Pattern Recognition Assignment 1

Group 24

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1.1 a) SVD performed on a Square Image in Grayscale

The following is the Plot for error to choice of singular values for the given square image, in grayscale.

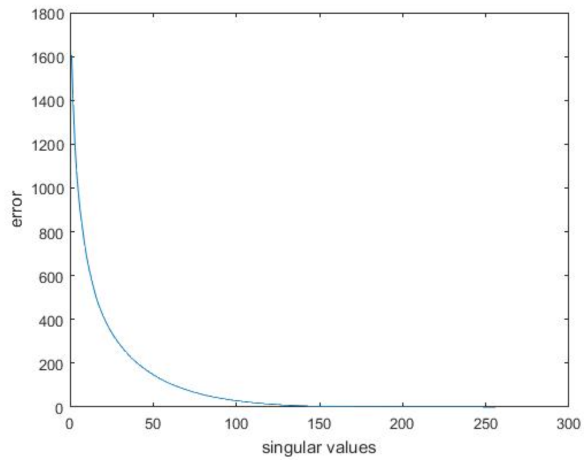


Figure : showing variation of error to choice of top N singular values.

Reconstructed image and corresponding error image for different choices of top N singular values.

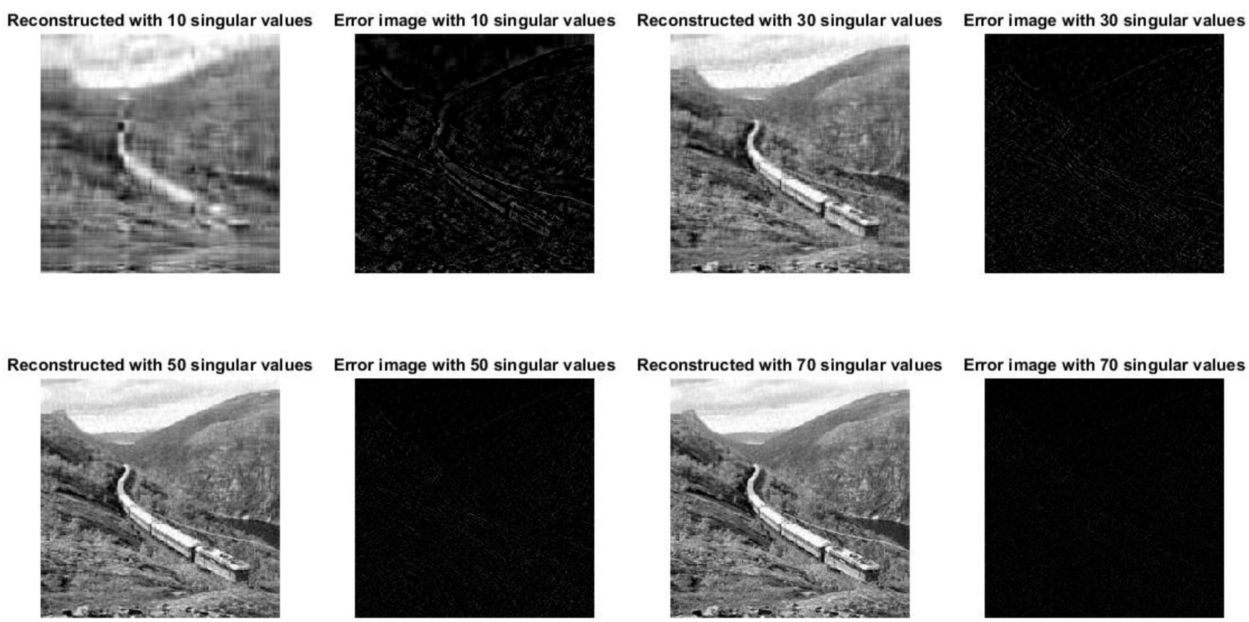


Figure : showing different image reproduction for different choice of top N singular values.

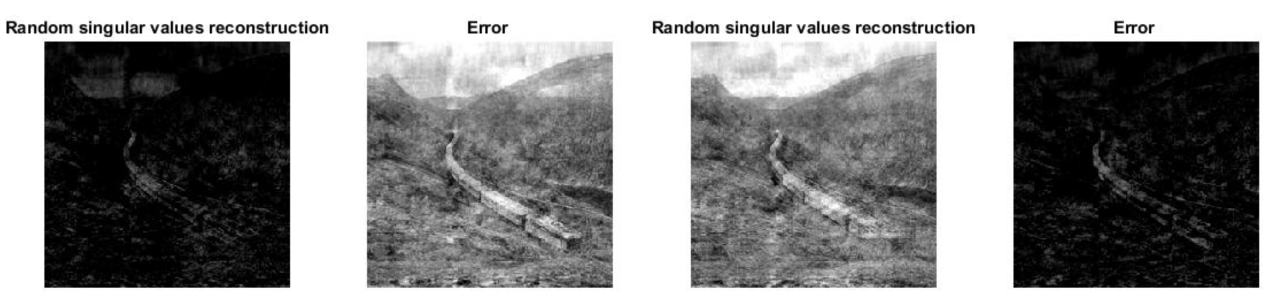


Figure: Error due to random N Singular Values

1.1 b) SVD performed on Square Image in Red, Blue and Green Images separately.

For this experiment, the original image was decomposed into its 3 color Images, red, green, blue, each of which was used for Singular Value Decomposition.

Red = double(image(:,:,1)); Green = double(image(:,:,2)); Blue = double(image(:,:,3));

Red Image

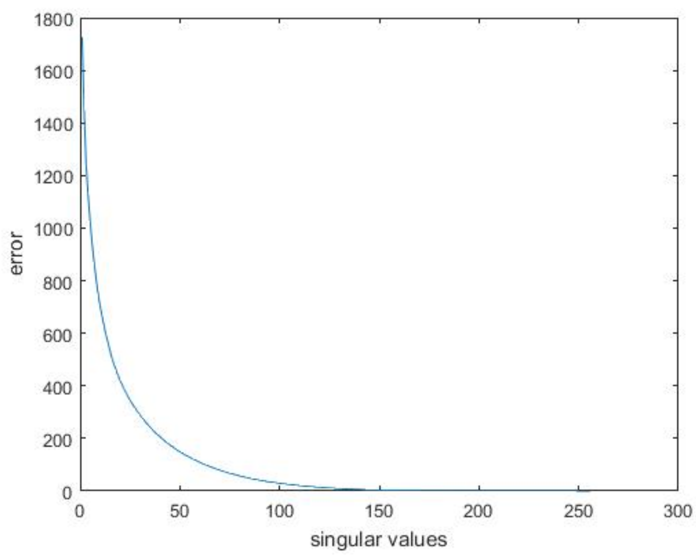


Figure : showing variation of error in Red Image to choice of top N singular values

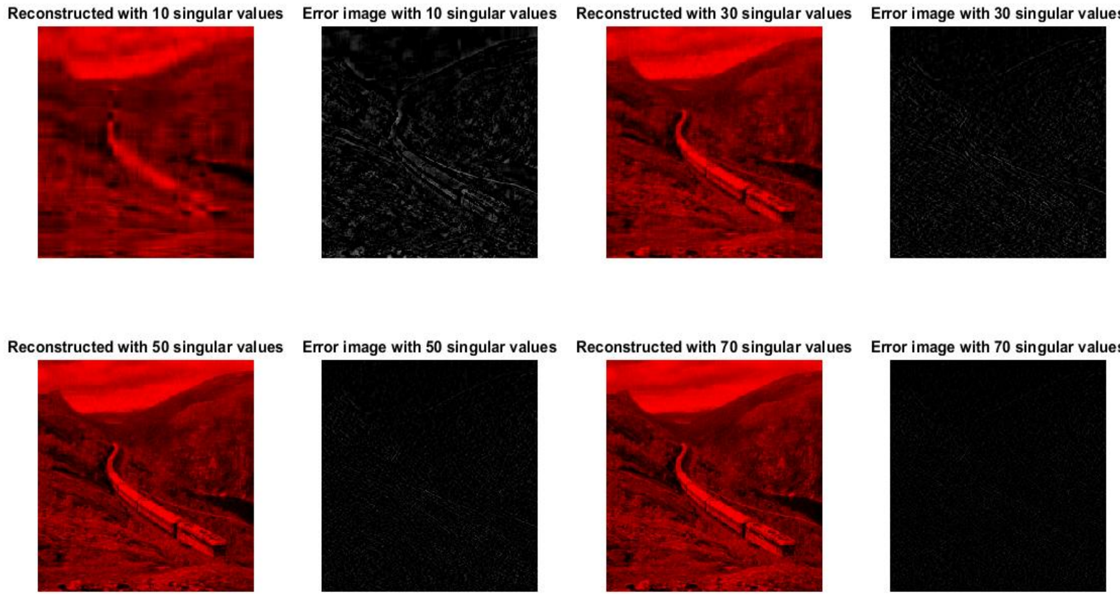


Figure 5: showing different image reproduction using the red Image for different choice of top N singular values.

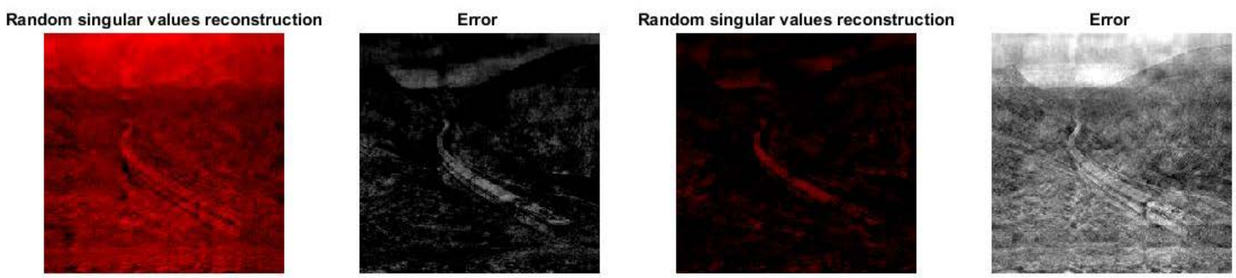


Figure: Error due to random N singular values

Green Image

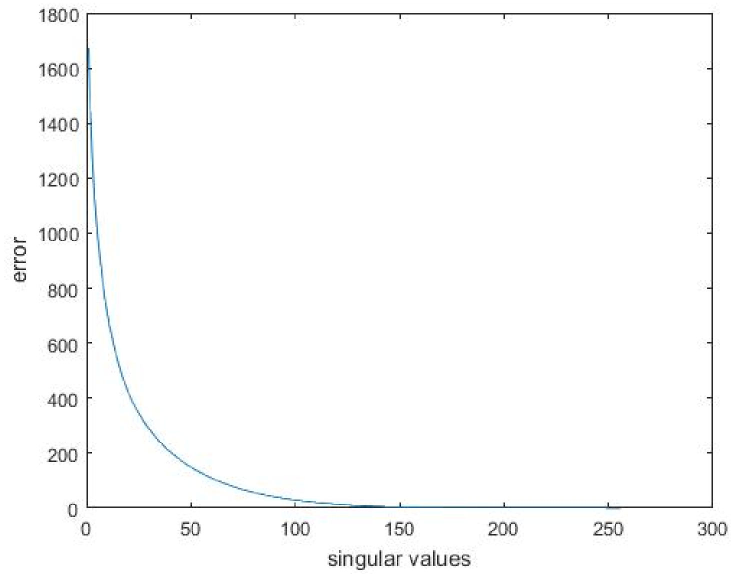


Figure 5 showing variation of error in Green Image to choice of top N singular values

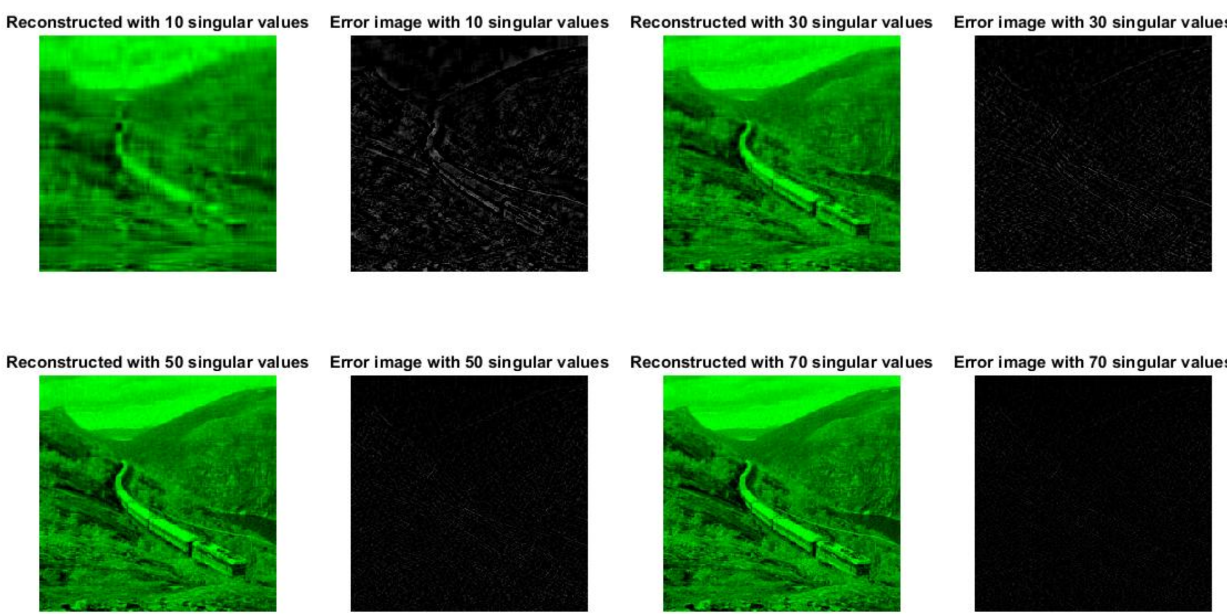


Figure 6 showing different image reproduction using the Green Image for different choice of top N singular values.

