SET-II

DESIGN OF THE QUESTION PAPER

Mathematics

Class X

Time : 3 Hours **Maximum Marks : 80**

Weightage and the distribution of marks over different dimensions of the question shall be as follows:

(A) Weightage to Content/ Subject Units :

S.No.	Content Unit	Marks
1.	Number Systems	04
2.	Algebra	20
3.	Trigonometry	12
4.	Coordinate Geometry	08
5.	Geometry	16
6.	Mensuration	10
7.	Statistics and Probability	10
		Total : 80

(B) Weightage to Forms of Questions :

S.No.	Form of Questions	Marks for each Question	Number of Questions	Total Marks
1.	MCQ	01	10	10
2.	SAR	02	05	10
3.	SA	03	10	30
4.	LA	06	05	30
		Total	**30**	**80**

(C) Scheme of Options

All questions are compulsory, i.e., there is no overall choice. However, internal choices are provided in one question of 2 marks, three questions of 3 marks each and two questions of 6 marks each.

(D) Weightage to Difficulty level of Questions

S.No.	Estimated Difficulty Level of Questions	Percentage of Marks
1.	Easy	20
2.	Average	60
3.	Difficult	20

Note : A question may vary in difficulty level from individual to individual. As such, the assessment in respect of each will be made by the paper setter/ teacher on the basis of general anticipation from the groups as whole taking the examination. This provision is only to make the paper balanced in its weight, rather to determine the pattern of marking at any stage.

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MATHEMATICS

CLASS X

Form of Question / **Units**	MCQ	SAR	SA	LA	Total
Number Systems	2(2)	2(1)	-	-	4(3)
Algebra Polynomials, Pair of Linear Equations in Two Variables, Quadratic Equations, Arithmatic Progressions	3(3)	2(1)	9(3)	6(1)	20(8)
Trigonometry Introduction to Trigonometry, Some Applications of Trigonometry	1(1)	2(1)	3(1)	6(1)	12(4)
Coordinate Geometry	1(1)	4(2)	3(1)	-	8(4)
Geometry Triangles, Circles, Constructions	1(1)	-	9(3)	6(1)	16(5)
Mensuration Areas related to Circles, Surface Areas and Volumes	1(1)	-	3(1)	6(1)	10(3)
Statistics & Probability	1(1)	-	3(1)	6(1)	10(3)
Total	**10(10)**	**10(5)**	**30(10)**	**30(5)**	**80(30)**

SUMMARY

Multiple Choice Questions (MCQ)	Number of Questions : 10	Marks : 10
Short Answer Questions with Resasoning (SAR)	Number of Questions : 05	Marks : 10
Short Answer Questions (SA)	Number of Questions : 10	Marks : 30
Long Answer Qustions (LA)	Number of Questions : 05	Marks : 30
Total	**30**	**80**

Mathematics
Class X

Maximum Marks : 80 **Time : 3 Hours**

General Instructions

1. All questions are compulsory.
2. The question paper consists of 30 questions divided into four sections A, B, C, and D. Section A contains 10 questions of 1 mark each, Section B contains 5 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 5 questions of 6 marks each.
3. There is no overall choice. However, an internal choice has been provided in one question of 2 marks, three questions of 3 marks and two questions of 6 marks each.
4. In questions on construction, the drawing should be neat and exactly as per given measurements.
5. Use of calculators is not allowed.

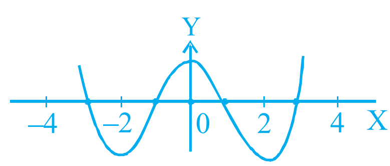
Section A

1. The largest number which divides 318 and 739 leaving remainders 3 and 4, respectively is

(A) 110 (B) 7 (C) 35 (D) 105

2. The number of zeroes lying between –2 to 2 of the polynomial $f(x)$, whose graph is given below, is

(A) 2 (B) 3 (C) 4 (D) 1



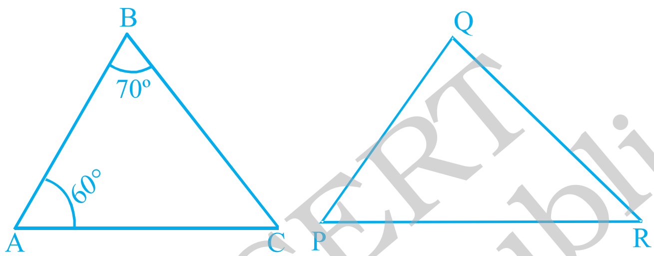
3. The discriminant of the quadratic equation $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$ is

