MATHEMATICS
EXEMPLAR PROBLEMS

Class X

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

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FOREWORD

The National Curriculum Framework (NCF) – 2005 initiated a new phase of development of syllabi and textbooks for all stages of school education. Conscious effort has been made to discourage rote learning and to diffuse sharp boundaries between different subject areas. This is well in tune with the NPE – 1986 and *Learning Without Burden-1993* that recommend child centred system of education. The textbooks for Classes IX and XI were released in 2006 and for Classes X and XII in 2007. Overall the books have been well received by students and teachers.

NCF–2005 notes that treating the prescribed textbooks as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. It further reiterates that the methods used for teaching and evaluation will also determine how effective these textbooks proves for making children's life at school a happy experience, rather than source of stress or boredom. It calls for reform in examination system currently prevailing in the country.

The position papers of the National Focus Groups on *Teaching of Science*, *Teaching of Mathematics and Examination Reform* envisage that the mathematics question papers, set in annual examinations conducted by the various Boards do not really assess genuine understanding of the subjects. The quality of question papers is often not up to the mark. They usually seek mere information based on rote memorization, and fail to test higher-order skills like reasoning and analysis, let along lateral thinking, creativity, and judgment. Good unconventional questions, challenging problems and experiment-based problems rarely find a place in question papers. In order to address to the issue, and also to provide additional learning material, the Department of Education in Science and Mathematics (DESM) has made an attempt to develop resource book of exemplar problems in different subjects at secondary and higher-secondary stages. Each resource book contains different types of questions of varying difficulty level. Some questions would require the students to apply simultaneously understanding of more than one chapters/units. These problems are *not* meant to serve merely as question bank for examinations but are primarily meant to improve the quality of teaching/learning process in schools. It is expected that these problems would encourage teachers to design quality questions on their own. Students and teachers should always keep in mind that examination and assessment should test

comprehension, information recall, analytical thinking and problem-solving ability, creativity and speculative ability.

A team of experts and teachers with an understanding of the subject and a proper role of examination worked hard to accomplish this task. The material was discussed, edited and finally included in this source book.

NCERT will welcome suggestions from students, teachers and parents which would help us to further improve the quality of material in subsequent editions.

Professor Yash Pal
Chairperson
National Steering Committee
National Council of Educational
Research and Training

New Delhi
21 May 2008

PREFACE

The Department of Education in Science and Mathematics (DESM), National Council of Educational Research and Training (NCERT), initiated the development of 'Exemplar Problems' in science and mathematics for secondary and higher secondary stages after completing the preparation of textbooks based on National Curriculum Framework–2005.

The main objective of the book on 'Exemplar Problems in Mathematics' is to provide the teachers and students a large number of quality problems with varying cognitive levels to facilitate teaching learning of concepts in mathematics that are presented through the textbook for Class X. It is envisaged that the problems included in this volume would help the teachers to design tasks to assess effectiveness of their teaching and to know about the achievement of their students besides facilitating preparation of balanced question papers for unit and terminal tests. The feedback based on the analysis of students' responses may help the teachers in further improving the quality of classroom instructions. In addition, the problems given in this book are also expected to help the teachers to perceive the basic characteristics of good quality questions and motivate them to frame similar questions on their own. Students can benefit themselves by attempting the exercises given in the book for self assessment and also in mastering the basic techniques of problem solving. Some of the questions given in the book are expected to challenge the understanding of the concepts of mathematics of the students and their ability to applying them in novel situations.

The problems included in this book were prepared through a series of workshops organised by the DESM for their development and refinement involving practicing teachers, subject experts from universities and institutes of higher learning, and the members of the mathematics group of the DESM whose names appear separately. We gratefully acknowledge their efforts and thank them for their valuable contribution in our endeavour to provide good quality instructional material for the school system.

I express my gratitude to Professor Krishna Kumar, *Director* and Professor G. Ravindra, *Joint Director*, NCERT for their valuable motivation and guidiance from time to time. Special thanks are also due to Dr. R.P.Maurya, *Reader* in Mathematics, DESM for coordinating the programme, taking pains in editing and refinement of problems and for making the manuscript pressworthy.

We look forward to feedback from students, teachers and parents for further improvement of the contents of this book.

