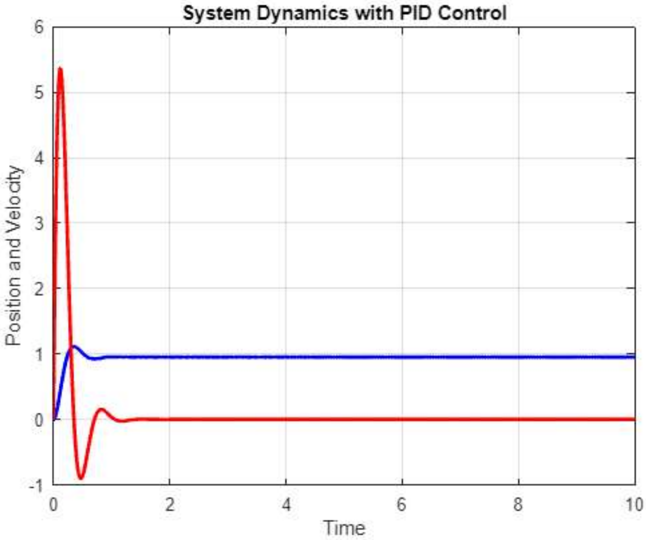
```
clear; clc; close all
```

Manual PID Tuning

```
clear compute_pid_control

% Defining System Parameters
dynamics_params.mass = 1.0;                   % Mass of the object
dynamics_params.spring_constant = 5.0;        % Spring constant of the spring
dynamics_params.damping_coefficient = 0.1;    % Damping coefficient of the damper

% Initialize PID gains
gains = [100; 0.001; 10];

% Initial state [position; velocity]
initial_state = [0; 0];

% Desired state [desired_position; desired_velocity]
desired_state = [1; 0];

% Time span for simulation
tspan = [0, 10];  % Start time and end time

% Use numerical integration (ode45) to simulate the system dynamics with PID control
[t, y] = ode45(@(t, y) dynamics(t, y, compute_pid_control(gains, y, desired_state), dynamics_params), tspan, initial_state);

% Extract position and velocity for plotting
position = y(:, 1);
velocity = y(:, 2);

% Plot the results
figure;
plot(t, position, 'b', 'LineWidth', 2);
hold on;
plot(t, velocity, 'r', 'LineWidth', 2);
xlabel('Time');
ylabel('Position and Velocity');
title('System Dynamics with PID Control');
grid on;
```




```
cost_manual = cost(gains,desired_state,initial_state,dynamics_params,tspan,ones(2))
```

```
cost_manual = 612.0727
```

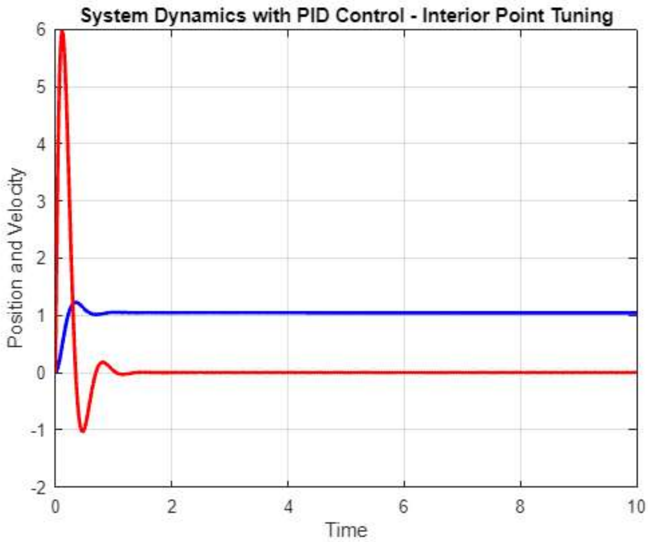
PID Gain Optimisation - ODE45

```
% interior point
options = optimoptions("fmincon","Display","iter",...
    "Algorithm","interior-point",...
    "EnableFeasibilityMode",true,...
    "SubproblemAlgorithm","cg");
gains_interiorP = fmincon(@(gains)cost(gains,desired_state,initial_state,dynamics_params,tspan,ones(2)),[100;0.01;10],[],[],[],[],[],
```

```
                                            First-order      Norm of
 Iter F-count            f(x)  Feasibility   optimality         step
    0       4    1.323707e+03    1.519e-02    2.395e+08
    1       8    6.583301e+02    6.949e-03    2.778e+07    1.732e+00

Converged to an infeasible point.
```

```
fmincon stopped because the size of the current step is less than
the value of the step size tolerance but constraints are not
satisfied to within the value of the constraint tolerance.

<stopping criteria details>
gains_interiorP = 3×1

  101.7317
    0.0055
    9.9655
```

```
% Use numerical integration (ode45) to simulate the system dynamics with PID control
[t, y] = ode45(@(t, y) dynamics(t, y, compute_pid_control(gains_interiorP, y, desired_state), dynamics_params), tspan, initial_state)

% Extract position and velocity for plotting
position = y(:, 1);
velocity = y(:, 2);

% Plot the results
figure;
plot(t, position, 'b', 'LineWidth', 2);
hold on;
plot(t, velocity, 'r', 'LineWidth', 2);
xlabel('Time');
ylabel('Position and Velocity');
title('System Dynamics with PID Control - Interior Point Tuning');
grid on;
```




```
cost_interiorP = cost(gains_interiorP,desired_state,initial_state,dynamics_params,tspan,ones(2))
```

```
cost_interiorP = 1.4607e+03
```

