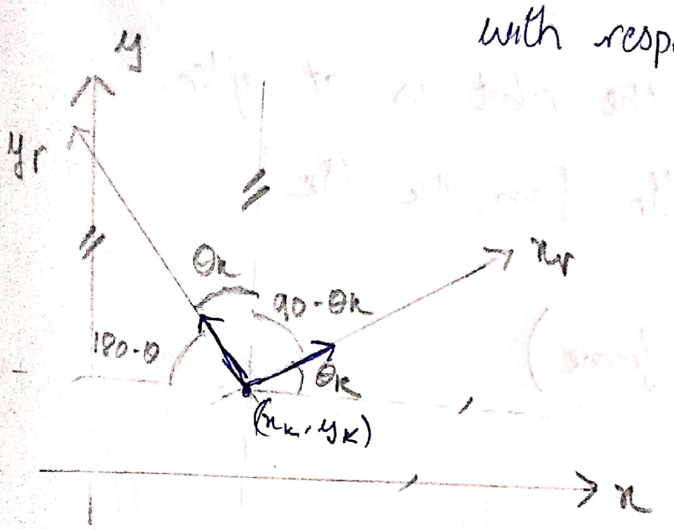
Geometric model (interpretation)

We know that $\Delta x, \Delta y$ are the control inputs to the robot in its frame (not equal to the world frame).

Let us denote: x, y as axis of the world frame.

x_r, y_r. the robot's' frame at (x_k, y_k).

θ_k is the rotation of the robot's frame with respect to x, y (world frame)



as convention, anticlockwise rotation is considered +ve. rotation.

' ' → denotes parallel x-axis

" " → parallel to the y-axis

Δy is along y_r.

Δx is along x_r.

From these equations and the given orientation of (x_r, y_r) wrt (x, y), we get:

Displacement of the robot along the x-axis of the world frame.

$$= (\Delta x \cos\theta_k - \Delta y \sin\theta_k)\hat{i}$$

$$\Rightarrow x_{k+1} - x_k = \Delta x \cos\theta_k - \Delta y \sin\theta_k$$

Let this equation be equation (I).

→ Net displacement along y-axis in the world frame.

$$= (\Delta x \sin\theta_k + \Delta y \cos\theta_k)\hat{j}$$

$$y_{k+1} - y_k = \Delta x \sin\theta_k + \Delta y \cos\theta_k$$

Let this equation be denoted by (II).

→ Analysis ~~of~~ w.r.t the matrix:

Let P denote the point that the robot is at after moving $\Delta x, \Delta y$ w.r.t x_r, y_r from x_k, y_k.

G → ground world frame (base frame)

R → Robot's current frame.

$${}^{G}P = {}^{G}_{R}R \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} + \underbrace{\begin{bmatrix} x_k \\ y_k \end{bmatrix}}_{\text{translation between robot frames (world to robot)}}$$

We have ${}^{G}_{R}R = \begin{bmatrix} \cos\theta_k & -\sin\theta_k \\ \sin\theta_k & \cos\theta_k \end{bmatrix}$ (θ_k anticlockwise)

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} \cos\theta_k & -\sin\theta_k \\ \sin\theta_k & \cos\theta_k \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} + \begin{bmatrix} x_k \\ y_k \end{bmatrix}$$

which gives us:

$$x_{k+1} = \Delta x \cos\theta_k - \Delta y \sin\theta_k + x_k$$

$$y_{k+1} = \Delta x \sin\theta_k + \Delta y \cos\theta_k + y_k.$$

The rotation by $\Delta\theta$ is done on the z-axis.

If the robot is rotated withrespect to a frame (x_k, y_k) by the $\Delta\theta$, it is equivalent to rotation of $\Delta\theta$ in the world frame. They are analogus.

Thus $\theta_{k+1} = \theta_k + \Delta\theta$.