## Biomimicry of Bacterial Foraging for Distributed Optimization and Control

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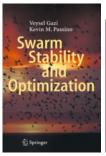
About the Author

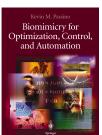
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### About the Author







### About the Author



Fuzzy control KM Passino, S Yurkovich, M Reinfrank Addison-wesley 42, 15-21, 1998	3599
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Stability analysis of swarms V Gazi, KM Passino IEEE transactions on automatic control 48 (4), 692-697, 2003	1125
Stable adaptive control using fuzzy systems and neural networks JT Spooner, KM Passino IEEE Transactions on Fuzzy Systems 4 (3), 339-359, 1996	728
Stability analysis of social foraging swarms  V Gazi, KM Passino IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics) 34, 2004	710

## Foraging

### Foraging

- searching for nutrients
- avoiding noxious stimuli (toxins, predators, etc)

### **Social Foraging**

- increases likelihood of finding nutrients
- better detection and protection from noxious stimuli
- gains can offset cost of food competition

#### How can we view foraging as an Optimization Process?

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- We have some parameters  $\theta$  and a loss function  $J(\theta)$  that we want to minimize
- $\bullet$   $\theta$  can represent the position of an organism in its environment
- J can represent the concentration of nutrients and noxious stimuli
  - $\triangleright$  smaller values of J = more nutrients, less noxious stimuli
  - $\blacktriangleright$  higher values of J= more noxious stimuli, less nutrients

#### How can we view foraging as an Optimization Process?

- We have some parameters  $\theta$  and a loss function  $J(\theta)$  that we want to minimize
- $\bullet$  d can represent the position of an organism in its environment
- J can represent the concentration of nutrients and noxious stimuli
  - $\triangleright$  smaller values of J= more nutrients, less noxious stimuli
  - $\blacktriangleright$  higher values of J= more noxious stimuli, less nutrients
- In general, J and  $\theta$  can be arbitrary
  - $\bullet$   $\theta \in \mathbb{R}^p$
  - $J: \mathbb{R}^p \to \mathbb{R}$

### E. coli

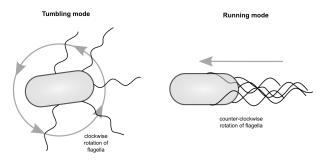
- Model organism
  - ► Highly studied
  - ▶ Well-characterized foraging behaviour
  - Probably won't feel bad about simplifying its behaviour

#### E. coli

- Model organism
  - Highly studied
  - Well-characterized foraging behaviour
  - Probably won't feel bad about simplifying its behaviour
- Social organism
  - Secretes signals to attract others nearby
  - ► Encourages "swarming" or "clumping"

### E. coli Behaviour

- Swims using left-handed helical flagella ("propellers")
  - ► Tumble: flagella all rotate clockwise → pull on cell in all directions → random movement
  - ▶ Run: flagella all rotate counterclockwise → flagella form a bundle
     → push on cell in one direction → directed movement



### E. coli Behaviour

- If during a tumble *E. coli* swims down a nutrient concentration gradient:
  - ▶ Prolongs time spent on a run
  - ▶ Continues moving in the same direction
- Otherwise:
  - ► Tends to switch to a tumble (search for more)
  - Moves randomly which searching for more nutrient gradients to exploit
- Call a tumble followed by a run a "chemotaxis step"

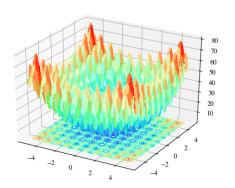
# Algorithm for a Single Bacterium

- 1: **for**  $j \leftarrow 1 \dots N_c$  **do**: 2:  $\phi \sim S^p$ 3:  $\theta \leftarrow \theta + c\phi$ 4: **while**  $J(\theta + c\phi) < J(\theta)$  **do**: 5:  $\theta \leftarrow \theta + c\phi$ 
  - $\theta$ : p-dimensional vector (randomly initialized)
  - $N_c$ : number of chemotaxis steps
  - $\phi \sim S^p$ : a random p-dimensional unit vector
  - c: a step-size



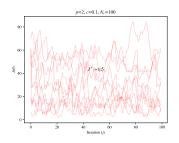
Passino; Van de Kleut (OST; UW)

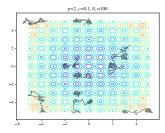
## Loss Function to Optimize



$$J(\theta) = An + \sum_{i=1}^{n} \left( x_i^2 - A\cos(2\pi x_i) \right)$$

## Results of Single Bacterium





• Relatively inconsistent performance for a highly nonconvex function.