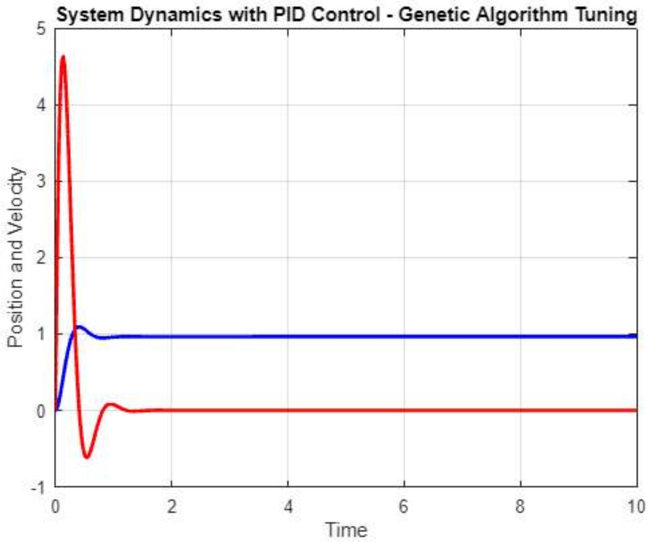
PID Gain Optimisation - Genetic Algorithm

```
% Define the GA options
ga_options = optimoptions('ga', 'Display', 'iter', 'PopulationSize', 20, 'MaxGenerations', 25);

% Set the lower and upper bounds for the PID gains
lb = [10, 0.0005, 1]; % Lower bounds for Kp, Ki, and Kd
ub = [200, 0.06, 20]; % Upper bounds for Kp, Ki, and Kd

% Perform the optimization using GA
gains_ga = ga(@(gains) cost(gains, desired_state, initial_state, dynamics_params, tspan, ones(2)), 3, [], [], [], [], lb, ub, @(gains
```

```
Single objective optimization:
3 Variable(s)
4 Nonlinear equality constraint(s)

Options:
CreationFcn:       @gacreationuniform
CrossoverFcn:      @crossoverscattered
SelectionFcn:      @selectionstochunif
MutationFcn:       @mutationadaptfeasible

                              Best       Max        Stall
Generation  Func-count        f(x)     Constraint  Generations
    1           540         164.061      0.3099       0
    2          1060         235.277     0.009584      0
    3          1580         577.877      0.09978      0
gains_ga = 1×3
```

```
    78.2568    0.0044    9.7559
```

```
% Use numerical integration (ode45) to simulate the system dynamics with PID control
[t, y] = ode45(@(t, y) dynamics(t, y, compute_pid_control(gains_ga, y, desired_state), dynamics_params), tspan, initial_state);

% Extract position and velocity for plotting
position = y(:, 1);
velocity = y(:, 2);

% Plot the results
figure;
plot(t, position, 'b', 'LineWidth', 2);
hold on;
plot(t, velocity, 'r', 'LineWidth', 2);
xlabel('Time');
ylabel('Position and Velocity');
title('System Dynamics with PID Control - Genetic Algorithm Tuning');
grid on;
```




```
cost_ga = cost(gains_ga,desired_state,initial_state,dynamics_params,tspan,ones(2))
```

```
cost_ga = 720.6366
```

