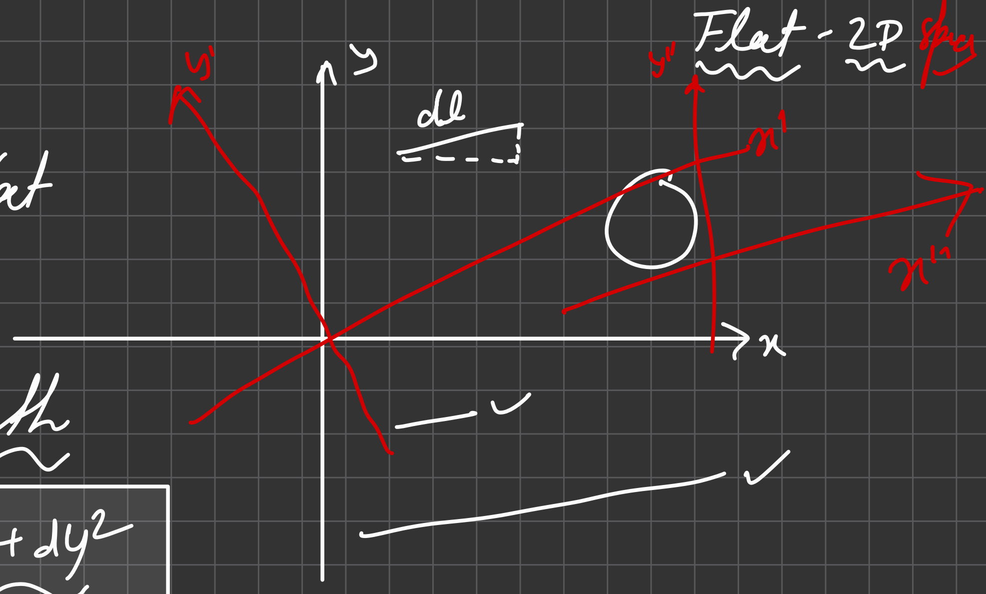
⇒ Metric :-

I) Geometry of this flat space

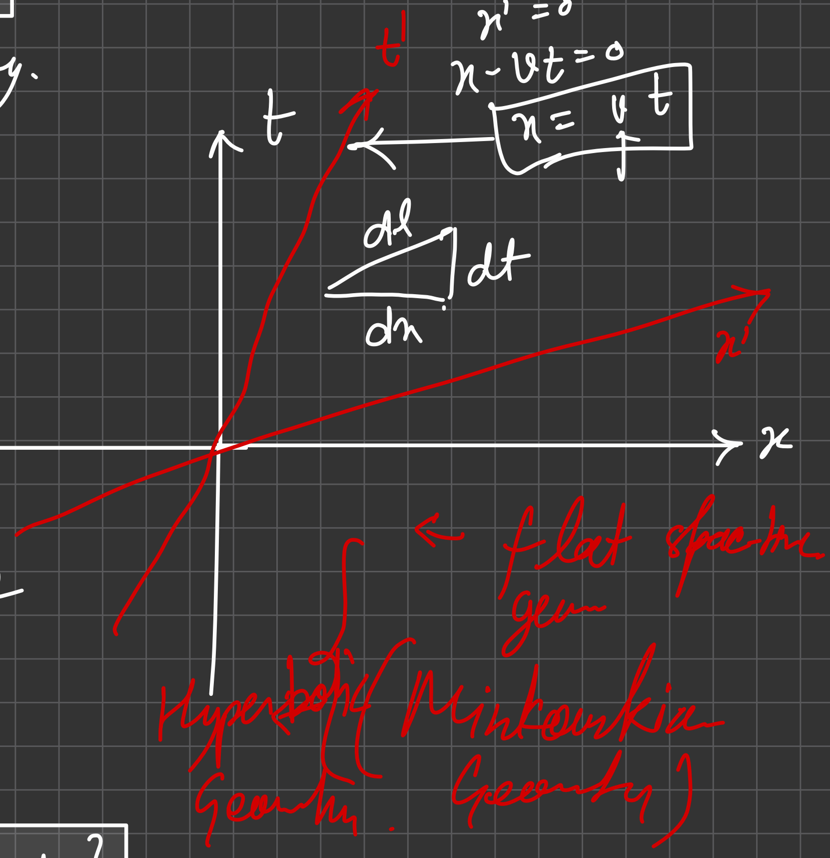


⇒ invariant length

$$\boxed{dl^2 = \underbrace{dx^2 + dy^2}}$$

↳ invariant quantity.

$\boxed{c=1}$



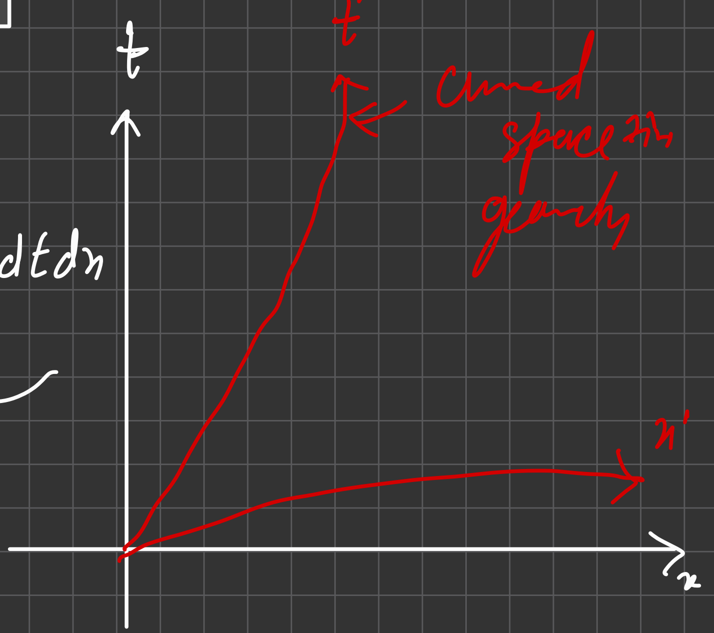
II) $\underbrace{dt^2 + dx^2}_{\text{not invariant}}$

$$ds^2 = -dt^2 + dx^2$$

↳ proper length

or

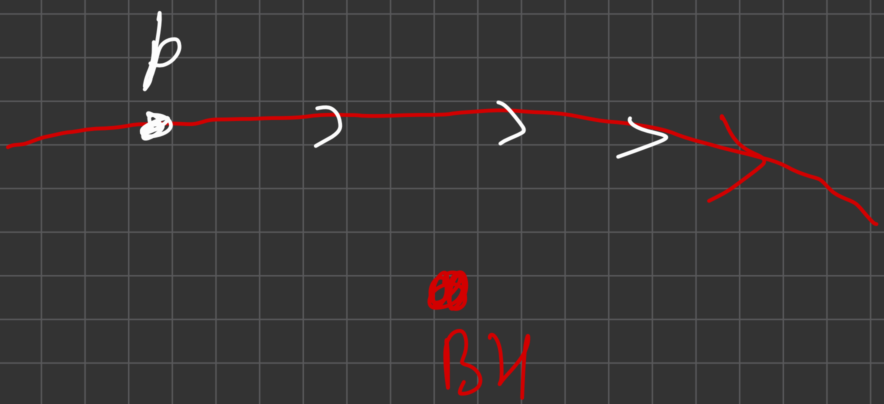
proper time $\boxed{d\tau^2 = -ds^2}$



$$ds^2 = \underbrace{g_{00}\,dt^2 + g_{11}\,dx^2 + g_{01}\,dt\,dx + g_{10}\,dt\,dx}$$

