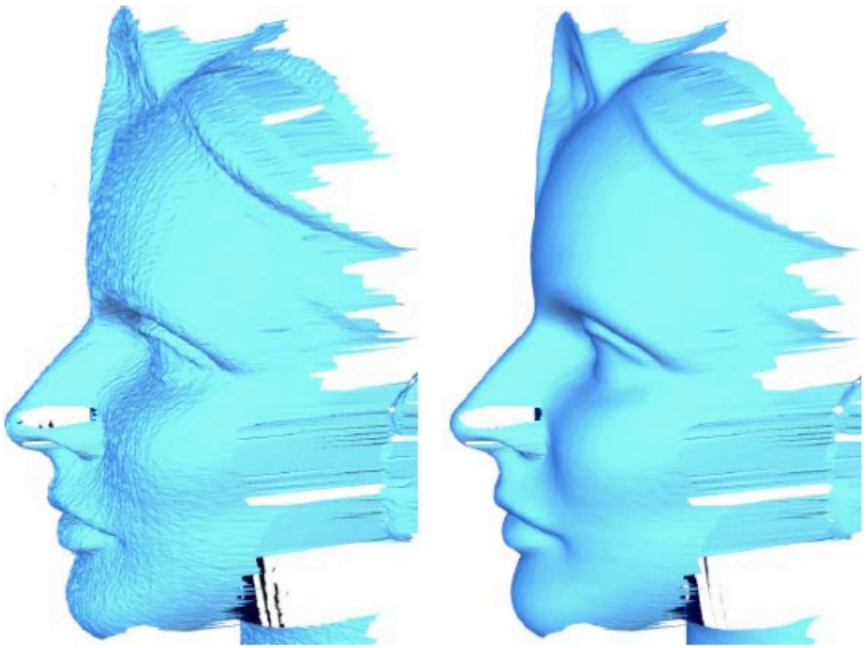
Mesh Smoothing

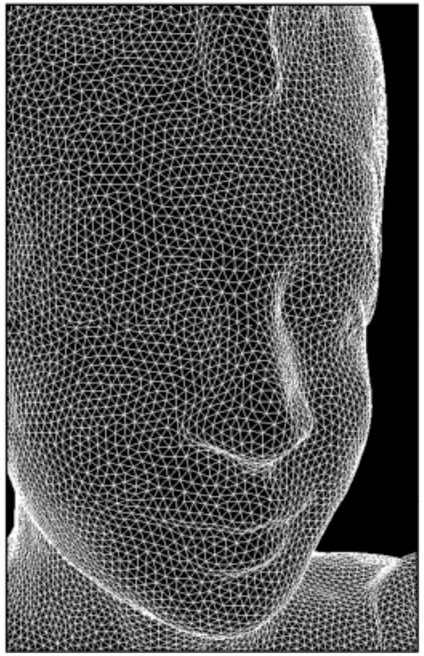


Mesh Processing Pipeline

Scan

Reconstruct

Clean



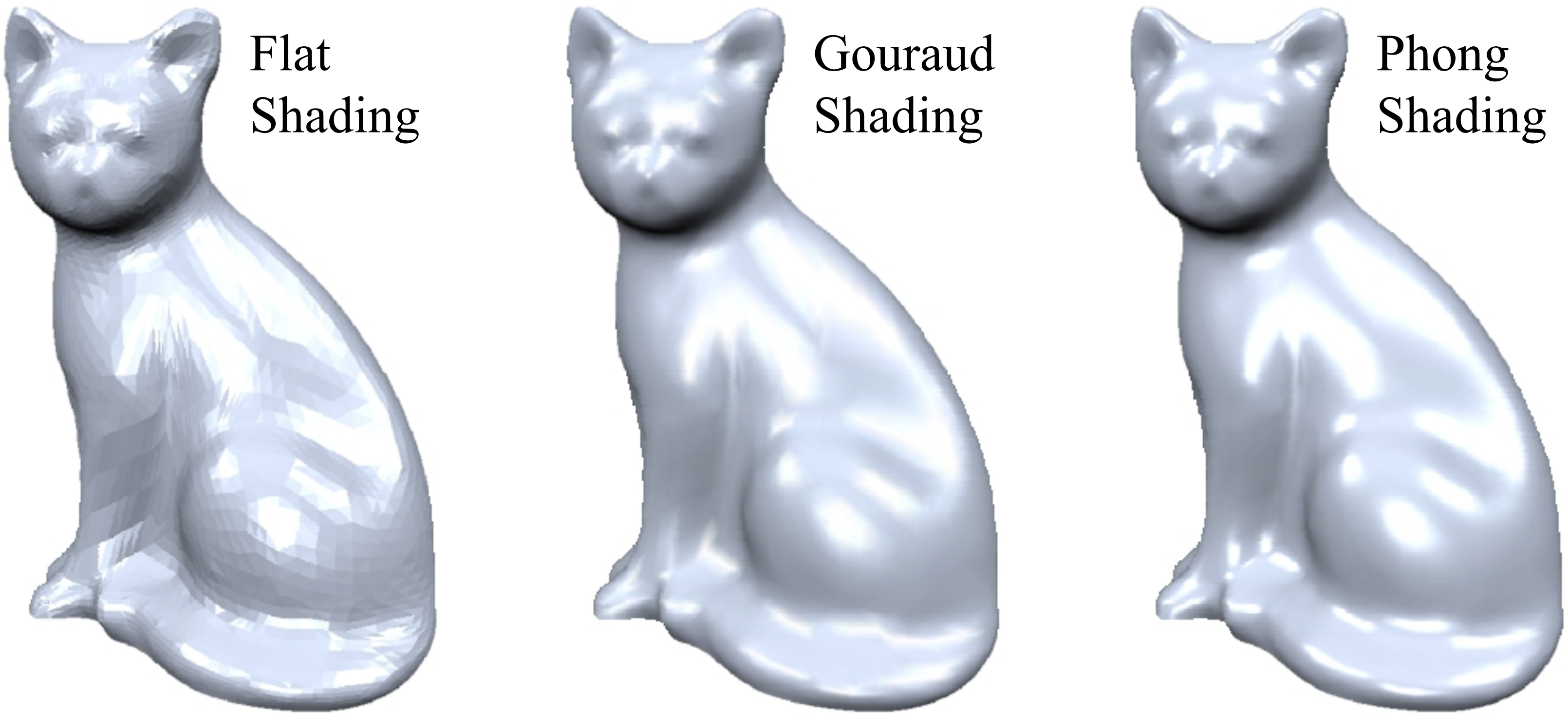
Remesh

...

