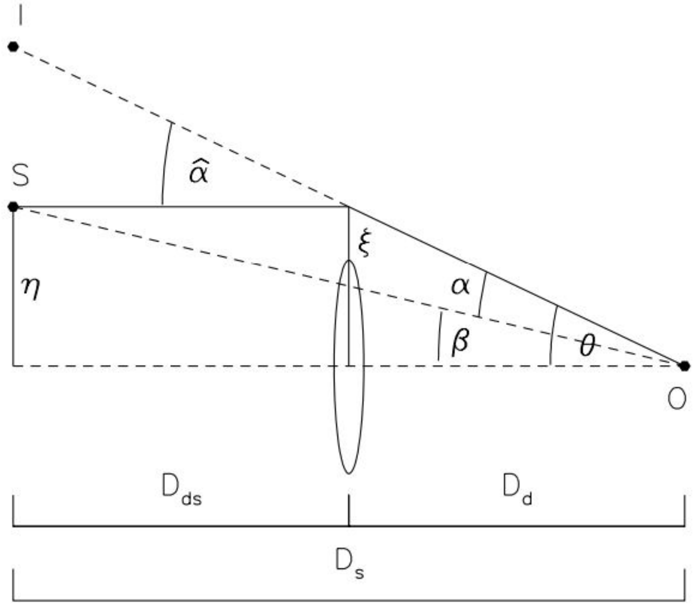
Presentation 2

Photometric Microlensing



The Lens Equation-

$$\theta - \beta = \frac{\theta_E^2}{\theta}$$

,where

$$\theta_E \equiv \left(\frac{4GM}{c^2} \frac{D_{LS}}{D_S D_L} \right)^{1/2}$$

Solution:

$$\theta_\pm = \frac{1}{2} \left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2} \right)$$

Magnification of the two Images:

$$\mu_\pm = \frac{y^2 + 2}{2y\sqrt{y^2 + 4}} \pm \frac{1}{2}$$

,where

$$y \equiv \beta / \theta_E$$

Total Magnification:

$$\mu = |\mu_+| + |\mu_-| = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}$$

Conditions:

Condition on Δr:

$$\Delta r \gtrsim r_E \quad \Leftrightarrow \quad \left(\frac{M}{10^{-7}M_\odot}\right) \lesssim \left(\frac{\Delta r}{\mathrm{AU}}\right)^2 \left(\frac{D}{\mathrm{Gpc}}\right)^{-1}$$

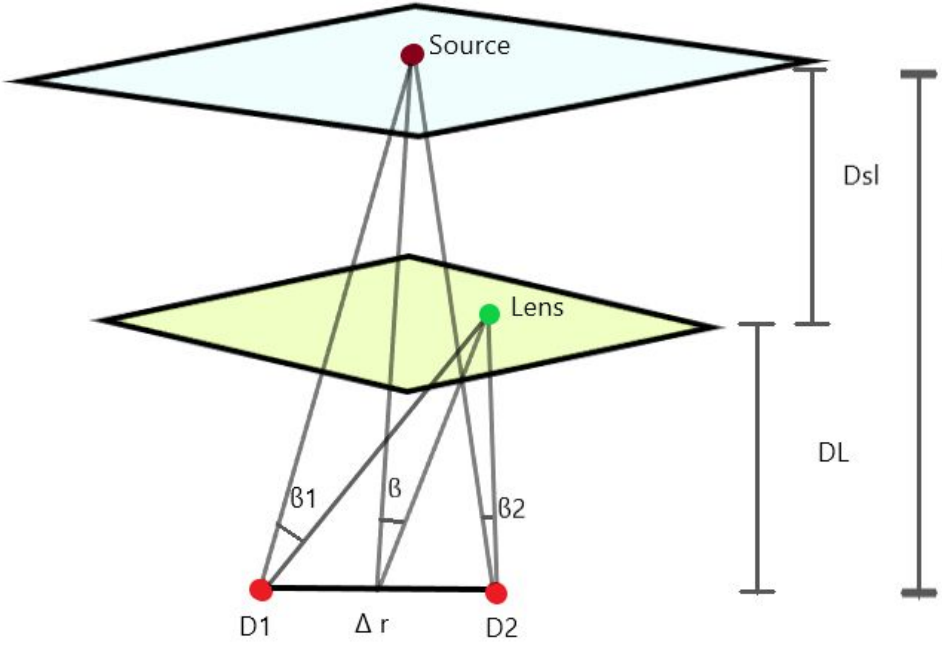
Condition on apparent source angular size:

$$\delta A \equiv \frac{|A_1 - A_2|}{(A_1 + A_2)/2} \gtrsim \epsilon$$

Condition on probe wavelength:

$$0.1\,\lambda \lesssim \frac{2GM}{c^2}(1 + z_L) \simeq 10^{-5}\,\mathrm{sec}\left(\frac{M(1 + z_L)}{M_\odot}\right)$$

Amplification



A1(u1), A2(u2)

