Biomimicry of Bacterial Foraging
for Distributed Optimization and Control

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IEEE Control Systems Magazine, 2002

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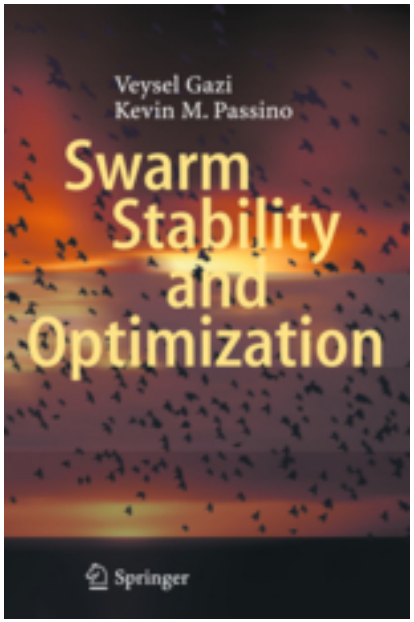
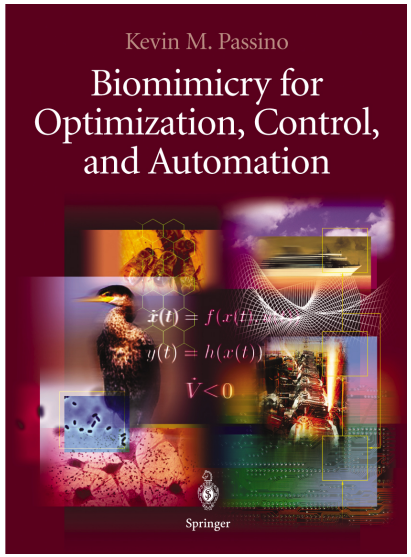
About the Author





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Foraging

Foraging

- searching for nutrients
- avoiding noxious stimuli (toxins, predators, etc)

Social Foraging

- increases likelihood of finding nutrients
- better detection and protection from noxious stimuli
- gains can offset cost of food competition

Foraging as Optimization

How can we view foraging as an Optimization Process?

- We have some parameters θ and a loss function $J(\theta)$ that we want to minimize

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- J can represent the concentration of nutrients and noxious stimuli
  - smaller values of J = more nutrients, less noxious stimuli
  - higher values of J = more noxious stimuli, less nutrients

Foraging as Optimization

How can we view foraging as an Optimization Process?

- We have some parameters θ and a loss function $J(\theta)$ that we want to minimize
- θ can represent the position of an organism in its environment
- J can represent the concentration of nutrients and noxious stimuli
  - smaller values of J = more nutrients, less noxious stimuli
  - higher values of J = more noxious stimuli, less nutrients
- In general, J and θ can be arbitrary
  - $\theta \in \mathbb{R}^p$
  - $J : \mathbb{R}^p \to \mathbb{R}$

E. coli

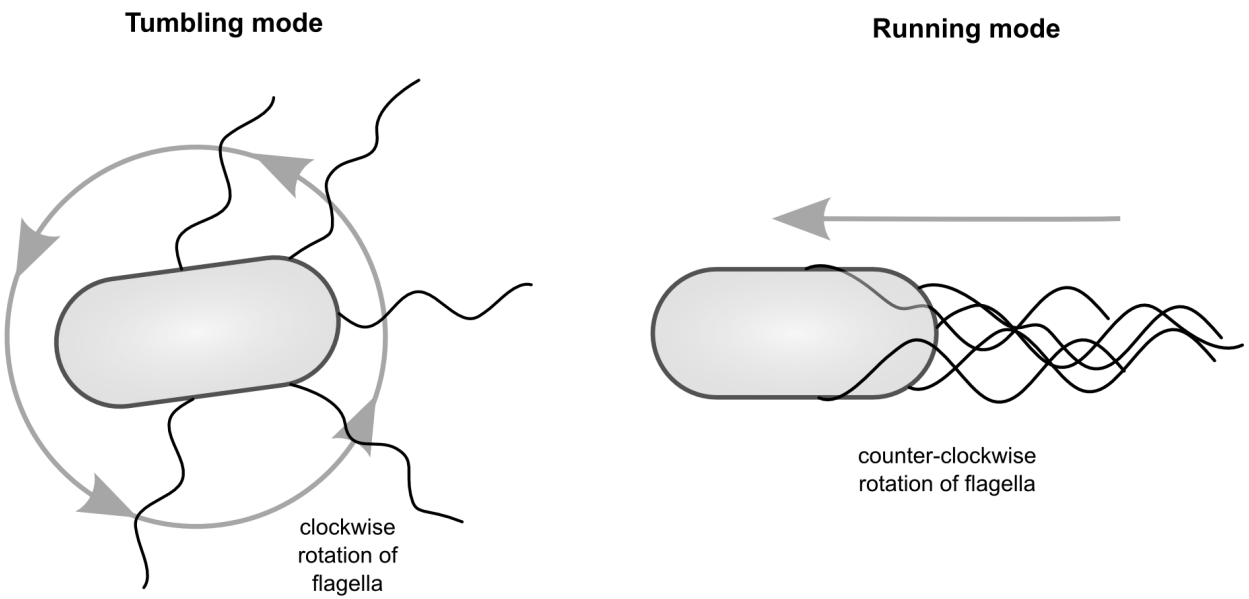
- Model organism
  - Highly studied
  - Well-characterized foraging behaviour
  - Probably won't feel bad about simplifying its behaviour

E. coli

- Model organism
  - Highly studied
  - Well-characterized foraging behaviour
  - Probably won't feel bad about simplifying its behaviour
- Social organism
  - Secretes signals to attract others nearby
  - Encourages "swarming" or "clumping"

E. coli Behaviour

- Swims using left-handed helical flagella ("propellers")
  - **Tumble**: flagella all rotate clockwise → pull on cell in all directions → random movement
  - **Run**: flagella all rotate counterclockwise → flagella form a bundle → push on cell in one direction → directed movement



