Pattern Recognition
Assignment 02

Group 24

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1: Plots For Linearly Separable Data

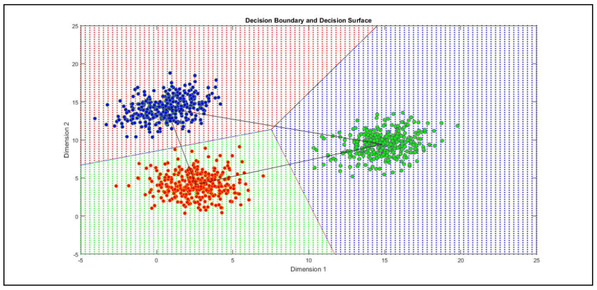

Figure 1(i): Bayes' Classifier with same Covariance

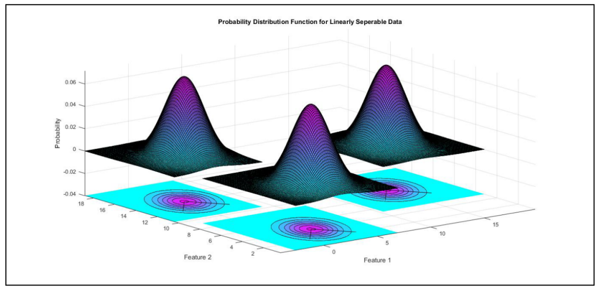

Figure 1(ii): PDF with Constant Density Curves and Eigen vectors

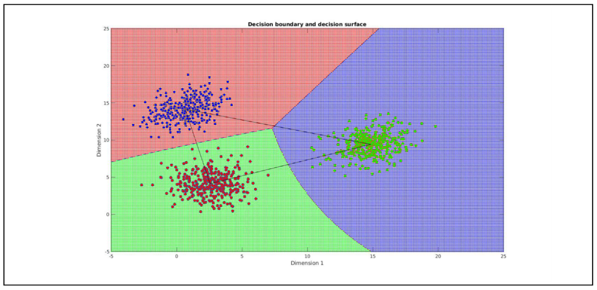

Figure 2(i): Bayes' Classifier with different Covariance

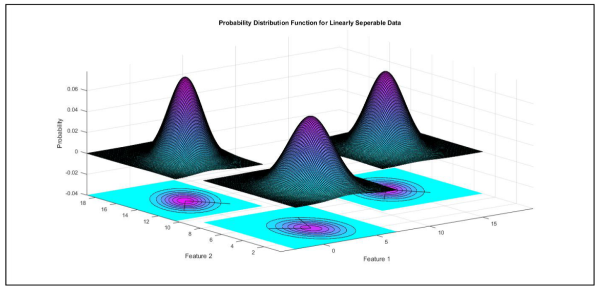

Figure 2(ii): PDF with Constant Density Curves and Eigen vectors

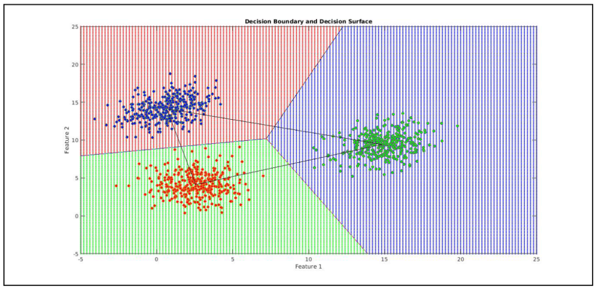

