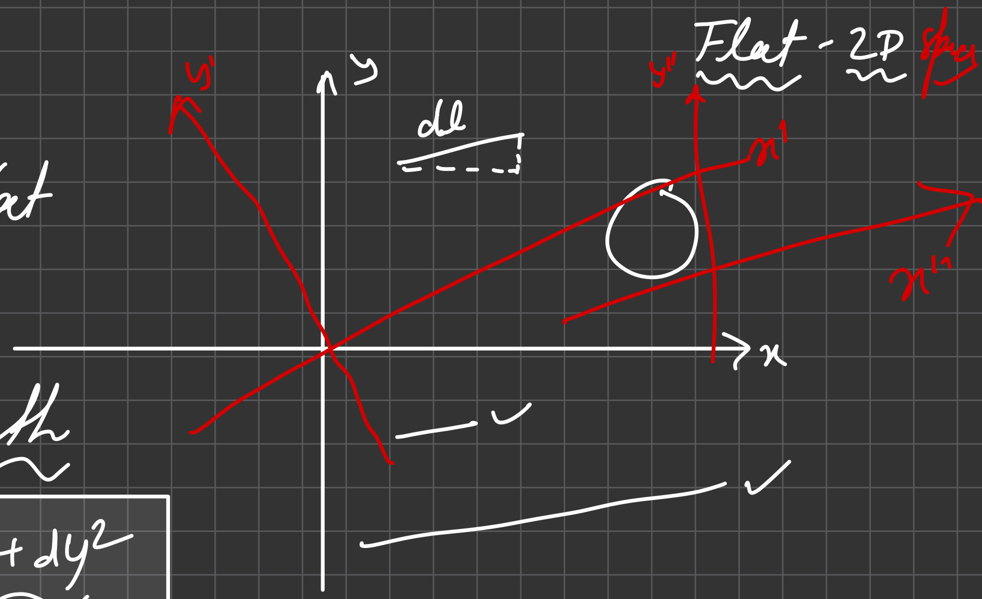
⇒ Metric :-

I) Geometry of this flat space



⇒ invariant length

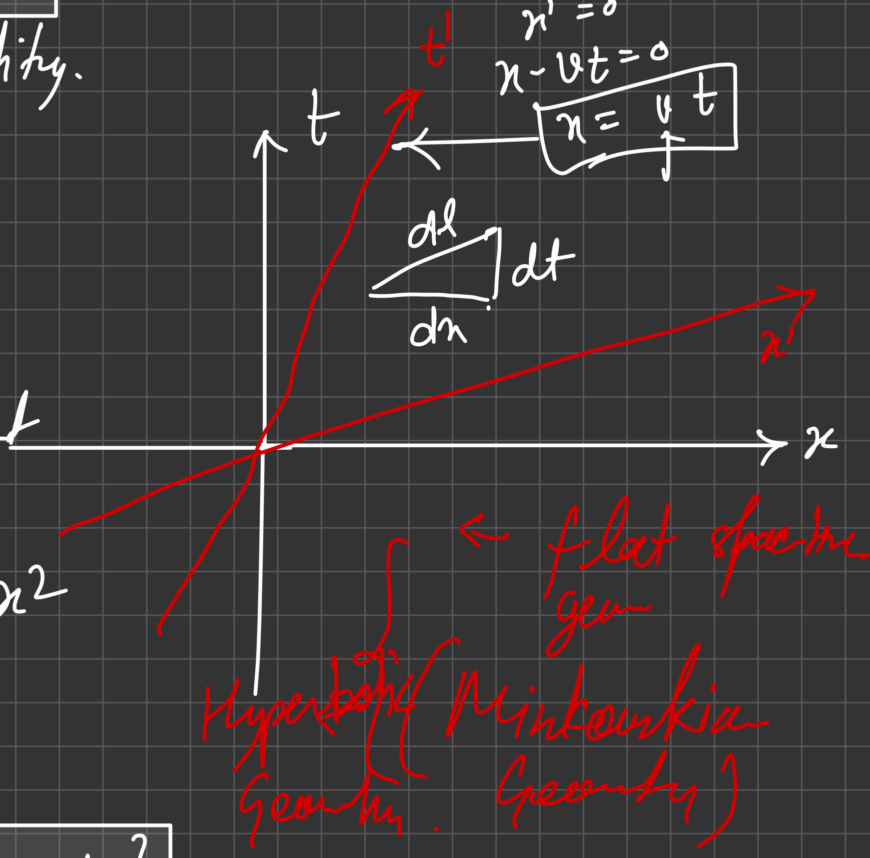
$$\boxed{dl^2 = \underbrace{dx^2 + dy^2}}$$

↳ invariant quantity.

$\boxed{c=1}$

II) $\underbrace{dt^2 + dx^2}$

not invariant



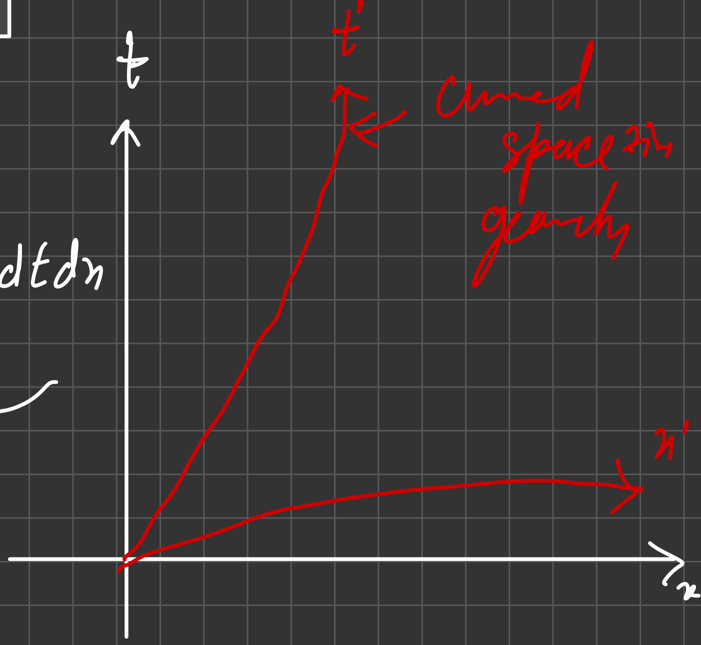
$$ds^2 = -dt^2 + dx^2$$

↳ proper length

or

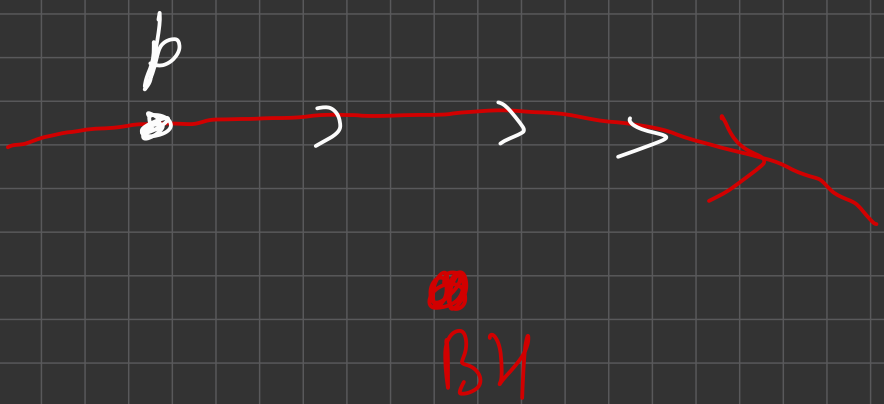
proper time $\boxed{d\tau^2 = -ds^2}$

$$ds^2 = \underbrace{g_{00}\,dt^2 + g_{11}\,dx^2 + g_{01}\,dt\,dx + g_{10}\,dt\,dx}$$



Equivalence principal

Locally (acceleration $\equiv$ Gravity)



Gravity $\Longrightarrow$ Curves the spacetime

$$ds^2 = g_{00}\,dt^2 + g_{11}\,dx^2 + g_{01}\,dt\,dx + g_{10}\,dx\,dt$$

$$\boxed{ds^2 = g_{\mu\nu}\,dx^\mu dx^\nu}$$

invariant length.

unknown

$\mu, \nu = (0, 1, 2, 3)$

$g_{\mu\nu} \leftarrow$ metric } it defines the space-time geometry around any massive object.

Q how to find out $g_{\mu\nu}$ for the given mass / Energy distribution.

A Einstein's Field Eqns.

$$R_{\mu\nu} \equiv f(g_{\mu\nu}, \partial g_{\mu\nu})$$

$$R = R_{\mu\nu} R^{\mu\nu}$$

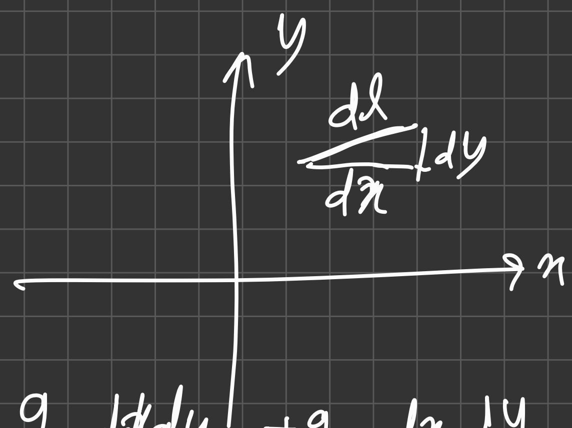
$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \# \, T_{\mu\nu}}$$

local Eq^n

Space-time tells matter how to move and matter tells space-time how to curve

Various forms of metric :-

(I) 2D flat space :-



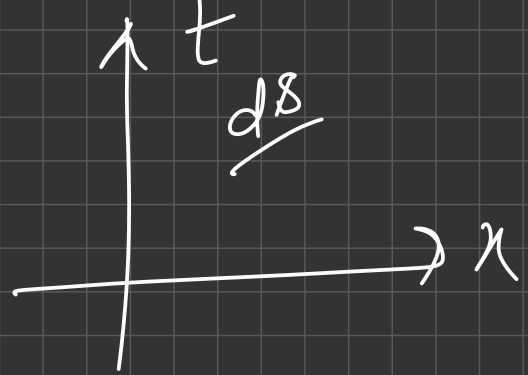
$$dl^2 = dx^2 + dy^2$$

$$\Updownarrow$$

$$= g_{11}\, dx^2 + g_{22}\, dy^2 + g_{12}\, dx\, dy + g_{21}\, dx\, dy$$

$$g = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(II) (1+1) D space-time (Minkowskian metric)



$$ds^2 = -dt^2 + dx^2$$

$$g_{1+1} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \leftarrow$$

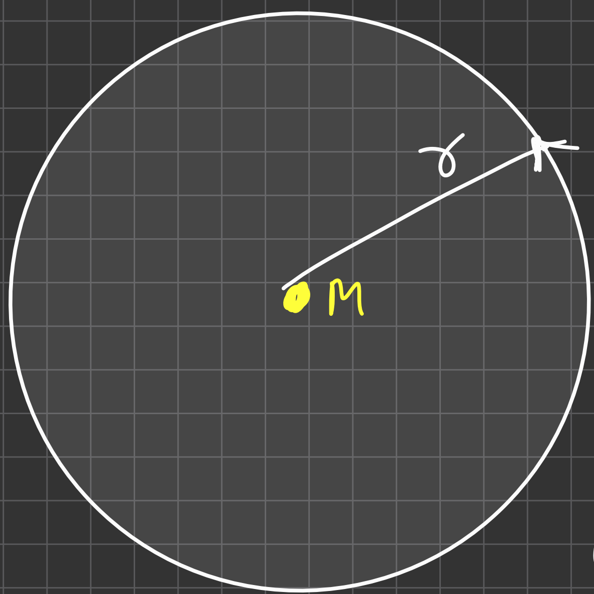
$$g = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{3+1} \equiv \eta \leftarrow \text{minkowskian metric}$$

$$\boxed{ds^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu}$$

III Schwarzschild Metric :-

$$g_{\mu\nu}(\vec{x}, t)$$

$$\boxed{G_{\mu\nu} \approx T_{\mu\nu}}$$



I

$g_{\mu\nu}(r) \rightarrow$ should be spherical symm.