E. coli Behaviour

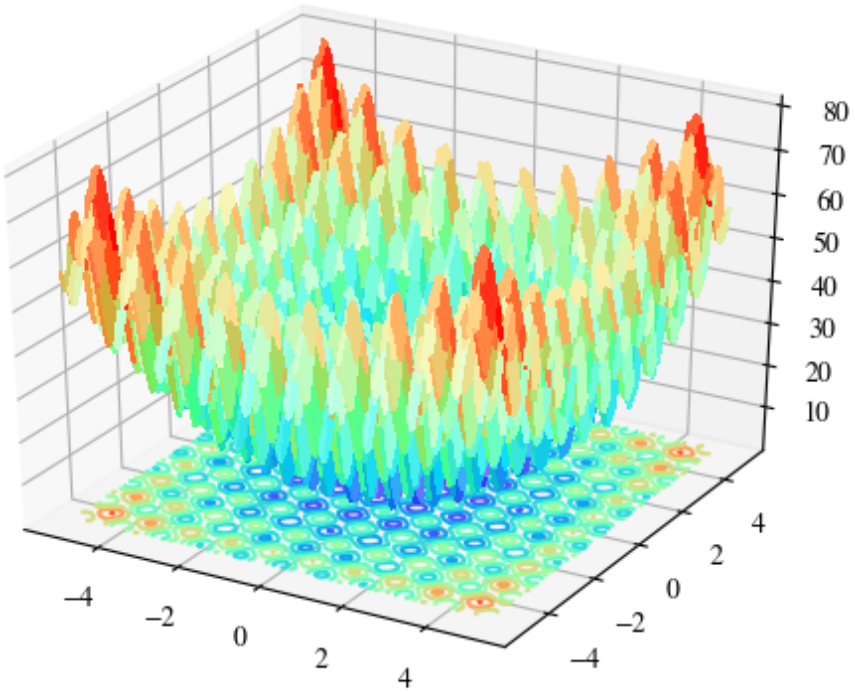
- If during a tumble *E. coli* swims down a nutrient concentration gradient:
  - Prolongs time spent on a run
  - Continues moving in the same direction
- Otherwise:
  - Tends to switch to a tumble (search for more)
  - Moves randomly which searching for more nutrient gradients to exploit
- Call a tumble followed by a run a "chemotaxis step"

Algorithm for a Single Bacterium

1: **for** $j \leftarrow 1 \ldots N_c$ **do**:
2: $\quad \phi \sim S^p$
3: $\quad \theta \leftarrow \theta + c\phi$
4: $\quad$ **while** $J(\theta + c\phi) < J(\theta)$ **do**:
5: $\quad\quad \theta \leftarrow \theta + c\phi$

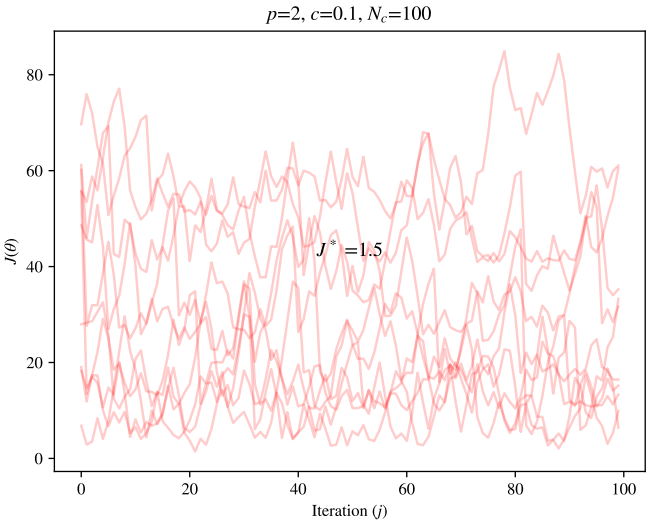
- θ: p-dimensional vector (randomly initialized)
- N_c: number of chemotaxis steps
- $\phi \sim S^p$: a random p-dimensional unit vector
- c: a step-size

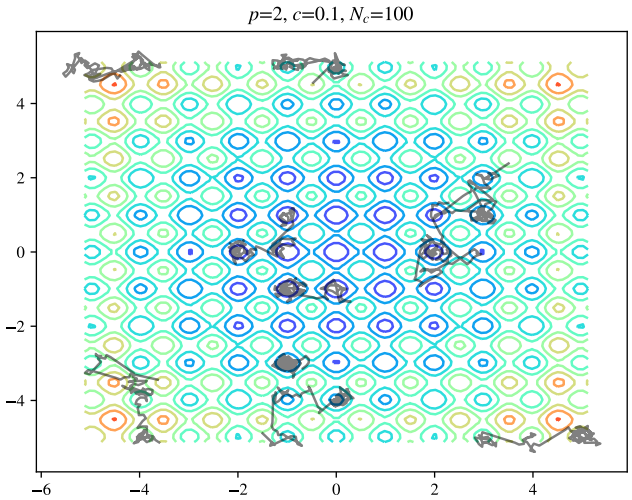
Loss Function to Optimize



$$J(\theta) = An + \sum_{i=1}^{n} \left(x_i^2 - A\cos(2\pi x_i) \right)$$

Results of Single Bacterium





- Relatively inconsistent performance for a highly nonconvex function

Algorithm for a Colony

1: **for** $j \leftarrow 1 \dots N_c$ **do**:
2: **for** $i \leftarrow 1 \dots S$ **do**:
3: $\phi \sim S^p$
4: $\theta_i \leftarrow \theta_i + c_i \phi$
5: **while** $J(\theta_i + c_i\phi) + J_{cc}(\theta_i + c_i\phi) < J(\theta_i) + J_{cc}(\theta_i)$ **do**:
6: $\theta_i \leftarrow \theta_i + c_i \phi$

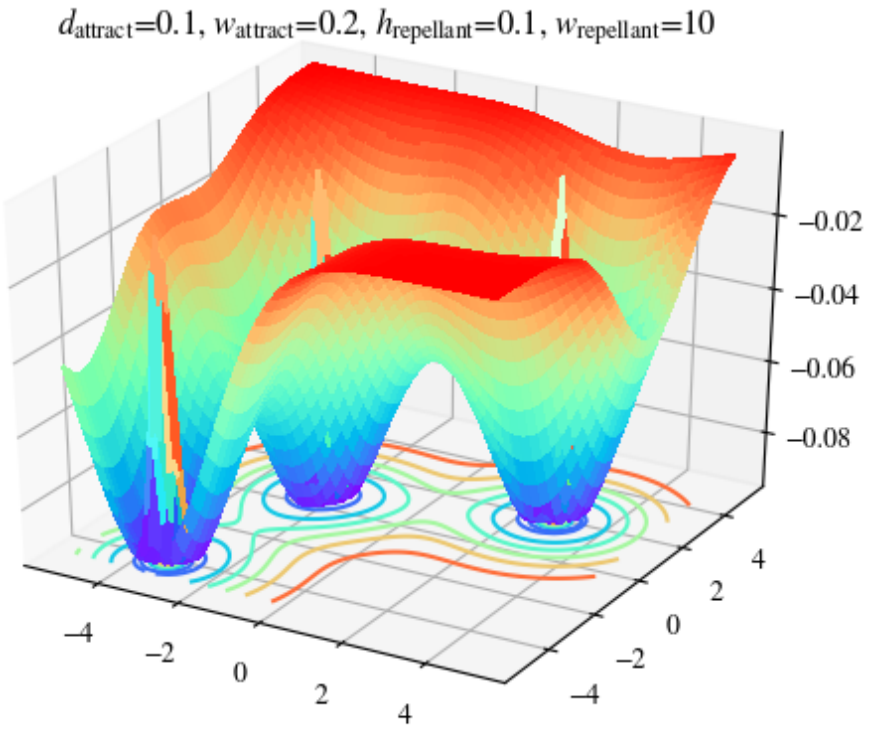
- θ_i: ith p-dimensional vector (randomly initialized)
- S: number of bacteria in the colony
- c_i: a step-size for bacterium i
- J_cc: cell-to-cell interactions