# Algorithm for a Colony

```
1: for j \leftarrow 1 \dots N_c do:

2: for i \leftarrow 1 \dots S do:

3: \phi \sim S^p

4: \theta_i \leftarrow \theta_i + c_i \phi

5: while J(\theta_i + c_i \phi) + J_{cc}(\theta_i + c_i \phi) < J(\theta_i) + J_{cc}(\theta_i) do:

6: \theta_i \leftarrow \theta_i + c_i \phi
```

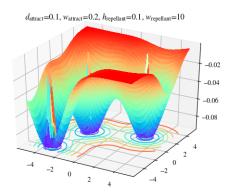
- $\theta_i$ : ith p-dimensional vector (randomly initialized)
- S: number of bacteria in the colony
- $c_i$ : a step-size for bacterium i
- $J_cc$ : cell-to-cell interactions

## $J_{cc}$ and swarming behaviour

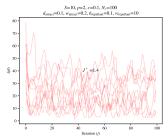
- E. coli do social foraging
- Secrete a substance to indicate to attract nearby *E. coli* and encourage swarming and biofilm formation
- Strength of signal diffuses over space
- Also want to avoid crowding
- Use sum of two Gaussian functions to model this

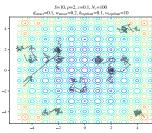
$$J_{cc}(\theta) = \sum_{i=1}^{S} -d_{\text{attract}} \exp\left(-w_{\text{attract}}(\theta - \theta_i)^T (\theta - \theta_i)\right) + h_{\text{repellant}} \exp\left(-w_{\text{repellant}}(\theta - \theta_i)^T (\theta - \theta_i)\right)$$

## $J_{cc}$ and swarming behaviour



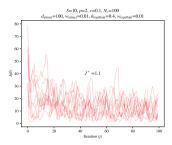
## Results of Colony with Swarming

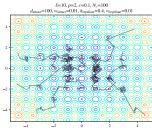




- Still relatively inconsistent performance for a highly nonconvex function.
- But wait... What if the problem is just the hyperparameters?

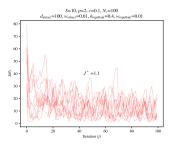
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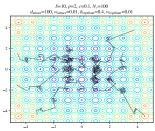




- By trying out different combinations of hyperparameters we can improve overall performance.
- Here we increased the depth and width of attraction as well as the depth and width of repellance to increase "global" behaviour.

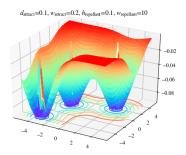
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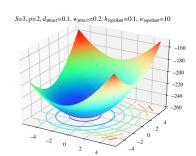




- By trying out different combinations of hyperparameters we can improve overall performance.
- Here we increased the depth and width of attraction as well as the depth and width of repellance to increase "global" behaviour.
- Important to know scale of J relative to scale of  $J_{cc}$  for tradeoff.
  - Can think of this like hyperparameters for PSO

# Comparing $J_{cc}$





## E. coli reproduction

- E. coli "reproduce" via
  - **1 Binary fission**: essentially creating a clone
  - **②** Horizontal Translation: merging genetic material with others

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  - ▶ More fit individuals more likely to survive
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## E. coli reproduction

- E. coli "reproduce" via
  - **1 Binary fission**: essentially creating a clone
  - **②** Horizontal Translation: merging genetic material with others
- Algorithm designed to mimic binary fission
  - ▶ More fit individuals more likely to survive
  - ▶ Less fit individuals more likely to die
- Horizontal translation could be incorporated (like a genetic algorithm)

