$$M_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & S_z & t_z \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \tilde{x}_e \\ \tilde{y}_e \\ \tilde{z}_e \\ 1 \end{bmatrix} = M_2 M$$

$$\begin{bmatrix} x_p \\ y_p \\ z_p \\ w \end{bmatrix} = M_3 \begin{bmatrix} \tilde{x}_e \\ \tilde{y}_e \\ \tilde{z}_e \\ 1 \end{bmatrix}$$

$$x_p = \tilde{x}_e$$

$$y_p = \tilde{y}_e$$

$$w = -\tilde{z}_e$$

$$z_p = S_z \tilde{z}_e + t_z$$

$$\tilde{z}_p = \frac{z_p}{w} = \frac{S_z \tilde{z}_e + t_z}{-z_e} \longleftarrow 1$$

$$\tilde{x}_p = x_p / w \qquad \tilde{y}_p = y_p / w$$

Lets map $z_e = -1$ to $\tilde{z}_p = -1$

& $z_e = -k$ to $\tilde{z}_p = +1$

for first mappn usin eq. 1 we get

$$-1 = \frac{S_z(-1) + t_z}{-(-1)} = -S_z + t_z \leftarrow 2$$

for second Mapping using eq. 2

$$+1 = \frac{S_z(-k) + t_z}{-(-k)} \Rightarrow -S_z k + t_z = k \leftarrow$$

Solve for $S_2$ & $t_2$ using eq. 2 & 3
(Two eq. in two unknown)

eq 2 minus eq 1

$$-S_2 k + \cancel{t_2} + S_2 - \cancel{t_2} = k+1$$

$$\boxed{S_2 = \frac{k+1}{1-k}} \leftarrow 4$$

$t_2$ = using eq. 4 & 2

$$t_2 = S_2 - 1 = \frac{k+1}{1-k} - 1$$

$$= \frac{k+1-1+k}{1-k} = \frac{2k}{1-k}$$

$$\boxed{t_2 = \frac{2k}{1-k}}$$

$$M_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1+k}{1-k} & \frac{2k}{1-k} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$