II for $r \rightarrow \infty$

$$g_{\mu\nu} \longrightarrow \eta_{\mu\nu}$$

↓

weak field approximation.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

perturbation.

$$g_{\mu\nu} = \begin{pmatrix} -1-\frac{2\Phi}{c^2} & 0 & 0 & 0 \\ 0 & 1-\frac{2\Phi}{c^2} & 0 & 0 \\ 0 & 0 & 1-\frac{2\Phi}{c^2} & 0 \\ 0 & 0 & 0 & 1-\frac{2\Phi}{c^2} \end{pmatrix}$$

$\Phi \leftarrow$ Gravitational potential

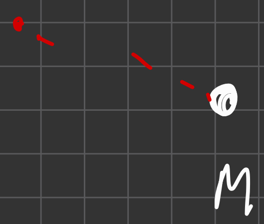
For point like object, $\Phi = -\dfrac{GM}{r}$

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu \quad , \quad g_{\mu\nu} = \begin{pmatrix} -1-\frac{2\Phi}{c^2} & 0 & 0 & 0 \\ 0 & 1-\frac{2\Phi}{c^2} & 0 & 0 \\ 0 & 0 & 1-\frac{2\Phi}{c^2} & 0 \\ 0 & 0 & 0 & 1-\frac{2\Phi}{c^2} \end{pmatrix}$$

$$\underset{\sim}{ds^2} = \left(1+\frac{2\Phi}{c^2}\right)c^2dt^2 - \left(1-\frac{2\Phi}{c^2}\right)(d\vec{x})^2$$

$c = \underset{\sim}{3\times10^8\, m/s}$

For light, $\underset{\sim}{ds^2 = 0}$



$$v = \frac{|d\vec{x}|}{dt}$$

$\downarrow$

$$n(\Phi) = \frac{c}{v(\Phi)}$$

$\uparrow$ r.i

$$\boxed{L = \int_{\text{path}} \underset{\sim}{n\, dl}}$$

$\boxed{\delta L = 0}$ for extrem.

$$S = \int \mathcal{L} \, d\lambda$$

$\mathcal{L} \leftarrow$ Lagrangian

$\downarrow$

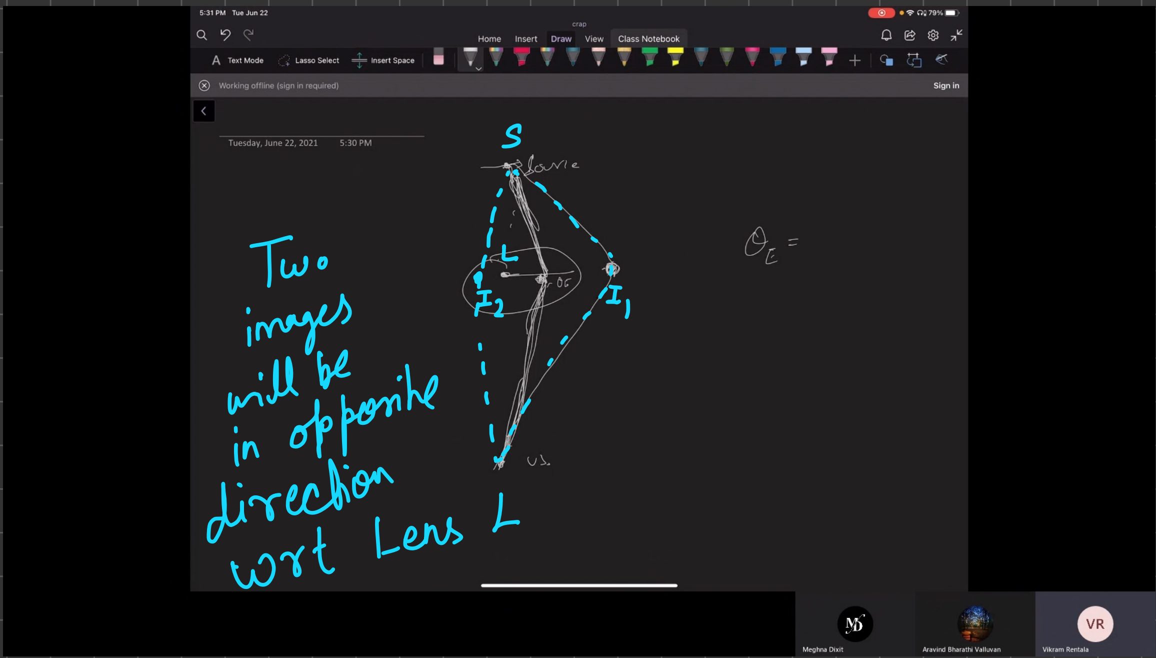
Euler Lagrange Eq^n

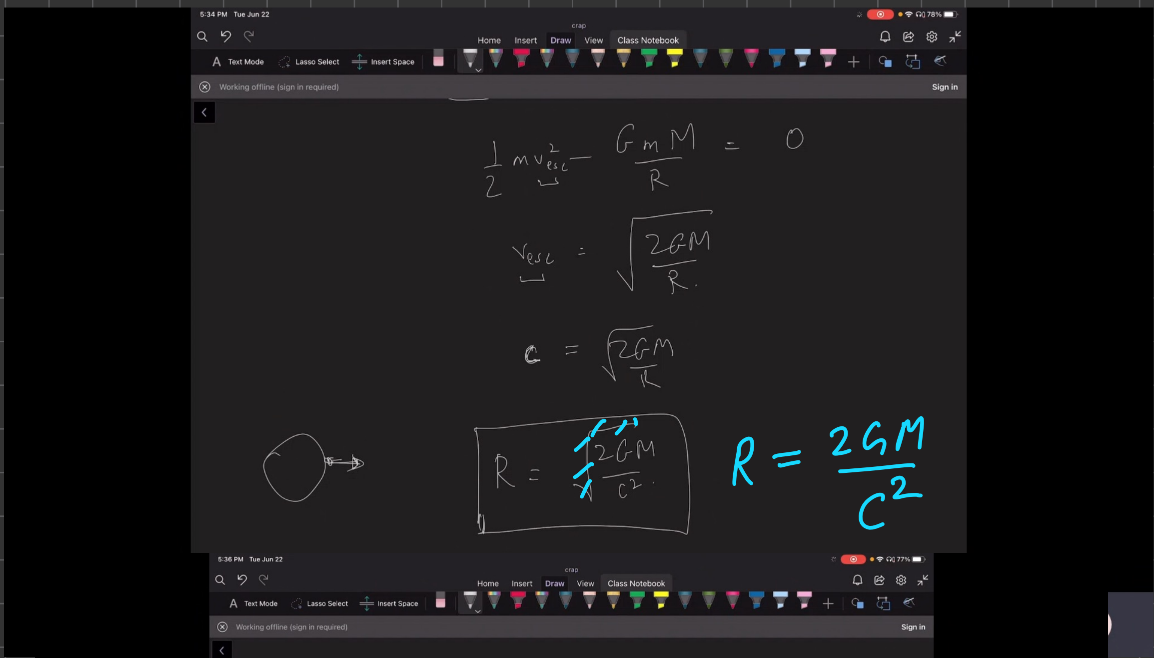
$\downarrow$

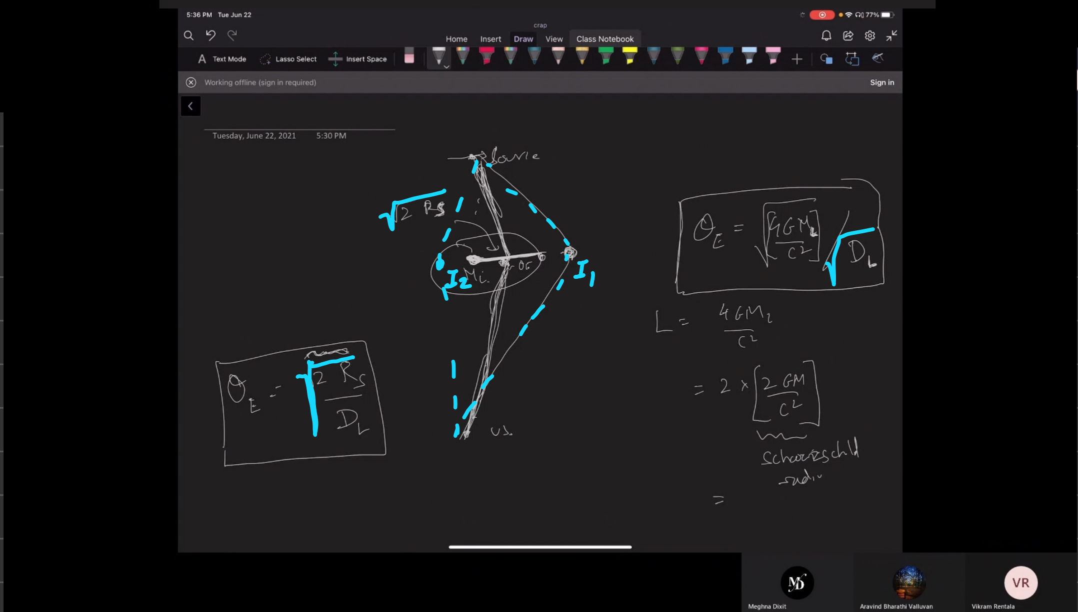
$$\boxed{\hat{\alpha} = \frac{2}{c} \int \nabla_\perp \Phi \, d\lambda}$$

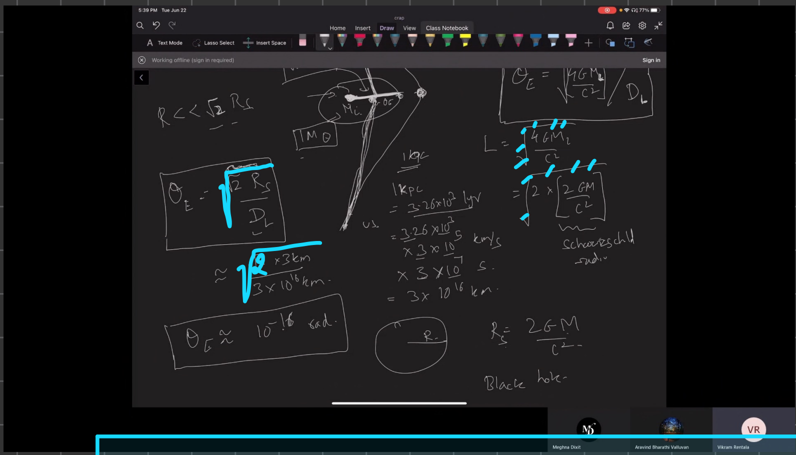
$\rightarrow$ Shapiro delay $\boxed{d\tau = \frac{dl}{c}}$

⇒ Last GRB lensing paper discussion Comments.

