{ mm}\uparrow \blacktriangleleft$$



PROBLEM 9.163

Before the uniformly distributed load w is applied, a gap, $\delta_0 = 1.2$ mm, exists between the ends of the cantilever bars AB and CD. Knowing that $E = 105$ GPa and $w = 30$ kN/m, determine (*a*) the reaction at A, (*b*) the reaction at D.

SOLUTION

$$I = \frac{1}{12}(50)(50)^3 = 520.833\times10^3\,\text{mm}^3 = 520.833\times10^{-9}\,\text{m}$$

$$EI = (105\times10^9)(520.833\times10^{-6}) = 54.6875\times10^3\,\text{N}\cdot\text{m}^2$$

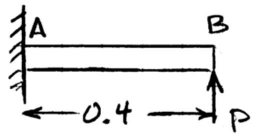
$$= 54.6875\,\text{kN}\cdot\text{m}^2$$

Units: Forces in kN; lengths in meters.

Compute deflection at B due to w. Case 8 of Appendix D.

$$(y_B)_1 = -\frac{wL^4}{8EI} = -\frac{(30)(0.400)^4}{(8)(54.6875)}$$

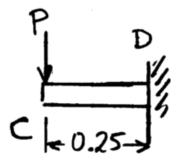
$$= -1.75543\times10^{-3} = -1.7553\,\text{mm}$$



The displacement is more than δ_0, the gap closes.

Let P be the contact force between points B and C.

Compute deflection of B due to P. Use Case 1 of Appendix D.



$$(y_B)_2 = \frac{PL^3}{3EI} = \frac{P(0.4)^3}{(3)(54.6875)}$$

$$= 390.095\times10^{-6}\,\text{P} \quad \text{m}$$

Compute deflection of C due to P.

$$y_C = -\frac{PL^3}{3EI} = -\frac{P(0.25)^3}{(3)(54.6875)} = -95.238\times10^{-6}\,\text{P} \quad \text{m}$$

Displacement condition:

$$y_B + \delta_0 = y_C$$

Using superposition, $$(y_B)_1 + (y_B)_2 - \delta_0 = y_C$$

$$-1.75543\times10^{-3} + 390.095\times10^{-6}\,\text{P} + 1.2\times10^{-3} = -95.238\times10^{-6}\,\text{P}$$

$$485.333\times10^{-6}\,\text{P} = 0.55543\times10^{-3}$$

$$\text{P} = 1.14443\,\text{kN}$$

PROBLEM 9.163 *(Continued)*

(*a*) Reaction at *A*: $+\uparrow \Sigma F_y = 0$: $R_A - 12 + 1.14443 = 0$

$$R_A = 10.86\text{ kN} \uparrow$$ ◀

$+\circlearrowleft \Sigma M_A = 0$: $M_A - (0.2)(12) + (0.4)(1.14443) = 0$

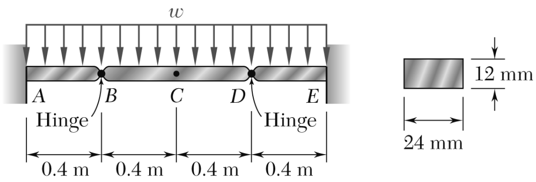
$$M_A = 1.942\text{ kN}\cdot\text{m} \circlearrowleft$$ ◀

(*b*) Reaction at *D*: $+\uparrow \Sigma F_y = 0$: $R_D - 1.14443 = 0$

$$R_D = 1.144\text{ kN} \uparrow$$ ◀

$+\circlearrowleft \Sigma M_D = 0$: $-M_D + (0.25)(1.14443) = 0$

$$M_D = 0.286\text{ kN}\cdot\text{m} \circlearrowright$$ ◀



PROBLEM 9.164

A central beam BD is joined at hinges to two cantilever beams AB and DE. All beams have the cross section shown. For the loading shown, determine the largest w so that the deflection at C does not exceed 3 mm. Use $E = 200$ GPa.

SOLUTION

Let $a = 0.4$ m

Cantilever beams AB and CD.

Cases 1 and 2 of Appendix D. $$y_C = -\frac{(wa)a^3}{3EI} - \frac{wa^4}{8EI} = \frac{11}{24}\frac{wa^4}{EI}$$

Beam BCD, with $L = 0.8$ m, assuming that points B and D do not move.