Equivalence principal

Locally (acceleration $\equiv$ Gravity)



Gravity $\Longrightarrow$ Curves the spacetime

$$ds^2 = g_{00}\,dt^2 + g_{11}\,dx^2 + g_{01}\,dt\,dx + g_{10}\,dx\,dt$$

$$\boxed{ds^2 = g_{\mu\nu}\,dx^\mu dx^\nu}$$

invariant length.

unknown

$\mu, \nu = (0, 1, 2, 3)$

$g_{\mu\nu} \leftarrow$ metric $\Big\}$ it defines the space-time geometry around any massive object.

<u>Q</u> How to find out $g_{\mu\nu}$ for the given mass / Energy distribution.

<u>A</u> Einstein's Field Eqns.

$R_{\mu\nu} \equiv f(g_{\mu\nu}, \partial g_{\mu\nu})$

$R = R_{\mu\nu} R^{\mu\nu}$

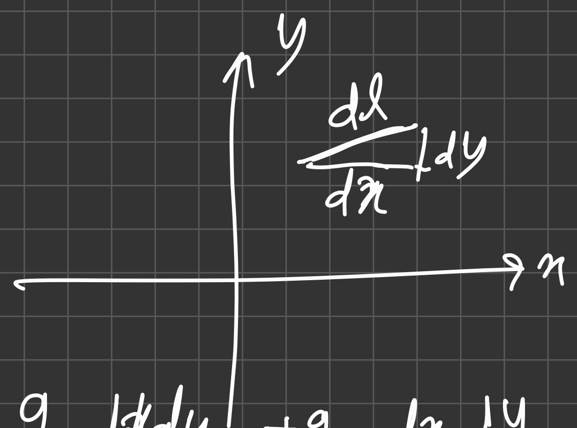
$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \# \, T_{\mu\nu}}$$

local Eq^n

Space-time tells matter how to move and matter tells space-time how to curve

Various forms of metric :-

(I) 2D flat space :-



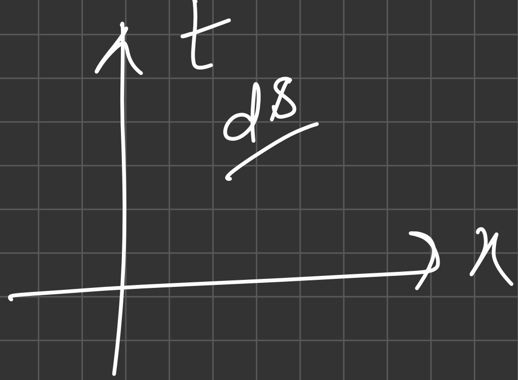
$$dl^2 = dx^2 + dy^2$$

$$\Updownarrow$$

$$= g_{11}\, dx^2 + g_{22}\, dy^2 + g_{12}\, dx\, dy + g_{21}\, dx\, dy$$

$$g = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(II) (1+1) D space-time (Minkowskian metric)



$$ds^2 = -dt^2 + dx^2$$

$$g_{1+1} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \leftarrow$$

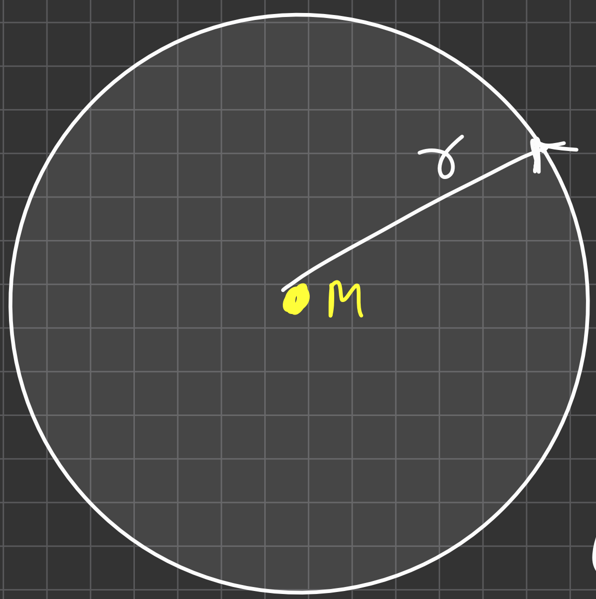
$$g = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{3+1} \equiv \eta \leftarrow \text{minkowskian metric}$$

$$\boxed{ds^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu}$$

III Schwarzschild Metric :-

$$g_{\mu\nu}(\vec{x}, t)$$

$$\boxed{G_{\mu\nu} \approx T_{\mu\nu}}$$



I $g_{\mu\nu}(r) \rightarrow$ should have spherical symmetry.

II for $r \to \infty$

$$g_{\mu\nu} \longrightarrow \eta_{\mu\nu}$$

weak field approximation.

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

perturbation.

$$g_{\mu\nu} = \begin{pmatrix} -1-\frac{2\Phi}{c^2} & 0 & 0 & 0 \\ 0 & 1-\frac{2\Phi}{c^2} & 0 & 0 \\ 0 & 0 & 1-\frac{2\Phi}{c^2} & 0 \\ 0 & 0 & 0 & 1-\frac{2\Phi}{c^2} \end{pmatrix}$$

$\Phi \leftarrow$ Gravitational potential

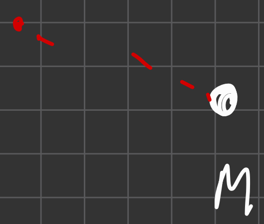
For point like object, $\Phi = -\frac{GM}{r}$

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu \;, \quad g_{\mu\nu} = \begin{pmatrix} -1-\frac{2\Phi}{c^2} & 0 & 0 & 0 \\ 0 & 1-\frac{2\Phi}{c^2} & 0 & 0 \\ 0 & 0 & 1-\frac{2\Phi}{c^2} & 0 \\ 0 & 0 & 0 & 1-\frac{2\Phi}{c^2} \end{pmatrix}$$

$$ds^2 = \left(1+\frac{2\Phi}{c^2}\right)c^2 dt^2 - \left(1-\frac{2\Phi}{c^2}\right)(d\vec{x})^2$$

$c = 3 \times 10^8\, m/s$

For light, $ds^2 = 0$



$$v = \frac{|d\vec{x}|}{dt}$$

$$\downarrow$$

$$n(\Phi) = \frac{c}{v(\Phi)}$$

r.i

$$\boxed{L = \int_{path} n\, dl}$$

$\boxed{\delta L = 0}$ for extrem.