J_{cc} and swarming behaviour

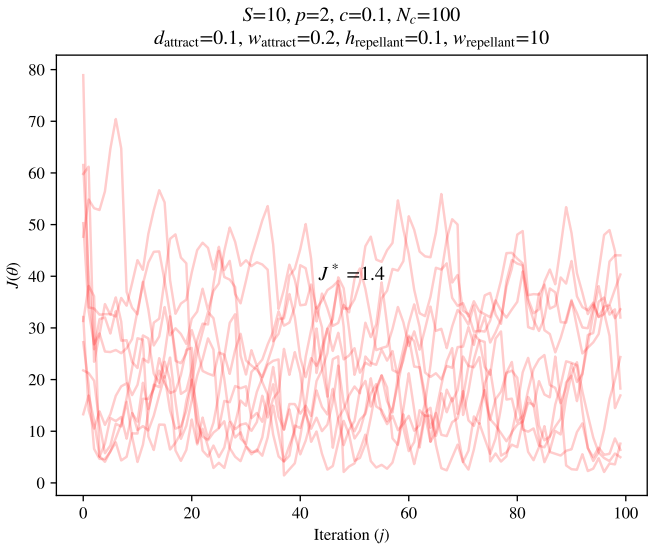
- *E. coli* do social foraging
- Secrete a substance to indicate to attract nearby *E. coli* and encourage swarming and biofilm formation
- Strength of signal diffuses over space
- Also want to avoid crowding
- Use sum of two Gaussian functions to model this

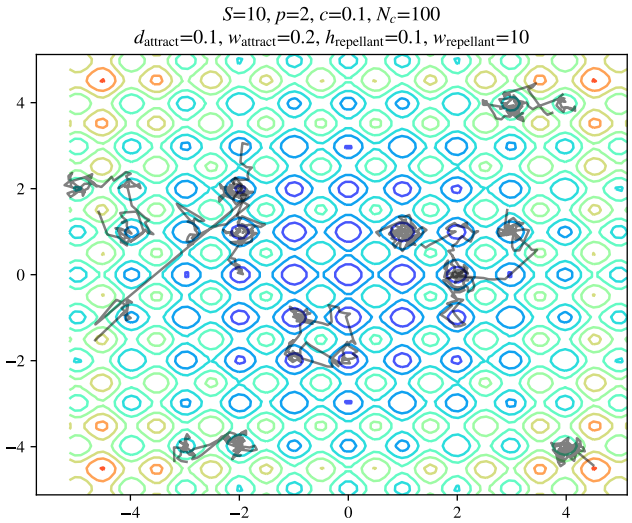
$$J_{cc}(\theta) = \sum_{i=1}^{S} - d_{\text{attract}} \exp\left(-w_{\text{attract}}(\theta - \theta_i)^T(\theta - \theta_i)\right) + h_{\text{repellant}} \exp\left(-w_{\text{repellant}}(\theta - \theta_i)^T(\theta - \theta_i)\right)$$

J_{cc} and swarming behaviour



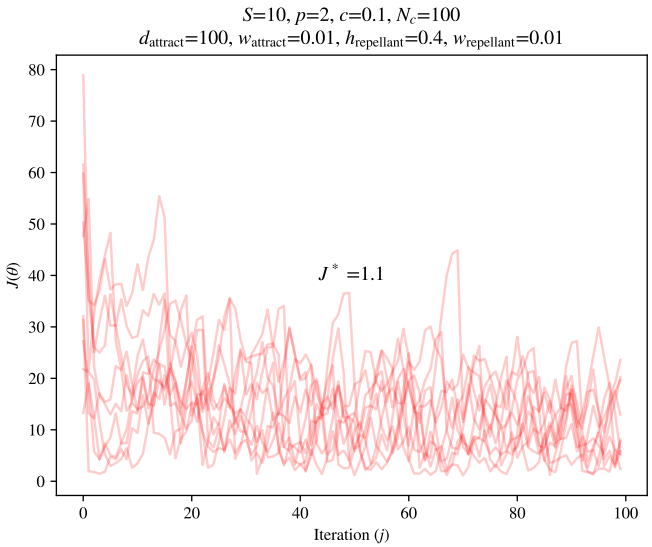
Results of Colony with Swarming

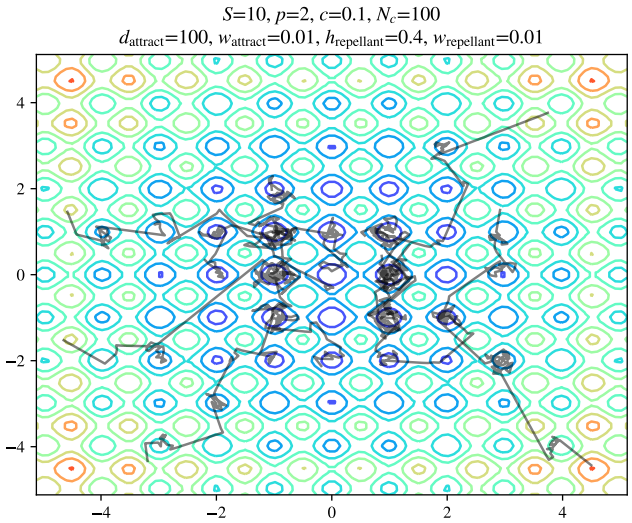




- Still relatively inconsistent performance for a highly nonconvex function
- But wait... What if the problem is just the hyperparameters?

