Discrete Mathematics

a) $A = \{\{1,2,3\}, \{4,5\}, \{6,7,8\}\}$

A → what are the elements of A?

→ $\{1,2,3\}$, $\{4,5\}$, $\{6,7,8\}$.

→ Is $1 \in A$?

↳ 1 belongs to A

No.

→ $\{6,7,8\} \in A$?

True.

→ $\{1,2,3\} \subseteq A$?

F

→ $\{1,2,3\} \subset A$?

↳ Proper subset

T

→ $\{\{1,2,3\}\} \subseteq A$?

T.

↳ subset.

→ $A \subset A$? F Proper subset.

→ $A \subseteq A$? T Subset

↱ all the elements are not a part of set, only some are there.

ϕ → null set

↳ is a subset of all sets.

→ $\phi \subset A$? (T), $\phi \subseteq A$? (T) $\phi = \{\ \}$

→ $\phi \in A$? (F)

→ universal set

$U = \{A1, A2, A3\}$

A1 complement

$A1' = U - A1 = \{A2, A3\}$

$A2 \cap A3$?

↳ Intersection

↗ Set - Builder Notation.

→ $A1 = \{ s \mid s \leq 24,$ s is a serial no.$\}$

↳ such that

↳ A1 is a set of students such that s is there serial no. ≤ 24.

$x \in Z^+$

→ $B = \{ x \mid x$ is a perfect square less than $50 \}$

$x \in Z^+$

→ $C = \{ x \mid x$ is an even no. less than $50 \}$

positive integers.

$B = \{1, 4, 9, 16, 25, 36, 49\}$

$C = \{4, 16, 36\}$

$B \cap C = \{4, 16, 36\}$.

Q) $A = \{ x \in N \mid x < 5 \}$

$B = \{ x \in N \mid x$ is even & $x \leq 10 \}$

find $A \cap B$ & $A \cup B$.

Q: Rational
R: Real
Z: Integers.

$A = \{1, 2, 3, 4\}$

$B = \{2, 4, 6, 8, 10\}$

$A \cap B = \{2, 4\}$

$A \cup B = \{1, 2, 3, 4, 6, 8, 10\}$

Q) which of the following pair of sets are equal

equal sets $\rightarrow$ same elements irrespective of order.

a) $A = \{x \mid x$ is an even no. less than $6\}$

$B = \{2, 4, 6\}$

$A = \{2, 4\}$
$B = \{2, 4\}$ } Not equal

b) $A = \{a, b, c\}$

$B = \{c, b, a, a\}$

equal
same elements.

c) $A = \{1, 2, 3\}$, $B = \{1, 2, 4\}$

not equal.

d) $A = \{x \mid x \in N$ & $x \leq 3\}$, $B = \{0, 1, 2, 3\}$

$A \Rightarrow \{1, 2, 3\}$, $B = \{0, 1, 2, 3\}$

not equal

(B) $A = \{1, 2, 3, 4, 5\}$

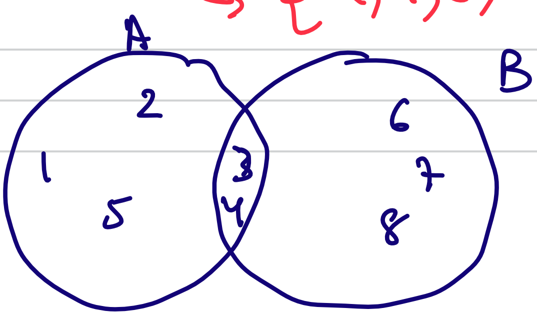
$B = \{4, 5, 6, 7\}$

$C = \{x \in N \mid x < 6\} \Rightarrow \{1, 2, 3, 4, 5\}$

$D = \{x \in N \mid x$ is even & $x \leq 8\}$.

$\hookrightarrow \{2, 4, 6, 8\}$

Q)



$A - B = \{1, 2, 5\}$
$B - A = \{6, 7, 8\}$.

$(1, 2, 5, 6, 7, 8)$

$\Rightarrow A \oplus B = (A - B) \cup (B - A)$

$\hookrightarrow$ XOR operator

$\Rightarrow (A \cap B)' \Rightarrow \{1, 2, 5, 6, 7, 8\}$.

1) $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

2) $B \cup C = \{1, 2, 3, 4, 5, 6, 7\}$

3) $A \cap B = \{4, 5\}$

4) $B \cap D = \{4, 6\}$

5) $C - D = \{1, 3, 5\}$

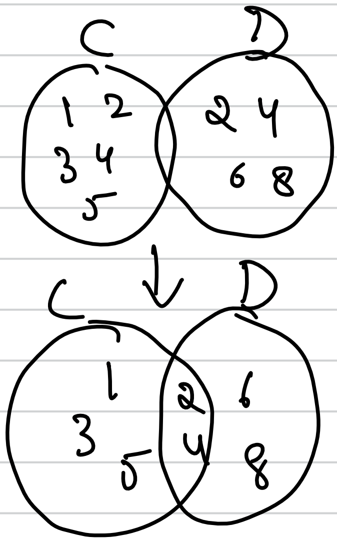
6) $D \oplus C = \{1, 3, 5, 6, 8\}$

7) $A' = \{B, C, D\}$

8) $(C \cup D)' =$

9) $(A \cup B) \cap C = \{1, 2, 3, 4, 5\}$

10) $B \cap D' =$



Tutorial

★ Set Equality.

$A \subseteq B$ & $B \subseteq A \Rightarrow A = B$
$\hookrightarrow$ subset.

$\rightarrow$ Every set is a subset of itself.

$\subset$?

→ Proper subset ⇒ $A \subseteq B$ But $A \neq B$ ⇒ then $A \subset B$

→ Cardinality → $|A|$ ⇒ no. of distinct elements in A.

→ ϕ is subset of

Q) Consider a universal set U such that

$$U = \{0,1,2,3,4,5,6,7,8,9,10\}.$$

$$A = \{1,2,3,4,5\}$$

$$B = \{4,5,6,7,8\}$$

$A \oplus B = (A-B) \cup (B-A)$

$\bar{A} \mid A^c = U - A$

$A \times B = \{x \mid (x,y)\ x \in A \text{ and } y \in B\}$

① $A \cup B = \{1,2,3,4,5,6,7,8\}$

② $A \cap B = \{4,5\}$

③ $\bar{A} = U - A = \{0,6,7,8,9,10\}$

④ $\bar{B} = \{0,1,2,3,9,10\}$

⑤ $A \oplus B = \{1,2,3,6,7,8\}$
↳ symmetric difference.

Power set - Is a set of all subsets which contains 2^n elements.

$A = \{1,2,3\} \longrightarrow n=3$ ↗ no. of elements in A.

$P(A) = 2^n = 2^3 = 8$

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

↳ null set is a subset of every set.

* Partition of sets:

conditions:→

→ $A_1 \cup A_2 \cup A_3 - - - - - = A$

↳ union of all subsets $= A$

→ $A_i \cap A_j = \phi$ for $i \neq j$ (subsets are mutually disjoint).

Q) $A = \{a, b, c, d\}$ → Determine Power set.

$P(A) = \{ \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{c,d\}, \{a,b,c,d\} \cdots -$

① $S = \{1,2,3,4,5,6,7,8,9\}$

i) $\{\{1,3,5\}, \{2,6\}, \{4,8,9\}\}$

$A_1 = \{1,3,5\}$

$A_2 = \{2,6\}$

$A_3 = \{4,8,9\}$.

$A_1 \cup A_2 \cup A_3 = \{1,2,3,4,5,6,8,9\} \neq S$

not

(ii) $\{\underbrace{\{1,3,5\}}_{A_1}, \underbrace{\{2,4,6,8\}}_{A_2}, \underbrace{\{5,7,9\}}_{A_3}\}$

⇒ union $= A_1 \cup A_2 \cup A_3 = \{1,2,3,4,5,6,7,8,9\}$

$= A$ ✓

$A_1 \cap A_2 = \phi$

$A_1 \cap A_3 = \phi$

$A_2 \cap A_3 = \{5\}$

~~Yes~~ No

iii) $\{\underbrace{\{1,3,5\}}_{A_1}, \underbrace{\{2,4,6,8\}}_{A_2}, \underbrace{\{7,9\}}_{A_3}\}$

$A_1 \cup A_2 \cup A_3 = \underbrace{\{1,2,3,4,5,6,7,8,9\}} = S$

$A_1 \cap A_2 = \phi$

$A_1 \cap A_3 = \phi$

$A_2 \cap A_3 = \phi$

Yes

* Laws of Set Theory:-

① Commutative Laws:-

$A \cup B = B \cup A$, $A \cap B = B \cap A$

② Associative laws

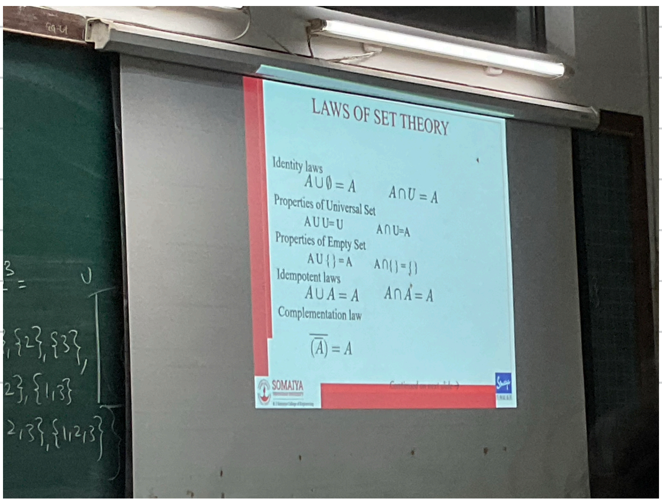
$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

③ Distributive Laws:-

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$



(9) De Morgan's law:-

$$\overline{A \cup B} = \overline{A} \cap \overline{B} \qquad \overline{A \cap B} = \overline{A} \cup \overline{B}$$

(10) Absorption laws:-

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

(11) Properties of complement law

$$A \cup \overline{A} = U \quad , \quad A \cap \overline{A} = \phi$$

★ Theorems.