$$S = \int \mathcal{L} \; d\lambda$$

$\mathcal{L} \leftarrow$ Lagrangian

$\downarrow$

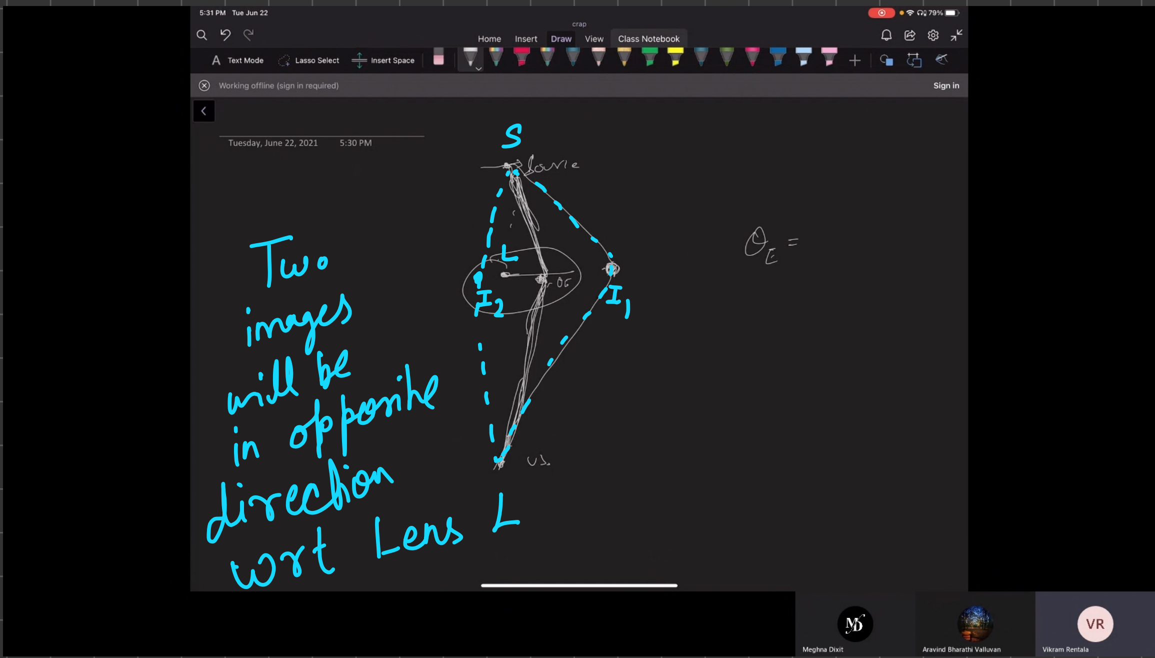
Euler Lagrange Eqn

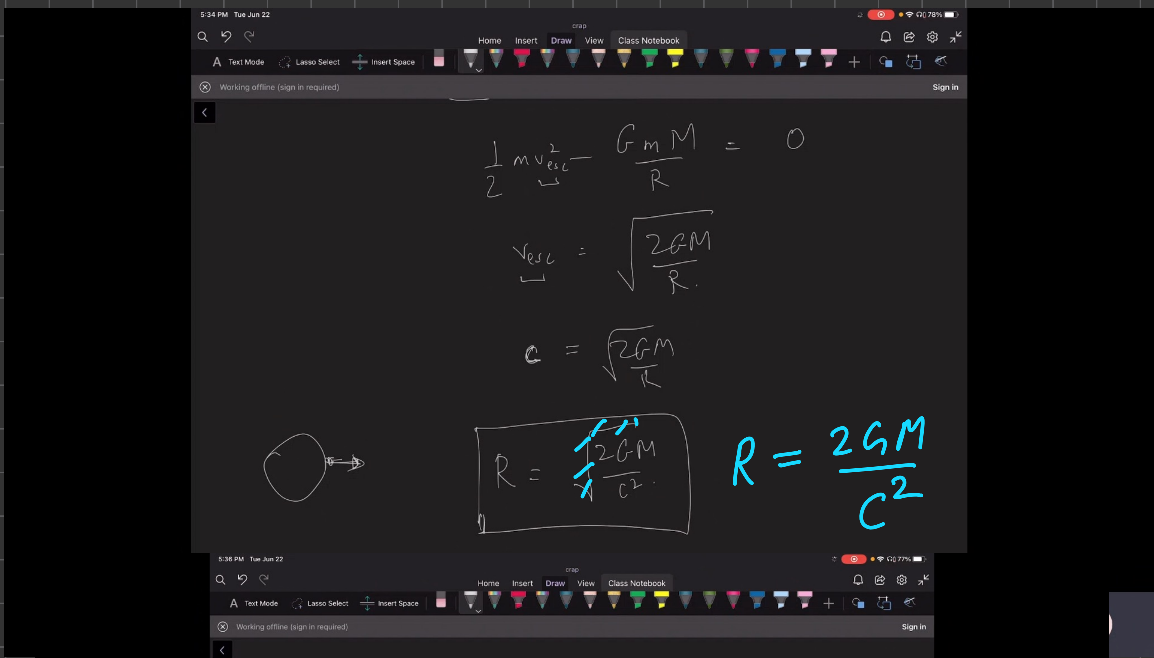
$\downarrow$

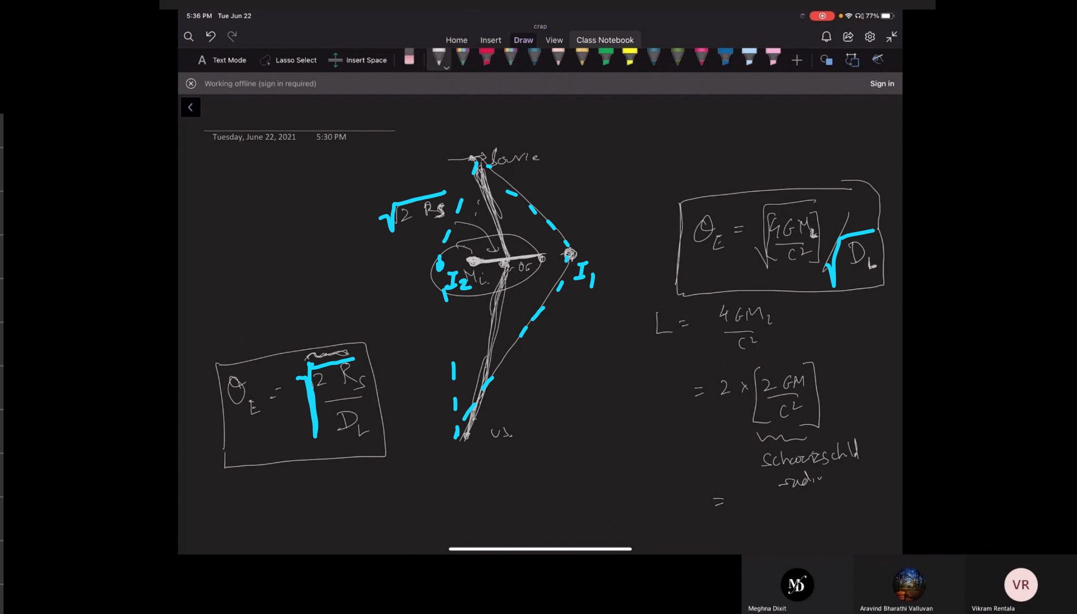
$$\boxed{\hat{\alpha} = \frac{2}{c} \int \nabla_\perp \Phi \, d\lambda}$$