Case 6 of Appendix D.

$$y'_C = -\frac{5wL^4}{384EI}$$

Additional deflection due to movement of points B and D.

$$y''_C = y_B = y_D = -\frac{11}{24}\frac{wa^4}{EI}$$

<u>Total deflection at C:</u> $$y_C = y'_C + y''_C$$

$$y_C = -\frac{w}{EI}\left\{\frac{5L^4}{384} + \frac{11a^4}{24}\right\}$$

<u>Data:</u> $$E = 200\times10^9\,\text{Pa},$$

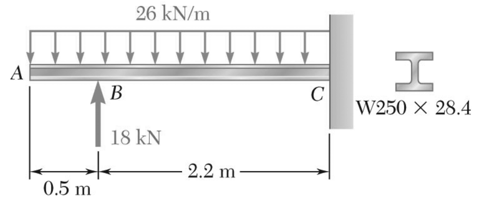
$$I = \frac{1}{12}(24)(12)^3 = 3.456\times10^{-3}\,\text{mm}^4 = 3.456\times10^{-9}\,\text{m}^4$$

$$EI = (200\times10^9)(3.456\times10^{-9}) = 691.2\ \text{N}\cdot\text{m}^2$$

$$y_C = -3\times10^{-3}\,\text{m}$$

$$-3\times10^{-3} = -\frac{w}{691.2}\left\{\frac{(5)(0.8)^4}{384} + \frac{(11)(0.4)^4}{24}\right\} = -24.69\times10^{-6}\,w$$

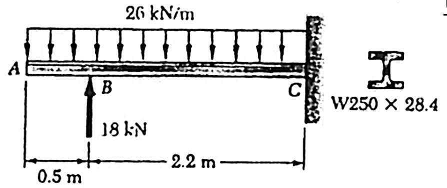
$w = 121.5$ N/m ◀

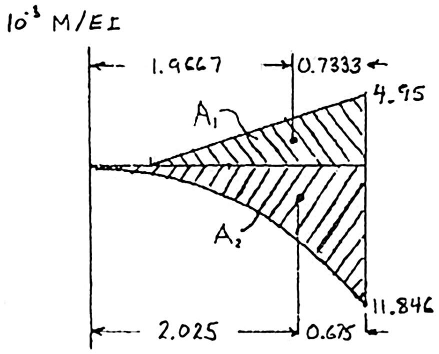


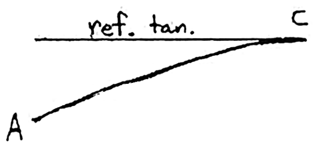
PROBLEM 9.165

For the cantilever beam and loading shown, determine (*a*) the slope at point *A*, (*b*) the deflection at point *A*. Use $E = 200$ GPa.

SOLUTION







Units: Forces in kN; lengths in m.

$$E = 200 \times 10^9 \text{ Pa}$$

$$I = 40.1 \times 10^6 \text{ mm}^4 = 40.1 \times 10^{-6} \text{ m}^4$$

$$EI = (200 \times 10^9)(40.1 \times 10^{-6}) = 8.02 \times 10^6 \text{ N} \cdot \text{m}^2$$

$$= 8020 \text{ kN} \cdot \text{m}^2$$

Draw M/EI diagram by parts.

$$\frac{M_1}{EI} = \frac{(18)(2.2)}{8020} = 4.9377 \times 10^{-3} \text{ m}^{-1}$$

$$A_1 = \frac{1}{2}(4.9377 \times 10^{-3})(2.2) = 5.4315 \times 10^{-3}$$

$$\bar{x}_1 = \frac{1}{3}(2.2) = 0.7333 \text{ m}$$

$$\frac{M_2}{EI} = -\frac{(26)(2.7)^2}{(2)(8020)} = -11.8167 \times 10^{-3} \text{ m}^{-1}$$

$$A_2 = \frac{1}{3}(-11.8167 \times 10^{-3})(2.7) = -10.6350 \times 10^{-3}$$

$$\bar{x}_2 = \frac{1}{4}(2.7) = 0.675 \text{ m}$$

Draw reference tangent at C.

$$\theta_C = \theta_A + \theta_{C/A} = \theta_A + A_1 + A_2 = 0$$

(*a*) Slope at A:

$$\theta_A = -A_1 - A_2 = -5.4315 \times 10^{-3} + 10.6350 \times 10^{-3}$$

$$= 5.20 \times 10^{-3} \text{ rad}$$

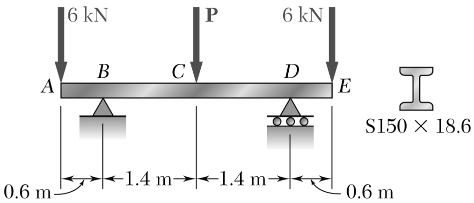
$$\theta_A = 5.20 \times 10^{-3} \text{ rad} \quad ◀$$

PROBLEM 9.165 *(Continued)*

(*b*) Deflection at *A*:

$$\begin{aligned} y_A &= y_C - \theta_C L + t_{A/C} \\ &= 0 - 0 + A_1\bar{x}_1 + A_2\bar{x}_2 \\ &= 0 - 0 + (5.4315\times 10^{-3})(1.9667) - (10.6350\times 10^{-3})(2.025) \\ &= -10.85\times 10^{-3}\text{ m} \end{aligned}$$

