Biomimicry of Bacterial Foraging

for Distributed Optimization and Control

Kevin M. Passino[1]
Presented by: Alexander Van de Kleut[2]

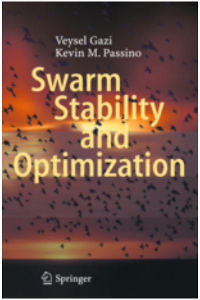
[1] The Ohio State University
Electrical and Computer Engineering

[2] University of Waterloo
Centre for Theoretical Neuroscience

IEEE Control Systems Magazine, 2002

Table of Contents

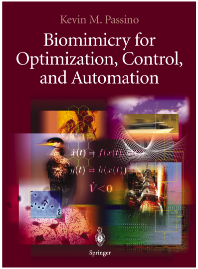
About the Author



Veysel Gazi
Kevin M. Passino

Swarm Stability and Optimization

Springer



Kevin M. Passino

Biomimicry for Optimization, Control, and Automation

Springer

About the Author

Fuzzy control 3599
KM Passino, S Yurkovich, M Reinfrank
Addison-wesley 42, 15-21, 1998

Biomimicry of bacterial foraging for distributed optimization and control 3023
KM Passino
IEEE control systems magazine 22 (3), 52-67, 2002

Stability analysis of swarms 1125
V Gazi, KM Passino
IEEE transactions on automatic control 48 (4), 692-697, 2003

Stable adaptive control using fuzzy systems and neural networks 728
JT Spooner, KM Passino
IEEE Transactions on Fuzzy Systems 4 (3), 339-359, 1996

Stability analysis of social foraging swarms 710
V Gazi, KM Passino
IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics) 34 …, 2004

Foraging

Foraging

- searching for nutrients
- avoiding noxious stimuli (toxins, predators, etc)

Social Foraging

- increases likelihood of finding nutrients
- better detection and protection from noxious stimuli
- gains can offset cost of food competition

Foraging as Optimization

How can we view foraging as an Optimization Process?

- We have some parameters θ and a loss function $J(\theta)$ that we want to minimize

Foraging as Optimization

How can we view foraging as an Optimization Process?

- We have some parameters θ and a loss function $J(\theta)$ that we want to minimize
- θ can represent the position of an organism in its environment

Foraging as Optimization

How can we view foraging as an Optimization Process?

- We have some parameters θ and a loss function $J(\theta)$ that we want to minimize
- θ can represent the position of an organism in its environment
- J can represent the concentration of nutrients and noxious stimuli
    - smaller values of J = more nutrients, less noxious stimuli
    - higher values of J = more noxious stimuli, less nutrients

Foraging as Optimization

How can we view foraging as an Optimization Process?

- We have some parameters θ and a loss function $J(\theta)$ that we want to minimize
- θ can represent the position of an organism in its environment
- J can represent the concentration of nutrients and noxious stimuli
  - smaller values of J = more nutrients, less noxious stimuli
  - higher values of J = more noxious stimuli, less nutrients
- In general, J and θ can be arbitrary
  - $\theta \in \mathbb{R}^p$
  - $J : \mathbb{R}^p \to \mathbb{R}$

E. coli

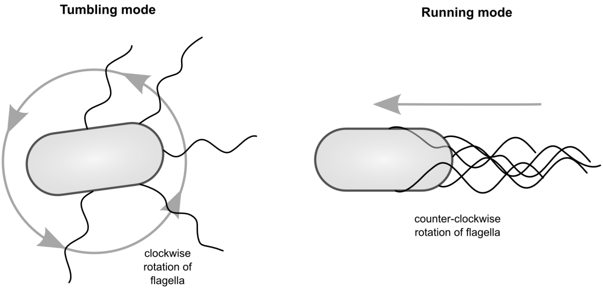
- Model organism
    - Highly studied
    - Well-characterized foraging behaviour
    - Probably won't feel bad about simplifying its behaviour

E. coli

- Model organism
  - Highly studied
  - Well-characterized foraging behaviour
  - Probably won't feel bad about simplifying its behaviour
- Social organism
  - Secretes signals to attract others nearby
  - Encourages "swarming" or "clumping"

E. coli Behaviour

- Swims using left-handed helical flagella ("propellers")
  - **Tumble**: flagella all rotate clockwise → pull on cell in all directions → random movement
  - **Run**: flagella all rotate counterclockwise → flagella form a bundle → push on cell in one direction → directed movement