Figure 3(i): Naive Bayes' Classifier with same Covariance

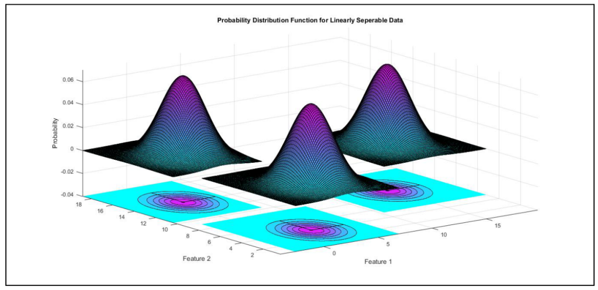

Figure 3(ii): PDF with Constant Density Curves and Eigen vectors

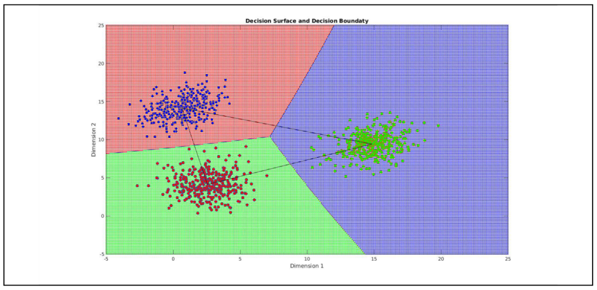

Figure 4(i): Naive Bayes' Classifier with different Covariance

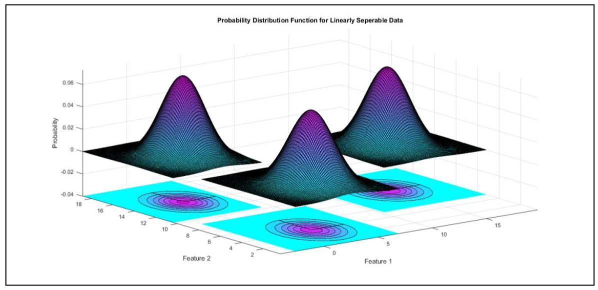

Figure 4(ii): PDF with Constant Density Curves and Eigen vectors

Data was already linearly separable, therefore, using linear or nonlinear classifiers did not make much of a difference. PDF plots have constant density curves, and Eigen vectors plotted below them. Figure 2, 4 are Non linear classifiers.