(1) Addition Principle.

$$|A \cup B| = |A| + |B|$$ mod is cardinality

$$|A - B| = |A| - |A \cap B|$$

(2) Principal of mutual inclusion exclusion.

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$$

Q) $|A_1 \cup A_2| = ?$

$|A_1| = 12, \quad |A_2| = 18$

$|A_1 \cup A_2| = |A_1| + |A_2|$

$= 12 + 18 = 30$

(i) $A_1 \cap A_2 = \phi$

$|A_1 \cup A_2| = |A_1| + |B_1| - |A_1 \cap$

Q no. of system programming jobs $= |A| = 25$

no. of application programming jobs $= |B| = 40$

$|A \cap B| = 10$ → no. of programmers who do both.

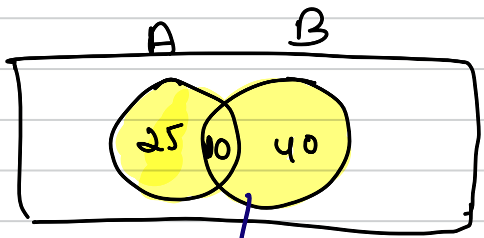
$|A \cup B| = ?$

$\Rightarrow |A| + |B| - |A \cap B|$

$\Rightarrow 25 + 40 - 10$

$\Rightarrow 65 - 10$

$= \underline{\underline{55}}$



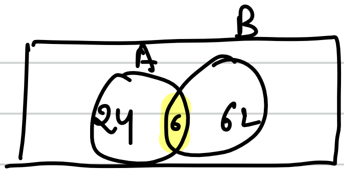
venn diagram.

highlighted

Q) $|A \cup B| = 80$

$|A| = 24$ cinema

$|B| = 62$ TV.



$|A \cap B| = ?$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$80 = 24 + 62 - |A \cap B|$$

$$|A \cap B| = 86 - 80 = \underline{\underline{6}}$$

Q// $|A \cup B| = 260$

$|A| = 64$ (maths)

$|B| = 94$ (CS)

$|C| = 58$ (BS)

$|A \cap C| = 28$

$|A \cap B| = 26$

$|B \cap C| = 22$

$|A \cap B \cap C| = 14$

i) $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |\cancel{A}\, B \cap C|$

$- |A \cap C| + |A \cap B \cap C|$

$\Rightarrow 64 + 94 + 58 - 26 - 22$

$- 28$

$+ 14$

$\Rightarrow \underline{\underline{154}}$

no. of students who have taken none of the

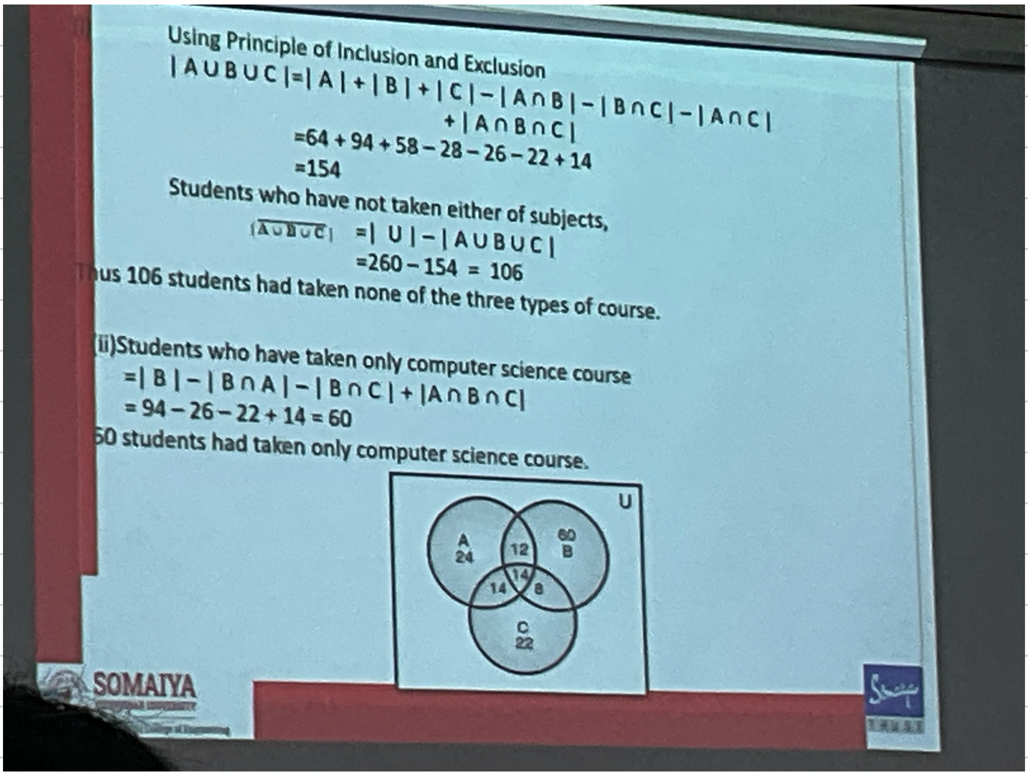
3 types of courses $= 260 - 154 = \underline{\underline{106}}$

ii) no. of students surveyed $= 260.$

only computer science

$\Rightarrow |B| - |B \cap A| - |B \cap C| + |A \cap B \cap C|$

$\Rightarrow \underline{\underline{60}}$



Q//

survey of 60 people,

$|A| = 25$ (Newsweek)

$|B| = 26$ (read time)

$|C| = 26$ (fortune)

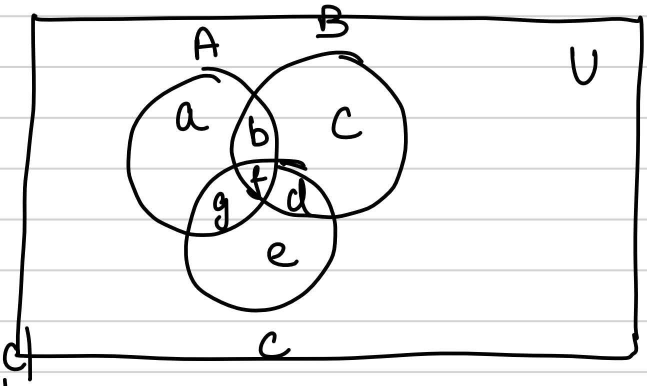
$|A \cap C| = 9$ $\qquad$ $|\overline{A \cup B \cup C}| = 8$

$|A \cap B| = 11$

$|B \cap C| = 8$

a) $|A \cap B \cap C| = ?$

$U = 60$



$|\overline{A \cup B \cup C}|$
$= a + b + c + d + e + f + g$

$|\overline{A \cup B \cup C}| = 60 - |A \cup B \cup C|$

$\Rightarrow 8 = 60 - |A \cup B \cup C|$

$|A \cup B \cup C| = 68$

$|A \cap C| = 9$

$\Rightarrow g + f = 9$

$|A \cap B| = 11$

$b + f = 11$

$|B \cap C| = f + d = 8$

$|A| = a + b + f + g = 25$

$|B| = b + c + f + d = 26$

$|C| = g + f + d + e = 26$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$

$68 \Rightarrow 25 + 26 + 26 - 11 - 8 - 9 + |A \cap B \cap C|$

$\Rightarrow 68 = 49 + |A \cap B \cap C|$

$\Rightarrow 68 - 49 \Rightarrow 29 = |A$

Q

$|M| = 120$ (mathematics student)

$|F \cup G \cup R| = 100$

$|F| = 65$

$|F \cap G| = 20$

$|G| = 45$

$|F \cap R| = 25$

$|R| = 42$

$|G \cap R| = 15$

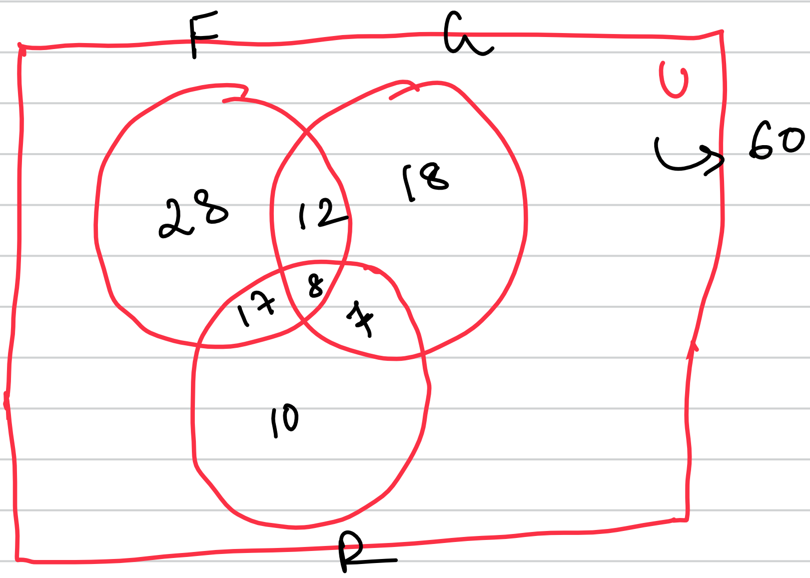
a) $|F \cap G \cap R| = ?$

$$|F \cup G \cup R| = |F| + |G| + |R| - |F \cap G| - |G \cap R| - |F \cap R| + |F \cap G \cap R|$$

$$\Rightarrow \quad 100 = 65 + 45 + 42 - 20 - 15 - 25 + |F \cap G \cap R|$$

$$\Rightarrow \quad 100 - 92 = |F \cap G \cap R| = \underline{\underline{8}}$$

b) $65 - 17 - 12 - 8$



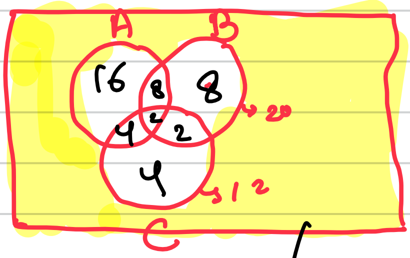
2)

Q) find how many integers b/w 1 & 60, which are not divisible by 2, nor by 3, nor by 5.