Mesh Quality

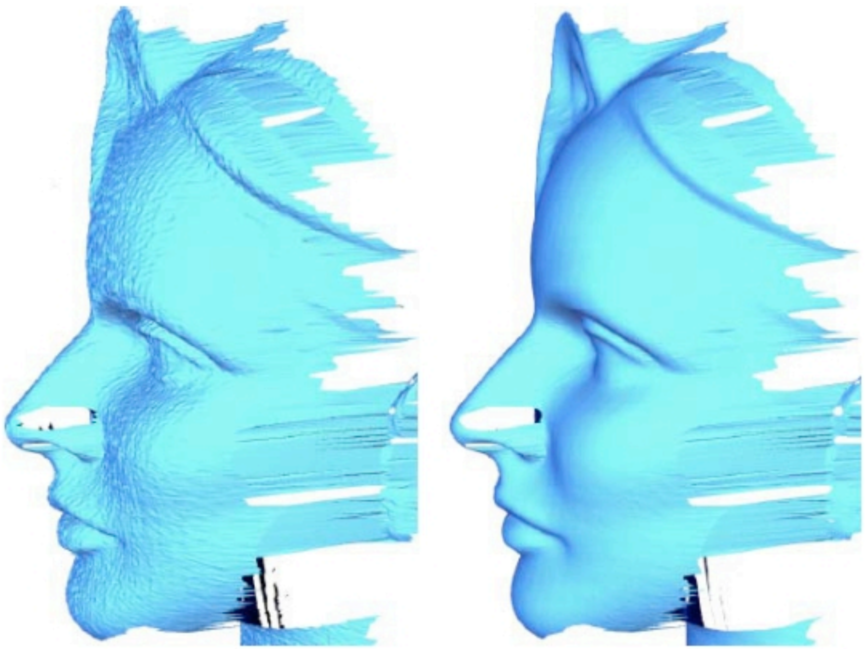
Visual inspection of "sensitive" attributes

- Specular shading



Motivation

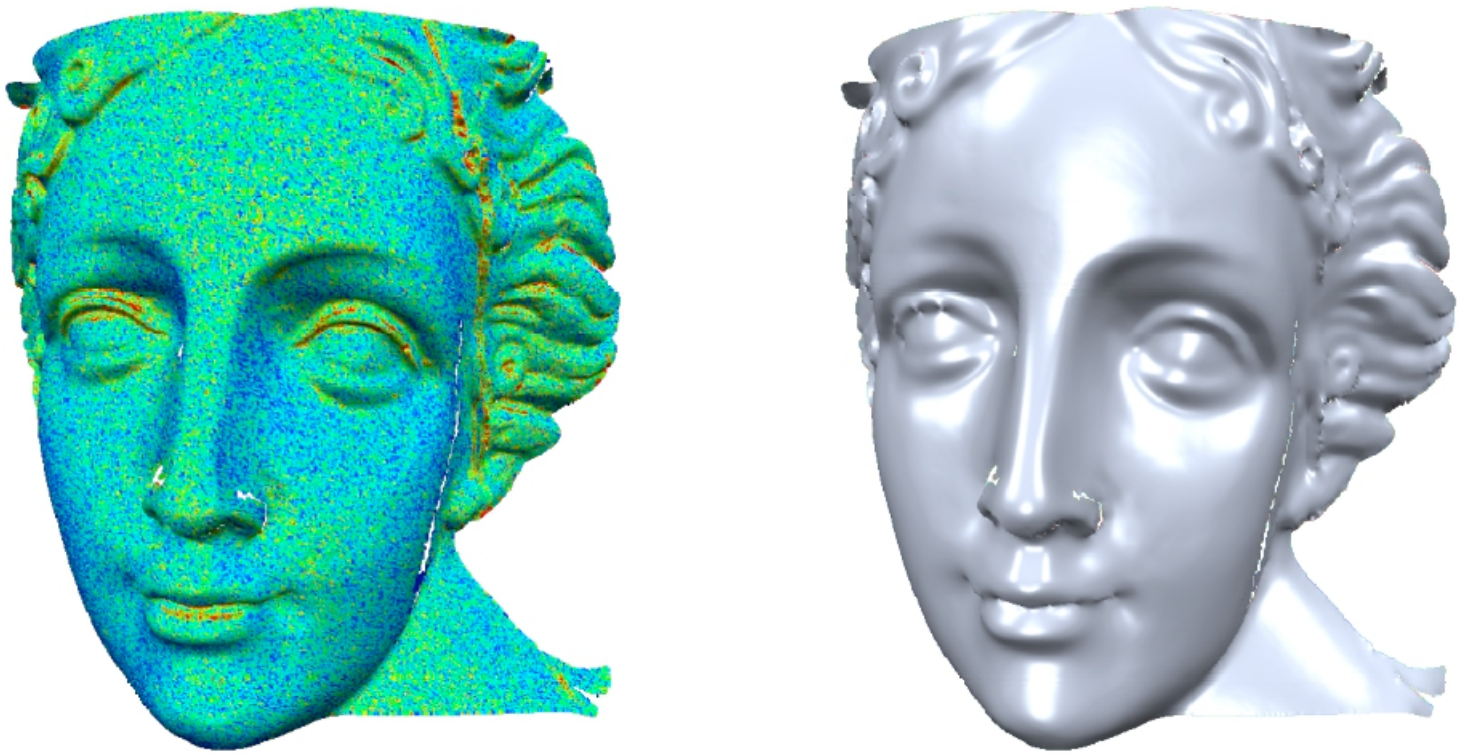
Filter out high frequency noise



Mesh Smoothing

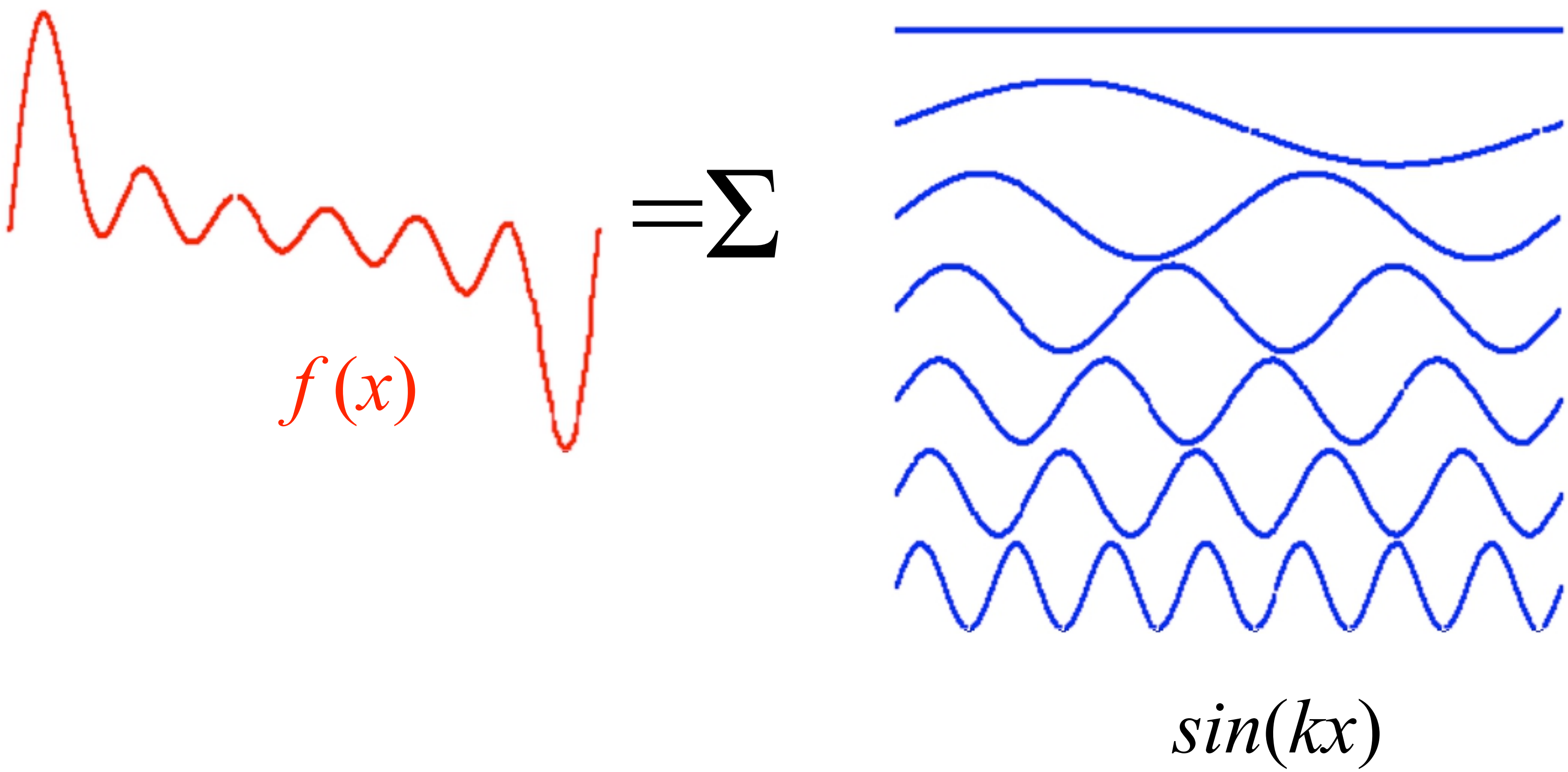
(aka Denoising, Filtering, Fairing)