2: Plots For Non Linearly Separable Data

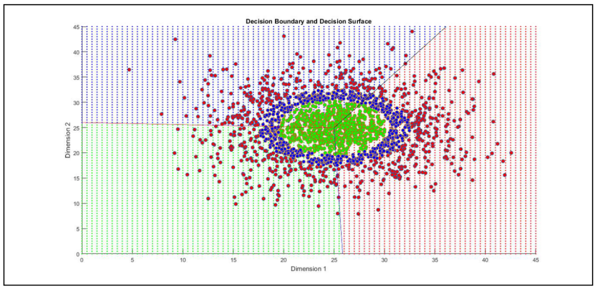

Figure 5(i): Bayes' Classifier with same Covariance

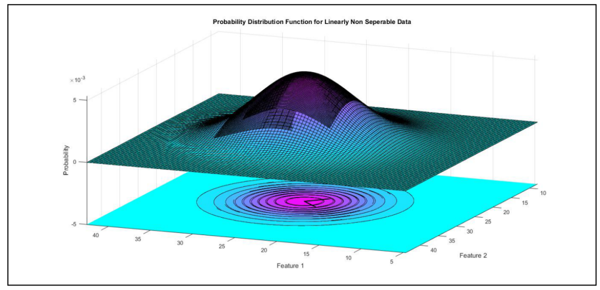

Figure 5(ii): PDF with Constant Density Curves and Eigen vectors

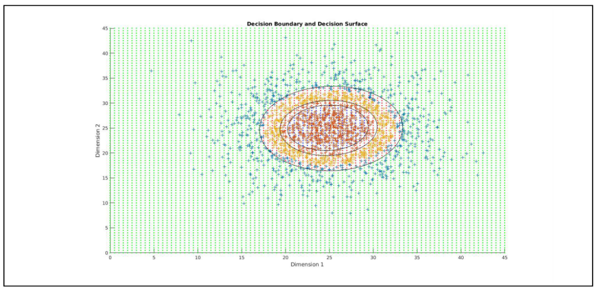

Figure 6(i): Bayes' Classifier with different Covariance

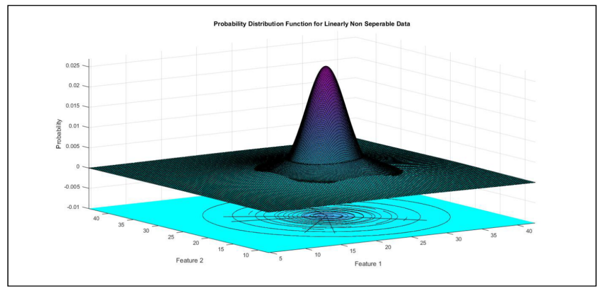

Figure 6(ii): PDF with Constant Density Curves and Eigen vectors

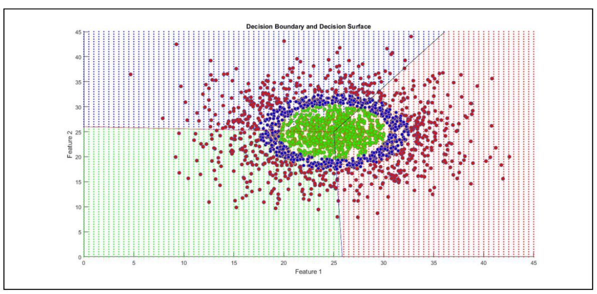

Figure 7(i): Naive Bayes' Classifier with same Covariance

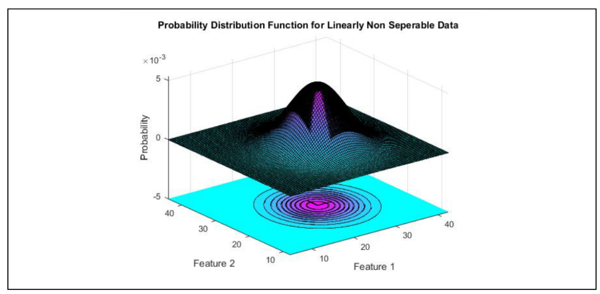

Figure 7(ii): PDF with Constant Density Curves and Eigen vectors

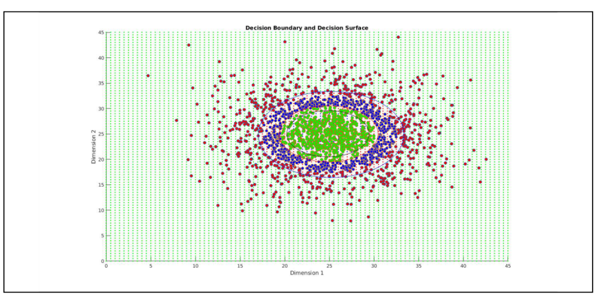

Figure 8(i): Naive Bayes' Classifier with different Covariance

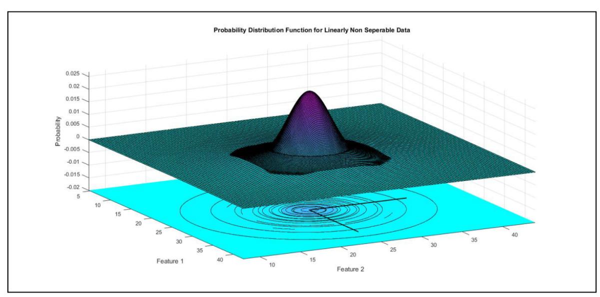

Figure 8(ii): PDF with Constant Density Curves and Eigen vectors

Data points were scattered in a non linearly separable fashion, in concentric circles. Linear classifiers failed to obtain desired results, but non Linear Classifiers succeeded. Figure 6, 8 are non linear classifiers.

