

# Biomimicry of Bacterial Foraging for Distributed Optimization and Control

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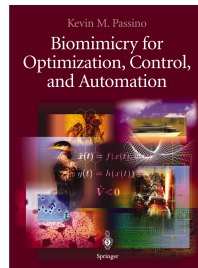
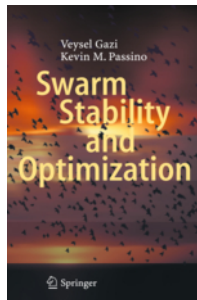
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IEEE Control Systems Magazine, 2002

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# About the Author



# About the Author



## Fuzzy control

KM Passino, S Yurkovich, M Reinfrank  
Addison-wesley 42, 15-21, 1998

3599

## Biomimicry of bacterial foraging for distributed optimization and control

KM Passino  
IEEE control systems magazine 22 (3), 52-67, 2002

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## Stability analysis of swarms

V Gazi, KM Passino  
IEEE transactions on automatic control 48 (4), 692-697, 2003

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## Stable adaptive control using fuzzy systems and neural networks

JT Spooner, KM Passino  
IEEE Transactions on Fuzzy Systems 4 (3), 339-359, 1996

728

## Stability analysis of social foraging swarms

V Gazi, KM Passino  
IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics) 34 ..., 2004

710

## Foraging

- searching for nutrients
- avoiding noxious stimuli (toxins, predators, etc)

## Social Foraging

- increases likelihood of finding nutrients
- better detection and protection from noxious stimuli
- gains can offset cost of food competition

# Foraging as Optimization

## How can we view foraging as an Optimization Process?

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- $J$  can represent the concentration of nutrients and noxious stimuli
  - ▶ smaller values of  $J$  = more nutrients, less noxious stimuli
  - ▶ higher values of  $J$  = more noxious stimuli, less nutrients



# Foraging as Optimization

## How can we view foraging as an Optimization Process?

- We have some parameters  $\theta$  and a loss function  $J(\theta)$  that we want to minimize
- $\theta$  can represent the position of an organism in its environment
- $J$  can represent the concentration of nutrients and noxious stimuli
  - ▶ smaller values of  $J$  = more nutrients, less noxious stimuli
  - ▶ higher values of  $J$  = more noxious stimuli, less nutrients
- In general,  $J$  and  $\theta$  can be arbitrary
  - ▶  $\theta \in \mathbb{R}^p$
  - ▶  $J : \mathbb{R}^p \rightarrow \mathbb{R}$

- Model organism

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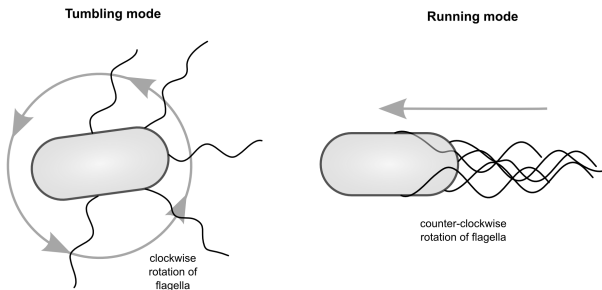
- Model organism
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- Model organism
  - ▶ Highly studied
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- Social organism
  - ▶ Secretes signals to attract others nearby
  - ▶ Encourages “swarming” or “clumping”



# *E. coli* Behaviour

- Swims using left-handed helical flagella (“propellers”)
  - ▶ **Tumble:** flagella all rotate clockwise → pull on cell in all directions → random movement
  - ▶ **Run:** flagella all rotate counterclockwise → flagella form a bundle → push on cell in one direction → directed movement



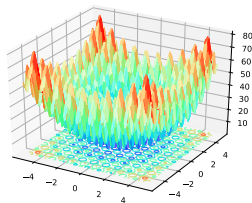
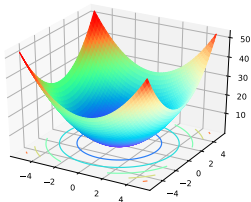
- If during a tumble *E. coli* swims down a nutrient concentration gradient:
  - ▶ Prolongs time spent on a run
  - ▶ Continues moving in the same direction
- Otherwise:
  - ▶ Tends to switch to a tumble (search for more)
  - ▶ Moves randomly while searching for more nutrient gradients to exploit
- Call a tumble followed by a run a “chemotaxis step”

# Algorithm for a Single Bacterium

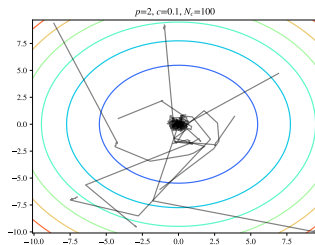
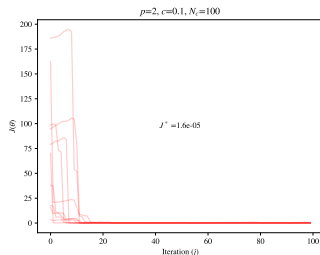
```
1: for  $j \leftarrow 1 \dots N_c$  do:  
2:    $\phi \sim \mathcal{U}$   
3:    $\theta \leftarrow \theta + c\phi$   
4:   while  $J(\theta + c\phi) < J(\theta)$  do:  
5:      $\theta \leftarrow \theta + c\phi$ 
```

- $\theta$ :  $p$ -dimensional vector (randomly initialized)
- $N_c$ : number of chemotaxis steps
- $\phi \sim \mathcal{U}$ : a random unit vector
- $c$ : a step-size

# Loss Function to Optimize

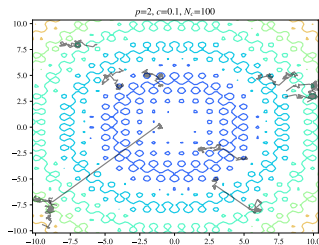
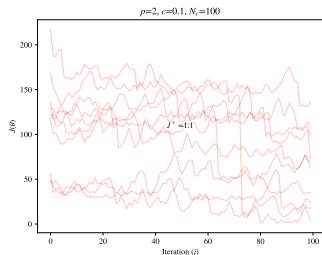


# Results of Single Bacterium



- Relatively consistent performance for a convex function.