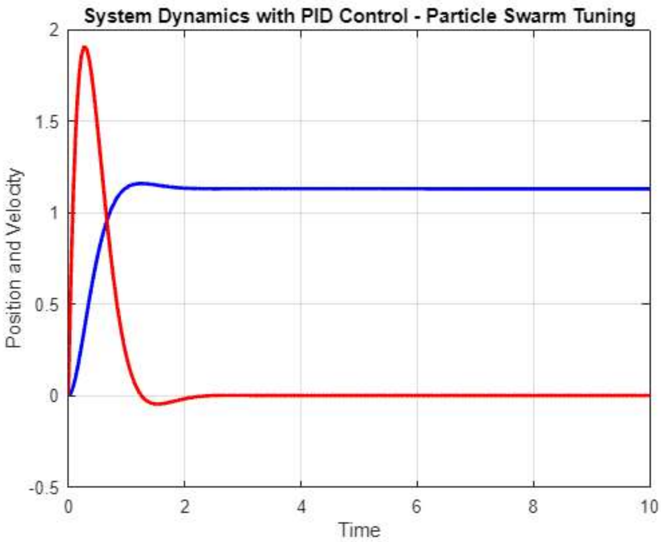
PID Gain Optimisation - Particle Swarm

```
% Define the GA options
ps_options = optimoptions('particleswarm', 'Display', 'iter', 'SwarmSize', 100);

% Set the lower and upper bounds for the PID gains
lb = [10, 0.0005, 1]; % Lower bounds for Kp, Ki, and Kd
ub = [200, 0.06, 20]; % Upper bounds for Kp, Ki, and Kd

% Perform the optimization using GA
gains_ps = particleswarm(@(gains) cost(gains, desired_state, initial_state, dynamics_params, tspan, ones(2)), 3, lb, ub, ps_options)
```

```
                                 Best            Mean     Stall
Iteration     f-count            f(x)            f(x)    Iterations
    0             100           412.9            1863        0
    1             200           160.2            1880        0
    2             300           160.2            1331        1
    3             400           159.5            1075        0
    4             500           159.5           989.1        1
    5             600           159.5           834.2        2
    6             700           159.5           608.7        3
    7             800           148.7           571.5        0
    8             900           148.7           538.9        1
    9            1000           148.7           376.6        2
   10            1100           148.7           655.6        3
   11            1200           148.7           392.5        4
   12            1300           148.7           464.8        5
   13            1400           148.7           388.8        6
   14            1500           148.7            1442        7
   15            1600           148.7           249.7        8
   16            1700           148.7           374.2        9
   17            1800           148.7           404.9       10
   18            1900           148.7           430.9       11
   19            2000           148.7             418       12
   20            2100           148.7           412.8       13
   21            2200           148.7           427.4       14
   22            2300           148.7             415       15
```

```
    23          2400          148.7          419.6      16
    24          2500          148.7            410      17
gains_ps = 1×3

   10.0000    0.0005    5.8126
```

```
% Use numerical integration (ode45) to simulate the system dynamics with PID control
[t, y] = ode45(@(t, y) dynamics(t, y, compute_pid_control(gains_ps, y, desired_state), dynamics_params), tspan, initial_state);

% Extract position and velocity for plotting
position = y(:, 1);
velocity = y(:, 2);

% Plot the results
figure;
plot(t, position, 'b', 'LineWidth', 2);
hold on;
plot(t, velocity, 'r', 'LineWidth', 2);
xlabel('Time');
ylabel('Position and Velocity');
title('System Dynamics with PID Control - Particle Swarm Tuning');
grid on;
```




```
cost_ps = cost(gains_ps,desired_state,initial_state,dynamics_params,tspan,ones(2))
```

```
cost_ps = 394.9282
```