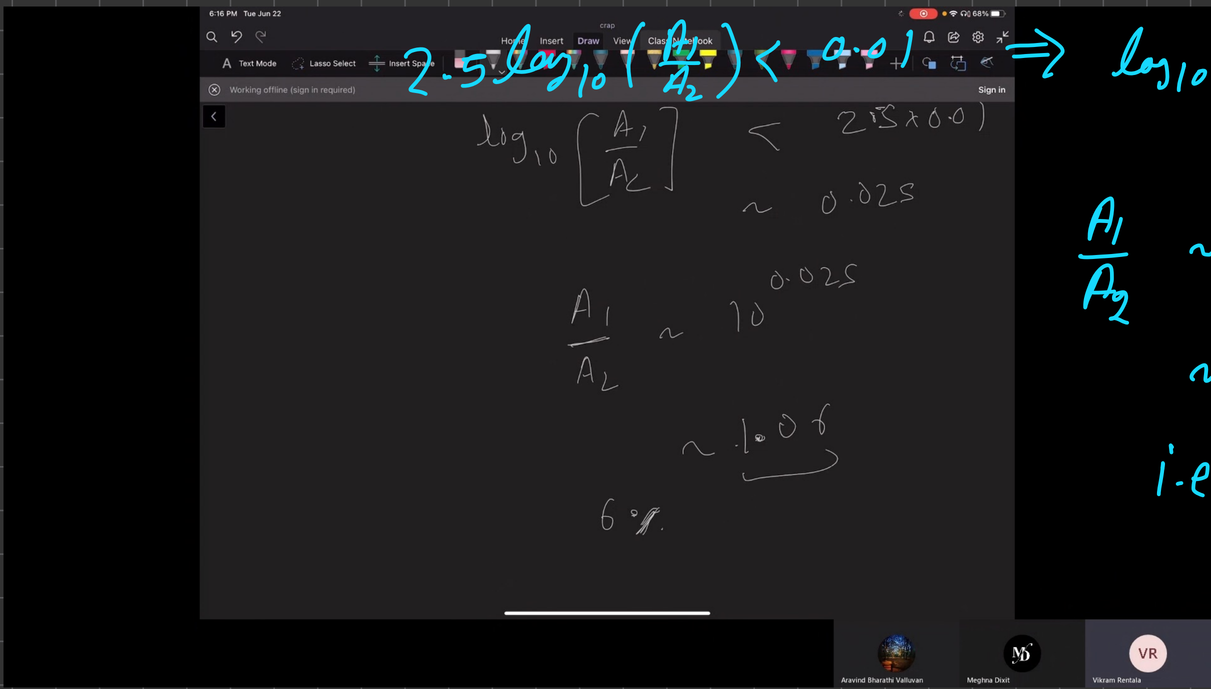








$$\theta_E = \sqrt{\frac{2\, R_s}{D_L}} = \sqrt{\frac{2 \times 3\ \text{km}}{3 \times 10^{16}\ \text{km}}} \approx 10^{-8}\ \text{rad}$$



$$2.5 \log_{10}\left(\frac{A_1}{A_2}\right) < 0.01 \Rightarrow \log_{10}\left(\frac{A_1}{A_2}\right) < \frac{0.01}{2.5}$$

$$\frac{A_1}{A_2} \sim 10^{0.004}$$

$$\sim 1.009$$

i.e. 0.9 %

Issue of resolution :-

We don't have to resolve the two images of stars/GRB rather as

$$A(u) = \frac{u^2+2}{u\sqrt{u^2+4}}$$

$$u = \frac{\theta}{\theta_E}$$

Hence, A changes from two positions bcoz u changes.

$$\mu(u_+) = \frac{1}{2}\left[1 \pm \frac{u^2+2}{u\sqrt{u^2+4}}\right]$$

$$u = \frac{\theta}{\theta_E}$$

$$A(u) = |\mu(u_+)| + \mu(u_-)|$$

θ_E

θ_E

$\underbrace{M \sim 10^{-10} \sim 10^{3}}$

$$\theta_1 \rightarrow \theta_2$$

$$u_1 \rightarrow u_2$$

$$A_1(u_1) \rightarrow A_2(u_2)$$

$FA_2 = F_2$ $\qquad$ $F_1 = FA_1$

$$\boxed{O \equiv \frac{A_1 - A_2}{\left|\frac{A_1 + A_2}{2}\right|} > \epsilon}$$

$\leftarrow$ detection efficiency.

$$R_s = \frac{2GM}{c^2}$$

$$\theta_E = \sqrt{\frac{2R_s}{D_s}\left(\frac{1}{x} - 1\right)}$$

$$x = \frac{D_L}{D_S}$$

$$O(\Delta r, \theta_E)$$

$$\downarrow$$

$$O(\Delta r, M, D_L, D_S)$$

$\Rightarrow$ point lens,

true position of star

$$u = \frac{\beta}{\theta_E} \qquad \tilde{u} = \frac{\theta}{\theta_E}$$

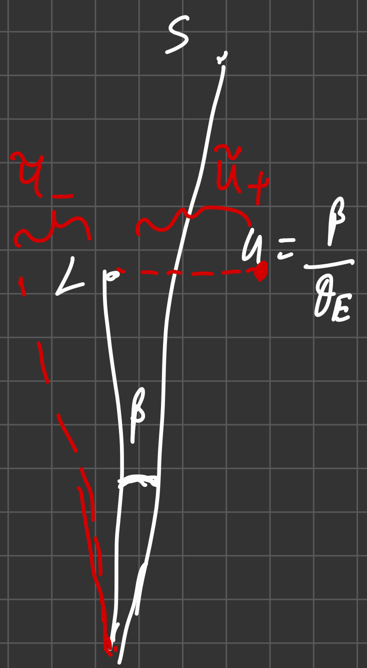
image position

$$\beta = \theta - \frac{\theta_E^2}{\theta}$$

$$\beta\theta = \theta^2 - \theta_E^2$$

$$\theta^2 - \beta\theta - \theta_E^2 = 0$$

$$\tilde{u}^2 - u\tilde{u} - 1 = 0$$



$$\boxed{\underbrace{\tilde{u}_\pm}_{} = \frac{u \pm \sqrt{u^2+4}}{2}}$$

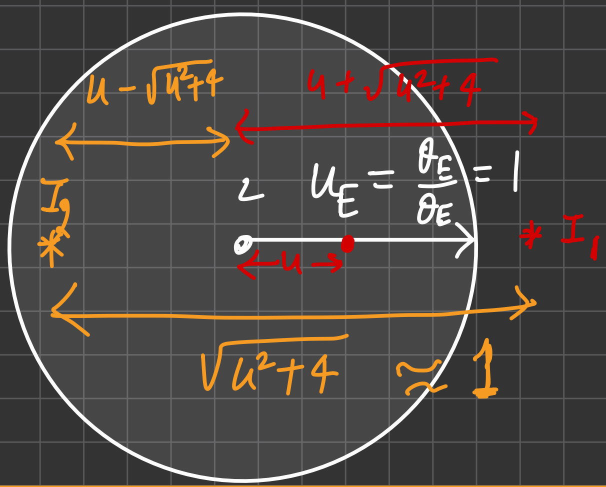
$$\mu(u) = \frac{1}{2}\left[1 \pm \frac{u^2+2}{u\sqrt{u^2+4}}\right]$$

$$\sqrt{u^2+4} > u$$

θ_E

$$u_{sep} = \tilde{u}_+ - \tilde{u}_- = \sqrt{u^2+4}$$

$$= \sqrt{\left(\frac{\beta}{\theta_E}\right)^2 + 4}$$



$$\boxed{\theta_E \approx 2.854\ \text{mas}\ \sqrt{\frac{M}{1M_\odot}\ \frac{1\,\text{kpc}}{D_s}\left(\frac{1}{x} - 1\right)}}$$