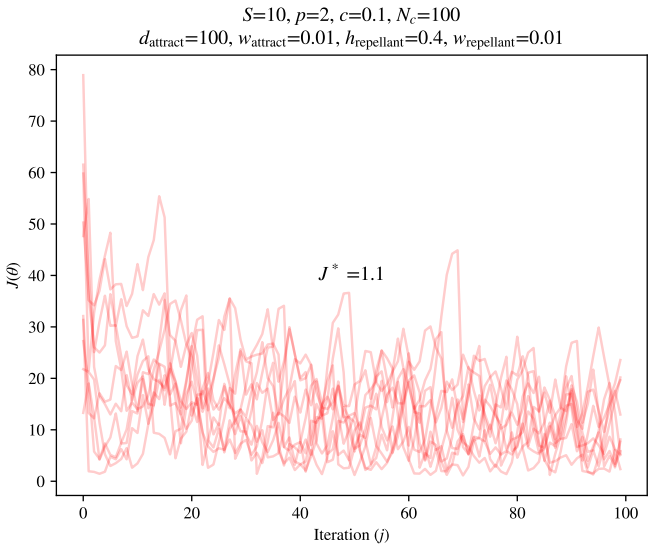
Results of Colony with Swarming

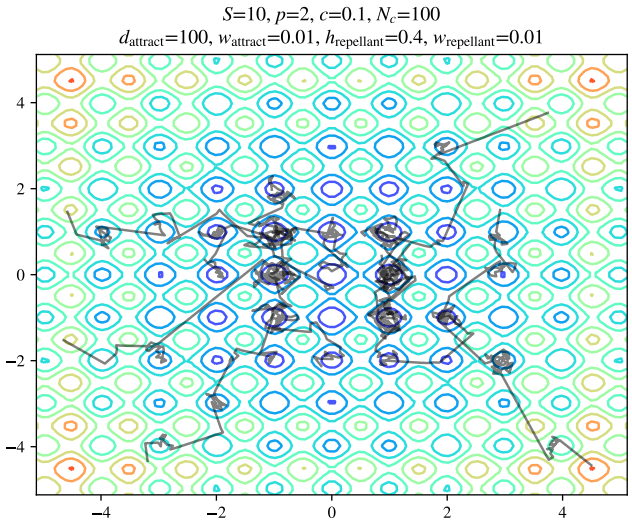




- By trying out different combinations of hyperparameters we can improve overall performance
- Here we increased the depth and width of attraction as well as the depth and width of repellance to increase "global" behaviour

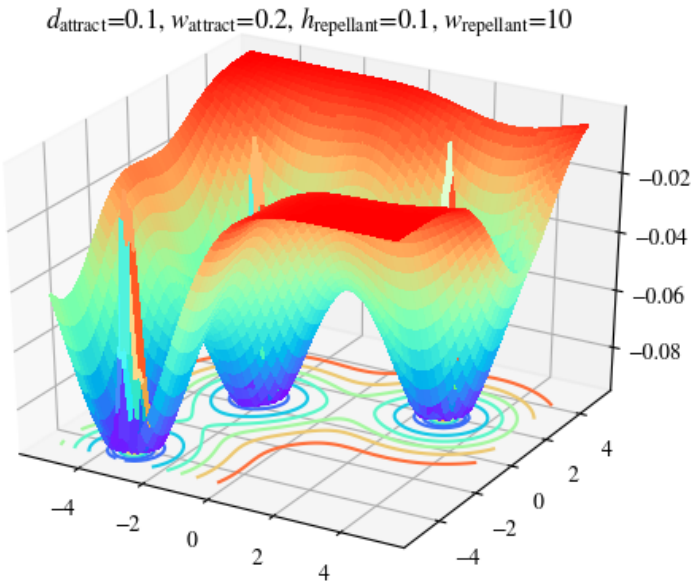
Results of Colony with Swarming

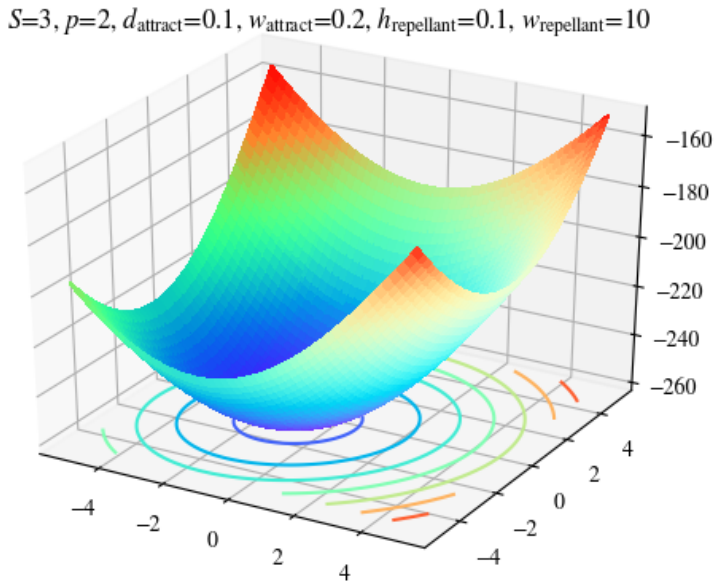




- By trying out different combinations of hyperparameters we can improve overall performance
- Here we increased the depth and width of attraction as well as the depth and width of repellance to increase "global" behaviour
- Important to know scale of J relative to scale of J_{cc} for tradeoff
  - Can think of this like hyperparameters c_1 and c_2 for PSO

Comparing J_{cc}





E. coli reproduction

- *E. coli* "reproduce" via
  1. **Binary fission**: essentially creating a clone
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- Algorithm designed to mimic binary fission
  - More fit individuals more likely to survive
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E. coli reproduction

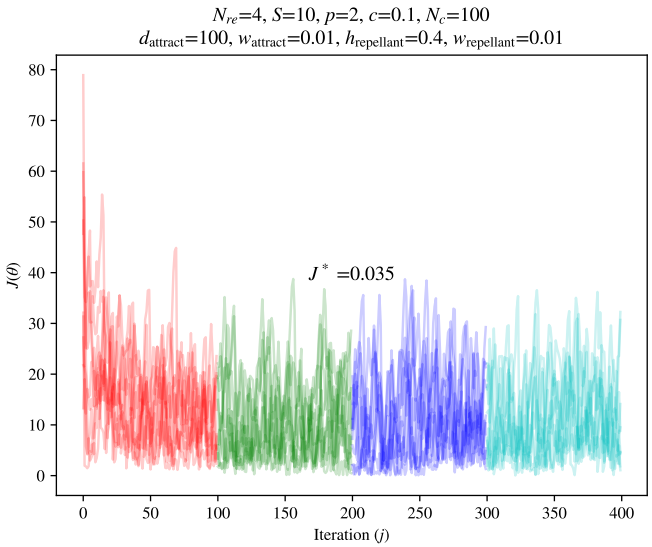
- *E. coli* "reproduce" via
  1. **Binary fission**: essentially creating a clone
  2. **Horizontal Translation**: merging genetic material with others
- Algorithm designed to mimic binary fission
  - More fit individuals more likely to survive
  - Less fit individuals more likely to die
- Horizontal translation could be incorporated (like a genetic algorithm)