Hu kum Singh

Professor and Head

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Students' Evaluation in Mathematics at the Secondary Stage

A. Introduction

The fascinating world of mathematics provides an unlimited scope to mathematicians to perceive problems pertaining to three situations visualised in the forms of concrete, abstraction and intuition. However, due to abstraction and intuition, sometimes some of the mathematical concepts become quite complicated even for teachers who are actively engaged in mathematics teaching at various stages. This needs the exhaustive training in methods/pedagogy as well as in contents. This also needs the clarifications of mathematical concepts using instructional materials, experimentation, observation and practicals etc. to avoid the abstraction at various stages of schooling. Good mathematics instruction requires good teachers, and good teachers are those with pedagogical content knowledge who, in turn, are predominantly those with good content. Improvement of school mathematics education therefore begins with teaching teachers the mathematics they need. In other words, the most difficult demand for becoming a good teacher is to achieve a firm mastery of the mathematical content . Without such a mastery, good pedagogy is difficult. A firm mastery of the content opens up the world of pedagogy and offers many more effective pedagogical possibilities. Even best pedagogy lavished on incorrect mathematics may result in poor quality in teaching.

Mathematics as a science of abstract objects, relies on *logic* rather than on observation, yet it employs observation, simulation, and even experiments as means of discovering truth. The ability to reason and think clearly is extremely useful in our daily life, that is, developing children's abilities for *mathematisation* is the main goal of mathematics education as has been emphasised in National Curriculum Framework-2005 (NCF-2005). It is in this context that NCF-2005 has set two distinct targets for mathematics education at school level viz. *narrow* and *higher*. The narrow aim of school mathematics is to develop useful capabilities, particularly those relating to numeracy- number, number operations, measurements, decimals and percentages. The higher aim is to develop the child's resources to think and reason mathematically, to pursue assumptions to their logical conclusions and to handle abstractions. It includes a way of doing things, and the ability and the attitude to formulate and solve problems. This calls for curriculum to be ambitious in the sense that it seeks to achieve the higher aim mentioned above, rather than only the narrow aim. It should be coherent in the

sense that the variety of methods and skills available piecemeal (in arithmetic, algebra, geometry) cohere into an ability to address problems that come from other domains such as sciences and in social studies at secondary stage. It should be important in the sense that students feel the need to solve such problems.

Evaluation is a very comprehensive term which, in general, includes evaluating any object, individual, event, trend, etc. A most common type of individual evaluation is the evaluation of a student. It includes the assessments of the performance of the student in the areas of her personality development in terms of intellectual, social and emotional developments after she has been provided learning experiences through classroom processes. Besides the factors like quality of teaching curricular materials, instructional technology, school infrastructure and societal support also influence the learning and experiences. In educational terminology, these areas of personality development are called scholastic and co-scholastic areas. Due to its wider applications in various other fields, mathematics is the most important scholastic area. It is for this reason, mathematics is a compulsory subject up to the secondary stage from quite a long time. This is the stage which acts as a bridge between the students who will continue with Mathematics in higher classes. Therefore, evaluation of Mathematics at this stage requires special attention. This evaluation is done to assess whether the main aim or objectives laid down in NCF-2005 have been achieved by the students or not?

B. Purposes of Evaluation

There are various purposes of evaluation. Some of these are to know the answers for the following questions:

(i) How has the teaching been effective?

(ii) Which method is more suitable for teaching a particular topic or concept?

(iii) To what extent students are ready to learn a particular topic?

(iv) What type of learning difficulties are faced by the students?

(v) Do the students require remedial measures?

(vi) Which students are to be provided some enrichment materials?

(vii) Which topics are more difficult for the student?

(viii) Is there a need to make a change in the teaching strategy for a particular topic?

(ix) How can the result of the evaluation can be utilised for the all round development of students?

C. Types of Evaluation

Evaluation is mainly of two types namely

(i) **Summative** and (ii) **Formative**

(i) **Summative Evaluation:** It is done at the end of the course or a term. It involves a formal testing of the student's achievements and is used for grading, ranking and certifying the achievements of the students.