$$A(u) = |\mu_1| + |\mu_2| = \frac{u^2+2}{u\sqrt{u^2+4}}$$

apparent magnitude,

$$\underbrace{m_G}_{\sim} = -2.5 \log_{10}\left(\frac{F}{F_0}\right)$$

$\rightarrow \sigma(m_G)$

$$\sigma = \frac{A_1 - A_2}{\left(\frac{A_1+A_2}{2}\right)} > \epsilon(m_G)$$

$F_1 = F A_1$

$F_2 = F A_2$

$.S$

i

$$m_1 = -2.5 \log_{10}\left(\frac{F_1}{F_0}\right)$$

$$m_2 = -2.5 \log_{10}\left(\frac{F_2}{F_0}\right)$$

$$(m_1 - m_2) = -2.5 \log_{10}\left(\frac{F_1}{F_2}\right)$$

F_2 2 1 F_1

$$(m_1 - m_2) = -2.5 \log_{10}\left(\frac{A_1}{A_2}\right)$$

detection criteria

$$\boxed{\Delta m > \sigma}$$

$$-2.5 \log_{10} \frac{A_1}{A_2} > \sigma$$

$$\frac{A_1}{A_2} - 1 > 10^{-\frac{\sigma}{2.5}} - 1$$

$$\frac{A_1 - A_2}{A_2} > \left(10^{-\frac{2\sigma}{5}} - 1\right)$$

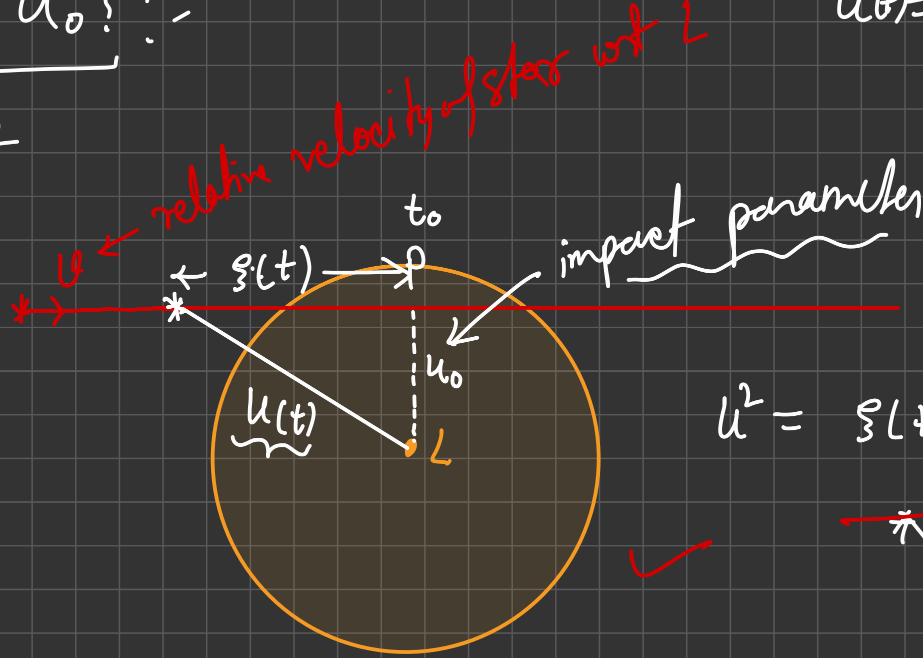
$\underbrace{\sigma(m_G)}_{\sim}$

$$\rightarrow \boxed{(A_1 - A_2) > \left(10^{-2\sigma/5} - 1\right) A_2}$$

what is u_0? :-

$$u(t) = \frac{\beta(t)}{\theta_E}$$

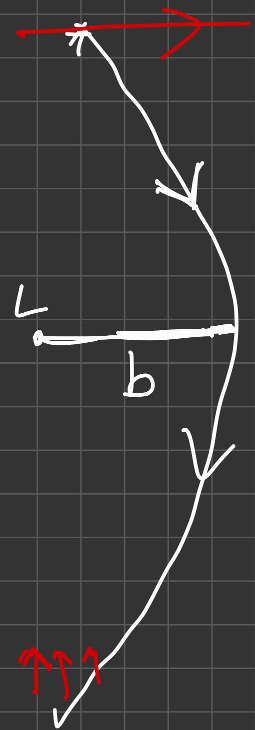
$$D_{SL} = D_S - D_L$$



$$\mu = \frac{v}{D_{SL}}$$

$$u^2 = \xi(t)^2 + u_0^2$$

$$u_0 \equiv u(t_0)$$



$$\hat{\alpha} = \frac{4GM}{c^2 b}$$

optical depth :-

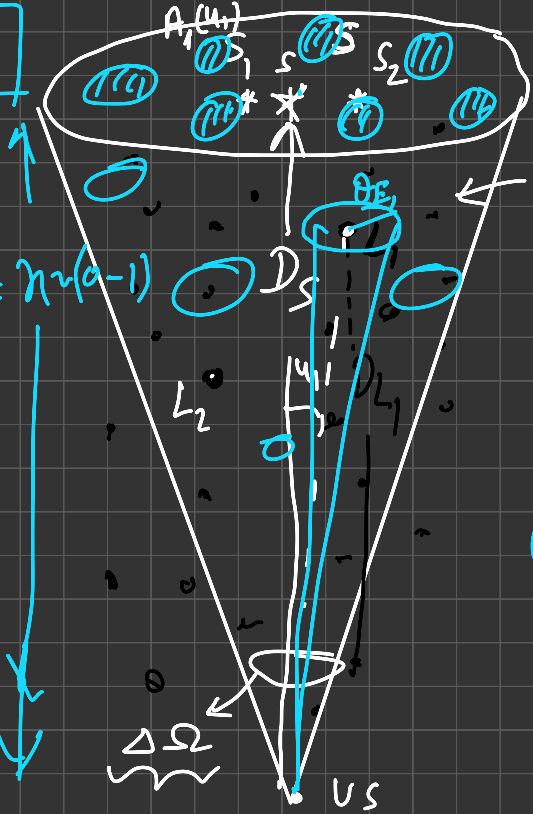
"for the given stars (m_q) what is the probability that it will get lensed and lensing would be detected."

II $\Delta A = |A_1 - 1| > \epsilon \leftarrow \boxed{\theta_1 = \theta_E K}$

OD for PBH

I u_1

$\frac{D_L}{D_S} = x \sim (a-1)$



$$\frac{f\rho_{DM}}{M_{PBH}} = \frac{\rho_{PBH}}{M_{PBH}} = n_{PBH}$$

if star lies inside θ_E then → detected

$$\theta_E = \sqrt{\frac{2R_s}{D_s}\left(\frac{1}{x}-1\right)}$$

$$\tau = \frac{\Delta A}{A} = \frac{\sum \pi \theta_E^2 K^2}{\Delta\Omega}$$

$$= \frac{\int dn \, \pi \theta_E^2 K^2}{\Delta\Omega}$$

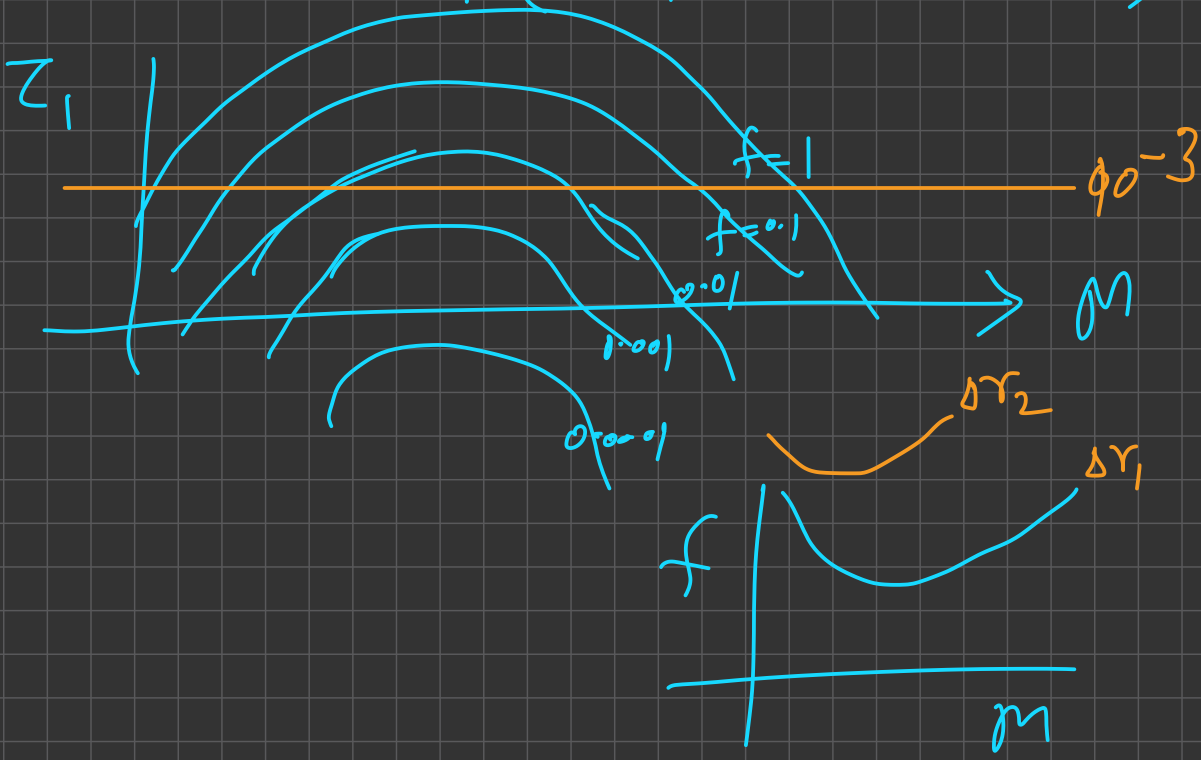
$$dn = \frac{f\rho_{DM}}{M_{BH}} D_S^3 \, dD_S \, \Delta\Omega$$

$$\boxed{\tau = \int \frac{f\rho_{DM}}{M_{BH}} D_S^3 \, \pi \theta_E^2 K \, dD_S}$$

$\underline{\underline{f=1}}$ $O_g(\Delta r_2 \ldots)$ $O(\Delta r_1, M_1, \text{—} \text{—})$

$\downarrow$ K_1 $\downarrow$

τ_2 $\tau_1(\Delta r_1, M_1, \text{—})$



Jul 5, 2021 $0 < x < 1$ $x = D_L/D_S$

lens plane

$$R_E = D_l \theta_E = D_L \sqrt{\frac{4GM}{c^2}\left(\frac{1}{D_L} - \frac{1}{D_S}\right)}$$

$$= \sqrt{2 R_S D_S x^2 \left(\frac{1}{x} - 1\right)}$$

$$R_E(M, D_S, x) = \sqrt{2 R_S D_S x(1-x)}$$

$$R_E(R_S, D_S, x)$$

$$R_S = \frac{2GM}{c^2}$$

$$= \underbrace{\frac{2G(1M_\odot)}{c^2}}\frac{M}{1M_\odot}$$

$$\boxed{R_S \approx \underbrace{3\,\text{km}}_{\text{preR}_S}\left(\frac{M}{1M_\odot}\right)}$$

$\text{preR}_S = 3$ km

AM = np.logspace(-6, 6, num = 100) ← should be $M_\odot$

$AR_S = R_S(AM)$ ← would be km

$$R_E = \sqrt{2 \times 3\,\text{km} \times \frac{R_S}{3\,\text{km}} \times 1\,\text{kpc}\,\frac{D_S}{1\,\text{kpc}}\,x(1-x)}$$

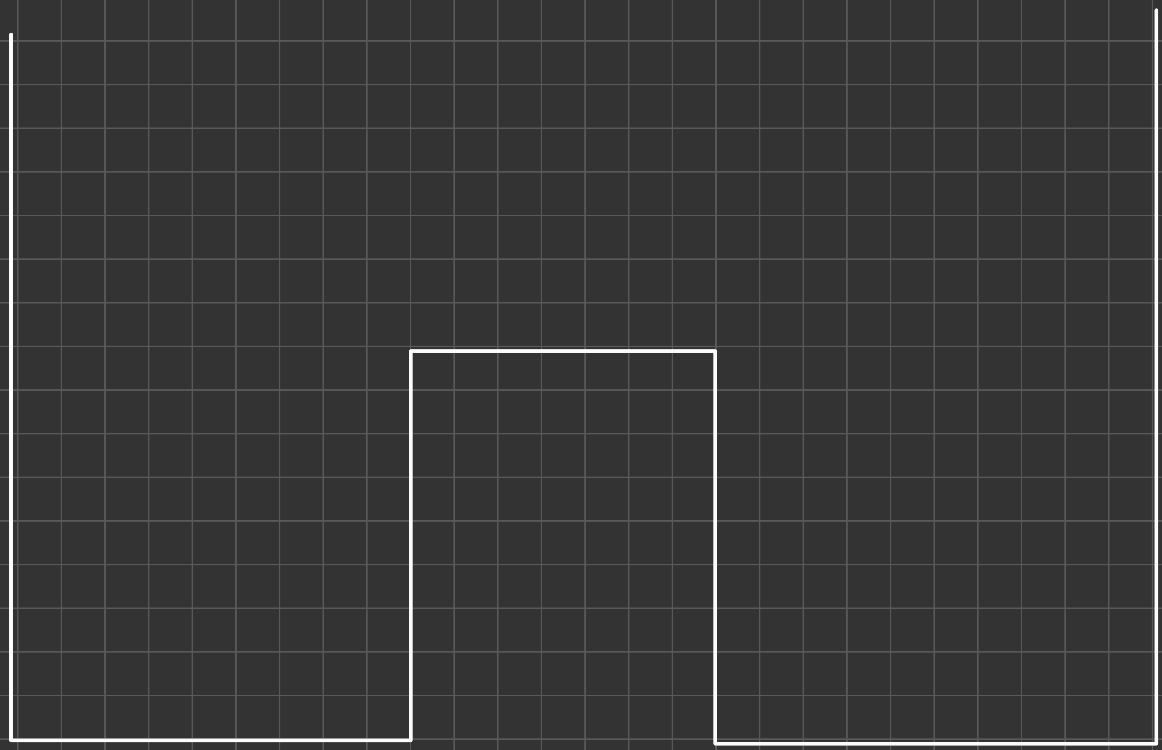
$$= \underbrace{\sqrt{2 \times 3\,\text{km} \times 1\,\text{kpc}}}\sqrt{\frac{R_S}{3\,\text{km}}\frac{D_S}{1\,\text{kpc}}\,x(1-x)}$$

$$= \underset{\text{kpc}}{\text{pre}R_E}\;\underbrace{\sqrt{\frac{R_S}{3\,\text{km}}\frac{D_S}{1\,\text{kpc}}\,x(1-x)}}$$