3: Plots For Real Data

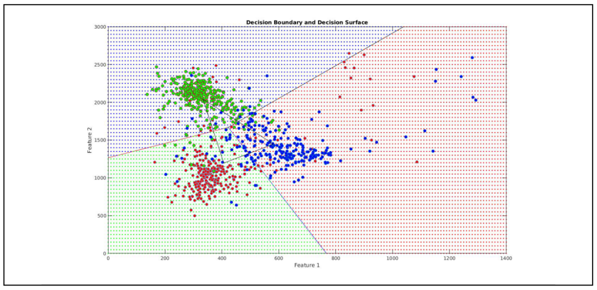

Figure 9(i): Bayes' Classifier with same Covariance

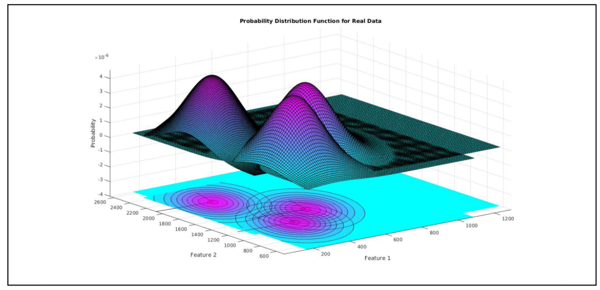

Figure 9(ii): PDF with Constant Density Curves and Eigen vectors

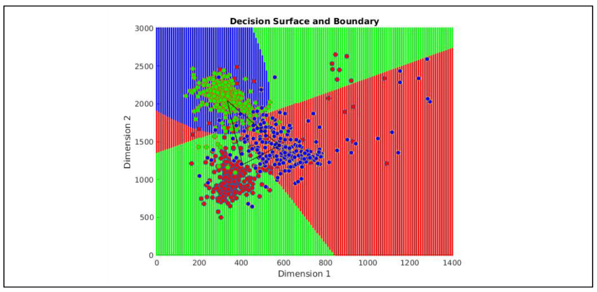

Figure 10(i): Bayes' Classifier with different Covariance

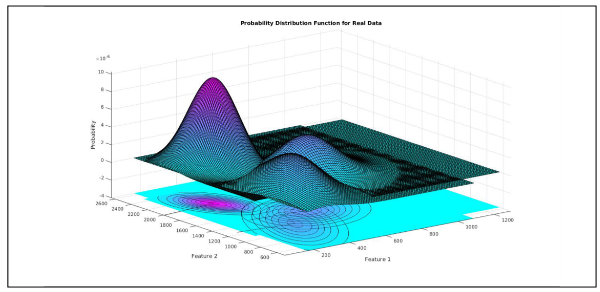

Figure 10(ii): PDF with Constant Density Curves and Eigen vectors

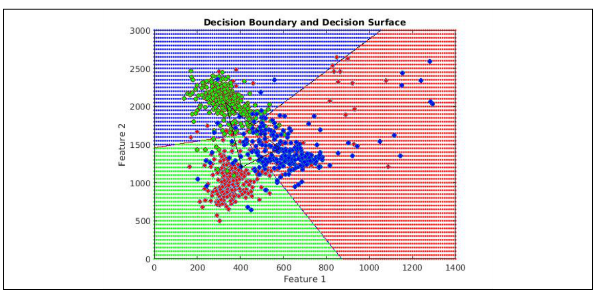

Figure 11(i): Naive Bayes' Classifier with same Covariance

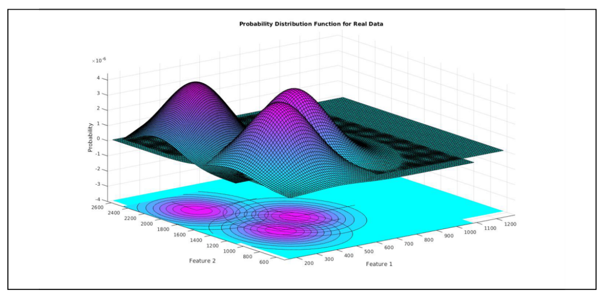

Figure 11(ii): PDF with Constant Density Curves and Eigen vectors

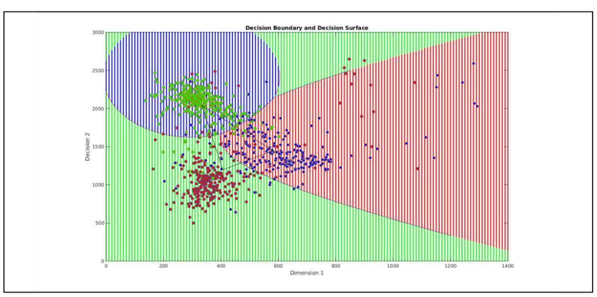

Figure 12(i): Naive Bayes' Classifier with different Covariance

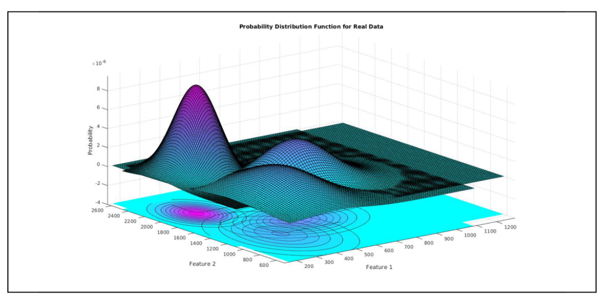

Figure 12(ii): PDF with Constant Density Curves and Eigen vectors