Input: Noisy mesh (scanned or other)
Outut: Smooth mesh
How: Filter out high frequency noise



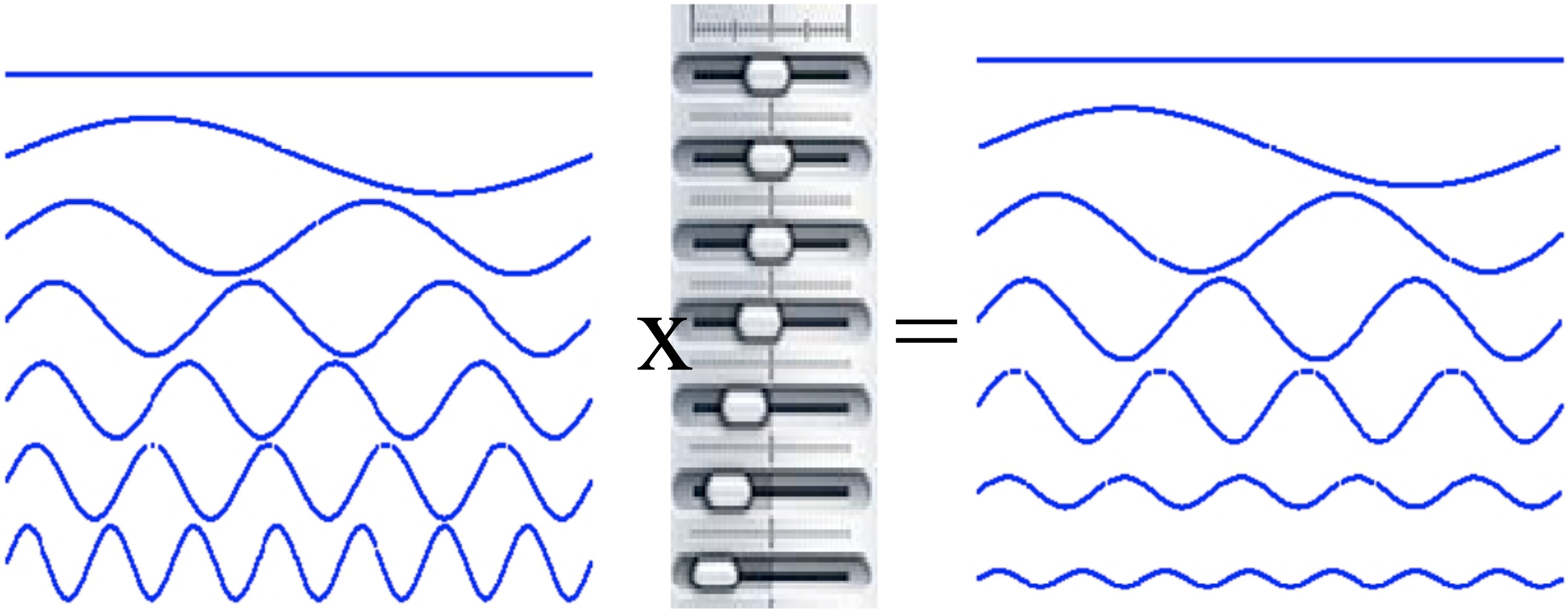
Smoothing Filtering

Fourier Transform



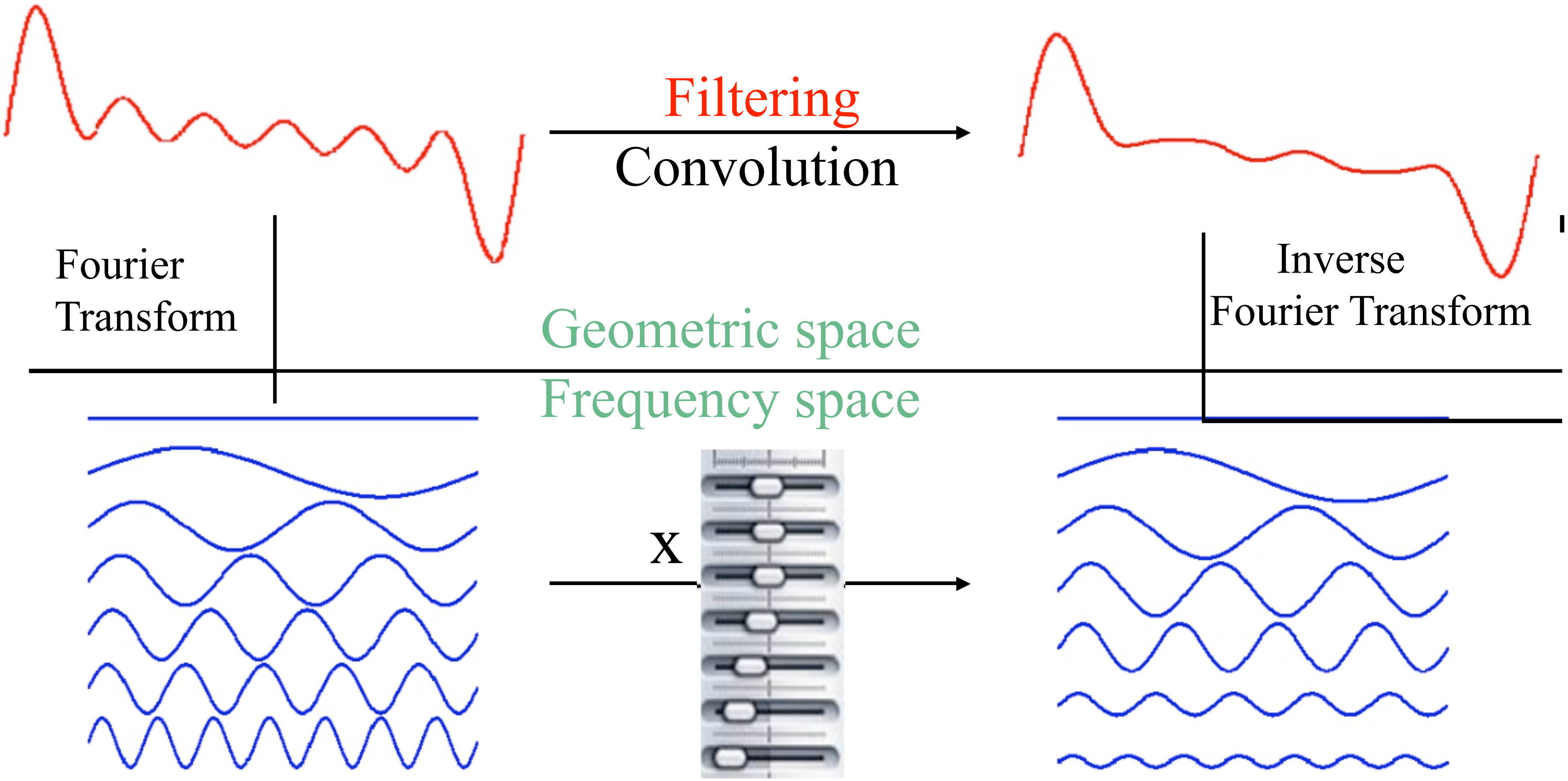
Smoothing Filtering