E. coli Behaviour

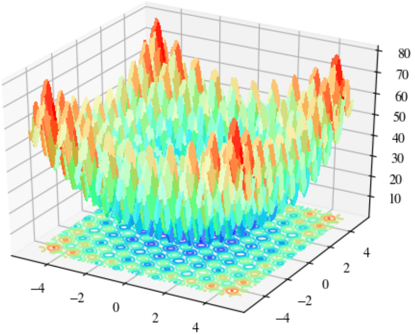
- If during a tumble *E. coli* swims down a nutrient concentration gradient:
    - Prolongs time spent on a run
    - Continues moving in the same direction
- Otherwise:
    - Tends to switch to a tumble (search for more)
    - Moves randomly which searching for more nutrient gradients to exploit
- Call a tumble followed by a run a "chemotaxis step"

Algorithm for a Single Bacterium

```
for $j \leftarrow 1 \ldots N_c$ do:
    $\phi \sim S^p$
    $\theta \leftarrow \theta + c\phi$
    while $J(\theta + c\phi) < J(\theta)$ do:
        $\theta \leftarrow \theta + c\phi$
```

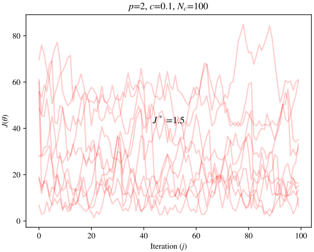
- θ: p-dimensional vector (randomly initialized)
- N_c: number of chemotaxis steps
- $\phi \sim S^p$: a random p-dimensional unit vector
- c: a step-size

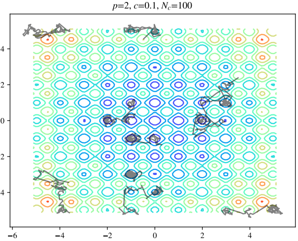
Loss Function to Optimize



$$J(\theta) = An + \sum_{i=1}^{n} \left(x_i^2 - A\cos(2\pi x_i)\right)$$

Results of Single Bacterium





- Relatively inconsistent performance for a highly nonconvex function.

Algorithm for a Colony

```
for $j \leftarrow 1 \ldots N_c$ do:
    for $i \leftarrow 1 \ldots S$ do:
        $\phi \sim S^p$
        $\theta_i \leftarrow \theta_i + c_i\phi$
        while $J(\theta_i + c_i\phi) + J_{cc}(\theta_i + c_i\phi) < J(\theta_i) + J_{cc}(\theta_i)$ do:
            $\theta_i \leftarrow \theta_i + c_i\phi$
```

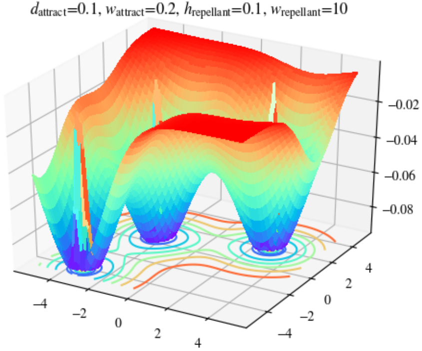
- θ_i: ith p-dimensional vector (randomly initialized)
- S: number of bacteria in the colony
- c_i: a step-size for bacterium i
- $J_c c$: cell-to-cell interactions

J_{cc} and swarming behaviour

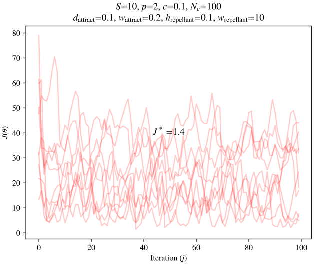
- *E. coli* do social foraging
- Secrete a substance to indicate to attract nearby *E. coli* and encourage swarming and biofilm formation
- Strength of signal diffuses over space
- Also want to avoid crowding
- Use sum of two Gaussian functions to model this

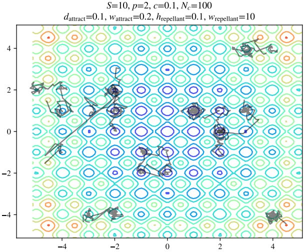
$$
\begin{aligned}
J_{cc}(\theta) = \sum_{i=1}^{S} & - d_{\text{attract}} \exp\left(-w_{\text{attract}}(\theta - \theta_i)^T(\theta - \theta_i)\right) \\
& + h_{\text{repellant}} \exp\left(-w_{\text{repellant}}(\theta - \theta_i)^T(\theta - \theta_i)\right)
\end{aligned}
$$

