# Midterm 1 Guide

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## 1 Lecture 3

### 1.1 Models for decision making under uncertainty

1. State Preference Model
2. Expected Utility Hypothesis (EUH)
3. Mean-Variance Model

Note that the state preference model is the most general model. The EUH is considered to be a special case of the State preference model. The mean-variance model can be considered to be a special case of EUH.

**These imply:**

- A decision maker acts as if a probability is assigned to each state
- $\exists$ a function, called the *Von-Neumann-Morgenstern* (vNM) utility function -> this is dependent only on the outcomes
- the decision maker orders the actions according to the expected value of the vNM utility function

***Von-Neumann-Morgenstern:*** In decision theory, the von Neumann-Morgenstern utility theorem shows that, under certain axioms of rational behavior, a decision-maker faced with risky (probabilistic) outcomes of different choices will behave as if he or she is maximizing the expected value of some function defined over the potential outcomes at some specified point in the future. This function is known as the von Neumann-Morgenstern utility function. The theorem is the basis for expected utility theory.

#### 1.1.1 Expected Utility Hypothesis:

**The EUH implies**:

$$\mathbb{U} = U(W_1, W_2, ..., W_l)$$
$$= p_1 u(W_1) + p_2 u(W_2) + ... + p_l u(W_l)$$

where $p_k$ is the probability assigned for state $k(s_k)$

$$\mathbb{E}[u(W)] = p_1 u(W_1) + p_2 u(W_2) + ... + p_l u(W_l)$$

Note that for the vNM - A utility function, u is said to have the expected utility property if:
For a gamble g, with outcomes $\{a_1, a_2, ..., a_n\}$ with effective probabilities $p_1, p_2, ..., p_n$ respectively we have:

$$u(g) = p_1 u(a_1) + p_2 u(a_2) + ... + p_n u(a_n)$$

***Note:*** $u()$ is the same across all states but the $W$ differs. Both probabilities and the vNM utility functions are allowed to differ across investors
*Assume:* u'(W)>0 for all levels of wealth - i.e investors prefer more wealth to less;
**EUH**:

- The EUH is a special form of the State preference approach that:
  - enables the use of probabilities to express beliefs and:
  - characterizes preferences about uncertain outcomes with a vNM utility function that depends on the outcome -> level of wealth -> but that is the same function for all states
- EUH is the cornerstone of decision making under uncertainty but evidence on individual behavior contradicts its predictions.
- Usually a specific form of the vNM utility function is assumed.

### 1.1.2 Mean-Variance Model

***Assumptions and General ideas:***

- We assume a specific form of the vNM utility function, the simplest:$u(\cdot)$ is quadratic in wealth $\longrightarrow$ $u(W) = W - bW^2$
- $b > 0$ is a parameter expressing preference
- If the vNM function is quadratic, then expected utility can be written as a function of the expected value (mean) of terminal wealth and and the variance (or its standard deviation) of terminal wealth
- Thus, the name: ***Mean-Variance Model***
- As investors we're trying to optimize a mean-variance objective function
  - We can construct portfolios
  - We find the foundation for CAPM
- Mean-variance analysis provides theory of individual behavior, regardless if market is in equilibrium
- CAPM provides theory of asset prices in market equilibrium
- Mean-Variance Analysis
  - The process of weighing risk (variance) against expected return
  - By looking at the expected return and variance of an asset, investors attempt to make more efficient investment choices
  - They seek the lowest var for a given exp. return
  - They could also seek the highest exp. return for a given
- This model is practical because means, vars and covars can be estimated from past observations
- Mean-Variance Analysis is a component of Modern Portfolio Theory (MPT), which assumes investors make rational decisions, and that for increased risk, they expect a higher return

***The Model:***

- Each investor who acts according to m-v objective is assumed to choose a portfolio that maximizes:

$$\mathbb{G} = G(\mu_p, \sigma_p^2)$$

- Subject to the constraint that total value of assets at total value of assets at initial prices does not exceed initial wealth.