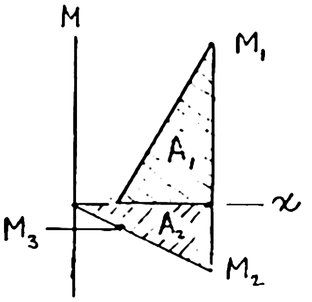
$y_A = 10.85\text{ mm}\downarrow$ ◀

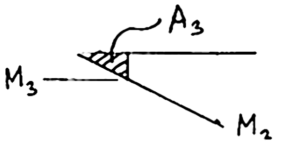


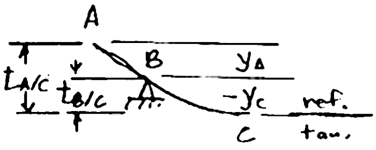
PROBLEM 9.166

Knowing that the magnitude of the load **P** is 30 kN, determine (*a*) the slope at end *A*, (*b*) the deflection at end *A*, (*c*) the deflection at midpoint *C* of the beam. Use $E = 200$ GPa.

SOLUTION







Use units of kN and m $P = 30$ kN

For $S150\times18.6$ $\qquad I = 9.11\times10^{-6}\,\text{m}$

$$EI = (200\times10^{6})(9.11\times10^{-6}) = 1822\ \text{kN}\cdot\text{m}$$

Symmetric beam with symmetric loading. Place reference tangent at midpoint C where $\theta_C = 0$.

$$R_B = R_D = \frac{1}{2}(6+30+6) = 21\ \text{kN}\uparrow$$

Draw the bending moment diagram by parts for the left half of the beam.

$$M_1 = (1.4)(21) = 29.4\ \text{kN}\cdot\text{m}$$

$$A_1 = \frac{1}{2}(1.4)(29.4) = 20.58\ \text{kN}\cdot\text{m}^2$$

$$M_2 = -(0.6+1.4)(6) = -12\ \text{kN}\cdot\text{m}$$

$$A_2 = \frac{1}{2}(2)(-12) = -12\ \text{kN}\cdot\text{m}^2$$

$$M_3 = -(0.6)(6) = -3.6\ \text{kN}\cdot\text{m}$$

$$A_3 = \frac{1}{2}(0.6)(-3.6) - 1.08\ \text{kN}\cdot\text{m}^2$$

Formulas: $\quad \theta_A = -\theta_{C/A}, \quad y_A - y_C = t_{A/C} \quad y_C = y_A - t_{A/C}$

$$y_B = y_A - 0.6\theta_A + t_{B/A} = 0, \qquad y_A = -2\theta_A - t_{B/A}$$

$$\theta_{C/A} = \frac{1}{EI}(A_1 + A_2) = \frac{20.58 - 12}{1822} = 4.709\times10^{-3}$$

$$t_{A/C} = \frac{1}{EI}\left\{(0.6 + 0.433)A_1 + \frac{2}{3}(2)A_2\right\} = \frac{15.549}{1822} = 8.534\times10^{-3}\ \text{m}$$

$$t_{B/A} = \frac{1}{EI}\left\{\frac{1}{3}(0.6)A_3\right\} = \frac{-0.216}{1822} = -0.1186\times10^{-3}\ \text{m}$$

PROBLEM 9.166 *(Continued)*

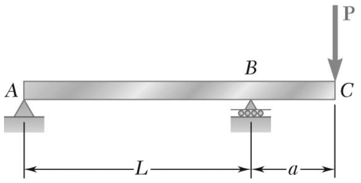
(*a*) Slope at end *A*: $\theta_A = -4.71 \times 10^{-3}$ ◀

(*b*) Deflection at *A*: $y_A = -(0.6)(-4.709 \times 10^{-3}) - (-0.1186 \times 10^{-3})$

$= 2.944 \times 10^{-3}$ m $y_A = 2.94$ mm$\uparrow$ ◀

(*c*) Deflection at *C*: $y_C = 2.944 \times 10^{-3} - 8.534 \times 10^{-3}$

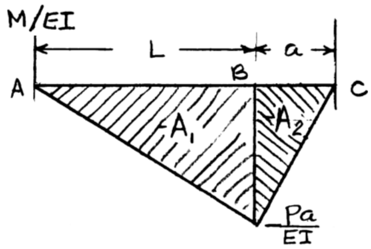
$y_C = -5.59 \times 10^{-3}$ m $y_C = 5.6$ mm$\downarrow$ ◀



PROBLEM 9.167

For the beam and loading shown, determine (*a*) the slope at point *C*, (*b*) the deflection at point *C*.