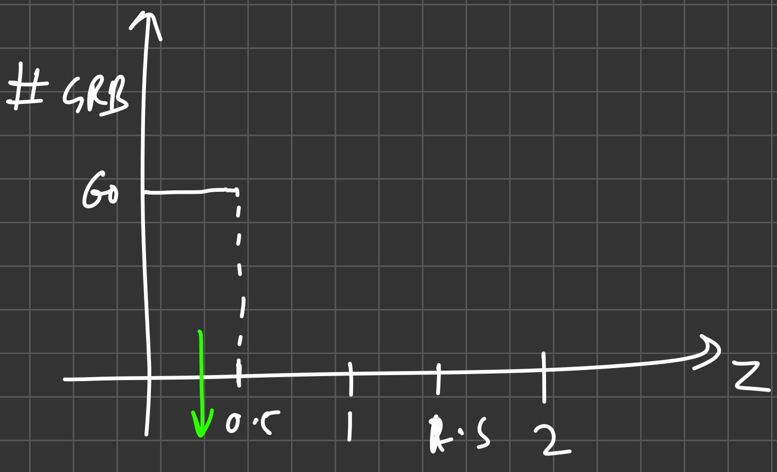
R_S = np.linspace(1, 10, num = 100) #3 km

D_S = -- (-- --) #kpc

$R_E(R_S, D_S, x) \longrightarrow$ kpc



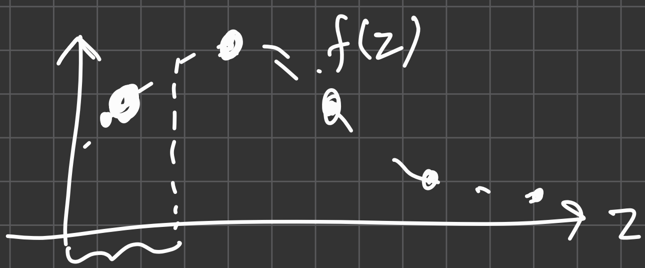
Jul 12, 2021



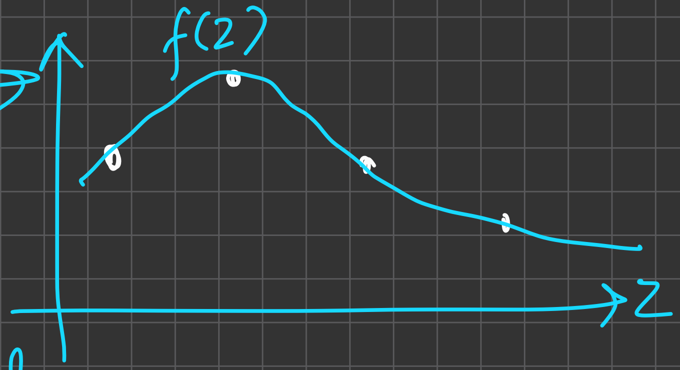
$$\int_0^{0.5} f(z)\,dz = 60$$

$$f(z)(0.5) = 60$$

$$\underbrace{f(z)} = \frac{60}{0.5}$$



I dist of GRB



I) 1000

rand

$f(z)$ → 1000 random value

II)

III)

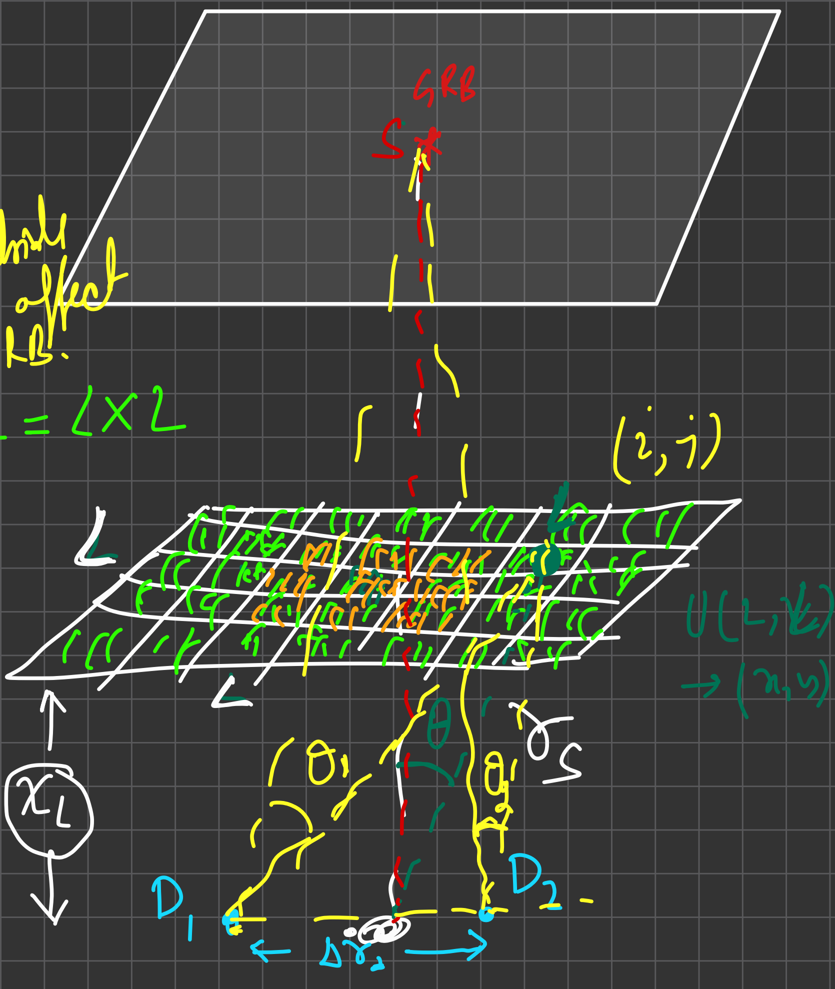
for each γ_s

$\sigma(x_L, \Delta\gamma_\perp, \gamma_s, \epsilon)$

a) $A_1 > 1+\epsilon$ or $A_2 > 1+\epsilon$ $\Rightarrow \delta\Omega = L \times L$

$\delta\Omega$ any lens outside won't affect GRB.

b) $\delta = \dfrac{A_1 - A_2}{\frac{A_1 + A_2}{2}} > \epsilon$



$$\boxed{\sigma = 9 \times \left(\frac{\mathcal{L}}{\theta_0}\right)^2}$$

$\theta(i,j)$

$\theta_1(\theta) \qquad \theta_2(\theta)$

$$u_1 = \frac{\theta_1(i,j)}{\theta_E} \qquad u_2 = \frac{\theta_2(i,j)}{\theta_E}$$

$$\boxed{A_1 = \frac{u_1^2+2}{u_1\sqrt{u_1^2+4}}} > 1+\epsilon \qquad A_2 = - - -$$

$$\rightarrow \quad u_1^T(\epsilon)\,\theta_E = \theta^T$$

$$\Delta\Omega_2$$

$$\frac{\mathcal{L}}{2} = \theta^T x_L$$

$$\Delta\Omega_1 = \mathcal{L}^2 = \left(2\theta^T x_L\right)^2$$

$$= \left(2u_1^T\,\theta_E(M, x_L, x_S)\,x_L\right)^2$$

$$\boxed{\Delta\Omega = \max(\Delta\Omega_1, \Delta\Omega_2)}$$

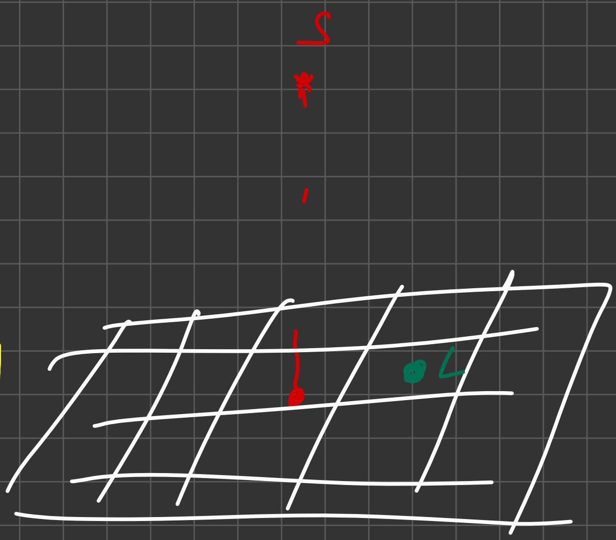
$(M, r_s, x_s, \delta r_\perp, \epsilon)$

$\Downarrow$

$\delta\Omega = \max(\delta\Omega_1, \delta\Omega_2)$

$\Downarrow$

divid $\delta\Omega = 10 \times 10 = N \times N$



$\Downarrow$

in each square put a Lens at at Distance x_L

$\Downarrow$

θ

$\downarrow$

$\theta_1 \, , \, \theta_2$

$\downarrow$

$A_1 \, , \, A_2$

$\downarrow$

$\delta > \epsilon$

$\downarrow$

n_s

$$\rho_{PB} = f \rho_{DM}$$

$$n_{PBM} = \frac{f}{M} \rho_{DM}$$

$$\boxed{\sigma(r_s, x_L, \delta r_\perp, M, \epsilon) = n_s \left(\frac{L}{N}\right)^2}$$

r_s : fro dist of GRB

$x = \{0, 1\} \leftarrow x_L$: Uniformly distributed $(0.01, 100)$ Kpc

$x = \frac{x_L}{x_S}$, $x_L = (0,1)$

Δr_1 : ✓ $\qquad \underline{\Delta r}$

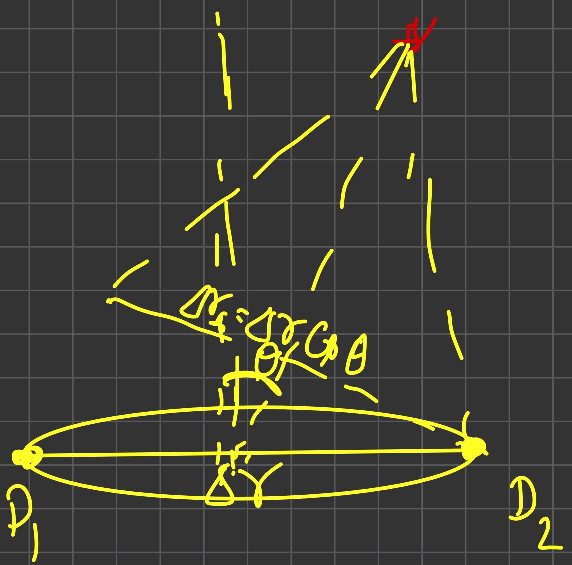
$M : (10^{-4}, 10^{-3}) M_\odot$

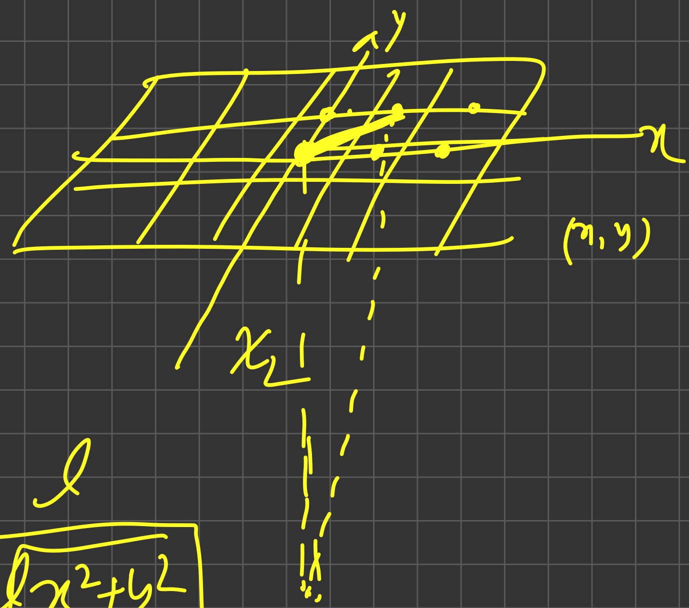
ϵ : ✓

x_S	x_L	M	σ

$$\underbrace{\hspace{6em}}$$

x_S	M	v_L





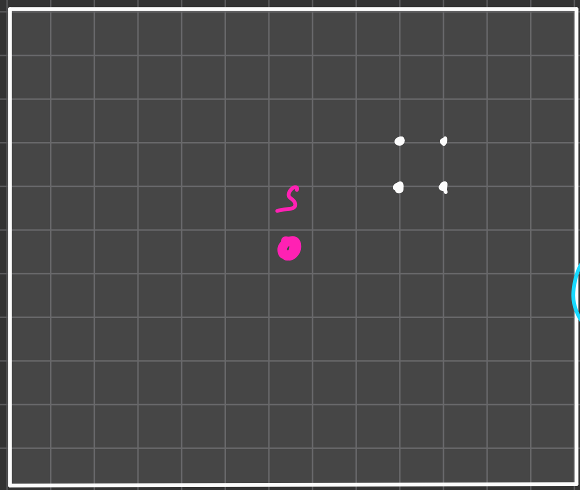
$$l = \sqrt{x^2+y^2}$$

$$x_2 \theta = l$$

$$\boxed{\theta = \frac{\sqrt{x^2+y^2}}{x_2}} \longrightarrow \begin{array}{l} \theta_1(\theta) \\ \theta_2(\theta) \end{array}$$

f, M

D_s



$$\boxed{O = \frac{A_1 - A_2}{A} > \epsilon}$$

0

1 (sensitive area?)