- The expected ( mean) return on the portfolio is denoted $\mu_p$
- Risk is measured with $\sigma_p$, the standard deviation (sd)

A mean-variance investor's optimal portfolio selection is split into two steps:

1. *Portfolio Frontier* - the portfolios for which $\sigma_p$ is minimized for each $\mu_p$.
2. We're looking to maximize the object $\mathbb{G}$, in accordance accordance with preferences, expressed by the investor's own $\mathbb{G}(\mu_p, \sigma_p^2)$
3. Assuming the objective function is increasing in expected return and decreasing in risk , only a portion of the portfolio frontier – the efficient portfolio set - is relevant in the second step

### 1.1.3 Modern Portfolio Theory - MPT model

***The Modern Portfolio Theory Model:***

- Given the same level of expected return, an investor will choose the investment with the lowest amount of risk.
- Investors measure risk in terms of an investment's variance or standard deviation.
- For each investment, the investor can quantify the investment's expected return and the probability of those returns over a specified time horizon.
- Investors seek to maximize their utility.
- Investors make decisions based on an investment's risk and return, therefore, an investor's utility curve is based on risk and return.

***General Definitions and Ideas:***

- *Stocks* - part of the equity class
- *Bonds* - fixed income
- *ETF - Exchange Traded Fund* - SPDR Gold Trust (GLD) is an example it is a fund that trades the gold spot price and you as an investor can be part of gold spot trading without participating in the actual spot market.
- *Mutual Fund* - a fund operated by a money manager who invests the fund's capital and attempts to produce capital gains and income for the fund's investors.
- *Asset classes* - stocks/bond/money market instruments
- *Index Fund* - A type of mutual fund with a portfolio constructed to match or track the components of a market index, such as the Standard & Poor's 500 Index (S&P 500)

## 1.2 Portfolio Basics

***General Definitions and Ideas:***

*Portfolio*

- vector/list/array of asset holdings
- Can be visualized as: $\{x_0, x_1, x_2, ..., x_n\}$
- These asset holdings are subject to the constraint of total invested.
- This is also seen as: assets=initial wealth.
- we can't purchase more assets than our initial wealth allows

***How to get the Expected Return of the portfolio:***

1. Suppose the market contains the following:
   - $n$ securities
   - Random returns: $r_i, \forall i \in \mathbb{N}$
   - Expected Returns: $\mu_i = \mathbb{E}[r_i]$ item Variances: $\sigma_i^2 = Var[r_i]$
   - Initial Prices: $P_{0,i}$
   - Terminal Prices (after *one* period): $P_{T,i}$
2. Let $V_0$ be the initial value of the portfolio and $V_T$ be the terminal value
3. Let $n_i$ be the number of shares of a security, $i$ held in the portfolio
4. The number of shares of a security $i$ must be the initial amount of money $V_{0,i}$ spent on buying the security divided by the initial price, i.e., $n_i = \frac{V_{0,i}}{P_{0,i}}$
5. The initial value of the portfolio must be the number of shares of *each* security multiplied by the security's initial price, i.e.,
$$V_0 = \sum_{i=1}^{n} [n_i \cdot P_{0,i}]$$
6. Assuming that no shares are bought or sold, the terminal value is:
$$V_T = \sum_{i=1}^{n} [n_i \cdot P_{T,i}]$$
7. The portfolio return is:
$$r_p = \frac{V_T - V_0}{V_0}$$
8. Now, substituting $n_i$ and using the expressions for $V_0$ and $V_T$ we have:
$$r_p = \frac{1}{V_0} \sum_{i=1}^{n} \frac{V_{0,i}}{P_{0,i}} (P_{T,i} - P_{0,i})$$
9. This is equal to:
$$\sum_{i=1}^{n} \frac{V_{0,i}}{V_0} \left( \frac{P_{T,i} - P_{0,i}}{P_{0,i}} \right)$$
10. Note that this is equal to:
$$\sum_{i=1}^{n} \frac{V_{0,i}}{V_0} (r_i)$$
    If we set $\frac{V_{0,i}}{V_0}$ to $w_i$ (*weight of the security*) then the equation (random portfolio return) is:
$$r_p = \sum_{i=1}^{n} w_i r_i$$
    Taking expected values, then we have:

***Variance of the Portfolio:***

- By definition: $\sigma_p^2 = \mathbb{E}[(r_p - \mu_p)^2]$

- But portfolio variance is a function of the variance of the component assets as well as the *covariance* between each of them.
- Portfolio variance looks at the *covariance* or *correlation coefficient* for the securities in the portfolio.
- Simply a portfolio with 2 assets : security i and security j:

$$\sigma_p^2 = w_i^2 \cdot \sigma^2(r_i) + w_j^2 \cdot \sigma^2(r_j) + 2 \cdot w_i \cdot w_j \cdot Cov(r_i, r_j)$$

- Covariance is a measure of the degree to which returns on two risky assets move in tandem. A positive covariance means that asset returns move together. A negative covariance means returns move inversely.
- Covariance is closely related to "correlation," wherein the difference between the two is that the latter factors in the standard deviation.

***Risk Reduction:***

> ***Modern portfolio theory*** says that portfolio variance can be reduced by choosing asset classes with a low or negative covariance, such as stocks and bonds. This type of diversification is used to *reduce risk*

***Covariance Matrix:***

$$Var[X] = \begin{array}{|c|c|c|c|} \hline Var[X_1] & Cov[X_1, X_2] & ... & Cov[X_1, X_k] \\ \hline Cov[X_1, X_2] & Var[X_1] & ... & Cov[X_1, X_k] \\ \hline \vdots & \vdots & \ddots & \vdots \\ \hline Cov[X_1, X_k] & Cov[X_2, X_k] & ... & Var[X_k] \\ \hline \end{array}$$

# 2 Lecture 4:

## 2.1 More Portfolio Concepts [Basics]

### 2.1.1 Efficient Frontier

- According to the theory, it's possible to construct an "efficient frontier" of optimal portfolios offering the maximum possible expected return for a given level of risk.
- Simply because you can use this approach to lower your risk (portfolio variance) while maintaining (or increasing) your expected returns.
- MPT - A theory on how risk-averse investors can construct portfolios to optimize or maximize expected return based on a given level of market risk, emphasizing that risk is an inherent part of higher reward.
- A set of optimal portfolios that offers the highest expected return for a defined level of risk or the lowest risk for a given level of expected return.
- Portfolios that lie below the efficient frontier are sub-optimal, because they do not provide enough return for the level of risk.
- Portfolios that cluster to the right of the efficient frontier are also sub-optimal, because they have a higher level of risk for the defined rate of return

### 2.1.2 Risk, Diversification and Correlation

***Risk Types:***

- *Systematic Risk* - markets risks that cannot be diversified away, i.e. interest rates, recessions and wars, climate

- *Unsystematic* or *Idiosyncratic* - known as "specific" risk , this risk is specific to the individual stock and can be diversified away as you increase the number of stocks in your portfolio. It represents the component of a stocks return that is not correlated with the general market moves

***Diversification and Correlation:***

- The underlying intuitive appeal of diversification is the Correlation
- Correlation - A measure that determines the degree to which two variable's movements are associated.
- To reduce the overall risk in a portfolio , it is best to combine assets that have negative (or low positive) correlation
- ***Correlation impact on risk-return***
  - In general, the lower(less positive and more negative) the correlation between asset returns, the greater the potential to diversify risk.
  - The amount of potential risk reduction for this combo depends on the degree of correlation of the two assets

***Correlation:***

- For each correlation coefficient there exists an infinite number of combos of two assets that result in many possible risk-return combinations

| Correlation Coefficient | Range of Return | Range of Risk |
|---|---|---|
| +1 | Between the returns of two assets held in isolation | Between the risk of the two assets held in isolation |
| 0 | Between the returns of two assets held in isolation | Between the risk of the most risky asset and less than the risk of the least risky asset , but greater than 0 |
| -1 | Between the returns of two assets held in isolation | Between the risk of the most risky asset and 0 |