J_{cc} and swarming behaviour



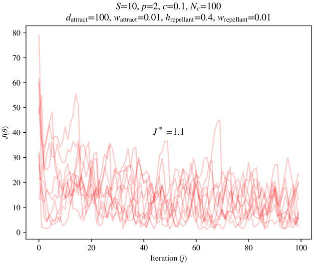
Results of Colony with Swarming





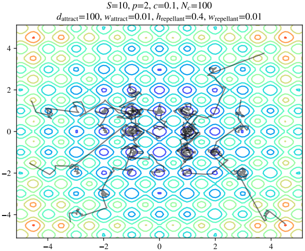
- Still relatively inconsistent performance for a highly nonconvex function.
- But wait... What if the problem is just the hyperparameters?

Results of Colony with Swarming



S=10, p=2, c=0.1, N_c=100

d_{attract}=100, w_{attract}=0.01, $h_{\text{repellant}}$=0.4, $w_{\text{repellant}}$=0.01

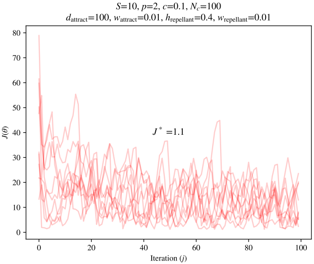


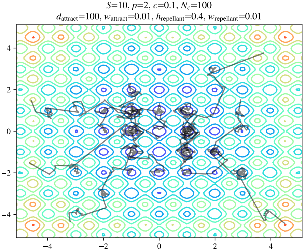
S=10, p=2, c=0.1, N_c=100

d_{attract}=100, w_{attract}=0.01, $h_{\text{repellant}}$=0.4, $w_{\text{repellant}}$=0.01

- By trying out different combinations of hyperparameters we can improve overall performance.
- Here we increased the depth and width of attraction as well as the depth and width of repellance to increase ”global” behaviour.

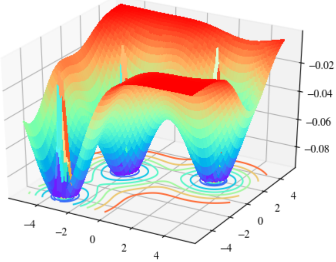
Results of Colony with Swarming


S=10, p=2, c=0.1, N_c=100
d_{attract}=100, w_{attract}=0.01, $h_{\text{repellant}}$=0.4, $w_{\text{repellant}}$=0.01

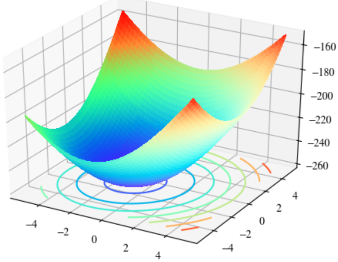

S=10, p=2, c=0.1, N_c=100
d_{attract}=100, w_{attract}=0.01, $h_{\text{repellant}}$=0.4, $w_{\text{repellant}}$=0.01

- By trying out different combinations of hyperparameters we can improve overall performance.
- Here we increased the depth and width of attraction as well as the depth and width of repellance to increase "global" behaviour.
- Important to know scale of J relative to scale of J_{cc} for tradeoff.
    - Can think of this like hyperparameters for PSO

Comparing J_{cc}



d_{attract}=0.1, w_{attract}=0.2, $h_{\text{repellant}}$=0.1, $w_{\text{repellant}}$=10



S=3, p=2, d_{attract}=0.1, w_{attract}=0.2, $h_{\text{repellant}}$=0.1, $w_{\text{repellant}}$=10

E. coli reproduction

- *E. coli* “reproduce” via
  1. **Binary fission**: essentially creating a clone
  2. **Horizontal Translation**: merging genetic material with others

E. coli reproduction

- *E. coli* "reproduce" via
  1. **Binary fission**: essentially creating a clone
  2. **Horizontal Translation**: merging genetic material with others
- Algorithm designed to mimic binary fission
  - More fit individuals more likely to survive
  - Less fit individuals more likely to die

E. coli reproduction

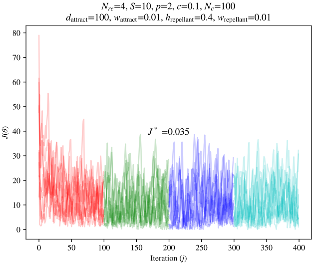
- *E. coli* “reproduce” via
  1. **Binary fission**: essentially creating a clone
  2. **Horizontal Translation**: merging genetic material with others
- Algorithm designed to mimic binary fission
  - More fit individuals more likely to survive
  - Less fit individuals more likely to die
- Horizontal translation could be incorporated (like a genetic algorithm)