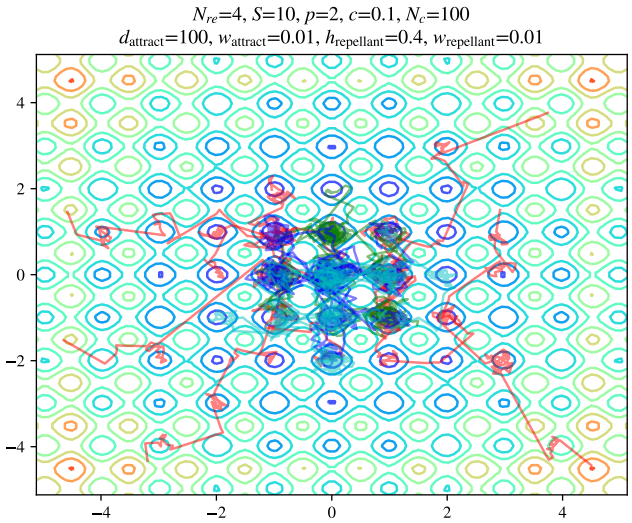
Algorithm for a Reproducing Colony

1: **for** $k \leftarrow 1 \ldots N_{re}$ **do**:
2: **for** $j \leftarrow 1 \ldots N_c$ **do**:
3: **for** $i \leftarrow 1 \ldots S$ **do**:
4: $\phi \sim S^p$
5: $\theta_i \leftarrow \theta_i + c_i \phi$
6: **while** $J(\theta_i + c_i\phi) + J_{cc}(\theta_i + c_i\phi) < J(\theta_i) + J_{cc}(\theta_i)$ **do**:
7: $\theta_i \leftarrow \theta_i + c_i \phi$
8: delete worst $S/2$ and reproduce best $S/2$

- N_{re}: number of reproduction steps

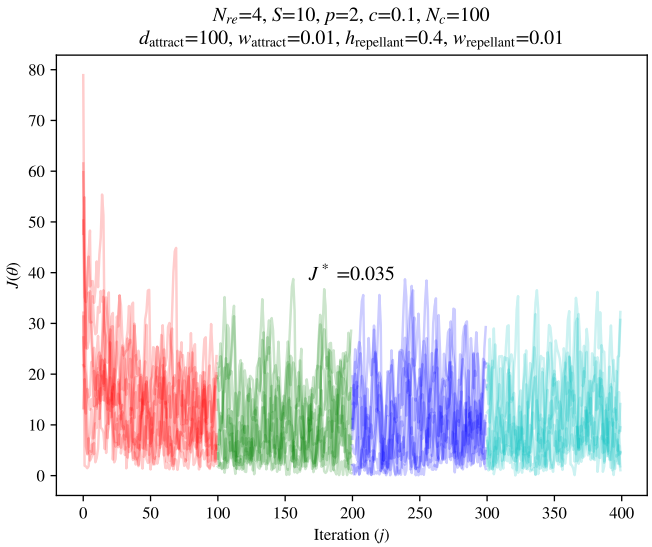
Results of Reproducing Colony

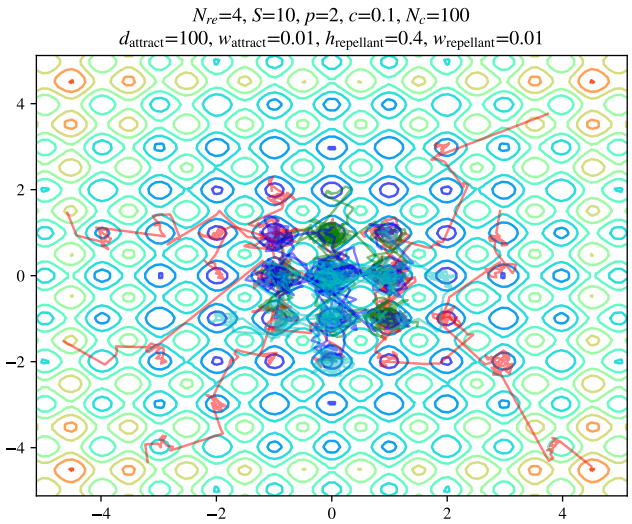




- Individuals with higher values of J killed off

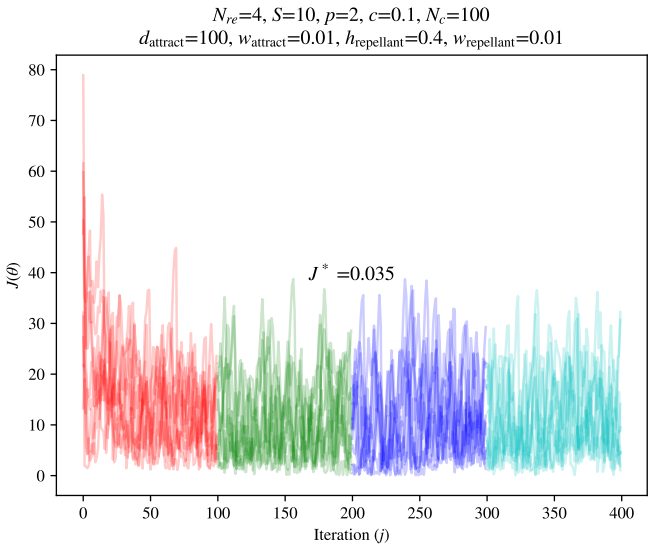
Results of Reproducing Colony

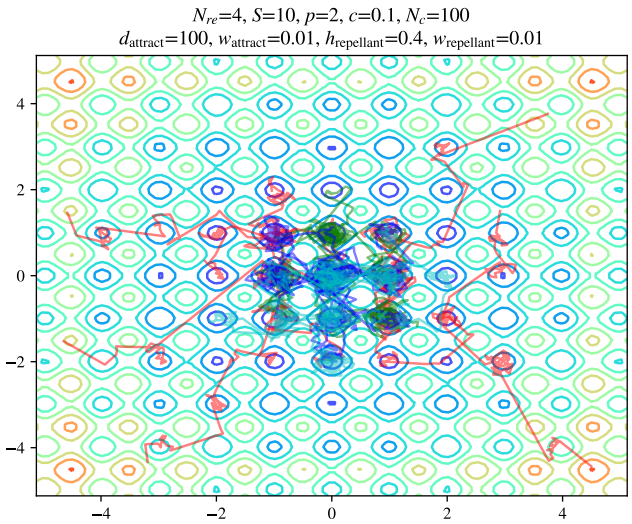




- Individuals with higher values of J killed off
- Individuals with lower values of J duplicated
  - Ideally move away due to repellance

Results of Reproducing Colony





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- Idea is to encourage searching in space nearby "best" individuals