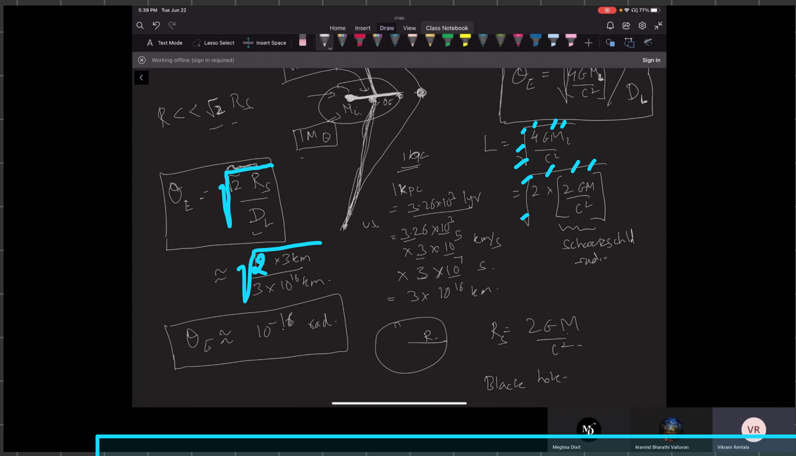
$\rightarrow$ Shapiro delay $\boxed{d\tau = \frac{dl}{c}}$

⇒ Last GRB lensing paper discussion Comments.



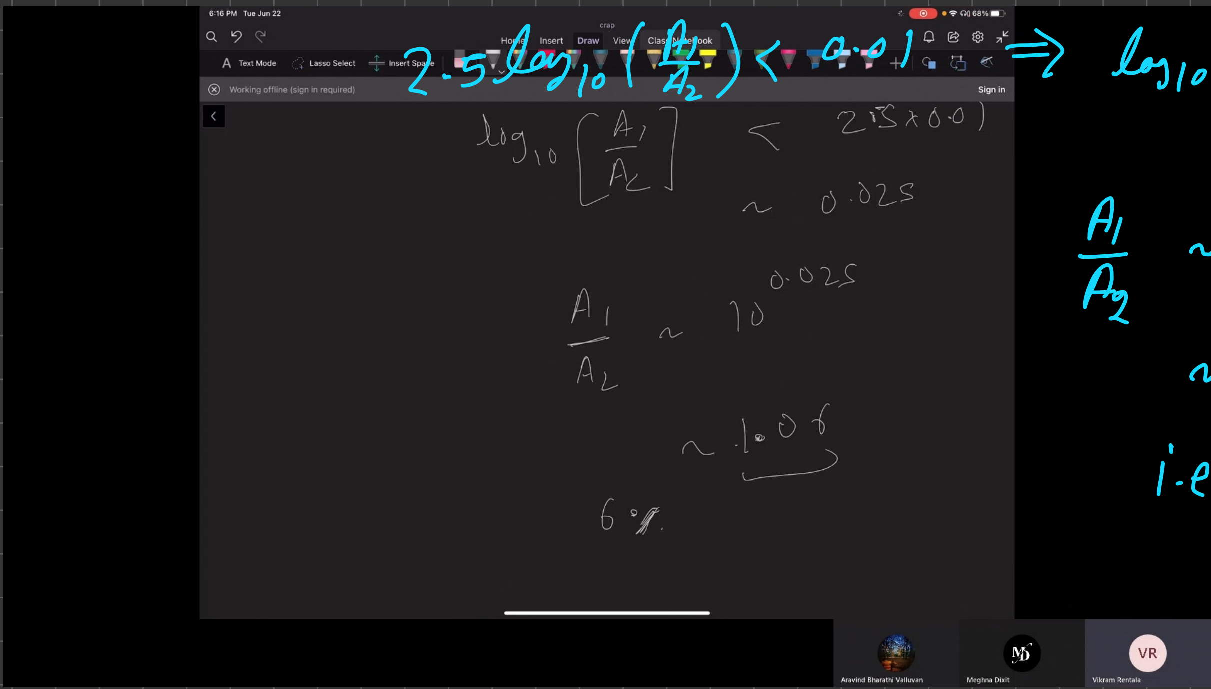






$$\boxed{\theta_E = \sqrt{\frac{2R_s}{D_L}} = \sqrt{\frac{2\times 3\text{ km}}{3\times 10^{16}\text{ km}}} \approx 10^{-8}\text{ rad}}$$

$$2.5\log_{10}\left(\frac{A_1}{A_2}\right) < 0.01 \Rightarrow \log_{10}\left(\frac{A_1}{A_2}\right) < \frac{0.01}{2.5}$$



$$\frac{A_1}{A_2} \sim 10^{0.004}$$

$$\sim 1.009$$

i.e. 0.9 %

Issue of resolution :-

We don't have to resolve the two images of stars/GRB rather as

$$A(u) = \frac{u^2+2}{u\sqrt{u^2+4}}$$

$$u = \frac{\theta}{\theta_E}$$

Hence, A changes from two positions bcoz u changes.

$$\mu(u_\pm) = \frac{1}{2}\left[1 \pm \frac{u^2+2}{u\sqrt{u^2+4}}\right]$$

$$u = \frac{\theta}{\theta_E}$$

$$A(u) = |\mu(u_+)| + \mu(u_-)|$$

θ_E

θ_E

$\nearrow$ $\underbrace{M \sim 10^{-10} \sim 10^{3}}$

$$\theta_1 \rightarrow \theta_2$$

$$u_1 \rightarrow u_2$$

$$A_1(u_1) \rightarrow A_2(u_2)$$

$FA_2 = F_2$ 2 Δr $\underset{\sim}{1}$ $\leftarrow F_1 = FA_1$

$$\boxed{O \equiv \frac{A_1 - A_2}{\frac{|A_1 + A_2|}{2}} > \underset{\sim}{\epsilon} \leftarrow \text{detection efficiency.}}$$

$$R_s = \frac{2GM}{c^2}$$

$$\theta_E = \sqrt{\frac{2R_s}{D_s}\left(\frac{1}{x} - 1\right)}$$

$$x = \frac{D_L}{D_S}$$

$$O(\Delta r, \theta_E)$$

$\downarrow$

$$O(\Delta r, M, D_L, D_S)$$

$\Rightarrow$ point lens, $\quad u = \frac{\beta}{\theta_E}$ $\leftarrow$ true position of star $\qquad \tilde{u} = \frac{\theta}{\theta_E}$ $\leftarrow$ image position