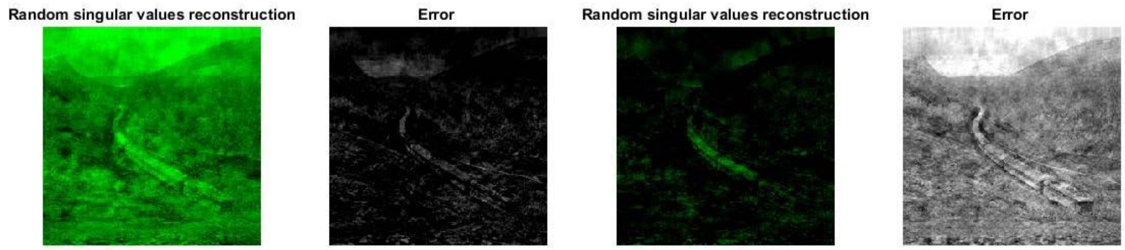


Figure 3: Error due to random N Singular Values

Blue Image

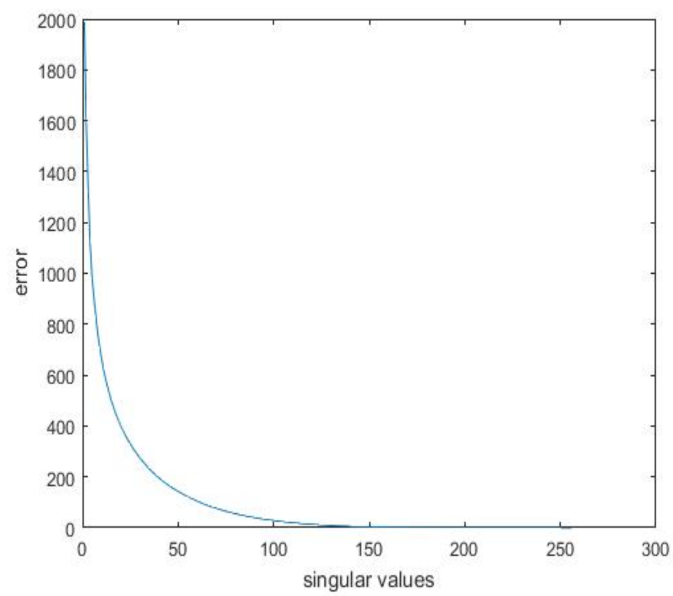


Figure: Error vs Top N singular value in Blue Image

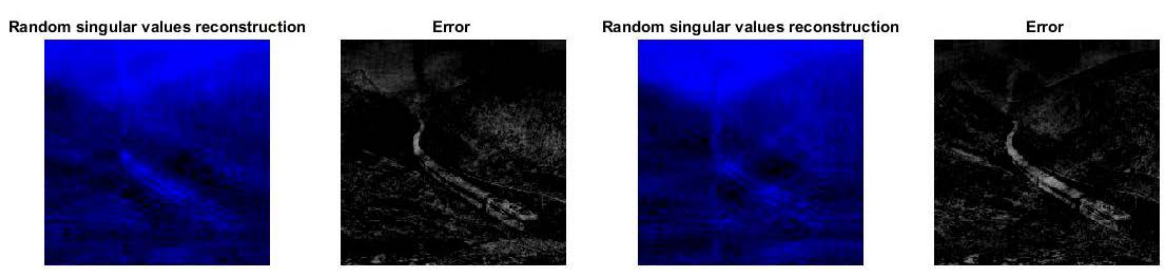


Figure 3: Error due to random N Singular Values

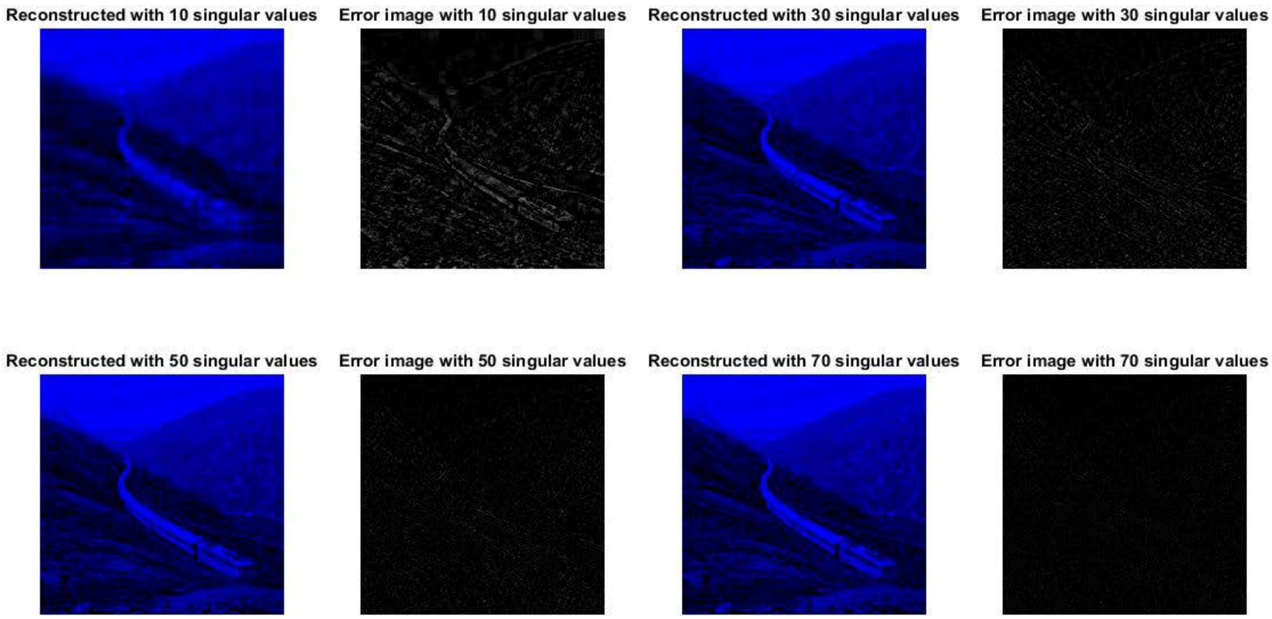


Figure: Reconstructed and error image for top N singular values

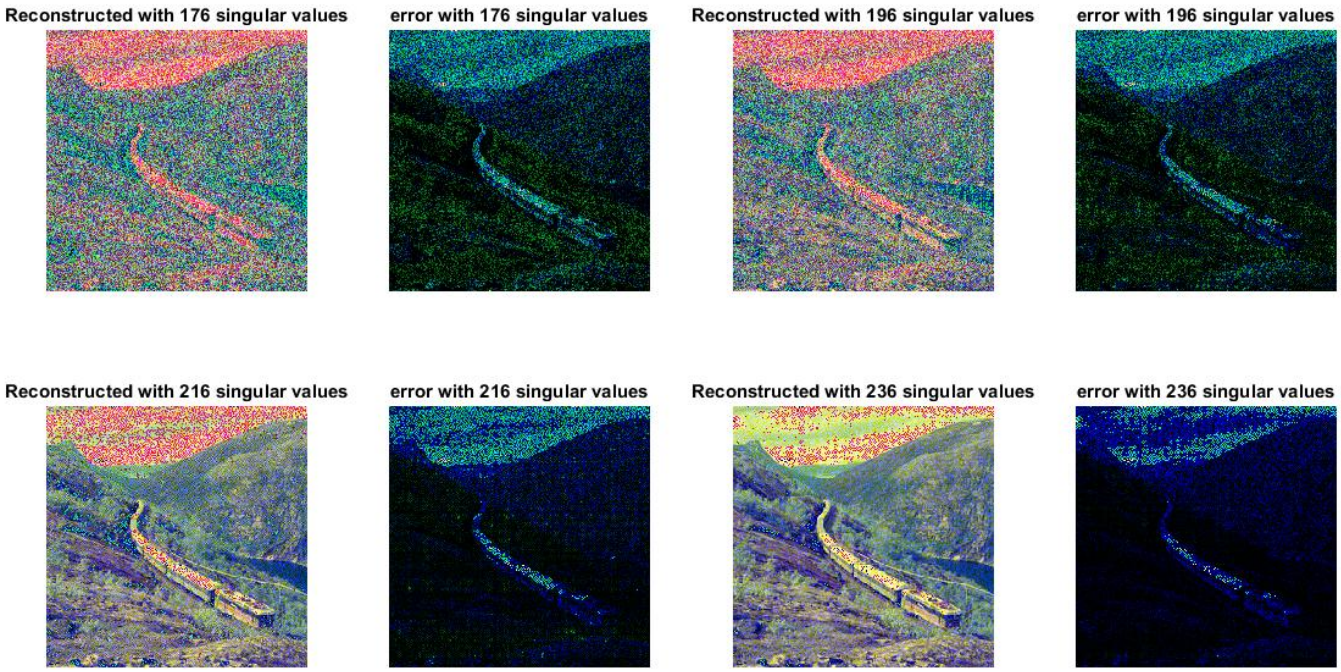
1.1 c) SVD on a square image after concatenating 8-bit R, G, B image

For the purpose of this experiments, the following steps were followed.

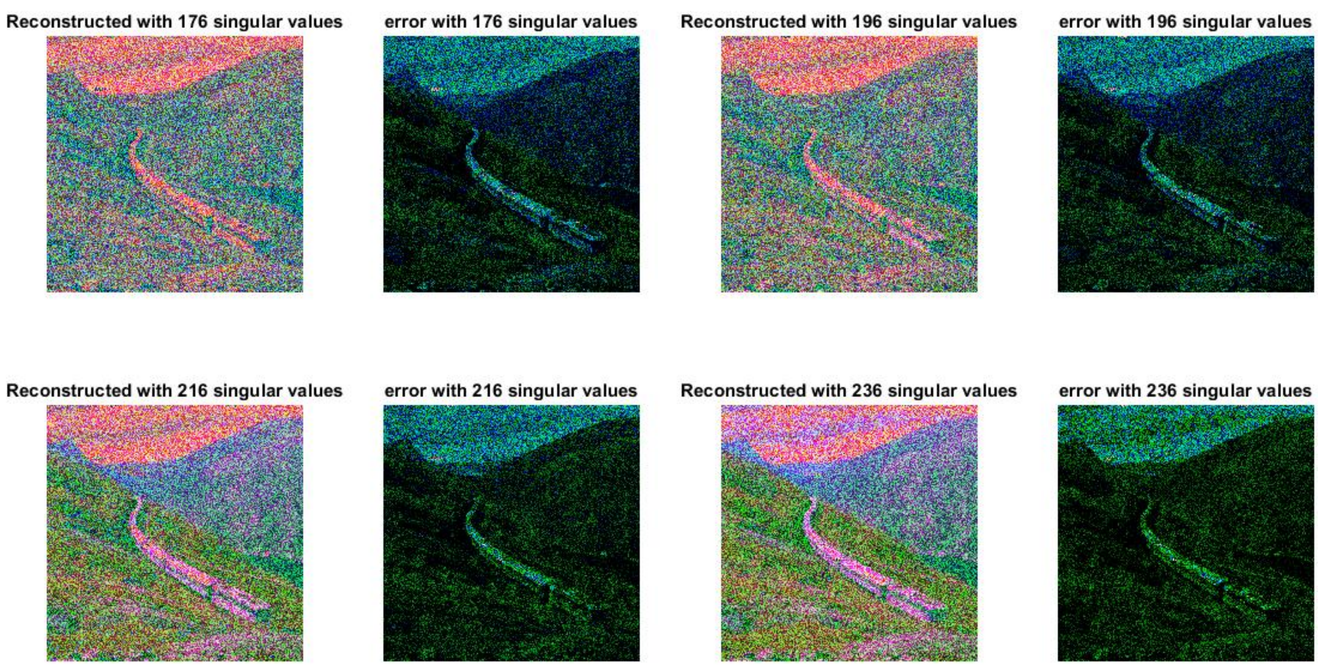
The red, green and blue color Images were separated, each corresponding 8-bit entry was concatenated together to form a 24bit entry in a new matrix, on which SVD was performed.

Every entry in the reconstructed matrix was segregated into three 8-bit component, and the red, blue and green Images were concatenated together.