### 2.1.3 More on Efficient Frontier:

- Since investors prefer *more* **return** to *less* and *less* **risk** as opposed to more
- Only the 'edge-line' of the hyperbola is considered to the investors, where the upper edge ***defines*** the efficient frontier
- The Efficient Frontier is as follows:
  - No lower risk can be obtained for any given level of return
  - No higher level of return can be obtained for any given level of risk
  - The efficient frontier draws out a locus of points in the northwest quadrant of the risk return-space
  - Portfolios that lie along the efficient frontier offer investors the optimal risk return combinations ; these portfolios are called: ***Mean Variance Efficient Portfolios***

***Minimum Variance Portfolio:*** *- See Excel for formulas + slides*
***Summary:***

- We have learned that we must invest in several stocks to benefit from diversification
- The lower the correlation coeff (lower than 1.0) between securities gives us greater diversification benefits
- We can choose stocks randomly and through construction of portfolios still achieve most the of the benefits of diversification - minimizing risk
- Remember the choice of the risky portfolio is still based on investor risk preference

# 3 Lecture 5:

## 3.1 Use Matrix Algebra for Portfolios

***Return*** - $r_p = \sum_{i=1}^{N} w_i r_i = w^T r$
***Expected Return*** - $\mu_p = \sum_{i=1}^{N} w_i \mu_i = w^T \mu$

$$w = \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix} \text{ and } \mu = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_N \end{bmatrix}$$

***How to calculate Matrix Q:***

1. Calculate the average vector
2. Matrix Subtraction: [return matrix]-[average vector], called excess return matrix
3. Multiplication + Summation
4. Divide by M

*See Excel Template*

# 4 Lecture 6:

## 4.1 Passive Management:

- Index style investments - believe in the efficient market hypothesis. It states that at all times markets incorporate and reflect all information, rendering individual stock picking futile. As a result, the best investing strategy is to invest in index funds, which have historically outperformed the majority of actively managed funds.

  ***Efficient Market Hypothesis:*** The EMH is the theory that beating the market is *impossible* because stocks are already accurately priced and reflect all available information. Thus, it is theoretically impossible to profit from any trading strategy, such as arbitrage. As a result, since there is no way to identify a "bargain stock" [fundamental analysis], or use past trends [technical analysis], the only way to earn higher returns than those of an index is to invest into stocks with higher risk.
  How could the EMH be ***incorrect***?

  1. All investors view information differently and thus will have different stock valuations.
  2. Stocks take time to respond to new information. Investors who react on this information *first* can take advantage of it.
  3. Stock prices can be affected by human error and emotional decision making.
  4. Investors have proven that they can profit from market anomalies.

  That is to say, if the EMH is true, then investors should place their assets in index funds (SP500, Russell 2000), so they will earn the same returns as the overall market. Also, **low management fees**. They will focus on minimizing their expenses to maximize their returns for any asset type and any given level of risk rather than try to outdo the index

- Buy and Hold Strategies

  - Tax advantages - because the index fund's buy-and-hold style does not trigger large annual capital gains tax.
  - Transparency - because investors know at all times what stocks or bonds an indexed investment contains
  - Low management fees (low expense ratio) $< 2\%$ - since there is no need to analyze securities in the index

- Popular Indices:
  - ***SP500*** - The Standard & Poor's 500 Index (S&P 500) is an index of 505 stocks issued by 500 large companies with market capitalizations of at least $6.1 billion. It is seen as a leading indicator of U.S. equities and a reflection of the performance of the large-cap universe.
  - ***DJIA*** - The Dow Jones Industrial Average (DJIA) is a price-weighted average of 30 significant stocks traded on the New York Stock Exchange (NYSE) and the NASDAQ.
  - ***FANG*** - Facebook Inc. (FB) , Apple Inc. (AAPL) , Amazon.com Inc. (AMZN) , Netflix Inc. (NFLX) and Alphabet Inc.'s (GOOGL) Google.

- Assets:
  - Stocks - a share in the ownership of a company. Stock represents a claim on the company's assets and earnings. As you acquire more stock, your ownership stake in the company becomes greater

    ***Random Walk Theory:*** **Random Walk Theory**

    * Stock Prices are independent of other factors. Thus past movements cannot predict the future
    * *Theory:* Stocks take a random path. The theory backs the EMH, that it's impossible to beat the stock prices because they already reflect all available information.
    * Opponents say that Technical Analysis contradicts this

    Stock prices are Independent and identically distributed (IID), so the past movement or trend of a stock price or market cannot be used to predict its future movement. In short, this is the idea that stocks take a random and unpredictable path.