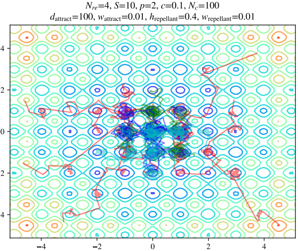
Algorithm for a Reproducing Colony

```
for $k \leftarrow 1 \ldots N_{re}$ do:
    for $j \leftarrow 1 \ldots N_c$ do:
        for $i \leftarrow 1 \ldots S$ do:
            $\phi \sim S^p$
            $\theta_i \leftarrow \theta_i + c_i \phi$
            while $J(\theta_i + c_i\phi) + J_{cc}(\theta_i + c_i\phi) < J(\theta_i) + J_{cc}(\theta_i)$ do:
                $\theta_i \leftarrow \theta_i + c_i \phi$
    delete worst $S/2$ and reproduce best $S/2$
```

- N_{re}: number of reproduction steps

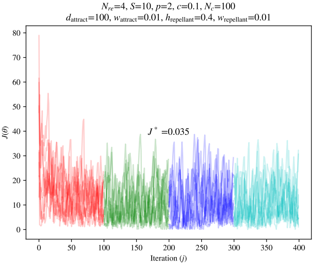
Results of Reproducing Colony

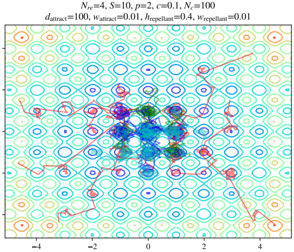




- Individuals with higher values of J killed off

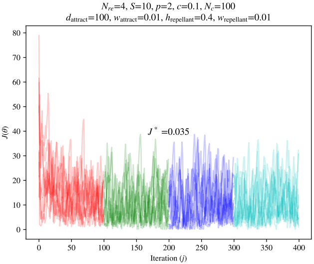
Results of Reproducing Colony

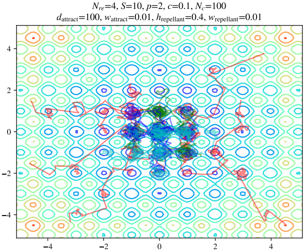




- Individuals with higher values of J killed off
- Individuals with lower values of J duplicated
  - Ideally move away due to repellance

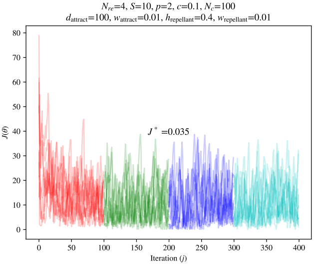
Results of Reproducing Colony

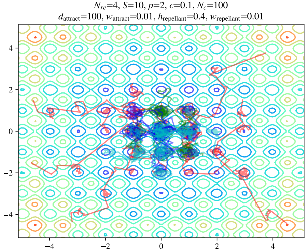




- Individuals with higher values of J killed off
- Individuals with lower values of J duplicated
  - Ideally move away due to repellance
- Idea is to encourage searching in space nearby "best" individuals

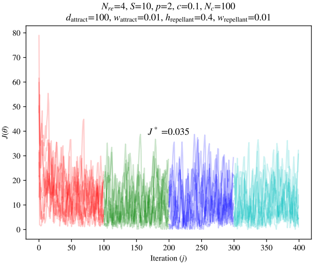
Results of Reproducing Colony

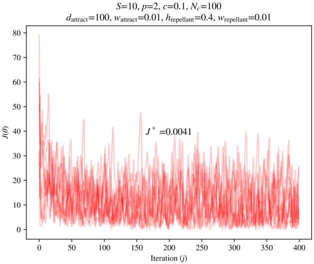

N_{re}=4, S=10, p=2, c=0.1, N_c=100
d_{attract}=100, w_{attract}=0.01, $h_{\text{repellant}}$=0.4, $w_{\text{repellant}}$=0.01

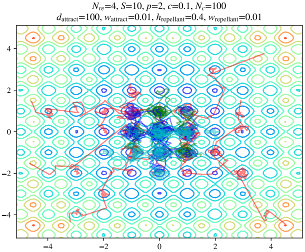

N_{re}=4, S=10, p=2, c=0.1, N_c=100
d_{attract}=100, w_{attract}=0.01, $h_{\text{repellant}}$=0.4, $w_{\text{repellant}}$=0.01

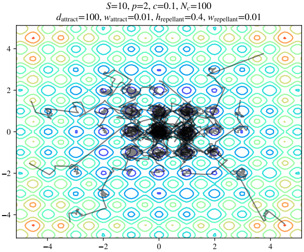
- Individuals with higher values of J killed off
- Individuals with lower values of J duplicated
  - Ideally move away due to repellance
- Idea is to encourage searching in space nearby "best" individuals
- If repellance isn't high enough then repeated iterations of evolution can concentrate colony in local minimum

Does Reproduction Help?