$|A| = \frac{60}{2} = 30$ {divisible by 2}

$|B| = \frac{60}{3} = 20$ (" by 3 }

$|C| = \frac{60}{5} = 12$ (" by 5)



$\overline{|A \cup B \cup C|} = 16$

$|A \cap B \cap C| = ?$

$|A \cap B| = ?$ $\quad |B \cap C| = ?$ $\quad |A \cap C| = ?$

$\frac{60}{2 \times 3} \Rightarrow \frac{60}{6} = 10$ $\quad \frac{\cancel{60}^{20}}{\cancel{3} \times 5} = 4$ $\quad \frac{\cancel{60}^{30}}{\cancel{2} \times 5} = \underline{\underline{6}}$

$|A \cap B \cap C| = \frac{\cancel{60}^{\cancel{30}^{10}}}{\cancel{2} \times \cancel{3} \times 5} = \underline{\underline{2}}$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

$\Rightarrow 30 + 20 + 12 - 10 - 4 - 6 + 2$

$\Rightarrow 44$

$\overline{|A \cup B \cup C|} = 60 - 44 = \underline{\underline{16}}$.

Q) $A = \{x : x^2 - 4x + 3 = 0\} \Rightarrow A = \{1, 3\}$

$B = \{x : x^2 - 3x + 2 = 0\} \Rightarrow B = \{1, 2\}$

$C = \{x : x \in N, x < 3\} \Rightarrow C = \{1, 2\}$

$D = \{x : x \in N, x \text{ is odd}, x < 5\} \Rightarrow D = \{1, 3\}$

$E = \{1, 2\}$

$F = \{1, 2, 1\}$

$G = \{3, 1\}$

which sets are equal.

$A = D$ $\quad B = C$ $\quad C = E$

$A = G$ $\quad B = E$ $\quad B = F$

$C = F$ $\quad E = F$

Q. Let A, B and C are subset of U (universal set), prove that

$A \times (B \cup C) = (A \times B) \cup (B \times C)$

LHS $= \{x, y \mid x \in A \ \& \ y \in (B \cup C)\}$

RHS $= \{x \in A \ \& \ (y \in B \text{ or } y \in C)\}$

$\Rightarrow \quad (x,y) \in (A \times B) \cup (x,y) \in (A \times C)$

Q. Show that:-

a) $A \cup (A^c \cap B) = A \cup B$

→ LHS $= A \cup (A^c \cap B)$

$\Rightarrow A \cup A^c \cap (A \cup B)$

$\Rightarrow U \cap (A \cup B)$

$\Rightarrow A \cup B$

intersection.

universal set.

b) $A \cap (A^c \cup B) = A \cap B$

$\Rightarrow (A \cap A^c) \cup (A \cap B)$

$\Rightarrow \phi \cup (A \cap B)$

null set is a subset of every set

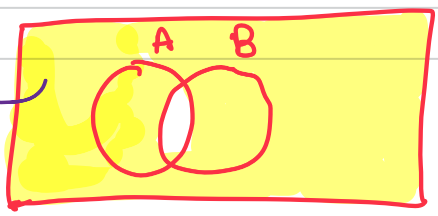
$\Rightarrow A \cap B$

like it is a subset of $(A \cap B)$ also

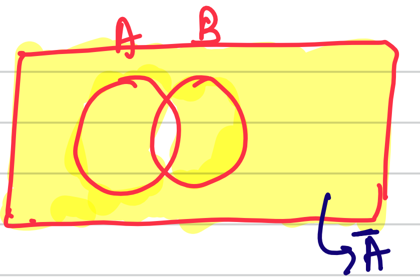
$\therefore$ union $= A \cap B$

Q. $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

LHS $= \overline{(A \cap B)}$



LHS = $\bar{A} \cup \bar{B}$



factorial =

Logic

* Logical operations:-

1. Negation ($\sim a$ or $\tilde{a}$ or $!a$)
2. AND ($\wedge$) (conjunction)
3. OR ($\vee$) (Disjunction)
4. XOR ($\oplus$)
5. Imply ($\Rightarrow$ or $\rightarrow$)
6. Bi-conditional ($\Leftrightarrow$ or $\leftrightarrow$) — if and only if.

→ Truth Table

1) Negation.

p	~p
T	F
F	T

2) Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

3) Disjunction.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

T F F

④ $p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

⑤ $p \leftrightarrow q$ (if & only if) $\rightarrow$ Biconditional.

p $\quad$ q $\quad$ $p \leftrightarrow q$

If $p \rightarrow q$ is the conditional statement then

$q \rightarrow p$ $\Rightarrow$ converse.

$\sim p \rightarrow \sim q$ = inverse.

$\sim q \rightarrow \sim p$ = Contrapositive.

p	q	$p \leftrightarrow q$
T	T	T
F	F	T
T	F	F
F	T	F

Q) Let:-

p denotes Raju is rich

q denotes Raju is happy.

i) Raju is poor but happy.

either $\rightarrow$ or

$\sim p \wedge q$

ii) $\sim p \wedge \sim q$ (Raju is neither rich nor happy)

iii) $p \vee \sim q$ (Raju is either rich or unhappy)

iv) $\sim p \vee (p \wedge \sim q)$ (Raju is poor or else he is rich and unhappy).

Q)

p: I will study discrete structures.
q: I will go to a movie.

r: I am in a good mood.

i) If I am not in a good mood, then I will go to a movie.

$\sim r \rightarrow q$

ii) $\sim q \wedge p$

iii) ~~[illegible]~~ $\sim p \rightarrow q$

iv) $\sim p \rightarrow \sim r$

Q)

$(p \vee q) \rightarrow (r \leftrightarrow \sim s)$.

then $\rightarrow$ $(\rightarrow)$ conditional
if and only if $\rightarrow (\leftrightarrow)$ bi conditional.

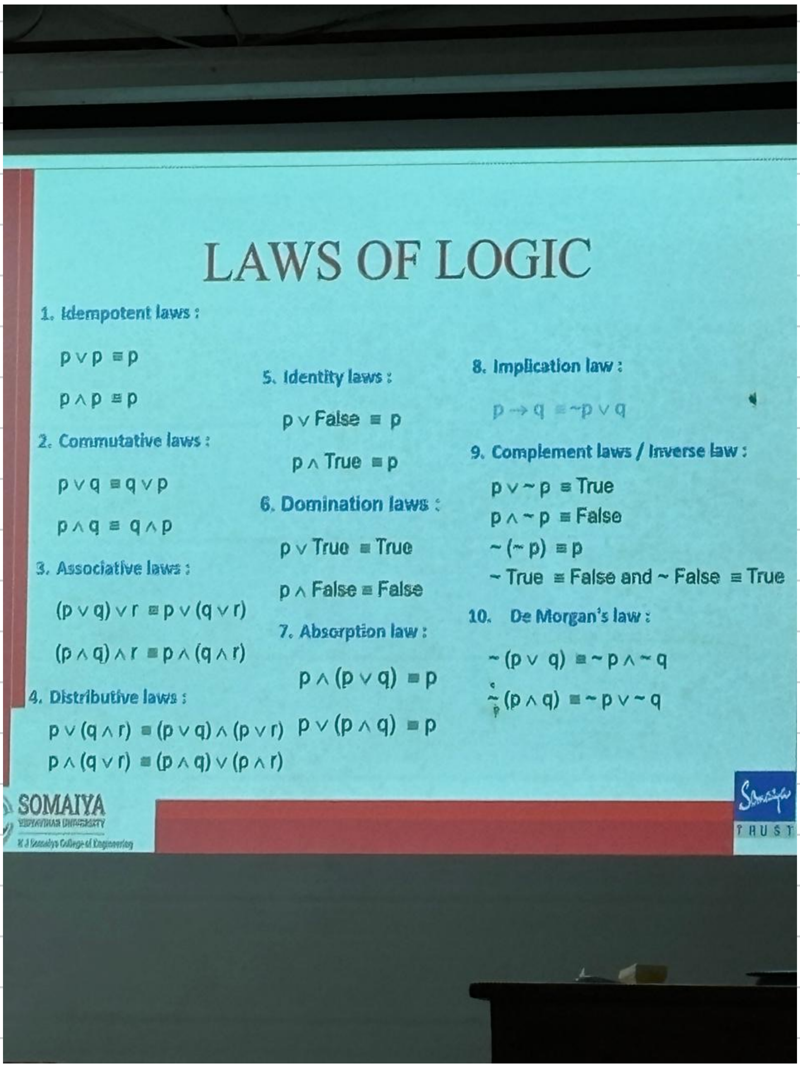
Q) $(p \vee \sim q) \rightarrow (p \wedge q)$

p	q	$\sim q$	$p \vee \sim q$	$p \wedge q$	$(p \vee \sim q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	F	F	F	F
F	T	T	T	F	T
F	F	T	T	F	F

