Date: ☐☐/☐☐/☐☐☐☐ ASB -9

Q No. Exercise No. ☐☐ Solved Problems: Sub ☐ Obj ☐

GRAVITATIONAL LENSING FORMALISM

14.6.21 In general relativity, a point mass deflects a light ray with impact parameter b by an angle $\hat{\alpha}$

θ angle of closest approach

$$\boxed{\hat{\alpha} = \frac{4GM}{bc^2}}$$

G → Gravitational constant; M → mass of lens; c → speed of light

approximation is valid when $\hat{\alpha}$ is small

In weak field limit general relativity / $g_{\mu\nu}$ → linearised gravity / $g'_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, deflection due to a spatially extended mass: vector sum over point masses

In the continuum limit → $\Sigma \rightarrow \int$ over density ρ

$$\Rightarrow \hat{\alpha}(\bar{\xi}) = \frac{4G}{c^2}\int d^2\xi' \int dz\, \rho(\bar{\xi}', z)\, \frac{\bar{b}}{|\bar{b}|^2}$$

→ for infinitesimal mass at $(\bar{\xi}', z)$

dz → line of sight coordinate

where $\bar{b} = \bar{\xi} - \bar{\xi}'$

→ vector impact parameter

Thin Lens Approximation

distances:

D_L → lens to observer

D_S → source to observer

D_{SL} → source to lens

$D_L, D_S, D_{LS} \gg$ size of lens

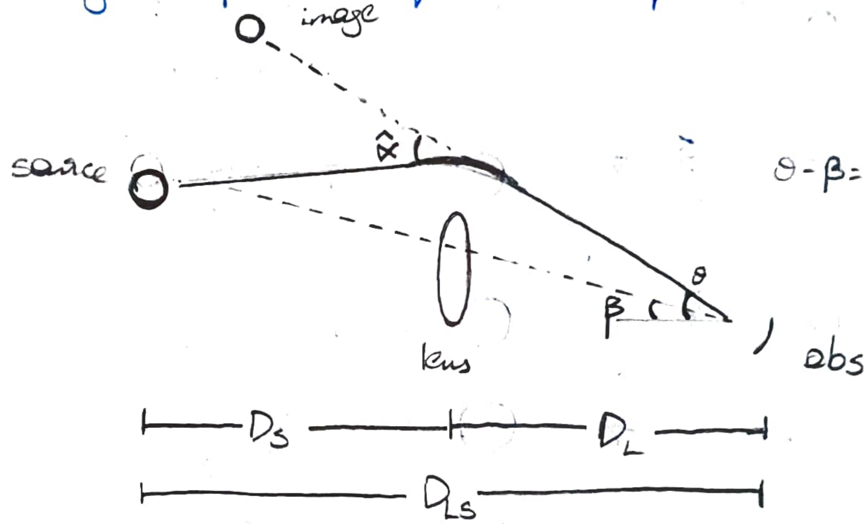
⇒ thin lens approximation is valid

Projected mass density, $\Sigma(\bar{\xi}) = \int \rho(\bar{\xi}', z)\, dz$

?? → vector in plane of the sky

$$\Rightarrow \hat{\alpha}(\bar{\xi}) = \frac{4G}{c^2}\int \frac{(\bar{\xi} - \bar{\xi}')}{|\bar{\xi} - \bar{\xi}'|^2}\, \Sigma(\bar{\xi}')\, d^2\xi'$$

Unlensed angular position of source $= \beta$



$\theta - \beta =$ reduced deflection angle $= \frac{D_{LS}}{D_S}\hat{\alpha}$

$$\boxed{\beta = \theta - \cancel{}\,\hat{\alpha}(D_L\theta)\,\frac{D_{SL}}{D_S}}$$

$\frac{D_{SL}}{D_S}$

Magnification: For a circularly symmetric lens,

magnification factor $\boxed{\mu_* = \frac{\theta}{\beta}\frac{d\theta}{d\beta}} = \frac{1}{\det(A_{ij})} = \frac{1}{((1-\kappa)^2 - \gamma^2)}$

$\det(A_{ij})$ ↳ inverse magnification matrix

Convergence, $\kappa(\theta) = \frac{\Sigma(\theta)}{\Sigma_{cr}}$ where $\Sigma_{cr} = \frac{c^2 D_S}{4\pi G D_{LS} D_L}$
??

Deflection potential, $\psi(\theta) = \frac{2D_{LS}}{D_L D_S c^2}\int \phi(D_L\theta, z)\,dz$
??

ϕ ↳ gravitational potential

Lensing Jacobian, $A_{ij} = \frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \psi}{\partial\theta_i \partial\theta_j}$ → symmetric
⇓
diagonal + traceless

$$= (1-\kappa)\,\mathbb{I}_{2\times2} - \gamma\underbrace{\begin{bmatrix}\cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi\end{bmatrix}}_{\text{shear}}$$

Since $\phi = \beta - \theta$ along x axis

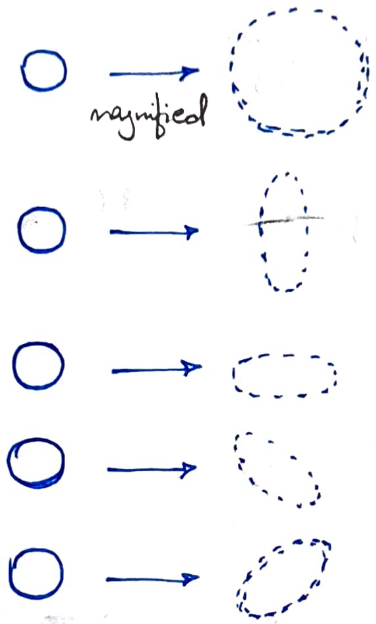
$K > 0 \Rightarrow$

$Re[\gamma] \lessgtr 0 \Rightarrow$

$Re[\gamma] \gtrless 0 \Rightarrow$

$Im[\gamma] < 0 \Rightarrow$

$Im[\gamma] > 0 \Rightarrow$



EINSTEIN RADIUS

- The radius of ~~the ring~~ an Einstein ring
- A characteristic for gravitational lensing
- Typical distances between images referred in units of R_E

$$\boxed{b = \theta D_L}$$ for small angles

$$\Rightarrow \hat{\alpha}(\theta) = \frac{4GM}{c^2} \frac{1}{D_L \theta}$$

Let $\theta_s = \beta$ (not observable)

Lens equation: $\boxed{\theta D_S = \theta_s D_S + \hat{\alpha} D_{LS}}$

$$\Rightarrow \hat{\alpha}(\theta) = \frac{D_S}{D_{LS}} (\theta - \theta_s)$$

Equating the two expressions for $\hat{\alpha}$

$$\Rightarrow (\theta - \theta_s) \frac{D_s}{D_{LS}} = \frac{4GM}{c^2} \frac{1}{\theta D_L}$$

$$\Rightarrow \boxed{R_E = \theta_E D_L}$$

where $\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_s D_L}}$

Lens equation $\Rightarrow \theta = \theta_s + \frac{\theta_E^2}{\theta}$.

θ_E provides a convenient linear scale to make dimensionless lensing variables