PID Gain Optimisation - with Pseudospectral Formulation

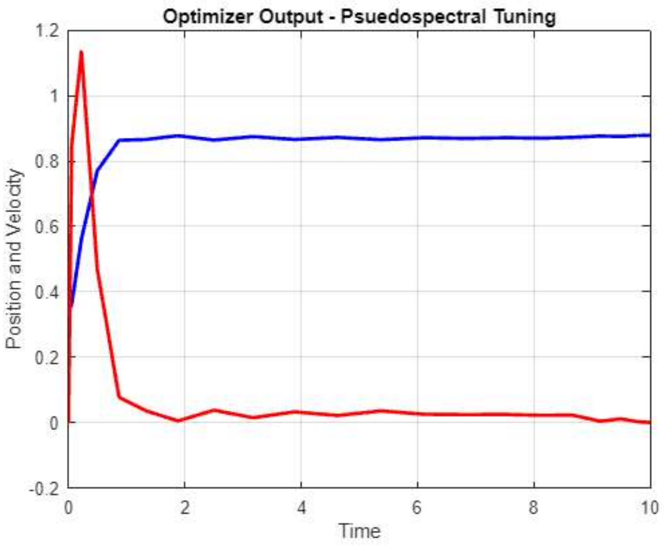
```
N = 21;

init = zeros(5, N+1);
options = optimoptions('fmincon','Display','iter','MaxFunctionEvaluations',1e11,'Algorithm','sqp','MaxIterations',5e3);

tf = 10;
desired_state = [1; 0];
initial_state = [0; 0];
Q = ones(2);

[Z,~,flg] = fmincon(@(Z)cost_pseudospectral(Z,desired_state,Q,N,tf),init,[],[],[],[],[],[],@(Z)NonLinCon_PseudoSpectral(Z,initial_sta
```

```
 Iter  Func-count            Fval   Feasibility   Step Length       Norm of   First-order
                                                                       step    optimality
    0         111    9.988662e+00     1.000e+01     1.000e+00     0.000e+00     1.492e+00
    1         232    8.840548e+00     9.718e+00     2.825e-02     4.041e-01     1.151e+01
    2         353    7.699081e+00     9.376e+00     2.825e-02     4.659e-01     1.175e+01
    3         475    6.480701e+00     8.929e+00     1.977e-02     5.869e-01     1.205e+01
    4         596    5.697973e+00     8.571e+00     2.825e-02     4.510e-01     1.179e+01
    5         715    4.818754e+00     8.077e+00     5.765e-02     5.985e-01     1.061e+01
    6         833    3.974559e+00     7.412e+00     8.235e-02     7.683e-01     8.465e+00
    7         951    3.407723e+00     6.802e+00     8.235e-02     6.850e-01     8.579e+00
    8        1067    2.587968e+00     5.659e+00     1.681e-01     1.315e+00     8.489e+00
    9        1179    1.099479e+00     1.698e+00     7.000e-01     4.556e+00     9.316e+00
   10        1290    1.037361e+00     3.179e-01     1.000e+00     1.745e+00     7.964e+00
   11        1401    1.000133e+00     3.079e-01     1.000e+00     1.078e+00     7.344e+00
   12        1514    9.562048e-01     2.898e-01     4.900e-01     1.105e+00     5.374e+00
```