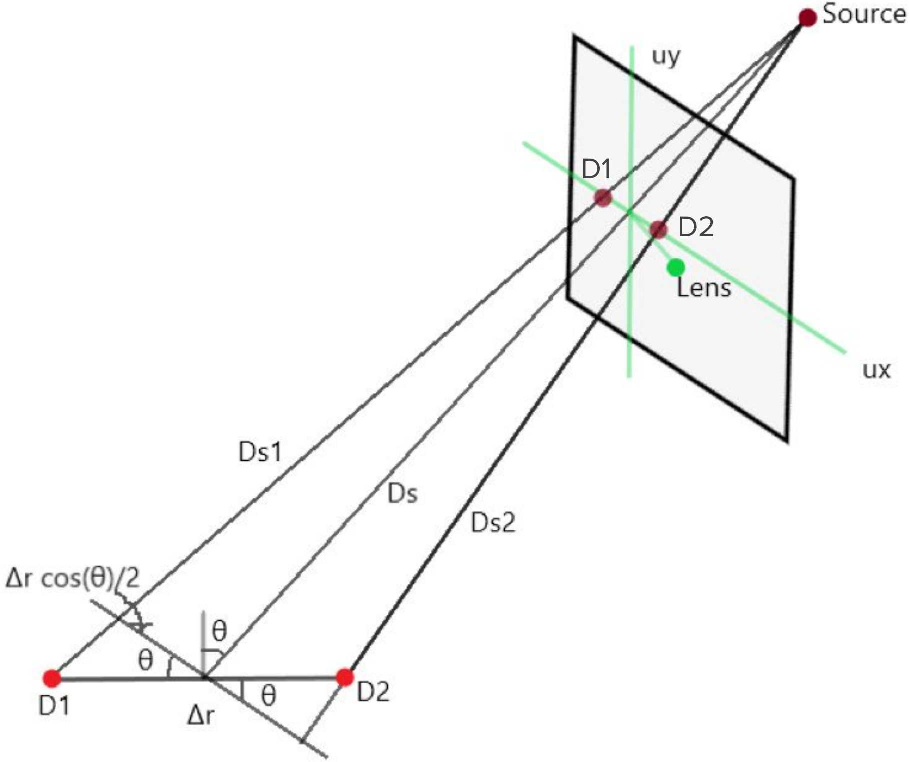
Finite Source Size Effect:

$$A(y,\delta) \approx \begin{cases} A_{in}(y,\delta) & \text{for } y<\delta, \\ A_{out}(y,\delta) & \text{for } y>\delta, \end{cases}$$

$$A_{in}(y,\delta) = \sqrt{1+\frac{4}{\delta^2}} - \frac{8}{\delta^3(\delta^2+4)^{3/2}}\frac{y^2}{2} - \frac{144(\delta^4+2\delta^2+2)}{\delta^5(\delta^2+4)^{7/2}}\frac{y^4}{24},$$

$$A_{out}(y,\delta) = \frac{2+y^2}{y\sqrt{y^2+4}} + \frac{8(y^2+1)}{y^3(y^2+4)^{5/2}}\frac{\delta^2}{2} + \frac{48(3y^6+6y^4+14y^2+12)}{y^5(y^2+4)^{9/2}}\frac{\delta^4}{24},$$

Angular Separation



$Y = u_x(i) + u_y(j)$

$D_{S2} = D_S - \Delta r \sin(\theta)/2$

$D_{L2} = D_L - \Delta r \sin(\theta)/2$

$D_{S1} = D_S + \Delta r \sin(\theta)/2$

$D_{L1} = D_L + \Delta r \sin(\theta)/2$

$\Delta r \cos(\theta)/2D_S = D1/ D_{LS}$

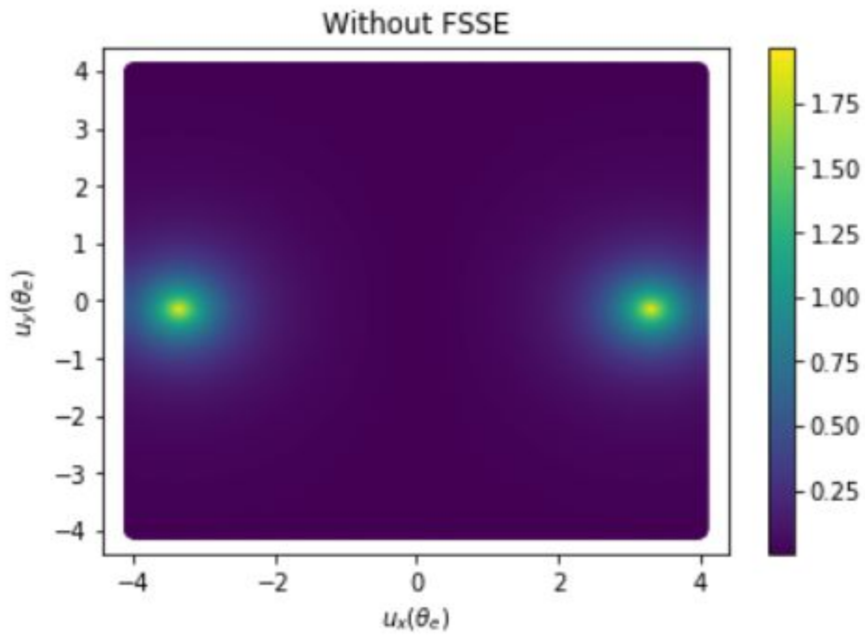
$\beta 1 = D1/D_L$

$Y1 = \beta 1/\theta_E$

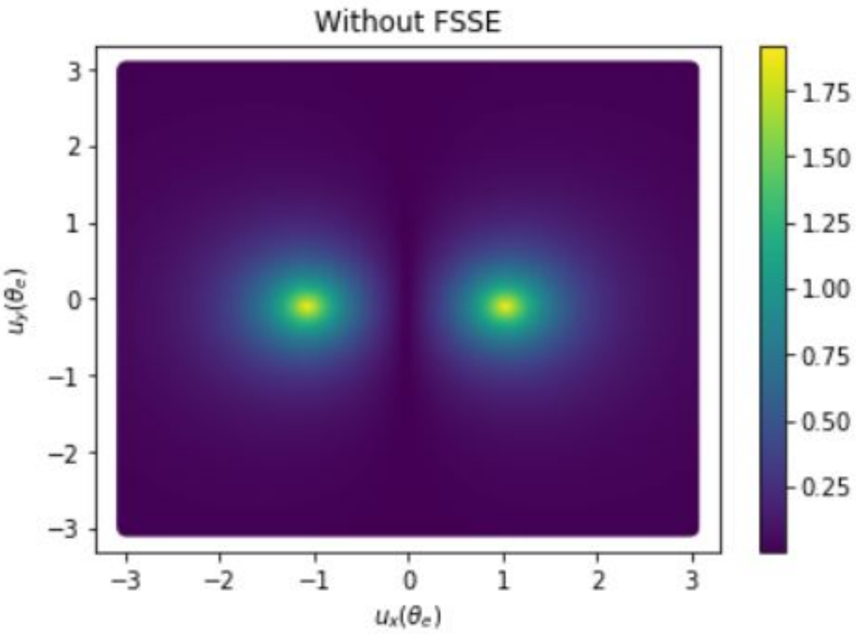
$Y1 = [u_x - \beta 1/\theta_E](i) + u_y(j)$