Real data had point clusters with no discrete boundary, due to which linear classifies performed poorly compared to nonlinear classifiers. Figure 10, 12 are non linear classifiers

4: Receiver operating characteristic Curves

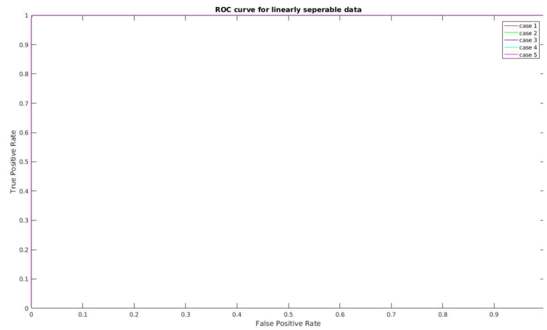

Figure 13: ROC Curve for Linearly Separable Data

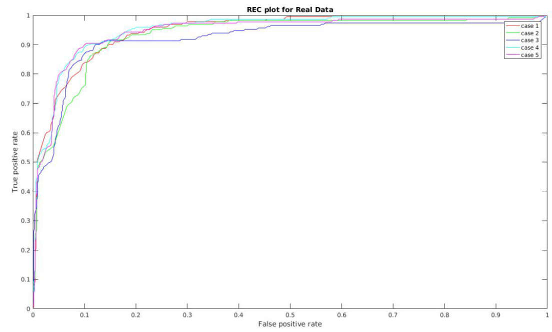

Figure 14: ROC Curve for Real Data

5: Detection Error Trade-off Curves

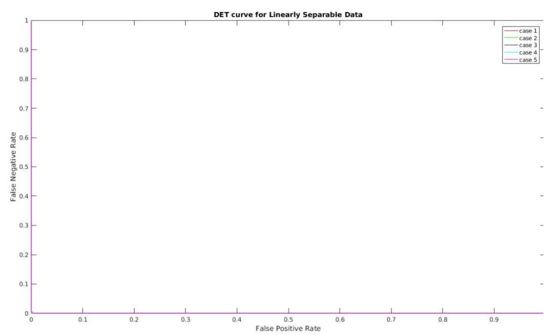

Figure 15: DET Curve for Linearly Separable Data

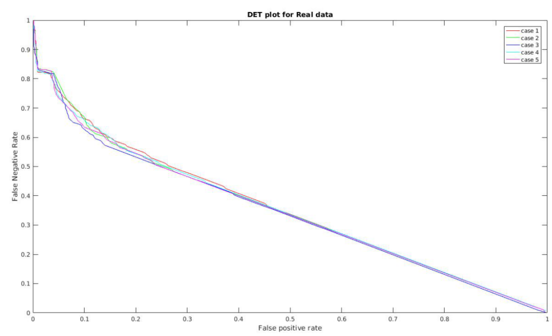

Figure 16: DET Curve for Real Data

Legend

Case 1: Bayes with Covariance same for all classes
Case 2: Bayes with Covariance different for all classes
Case 3: Naive Bayes with C = $\sigma^2 \times I$.
Case 4: Naive Bayes with C same for all classes
Case 5: Naive Bayes with C different for all classes