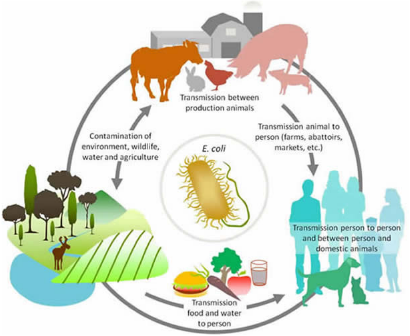

N_{re}=4, S=10, p=2, c=0.1, N_c=100
d_{attract}=100, w_{attract}=0.01, $h_{\text{repellant}}$=0.4, $w_{\text{repellant}}$=0.01


S=10, p=2, c=0.1, N_c=100
d_{attract}=100, w_{attract}=0.01, $h_{\text{repellant}}$=0.4, $w_{\text{repellant}}$=0.01


N_{re}=4, S=10, p=2, c=0.1, N_c=100
d_{attract}=100, w_{attract}=0.01, $h_{\text{repellant}}$=0.4, $w_{\text{repellant}}$=0.01

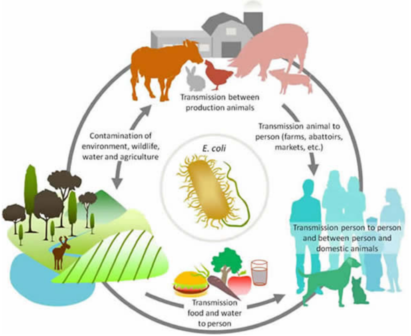

S=10, p=2, c=0.1, N_c=100
d_{attract}=100, w_{attract}=0.01, $h_{\text{repellant}}$=0.4, $w_{\text{repellant}}$=0.01

Elimination-Dispersal Events



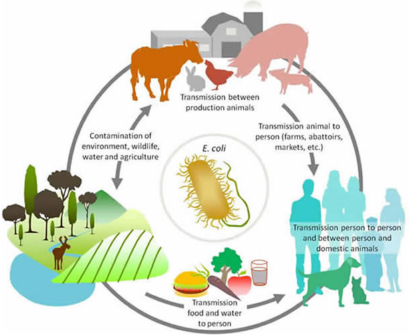
- Over time, random events disperse populations of *E. coli*
  - Water, animal activity, human intervention

Elimination-Dispersal Events



- Over time, random events disperse populations of *E. coli*
    - Water, animal activity, human intervention
- May destroy chemotactic progress
    - But may also bring *E. coli* to good food sources

Elimination-Dispersal Events



- Over time, random events disperse populations of *E. coli*
  - Water, animal activity, human intervention
- May destroy chemotactic progress
  - But may also bring *E. coli* to good food sources
- For optimization, this is a method to prevent stagnation and move out from local minima

Algorithm for a Dispersing Colony

```
for $l \leftarrow 1 \ldots N_{ed}$ do:
    for $k \leftarrow 1 \ldots N_{re}$ do:
        for $j \leftarrow 1 \ldots N_c$ do:
            for $i \leftarrow 1 \ldots S$ do:
                $\phi \sim S^p$
                $\theta_i \leftarrow \theta_i + c_i\phi$
                while $J(\theta_i + c_i\phi) + J_{cc}(\theta_i + c_i\phi) < J(\theta_i) + J_{cc}(\theta_i)$ do:
                    $\theta_i \leftarrow \theta_i + c_i\phi$
        delete worst $S/2$ and reproduce best $S/2$
    for $i \leftarrow 1 \ldots S$ do:
        if $\epsilon \sim \mathcal{U}(0,1) < p_{ed}$ then:
            $\theta_i \sim d(\theta)$
```

- N_{ed}: number of elimination-dispersal events
- p_{ed}: probabilty of a single elimination-dispersal event
- $d(\theta)$: initial distribution of θ

Does Elimination-Dispersal Help?