Fourier Transform

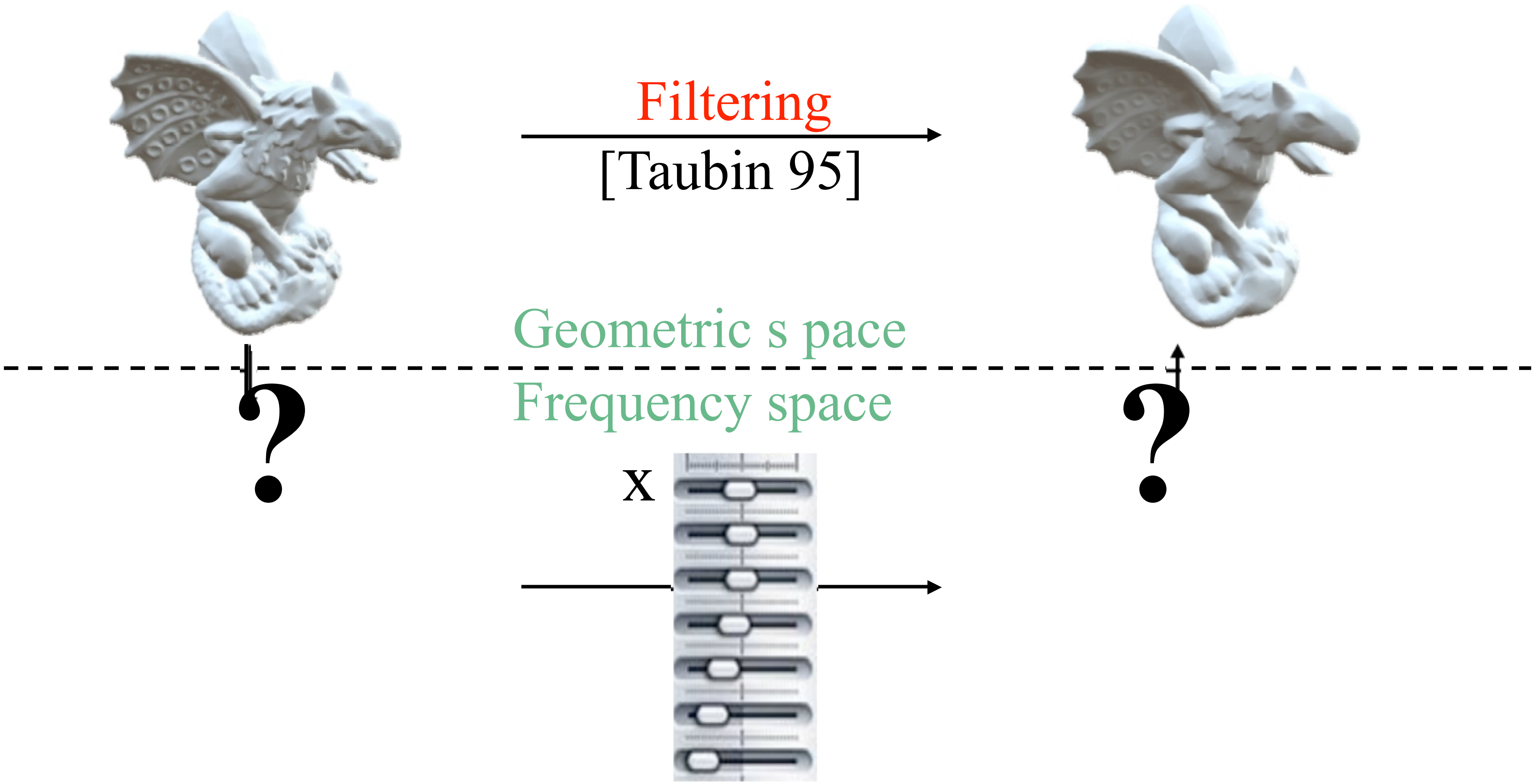


Smoothing Filtering

Fourier Transform

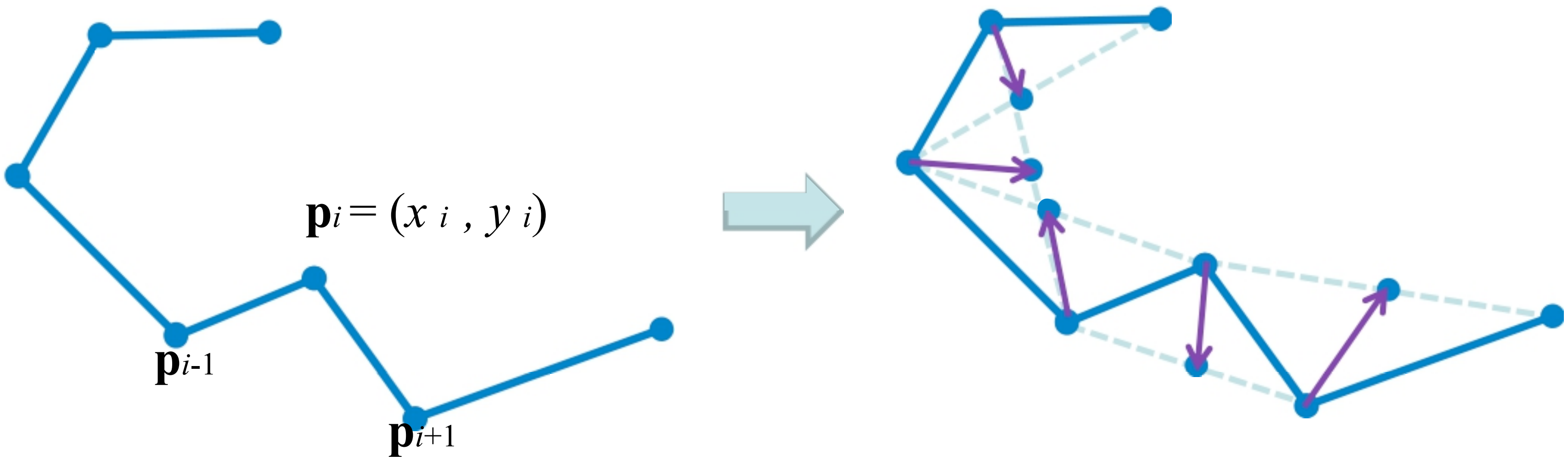


Filtering on a Mesh



Laplacian Smoothing

An easier problem: How to smooth a curve?