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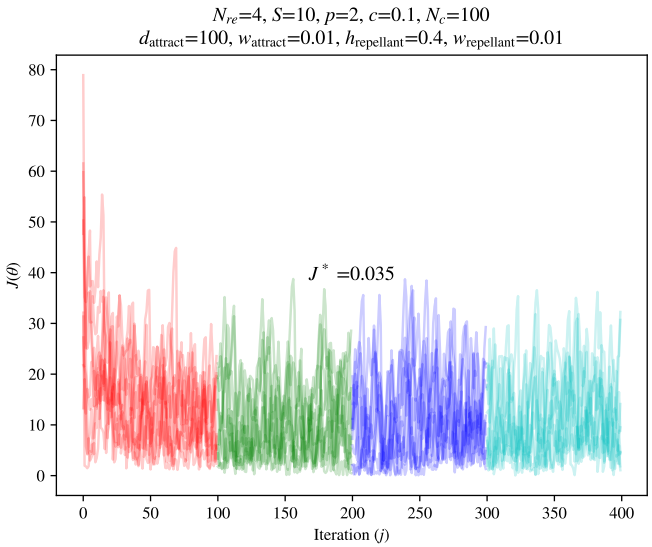
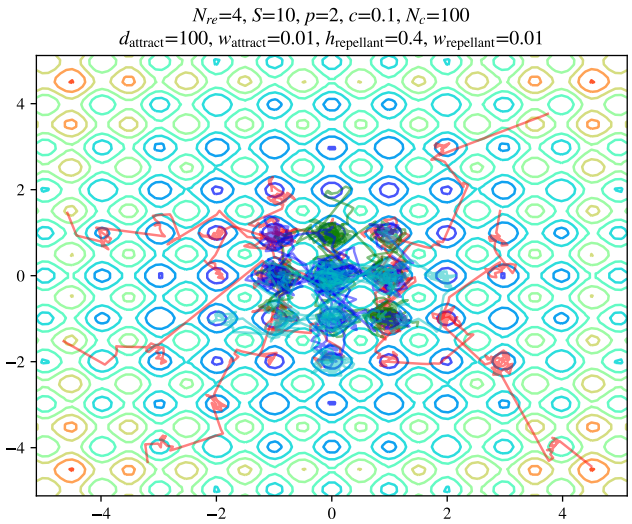
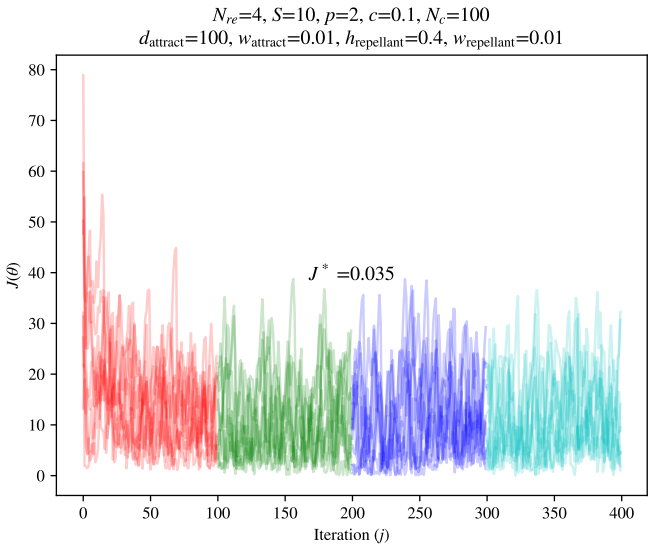
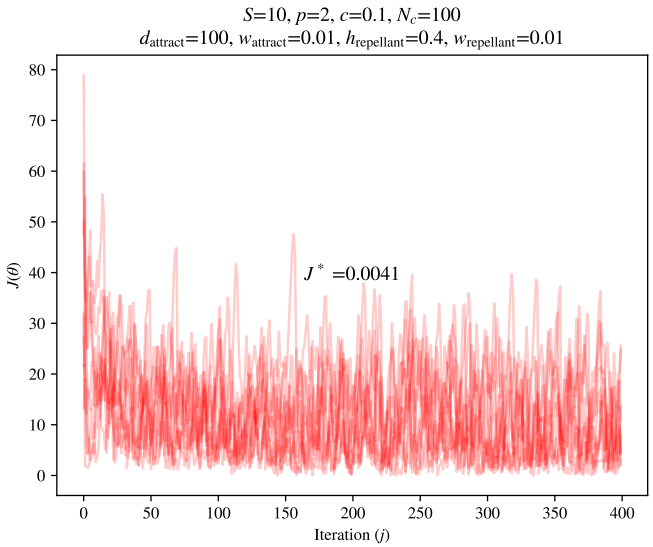


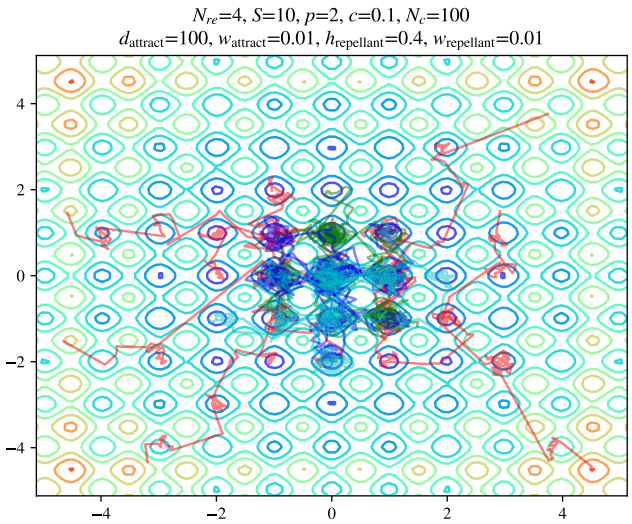


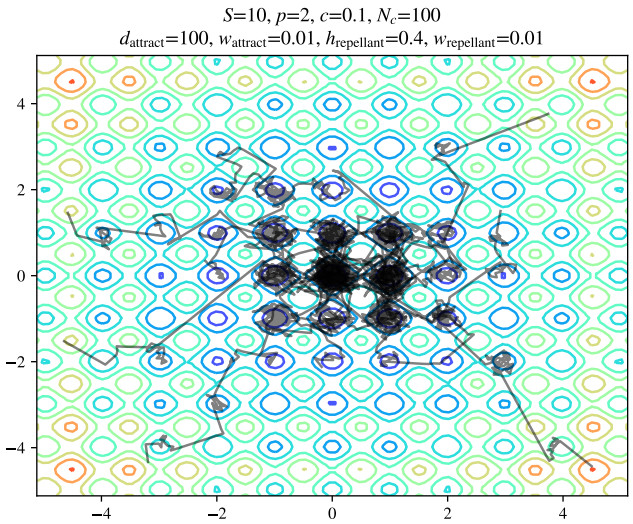
- Individuals with higher values of J killed off
- Individuals with lower values of J duplicated
  - Ideally move away due to repellance
- Idea is to encourage searching in space nearby "best" individuals
- If repellance isn't high enough then repeated iterations of evolution can concentrate colony in local minimum

Does Reproduction Help?