Amplitude plots as a function of Angular Separation

M = 1e-07 Distance of Source = 1 Epsilon = 0.1

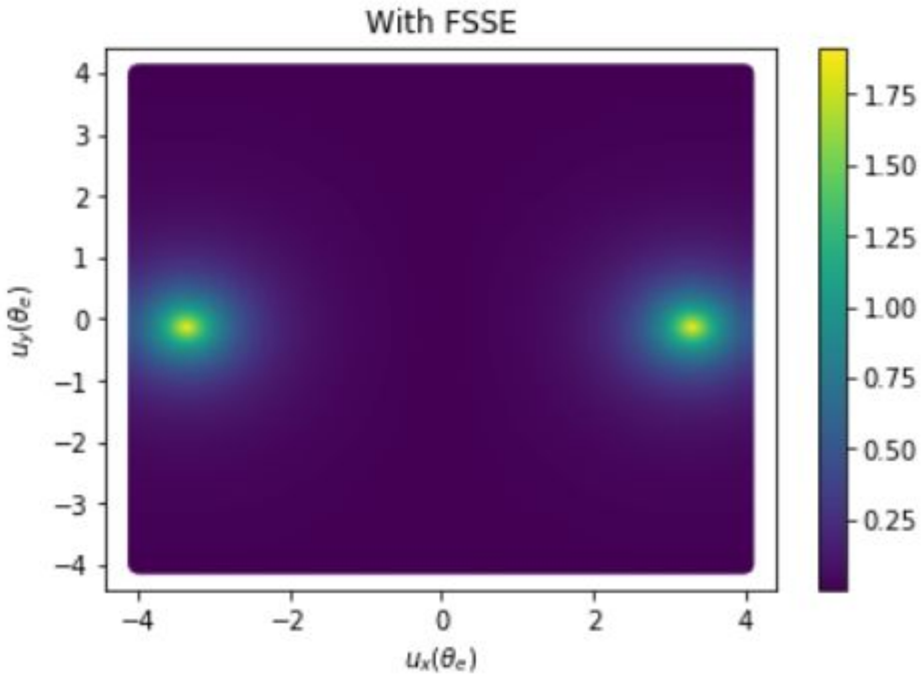


M = 1e-06 Distance of Source = 1 Epsilon = 0.1



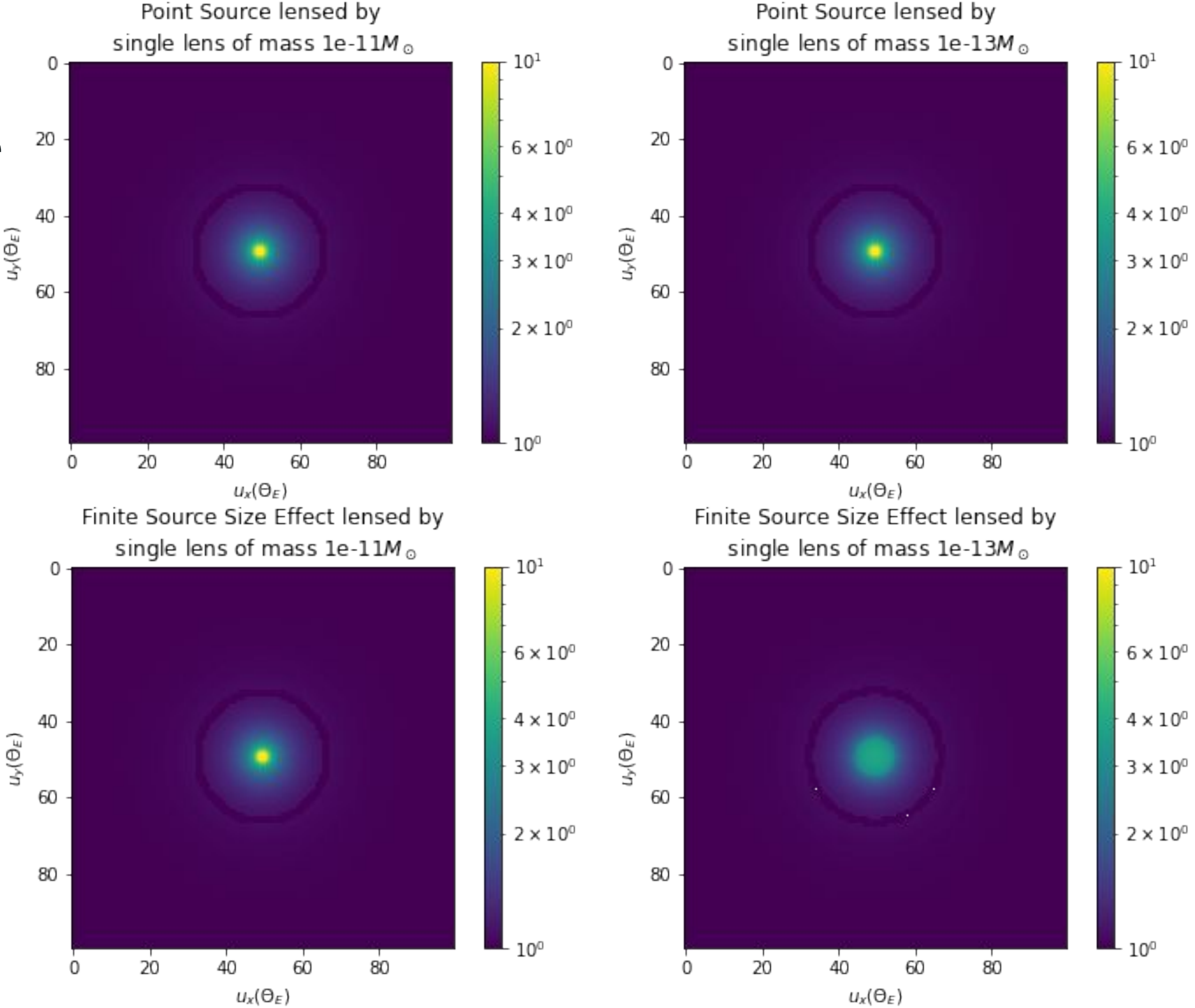
Finite Source Size Effect

M = 1e-07 Distance of Source = 1 Epsilon = 0.1



Point Source vs Finite Source Size Effect

Faint circle represents contour of detectable regions of observable

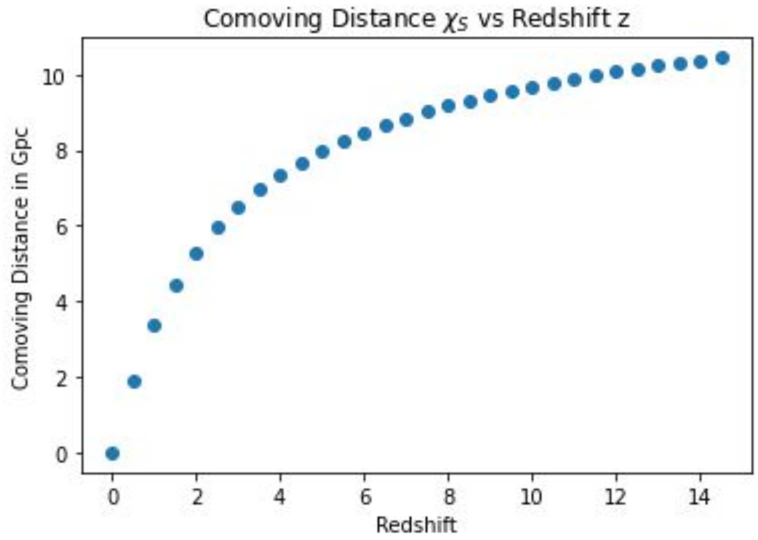


Deriving bounds on PBH composition in DM

Comoving distance