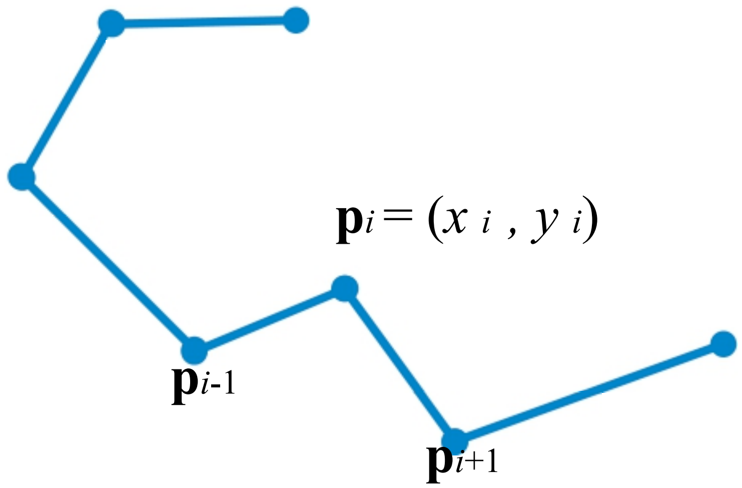


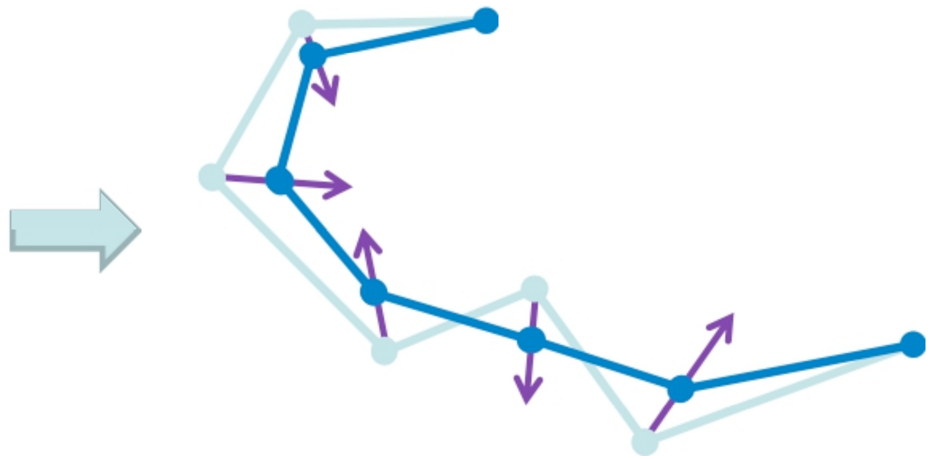
$(\mathbf{p}_{i-1} + \mathbf{p}_{i+1})/2 - \mathbf{p}_i$

$$L(\mathbf{p}_i) = \frac{1}{2}(\mathbf{p}_{i+1} - \mathbf{p}_i) + \frac{1}{2}(\mathbf{p}_{i-1} - \mathbf{p}_i)$$

Laplacian Smoothing

An easier problem: How to smooth a curve?





$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \frac{1}{2} L(\mathbf{p}_i)$$

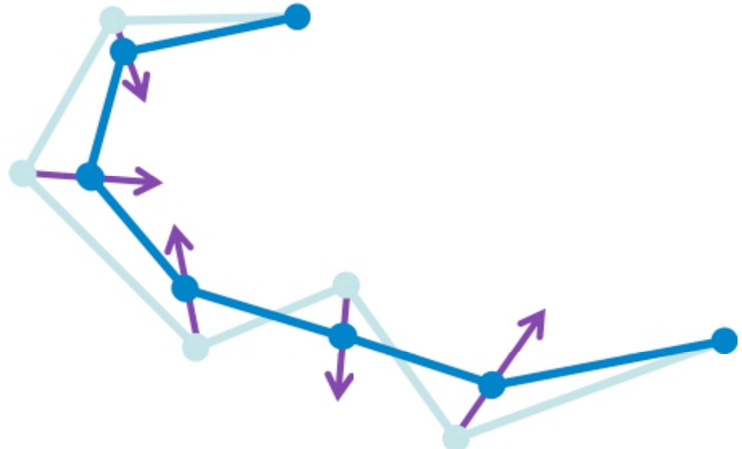
Finite difference discretization of second derivative
= Laplace operator in one dimension

$$L(\mathbf{p}_i) = \frac{1}{2}(\mathbf{p}_{i+1} - \mathbf{p}_i) + \frac{1}{2}(\mathbf{p}_{i-1} - \mathbf{p}_i)$$

Laplacian Smoothing

Algorithm:
Repeat for m iterations (for non boundary points) :

$$\mathbf{p}_i \leftarrow \mathbf{p}_i + \lambda L(\mathbf{p}_i)$$



For which λ ?
$0 < \lambda < 1$

Closed curve conver ges to?
Single point

Spectral Analysis
Closed Curve

Re-write $\mathbf{p}_i^{(t+1)} = \mathbf{p}_i^{(t)} + \lambda L\left(\mathbf{p}_i^{(t)}\right)$

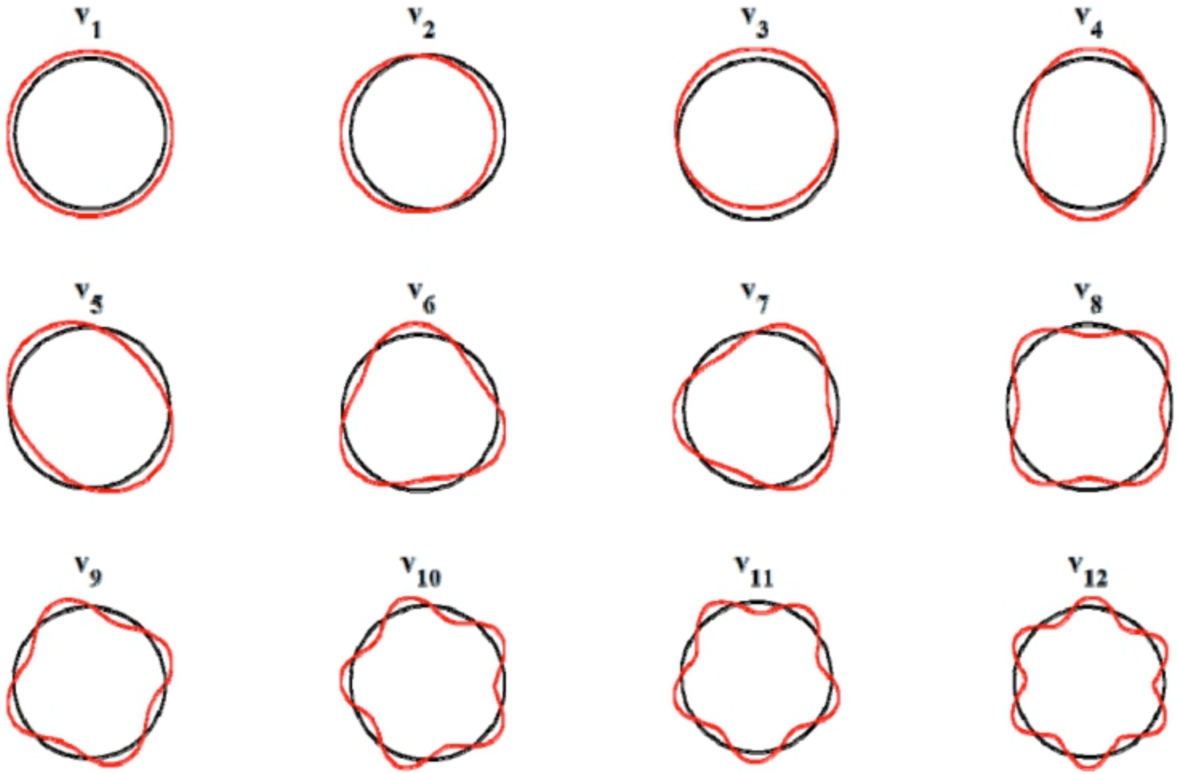
$$L(\mathbf{p}_i) = \frac{1}{2}(\mathbf{p}_{i+1} - \mathbf{p}_i) + \frac{1}{2}(\mathbf{p}_{i-1} - \mathbf{p}_i)$$