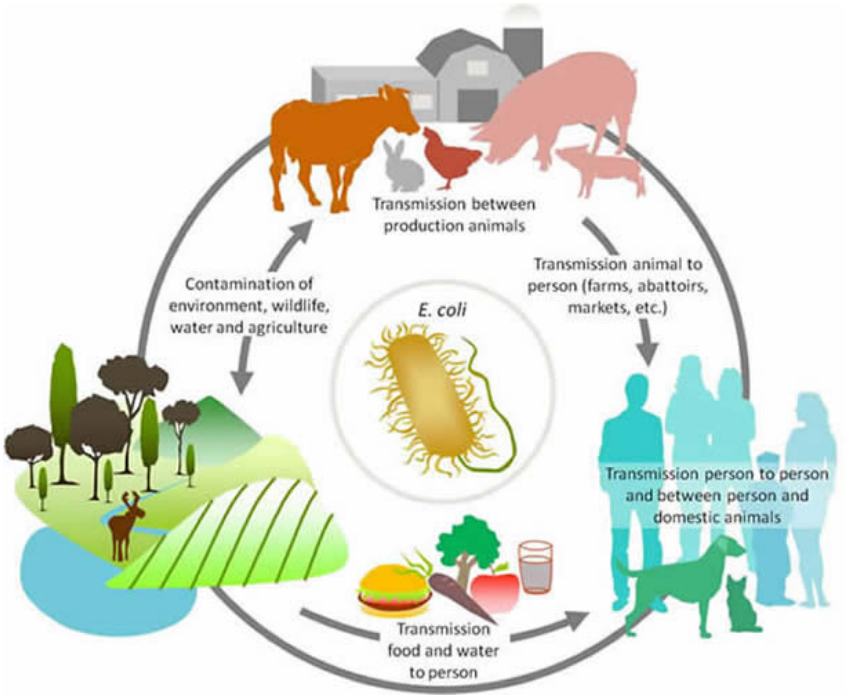






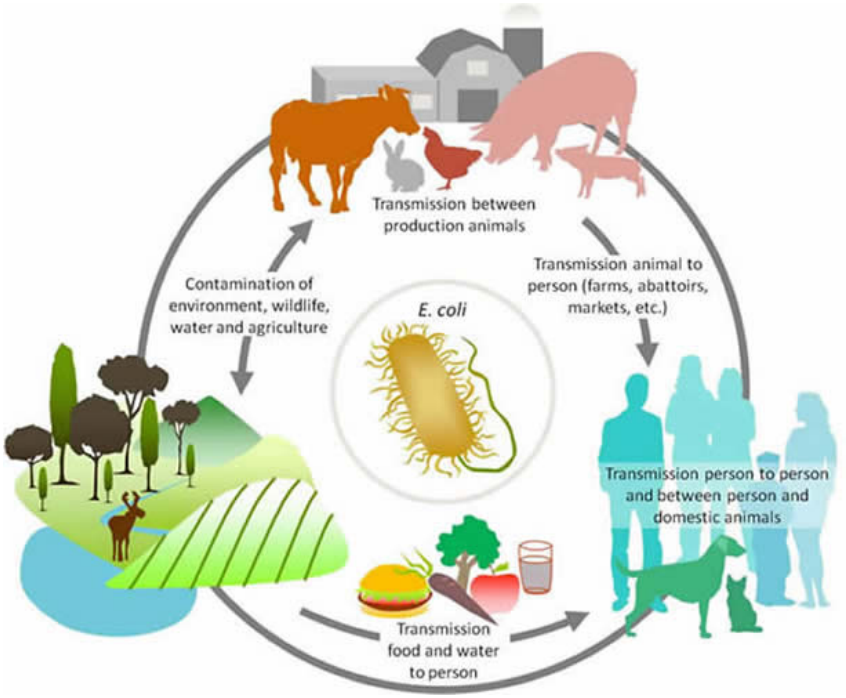


Elimination-Dispersal Events



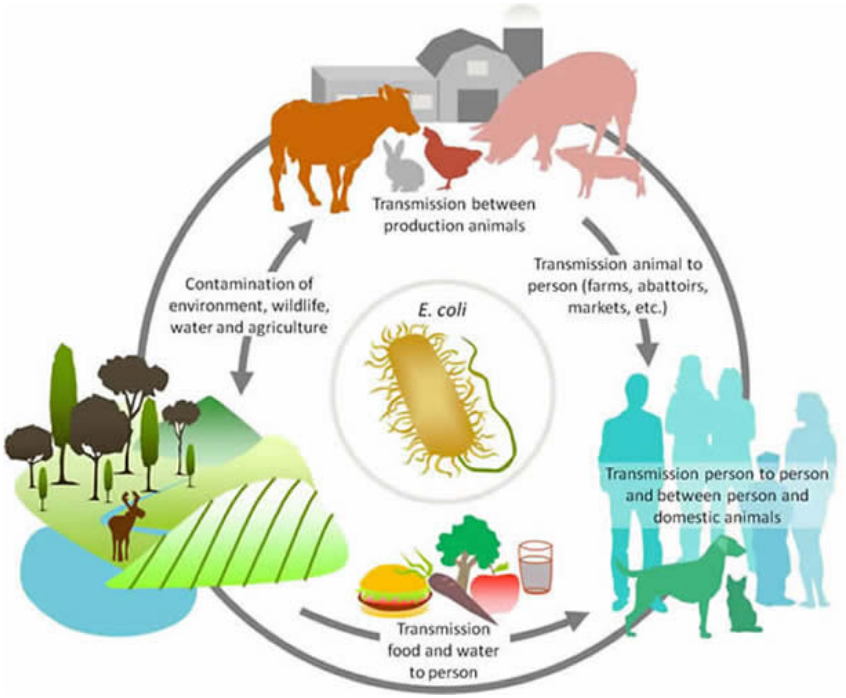
- Over time, random events disperse populations of *E. coli*
  - Water, animal activity, human intervention

Elimination-Dispersal Events



- Over time, random events disperse populations of *E. coli*
  - Water, animal activity, human intervention
- May destroy chemotactic progress
  - But may also bring *E. coli* to good food sources

Elimination-Dispersal Events



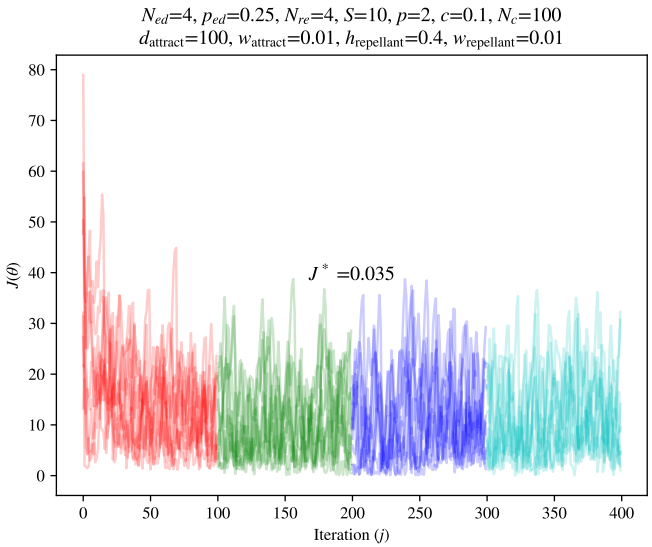
- Over time, random events disperse populations of *E. coli*
  - Water, animal activity, human intervention
- May destroy chemotactic progress
  - But may also bring *E. coli* to good food sources
- For optimization, this is a method to prevent stagnation and move out from local minima