Q $((p \wedge q) \vee \sim q)$

p	q	$\sim q$	$p \wedge q$	$((p \wedge q) \vee \sim q)$
T	T	F	T	T
T	F	T	F	T
F	T	F	F	F
F	F	T	F	T

→ Tautology → T (Truthvalues ⇒ T)
→ contradiction → F (Truthvalue ⇒ F)
→ Contingency → neither a Tautology nor a contradiction



$p \leftrightarrow q$

$\Rightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

Q) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

p	q	r	p → q	q → r	(p → q) ∧ (q → r)	p → r
T	T	T	T	T	T	T
T	F	T	F	T	F	T
T	T	F	T	F	F	F
T	F	F	F	T	F	F
F	T	T	T	T	T	T
F	F	T	T	T	T	T
F	T	F	T	F	F	
F	F	F	T	T	T	

Q) $((p \wedge q) \vee \sim q)$

p	q	$p \wedge q$	$\sim q$	$(p \wedge q) \vee \sim q$
T	T	T	F	T
T	F	F	T	T
F	T	F	F	F
F	F	F	T	T

Q// $\sim[p \wedge (p \vee \sim q)]$

p	q	$\sim q$	$p \vee \sim q$	$p \wedge (p \vee \sim q)$	$\sim$
T	T	F	T	T	F
T	F	T	T	T	F
F	T	F	F	F	T
F	F	T	T	F	T

$\sim[p \vee (p \wedge \sim q)]$

p	q	$\sim q$	$p \wedge \sim q$	$p \vee (p \wedge \sim q)$	$\sim$
T	T	F	F	T	F
T	F	T	T	T	F
F	T	F	F	F	T
F	F	T	F	F	T

Q// $(p \wedge q) \rightarrow (p \vee q)$

$\Rightarrow \sim(p \wedge q) \vee (p \vee q)$

$\Rightarrow (\sim p \vee \sim q) \vee (p \vee q)$

$\Rightarrow (\sim p \vee p) \vee (\sim q \vee q)$

$\Rightarrow T \vee T$

$\Rightarrow T$

Q) $\sim(p \vee (\sim p \wedge q)) \Leftrightarrow \sim p \wedge \sim q$ ↳ double implication.

$\Rightarrow \sim((p \vee \sim p) \wedge (p \vee q)) \Leftrightarrow \sim(p \vee q)$

$\Rightarrow \sim(T \wedge (p \vee q)) \Leftrightarrow \sim(p \vee q)$

$\Rightarrow \sim T \vee \sim(p \vee q) \Leftrightarrow \sim(p \vee q)$

$\Rightarrow F \vee \sim(p \vee q) \Leftrightarrow \sim(p \vee q)$

$\Rightarrow \sim(p \vee q) \Leftrightarrow \sim(p \vee q)$

Q Simplify

$(P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R))$

$\vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$

$\Rightarrow \underbrace{\sim(P \vee Q)}_{a} \vee \underbrace{\sim(P \vee R)}_{b}$

$\Rightarrow \sim a \vee \sim b$

$\Rightarrow \sim(a \wedge b)$

$\Rightarrow \sim((P \vee Q) \wedge (P \vee R))$

ans = T. ↳ $((P \vee Q) \wedge (P \vee R)) \vee \sim((P \vee Q) \wedge (P \vee R))$

Q. $(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$

$((\sim P \wedge \sim Q) \wedge R) \vee (Q \wedge R) \vee (P \wedge R)$

$\Rightarrow (\sim (P \vee Q) \wedge R) \vee (Q \wedge R) \vee (P \wedge R)$

$\Rightarrow (\sim (P \vee Q) \wedge R) \vee (Q \wedge (R \vee P))$

$\Rightarrow$

Q. Using law of logic.

$[(p \to q) \wedge \sim q] \to \sim p$ is a tautology

$p \to q \equiv \sim p \vee q$

$\Rightarrow [(\sim p \vee q) \wedge \sim q] \to \sim p$

$\Rightarrow \sim [(\sim p \vee q) \wedge \sim q] \vee \sim p$

Implication law.

$\Rightarrow \sim [\sim q \wedge (\sim p \vee q)] \vee \sim p$

commutative law

$\Rightarrow \sim [(\sim q \wedge \sim p) \vee (\sim q \wedge q)] \vee \sim p$

$\Rightarrow \sim [(\sim q \wedge \sim p) \vee (q \wedge \sim q)] \vee \sim p$

$\Rightarrow \sim [(\sim q \wedge \sim p) \vee F] \vee \sim p$

Identity complement law.

$\Rightarrow \sim (\sim q \wedge \sim p) \vee \sim p$

$\Rightarrow (q \vee p) \vee \sim p$ Demorgans' law.

Complement law.

$\Rightarrow q \vee (p \vee \sim p)$ Associative law.

$\Rightarrow q \vee T$ Inverse law

$\Rightarrow T$ Identity law.

Q) Show that the following statement is tautology.

$(p \wedge (p \rightarrow q)) \rightarrow q$.

$\Rightarrow \sim [p \wedge (p \rightarrow q)] \vee q$ (Implication)

$\Rightarrow \sim [p \wedge (\sim p \vee q)] \vee q$ (Implication)

$\Rightarrow \sim [(p \wedge \sim p) \vee (p \wedge q)] \vee q$ (Distributive)

$\Rightarrow \sim [F \vee (p \wedge q)] \vee q$

$\Rightarrow \sim [(p \wedge q) \vee F] \vee q$ commutative.

$\Rightarrow \sim [p \wedge q] \vee q$

$\Rightarrow (\sim p \vee \sim q) \vee q$ Demorgan.

$\Rightarrow \sim p \vee (\sim q \vee q)$ Inverse law.

$\Rightarrow \sim p \vee T$ Domination.

$\Rightarrow T$

↳ Tautology

Q) Prove:→

$[(\sim p \vee \sim q) \rightarrow (p \wedge q \wedge r)] = p \wedge q$

LHS $\Rightarrow [(\sim p \vee \sim q) \rightarrow (p \wedge q \wedge r)]$

$\Rightarrow [\sim(p \wedge q) \rightarrow (p \wedge q \wedge r)]$ Demorgan.

$\Rightarrow \sim(\sim(p \wedge q)) \vee (p \wedge q \wedge r)$ Implication.

$\Rightarrow (p \wedge q) \vee (p \wedge q \wedge r)$

$\Rightarrow p \wedge q = x$

$\Rightarrow x \vee (x \wedge r)$ Absorption law

$\Rightarrow x$

$\Rightarrow p \wedge q$

Q) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Truth Table prove.

p	q	r	$(q \wedge r)$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	F	T	F	T	T	T	T
T	T	F	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	F	T	F	F	F	T	F
F	T	F	F	F	T	F	F
F	F	F	F	F	F	F	F

proved

* predicate: → An assertion that contains one or more variables is called a predicate.

* Quantifiers:-
  - universal ⇒ for all values of x $p(x) \Rightarrow$ always true.
  - existential ⇒ atleast one value of x, for which $p(x) \rightarrow$ True, & others false.

there exists ⇒ $\exists$

for all ⇒ $\forall$

for ex:- $x+3=5$
$x=2$ } only true for one value $x=2$
other than that false.

Q i) for any value of x, x^2 is non-negative.

universal $\forall x \, (x^2 \geq 0)$ $p(x) = x^2$

ii) for every value of x, there is some value of y such that $x \cdot y = 1$

$\forall x \, \exists y \, [x \cdot y = 1]$

$5 - y = 1$

iii) There are positive $-ve \times -ve > 0$.

$\exists x \, \exists y \, ((x>0) \wedge (y>0) \wedge (x \cdot y > 0))$

iv) $\exists x \, \forall y \, [(y>0) \rightarrow (x+y) < 0]$
↳ implication

I

* Normal forms:-

When the statements consist of more variables or are of complex form, then the method of employing truth table is not efficient.

Thus, we will have to reduce the statement to normal forms

① Disjunctive (OR) $\vee$
② Conjunctive (And) $\wedge$

$(x_1 \wedge x_2 \wedge x_3) \vee (x_1 \wedge x_2 \wedge x_3)$ → Disjunctive normal form. (DNF)

joining 2 or more and terms with or.

Ex:-

① $(p \wedge q) \vee \sim q$

② $(\sim p \wedge q) \vee (p \wedge q) \vee q$

③ $(p \wedge q \wedge r) \vee (p \wedge \sim r) \vee (q \wedge r)$ — mean ~~add~~ and terms

④ $(p \wedge \sim q) \vee (p \wedge r)$

⑤ $((p \wedge q \wedge r) \vee \sim r)$

Q) Obtain the DNF of the form $p \wedge (p \rightarrow q)$

Try to combine min^m and terms with or.