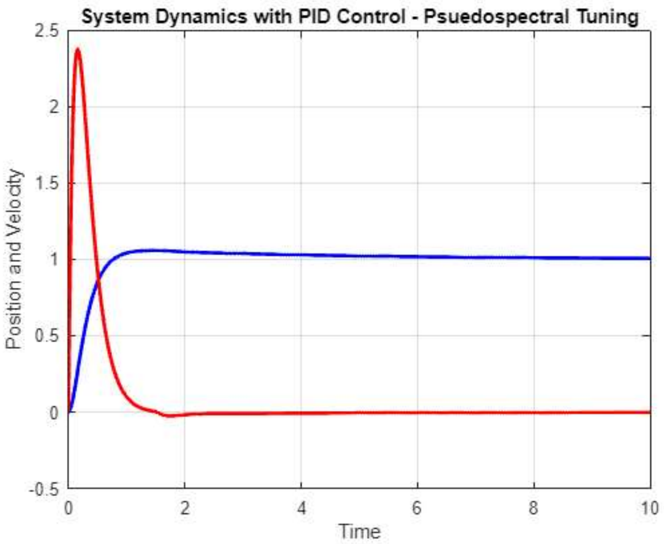
```
gains_psuedospectral = Z(3:5,1).'
```

```
gains_psuedospectral = 1×3

   35.4834    0.0600   13.1260
```

```
[nodes,wk] = clencurt(N);
tspan_ = ((flip(nodes)+1)/2).*tf;

% Extract position and velocity for plotting
position = Z(1, :);
velocity = Z(2, :);

% Plot the results
figure;
plot(tspan_, position, 'b', 'LineWidth', 2);
hold on;
plot(tspan_, velocity, 'r', 'LineWidth', 2);
xlabel('Time');
ylabel('Position and Velocity');
title('Optimizer Output - Psuedospectral Tuning');
grid on;
```




```
% Use numerical integration (ode45) to simulate the system dynamics with PID control
[t, y] = ode45(@(t, y) dynamics(t, y, compute_pid_control(gains_psuedospectral, y, desired_state), dynamics_params), tspan, initial_s

% Extract position and velocity for plotting
position = y(:, 1);
velocity = y(:, 2);

% Plot the results
figure;
plot(t, position, 'b', 'LineWidth', 2);
hold on;
plot(t, velocity, 'r', 'LineWidth', 2);
xlabel('Time');
ylabel('Position and Velocity');
title('System Dynamics with PID Control - Psuedospectral Tuning');
grid on;
```



```
cost_pseudospec = cost(gains_psuedospectral,desired_state,initial_state,dynamics_params,tspan,ones(2))
```

```
cost_pseudospec = 1.3262e+03
```

Non-Linear Constraint for Pseudospectral Optimisation

```
function [c,ceq] = NonLinCon_PseudoSpectral(Z,initial_state,desired_state,dynamics_params,N,tf)

    state = Z(1:2,:);
    gains = Z(3:5,1);

    Kp = gains(1);
    Ki = gains(2);
    Kd = gains(3);

    c(1) = Kp - 200;
    c(2) = Ki - 0.06;
    c(3) = Kd - 20;
    c(4) = 10 - Kp;
    c(5) = 0.0005 - Ki;
    c(6) = 1 - Kd;

    % Use Guass-Quadrature integration to simulate system dynamics with PID
    error = desired_state(1,:) - state(1,:);
    integral = zeros(size(error));
    integral(:,1) = error(:,1);

    velocity_error = desired_state(2,:) - state(2,:);

    [~,wk] = clencurt(N);

    for i = 2:size(state,2)
        integral(:,i) = integral(:,i-1) + wk(i)*error(:,i);
    end

    D = -DCPeyret(N+1);

    % Unpack state variables
    position = state(1,:);
    velocity = state(2,:);

    % Unpack dynamics parameters from the struct
    mass = dynamics_params.mass;
    spring_constant = dynamics_params.spring_constant;
    damping_coefficient = dynamics_params.damping_coefficient;

    input = Kp*error + Ki*integral + Kd*velocity_error;

    % Compute the acceleration (second derivative of position)
    acceleration = (input - damping_coefficient * velocity - spring_constant * position) / mass;

    c_dynamics = ((2/tf)*D*state.').' - ([velocity; acceleration]);
    reshape(c_dynamics, [], 1);

    ceq_init = state(:,1) - initial_state;
    ceq_des = state(:,end) - desired_state;

    ceq = vertcat(reshape(ceq_init,[2,1]),reshape(ceq_des,[2,1]),reshape(c_dynamics, [], 1));
end

%% Cost Pseudospectral Optimisation
function J = cost_pseudospectral(Z,desired_state,Q,N,tf)

    [~,wk] = clencurt(N);

    state = Z(1:2,:);
    J = 0;
    for i = 1:N
        J = J + wk(i)*((state(:,i)-desired_state).'*Q*(state(:,i)-desired_state));
    end
    J = J.*(tf/2);
end
```