I Image → plot $A(x,y)$

scatter

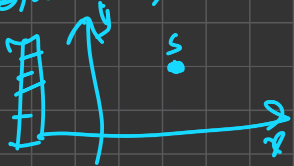
$A = 80 \times 80$

$A[i][j] = \begin{cases} 0 \\ 1 \end{cases}$

$\Delta r_\perp, r_s, \epsilon$

(f, M)

II Im contour of $O(x,y)$



$\sigma = \underbrace{\mathrm{sum}(A == 1)} \times \Delta A$

D_s	σ

$\Longrightarrow$ $\boxed{\tau = n(f, M)\sigma}$

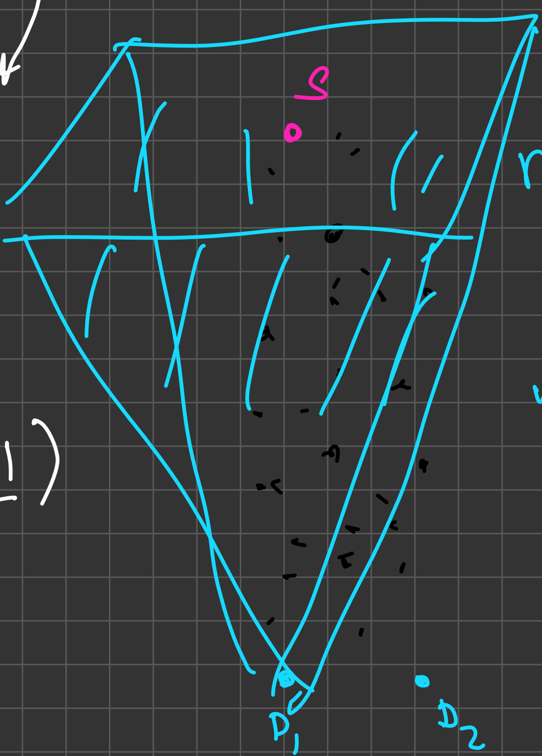
$\theta_E = \sqrt{\frac{2R_s}{D_s}\left(\frac{1}{x} - 1\right)}$

D_s

$x = \frac{D_L}{D_S}$

$x = (0, 1)$

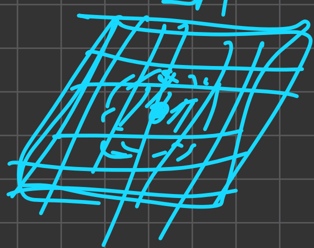
$x =$



$n = \frac{\rho_0}{M}$

$N_l = n \Delta V$

ΔA



$\sigma = A$

$p = \frac{\sigma}{\Delta A}$

$p N_l = \Sigma$

$$V_L = \int_0^{x_S} \sigma \, d\cancel{x}_L \qquad x = \frac{x_L}{\cancel{D}_S}$$

$$dx_L = x_S \, dx$$

$$V = x_S \int_0^1 \sigma \, dx$$

$\theta_E(m, D_S, x)$

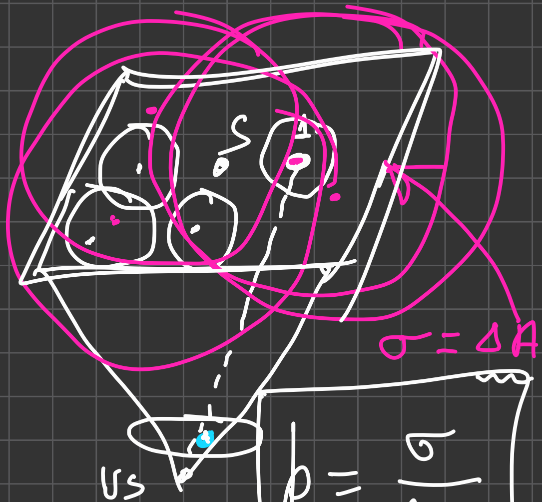
$$\sigma(x_L; \Delta r_\perp, r_S, \epsilon)$$

$$\sigma(x, \Delta r_\perp, r_S, \epsilon)$$

f, M

D_S

x	σ
0.0001	
0.1	
0.2	
⋮	
0.9	



$$\boxed{\rho_{det} = \frac{\sigma}{\Delta A}}$$

$$\Delta\Omega = \frac{\Delta A}{D_S^2}$$

$$\rho_{PBH} = f \rho_{DM}$$

$$n_{PBH} = \frac{\rho_{PBH}}{M} = \frac{f \rho_{DM}}{M}$$

$$N_{PBH} = \underbrace{n_{PBH} \Delta\Omega \, D_l^2 \, dD_l}$$

$$= \frac{1}{3} D_l^3 \Delta\Omega \, n_{PBH} \Big|_0^{D_S}$$

$$\boxed{N_{PBH} = \frac{1}{3} D_S^3 \, x^3 \Delta\Omega \, n_{PBH} \Big|_0}$$

f, M, D_S

$x \to \theta_E$	$\frac{\sigma}{\Delta A}$
0.0001	
⋮	
0.9991	

$$\frac{\Delta A}{\Delta A} + () + \cdots$$

$$= \frac{1}{3} D_s^3 \, \Delta \Omega \, n_{PBN}$$

$$\sigma = \cancel{N}_{PBN} \frac{\Delta A}{\Delta A} = \frac{1}{3} D_s^3 \frac{\cancel{\Delta A} \, n_{PBN}}{D_s^2} \quad \frac{\Delta A}{\cancel{\Delta A}}$$

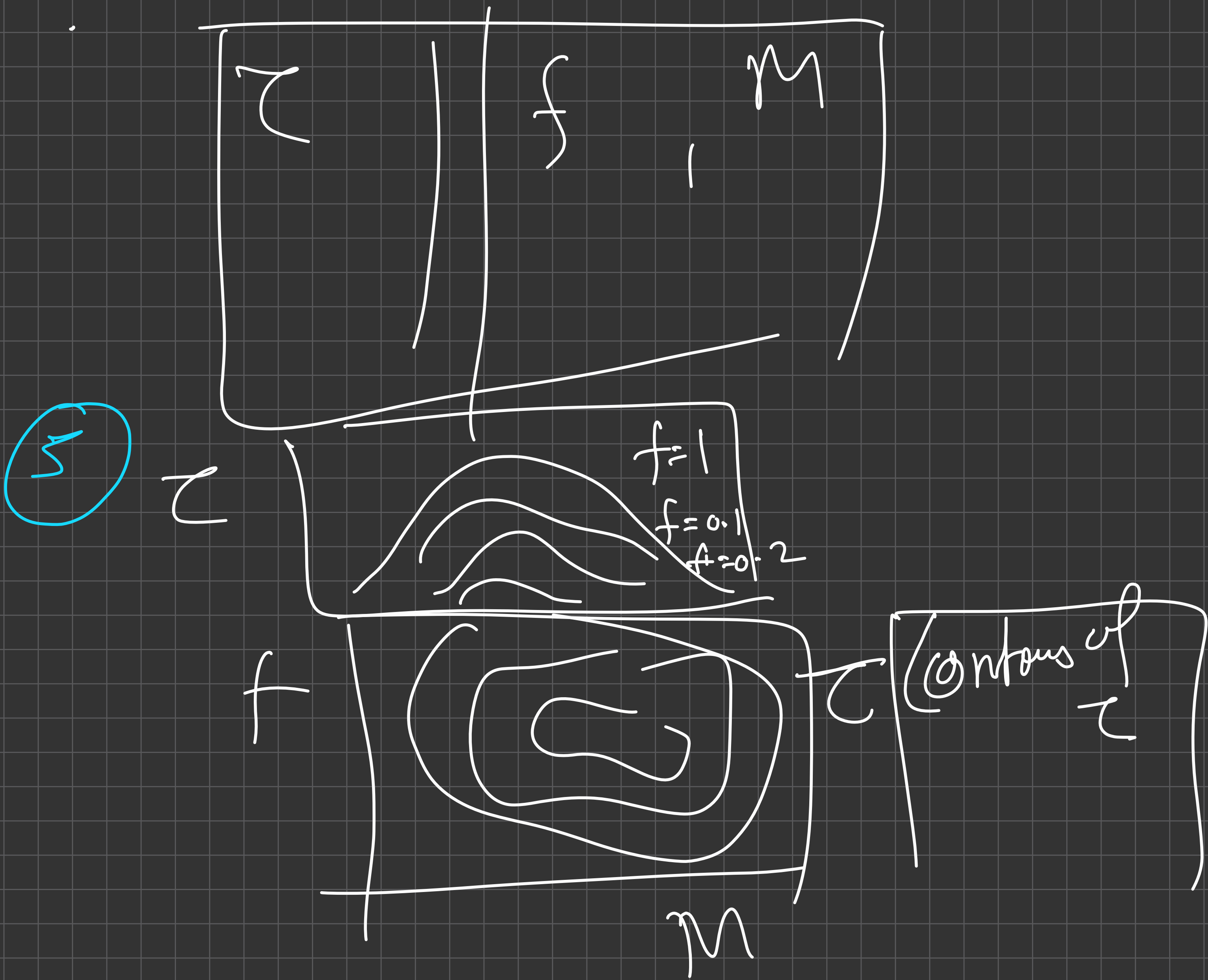
$$\sigma = \frac{1}{3} D_s \, n_{PBN} \, \Delta A$$

3 V_L vs D_s

D_s	V_L

4 ~~Contour~~ (pairplot) → seaborn(csv)