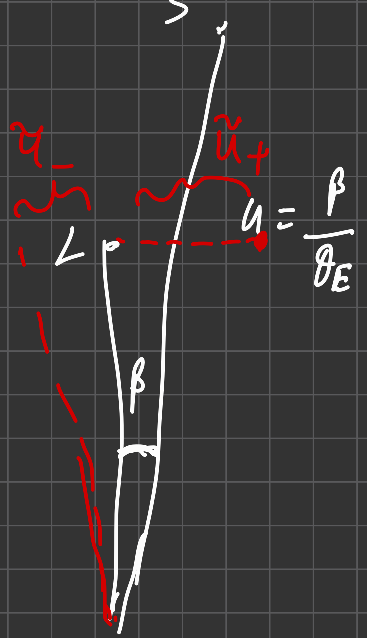
$$\beta = \theta - \frac{\theta_E^2}{\theta}$$

$$\beta\theta = \theta^2 - \theta_E^2$$

$$\theta^2 - \beta\theta - \theta_E^2 = 0$$

$$\tilde{u}^2 - u\tilde{u} - 1 = 0$$

$$\boxed{\tilde{u}_\pm = \frac{u \pm \sqrt{u^2+4}}{2}}$$



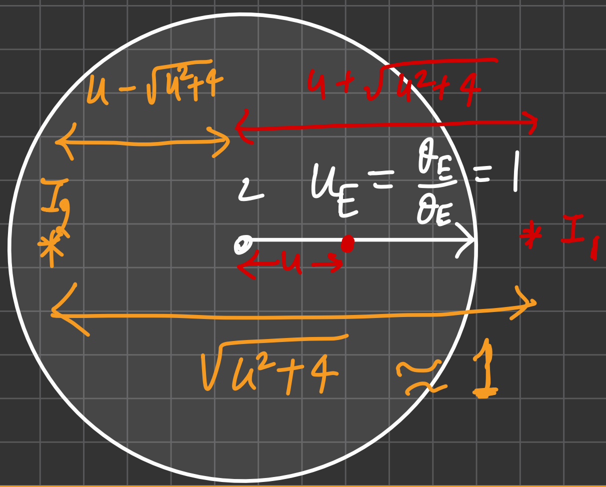
$$\mu(u) = \frac{1}{2}\left[1 \pm \frac{u^2+2}{u\sqrt{u^2+4}}\right]$$

$$\sqrt{u^2+4} > u$$

θ_E

$$u_{sep} = \tilde{u}_+ - \tilde{u}_- = \sqrt{u^2+4}$$

$$= \sqrt{\left(\frac{\beta}{\theta_E}\right)^2+4}$$



$$\boxed{\theta_E \approx 2.854 \text{ mas} \sqrt{\frac{M}{1M_\odot}\frac{1\,\text{kpc}}{D_S}\left(\frac{1}{x}-1\right)}}$$

$$A(u) = |\mu_1| + |\mu_2| = \frac{u^2+2}{u\sqrt{u^2+4}}$$

apparent magnitude,

$$\underbrace{m_G}_{} = -2.5 \log_{10}\left(\frac{F}{F_0}\right)$$

$\rightarrow \sigma(m_G)$

$$\sigma = \frac{A_1 - A_2}{\left(\frac{A_1 + A_2}{2}\right)} > \epsilon(m_G)$$

$F_1 = F A_1$

$F_2 = F A_2$

$.S$

$$m_1 = -2.5 \log_{10}\left(\frac{F_1}{F_0}\right)$$

$$m_2 = -2.5 \log_{10}\left(\frac{F_2}{F_0}\right)$$

i

$$(m_1 - m_2) = -2.5 \log_{10}\left(\frac{F_1}{F_2}\right)$$

F_2 $\dot{2}$ $\dot{1}$ F_1

$$(m_1 - m_2) = -2.5 \log_{10}\left(\frac{A_1}{A_2}\right)$$

detection criteria

$$\boxed{\Delta m > \sigma}$$

$$-2.5 \log_{10} \frac{A_1}{A_2} > \sigma$$

$$\frac{A_1}{A_2} - 1 > 10^{-\frac{\sigma}{2.5}} - 1$$

$$\frac{A_1 - A_2}{A_2} > \left(10^{-\frac{20}{5}} - 1\right)$$

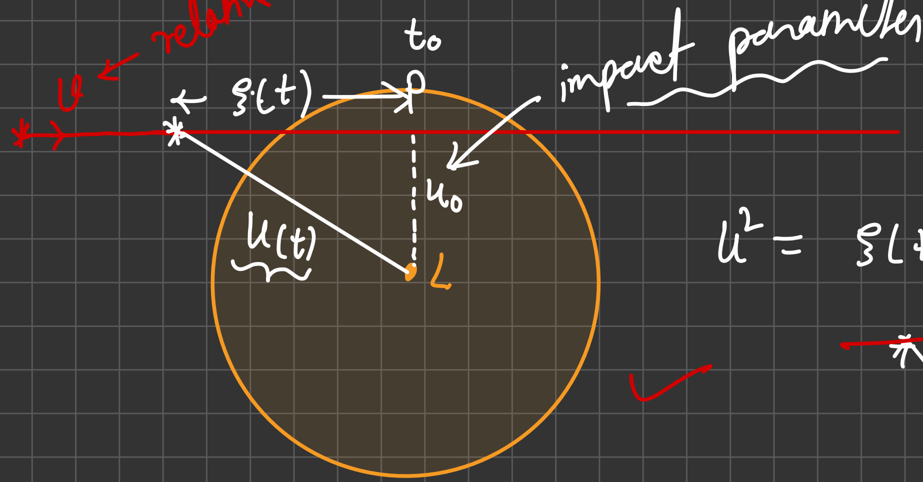
$\underbrace{\sigma(m_G)}_{}$

$$\rightarrow \boxed{(A_1 - A_2) > \left(10^{-20/5} - 1\right) A_2}$$

what is u_0?:-

$$D_{SL} = D_S - D_L$$

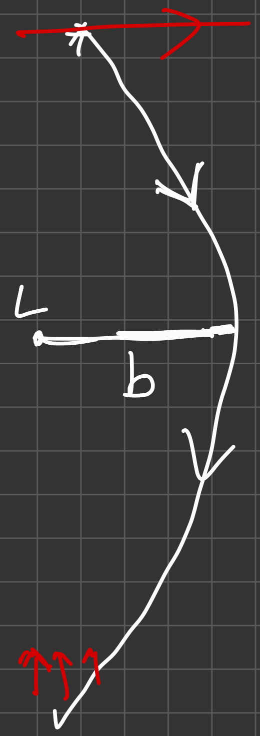
$$u(\theta) = \frac{\beta(t)}{\theta_E}$$