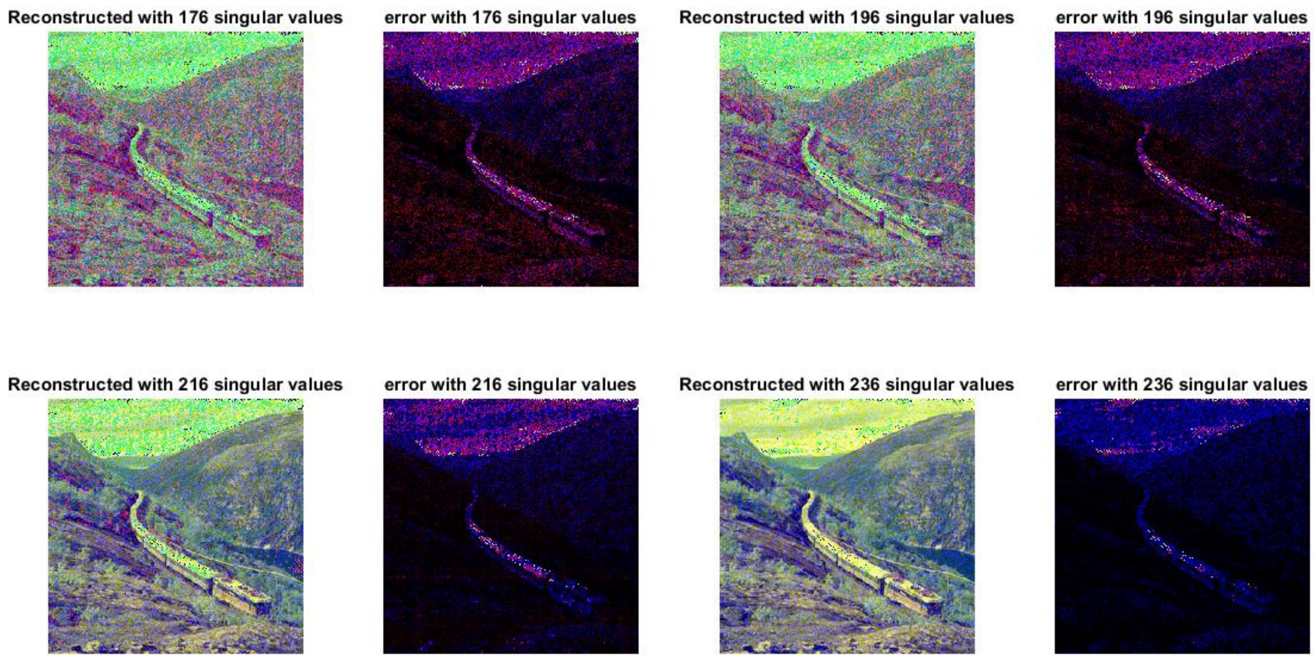
Order 1: RGB



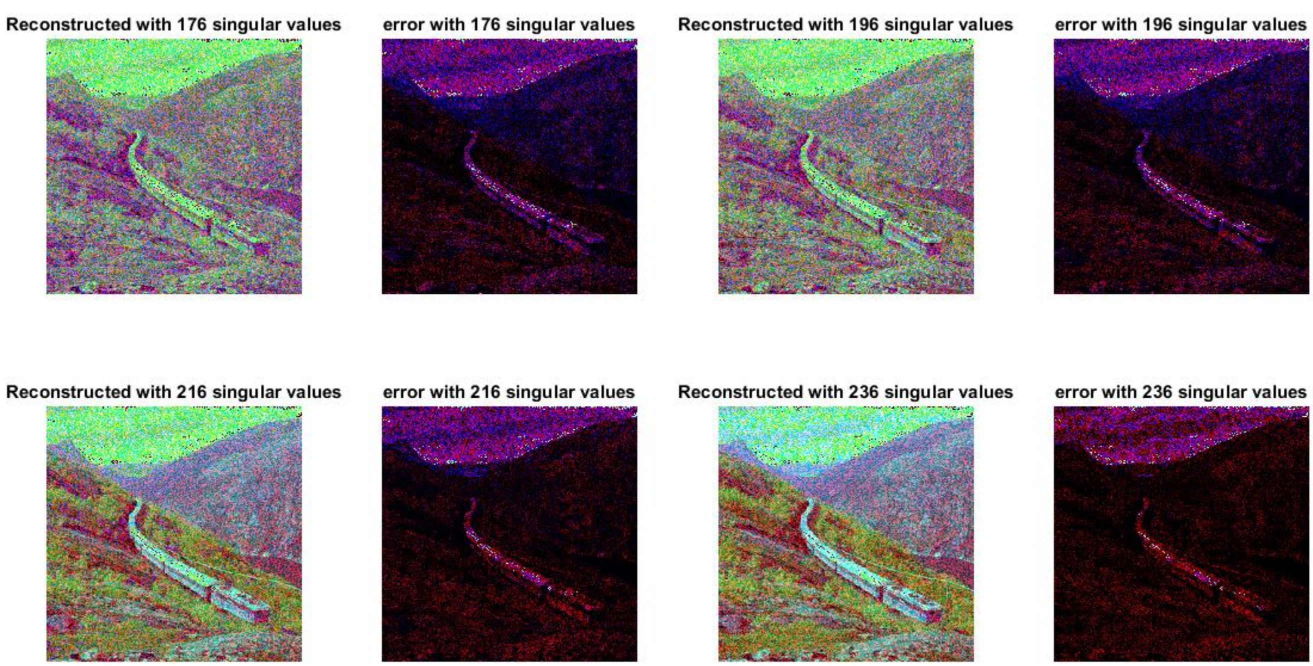
Order 2: RBG



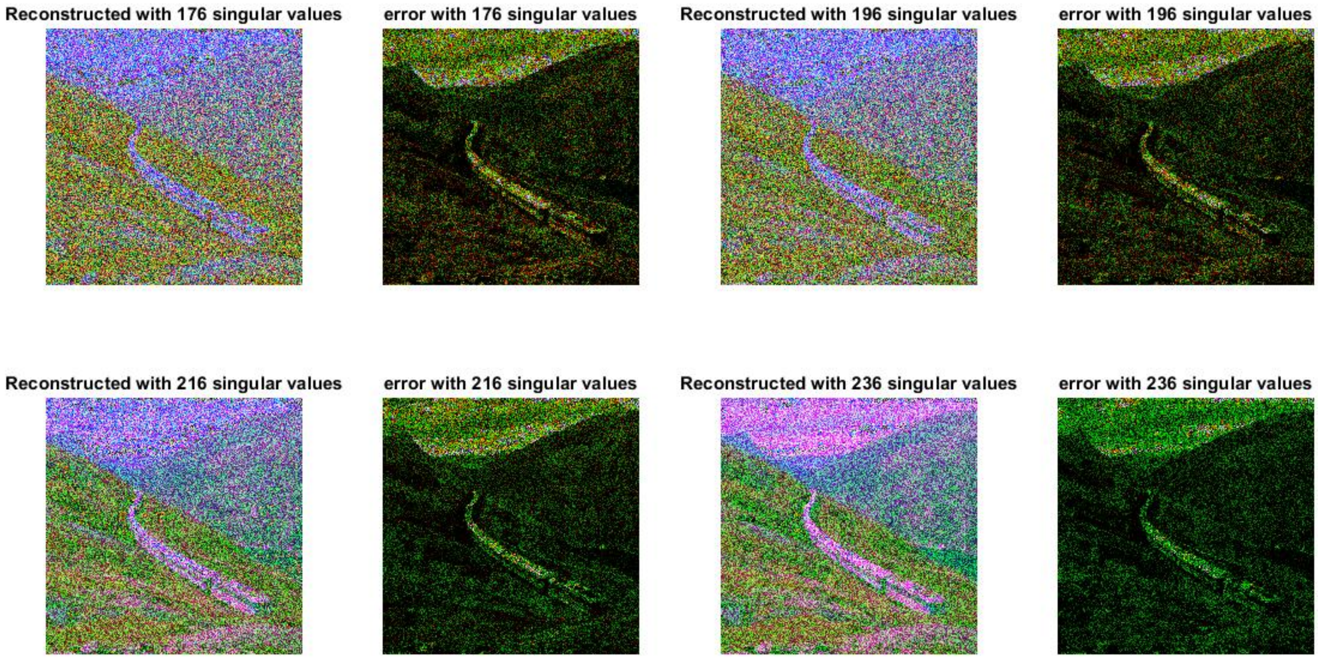
Order 3: GRB



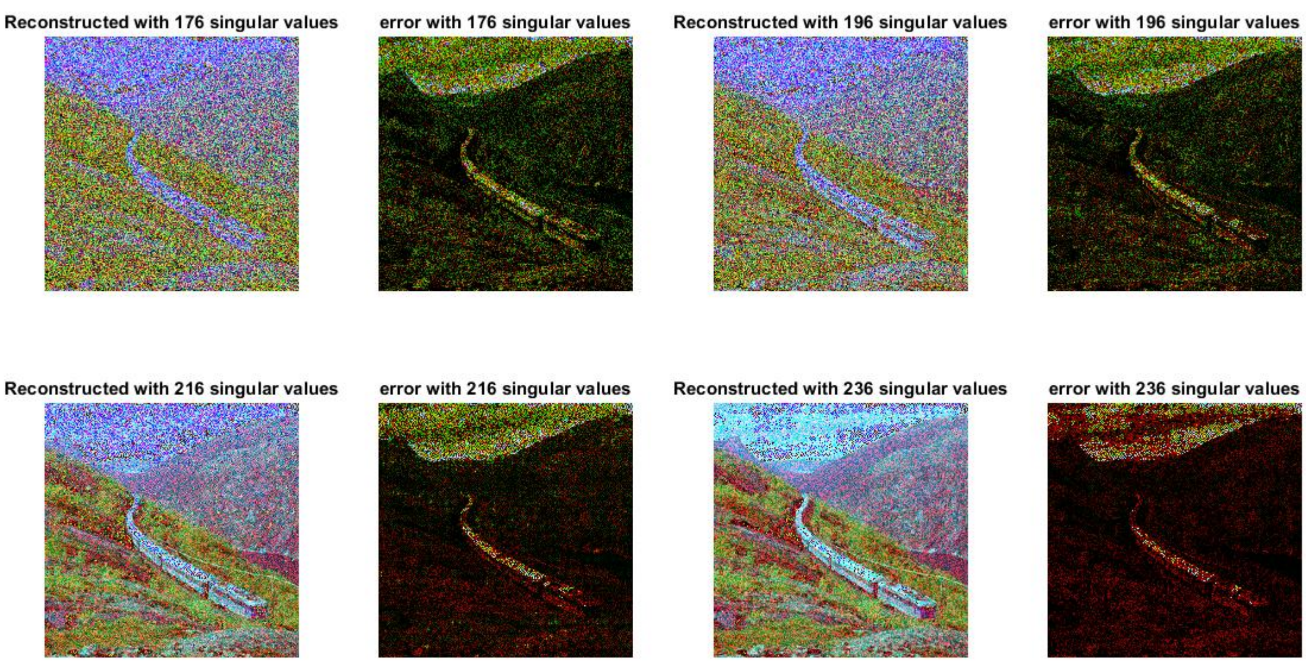
Order 4: GBR



Order 5: BRG



Order 6: BGR



The best combination was found out to be RGB, with the corresponding error graph being as shown below.

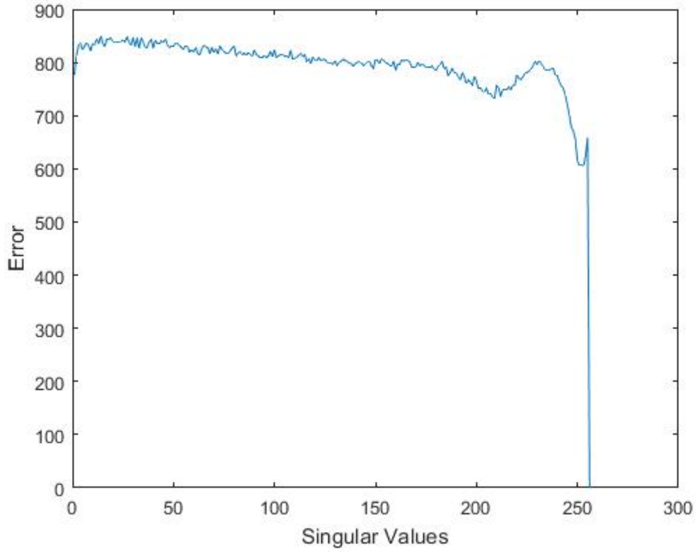


Figure: Error against Singular values for Image reconstruction using 8-bit R-G-B Images

Figure 3: Error due to random N Singular Values

<u>*1.2 a) SVD on rectangular image in Grayscale*</u>

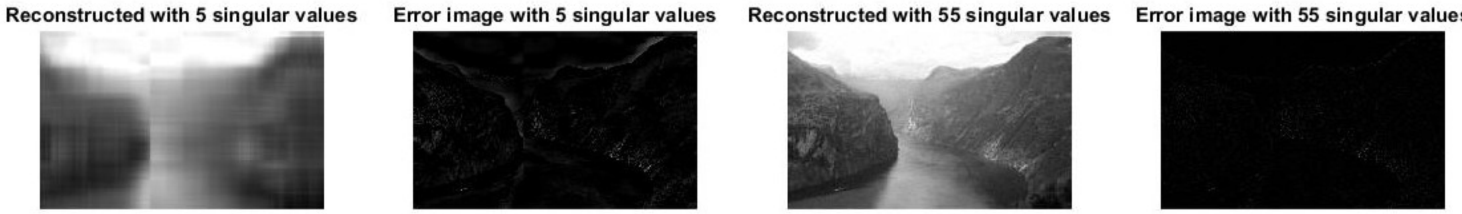


Figure: Variation of reconstructed Grayscale Image with Top N Singular Values

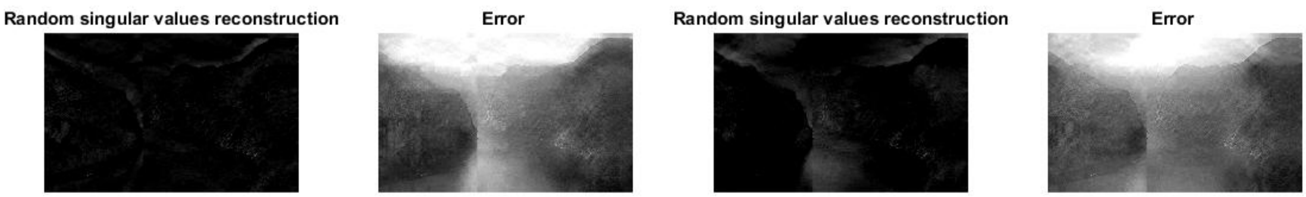


Figure: Error due to random N Singular Values

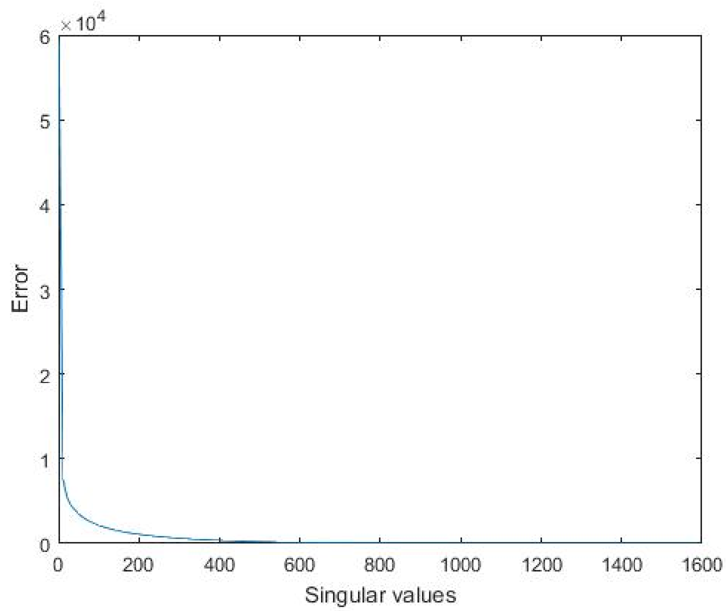


Figure: Error vs Top N Singular values for rectangular grayscale image

1.2 b) SVD on R, G, B Images separately in a rectangular image

Red Image

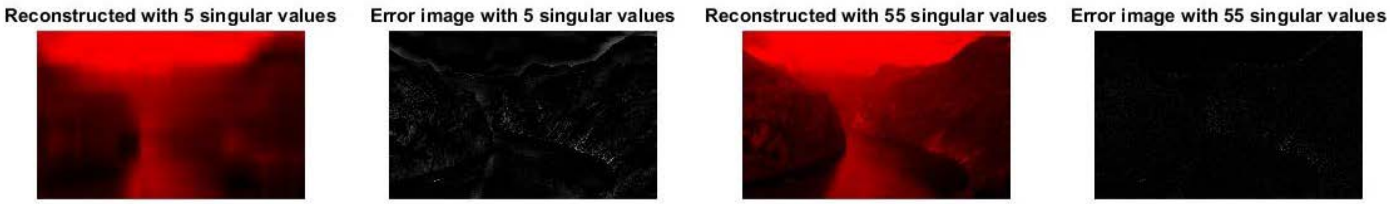


Figure: Error caused while taking 5, and 55 N singular values.