τ	f, M, D_s



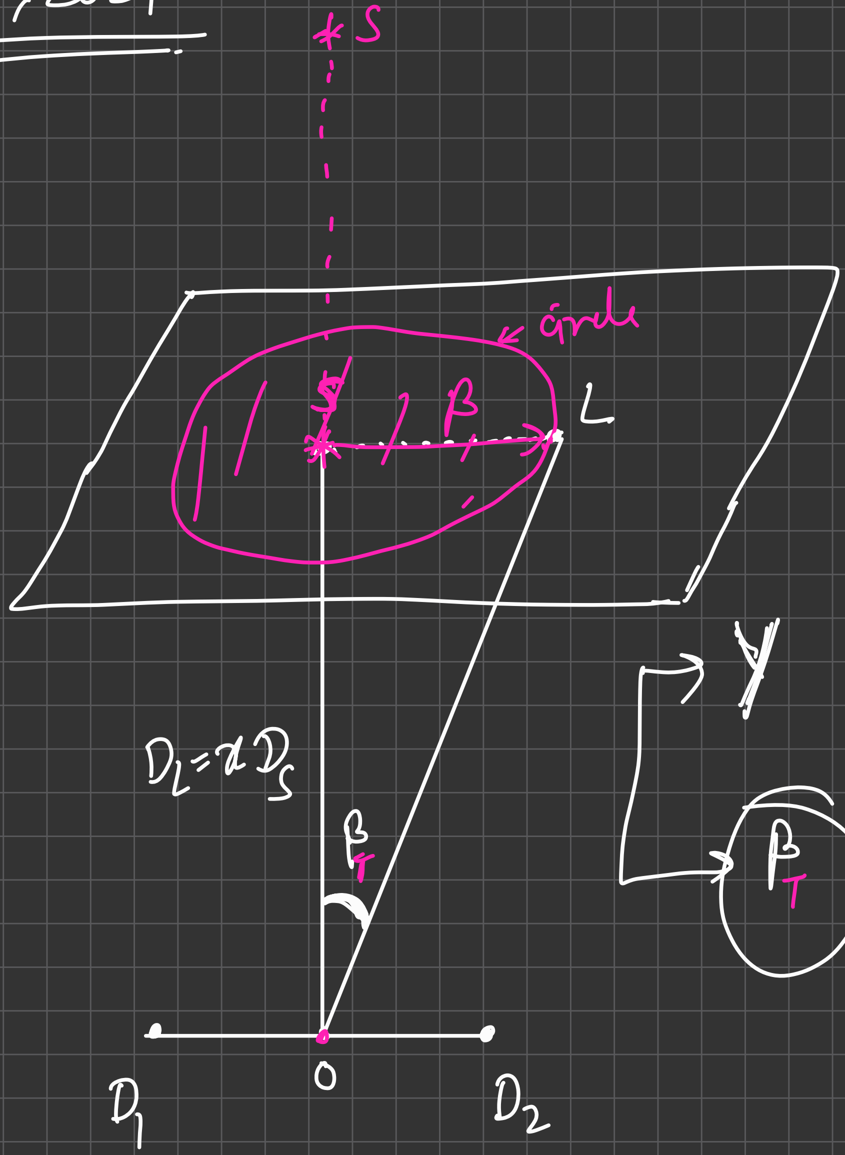
τ
f
M
ξ
τ
$f=1$
$f=0.1$
$f=0.2$
f
τ
Contours of τ
M

Jul 29, 2021

(1)

Lens plane

$x = \frac{D_L}{D_S}$



$A^2 = \pi y^2$

$A^2 = \pi \beta^2 D_L^2$

$\pi \beta^2$

$y = \beta D_L$

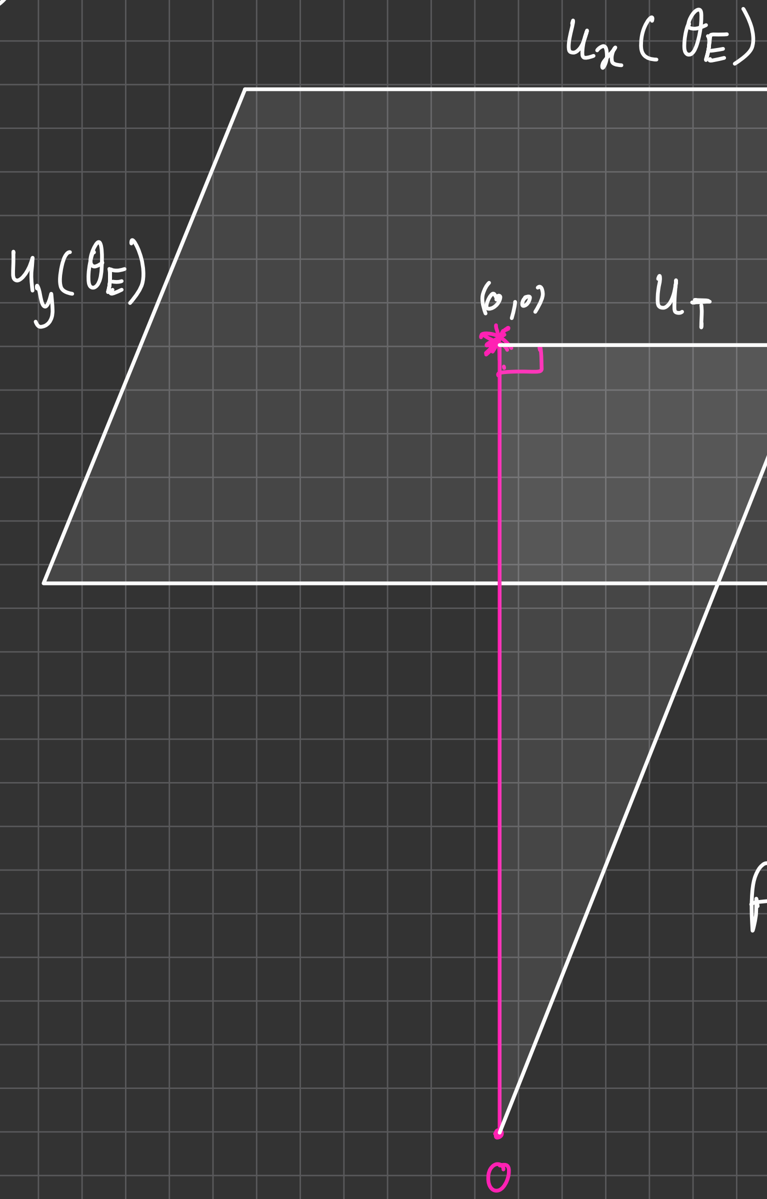
β_T

(2)

$$\theta_E = \sqrt{\frac{4GM}{c^2}\frac{1}{D_S}\left(\frac{1}{x}-1\right)} \quad \Bigg| \quad r_E = D_L \theta_E$$

$$\boxed{u \equiv \frac{\beta}{\theta_E}}$$

3



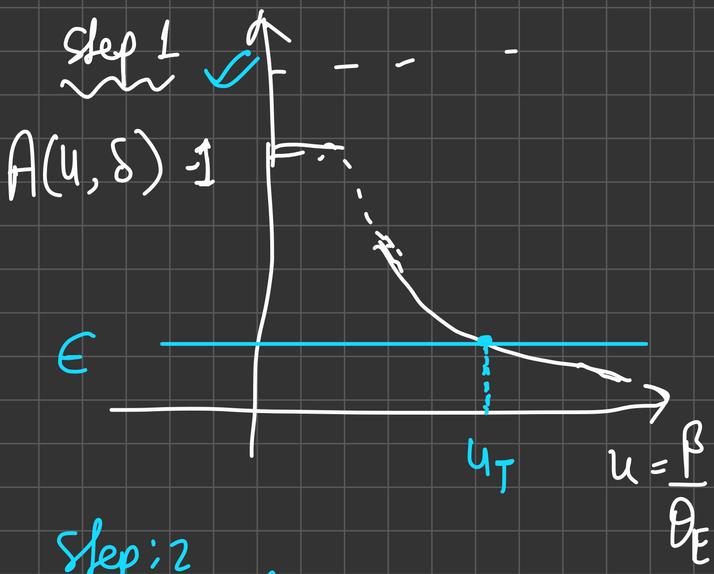
$\rightarrow \delta = \dfrac{\theta_S}{\theta_E}$

$\rightarrow u_T = \dfrac{\beta_T}{\theta_E}$

$A(u, \delta)$

$A(u_{T_0}, \delta) - 1 > \epsilon$

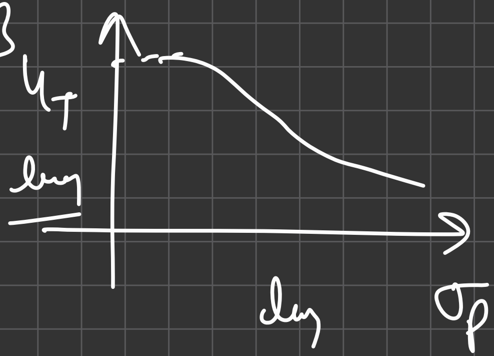
step 1 ✓



step: 2

find u_T

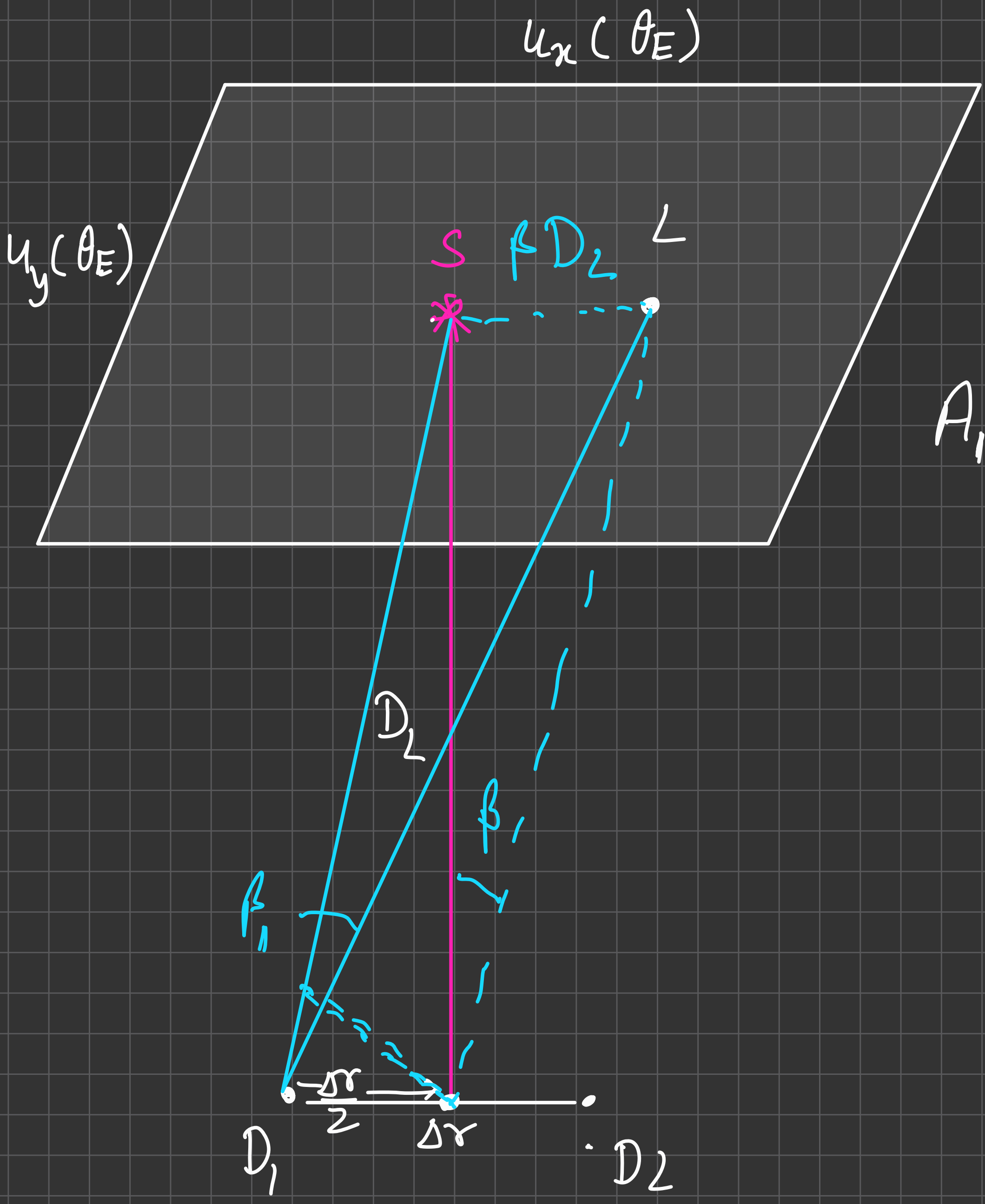
step: 3



(4)