in matrix notation: $\mathbf{P}^{(t+1)} = \mathbf{P}^{(t)} - \lambda \mathbf{L}\mathbf{P}^{(t)}$

$$\mathbf{P} = \begin{pmatrix} x_1 & y_2 \\ \dots & \dots \\ x_n & y_n \end{pmatrix} \in \mathbb{R}^{n \times 2} \quad \mathbf{L} = \frac{1}{2}\begin{pmatrix} 2 & -1 & & & & -1 \\ -1 & 2 & -1 & & & \\ & & \dots & & & \\ & & & -1 & 2 & -1 \\ & & & & & \\ -1 & & & & -1 & 2 \end{pmatrix} \in \mathbb{R}^{n \times n}$$

The Eigen vectors of $\mathbf{L}$

$$\mathbf{L} = \mathbf{V}\mathbf{D}\mathbf{V}^T \qquad \mathbf{V} = \begin{pmatrix} | & | & & | \\ \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_n \\ | & | & & | \end{pmatrix}, \qquad \mathbf{D} = \begin{pmatrix} k_1 & & & \\ & k_2 & & \\ & & \cdots & \\ & & & k_n \end{pmatrix}$$



Spectral Analysis

Then: $\mathbf{P}^{(t+1)} = \mathbf{P}^{(t)} - \lambda \mathbf{L} \mathbf{P}^{(t)} = (\mathbf{I} - \lambda \mathbf{L}) \mathbf{P}^{(t)}$

After m iterations: $\mathbf{P}^{(m)} = (\mathbf{I} - \lambda \mathbf{L})^m \mathbf{P}^{(0)}$

Can be described using eigen-decomposition of **L**

Filtering high frequencies

$$\mathbf{L} = \mathbf{V}\mathbf{D}\mathbf{V}^T$$

$$\mathbf{V} = \begin{pmatrix} | & | & & | \\ \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_n \\ | & | & & | \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} k_1 & & & \\ & k_2 & & \\ & & \cdots & \\ & & & k_n \end{pmatrix}$$

$$\Rightarrow \quad \mathbf{P}^{(m)} = \mathbf{V} \boxed{(\mathbf{I} - \lambda \mathbf{D})^m} \mathbf{V}^T \mathbf{P}^{(0)}$$

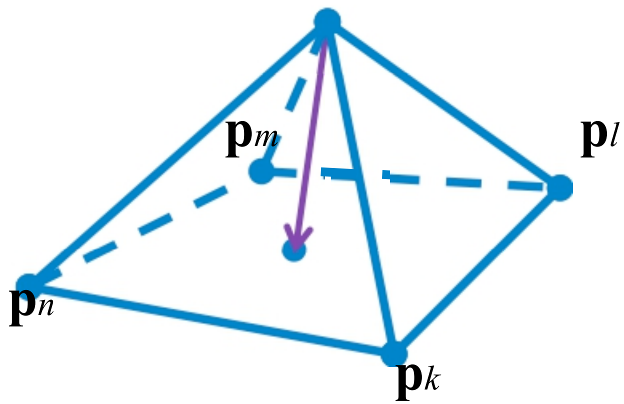
Laplacian Smoothing on Meshes

Same as for curves:

$$\mathbf{p}_i^{(t+1)} = \mathbf{p}_i^{(t)} + \lambda \Delta \mathbf{p}_i^{(t)}$$

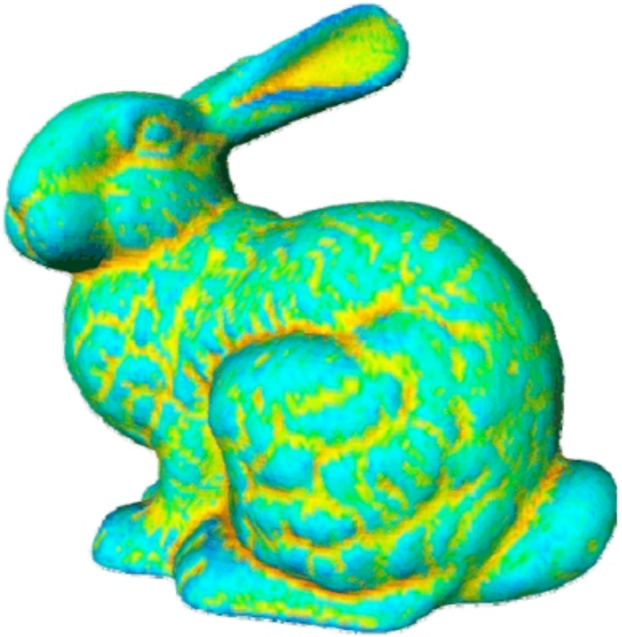
$N = \{ k, l, m, n \}$
$\mathbf{p}_i = (x_i, y_i, z_i)$

What is $\Delta \mathbf{p}_i$?

