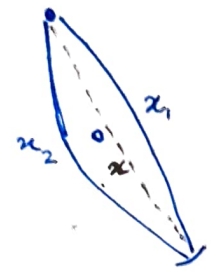
Date: ☐☐/☐☐/☐☐☐☐ ASB -13

Q No. Exercise No. ☐☐ Solved Problems: Sub ☐ Obj ☐

28.6.21



$\Delta x = c\Delta t$

Light travel time along the deflected light path is longer than along the undeflected path for two reasons:

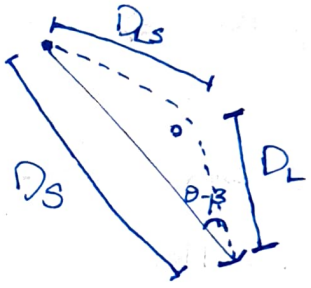
1. Deflected path is geometrically longer Δt_{geo}
2. Gravitational time delay (dilation) due to the local gravitational potential causing the deflection Δt_{grav} $= \Delta T_{shapiro}$

$\Delta x_1 = x_1 - x$

$\Delta x_2 = x_2 - x$

$\Delta x = |\Delta x_1 - \Delta x_2|$

Δt_{geo} in static, flat metric



$\Delta x = (D_{LS} + D_L) - D_S$

$D_S^2 + D_L^2 - 2D_S D_L \cos(\theta-\beta) = D_{LS}^2$

Taking small angle approximation $\cos x = 1 - \frac{x^2}{2}$

$\Rightarrow D_{LS}^2 = D_S^2 + D_L^2 - \left(2D_S D_L - D_S D_L(\theta-\beta)^2\right)$

$= (D_S - D_L)^2 + D_S D_L(\theta-\beta)^2$

Taking square root and for $x \ll 1$, $\sqrt{1+x} = 1 + \frac{1}{2}x$

$$D_{LS} = (D_S - D_L)\left[1 + \frac{\frac{1}{2}D_S D_L(\theta-\beta)^2}{(D_S-D_L)^2}\right]$$

$$= D_S - D_L + \frac{\frac{1}{2}D_S D_L(\theta-\beta)^2}{D_S - D_L} \quad \ldots$$

$$= \frac{2(D_S-D_L)^2 + D_S D_L(\theta-\beta)^2}{2(D_S-D_L)} = \frac{x^2 + \frac{1}{2}D_L D_S(\theta-\beta)^2}{x} = D_{LS}$$

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$$x^2 - D_{LS}\, x + \frac{1}{2} D_L D_S (\theta-\beta)^2 = 0$$

$$x = \frac{D_{LS}}{2} \pm \sqrt{D_{LS}^2 - 2 D_L D_S (\theta-\beta)^2} = D_S - D_L$$

→ Eq 8A Bartelmann M (2010) ??

$$\boxed{\Delta t_{geo} = \frac{1}{c} \frac{D_S D_L}{D_{LS}} \frac{(\theta-\beta)^2}{2}}$$

$$\Delta t_{grav} = \Delta t_{Shapiro} = -\frac{2}{c^3} \int \phi \, dl = -\frac{1}{c} \int \frac{2\phi}{c^2} dl$$

$$\Psi(\theta) = \frac{2 D_{LS}}{D_L D_S} \int \frac{\phi}{c^2} dl$$

$$\Rightarrow \boxed{\Delta t_{grav} = -\frac{1}{c} \frac{D_L D_S}{D_{LS}} \Psi(\theta)}$$

$$\Delta t(\theta) = \Delta t_{geo} + \Delta t_{grav} = \frac{1}{c} \frac{D_L D_S}{D_{LS}} \left[\frac{(\theta-\beta)^2}{2} - \Psi(\theta) \right]$$

Eq : 2.3 ?? $\Psi(\theta) = \frac{4GM}{c^2} \frac{D_{LS}}{D_S D_L} \log \theta$