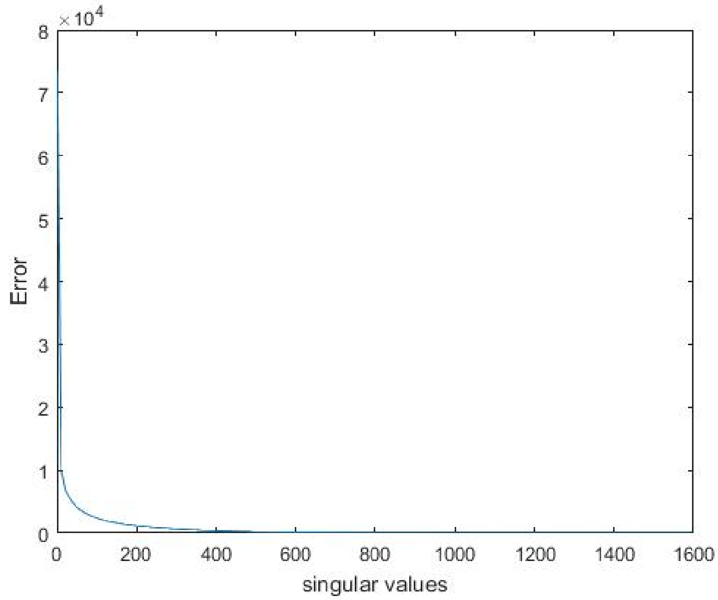


Figure: Graph for Error vs Singular values in Red Image

<u>Green Image</u>

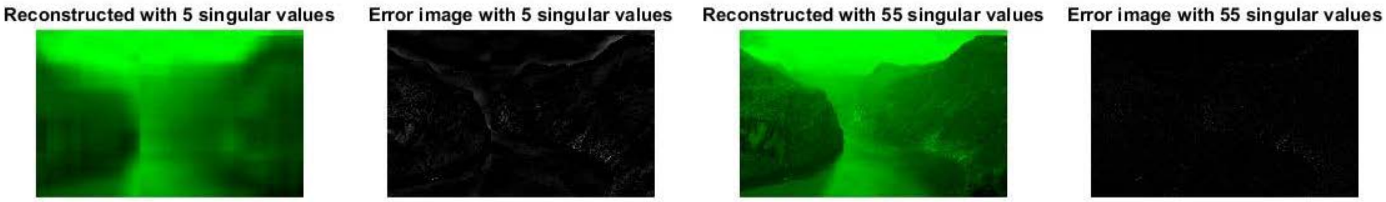


Figure: Reconstructed and error image for 5 and 55 singular values;

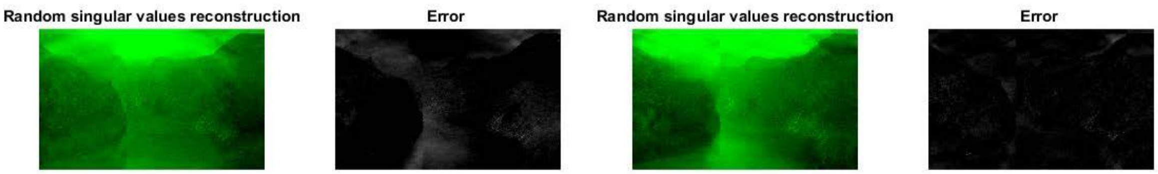


Figure: Error caused due to random N singular values.

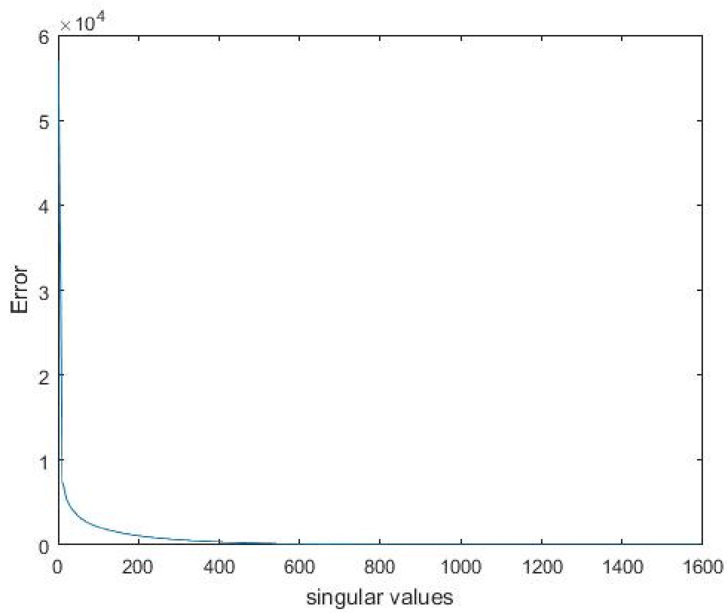


Figure: Graph for Error Vs Singular values in Green Image

Blue Image

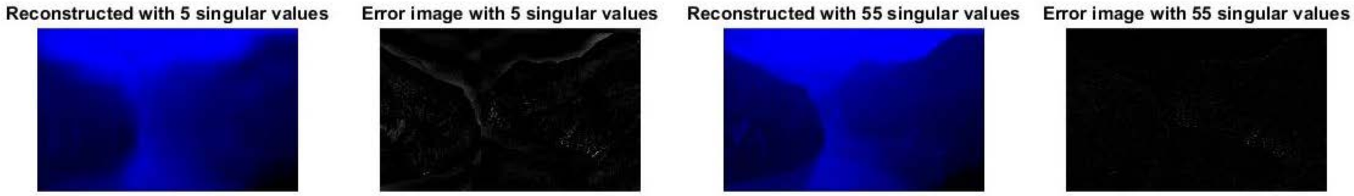


Figure: Reconstructed and error image for and 55 singular values;

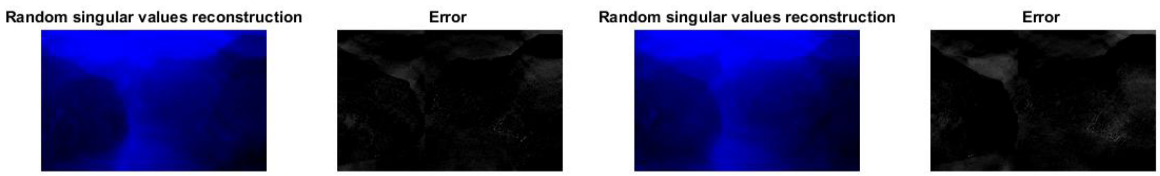


Figure: Error caused due to random N singular values.

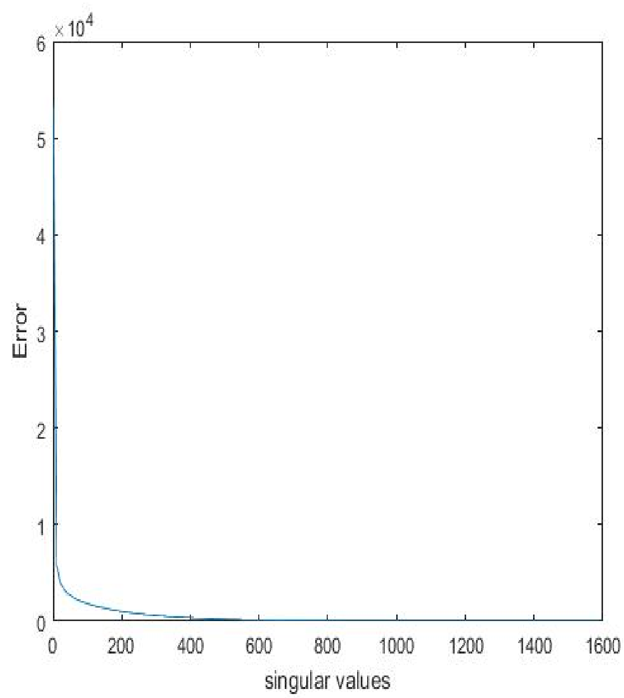
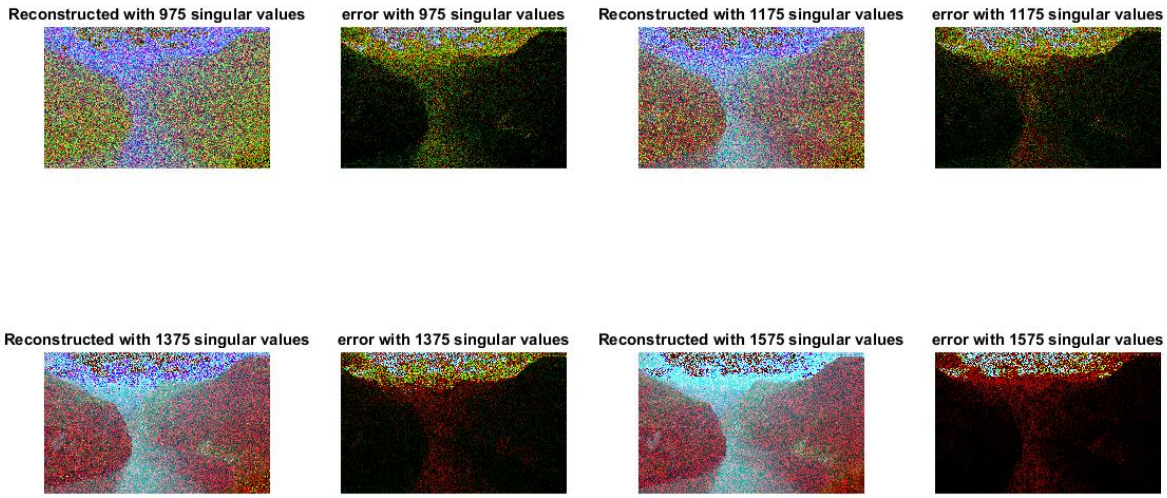


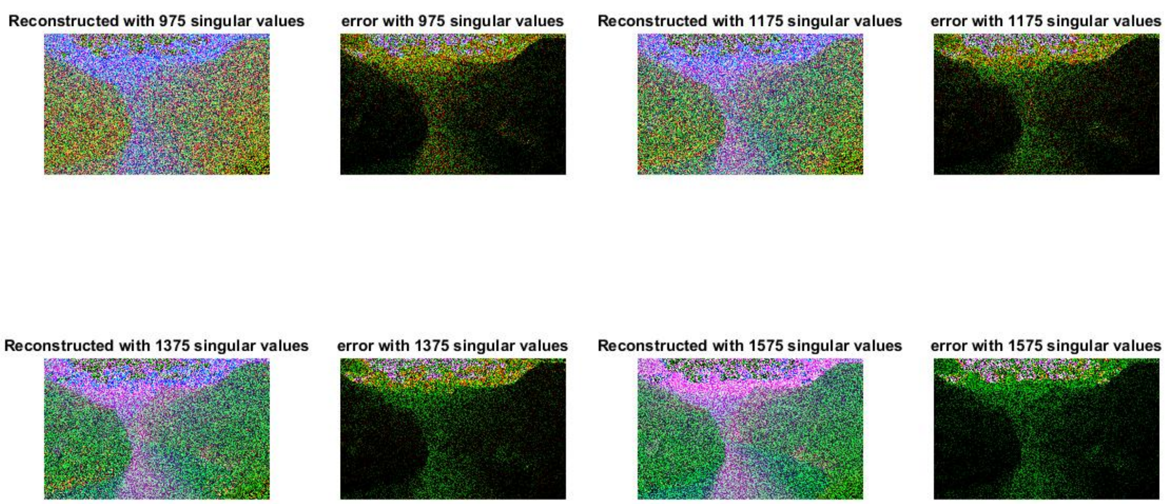
Figure: Graph for Error Vs Singular values in Blue Image