SOLUTION

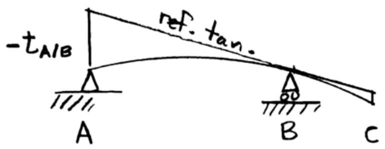


$$A_1 = -\frac{PaL}{2EI},$$

$$A_2 = -\frac{Pa^2}{2EI}$$

$$t_{A/B} = A_1\left(\frac{2}{3}L\right) = -\frac{PaL^2}{3EI}$$

$$\theta_B = \frac{t_{A/B}}{L} = -\frac{PaL}{3EI}$$



(*a*) Slope at *C*:

$$\theta_C = \theta_B + \theta_{C/B}$$

$$\theta_C = -\frac{PaL}{3EI} - \frac{Pa^2}{2EI}$$

$$= -\frac{Pa(2L+3a)}{6EI}$$

$$\theta_C = \frac{Pa(2L+3a)}{6EI} \blacktriangleleft$$

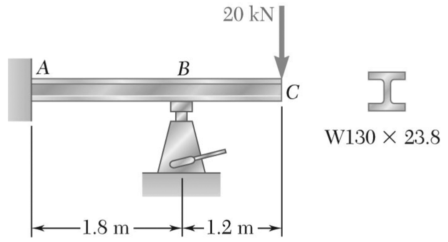
(*b*) Deflection at point *C*:

$$y_C = a\theta_B + t_{C/B}$$

$$y_C = -\frac{Pa^2L}{3EI} + \left(-\frac{Pa^2}{2EI}\right)\left(\frac{2}{3}a\right)$$

$$= -\frac{Pa^2(L+a)}{3EI}$$

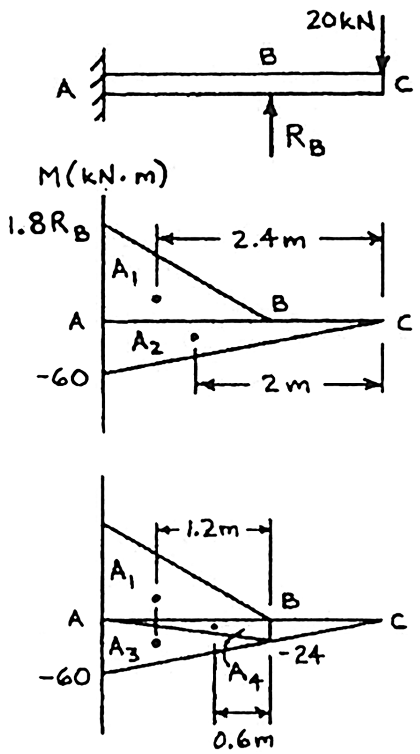
$$y_C = \frac{Pa^2(L+a)}{3EI} \downarrow \blacktriangleleft$$



PROBLEM 9.168

A hydraulic jack can be used to raise point B of the cantilever beam ABC. The beam was originally straight, horizontal, and unloaded. A 20-kN load was then applied at point C, causing this point to move down. Determine (*a*) how much point B should be raised to return point C to its original position, (*b*) the final value of the reaction at B. Use $E = 200$ GPa.

SOLUTION



For W130 × 23.8, $I_x = 8.91\times 10^6\ \text{mm}^4$

$$EI = (200\times 10^6\ \text{kPa})(8.91\times 10^{-6}\ \text{m}^4) = 1782\ \text{kN}\cdot\text{m}^2$$

Let R_B be the jack force in kN.

$$A_1 = \frac{1}{2}(1.8R_B)(1.8) = 1.62R_B$$

$$A_2 = \frac{1}{2}(-60)(3) = -90\ \text{kN}\cdot\text{m}^2$$

$$EI\,t_{C/A} = (2.4)A_1 + (2)A_2$$

$$0 = (2.4)(1.62R_B) + (2)(-90)$$

$$R_B = 46.296\ \text{kN}$$

$$A_1 = 75\ \text{kN}\cdot\text{m}^2$$

$$A_3 = \frac{1}{2}(-60)(1.8) = -54\ \text{kN}\cdot\text{m}^2$$

$$A_4 = \frac{1}{2}(-24)(1.8) = -21.6\ \text{kN}\cdot\text{m}^2$$

$$EIt_{B/A} = (1.2)A_1 + (1.2)A_3 + (0.6)A_4$$

$$= (1.2)(75) + (1.2)(-54) + (0.6)(-21.6)$$

$$= 12.24\ \text{kN}\cdot\text{m}^2$$

(*a*) <u>Deflection at *B*</u>: $y_B = t_{B/A} = \dfrac{EI\,t_{B/A}}{EI} = \dfrac{12.24}{1782} = 6.8687\times 10^{-3}\ \text{m}$ $\qquad y_B = 6.87\ \text{mm} \uparrow$ ◀

(*b*) <u>Reaction at *B*</u>: $\qquad R_B = 46.3\ \text{kN} \uparrow$ ◀