(ii) **Formative Evaluation:** It is in-built in the teaching learning process. It is a continuous process going on throughout the course. The purpose of such evaluation is to obtain feedback so that teaching or instructional strategies could be improved. Further, on the basis of the feedback, strategies and weaknesses of the students can be assessed.

NCF-2005 has also given more stress on continuous and comprehensive evaluation in comparison to the summative evaluation. For this, a mathematics teacher may

(i) ask some questions to know to what extent the students understand about the new concept to be taught before it is started.

(ii) ask questions at regular intervals to check the understanding of students during the presentation of a concept.

(iii) assess students by the questions asked by them during the teaching of a chapter.

(iv) assess the students during class work.

(v) assess students on the basis of the home assignments given to them.

(vi) assess students by asking some questions at the end of the chapter.

(vii) encourage peer group members (students) to evaluate one another. This may be called as **Peer Evaluation**. This evaluation can bring out the hidden talents among the students.

Thus, whatever may be the way of evaluation, it is done through some well thought questions, which may be referred to as **good questions**.

D. Characteristics of a Good Question

Quality of a question depends on the situation where it is to be used. In general, following are some of the characteristics of a 'good question':

(i) **Validity:** A question is said to be valid, if it serves the purpose for which it has been framed.

Thus, for a question to be valid, it must be based on (a) a specified content area and also on (b) a predetermined aim or objective.

In case it is not valid, it will be treated as a question 'out of course or syllabus'.

(ii) **Reliability:** A question is said to be reliable, if its answer gives the true achievement of the student. In other words, the achievement of the student must be free from chance errors. These errors, generally, occur due to vagueness of language or direction provided in the question. They may occur (1) at the time when the student is answering the question and (2) at the time when the teacher is evaluating the answer. In view of the above, following steps can ensure higher reliability of a question:

(a) The question should admit of one and only one interpretation.

(b) The scope of the answer must be clear.

(c) The directions to the question must be clear.

(d) A well thought marking scheme should be provided for the question.

(iii) **Difficulty Level:** Difficulty level is a very important characteristic of a question. In different situations, questions of different difficulty levels are needed. For example, for assessing the achievement of Minimum Level of Learning, there will always be a need of questions of lower difficulty level. Difficulty level of a question may be categorised in the following three types:

(a) **Difficult**: Which could be done by about less than 30% of the students.

(b) **Average**: Which could be done by $\geq 30\%$ but $\leq 70\%$ of the students.

(c) **Easy**: Which could be done by more than 70% of the students.

These levels can be decided by the question framer herself on the basis of her own experiences.

(iv) **Language:** Language of a question must be simple and within the comprehension level of the student's vocabulary. It should not lead to different answers. However, if necessary, the same question can be presented before the students at different difficulty levels, by using a little different language or wordings.

(v) **Form:** There are different forms of questions and each form is more suitable than the other depending upon the situations. There may be several factors for choosing a particular form of questions. These may be one or more of the following: (a) Economy (b) Facility in printings (c) Ease in scoring and so on.

E. Different Forms of Questions

In general, the questions are of the following two forms:

(1) Free Response Type and (2) Fixed Response Type

1. Free Response Questions: In a free response question, a student formulates and organizes her own answer. These type of questions are very much in use in the present system of examination. These are of two types, namely

(a) **Long Answer Questions**

A question which requires comparatively a lengthy answer is called a long answer type question. These questions require the student to select relevant facts, organise them and write answers in her own words. In these type of questions, there is a very little scope of guessing. However, if there are more number of long answer questions, then the possibility of covering the whole content area in the examination will become less. To overcome this difficulty, we may choose such long answer type questions which involve more than one content areas.

(b) **Short Answer Questions**

A question in which a student is expected to write the answer in 3 or 4 lines is called a short answer type question. In these questions, the coverage of content areas is more specific and definite. It may be noted that a question whose answer may be a simple diagram is also considered to be a short answer type question.

2. Fixed Response Questions: In these type of questions, the answer is fixed and definite. These type of questions are being encouraged due to their objectivity in scoring. They are also of two types, namely

(a) **Very Short Answer Questions**

A question in which a student is expected to give the answer in just one word or a phrase is called a very short answer type question. In mathematics, by a word or a phrase, we generally mean a group of symbols or numbers (numerals). It is expected to take 1 to 3 minutes to answer such a question. Fill in the blanks question is one of the examples of such type of questions.