SVD on a rectangular image after concatenating 8-bit rectangular R, G, B image

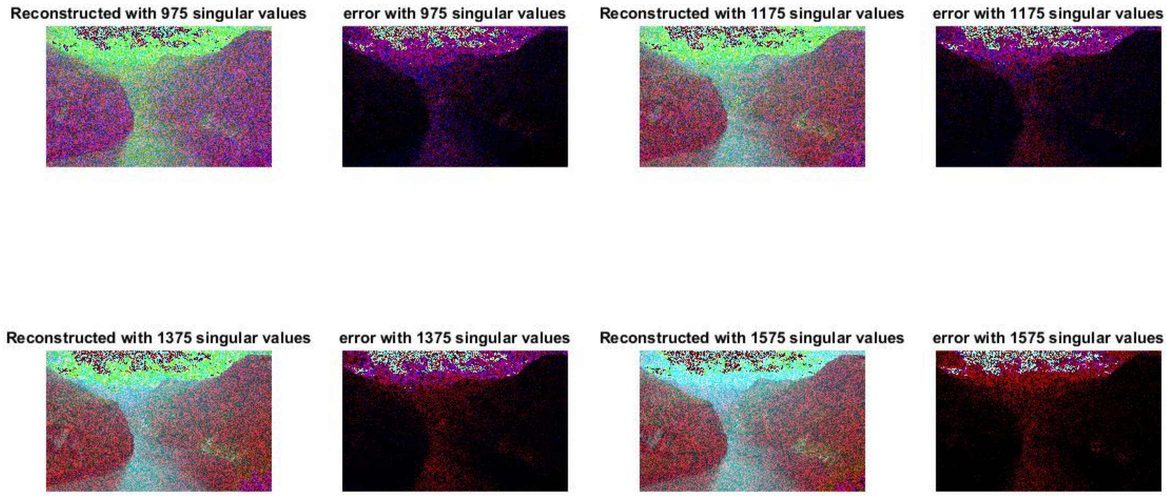
Order 1: BGR



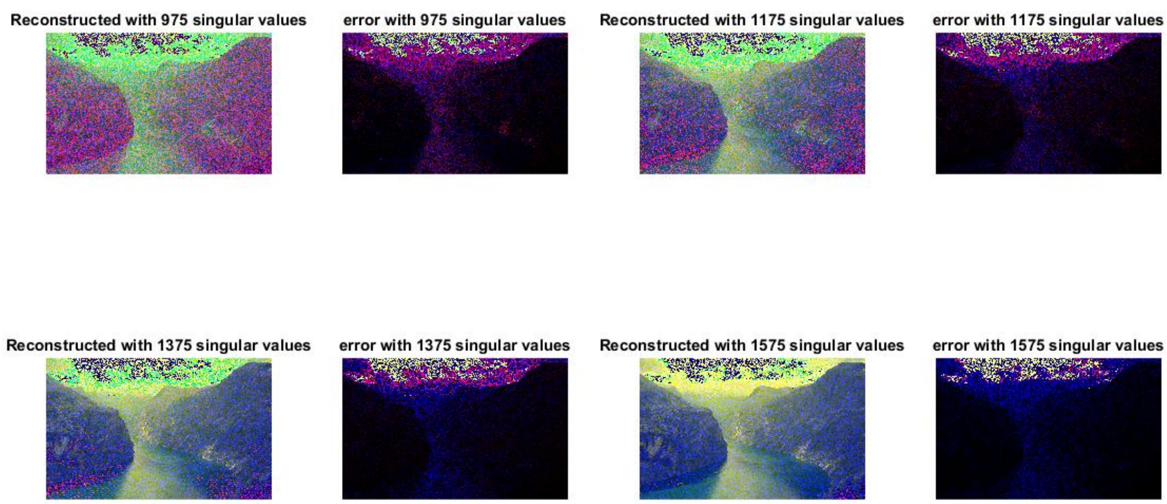
Order 2: BRG



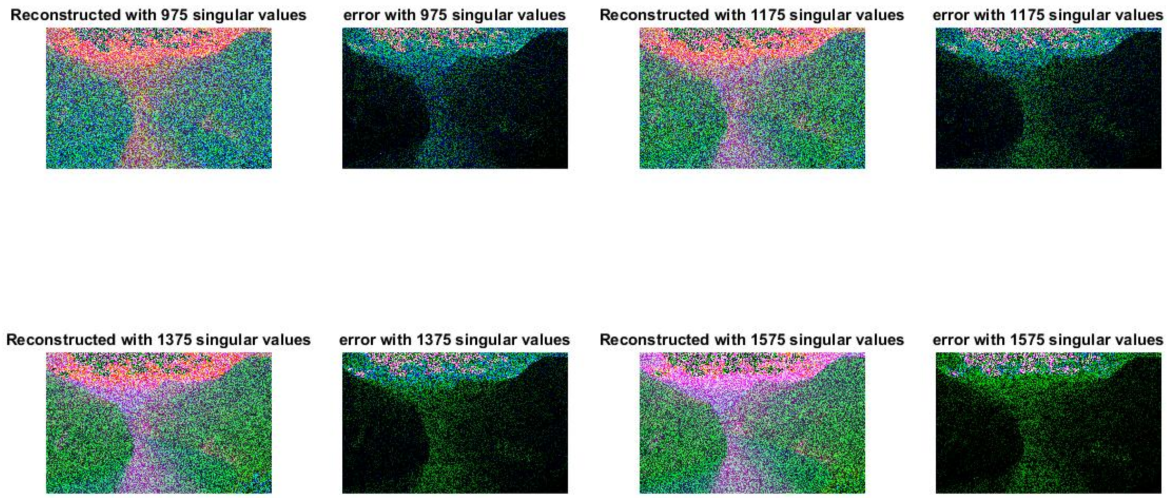
Order 3: GBR



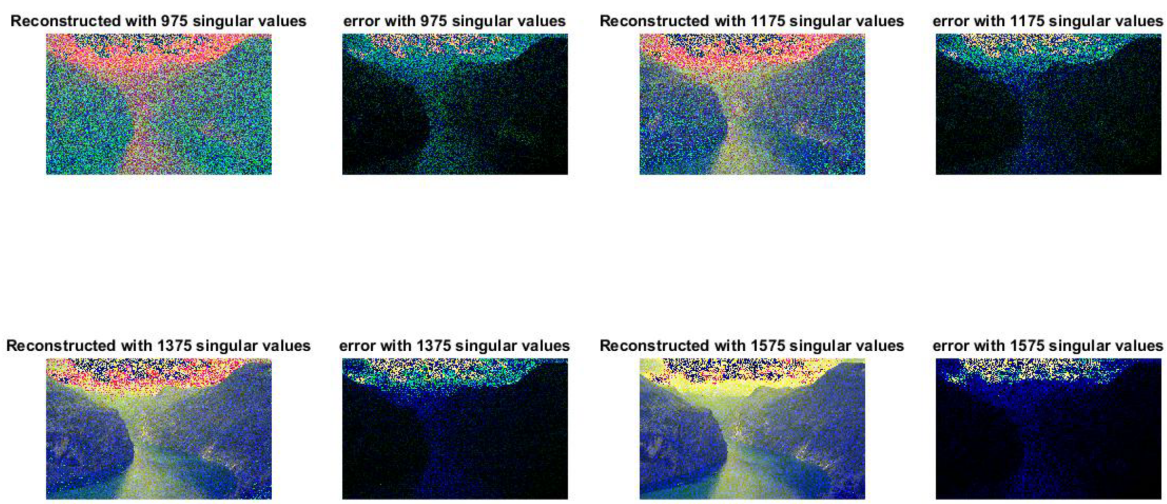
Order 4: GRB



Order 5: RBG



Order 6: RGB



Conclusion: Both the RGB and BRG versions came very close to replicating the original image.

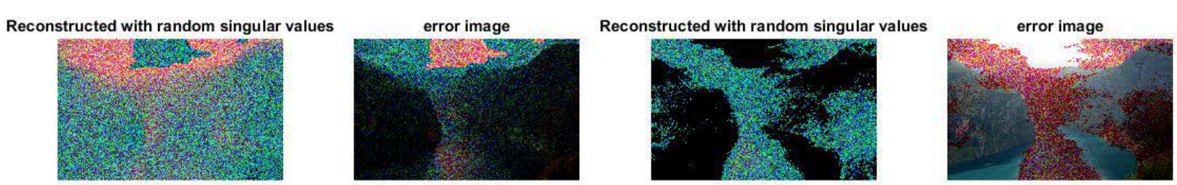


Figure: Error caused due to random N singular values.

2.1 a) Eigen Decomposition performed in Square Image in Grayscale

Eigenvectors and *eigenvalues* are numbers and vectors associated to square matrices, and together they provide the *eigen-decomposition* of a matrix which analyzes the structure of this matrix. Even though the eigen-decomposition does not exist for all square matrices, it has a particularly simple expression for a class of matrices often used in multivariate analysis such as correlation, covariance, or cross-product matrices.

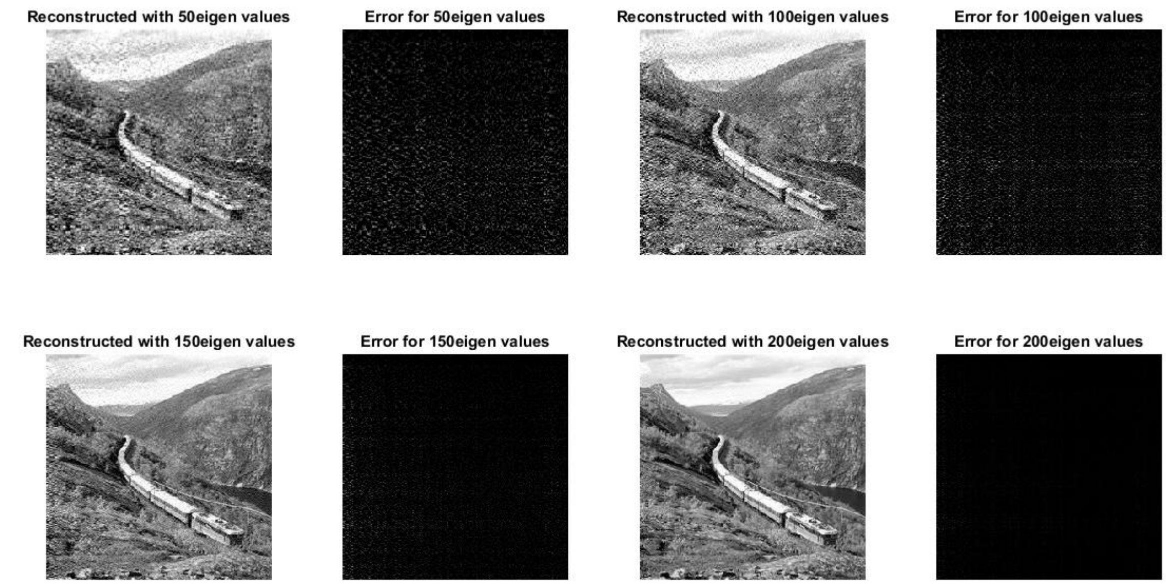


Figure: Eigen reconstruction for a grayscale image and its error for 50,100,150,200 Eigen values

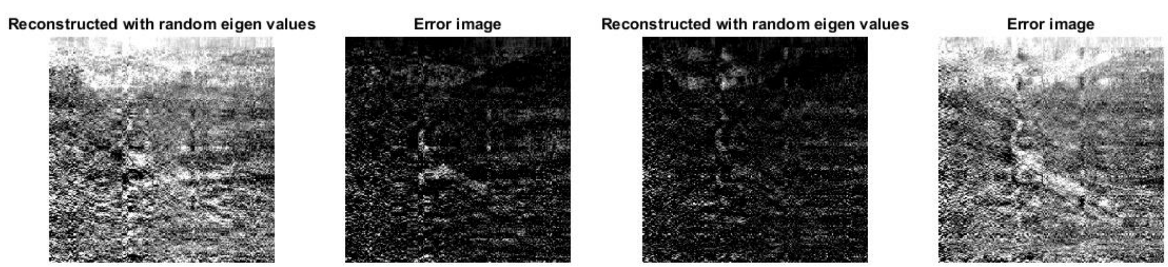


Figure: Eigen Decomposition and Error image for Random Eigen values.

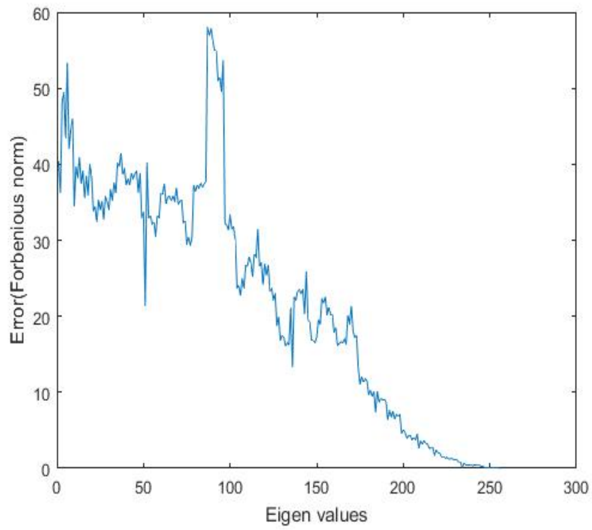


Figure: Graph for Error vs top N Eigen values, during EVD on square grayscale image

2.1 b) EVD performed on Square Image in Red, Blue and Green Images separately.

Red Image

The following images are of EVD reconstruction on red Image

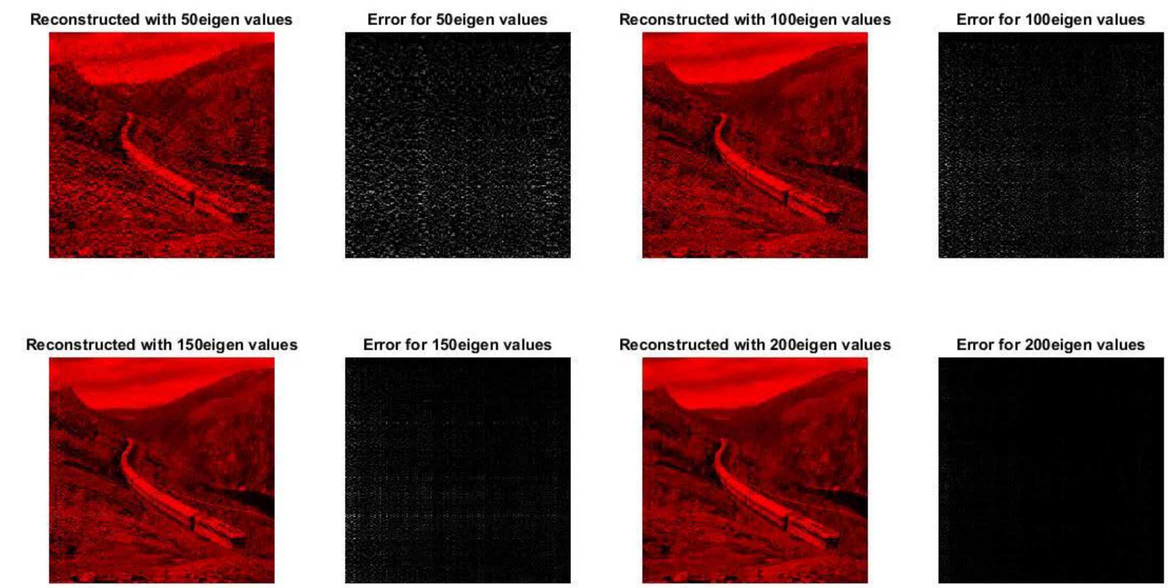


Figure: Evd reconstruction on a square image and its error image for 50,100,150,200 Eigen values

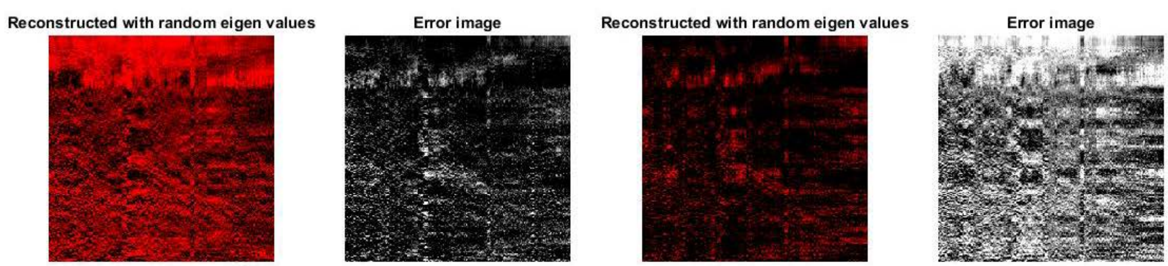


Figure: Image reconstruction for random N Eigen values

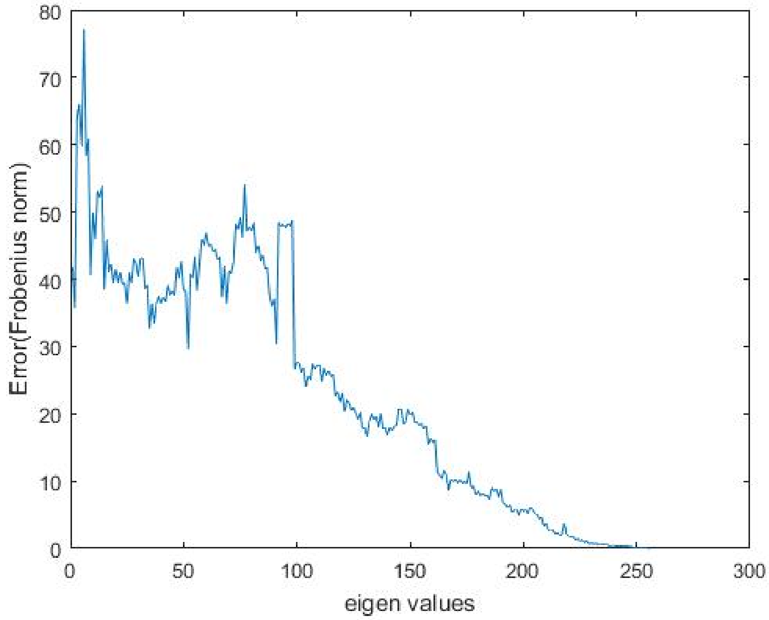


Figure: Graph for error vs Top N Eigen values chosen during reconstruction

Green Image

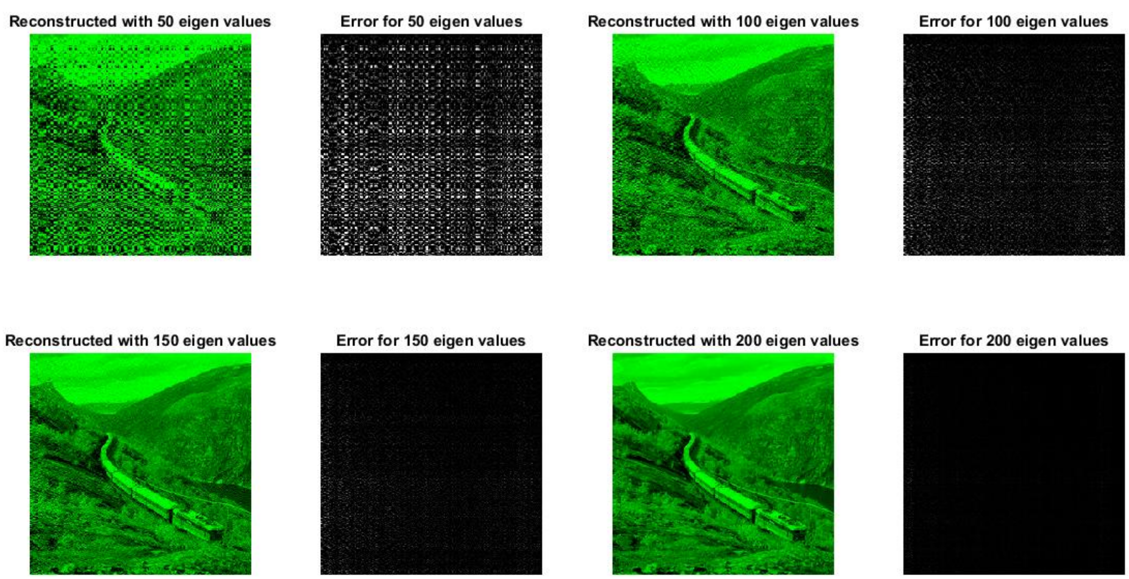


Figure: Evd and Error on green Image of a square image, for 50,100,150,200 Eigen values