(A) 8 (B) 64 (C) $-\frac{1}{3\sqrt{3}}$ (D) $-\sqrt{3}$

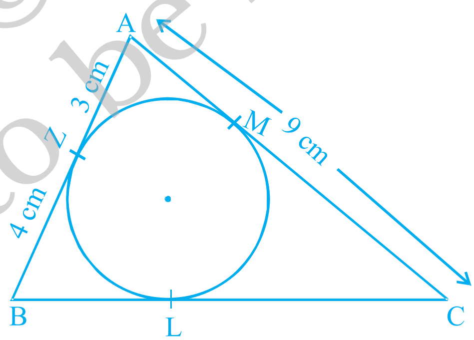
4. If $\frac{6}{5}$, a, 4 are in AP, the value of a is

(A) 1 (B) 13 (C) $\frac{13}{5}$ (D) $\frac{26}{5}$

5. If in the following figure, Δ ABC : Δ QPR, then the measure of $\angle$Q is
(A) 60° (B) 90° (C) 70° (D) 50°



6. In the adjoining figure, Δ ABC is circumscribing a circle. Then, the length of BC is
(A) 7 cm (B) 8 cm (C) 9 cm (D) 10 cm



7. If $\sin\theta = \frac{1}{3}$, then the value of $(9\cot^2\theta + 9)$ is

(A) 1 (B) 81 (C) 9 (D) $\frac{1}{81}$

8. The radii of the ends of a frustum of a cone 40 cm high are 20 cm and 11 cm. Its slant height is

(A) 41 cm (B) $20\sqrt{5}$ cm (C) 49 cm (D) $\sqrt{521}$ cm

9. A bag contains 40 balls out of which some are red, some are blue and remaining are black. If the probability of drawing a red ball is $\frac{11}{20}$ and that of blue ball is $\frac{1}{5}$, then the number of black balls is

(A) 5 (B) 25 (C) 10 (D) 30

10. Two coins are tossed simultaneously. The probability of getting at most one head is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1

SECTION B

11. Which of the following can be the n^{th} term of an AP?
$3n + 1$, $2n^2 + 3$, $n^3 + n$.
Give reasons.

12. Are the points (–3, –3), (–3, 2) and (–3, 5) collinear? Give reasons.

13. ABC and BDE are two equilateral triangles such that D is the mid point of BC. What is the ratio of the areas of triangles ABC and BDE? Justify your answer.

14. $\cos(A + B) = \frac{1}{2}$ and $\sin(A - B) = \frac{1}{2}$, $0° < A + B < 90°$ and $A - B > 0°$. What are the values of $\angle A$ and $\angle B$? Justify your answer.

15. A coin is tossed twice and the outcome is noted every time. Can you say that head must come once in two tosses? Justify your answer.

OR

A die is thrown once. The probability of getting a prime number is $\frac{2}{3}$. Is it true? Justify your answer.

SECTION C

16. Show that square of an odd positive integer is of the form $8q + 1$, for some positive integer q.

OR

Write the denominator of the rational number $\frac{357}{5000}$ in the form of $2^m\, 5^n$, m, n are non-negative integers and hence write its decimal expansion, without actual division.

17. If $(x - 2)$ is a factor of $x^3+ax^2+bx+16$ and $b = 4a$, then find the values of a and b.

18. The sum of reciprocals of a child's age (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

OR

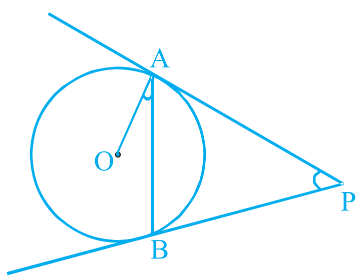
Solve for x : $6\, a^2x^2 - 7abx - 3b^2 = 0$, $a \neq 0$, using the quadratic formula.

19. Find the sum of all two digit natural numbers which are divisible by 7.

20. Find the ratio in which the line $x + 3y - 14 = 0$ divides the line segment joining the points A (–2, 4) and B (3, 7).