$$\mu = \frac{v}{D_{SL}}$$

$$u^2 = \xi(t)^2 + u_0^2$$

$$u_0 \equiv u(t_0)$$



$$\hat{\alpha} = \frac{4GM}{c^2 b}$$

optical depth:-

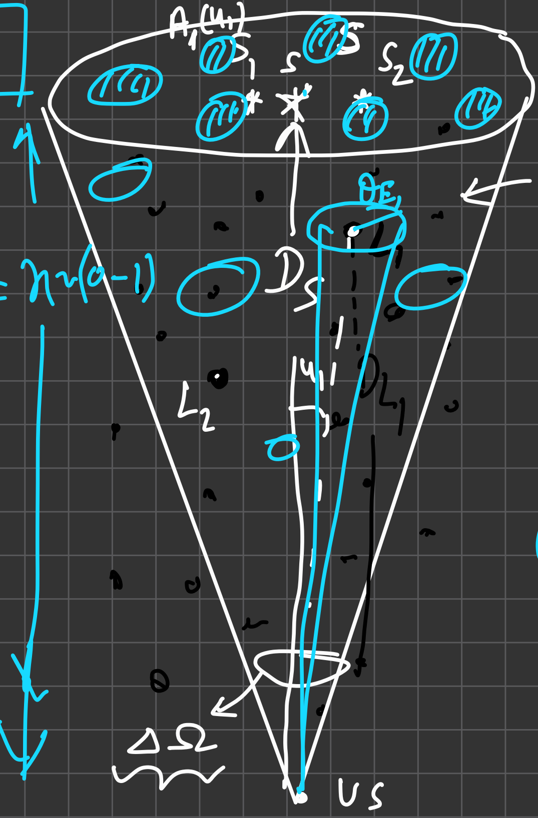
"for the given stars (m_q) what is the probability that it will get lensed and lensing would be detected."

(II) $\Delta A = |A_1 - 1| > \epsilon \leftarrow \boxed{\theta_1 = \theta_E K}$

(I) u_1

→ if star lies inside θ_E then → detected

$\frac{D_L}{D_S} = x \sim (0-1)$



OD for PBH

$\frac{f \rho_{DM}}{M_{PBH}} = \frac{\rho_{PBH}}{M_{PBH}} = n_{PBH}$

$\theta_E = \sqrt{\frac{2R_s}{D_s}\left(\frac{1}{x} - 1\right)}$

$$\tau = \frac{\Delta A}{A} = \frac{\sum \pi \theta_E^2 K^2}{\Delta\Omega}$$

$$= \frac{\int dn \, \pi \theta_E^2 K^2}{\Delta\Omega}$$

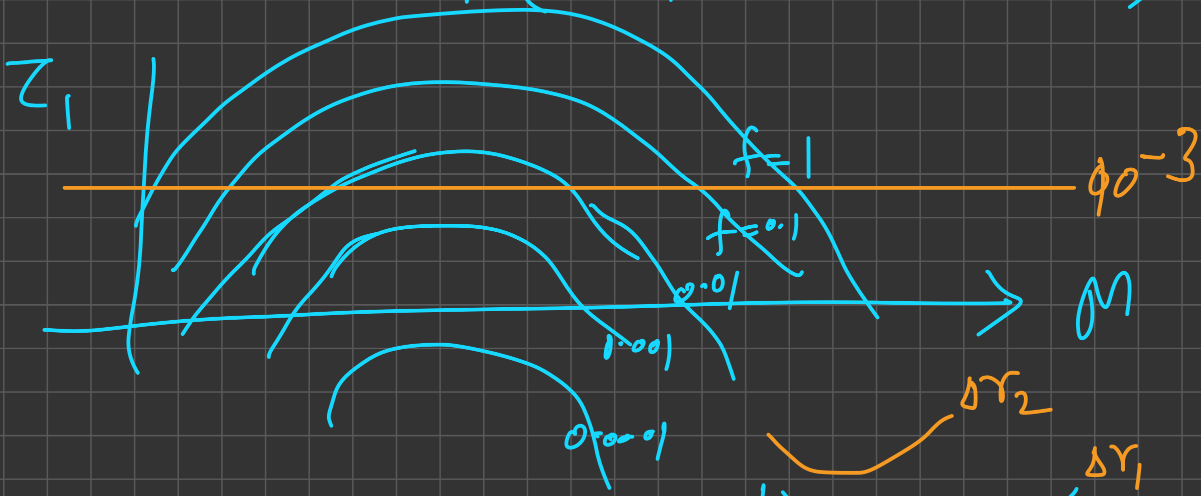
$$dn = \frac{f \rho_{DM}}{M_{BH}} D_S^3 \, dD_S \, \Delta\Omega$$

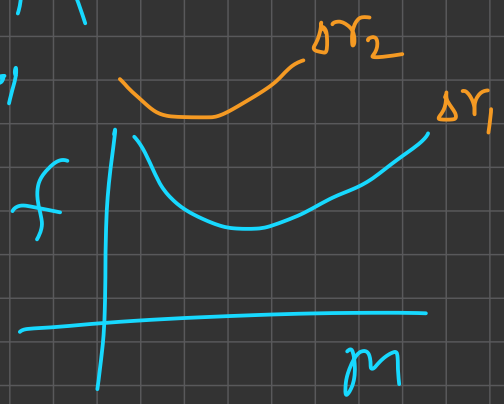
$$\boxed{\tau = \int \frac{f \rho_{DM}}{M_{BH}} D_S^3 \pi \theta_E^2 K \, dD_S}$$

$\underline{\underline{f=1}}$ $\quad O_g(\Delta r_2 \ldots) \; O(\Delta r_1, M_1, \text{———})$

$$O_g \longrightarrow \tau_2$$

$$O \longrightarrow K_1 \longrightarrow \tau_1(\Delta r_1, M_1, \text{———})$$





Jul 5, 2021

$0 < x < 1$ $\qquad x = D_L/D_S$

lens plane

$$R_E = \underbrace{D_l}_{\text{lens plane}} \theta_E = D_L \sqrt{\frac{4GM}{c^2}\left(\frac{1}{D_L} - \frac{1}{D_S}\right)}$$

$$= \sqrt{2 R_S D_S x^2 \left(\frac{1}{x} - 1\right)}$$

$$R_E(\tilde{M}, D_S, \overset{\downarrow}{x}) = \sqrt{2 R_S D_S x(1-x)}$$

$$R_E(R_S, D_S, x)$$

$$R_S = \frac{2GM}{c^2}$$

$$= \underbrace{\frac{2G(1M_\odot)}{c^2}}\ \frac{M}{1M_\odot}$$

$$\boxed{R_S \approx \underbrace{3\,\text{km}}_{\text{preR}_S} \left(\frac{M}{1M_\odot}\right)}$$

$\text{pre}R_S = 3\,\text{km}$

AM = np.logsp(-6,6,num=100) ← should be $M_\odot$

$AR_S = R_S(AM)$ ← would be km

$$R_E = \sqrt{2 \times 3\,\text{km} \times \frac{R_S}{3\,\text{km}} \times 1\,\text{kpc}\ \frac{D_S}{1\,\text{kpc}}\ x(1-x)}$$