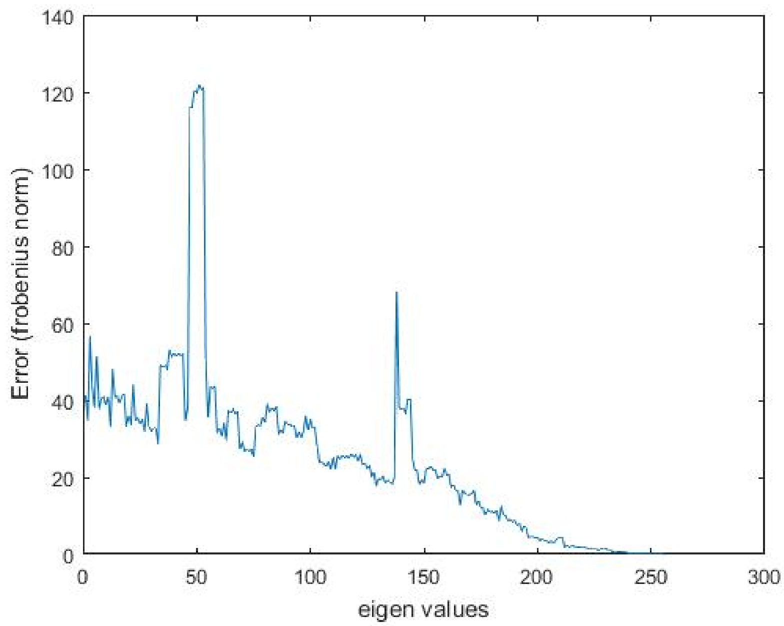


Figure : Graph for Error vs top N Eigen values during EVD on green Image.

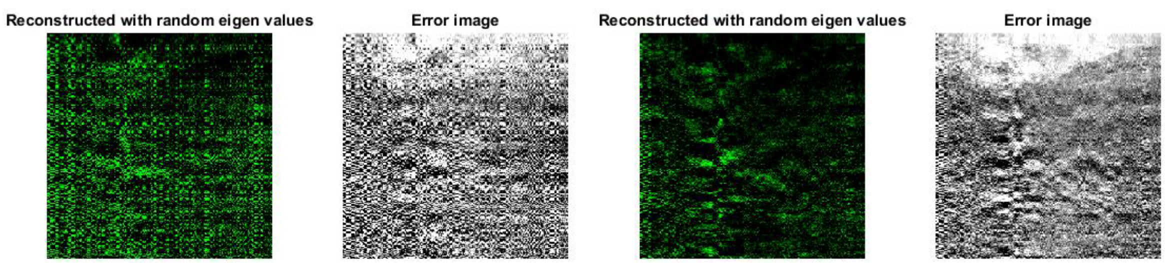


Figure: Reconstructed and Error image on green Image for random N Eigen values

Blue Image

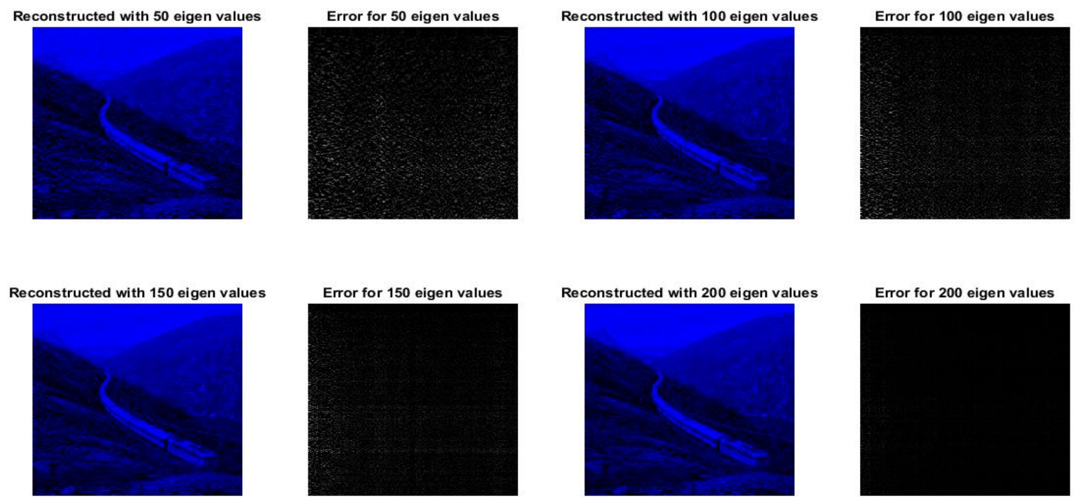


Figure: reconstructed and error image on blue Image for N=50,100…

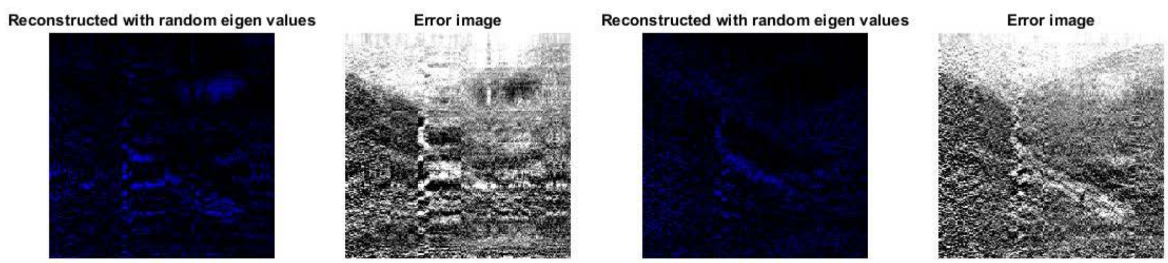


Figure: Reconstruction for Random N Eigen values

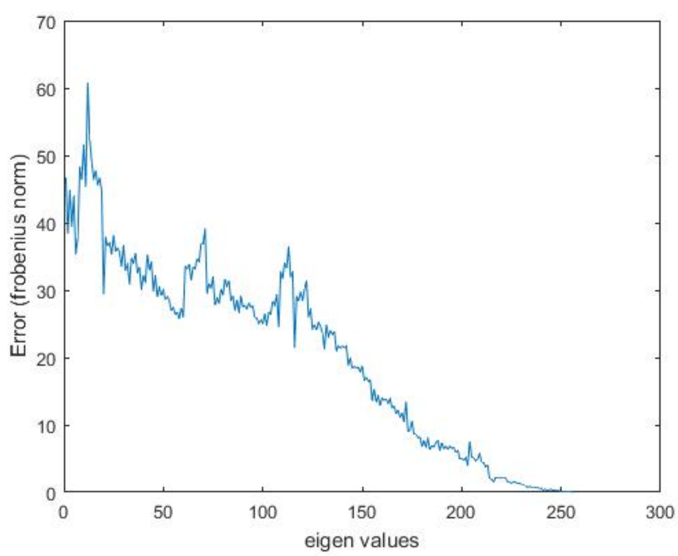
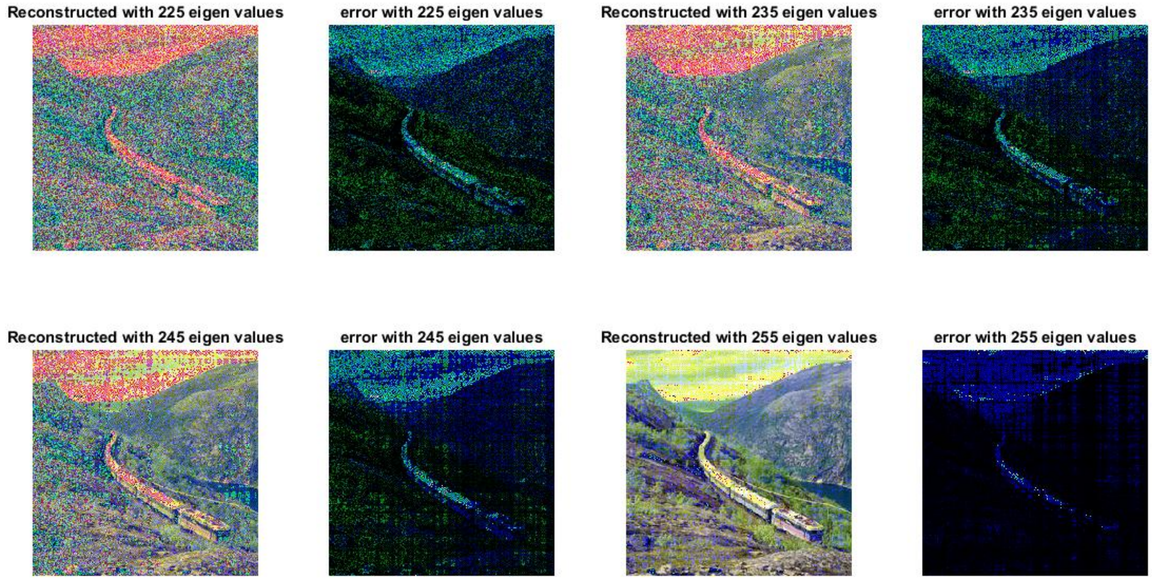


Figure: Graph for Error against Eigen values

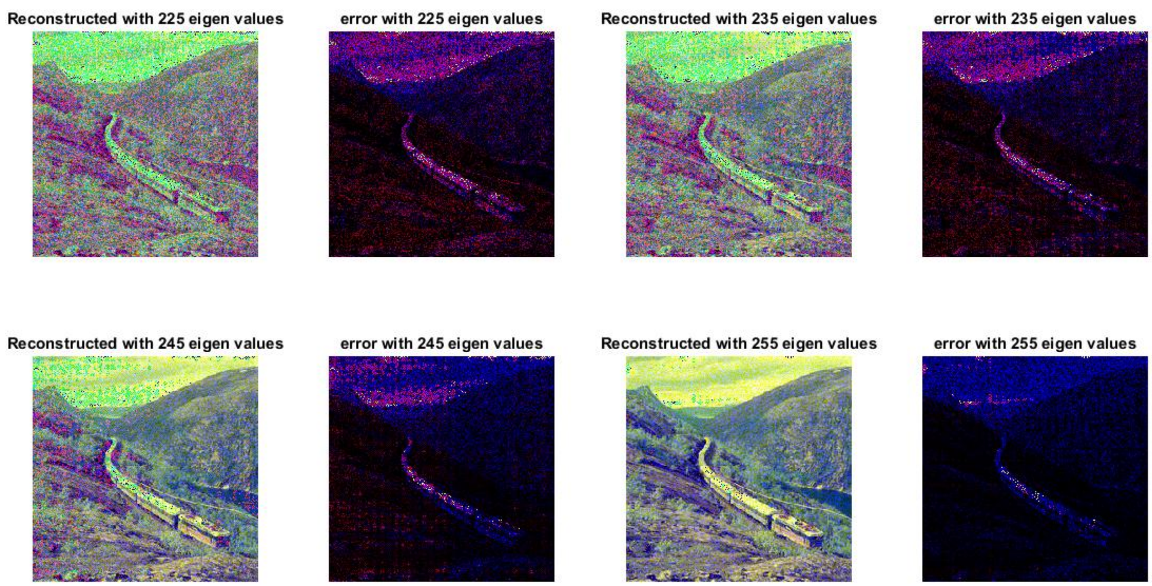
2.1 c) EVD on 8 Bit to 24-bit concatenated square image

After trying different orders of concatenation, the following two combinations, came close to replicating the image

Order: RGB



Order: GRB



GRB combination comes this close mostly pertaining to the predominance of green color in the image.