21. Find the area of the quadrilateral whose vertices in the same order are (–4, –2), (–3, –5), (3, –2) and (2, 3).

22. Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that $\angle$APB = 2 $\angle$OAB. (see the following figure).



23. Construct a triangle with sides 3 cm, 5 cm and 7 cm and then construct another triangle whose sides are $\frac{5}{3}$ of the corresponding sides of the first triangle.

24. Prove the identity $(1 + \cot\theta + \tan\theta)(\sin\theta - \cos\theta) = \frac{\sec\theta}{\operatorname{cosec}^2\theta} - \frac{\operatorname{cosec}\theta}{\sec^2\theta}$

OR

Find the value of

$$\frac{\cos^2 32° + \cos^2 58°}{\sec^2 50° - \cot^2 40°} - 4\tan 13° \tan 37° \tan 53° \tan 77°$$

25. The area of an equilateral triangle is $49\sqrt{3}$ cm². Taking each vertex as centre, circles are described with radius equal to half the length of the side of the triangle. Find the area of the part of the triangle not included in the circles. [Take $\sqrt{3} = 1.73$, $\pi = \frac{22}{7}$]

SECTION D

26. In a bag containing white and red balls, half the number of white balls is equal to the one third the number of red balls. Twice the total number of balls exceeds three times the number of red balls by 8. How many balls of each type does the bag contain?

27. Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of the other two sides.
Using the above theorem, prove that in a triangle ABC, if AD is perpendicular to BC, then $AB^2 + CD^2 = AC^2 + BD^2$.

28. A pole 5m high is fixed on the top of a tower. The angle of elevation of the top of the pole as observed from a point A on the ground is 60° and the angle of depression of point A from the top of the tower is 45°. Find the height of the tower. (Take $\sqrt{3}$ = 1.73)

29. The interior of a building is in the form of a cylinder of diameter 4 m and height 3.5 m, surmounted by a cone of the same base with vertical angle as a right angle. Find the surface area (curved) and volume of the interior of the building.

OR

A vessel in the form of an open inverted cone of height 8 cm and radius of its top is 5 cm. It is filled with water up to the brim. When lead shots, each of radius 0.5 cm are dropped into the vessel, one fourth of the water flows out. Find the number of lead shots dropped in the vessel.

30. Find the mean, median and mode of the following frequency distribution:

Class	0-10	10-20	20-30	30-40	40-50	50-60	60–70
Frequency	4	5	7	10	12	8	4

OR

The following distribution gives the daily income of 50 workers of a factory:

Daily income (in Rs)	100-120	120-140	140-160	160-180	180-200
Number of workers	12	14	8	6	10

Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive. Find the median from this ogive.

MARKING SCHEME

SECTION A

MARKS

1. (D)	**2.** (A)	**3.** (B)	**4.** (C)	**5.** (A)
6. (D)	**7.** (B)	**8.** (A)	**9.** (C)	**10.** (C)

(1 × 10 = 10)

SECTION B

11. n^{th} term is $3n + 1$, $(\frac{1}{2})$

because, n^{th} term of an AP can only be a linear relation in n. $(1\frac{1}{2})$

12. Yes, $(\frac{1}{2})$

Since all the three points are on the line $x = -3$. $(1\frac{1}{2})$

13. 4 : 1 $(\frac{1}{2})$

$$\frac{\text{ar ABC}}{\text{ar BDE}} = \frac{BC^2}{BD^2} = \frac{BC^2}{\left[\frac{1}{2}(BC)\right]^2} = \frac{4}{1}$$ $(1\frac{1}{2})$

14. $\angle A = 45°$, $\angle B = 15°$ $(\frac{1}{2})$

$A + B = 60°$ and $A - B = 30°$, solving, we get $\angle A = 45°$, $\angle B = 15°$ $(1\frac{1}{2})$

15. No. $(\frac{1}{2})$

Head may come and head may not come. In every toss, there are two equally likely outcomes. $(1\frac{1}{2})$

OR

No. $(\frac{1}{2})$

P (a pirme number) = P (2, 3, 5) = $\frac{3}{6} = \frac{1}{2}$ $(1\frac{1}{2})$

SECTION C

16. An odd positive integer can be of the form, $4n+1$ or $4n+3$ (1)

Therefore, $(4n+1)^2 = 16n^2 + 8n + 1 = 8(2n^2 + n) + 1 = 8q + 1.$ (1)