  - Bonds - A bond is a fixed income investment in which an investor loans money to an entity (typically corporate or governmental) which borrows the funds for a defined period of time at a variable or fixed interest rate. (Owners of bonds are debtholders, or creditors, of the issuer.)
  - ETFs - An ETF, or exchange-traded fund, is a marketable security that tracks an index, a commodity, bonds, or a basket of assets like an index fund. Unlike mutual funds, an ETF trades like a common stock on a stock exchange. ETFs experience price changes throughout the day as they are bought and sold. - ETF is a type of index fund but is listed and traded like a stock with brokerage commissions. ETF can track indexes that don't exist as index funds.
    * Exchange Traded fund evolved to specialize and use derivatives
  - Mutual Funds (MFs) *[Active]* - Mutual fund is a basket of stocks, bonds, and other assets that is professionally managed by an investment company. Pool all investor cash to construct and buy assets to create a portfolio and every investor gets return proportional to how much they put in.
    * management fees
    * priced once daily at the close
    * required to have cash reserves for liquidity

  - Index Fund *[Passive]*- An index fund is a type of mutual fund with a portfolio constructed to match or track the components of a market index, such as the Standard & Poor's 500 Index (S&P 500). An index mutual fund is said to provide broad market exposure, low operating expenses and low portfolio turnover. - Index fund is not generating alpha/out performing, it attempts to mimic the performance of that entire market.
    * much lower fees
    * you're getting an average return

## *Passive Management: Indexing*

- ***Cap-Weighted Index*** - A capitalization-weighted (or "cap-weighted") index, also called a market-value-weighted index is a stock market index whose components are weighted according to the total market value of their outstanding shares. Every day an individual stock's price changes and thereby changes a stock index's value. The impact that individual stock's price change has on the index is proportional to the company's overall market value (the share price multiplied by the number of outstanding shares), in a capitalization-weighted index.
  - ***Example:*** Suppose a capitalization-weighted index ZYXWV comprises five public companies. Company Z has a market capitalization of $50 million and 5 million shares outstanding. Company Y has a market capitalization of $30 million and 1 million shares outstanding. Company X has a market capitalization of $20 million and 500,000 shares outstanding. Company W has a market capitalization of $25 million and 1 million shares outstanding. Company V has a market capitalization of $100 million and 5 million shares outstanding.
  - Each component should have a different weight based on the size of its market capitalization. The percent each component should be weighted is calculated by dividing each individual market capitalization by the sum of all of the index components' market capitalizations. Company Z has a weight of 22.22%, or $50 million / $225 million. Company Y has a weight of 13.33%, Company X has a weight of 8.9%, Company W has a weight of 11.11%, and Company V has a weight of 44.44%.
  - The component with a higher market capitalization has a higher weight in a capitalization-weighted index. The divisor given for index ZYXWV is 225,000, and the index opens with a value of 1,000, or 225 million / 225,000.
- ***Price-Weighted Index*** - A price-weighted index is a stock index in which each stock influences the index in proportion to its price per share. The value of the index is generated by adding the prices of each of the stocks in the index and dividing them by the total number of stocks. Stocks with a higher price will be given more weight and, therefore, will have a greater influence over the performance of the index.
  - **Example:** - For example, assume that an index contains only two stocks, one priced at $1 and one priced at $10. The $10 stock is weighted nine times higher than the $1 stock. Overall, this means that this index is composed of 90% of the $10 stocks and 10% of $1 stock. In this case, a change in the value of the $1 stock will not affect the index's value by a large amount, because it makes up such a small percentage of the index.