# Results of Single Bacterium



- Relatively inconsistent performance for a highly nonconvex function.

# Algorithm for a Colony

```
1: for  $j \leftarrow 1 \dots N_c$  do:
2:   for  $i \leftarrow 1 \dots S$  do:
3:      $\phi \sim \mathcal{U}$ 
4:      $\theta_i \leftarrow \theta_i + c_i \phi$ 
5:     while  $J(\theta_i + c_i \phi) + J_{cc}(\theta_i + c_i \phi) < J(\theta_i) + J_{cc}(\theta_i)$  do:
6:        $\theta_i \leftarrow \theta_i + c_i \phi$ 
```

- $\theta_i$ :  $i$ th  $p$ -dimensional vector (randomly initialized)
- $S$ : number of bacteria in the colony
- $c_i$ : a step-size for bacterium  $i$

# $J_{cc}$ and swarming behaviour

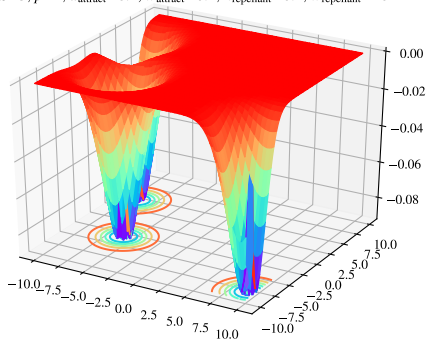
- *E. coli* do social foraging
- Secrete a substance to indicate to attract nearby *E. coli* and encourage swarming
- Strength of signal diffuses over space
- Use gaussian distribution to model this

$$J_{cc}(\theta) = \sum_{i=1}^S -d_{\text{attract}} \exp\left(-w_{\text{attract}}(\theta - \theta_i)^T(\theta - \theta_i)\right) \\ + h_{\text{repellant}} \exp\left(-w_{\text{repellant}}(\theta - \theta_i)^T(\theta - \theta_i)\right)$$

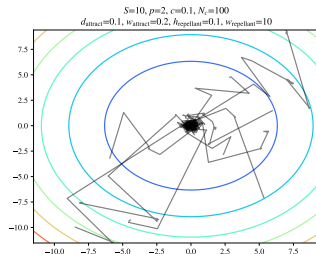
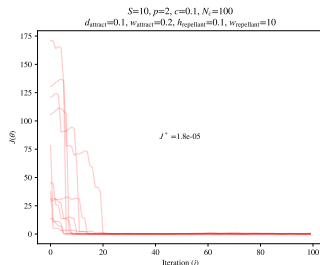


# $J_{cc}$ and swarming behaviour

$S=3, p=2, d_{\text{attract}}=0.1, w_{\text{attract}}=0.2, h_{\text{repellant}}=0.1, w_{\text{repellant}}=10$

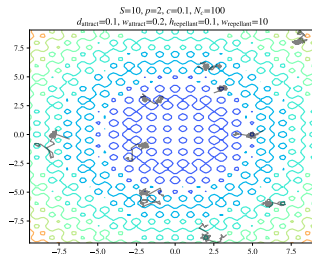
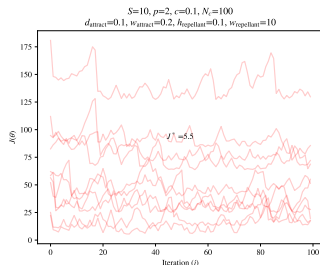


# Results of Single Bacterium



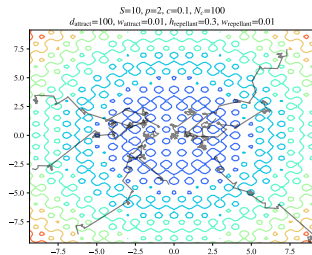
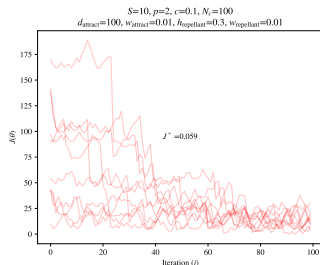
- Still relatively consistent performance for a convex function.
- Achieves similar performance to single bacterium.

# Results of Single Bacterium



- Still relatively inconsistent performance for a highly nonconvex function.
- But wait... What if the problem is just the hyperparameters?

# Results of Single Bacterium



- By trying out different combinations of hyperparameters we can improve overall performance.
- Here we increased the depth and width of attraction as well as the depth and width of repulsion to increase "global" behaviour.

# Algorithm for a Reproducing Colony

```
1: for  $k \leftarrow 1 \dots N_{re}$  do:
2:   for  $j \leftarrow 1 \dots N_c$  do:
3:     for  $i \leftarrow 1 \dots S$  do:
4:        $\phi \sim \mathcal{U}$ 
5:        $\theta_i \leftarrow \theta_i + c_i \phi$ 
6:       while  $J(\theta_i + c_i \phi) + J_{cc}(\theta_i + c_i \phi) < J(\theta_i) + J_{cc}(\theta_i)$  do:
7:          $\theta_i \leftarrow \theta_i + c_i \phi$ 
8:   delete worst  $S/2$  and reproduce best  $S/2$ 
```

- $N_{re}$ : number of reproduction steps

# Algorithm for a Reproducing Colony