$(4n+3)^2 = 16n^2 + 24n + 9 = 8(2n^2 + 3n + 1) + 1 = 8q + 1.$ (1)

OR

$$\frac{357}{5000} = \frac{357}{2^3 \times 5^4} \quad (1)$$

$$= \frac{357 \times 2}{2^4 \times 5^4} = \frac{714}{(10)^4} \quad (1)$$

$$= 0.0714 \quad (1)$$

17. $(x-2)$ is a factor of $x^3 + ax^2 + bx + 16$

Therefore, $(2)^3 + a(2)^2 + b(2) + 16 = 0$ (1)

$4a + 2b + 24 = 0$ or $2a + b + 12 = 0$ (1)

Given $b = 4a$, so $a = -2$ (1)

and $b = -8$

18. Let the present age be x years. (1)

Therefore, $\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

or $3[(x+5) + (x-3)] = (x-3)(x+5)$

or $6x + 6 = x^2 + 2x - 15.$

or $x^2 - 4x - 21 = 0$

or $(x-7)(x+3) = 0$ (1)

i.e., $x = 7$, $x = -3$ (rejected)

Therefore, present age = 7 years (1)

OR

$6a^2x^2 - 7abx - 3b^2 = 0$

$B^2 - 4AC = [(-7ab)^2 - 4(6a^2)(-3b^2)]$

$= 49a^2b^2 + 72a^2b^2 = 121a^2b^2$ (1)

Therefore, $x = \frac{-(-7ab) \pm 11ab}{12a^2}$ (1)

$$= \frac{18ab}{12a^2} \text{ or } \frac{-4ab}{12a^2}$$

$$= \frac{3b}{2a} \text{ or } -\frac{b}{3a} \qquad (1)$$

19. Numbers are

14, 21, ..., 98 (1)

98 = 14 + $(n - 1)$ 7 i.e., $n = 13$ (1)

$$S_{13} = \frac{13}{2}[14 + 98] = 728. \qquad (1)$$

20. Let C (x, y) be the point where the line $x + 3y - 14 = 0$ divides the line segment in the ratio k:1.

$$\text{So, } x = \frac{3k-2}{k+1}, y = \frac{7k+4}{k+1} \qquad (1)$$

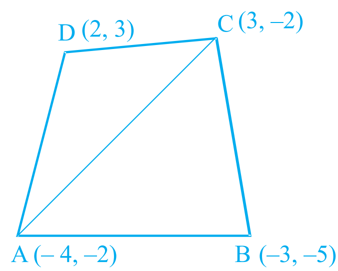
$$\text{and, } \frac{3k-2}{k+1} + 3.\frac{7k+4}{k+1} - 14 = 0 \qquad \left(\frac{1}{2}\right)$$

i.e., $3k - 2 + 21k + 12 - 14k - 14 = 0$,

i.e., $10k - 4 = 0$

$$\text{i.e., } k = \frac{4}{10} = \frac{2}{5} \qquad (1)$$

Therefore, ratio is 2 : 5 $\left(\frac{1}{2}\right)$



21. Area of Δ ABC

$$= \frac{1}{2}[-4(-5+2) - 3(-2+2) + 3(-2+5)]$$

$$= \frac{1}{2}[12 + 9] = \frac{21}{2} \text{ sq.units} \qquad (1)$$

$$\text{area of } \Delta \text{ ACD} = \frac{1}{2}[-4(-2-3) + 3(3+2) + 2(-2+2)]$$

$$= \frac{1}{2}[20 + 15] = \frac{35}{2} \text{ sq. units} \qquad (1)$$

Therefore, area of quadrilateral ABCD = $\frac{21+35}{2} = \frac{56}{2}$ = 28 sq. units (1)

22. AP = PB. So, $\angle PAB = \angle PBA = \frac{1}{2}[180^\circ - \angle APB]$

$$= 90^\circ - \frac{1}{2}\angle APB \quad (1)$$

$\angle OAB = 90^\circ - \angle PAB$ (1)

$$= 90^\circ - [90^\circ - \frac{1}{2}\angle APB] = \frac{1}{2}\angle APB$$

i.e., $2\angle OAB = \angle APB$ (1)