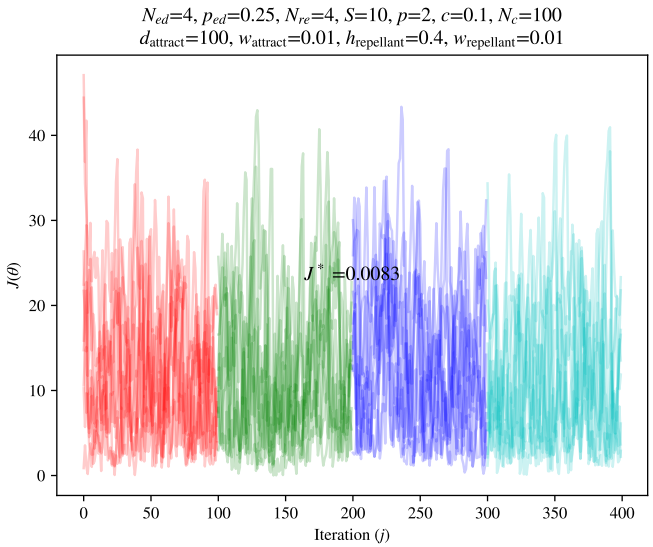
Algorithm for a Dispersing Colony

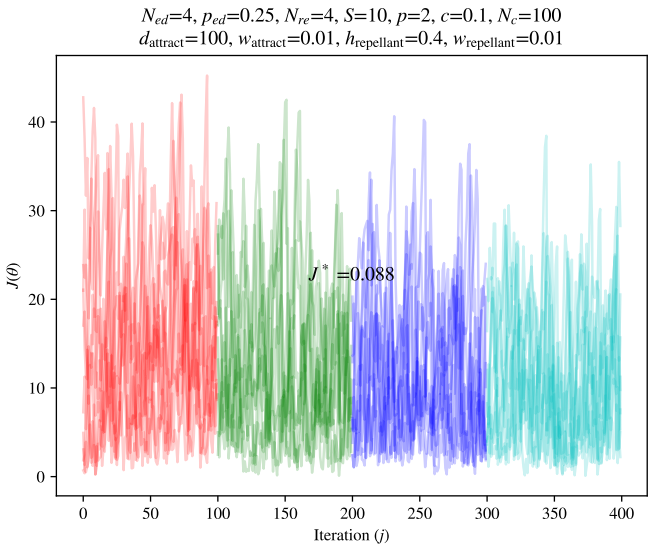
for $l \leftarrow 1 \ldots N_{ed}$ **do**:
  for $k \leftarrow 1 \ldots N_{re}$ **do**:
    for $j \leftarrow 1 \ldots N_c$ **do**:
      for $i \leftarrow 1 \ldots S$ **do**:
        $\phi \sim S^p$
        $\theta_i \leftarrow \theta_i + c_i \phi$
        while $J(\theta_i + c_i\phi) + J_{cc}(\theta_i + c_i\phi) < J(\theta_i) + J_{cc}(\theta_i)$ **do**:
          $\theta_i \leftarrow \theta_i + c_i \phi$
    delete worst $S/2$ and reproduce best $S/2$
  for $i \leftarrow 1 \ldots S$ **do**:
    if $\epsilon \sim \mathcal{U}(0,1) < p_{ed}$ **then**:
      $\theta_i \sim d^0(\theta)$

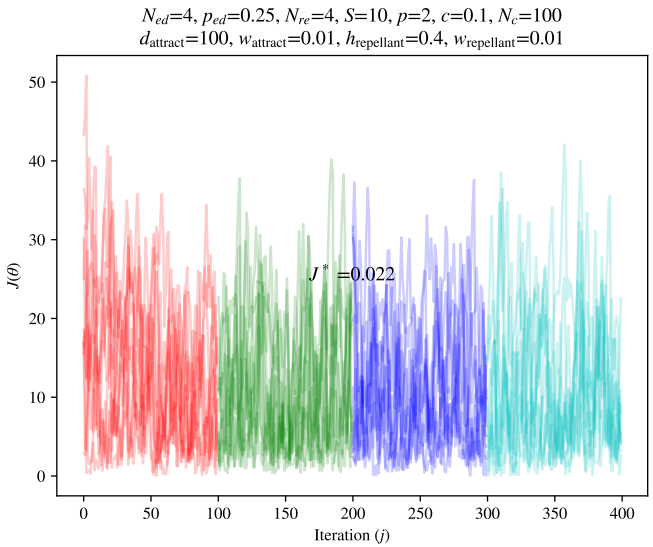
- N_{ed}: number of elimination-dispersal events
- p_{ed}: probabilty of a single elimination-dispersal event
- $d^0(\theta)$: initial distribution of θ

Does Elimination-Dispersal Help?









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- Chose a single seed arbitrarily to run experiments:
  - Results could vary if a different seed was chosen
- Hyperparameters were chosen by trying a few random combinations
  - Tried to avoid "overfitting" hyperparameters (abuse of terminology)

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  - (Ask me about the comparison I did afterwards!)

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- It can also explore the search space beyond the initial distribution $d^0(\theta)$ in case we do not know where the optimal value θ^* lies

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Is this a good model of *E. coli*?

Some important limitations from the authors:

- Ignore characteristics of actual biological processes in favor of simplicity and capturing the essence of chemotactic hill-climbing and swarming

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Some important limitations from the authors:

- Ignore characteristics of the chemical medium and assume that consumption does not affect the nutrient surface

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Some important limitations from the authors:

- They assume a constant population size, even if there are many nutrients and generations

Is this a good model of *E. coli*?

Some important limitations from the authors:

- They assume that the cells respond to nutrients in the environment in the same way that they respond to ones released by other cells for the purpose of signaling the desire to swarm

Is this a good paper?

Our objective is to explain how motile behaviors in both individual and groups of bacteria implement foraging and hence optimization.

to come up with a simple model that only represents certain aspects of the foraging behavior of bacteria.

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  - This information is later thrown away during development of the algorithm
- It is not clear if the goal is create a simulation of *E. coli* foraging behaviour or to create another biologically-inspired optimization algorithm

Discussion

Lessons Learned

- As with any stochastic algorithm, **your random seed matters**
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- As with any algorithm with lots of hyperparameters, tuning hyperparameters can be hard
  - For a fixed seed with a grid search you may find good hyperparameters
  - But those hyperparameters may not be robust so seed changes
- As with any algorithm, trying a single loss function is not usually sufficient
  - I chose rastrigin specifically for its difficulty
  - However, you run the risk of "overfitting" your algorithm and hyperparameters to a single problem

Comparison to PSO

- Bacterial foraging method:
  - At least $N_{ed} \times N_{re} \times S \times N_c$ values of θ seen
- PSO: $N \times$ iter values of θ seen
  - PSO strongest with large N, so presume iter $\equiv N_c$
  - Then $N = N_{ed} \times N_{re} \times S$

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