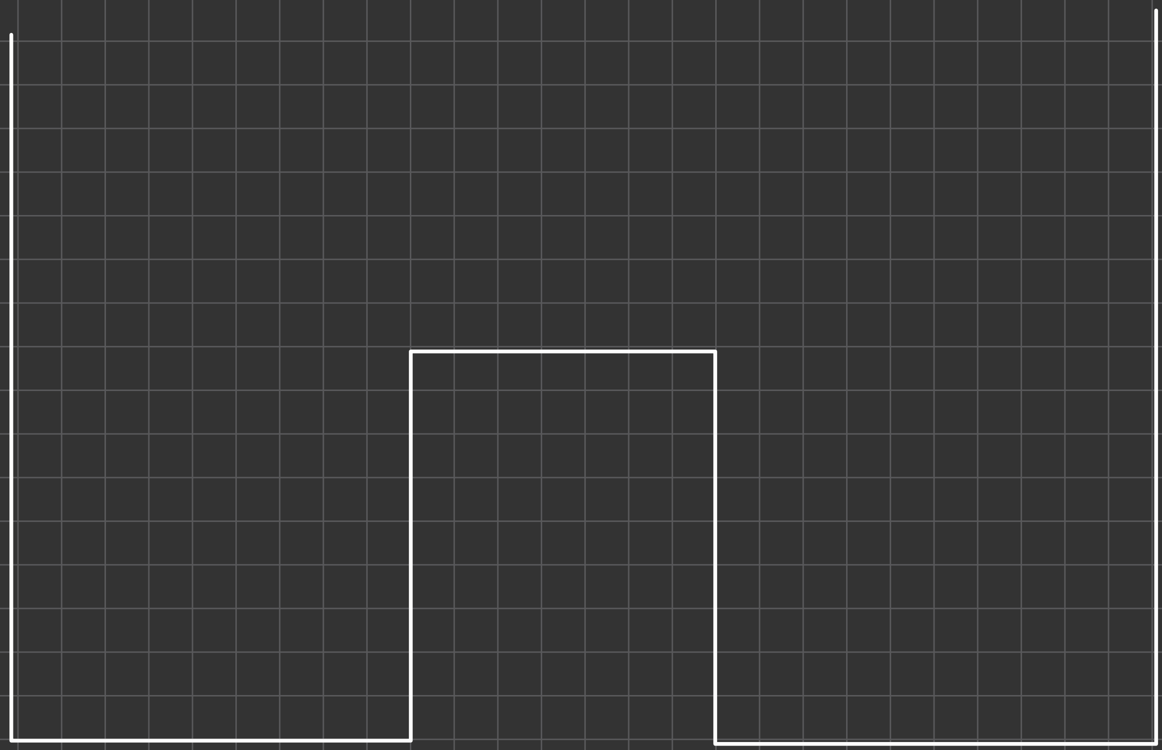
$$= \underbrace{\sqrt{2 \times 3\,\text{km} \times 1\,\text{kpc}}}\ \sqrt{\frac{R_S}{3\,\text{km}} \frac{D_S}{1\,\text{kpc}} \cdots x(1-x)}$$

$$= \underset{\text{kpc}}{\text{pre}R_E}\ \underbrace{\sqrt{\frac{R_S}{3\,\text{km}} \frac{D_S}{1\,\text{kpc}}\ x(1-x)}}$$

R_S = np.lins(1, 10, num=100) #3 km

D_S = - - (- - -) #kpc

$$R_E(R_S, D_S, x) \longrightarrow \underbrace{\text{kpc}}$$

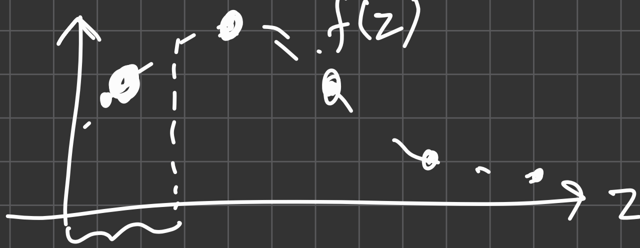


Jul 12, 2021

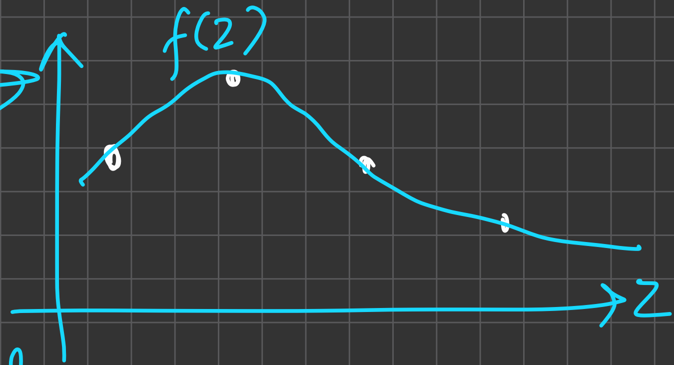
$$\int_0^{0.5} f(z)\,dz = 60$$

$$f(z)(0.5) = 60$$

$$f(z) = \frac{60}{0.5}$$



I dist of GRB



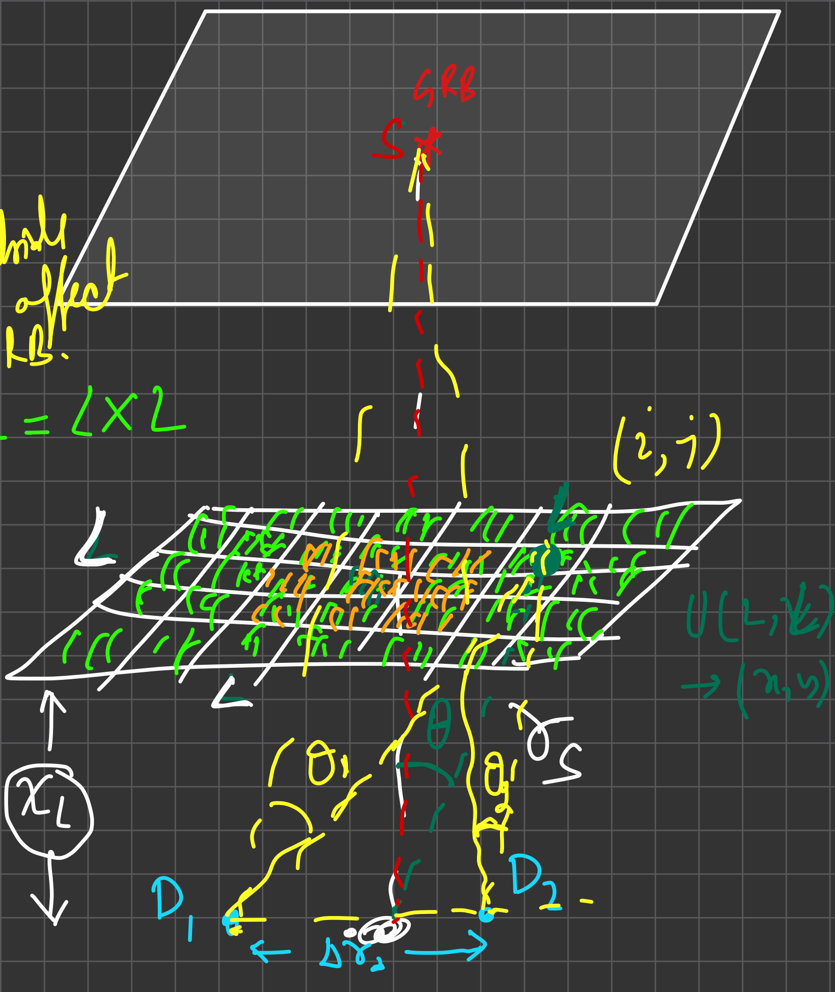
I) 1000

rand $f(z)$ → 1000 random value

II)