$\Rightarrow p \wedge (p \rightarrow q)$ — implication form

$\Rightarrow p \wedge (\sim p \vee q)$

$\Rightarrow \underbrace{(p \wedge \sim p)}_{\text{and terms}} \vee \underbrace{(p \wedge q)}_{\text{and terms}}$ — Distributive

DNF

$\Rightarrow F \vee (p \wedge q)$

$\Rightarrow p \wedge q$

CNF ⇒ Conjunctive Normal form.

combining or terms with and

$(x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee x_3)$

Max^m of OR we are conneciy with and.

Q) Obtain CNF $(\sim p \to r) \wedge (p \Leftrightarrow q)$

$p \Leftrightarrow q$

$\Rightarrow (p \to q) \wedge (q \to p)$

$\Rightarrow (\sim p \to r) \wedge (p \Leftrightarrow q)$

write law

$\Rightarrow (\sim(\sim p) \vee r) \wedge (p \Leftrightarrow q)$

$\Rightarrow (p \vee r) \wedge [(p \to q) \wedge (q \to p)]$

$\Rightarrow (p \vee r) \wedge [(\sim p \vee q) \wedge (\sim q \vee p)]$

$\Rightarrow (p \vee r) \wedge (\sim p \vee q) \wedge (\sim q \vee p)$

Q) CNF:-

$(p \wedge q) \vee (\underbrace{\sim p \wedge q \wedge r}_{S})$

$\Rightarrow$

$(p \wedge q) \vee (S)$

$S \vee (p \wedge q)$

$\Rightarrow (S \vee p) \wedge (S \vee q)$

$\Rightarrow [(\sim p \wedge \underbrace{q \wedge r}) \vee p] \wedge [(\sim p \wedge \underbrace{q \wedge r}) \vee q]$

$\Rightarrow$

$q \wedge r = t$

Distributive

$\Rightarrow [(\sim p \wedge t) \vee p] \wedge [(\sim p \wedge t) \vee q]$

$\Rightarrow [p \vee (\sim p \wedge t)] \wedge [q \vee (\sim p \wedge t)]$

$\Rightarrow (p \vee \sim p) \wedge (p \vee t) \wedge (q \vee \sim p) \wedge (q \vee t)$

$\Rightarrow$

$\Rightarrow (p \vee \sim p) \wedge [p \vee (q \wedge r)] \wedge (q \vee \sim p) \wedge [q \vee (q \wedge r)]$

$\Rightarrow (p \vee \sim p) \wedge (p \vee q) \wedge (p \vee r) \wedge (q \vee \sim p) \wedge (q \vee q) \wedge (q \vee r)$

$\Rightarrow (p \vee q) \wedge (p \vee r) \wedge (q \vee \sim p \wedge (q \vee r)$

$\wedge q \wedge (q \vee r)$

Q) CNF & DNF $\sim(p \vee q) \leftrightarrow (p \wedge q)$

CNF: $\rightarrow$

$\Rightarrow [\sim(p \vee q) \rightarrow (p \wedge q)] \wedge [(p \wedge q) \rightarrow \sim(p \vee q)]$

$\Rightarrow [\sim(\sim(p \vee q)) \vee (p \wedge q)] \wedge [\sim(p \wedge q) \vee \sim(p \vee q)]$

$\Rightarrow [(p \vee q) \vee (p \wedge q)] \wedge [(\sim p \vee \sim q) \vee (\sim p \wedge \sim q)]$

associative $\Rightarrow$

$\Rightarrow (p \vee q) \wedge (\sim p \vee \sim q) \wedge (\sim p \vee \sim q)$

$\Rightarrow (p \vee q) \wedge (\sim p \vee \sim q) \Rightarrow$ CNF.

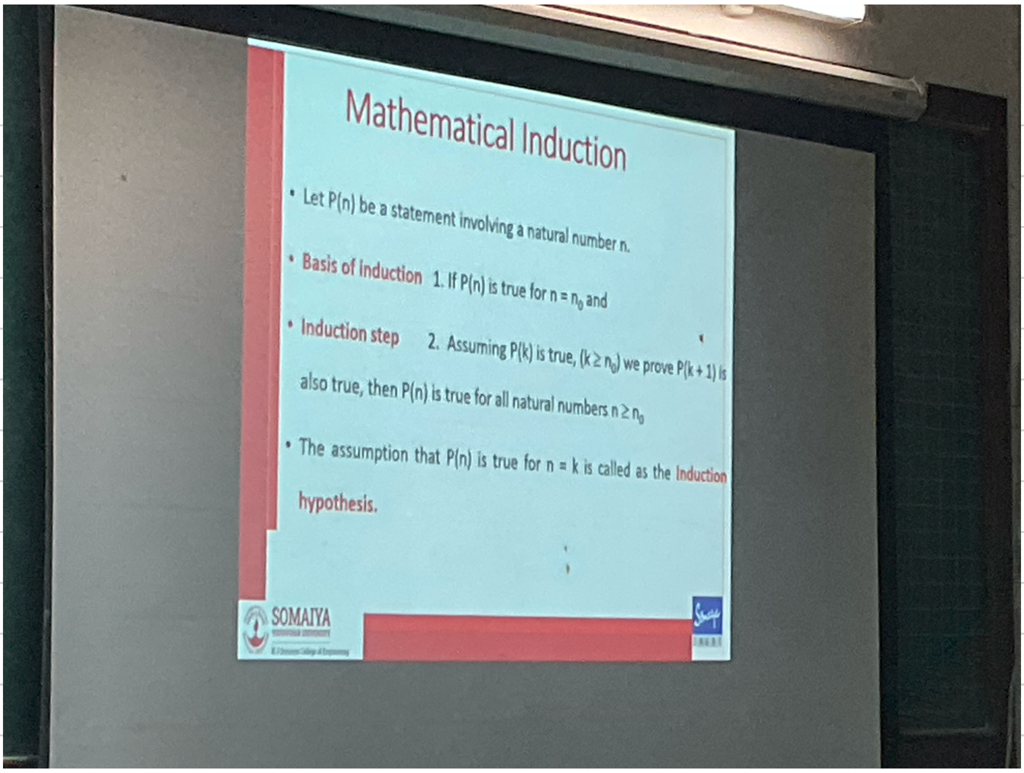
further $\rightarrow (p \vee q) \wedge (\sim p \vee \sim q)$

$\Rightarrow ((p \vee q) \wedge \sim p) \vee ((p \vee q) \wedge \sim q)$

$\vdots$

$\Rightarrow (q \wedge \sim p) \vee (p \wedge \sim q) \Rightarrow$ DNF.

Mathematical Induction.



Q) Show that $1^3 + 2^3 + 3^3 + \dots + n^3 = (1+2+\dots+n)^2$

LHS ⇒ for $n=1$:-

$P(1) = 1^3 = 1^2$

∴ True.

For $n=k$:-

Assuming for $n=k$ ⇒ LHS = RHS.

$P(k) = 1^3 + 2^3 + 3^3 + \dots + k^3 = (1+2+\dots k)^2$

LHS = RHS — (2)

therefore for $n=k+1$?

For $n=k+1$;

$P(k+1) = 1^3 + 2^3 + \dots + (k+1)^3 = (1+2+\dots (k+1))^2.$

$P(k+1) = 1^3+2^3+3^3+\dots+k^3+1+3k(k+1)$

$\Rightarrow (1+2+3+\dots+k+1)^2$

$\Rightarrow 1^3+2^3+3^3+\dots+k^3+1+3k^2+3k$

$\Rightarrow (1+2+3+\dots+(k+1))^2$

$\Rightarrow 1^3+2^3+3^3+\dots+k^3+3k^2+3k+1 = (1+2+3+\dots+k+1)^2$

$$\underbrace{\qquad\qquad}_{x}$$

$1+2+3+\cdots+K = \frac{K(K+1)}{2}$

$\hookrightarrow (x+1)^2$

$\Rightarrow x^2+1^2+2x$

$\Rightarrow (1+2+3+\cdots+K)^2 + 2(1+2+3+\cdots K) + 1$

$\Rightarrow 1^3+2^3+3^3+\cdots+K^3+3K^2+3K+\cancel{1} = (1+2+3+\cdots+K)^2 + \cancel{2}\left[\frac{K(K+1)}{\cancel{2}}\right] + \cancel{1}$

$\Rightarrow 1^3+2^3+3^3+\cdots+K^3+3K^2+3K = (1+2+3+\cdots+K)^2 + K^2 + K$

$\Rightarrow 1^3+2^3+3^3+\cdots+K^3+2K^2+2K = (1+2+3+\cdots+K)^2$

$\Rightarrow$ $\hookrightarrow$ again

$P(K+1) = 1^3+2^3+3^3+\cdots+K^3+(K+1)^3 = (1+2+\cdots+K)^2$

$\Rightarrow \big(\underbrace{1^3+2^3+3^3+\cdots}+K^3\big) + (K+1)^3 \Rightarrow (1+2+\cdots(K+1))^2$