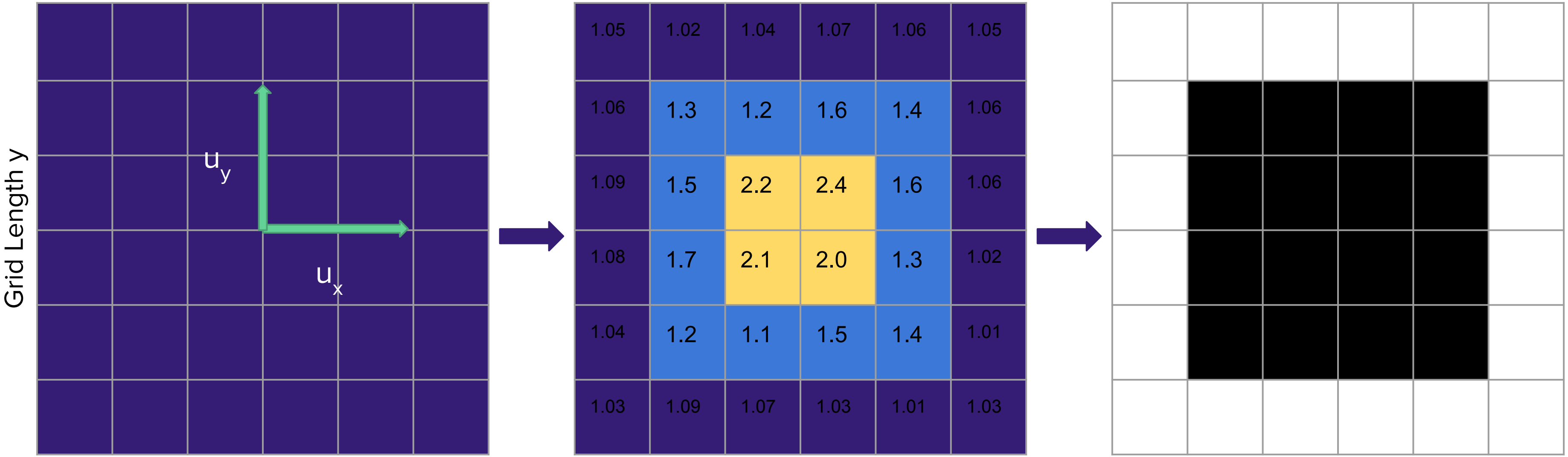
- Calculated using the FLRW metric
- Factors out the expansion of the universe, giving a distance that does not change in time due to the expansion of space
- Measures of distance, area, volume are calculated using this

- On the other hand, proper distance roughly corresponds to where a distant object would be at a specific moment of cosmological time



$$R_0 dr = \frac{c}{H(z)} dz$$
$$= \frac{c}{H_0} \left[(1-\Omega)(1+z)^2 + \Omega_v + \Omega_m(1+z)^3 + \Omega_r(1+z)^4\right]^{-1/2} dz.$$