# Algorithm for a Reproducing Colony

```
1: for k \leftarrow 1 \dots N_{re} do:

2: for j \leftarrow 1 \dots N_c do:

3: for i \leftarrow 1 \dots S do:

4: \phi \sim S^p

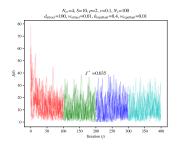
5: \theta_i \leftarrow \theta_i + c_i \phi

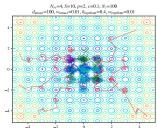
6: while J(\theta_i + c_i \phi) + J_{cc}(\theta_i + c_i \phi) < J(\theta_i) + J_{cc}(\theta_i) do:

7: \theta_i \leftarrow \theta_i + c_i \phi

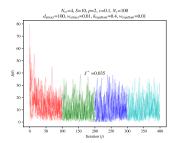
8: delete worst S/2 and reproduce best S/2
```

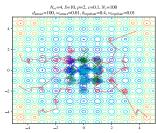
•  $N_{re}$ : number of reproduction steps



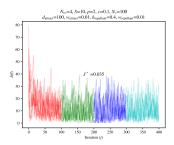


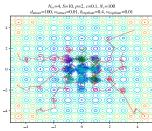
• Individuals with higher values of J killed off



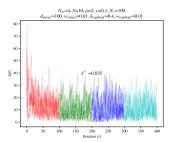


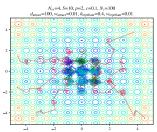
- Individuals with higher values of J killed off
- Individuals with lower values of J duplicated
  - ► Ideally move away due to repellance





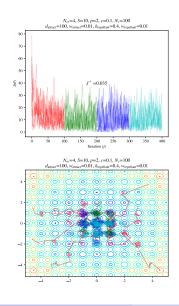
- Individuals with higher values of J killed off
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- Idea is to encourage searching in space nearby "best" individuals

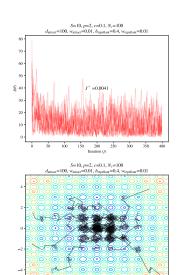




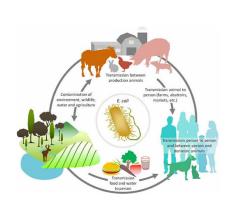
- Individuals with higher values of J killed off
- Individuals with lower values of *J* duplicated
  - ► Ideally move away due to repellance
- Idea is to encourage searching in space nearby "best" individuals
- If repellance isn't high enough then repeated iterations of evolution can concentrate colony in local minimum

## Does Reproduction Help?



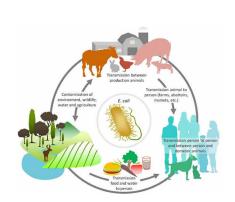


## Elimination-Dispersal Events



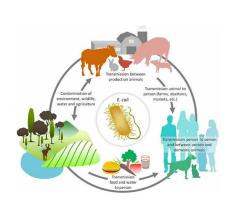
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  - ► Water, animal activity, human intervention

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- May destroy chemotactic progress
  - ▶ But may also bring *E. coli* to good food sources

## Elimination-Dispersal Events



- Over time, random events disperse populations of *E. coli* 
  - Water, animal activity, human intervention
- May destroy chemotactic progress
  - ▶ But may also bring *E. coli* to good food sources
- For optimization, this is a method to prevent stagnation and move out from local minima