2.2 a) Eigen Value Decomposition on Rectangular Image in grayscale

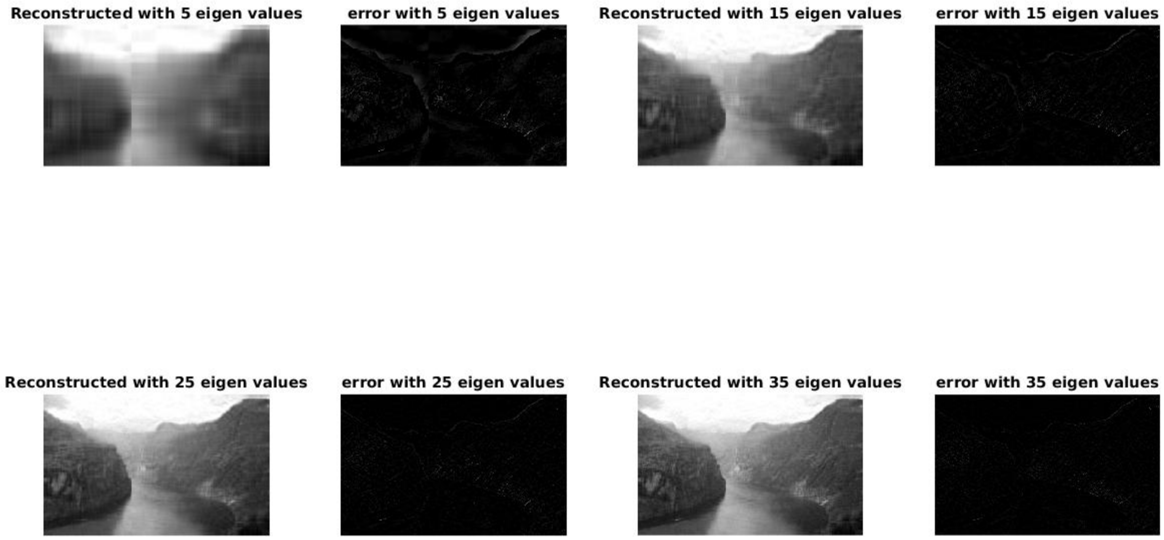


Figure: Rectangular image reconstruction using EVD, for 5,15,25,35 Eigen values

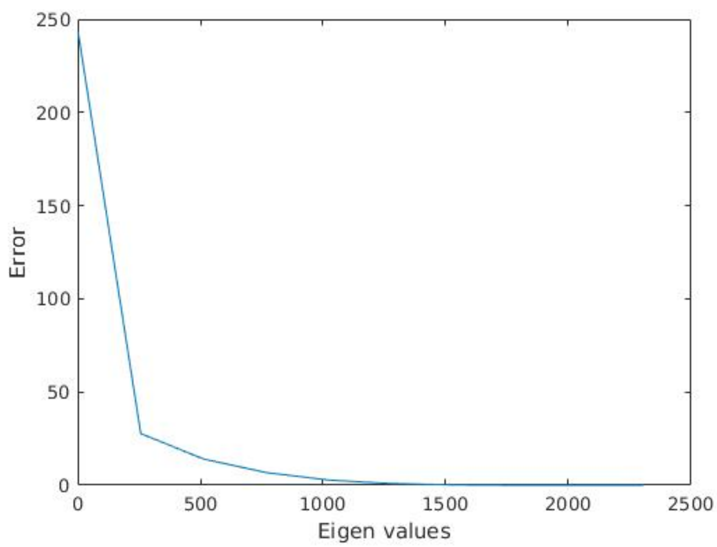


Figure: Graph for Error against top N Eigen values chosen during reconstruction;

2.2 b) EVD on each color band of rectangular image

Red Image

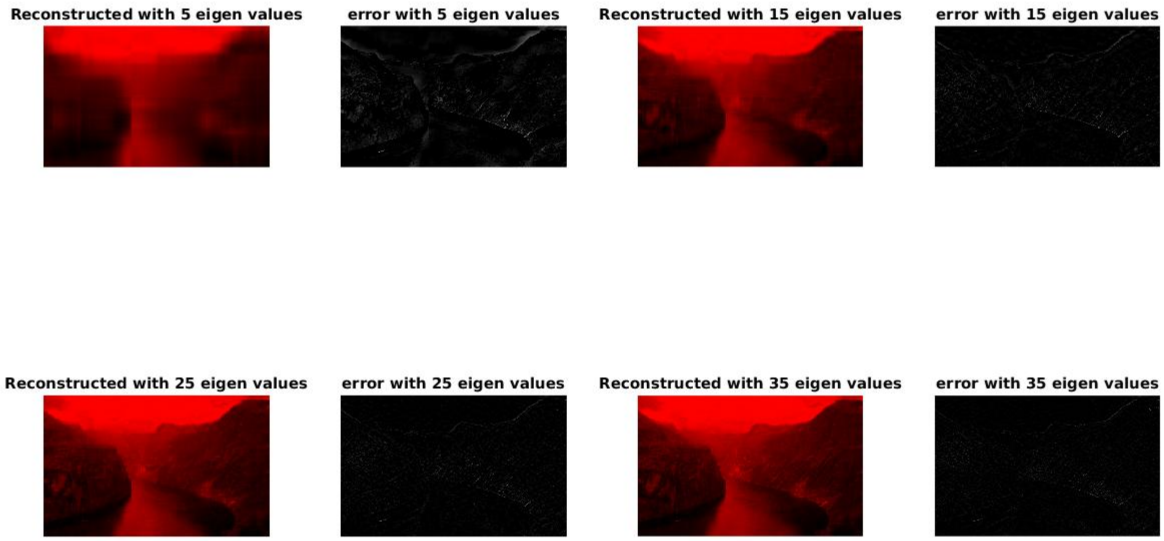


Figure: EVD on red Image of a rectangular image, for Eigen vectors = 5,15,25, 35,..

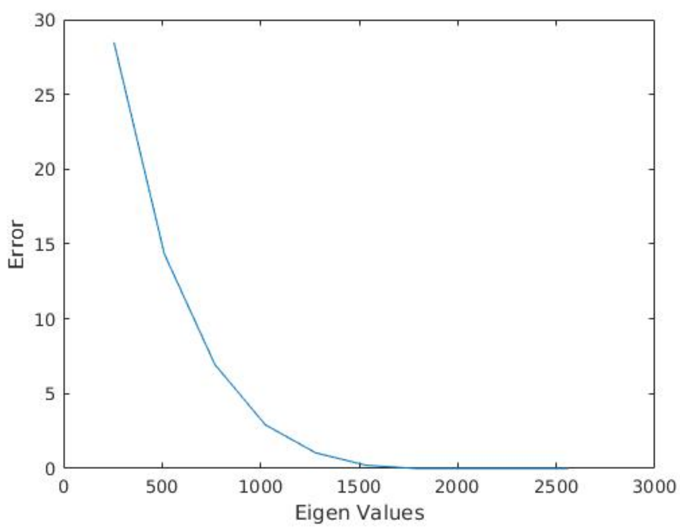


Figure: Graph for Error against Top N Eigen values chosen for reconstruction of red Image image

Green Image

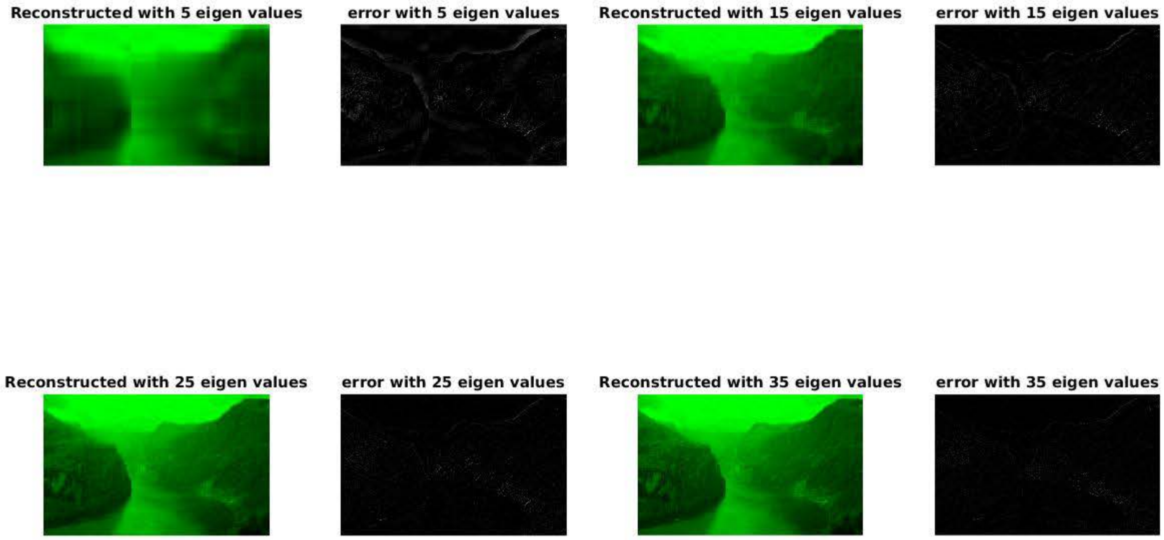


Figure: EVD on green Image of a rectangular image for different Eigen values

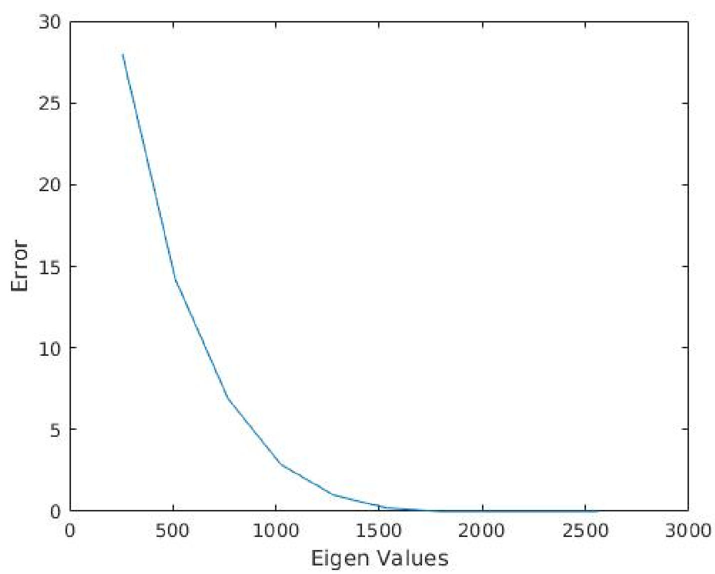


Figure: Graph for Error against Top N Eigen values chosen for reconstruction of green Image image

Blue Image

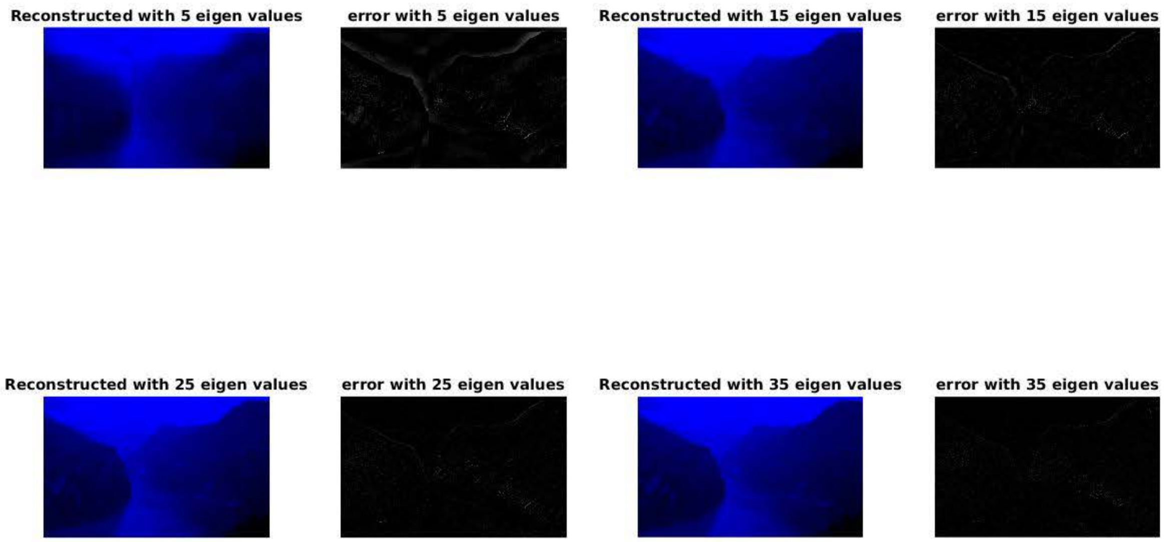


Figure: EVD on green Image of a rectangular image for different Eigen values

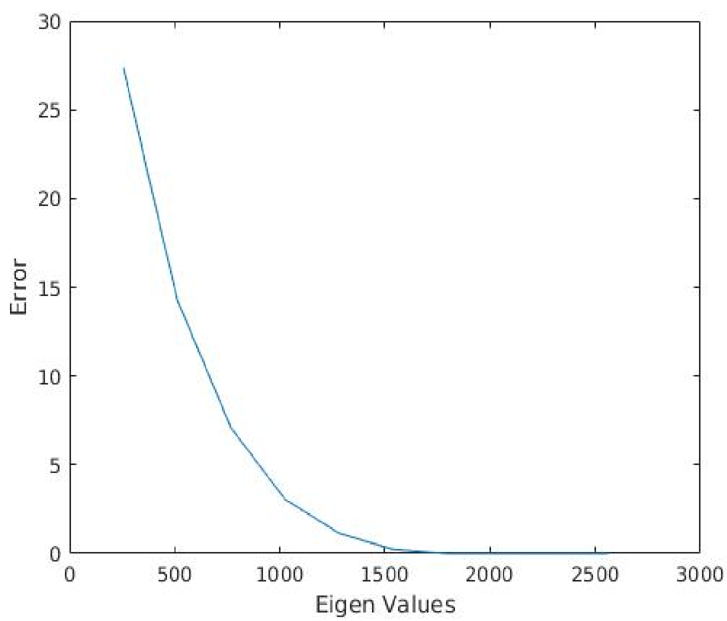


Figure: Graph for Error against Top N Eigen values chosen for reconstruction of green Image image

2.2 c) EVD after concatenating the 8bit R, G, B Image to form a 24bit number

The eigen value decomposition on a concatenated 8-bit R-G-B Image showed certain characteristics, First, being a rectangular image, (A'*A), was used the reconstructed matrix showed almost no improvement till a point beyond which it rapidly changed to our desired results. The following sequence of images exhibit that region of sudden change.

This image was the reconstructed matrix using 256 (out of 2560) Eigen values.

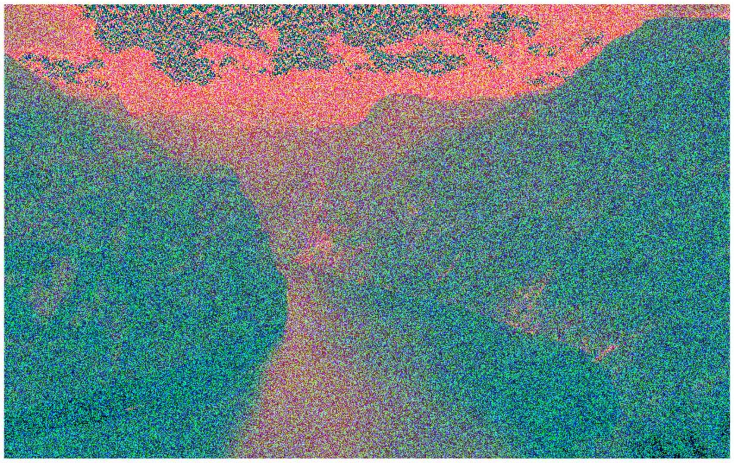


Figure: EVD on 24bit concatenated matrix using 256 eigen values,

This one below uses 1024, and is almost similar to the previous one.

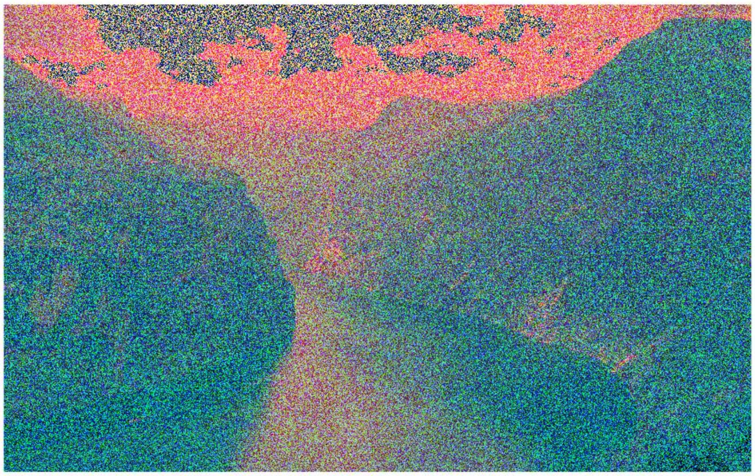


Figure: EVD on 24bit concatenated matrix using 1024 eigen values, After this however, using more eigenvalues shows drastic changes. This one below uses 1280.