23. Correct construction of Δ with sides 3, 5 and 7 cm (1)

Correct construction of similar triangle (2)

24. LHS = $\left(1+\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}\right)(\sin\theta-\cos\theta)$ $(\frac{1}{2})$

$$= \frac{(\sin\theta\cos\theta+\cos^2\theta+\sin^2\theta)(\sin\theta-\cos\theta)}{\sin\theta\cos\theta} = \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta} \quad (1\frac{1}{2})$$

$$= \frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta} = \frac{\sec\theta}{\mathrm{cosec}^2\theta} - \frac{\mathrm{cosec}\theta}{\sec^2\theta} \quad (1)$$

OR

$\cos^2 58^\circ = \sin^2 32^\circ$, $\tan 53^\circ = \cot 37^\circ$

$\sec^2 50^\circ = \mathrm{cosec}^2 40$, $\tan 77^\circ = \cot 13^\circ$ (2)

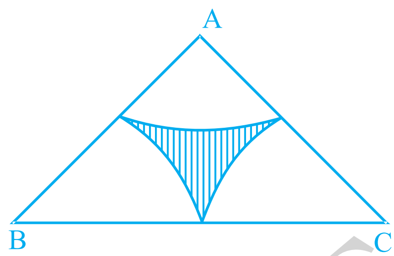
Given expression

$$= \frac{\cos^2 32^\circ + \sin^2 32^\circ}{\mathrm{cosec}^2 40^\circ - \cot^2 40^\circ} - 4\tan 13^\circ \tan 37^\circ \cot 37^\circ \cot 13^\circ \quad (1)$$

$= 1 - 4 = -3$

25. Area of Δ ABC $= 49\sqrt{3}$ cm^2 $= \sqrt{3}\,\frac{a^2}{4}$

So, $a = 14$ cm (1)



Area of one sector $= \pi \times 7^2 \frac{60}{360} = \frac{49\pi}{6}$ (1)

Therefore, required area $= 49\sqrt{3} - \frac{3 \times 49}{6} \times \left(\frac{22}{7}\right)$

$= 49\sqrt{3} - 77$

$= 84.77 - 77 = 7.77$ cm^2 (1)

SECTION D

26. Let the number of white balls be x and number of red balls be y

Therefore, $\frac{1}{2}x = \frac{1}{3}y$, i.e., $3x - 2y = 0$ (I) $(1\frac{1}{2})$

and $2(x + y) = 3y + 8$

i.e., $2x - y = 8$ (II) $(1\frac{1}{2})$

Solving (I) and (II), we get $x = 16$, $y = 24$ (2)

Therefore, number of white balls = 16

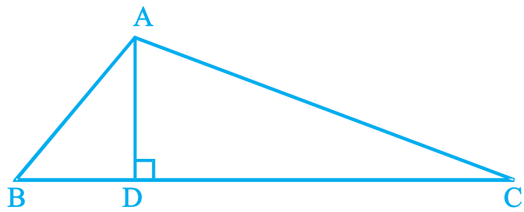
Number of red balls = 24 (1)

27. For correct given, to prove, construction and proof $(\frac{1}{2} \times 4 = 2)$

For correct proof (2)

$AD^2 = AB^2 - BD^2$ $(\frac{1}{2})$

and $AD^2 = AC^2 - CD^2$



$(\frac{1}{2})$

i.e., $AB^2 - BD^2 = AC^2 - CD^2$ $(\frac{1}{2})$

or $AB^2 + CD^2 = AC^2 + BD^2$ $(\frac{1}{2})$

28. For correct figure (1)

Let height of tower be h metres and AB = x metres. $(\frac{1}{2})$

Therefore, $\frac{x}{h} = \cot 45° = 1$ (1)

i.e., $x = h$. $(\frac{1}{2})$

Also, $\frac{h+5}{x} = \tan 60° = \sqrt{3}$ (1)