***The divisor*** is a number initially chosen at inception. It is frequently chosen so that the price index has a convenient initial value, such as 1,000. The index provider then adjusts the value of the divisor as necessary to avoid changes in the index value that are unrelated to changes in the prices of its constituent securities. For example, when changing index constituents, the index provider may adjust the divisor so that the value of the index with the new constituents equals the value of the index prior to the changes. The divisor can be:

- Proprietary
- Number of stocks

- as we have chosen a number to simplify
- primary goal: to scale
- ***It can Change***

Float weighted assigns a factor to the market cap calculation that accounts for how many shares are publicly held, or are "floating". This is multiplied by the market cap before the weighting is determined for the index.

Historically 100% shares o/s were used, now because of large cross-holdings or gov't ownership when indexing you really want how many "free" shares are o/s

An index is not exactly the same as a portfolio. For instance, when a stock is added to or deleted from an index, the index level should not jump up or drop down; while a portfolio's value would usually change as stocks are swapped in and out. To assure that the index's value, or level, does not change when stocks are added or deleted, the divisor is adjusted to offset the change in market value of the index.

## 4.2 Active Management

- Extensive research performed to beat the average market index return.
  - You pay more for this skill (Management fees are much more than in passive management)
  - It is difficult to outperform the market to cover the required fees.
- Assets:
  - ETFs
  - Formulated Portfolios
  - MFs
- Flexibility to adjust portfolios
- Hedging advantages due to short-selling ability
  - Short-Selling - Short selling stock is a fairly common way knowledgeable investors make money on stocks, even when share prices drop. To short a stock, you actually sell shares you don't own and then buy to cover the sale when the price drops. While that may sound odd, investors often choose this strategy over others for particular reasons in given situations.
- More actively re-formulate portfolios to keep up risk management
- Tax Advantages to liquidate at prime-time periods to offset taxes

## 4.3 Other Weighting Methods

- Equal Weighted
- Market Cap Weighted
- Price Weighted
- Fundamentally Weighted

## *Bloomberg 2: Buffet Says $100 Billion Wasted trying to beat the market*

- Passive investment is the way to go
- "Let the tide take you out and you can reap the rewards"

- Protege vs Buffet Fund, Buffet would wins (2.2% vs 7.1%)
- Protege invested in 5 hedge funds, with alright returns, but the management fees are too high (According to Buffet they take away about 60% of the gains)
- Passive Assets ==> 4.3 Trillion dollars
- Active Assets ==> 5.1 Trillion dollars
- Vanguard and BlackRock up by $\geq$ 20 trillion dollars

## 4.4 Asset Allocation vs Stock Picking

Your portfolio (401k, IRA, etc.) probably consists of a number of stocks, bonds, ETF's, and mutual funds.The mix of these assets constitutes your portfolio allocation. How your portfolio is allocated determines its performance. During the first quarter of every year, investors typically spend a few hours reallocating/rebalancing their retirement accounts.

Investors often don't appreciate that some of the factors that affect their investments have little to do with their actual investment choices – these systematic risks are borne by all investors. Accordingly, systematic risks are what influence asset allocation; idiosyncratic risk influences stock picking.

The idea is that using portfolio theory you should be able to diversify away all idiosyncratic risk and only be left with systematic risk. Only have to deal with market risk for different asset classes.

### 4.4.1 Strategic Asset Allocation

***Investopedia:***

- SAA - portfolio strategy that involves setting target allocations for various asset classes
- Then, periodically rebalance the portfolio to maintain the original allocations
  - Buying or selling asset classes which, due to market price fluctuations, no longer comply with the original target proportions within a portfolio. Typically, target ranges or values are set to trigger the rebalancing exercise which take into consideration the costs of rebalancing
- Allocations can deviate due to differing returns from various assets. Balanced diversification can reduce risk and improve portfolio returns
- SAA focused on the needs on the investor rather than the constant tracking of the markets & is thought to remove the emotion from investment strategies

### 4.4.2 Tactical Asset Allocation:

***Investopedia:***

- Active Management portfolio strategy that shifts the percentage of assets held in various categories to take advantage of market pricing anomalies or market sectors
- Allows Portfolio Managers to create extra value by taking advantage of certain situations in the marketplace
  - It's a moderately active strategy because managers return to the portfolio's original strategic asset mix when desired short-term profits are achieved