System Dynamics Constraint Function - ODE

```
function [c,ceq] = DynamicConstODE(gains,desired_state,initial_state,dynamics_params,tspan)
    c = [];

    % Use numerical integration (ode45) to simulate the system dynamics with PID control
    [t, y] = ode45(@(t, y) dynamics(t, y, compute_pid_control(gains, y, desired_state), dynamics_params), tspan, initial_state);

    ceq_init = y(1,:) - initial_state.';
    ceq_des = y(end,:) - desired_state.';

    ceq = vertcat(reshape(ceq_init,[2,1]),reshape(ceq_des,[2,1]));
end
```

Cost Function

```
function J = cost(gains,desired_state,initial_state,dynamics_params,tspan,R)
    % Use numerical integration (ode45) to simulate the system dynamics with PID control
    [t, y] = ode45(@(t, y) dynamics(t, y, compute_pid_control(gains, y, desired_state), dynamics_params), tspan, initial_state);

    J = 0;
    for timestep = 1:size(y,1)
        state = y(timestep,:);
        J = J + state*R*state.';
    end
end
```

Input Calculator - PID - persistent variables

```
function control_input = compute_pid_control(gains, state, desired_state)
    % gains: PID gain vector [Kp; Ki; Kd]
    % state: State vector [position; velocity]
    % desired_state: Desired state vector [position; velocity]

    % PID error computation
    position_error = desired_state(1) - state(1);
    velocity_error = desired_state(2) - state(2);

    % Initialize persistent integral error variable (only done on the first function call)
    persistent integral_error
    if isempty(integral_error)
        integral_error = 0;
    end

    % Compute the integral term using trapezoidal rule and update the persistent variable
    integral_error = integral_error + position_error;

    % PID control law
    P_term = gains(1) * position_error;
    I_term = gains(2) * integral_error;
    D_term = gains(3) * velocity_error;

    % Compute the PID control input
    control_input = P_term + I_term + D_term;
end
```

Spring Mass Damper System Dynamics

```
function dydt = dynamics(t, state, input, dynamics_params)
    % t: Time (not used in this function, but required for numerical integration)
    % state: State vector [position; velocity]
    % input: Control input applied to the system (force)
    % dynamics_params: struct containing system dynamic parameters (mass,
    % spring_constant, damping_coefficient)

    % Unpack state variables
    position = state(1);
    velocity = state(2);

    % Unpack dynamics parameters from the struct
    mass = dynamics_params.mass;
    spring_constant = dynamics_params.spring_constant;
    damping_coefficient = dynamics_params.damping_coefficient;

    % Compute the acceleration (second derivative of position)
    acceleration = (input - damping_coefficient * velocity - spring_constant * position) / mass;

    % Return the derivative of the state variables
    dydt = [velocity; acceleration];
end
```

Chebyshev Pseudospectral Method

```
function [x,w] = clencurt(N)
    % File to compute CGL Nodes x and
    % Clenshaw-Curtis Quadrature Weights w

    theta = pi*(0:N)'/N;
    x = cos(theta);

    w = zeros(N+1,1);
    i = 2:N;
    v = ones(N-1,1);

    if mod(N,2) == 0
        w(1) = 1/(N^2 -1);
        w(N+1) = w(1);
```

```
        for j = 1:0.5*N-1
            v = v - 2*cos(2*j*theta(i))/(4*j^2 - 1);
        end
        v = v - cos(N*theta(i))/(N^2 - 1);
    else
        % CHECK ODD CASE
        w(1) = 1/N^2;
        w(N+1) = w(1);

        for j = 1:0.5*(N-1)
            v = v - 2*cos(2*j*theta(i))/(4*j^2 - 1);
        end
    end
    w(i) = 2*v/N;
end

function D = DCPeyret(N)
    % Peyret's D matrix
    D = zeros(N);
    D(1,1) = (2*(N-1)^2 + 1)/6;
    D(N,N) = -(2*(N-1)^2 + 1)/6;
    m = 0:N-1;
    ndsP = cos(m*pi/(N-1));
    cbar = ones(N,1);
    cbar(1) = 2;
    cbar(N) = 2;

    for i = 1:N
        if i ~= 1 && i ~= N
            D(i,i) = -0.5*ndsP(i)/(1- ndsP(i)^2);
        end

        for j = setdiff(1:N,i)
            D(i,j) = cbar(i) * (-1)^(i + j) / (cbar(j) * (ndsP(i) - ndsP(j)));
        end
    end
end
```