# Algorithm for a Dispersing Colony

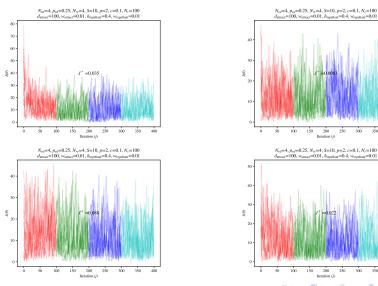
```
1: for l \leftarrow 1 \dots N_{ed} do:
            for k \leftarrow 1 \dots N_{re} do:
 2:
                 for i \leftarrow 1 \dots N_c do:
 3:
                       for i \leftarrow 1 \dots S do:
 4:
                            \phi \sim S^p
 5:
                            \theta_i \leftarrow \theta_i + c_i \phi
 6:
                             while J(\theta_i + c_i\phi) + J_{cc}(\theta_i + c_i\phi) < J(\theta_i) + J_{cc}(\theta_i) do:
 7:
                                   \theta_i \leftarrow \theta_i + c_i \phi
 8:
                 delete worst S/2 and reproduce best S/2
 9:
            for i \leftarrow 1 \dots S do:
10:
                 if \epsilon \sim \mathcal{U}(0,1) < p_{ed} then:
11:
```

- $N_{ed}$ : number of elimination-dispersal events
- $p_{ed}$ : probabilty of a single elimination-dispersal event
- $d^0(\theta)$ : initial distribution of  $\theta$

 $\theta_i \sim d^0(\theta)$ 

12:

## Does Elimination-Dispersal Help?



150 200 250 300

#### Caveats

- This is only a single application:
  - ► A single highly nonconvex function
  - Should try out other loss functions and applications (see post-presentation slides)

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### Caveats

- This is only a single application:
  - ▶ A single highly nonconvex function
  - Should try out other loss functions and applications (see post-presentation slides)
- Chose a single seed arbitrarily to run experiments:
  - ▶ Results could vary if a different seed was chosen
- Hyperparameters were chosen by trying a few random combinations
  - ► Tried to avoid "overfitting" hyperparameters (abuse of terminology)

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- The author draws a comparison to genetic algorithms (GAs) since both are hill-climbing algorithms, but asserts that they are really different algorithms based on their biological inspiration.
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- The author draws a comparison to genetic algorithms (GAs) since both are hill-climbing algorithms, but asserts that they are really different algorithms based on their biological inspiration.
  - ▶ Is this really enough to make it a novel algorithm?
- The method is also suspiciously similar to partical swarm optimization
  - ▶ Combines local and global information with stochastic hill-climbing algorithm.
  - ▶ The authors never mention this despite the popularity of PSOs (this paper publish 7 years later).

 $\bullet$  However, this algorithm is  ${\bf gradient\text{-}free}.$ 

- However, this algorithm is **gradient-free**.
  - ▶ We can minimize functions that we may not have access to the gradient for (or it may not exist)
  - ▶ For example (from the paper) fitting model parameters for dynamic control problem
  - ▶ Or for example fitting the parameters of a neural network to perform a task

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  - ▶ We can minimize functions that we may not have access to the gradient for (or it may not exist)
  - ► For example (from the paper) fitting model parameters for dynamic control problem
  - ▶ Or for example fitting the parameters of a neural network to perform a task
- It can also explore the search space beyond the initial distribution  $d^0(\theta)$  in case we do not know where the optimal value  $\theta^*$  lies.

• We don't know. The authors don't compare to any ecological data.

#### Some important limitations from the authors:

• Ignore characteristics of actual biological processes in favor of simplicity and capturing the essence of chemotactic hill-climbing and swarming.

Some important limitations from the authors:

• Ignore characteristics of the chemical medium and assume that consumption does not affect the nutrient surface.

Some important limitations from the authors:

• They assume a constant population size, even if there are many nutrients and generations.

Some important limitations from the authors:

• They assume that the cells respond to nutrients in the environment in the same way that they respond to ones released by other cells for the purpose of signaling the desire to swarm.

### Is this a good paper?

Our objective is to explain how motile behaviors in both individual and groups of bacteria implement foraging and hence optimization. to come up with a simple model that only represents certain aspects of the foraging behavior of bacteria. Is this a good paper?

 A significant portion of the paper is dedicated to discussing biology and foraging. This information is later thrown away during development of the algorithm.

## Is this a good paper?

- A significant portion of the paper is dedicated to discussing biology and foraging. This information is later thrown away during development of the algorithm.
- It is not clear if the goal is create a simulation of *E. coli* foraging behaviour or to create another biologically-inspired optimization algorithm.

### Discussion