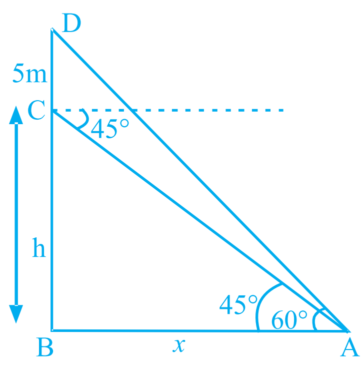
i.e., $h + 5 = \sqrt{3}x = \sqrt{3}h$ $(\frac{1}{2})$

i.e., $(\sqrt{3} - 1)h = 5$

$h = \frac{5}{\sqrt{3}-1} \cdot \frac{\sqrt{3}+1}{\sqrt{3}+1}$ $(\frac{1}{2})$

$= \frac{5(\sqrt{3}+1)}{2} = \frac{5(2.73)}{2}$

$= \frac{13.65}{2} = 6.825$ m (1)



29. For correct figure $(\frac{1}{2})$

Here, $\angle Q = 45°$, i.e., height of cone = radius = 2m (1)

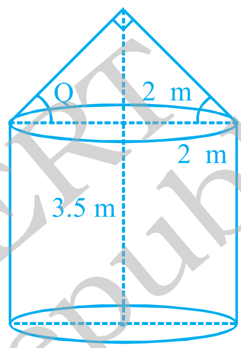
Therefore, surface area = $\pi rl + 2\pi rh$

$= \pi r\,(l + 2h)$ (1)

$= \pi \times 2 \times (2\sqrt{2} + 7)$ $(\frac{1}{2})$

$= (14 + 4\sqrt{2})\ \pi\ \text{m}^2$ (1)

Volume $= \frac{1}{3}\pi r^2 h_1 + \pi r^2 h$ $(\frac{1}{2})$



$= \pi r^2 \left[\frac{h_1}{3} + h\right]$

$= \pi \times 4 \times \left[\frac{2}{3} + 3.5\right] = 4\pi\left[\frac{2 + 10.5}{3}\right]$ $(\frac{1}{2})$

$= \frac{50\pi}{3}\ \text{m}^3$ (1)

OR

Volume of water $= \frac{1}{3}\pi \times (5)^2 \times 8$ $(\frac{1}{2})$

$= \frac{200\pi}{3}\ \text{cm}^3$ (1)

$\frac{1}{4}$th volume $= \frac{50\pi}{3}\ \text{cm}^3$ (1)

Volume of one lead shot $= \frac{4}{3}\pi(0.5)^3 = \frac{0.5\pi}{3}\ \text{cm}^3$ $(1\frac{1}{2})$

Let number of shots be n.

Therefore, $\frac{0.5\pi}{3} \times n = \frac{50\pi}{3}$ (1)

i.e., $n = 100$. (1)

30.

CI	0–10	10–20	20–30	30–40	40–50	50–60	60–70	Total
f_i	4	5	7	10	12	8	4	50
x_i	5	15	25	35	45	55	65	
u_i	–3	–2	–1	0	1	2	3	
$f_i\, u_i$	–12	–10	–7	0	12	16	12	11
cf	4	9	16	26	38	46	50	

$(1\frac{1}{2})$

$$\Sigma f_i = 50$$
$$\Sigma f_i u_i = 11$$

Mean $= 35 + \frac{11}{50} \times 10 = 35 + 2.2 = 37.2$ (1)

Median $= l + \left(\frac{\frac{n}{2} - \text{cf}}{f}\right) \times h$ $(\frac{1}{2})$

$= 30 + \frac{25 - 16}{10} \times 10 = 30 + 9 = 39$ (1)

Mode $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$ $(\frac{1}{2})$

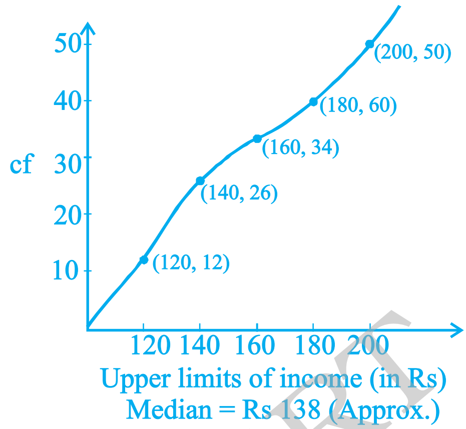
$= 40 + \frac{12 - 10}{24 - 10 - 8} \times 10$ $(\frac{1}{2})$

$= 40 + \frac{20}{6} = 43.33$ (1)

OR

Writing as (1)

Daily income (in Rs)	cf
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50



(5)

Note: Full credit should be given for alternative correct solution.