6: Confusion Matrix

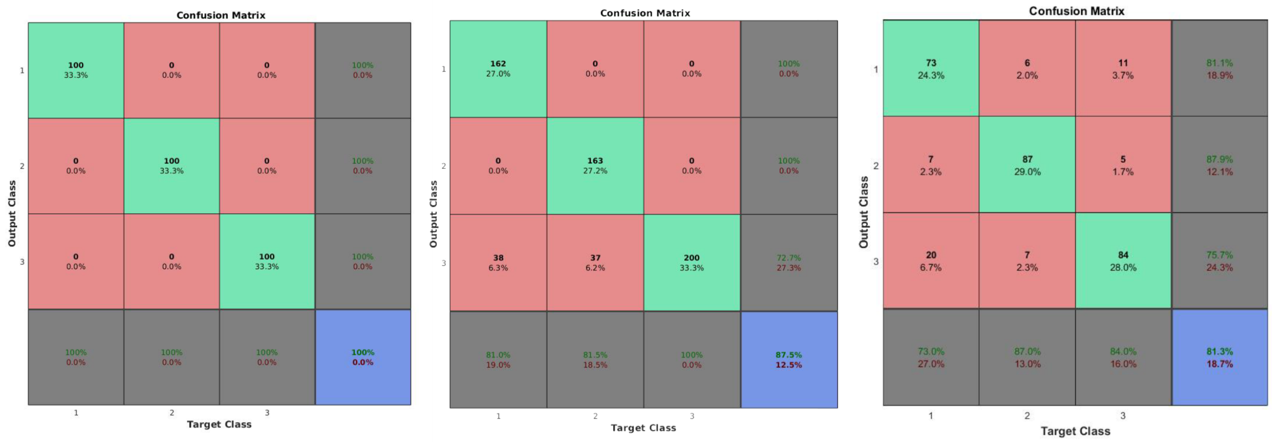

Figure 17: Confusion Matrix for Linearly Separable, Non Linearly Separable, Real Data

7: Conclusion

In each of the plots, Class 1 is represented by red scatter points on green background, Class 2 is represented by green scatter points on blue background and Class 3 is represented by blue scatter points on red background.

Case 1: Bayes with covariance same for all classes.
All classes have the same covariance matrix. Data points cluster into an hyper-ellipsoidal space. General equation for multivariate discriminant function (using minimum error rate approach) is :

$$g_i(x) = -\frac{1}{2}(x-\mu)^t\Sigma^{-1}(x-\mu_i) - \frac{d}{2}\ln 2\pi - \frac{1}{2}\ln|\Sigma_i| + \ln p(\omega_i)$$

boundary equation between class i and j is given by $g_i(x) - g_j(x) = 0$; when Σ is same for all, the quadratic term gets canceled, leading to a linear boundary equation.

Case 2: Bayes with covariance different for all classes here the quadratic terms don't cancel each other, leading to a non linear boundary. In general the boundary is an hyper-quadric.

Case 3: Naive Bayes, with covariance matrix in the form $\sigma^2 \times I$. Constant density function form concentric circles, axes are parallel to x and y axis. Non linear classifier.

Case 4: Naive Bayes with all covariance same, covariance matrix are diagonal, and the constant density curves form an ellipse whose axes are parallel to the x and y axes. Linear classifier.

Case 5: Naive Bayes with all covariances different, non linear classifier.

Receiver operating Characteristic (ROC) Curve, Detection Error Trade-off (DET), and Confusion Matrix depict the performance of our classifiers.

Inference, The best results were obtained using the classifier from question 2, as it was the most generic, and involved a non linear boundary between two classes.