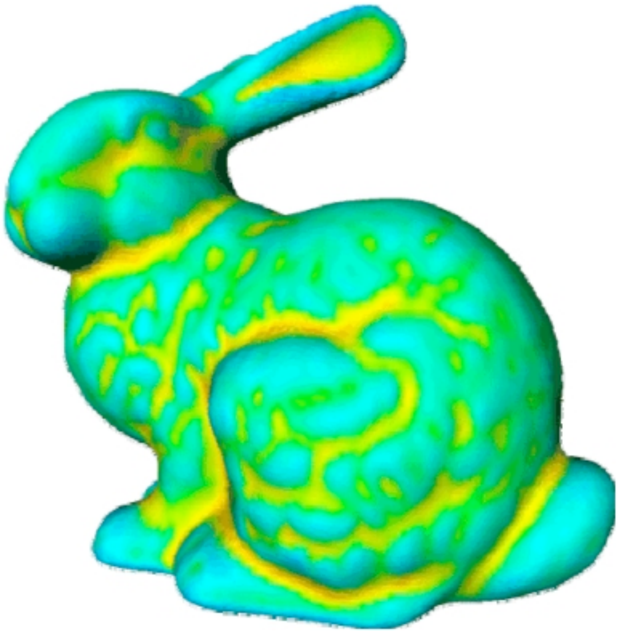


$$\frac{1}{2}\left(\mathbf{p}_{i+1} + \mathbf{p}_{i-1}\right) - \mathbf{p}_i \qquad \frac{1}{|N_i|}\left(\sum_{j \in N_i} \mathbf{p}_j\right) - \mathbf{p}_i$$

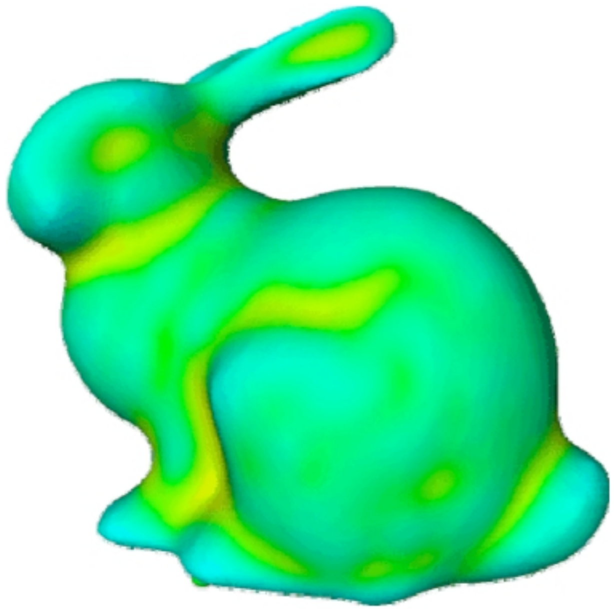
Laplacian Smoothing on Meshes



0 Iterations



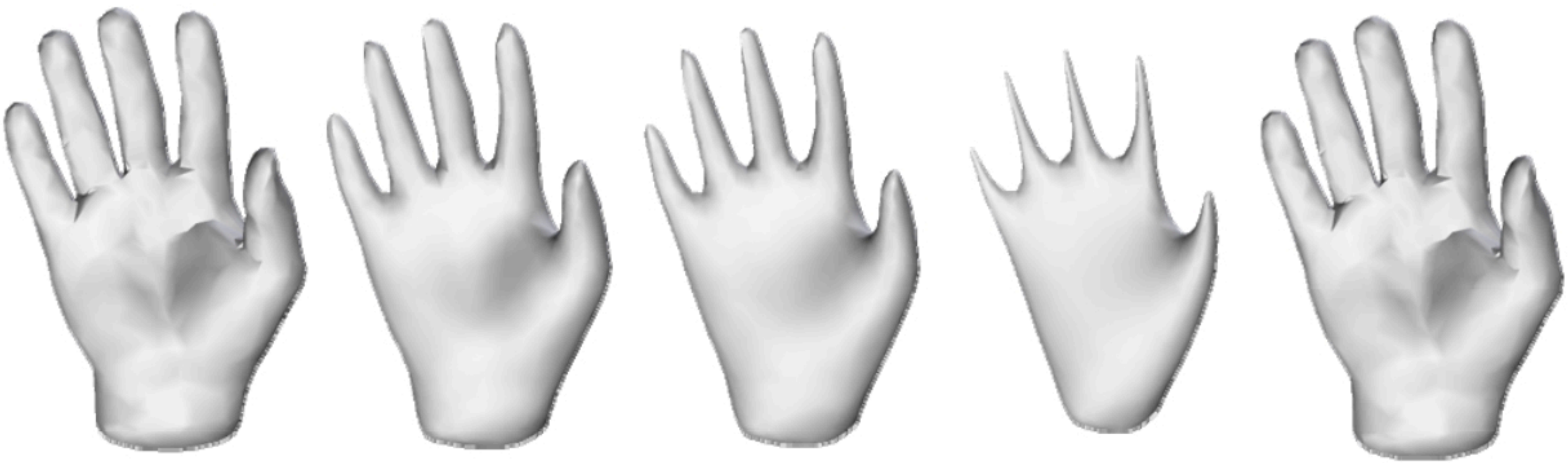
5 Iterations



20 Iterations

Problem - Shrinkage

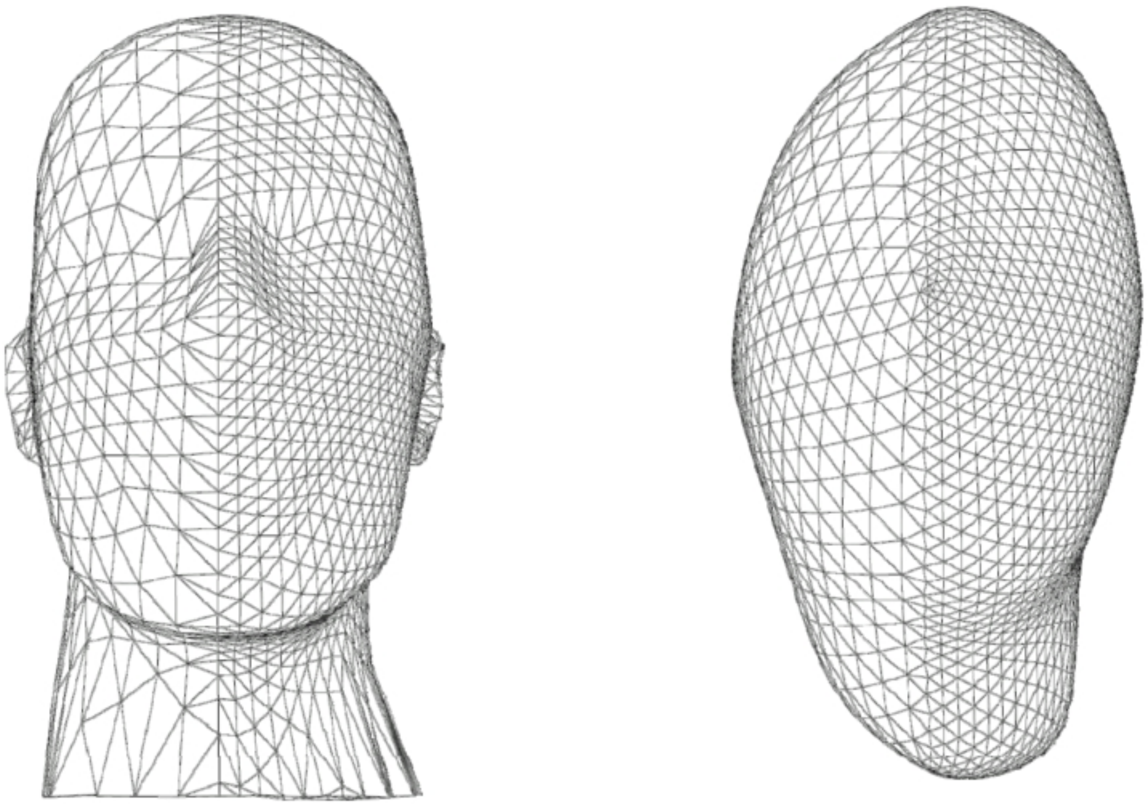
Repeated iterations of Laplacian smoothing shrinks the mesh



original 3 steps 6 steps 18 steps original

Lalace Operator Discretization
The Problem

Not good – The result should not depend on triangle si zes



From Desbrun et al., Siggraph 1999

Laplace Operator Discretization

What Went Wrong?