$u_1 = \frac{\beta_1}{\theta_E}$

$A_1(u_1, \delta)$

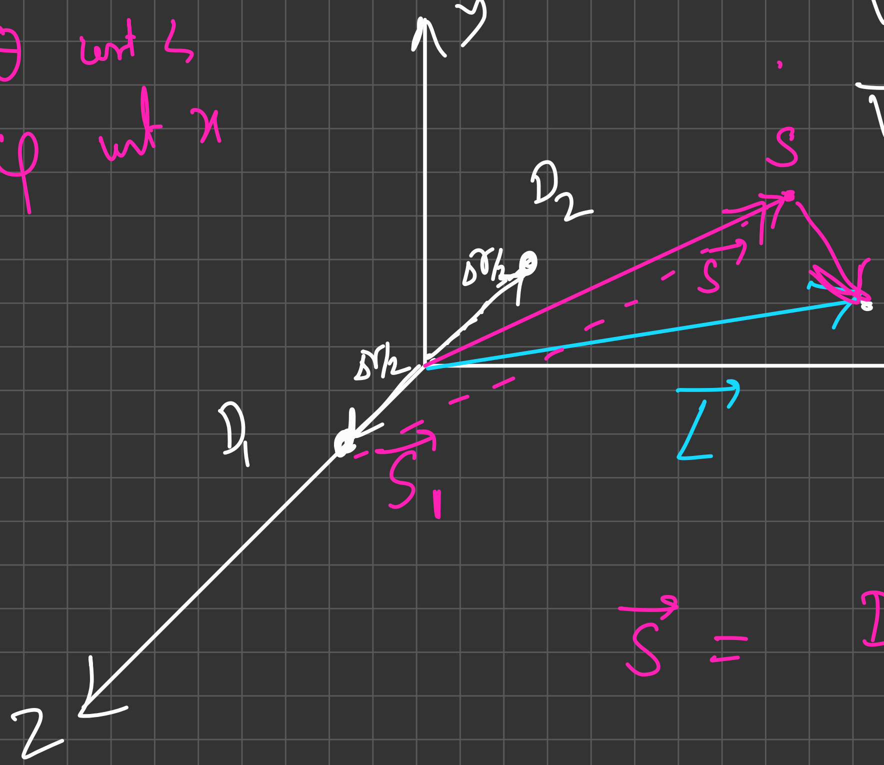


$$\beta_1 = \frac{\beta D_L}{D_L - \frac{\Delta r}{2}} \qquad \beta_2 = \frac{\beta D_L}{D_2 + \frac{\Delta r}{2}}$$

$$u_1 = \frac{u D_2}{D_2 - \frac{\Delta r}{2}} \qquad u_2 = \frac{u D_2}{D_2 + \frac{\Delta r}{2}}$$

θ wrt z

φ wrt x



$$\vec V_{D_1} = \frac{\Delta r}{2}\begin{pmatrix}0\\0\\-1\end{pmatrix}$$

$$\vec V_{D_2} = \frac{\Delta r}{2}\begin{pmatrix}0\\0\\1\end{pmatrix}$$

ℓ

$$\vec S = D_S \underbrace{\begin{pmatrix}\sin\theta_S\cos\phi_S\\ \sin\theta_S\sin\phi_S\\ \cos\theta_S\end{pmatrix}}_{\hat S}$$

$$\vec S_1 = \vec S - \vec V_{D_1}$$

$$\vec S_2 = \vec S - \vec V_{D_2}$$

$$\vec L = \cancel{D}D_S \underbrace{\begin{pmatrix}\sin\theta_L\cos\phi_L\\ \sin\theta_L\sin\phi_L\\ \cos\theta_L\end{pmatrix}}_{\hat L}$$

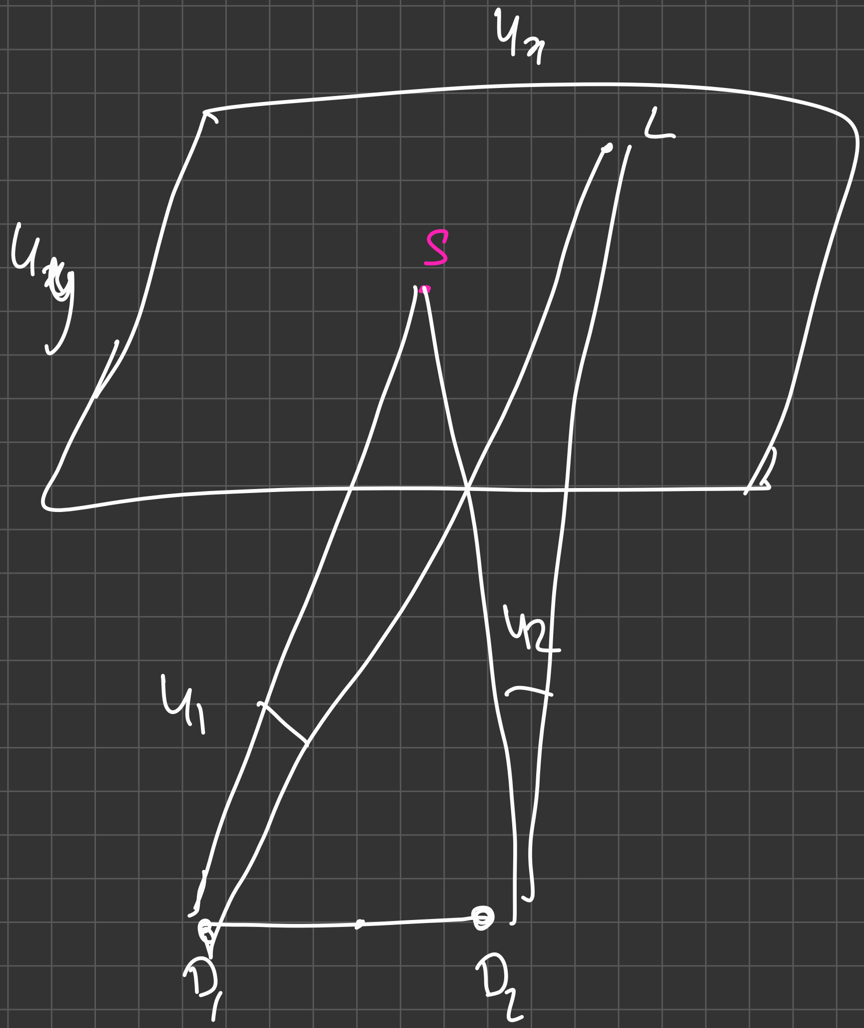
$$\frac{\vec\Delta_{S_2}}{D_2} = \frac{\vec S - \vec L}{D_2} = \underset{\sim}{\vec u}$$

$$\hat S \approx \hat L$$

$\vec L_1$, $\vec L_2$

$$\boxed{\frac{\vec S_1 - \vec L_1}{D_L} = \vec u_1}\ , \quad \boxed{\frac{\vec S_2 - \vec L_2}{D_2} = \vec u_2}$$

$u_1(\underset{\sim}{\theta}, \underset{\sim}{\phi})$, $u_2(\underset{\sim}{\theta}, \underset{\sim}{\phi})$

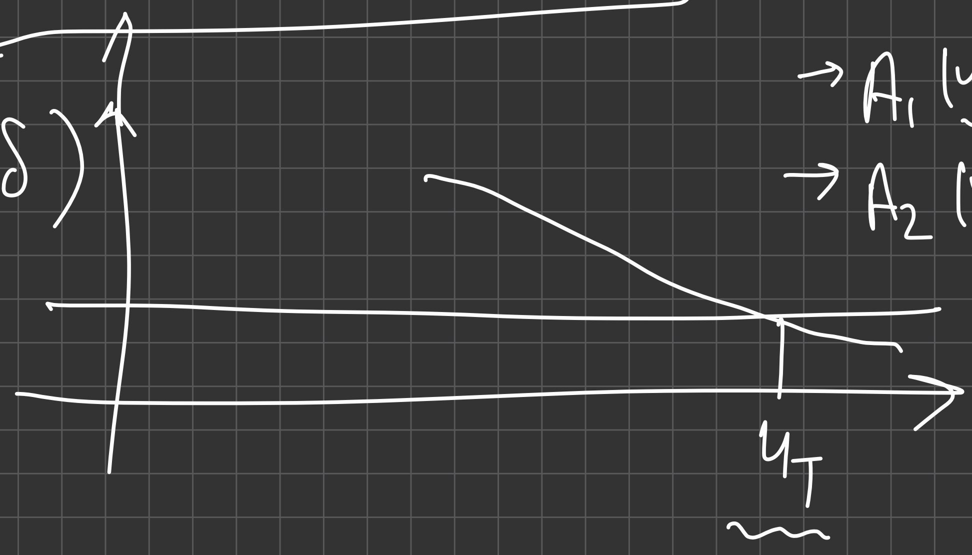


$$\boxed{O(u)\,\frac{A_1 - A_0}{\bar{A}} > \epsilon}$$

$u_1(u)$
$u_2(u)$

~~Drams~~

$O(u, \delta)$



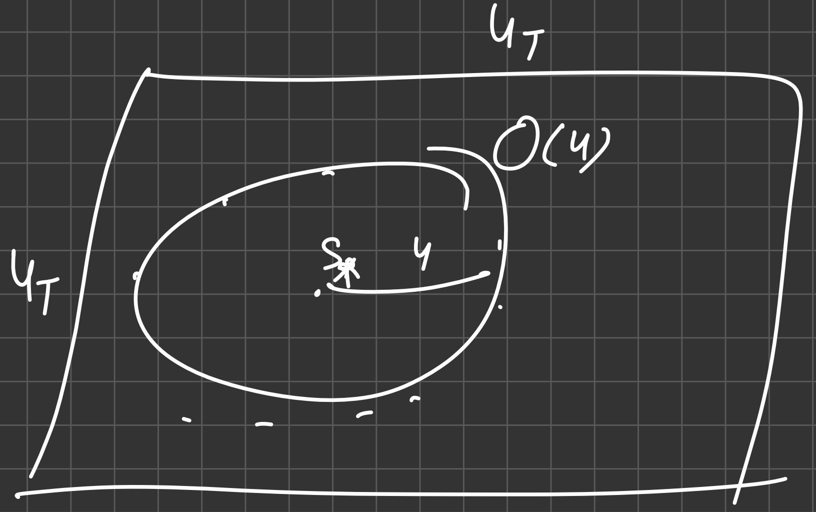
$\rightarrow A_1(\underset{\sim}{u_1})$

$\rightarrow A_2(\underset{\sim}{u_2})$

$\frac{dO}{du}$

$$\frac{dO}{dt} = \frac{dO}{du}\frac{du}{dt}$$

1



$\rightarrow$ $O_M(u) = \max\big(A_1(u),\ A_2(u)\big)$

$O_N(u)$