Cross Section Area within FoV

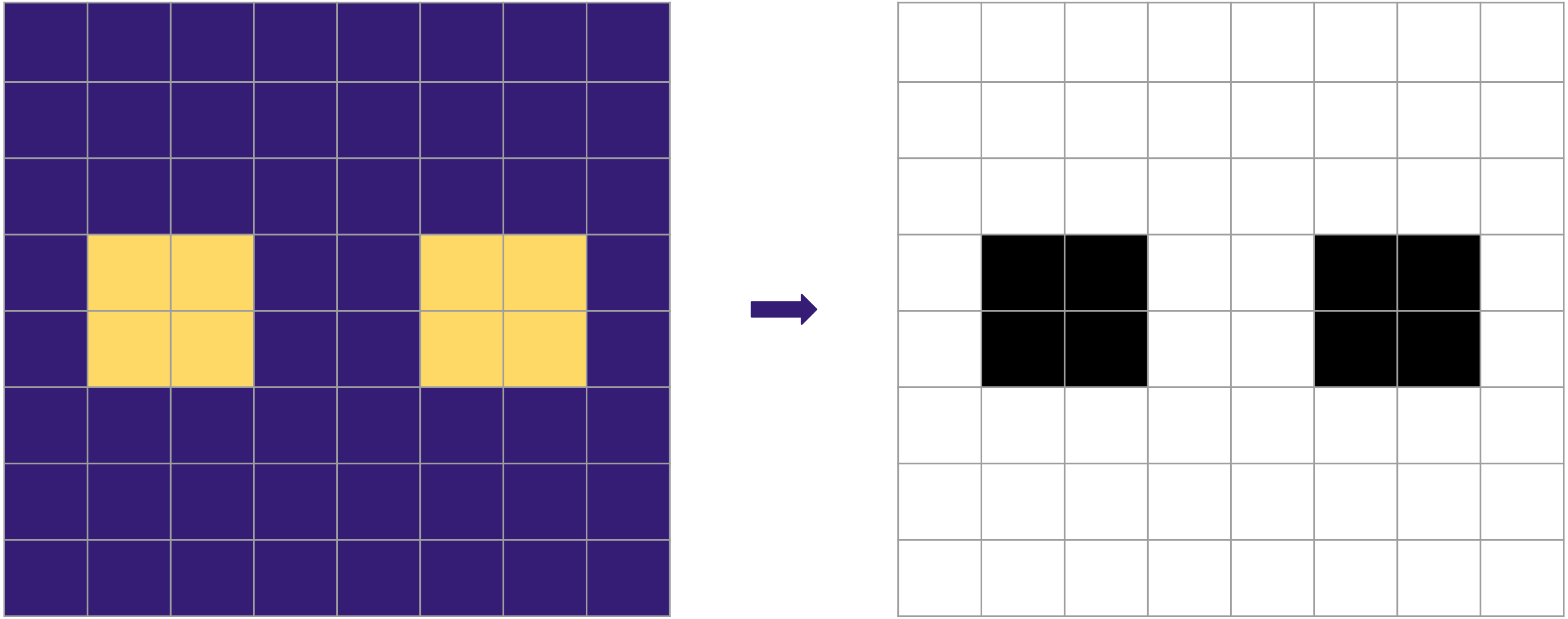


Grid Length x

The numbers represent the value of the observable

Count the number of cells exceeding value ϵ

Cross Section Area within FoV



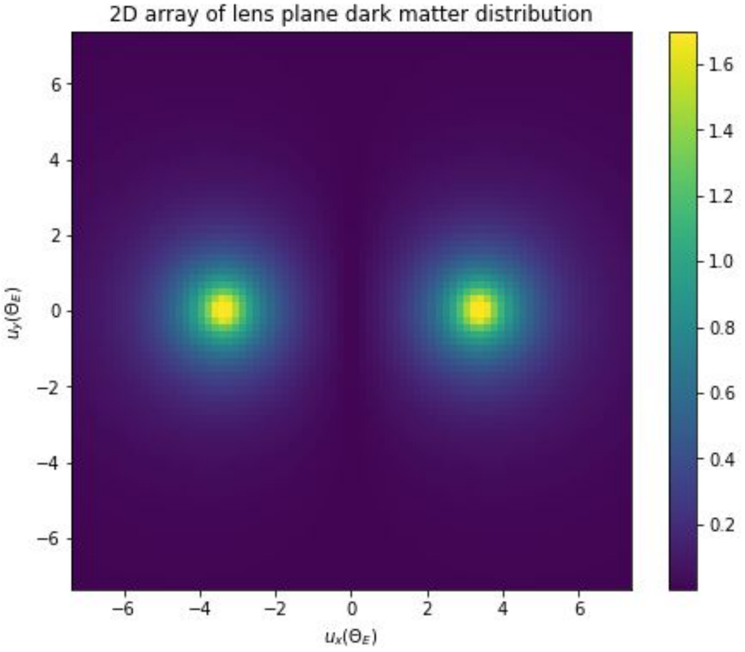
Simulated Result

Parameter values:

- ϵ = 0.01
- Grid length = $\Delta r/D + 4*\Theta_E$
- Number of cells = 80x80 = 6400

Area of ‘bright’ cells σ = #bright cells*area of one cell

```
count= [4988.]
```




```
Mass of lens = 1e-11
Maximum value of observable flux 1.6989357122225737
```

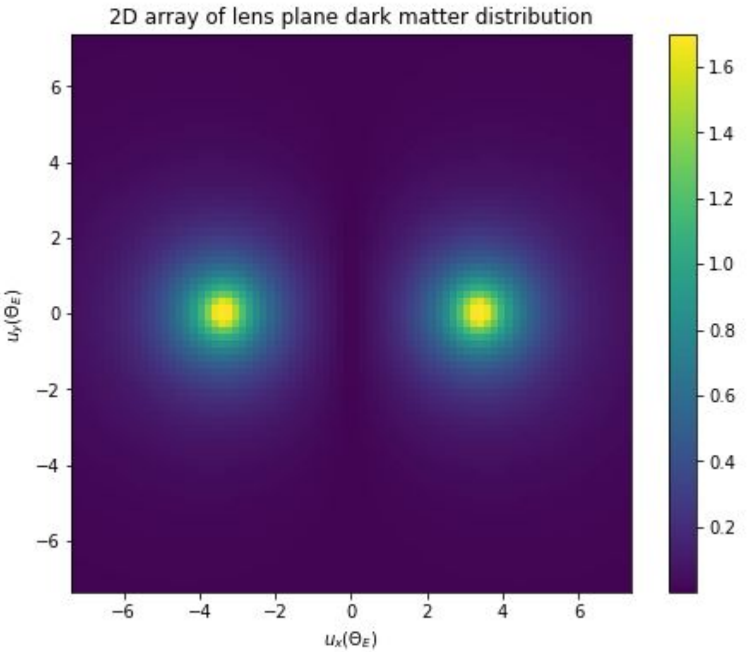
Simulated Result

Parameter values:

- ϵ = 0.1
- Grid length = $\Delta r/D + 4*\Theta_E$
- Number of cells = 80x80 = 6400

Area of ‘bright’ cells σ = #bright cells*area of one cell

```
count= [1776.]
```




```
Mass of lens = 1e-11
Maximum value of observable flux 1.6989357122225737
```

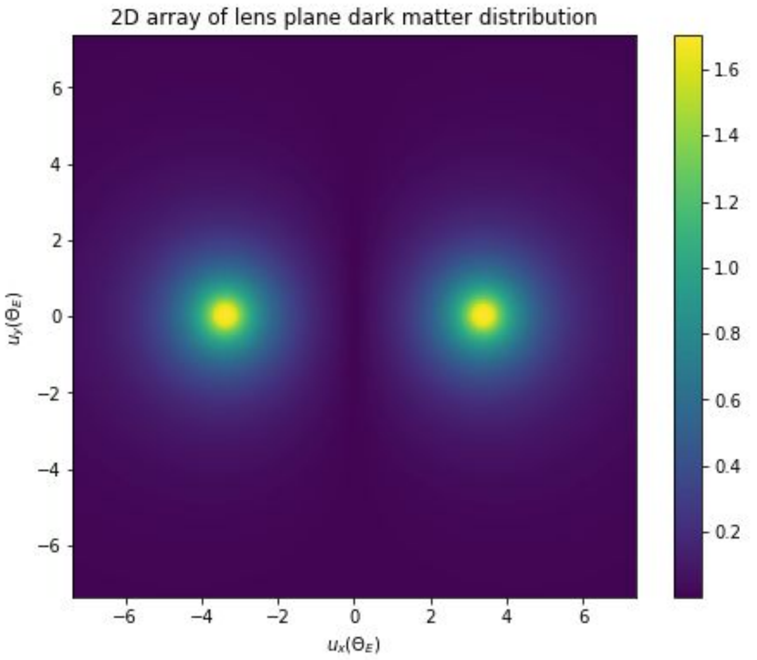
Simulated Result

Parameter values:

- ϵ = 0.1
- Grid length = $\Delta r/D + 4*\Theta_E$
- Number of cells = 200x200 = 40000

Area of ‘bright’ cells σ = #bright cells*area of one cell

```
count= [11136.]
```



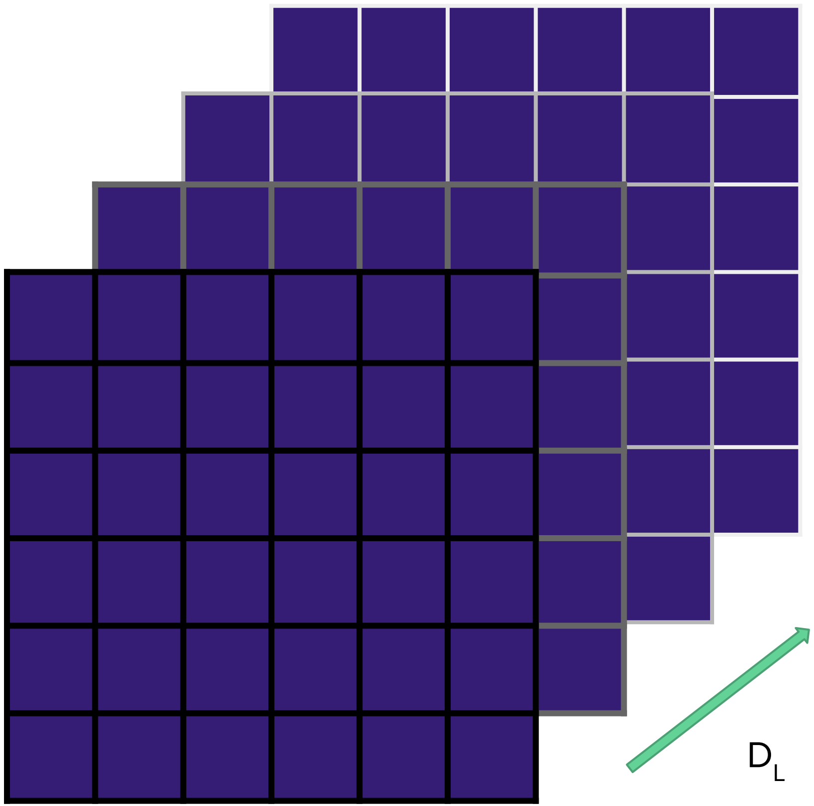

```
Mass of lens = 1e-11
Maximum value of observable flux 1.7046229858810007
```

Volume Occupied by DM

Stack multiple such cross sections at different distances D_L

Calculate 'bright' area in each cross section

Compute volume V_L

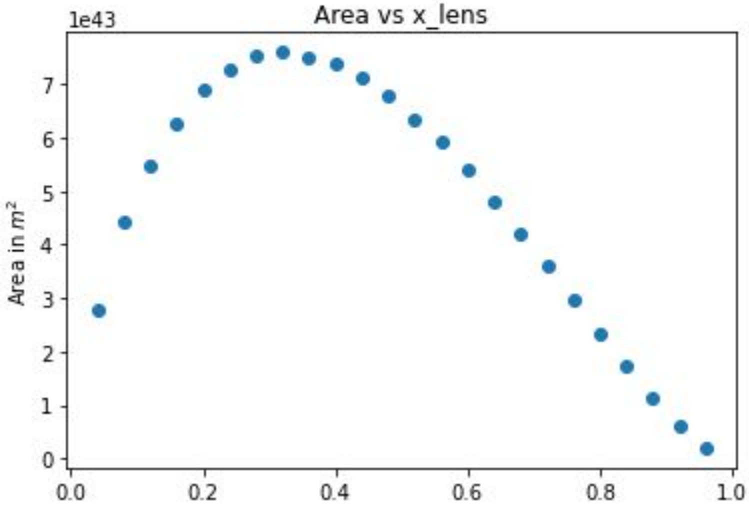


Area of cross section vs Distance from detector

Parameter values:

- Mass = 1e-11 M_{sun}
- ϵ = 0.1

Here, x = $D_L \cdot D_S / D_{LS}$



Optical Depth

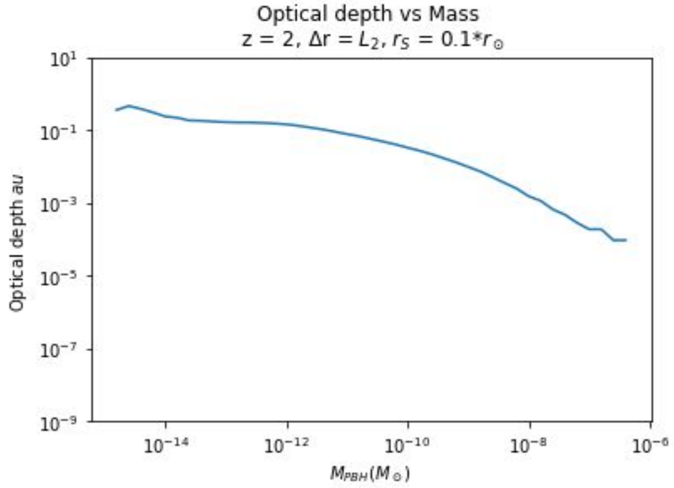
A measure of transparency of the medium

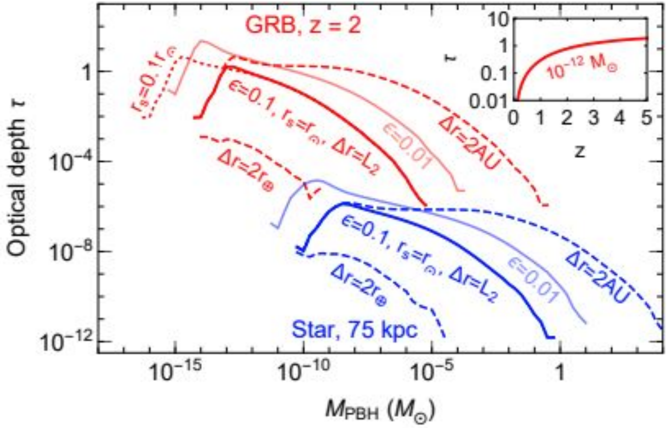
$$\tau = n \,.\, V_L$$

where comoving number density, $n = \rho_{crit,o}\Omega_{DM}f/M$ and PBH abundance, $f = \Omega_{PBH} / \Omega_{DM}$ *

Plot on top-right represents the solid red line on bottom-right

*Follows that $0 < f < 1$





Probability of Lensing

Follows as a Poisson distribution of optical depth

$$P(\alpha) = \frac{\tau^\alpha e^{-\tau}}{\alpha!}$$

where ɑ is the number of lensing events per detection

For maximising single lensing probability, i.e., $\alpha = 1 \rightarrow \tau = 1$

Probability of Null Detection

- Probability of more than 2 lensing events is expected to be negligibly small
- Probability depends on optical depth ➔ Depends on volume and PBH abundance ➔ $P \equiv P(\alpha, f)$
- Say $\tau = 1e\text{-}2$ $\Rightarrow$ $P(1, 1) = 0.0099$, $P(2, 1) = 4.95e\text{-}5$
- $\Rightarrow P(0, f) \cong (1 - P(1, f))$

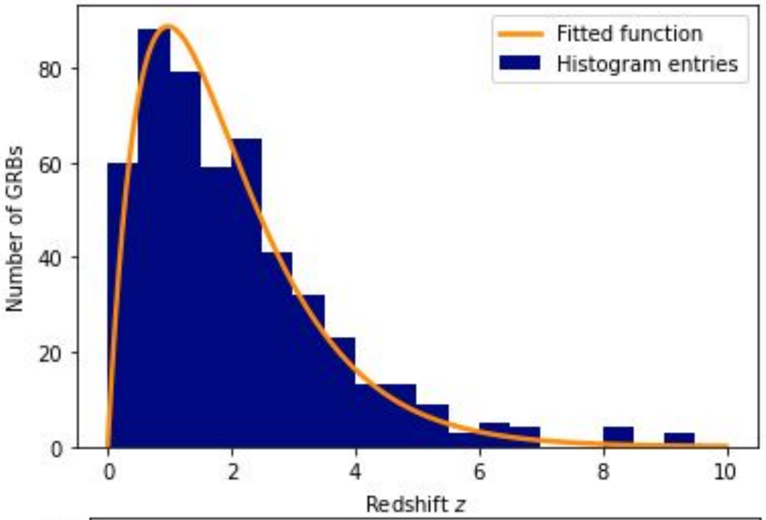
- Let $P_{Null}(f)$ represent the probability of no lensing event for multiple sources
- $\Rightarrow P_{Null}(f) = \prod P(0, f)$ where product is over multiple such observations of GRB progenitors
- Statistically, if $P_{Null} < 0.05$, no detections are reported

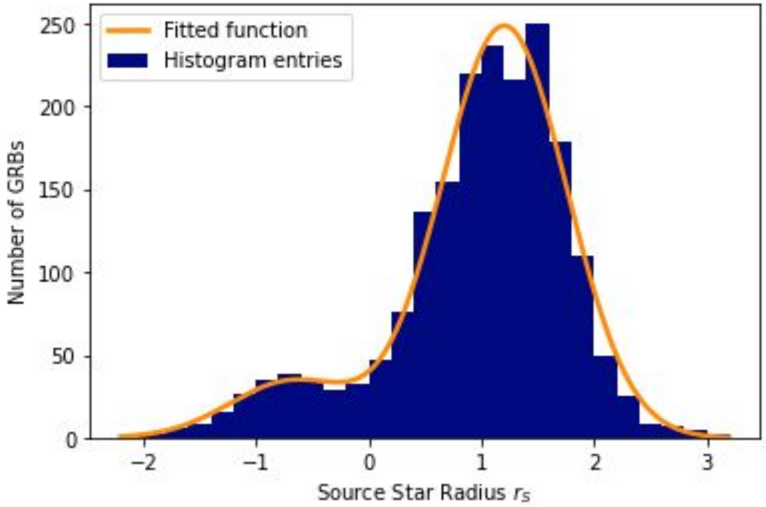
- Hence, bound f by finding $P_{Null}(f) \geq 0.05$
- Termed “criteria for exclusion”