(b) **Objective Questions**

An objective type question is one in which alternate answers are given and student has to just indicate the correct answer. These questions can also be answered in just 1 to 3 minutes. They can be further classified into the following forms:

(i) **True-False Type:** In these type of questions, a statement or formula is given and the student is expected to write whether it is 'True' or 'False'.

(ii) **Matching Type:** These type of questions consist of two columns. The student has to pair each item of first column with some item of the second column on the basis of some criterion. The number of items in the second column may be more than that of the first column.

(iii) **Sentence Completion Type:** In these type of questions, the student has to complete the given sentence using one or more words given in brackets along with the question.

(iv) **Multiple Choice Type:** In these type of questions, number of alternatives (usually called distracters), only one is appropriate or correct. The student is expected to write or tick (✓) the correct alternative.

In the fixed response questions, the scope of guess work is very high. However, this can be minimised by attaching some element of reasoning in such questions. We may call these questions as **Short Answer Questions with Reasoning**.

F. Instructional Objectives

As already stated, a question is said to be valid if it also based on a predetermined objective. The word 'objective' is a wider term. Objectives are divided into two groups, namely (1) educational objectives and (2) instructional objectives. Educational objectives play a directive role in the process of education, while instructional objectives are those goals for the achievement of which all educational efforts are directed. Mathematics is a special language with its own vocabulary and grammar. The vocabulary consists of concepts, terms, facts, symbols, assumptions, etc., while the grammar relates to principles, processes, functional relationships, etc. Knowledge and understanding of these and their applications to new situations have helped mankind to achieve tremendous progress in various fields. Therefore, the main instructional objectives for mathematics are as follows:

1. Knowledge with Specifications

The students

1.1 recall or reproduce terms, facts, etc.

1.2 recognise terms, symbols, concepts, etc.

2. Understanding with Specifications

The students

2.1 give illustrations for terms, definitions, etc.

2.2 detect conceptual errors (and correct) in definitions, statements, formulae, etc.

2.3 compare concepts, quantities, etc.

2.4 discriminate between closely related concepts

2.5 translate verbal statements into mathematical statements and vice-versa

2.6 verify the results arrived at

2.7 classify data as per criteria

2.8 find relationships among the given data

2.9 interpret the data

3. Application with Specification

The students

3.1 analyse and find out what is given and what is required to be done

3.2 find out the adequecy, superflousity and relevancy of data

3.3 estabish relationship among the data

3.4 reason out deductively

3.5 select appropriate methods for solutions of problems

3.6 suggest alternative methods for solving problems

3.7 generalise from particular situations

4. Skill with Specifications

The students

4.1 Carry out calculation easily and quickly

4.2 Handle geometrical instruments properly

4.3 Draw figure accurately and to the scale

4.4 Read tables and graphs properly

4.5 Interpret graphs correctly

As far as the main goal or objective in the NCF-2005 is concerned, it is to develop abilities in the student for mathematisation. It also states (1) the narrow aims of school mathematics, which concern with decimals and percents and (2) the higher aims, which are for developing the child resources to think and reason mathematically, to pursue assumption to their logical conclusions and to handle abstractions. Keeping this in view, at this stage, the stress is only on the higher aims. These higher aims may be considered as the instructional objectives. Objective based questions and objective type questions are often confused with each other. When a question is framed keeping a definite aim or objective in mind, it is called an objective based question, while if a question is framed to measure the students achievement which is objective rather than subjective is called objective type question. It may also be noted that determination of the objective of a question varies from person to person. For example, a question may appear to be of 'knowledge' type to one teacher who may think that the answer of the question is known to the students, but the same question may appear to be of understanding type to another teacher if she thinks that the question is completely unknown to the same group of students. In the light of the views expressed in NCF-2005, the following types of questions are suggested:

1. Long answer questions
2. Short answer questions
3. Short answer questions with reasoning
4. Multiple choice questions

It is hoped that these questions along with the questions in the textbook would be effectively able to evaluate the Classes IX and X students in mathematics.

Contents

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a **SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the unity and integrity of the Nation;

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949, do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**