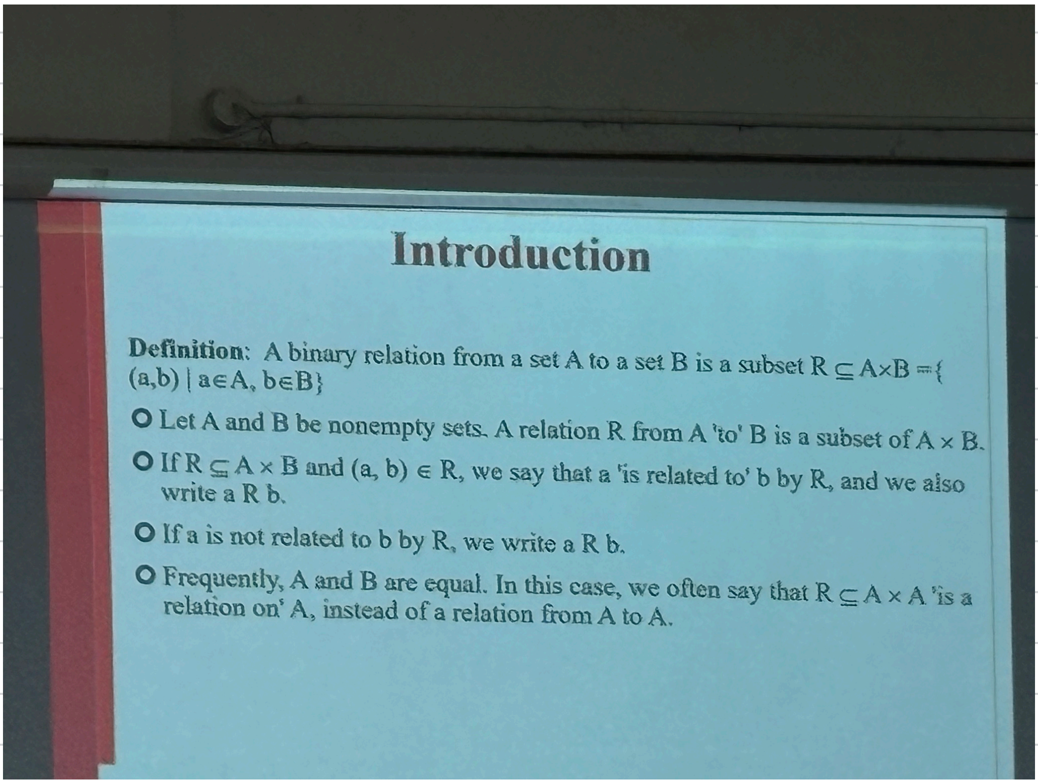
$\hookrightarrow$ from ② $(1+2+\cdots+K)^2$

$$\Rightarrow (1+2+\cdots+k)^2 + (k+1)^3$$
$$= (1+2+\cdots+(k+1))^2$$

$LHS = \underline{\underline{RHS}}$

$n^2 - 4n \rightarrow$ divisible by 3, $n \geq 2$.

Relations, Digraphs

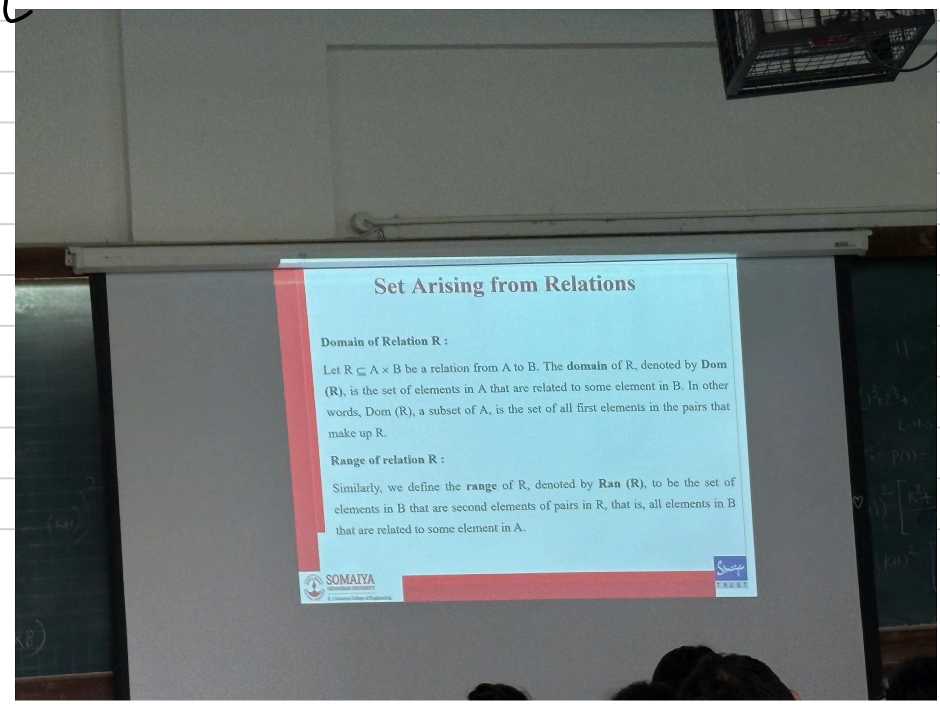


R will always be a subset of $(A \times B)$
↳ cartesian.

Q) $A = \{1, 2, 3\}$, $B = \{r, s\}$

then $R = \{(1,r), (2,s), (3,r)\}$ is a relation from A to B.
↳ subset

$A \times B = \{(1,r), (1,s), (2,r), (2,s), (3,r), (3,s)\}$



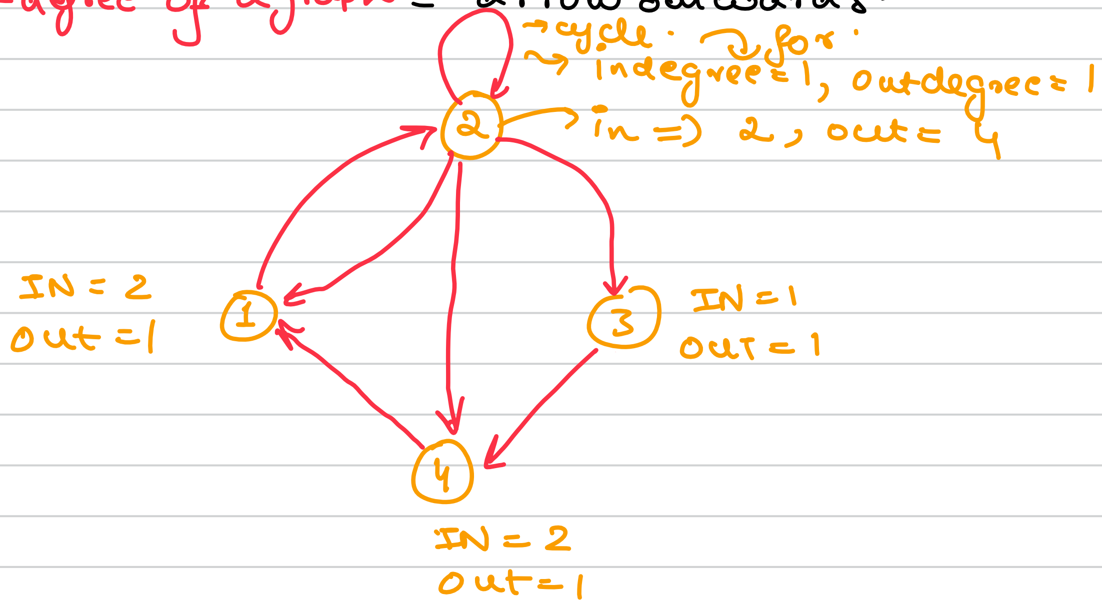
Domain - first element of pair in R.
Range - second element of pair in R.

a) $R = \{(a,\alpha), (b,\delta), (c,\alpha), (c,\delta), (d,\beta)\}$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

In-degree of a graph = No. of edges which are coming into the vertex v.

Out-degree of a graph = arrow outwards.



Q) Let $A = \{1,2,3,4,5,6\}$ & let R be the relation on A defined by 'x divides y'. Find R & draw the digraph of R. Find matrix of R & find Inverse Relation of R.

$A = \{1,2,3,4,5,6\} \to x$ $\qquad x \mid y$

$B = \{1,2,3,4,5,6\} \to y$.

$R = \{(1,2), (1,1), (1,3), (1,4), (1,6), (2,2), (2,4),$
$(2,6), (3,3), (3,6), (4,4), (6,6)\}$

$$A\left\{\begin{array}{c|c} & \begin{array}{ccccc}1 & 2 & 3 & 4 & 6\end{array} \xrightarrow{} B \\ \begin{array}{c}1\\2\\3\\4\\6\end{array} & \begin{bmatrix}1 & 1 & 1 & 1 & 1\\ 0 & 1 & 0 & 1 & 1\\ 0 & 0 & 1 & 0 & 1\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}\end{array}\right.$$

from rows to columns.

$R^{-1} = \{(1,1),(2,1),(3,1),(4,1),(6,1),(2,2),(4,2),(4,4),(6,6),(6,2),(3,3),(6,3)\}$

Q. Let $A = [1,2,3,4,6] = B$, aRb, if & only if a is a multiple of b. Find R & draw the digraph of R. Find Matrix of R.

$A = [1,2,3,4,6] \to a$

$B = [1,2,3,4,6] \to b$.

1) $R(3)$ $\quad 1,2$

$R = \{(1,1),(2,1),(3,1),(4,1),(6,1),(2,2),(4,2),(6,2),(3,3),(6,3),(4,4),(6,6)\}$

Domain$(R) = \{1,2,3,4,6\}$

Range$(R) = \{1,2,3,4,6\}$

→ 3 should be the first element.

Range of 3 $\quad R(3) = \{(3,1),(3,3)\} = 2$

↳ in relation, see where 3 is there.

→ Domain $\Rightarrow \{3\}$

Range $= \{1,3\}$

$R(3) = \{1,3\}$

since $(3,1) \in R$

& $(3,3) \in R$.