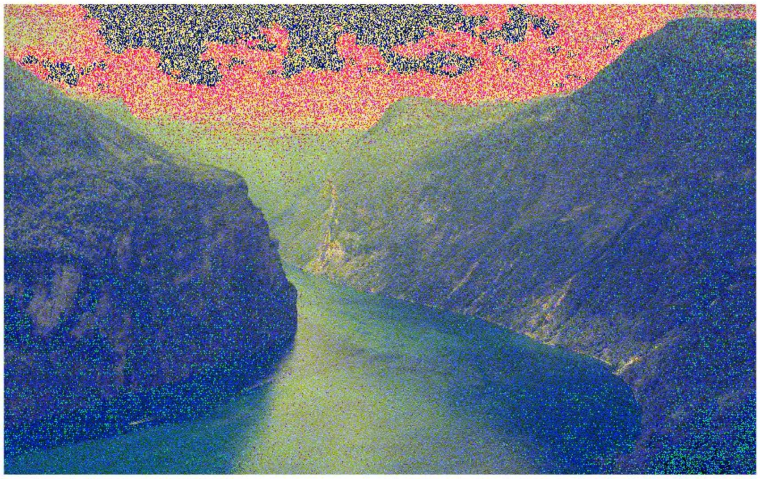
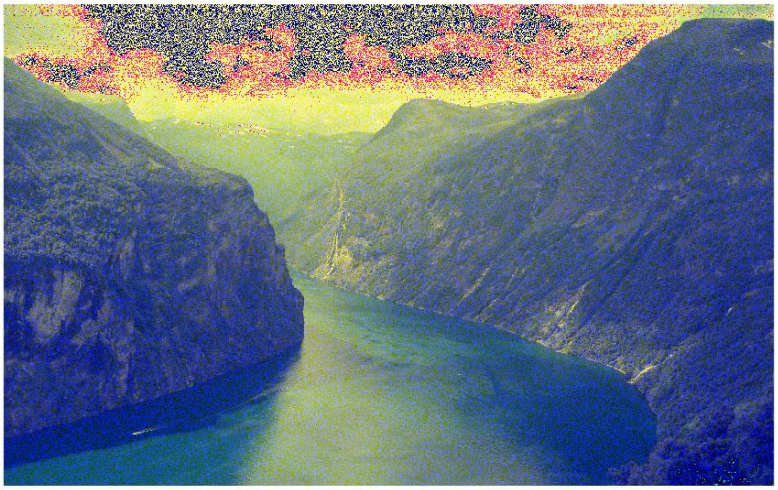
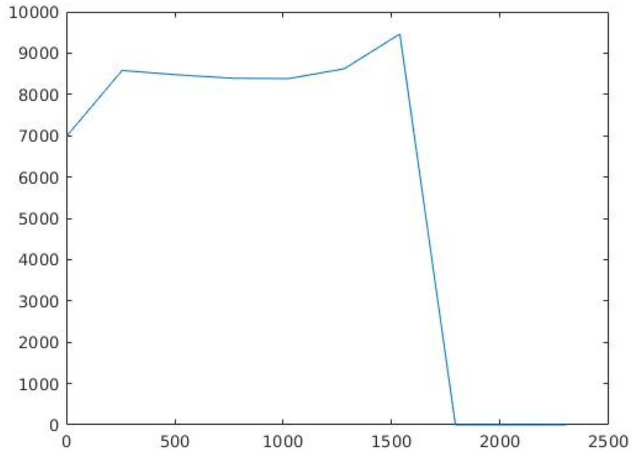


Figure: Using 1280 Eigen values.

While the next none, that uses 1536, is just a few Eigen vectors away from a perfect reconstruction.



The following graph denotes the Error VS Eigen value plot, during the process of reconstruction.



2: Polynomial Regression

(a) Polynomial Regression on 1D data

Result:

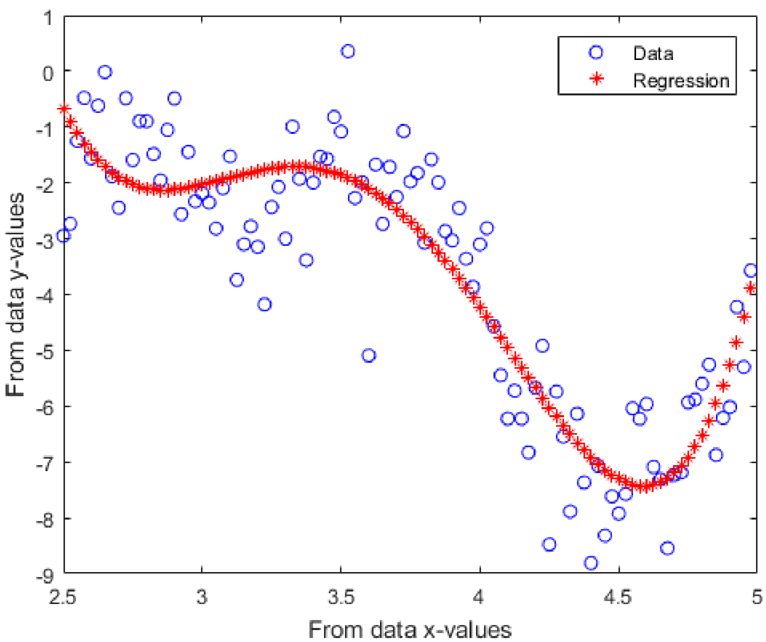


Figure 1: Single variable Polynomial Regression, with 1D data

The Regression curve is a result of 4^{th} order polynomial in x; Fitted using provided data, available in file: *q_24_1.txt*.

$$\begin{aligned}\text{Polynomial}(x) = p_0 \\ + p_1 * x \\ + p_2 * x^2 \\ + p_3 * x^3 \\ + p_4 * x^4\end{aligned}$$

the coefficients are found out to be:

p_0	586.6429
p_1	-691.2679
p_2	299.7572
p_3	-56.7550
p_4	3.9469

Goodness of this fit:
Used Sum of Residual Square to comparatively determine how good the fit is

Sum of Squares Error	108.5

Alternative Version(Python Code):
Final Test error = 47.8011873218
Error computed is Residual Squared Error.

Ridge Regression on 1D Data
Final Test error 79.7844009994

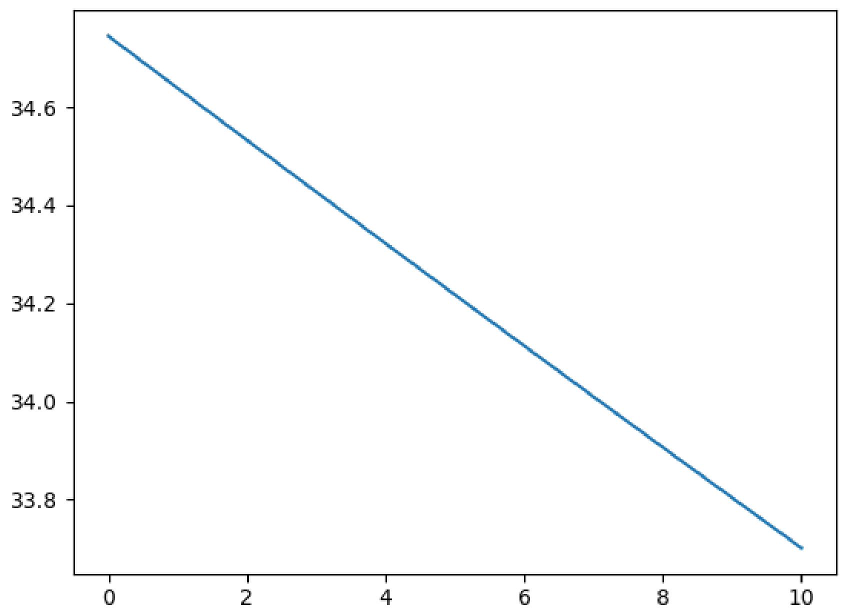


Figure 2: λ Vs Residual Square Error (1D Data)

(b) Polynomial Regression on 2D data

Result:

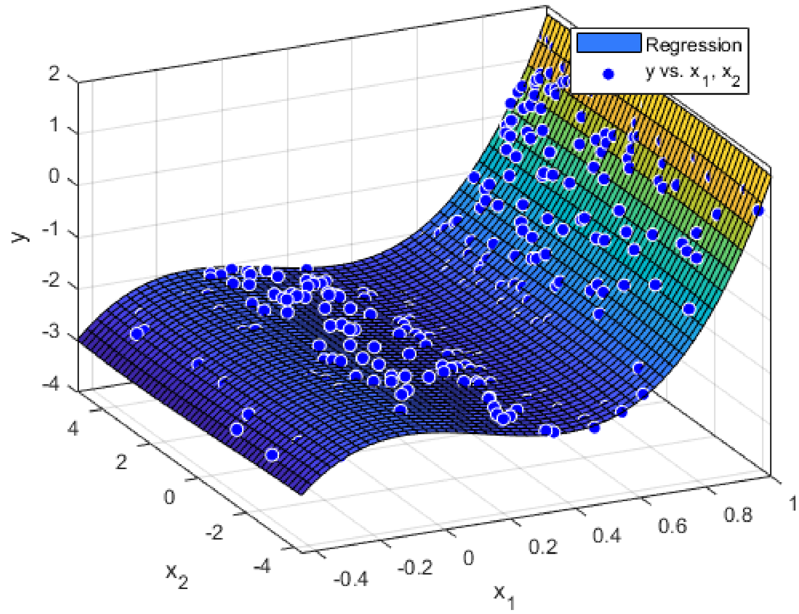


Figure 3: Polynomial Regression with 2 variables, using 2D data

The Regression curve is a result of 3^{rd} order in x_1 & 3^{rd} order polynomial in x_2; Fitted using provided 2D data, available in file: *q_24_2.txt*.

$$\begin{aligned}\text{Polynomial}(x_1, x_2) = p_{00} \\ &+ p_{10} * x_1 + p_{01} * x_2 \\ &+ p_{20} * x_1^2 + p_{11} * x_1 * x_2 + p_{02} * x_2^2 \\ &+ p_{30} * x_1^3 + p_{21} * x_1^2 * x_2 + p_{12} * x_1 * x_2^2 + p_{03} * x_2^3\end{aligned}$$

the coefficients are found out to be:

p_{00}	-2.089
p_{10}	-0.87
p_{01}	-0.007661
p_{20}	-2.969
p_{11}	0.03174
p_{02}	0.0003155
p_{30}	7.814
p_{21}	-0.03498
p_{12}	-0.004012
p_{03}	0.0006454

Goodness of this fit:
Used Sum of Residual Square to comparatively determine how good the fit is

Sum of Squares Error	3.3385

Alternative Version(Python Code):
Final Test error = 78.4547244395
Error computed is Residual Squared Error.

Ridge Regression on 2D Data
Final Test error 173.855102873

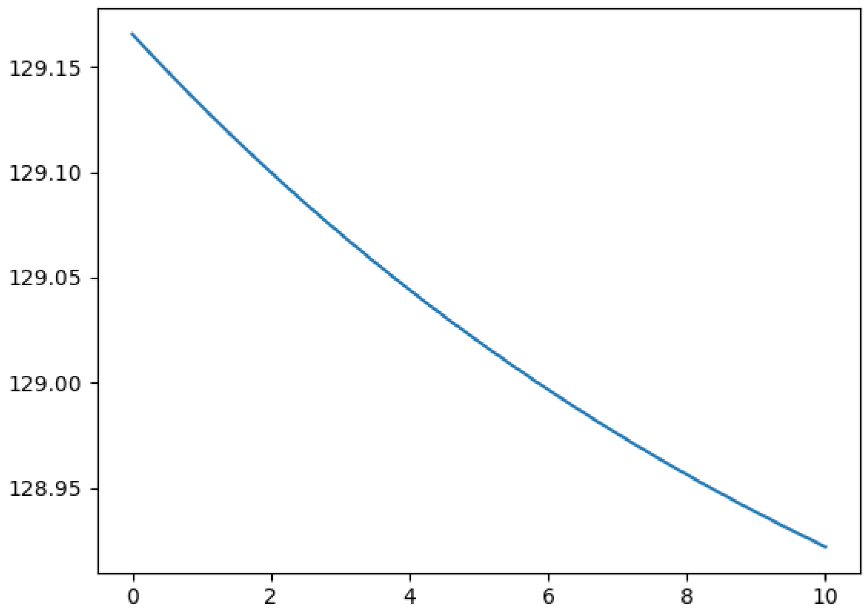


Figure 4: λ Vs Residual Square Error (2D Data)

(c) Polynomial Regression on Multi-Dimensional Data

It was found that the features are correlated.

Final Test error(Polynomial Regression) = 0.0552611556014
Error computed is Residual Squared Error.

Ridge Regression on Multi Dimensional data:
Final Test Error(Ridge Regression) = 0.61410718248

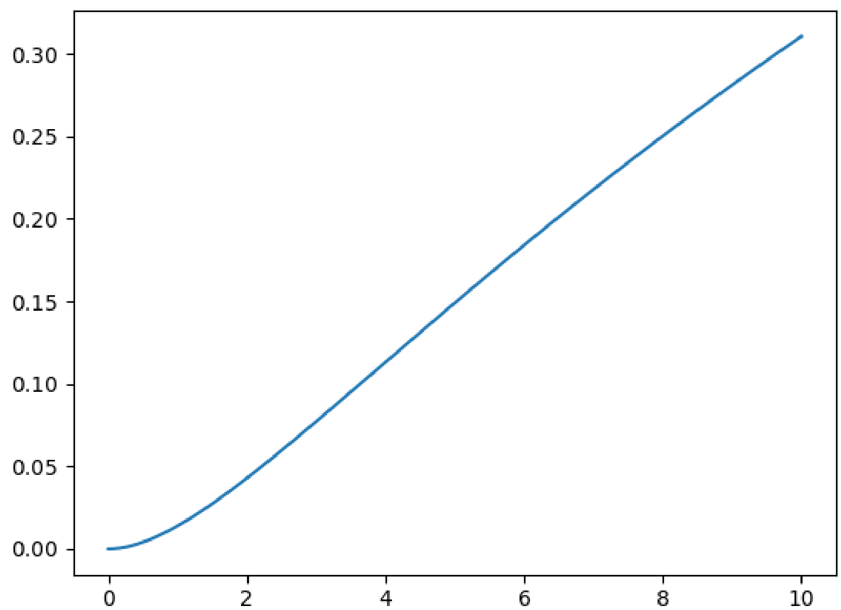


Figure 5: λ Vs Residual Square Error (Multi Dimensional Data)