III)

for each γ_s



$\sigma(x_L, \Delta\gamma_\perp, \gamma_s, \epsilon)$

any lens outside $\delta\Omega$ won't affect GRB.

(a)
$A_1 > 1+\epsilon$
or
$A_2 > 1+\epsilon$

$\Rightarrow \delta\Omega = L \times L$

(b) $\delta = \dfrac{A_1 - A_2}{\frac{A_1 + A_2}{2}} > \epsilon$

$$\boxed{\sigma = 9 \times \left(\frac{L}{80}\right)^2}$$

$\theta(i,j)$

$\theta_1(\theta)$ $\theta_2(\theta)$

$$u_1 = \frac{\theta_1(i,j)}{\theta_E} \qquad u_2 = \frac{\theta_2(i,j)}{\theta_E}$$

$$\boxed{A_1 = \frac{u_1^2+2}{u_1\sqrt{u_1^2+4}}} > 1+\epsilon \qquad A_2 = \text{- - -}$$

$\Delta\Omega_2$

$\rightarrow$ $u_1^T(\epsilon)\,\theta_E = \theta^T$

$$\frac{L}{2} = \theta^T x_L$$

$$\Delta\Omega_1 = L^2 = \left(2\theta^T x_L\right)^2$$

$$= \left(2u_1^T\,\theta_E(M, x_L, x_S)\,x_L\right)^2$$

$$\boxed{\Delta\Omega = \max(\Delta\Omega_1, \Delta\Omega_2)}$$

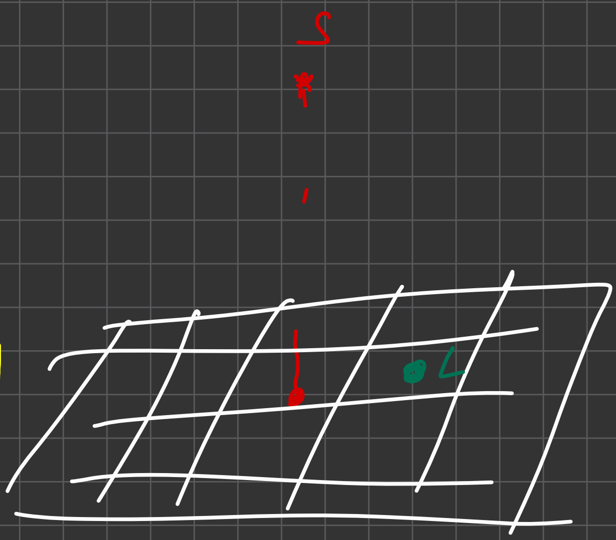
$(M, r_s, x_s, \delta r_\perp, \epsilon)$

$\Downarrow$

$\delta\Omega = \max(\delta\Omega_1, \delta\Omega_2)$

$\Downarrow$

divide $\delta\Omega = 10 \times 10 = N \times N$



$\Downarrow$

in each square put a Lens at at Distance x_L

$\Downarrow$

θ

$\downarrow$

θ_1 , θ_2

$\downarrow$

A_1 , A_2

$\downarrow$

$\delta > \epsilon$

$\downarrow$

n_s

$$\rho_{PB} = f \rho_{DM}$$

$$n_{PBH} = \frac{f}{M} \rho_{DM}$$

$$\boxed{\sigma(r_s, x_L, \delta r_\perp, M, \epsilon) = n_s \left(\frac{L}{N}\right)^2}$$

r_s : from dist of GRB

x_L : Uniformly distributed $(0.01, 100)$ kpc

$x = \dfrac{x_L = (0,1)}{x_S}$

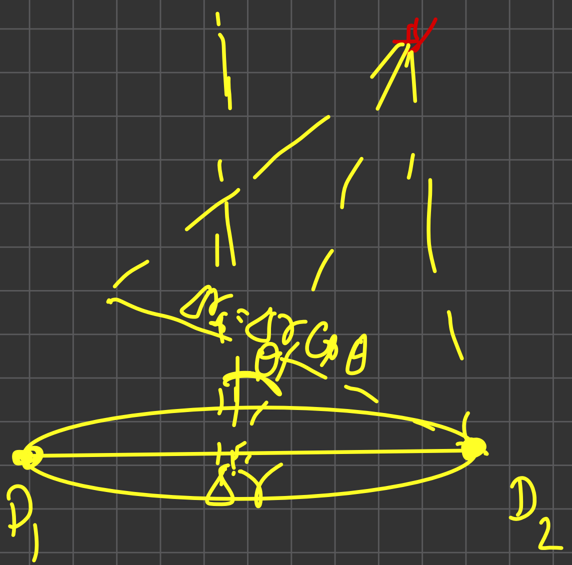
Δr_2 : ✓

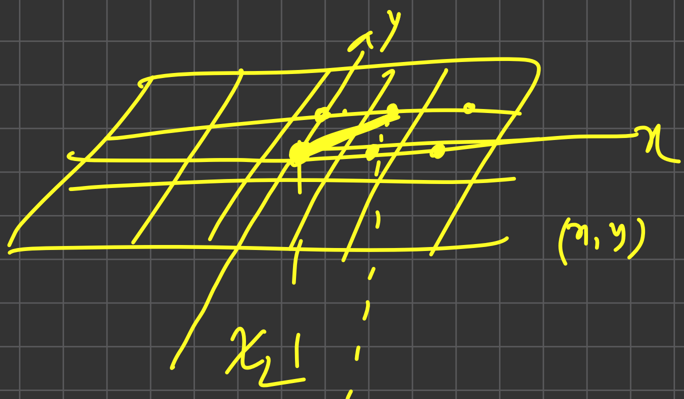
M : $(10^{-11}, 10^{-3}) M_\odot$

ϵ : ✓

x_S	x_L	M	σ

x_S	M	V_L





$$l = \sqrt{x^2+y^2}$$

$$x_2\,\theta = l$$

$$\boxed{\theta = \frac{\sqrt{x^2+y^2}}{x_2}}$$

$\rightarrow$ $\theta_1(\theta)$

$\theta_2(\theta)$