2) R(6)

$R(6) = \{(6,1), (6,2), (6,3), (6,6)\} \Rightarrow Y$

↳ Domain $= \{6\}$

Range $= \{1, 2, 3, 6\}$

3) R(2, 4, 6)

Range of 2, 4 & 6.

$R(2, 4, 6) = \{1, 2, 4, 3, 6\}$.

Since $(2,1) \in R$, $(4,2) \in R$, $(6,1) \in R$, $(6,2) \in R$,

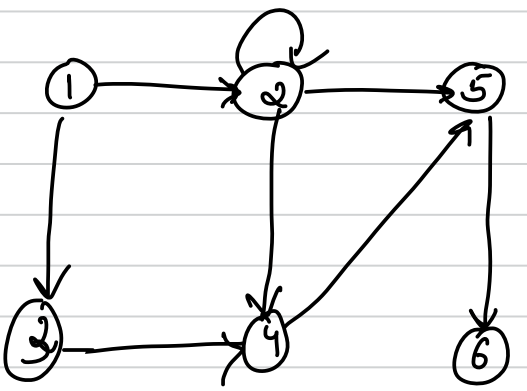
$(6,3) \in R$, $(6,6) \in R$, $(4,4) \in R$.

→ The length of a path is the no. of edges in the path, where the vertices need not all be distinct.

→ A path that begins and ends at the same vertex is called a cycle.

* Paths in Relations & Digraphs.

$A = \{1, 2, 3, 4, 5, 6\}$



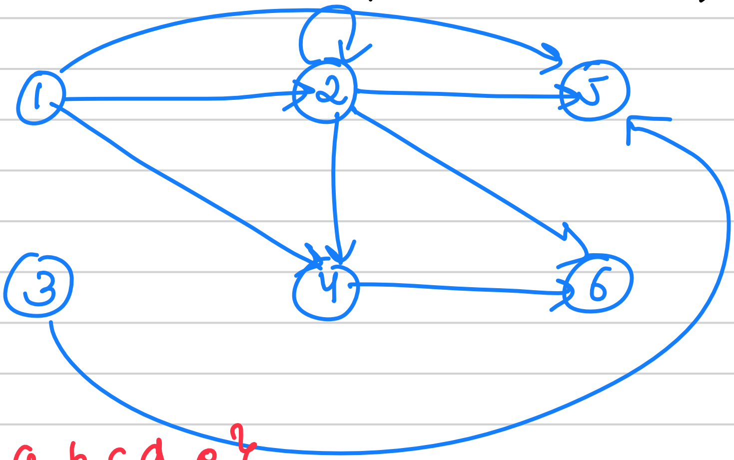
$\Rightarrow R$.

$R = \{(1,2), (1,3) (2,4), (2,5), (2,2), (4,5) (5,6)$
$(3,4)\}$

2

2edge

$R = \{(1,2), (1,4), (1,5), (2,5), (2,4), (2,2), (2,6), (3,5), (4,6)\}$



Q) let $A = \{a, b, c, d, e\}$

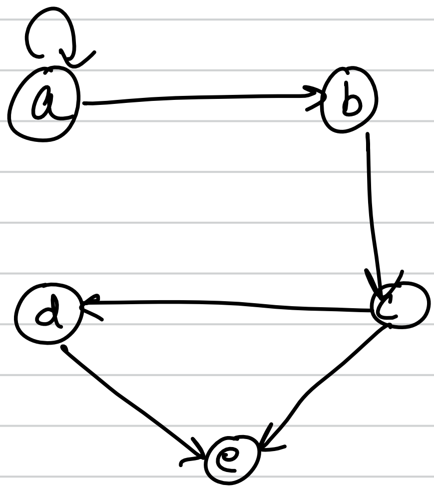
$R = \{(a,a), (a,b), (b,c), (c,e), (c,d), (d,e)\}$.

$R^\infty \Rightarrow$ connectivity relation of R.

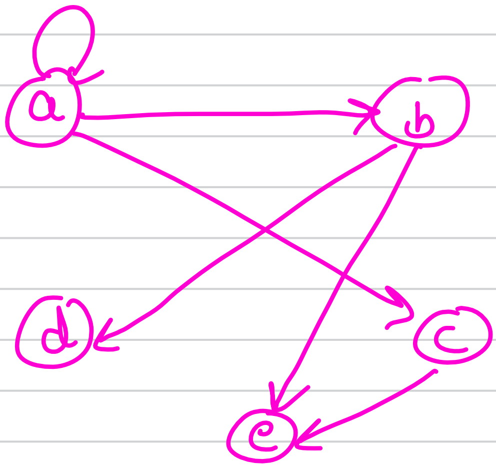
Compute

① R^2

$R \rightarrow$



$R^2 = \{(a,a), (a,b), (a,c), (b,e), (c,e), (b,d)\}$



all possible path lengths

$R^\infty = \{(a,a), (a,b), (a,c), (a,d), (a,e), (b,c), (c,e), (c,d), (b,e)\}$

$R^\infty \rightarrow$ Relation which includes all possible path lengths. (1, 2, - - - - -).

* Boolean Product $A \odot B$ $(m \times n)$

$C = [c_{ij}]$

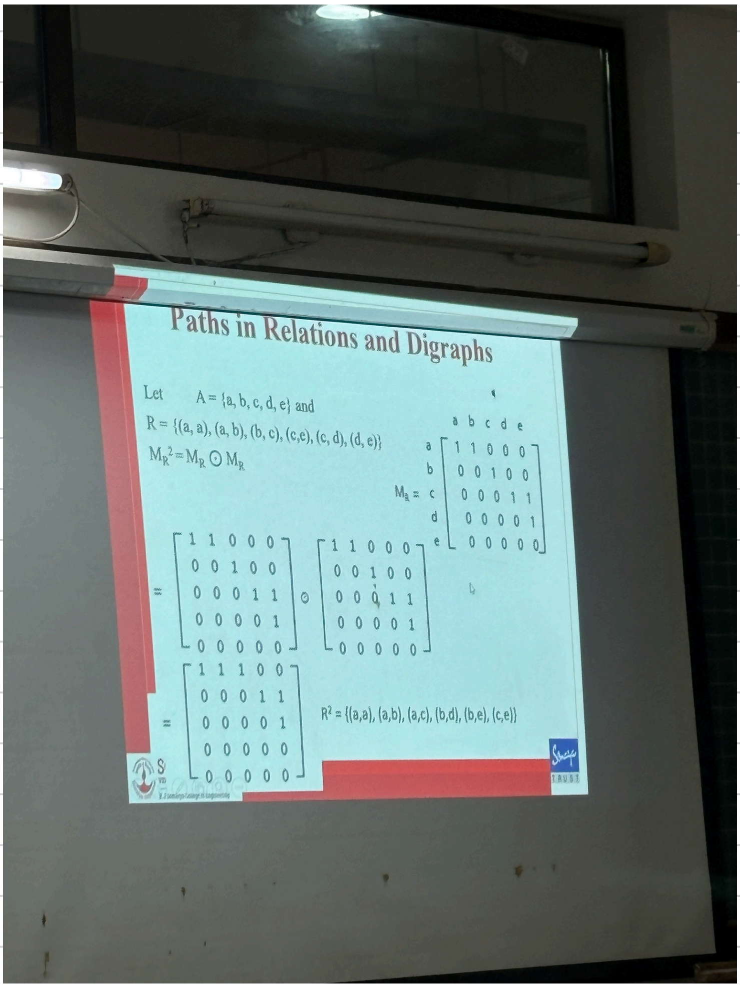
$$c_{ij} = \begin{cases} 1 & \text{if } a_{ik} = 1 \ \& \ b_{kj} = 1 \text{ for some } k,\ 1 \le k \le p \\ 0 & \text{otherwise.} \end{cases}$$

normal matrix multiplication.

$M_R^2 = M_R \odot M_R$

$M_R^n = M_R \odot M_R \odot \ldots\ldots\ldots \odot M_R$ (n factors)

R^2



rows

$A = \{a, b, c, d, e\}$

$R = \{(a,a), (a,b), (b,c), (c,e), (c,d), (d,e)\}$

set B columns.

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \left[\begin{array}{ccccc} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right. \end{array}$$

$e \; \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$M_R^2 = M_R \odot M_R$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = R^2 = \{(a,a),(a,b),(a,c),(b,d),(b,e),(c,e)\}$$

Properties / Types of Relations

1. Reflexive
2. Symmetry
3. Asymmetric
4. Antisymmetric.
5. Transitivity

① Reflexivity. → for every element $a \in A$, aRa i.e $(a,a) \in R$.

$A = \{a, b\}$

$R = \{(a,a),(a,b),(b,b)\}$. $\Rightarrow$ R is reflexive.

or

$R = \{(a,a),(a,b),(b,a),(b,b)\}$

If

$R = \{(a,a),(a,b),(b,a)\}$ $\Rightarrow$ not Reflexive.

eg:- $A = \{1,2\}$, $R = \{(1,1),(1,2)\}$

R is not reflexive since $(2,2) \notin R$

② Symmetry.

only if aRb, then bRa.

ex:- $A = \{1,2,3\}$.

$R = \{(1,2),(2,1),(2,3),(3,2),(1,1)\}$

↳ symmetric.

If $(1,2)$ is there, then $(2,1)$ must be there so that R is symmetry.

③ Asymmetric.

$(a,b) \in R$ & $(b,a) \notin R$

$R = \{(1,2),(3,4),(4,1)\}$.