Exclusion Curves

Incorporated various distributions such as size of sources and redshift (distance) of sources

Need to incorporate distribution of angle of plane of orbit vs GRB source





Exclusion Curves

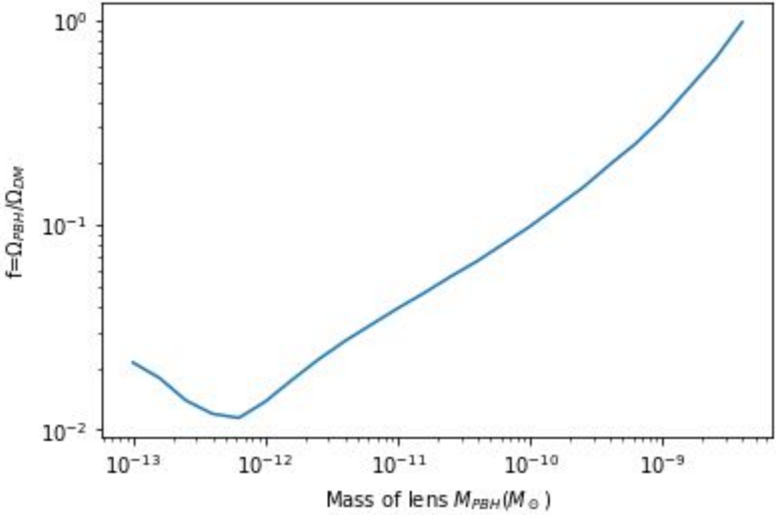
Incorporated various distributions such as size of sources and redshift (distance) of sources

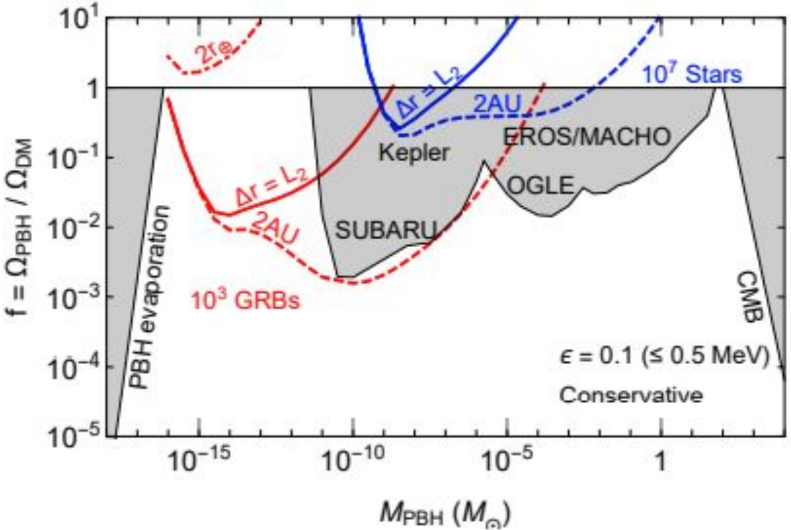
Need to incorporate distribution of angle of plane of orbit vs GRB source

Parameter values:

- ϵ = 0.1

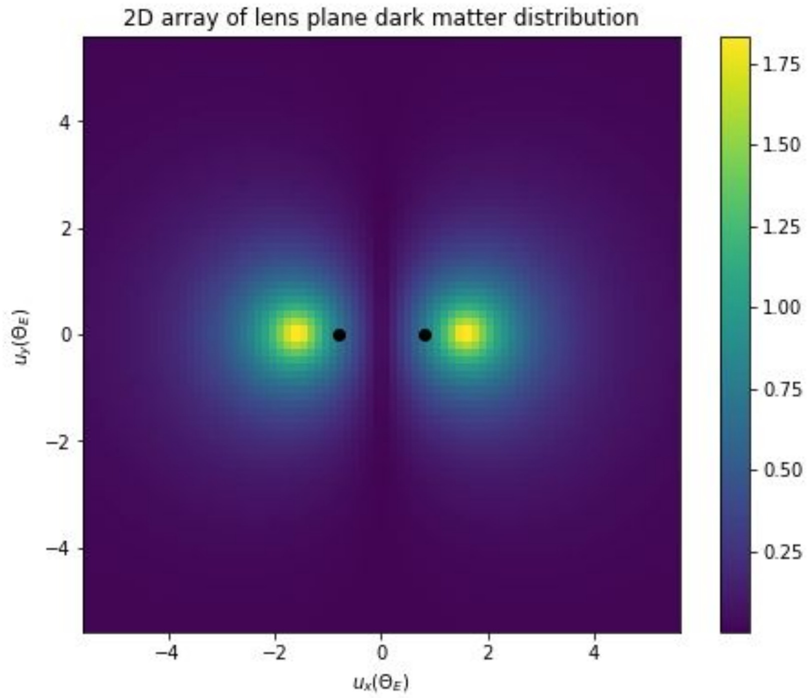
Plot on top-right represents solid red line on bottom-right





Future Work

- Fix a bug in the projection of detectors onto lens plane (Overall results may scale up or down)
- Black dots (detectors) are expected to overlap the brightest points



Future Work

- Fix a bug in volume calculation (Some mismatch in variable assignment)
- ‘Pythonify’ the code
- Obtain similar plots for other detector separations (r_{Earth} and AU)
- Investigate and address research gaps

Research Gaps

Research Gaps

- Incorporate distribution of tilt orbit vs GRB source
- Wave optic effects could be accounted for at lower masses of the lens

- Slightly outside the paper, investigate lensing events in dwarf galaxies apart from the uniformly distributed halo distributions