Back to curves:

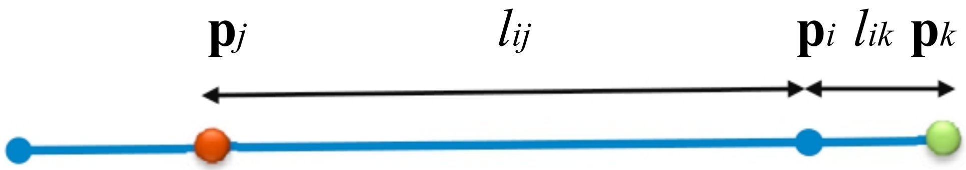
$$\boxed{\frac{1}{2}}\left(\mathbf{p}_{i+1}+\mathbf{p}_{i-1}\right)-\mathbf{p}_i$$

Same weight for both neighbors , although one is closer

Laplace Operator Discretization

The Solution

Use a weighted average to define Δ



$$w_{ij} = \frac{1}{l_{ij}} \qquad w_{ik} = \frac{1}{l_{ik}}$$

$$L(\mathbf{p}_i) = \frac{w_{ij}\mathbf{p}_j + w_{ik}\mathbf{p}_k}{w_{ij} + w_{ik}} - \mathbf{p}_i$$

Straight curves will be invariant to smoothing

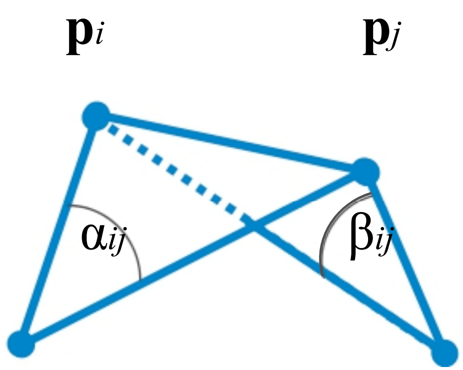
Laplace Operator Discretiztion

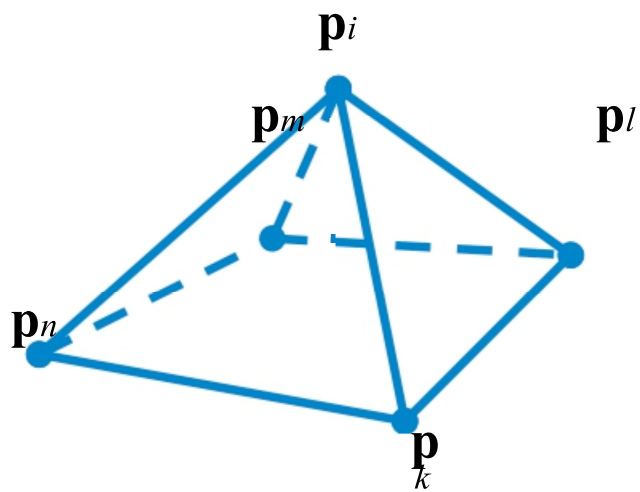
Cotangent Weights

Use a weighted average to define Δ

Which weights?

$N_i = \{ k, l, m, n \}$



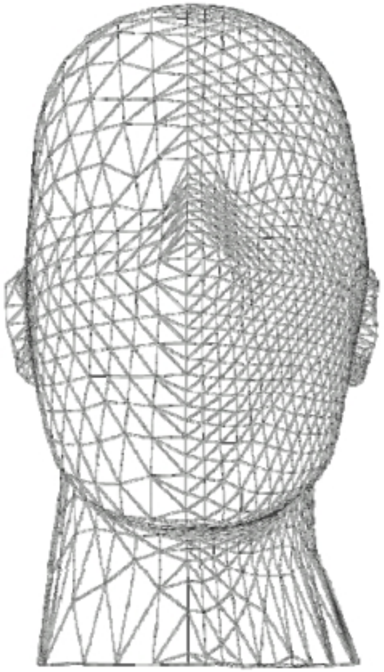


$$w_{ij} = \frac{h_{ij}^{1} + h_{ij}^{2}}{l_{ij}} = \frac{1}{2}\left(\cot\alpha_{ij} + \cot\beta_{ij}\right)$$

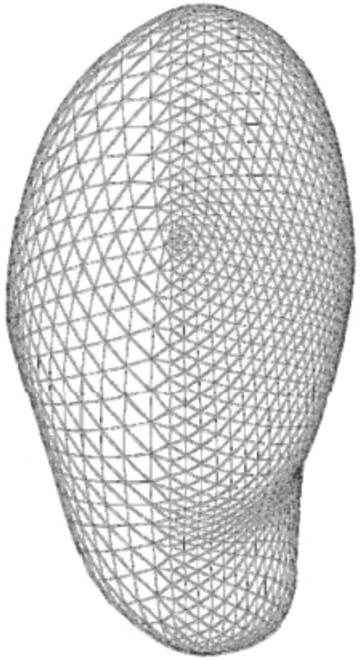
$$L(\mathbf{p}) = \frac{1}{\sum w_{ij}} \sum_{j \in N_i} w_j \left(\mathbf{p}_j - \mathbf{p}\right)$$

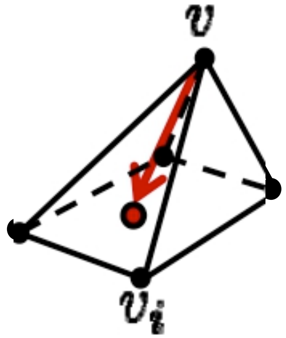
Planar meshes will be invariant to smoothing

Smoothing with the Cotangent Laplacian



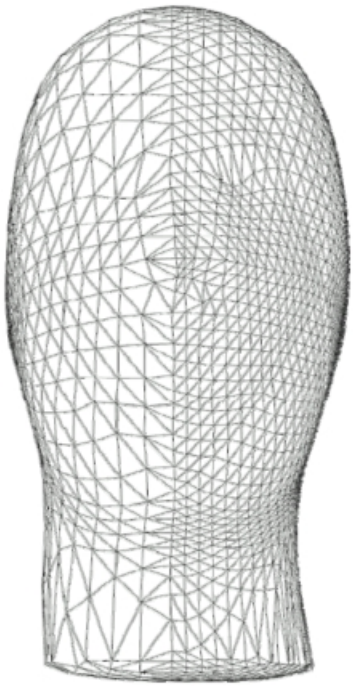
original

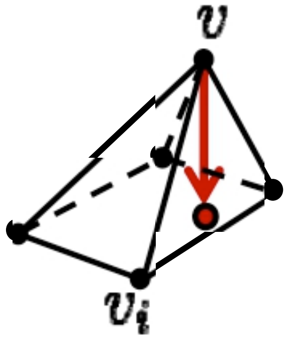




normal
and
tangential
movement

Uniform weights





normal
movement

Cotan gent wei ghts

From Desbrun et al., Siggraph 1999

References

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"An Intuitive Framework for Real -Time Freeform Modelin ", Botsch et al ., Siggraph '04

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