$R = \{(1,2), \underbrace{(2,2)}, (3,4),(4,1)\}$

aRb, bRa.

not asymmetric.

④ Antisymmetric.

aRb, bRa only if $a=b$.

is not necessarily a reflexive

$A = \{1,2,3\}$

$R = \{(1,1),(2,2)\}$.

Q) $A = \{1,2,3\}$, $R = \{(1,2),(2,1),(2,3)\}$

not reflexive, not asymmetric, not antisymmetric.

(5) Transitivity $\quad (1,2)(2,3) \Rightarrow (1,3)$

$(a,b) \in R$ & $(b,c) \in R$, then $(a,c) \in R$ for all

$a, b, c \in A$

$\forall a,b,c \in A \; ((aRb) \wedge (bRc)) \Rightarrow aRc.$

Special case

(1) $A = \{1,2,3,4\}$

$R = \{(1,2),(1,3),(4,2)\}$

Transitive.

$(2,?)$ $\quad (a,b)$, (b,c) $\quad (a,c) \notin R$

(2) $R = \{\}$

Transitive.

(3) $R = \{(1,1),(2,2)\}$ on the set $A = \{1,2,3\}$.

symmetric & anti-symmetric.

R on $A = \{1,2,3\}$.

(1) R is transitive but not symmetric.

$R = \{(1,2),(2,3),(1,3)\}$

(2) symmetric but not transitive.

$R = \{(1,2),(2,1)\}$

(3) both symmetric & anti-symmetric.

$R = \{(1,1),(2,2)\}$

(4) neither symmetric nor anti-symmetric

$R = \{(1,2),(2,3),(3,2)\}$

Q. $A = \{a,b,c,d\}$

i) transitive, reflexive & symmetric
ii) symmetric & transitive.

i) $\{(a,a), (b,b), (c,c), (d,d), (a,b), (b,a), (b,c), (c,b), (c,d), (d,c), (a,c), (b,d), (c,a), (a,d), (d,a), (d,b)\}$

ii) $\{(a,b), (b,a), (a,c), (c,a), (b,c), (c,b), (b,d), (d,b), (a,d), (d,a), (c,d), (d,c)\}$

Irreflexive Relations

$(a,a) \notin R$ for every $a \in A$

$A = \{1,2\}$, $R = \{(1,2),(2,1)\}$

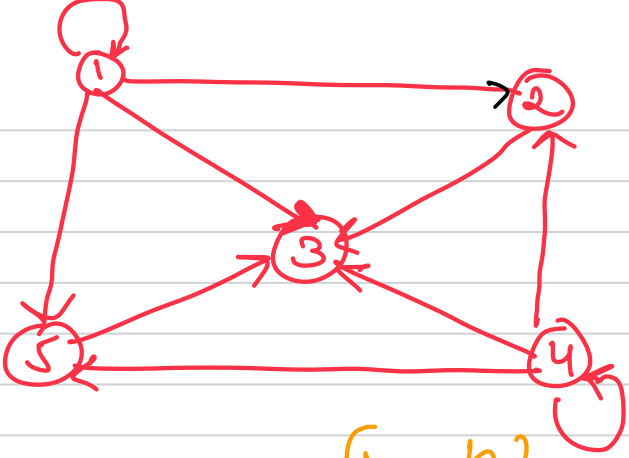
Universal Relation. (Equivalence relation).

universal relation from $A \to B \Rightarrow$ reflexive, symmetric, transitive.

Relation $R: A \to B$ such that
$R = A \times B \; (\subseteq A \times B)$

Q) $A = \{1, 2, 3, 4, 5\}$.

whether R is reflexive, $\longrightarrow$ all prop.



$R = \{(1,1), (1,2), (1,3), (1,5), (2,3), (4,4), (4,3), (4,2), (4,5), (5,3)\}$

irreflexive

not reflexive, ~~asymmetric~~, antisymmetric, transitive.

↳ b/c (1,1), (4,4)

a,b
b,a

∈R

$(a,b)\ \{(b,c), (a,c)\} \notin R$ check every pair for property.

↳ transitive special case.

Q) Let $A = \{a,b,c\}$, & let $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

Determine if R is equivalence relation.

$$M_R = \begin{array}{c|ccc} & a & b & c \\ \hline a & 1 & 0 & 0 \\ b & 0 & 1 & 1 \\ c & 0 & 1 & 1 \end{array}$$

$R = \{(a,a), (b,b), (b,c), (c,b), (c,c)\}$

reflexive, symmetric, transitive

↳ equivalence.

a b
c,b

b, c

c, b

b, b

special case

a, a

a=b=c

a b

b, c

c b

(b, b)

b c

Q) $A = \{1,2,3,4\}$

$R = \{(1,1), (1,2), (2,1), (2,2), (3,1), (3,3), (1,3), (4,1)$
$(4,4)\}$

Determine if R is equivalence relation.

reflexive, not symmetric → not equivalent

~~de~~ no need to check for transitive.

$(2,1), (1,3) \in R$
but $(2,3) \notin R$
↳ not transitive.

Equivalence class & Partitions:-

let $A = \{1,2,3,4\}$ & consider the partition

$P = \{\{1,2,3\}, \{4\}\}$ of A.

Each element in a Block is related to every other element in the same Block & only to those elements.

$R = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2)$
$(3,3), (4,4)\}$.

reflexive, sym ⟶ equivalence Relation.

How to find equivalence class.

$R = \{(1,1), (1,5), (2,2), (2,3), (2,6), (3,2), (3,3), (3,6), (4,4), (5,1), (5,5), (6,2), (6,3), (6,6)\}$

equivalence class :-

$R(1) = \{1,5\}$ $\quad$ $R(2) = \{2,3,6\}$

$R(3) = \{2,3,6\}$ $\quad$ $R(4) = \{4\}$

$R(5) = \{1,5\}$ $\quad$ $R(6) = \{2,3,6\}$.

Rank R (Number of distinct equivalence classes)
$= 3$

the partition of A induced By R i.e $A/R = \{\{1,5\}, \{2,3,6\}, \{4\}\}$

Q) find E-class, partition & rank.

$A = \{1,2,3\}$ $\quad$ $R = \{(1,1), (2,2), (1,3), (3,1), (3,3)\}$

find A/R.

$R(1) = \{1,3\}$ $\quad$ $A/R = \{\{1,3\}, \{2\}\}$

$R(2) = \{2\}$ $\quad$ Rank = 2.

$R(3) = \{1,3\}$

Q) let $A = \{1,2,3,4\}$

$R = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,3), (3,3), (4,4)\}$

$R(1) = \{1,2\}$ $\quad$ $A/R = \{\{1,2\}, \{3,4\}\}$

$R(2) = \{1,2\}$ $\quad$ rank = 2

$R(3) = \{3,4\}$

$R(4) = \{3,4\}$

Q) $A = \{1,2,3,4\}$

$R = \{(1,1),(1,2),(1,3),(2,1),(2,2),(3,1),(2,3),(3,2),(3,3),(4,4)\}$

$R(1) = \{1,2,3\}$
$R(2) = \{1,2,3\}$
$R(3) = \{1,2,3\}$
$R(4) = \{4\}$

$A|R = \{\{1,2,3\},\{4\}\}$

Rank = 2

Combining Relations :-

1. compliment $\overline{R}$
2. Intersection $(R_1 \cap R_2)$
3. Union $(R_1 \cup R_2)$
4. Set difference $(R_1 \setminus R_2)$
5. Inverse R^{-1}.

Q) $A = \{1,2,3\}$, $B = \{u,v\}$.

$R_1 = \{(1,u),(2,u),(2,v),(3,u)\}$ R

$R_2 = \{(1,v),(3,u),(3,v)\}$

$R_1 \cup R_2 = \{(1,u),(1,v),(2,u),(2,v),(3,u),(3,v)\}$

Q) A

Composite of Relations:-

The composite of R_1 & R_2 is the relation consisting of ordered pairs (a, c) where $a \in A$, $c \in C$ & for which there exists an element $b \in B$ such that $(a, b) \in R_1$ & $(b, c) \in R_2$. We denote the composite of R_1 & R_2 by

$$R_2 \circ R_1.$$

$$R_1 \subseteq A \times B \ \& \ R_2 \subseteq B \times C.$$

Q. $A = \{1, 2, 3\}$, $B = \{0, 1, 2\}$, $C = \{a, b\}$

$R = \{(1,0), (1,2), (3,1), (3,2)\}$ ⇒ $A \times B$

$S = \{(0,b), (1,a), (2,b)\}$ = $B \times C$.

find composition b/w R & S.

$\underline{S \circ R}$

$\{(1,b)$ ~~$\{$~~ $(3,a)$ $(3,b)\}$

Q) $A = \{1, 2, 3\}$

$R = \{(1,1), (1,3), (2,1), (2,2), (2,3), (3,2)\}$

$S = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$

find $M_{S \circ R}$.

$$S \circ R = \{(1,1), (1,3), (2,1), (2,2), (2,3), (3,2), (3,3)\}$$

$$M_{S \circ R} = \begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

Q) $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,2), (2,3), (2,4), (3,4), (4,1), (4,2)\}$

$S = \{(3,1), (4,4), (2,3), (2,4), (1,1), (1,4)\}$

Compute $S \circ R$, $R \circ S$, $R \circ R$, $S \circ S$.

$\underline{S \circ R} = \{(3,2), (4,1), (2,4), (2,1), (1,1), (1,2), (4,4),\}$

$R \circ R = \{(\cancel{1}, 2), (1,3), (2,2),$