Submission Deadline: November 07, 2024, 12:30 hrs

For the background material, refer to Glasserman (Sections 5.1 & 5.2) / Seydel-4e (Section 2.5)

1. Generate the first 25 values of the van der Corput sequence $x_1, x_2, \ldots, x_{25}$ using the radical inverse function $x_i := \phi_2(i)$ and list them in your report. Next, generate the first 1000 values of this sequence and plot the overlapping pairs (x_i, x_{i+1}) as a two dimensional plot. What do you observe?

2. Generate first 100 and 100000 values of the van der Corput sequence and plot the histogram (for sampled distributions) for both the cases. Compare these plots with the sampled distributions of 100 and 100000 values generated by an LCG, by plotting the sampled distributions in two graphs side by side for both the cases. Specify the LCG that you have used.

3. Generate the Halton sequence $x_i = (\phi_2(i), \phi_3(i))$ (as points in $\mathbb{R}^2$) and plot the first 100 and 100000 values. What are your observations? What are your observations if, instead of the bases 2 and 3, you use larger values (say, something close to 100) as the bases?

Recall that the radical inverse function is defined by $\phi_b(i) := \sum_{k=0}^{j} d_k b^{-k-1}$, *where* $i = \sum_{k=0}^{j} d_k b^k$.

Put all your observations in the report.

Submission Deadline: October 29, 2024, 23:00 hrs

For the background material (Q.2), refer to Glasserman (Chapter 3, Sub-Section 3.5.1)

1. Let us try to fit a geometric Brownian motion (GBM), which you have done in Q.4 in the previous assignment (Lab 09), to a stock price data. First download the daily price data of a stock (say, from yahoo finance) for as long as available. Let T denote the period of availability in years. Take the '*adjusted closing price*' as the daily price of the stock and call it P_i for the ith day, with the number of days as $n+1$. From these P_i values, compute the daily returns, say $R_i, i = 1, 2, \ldots, n$. Let A and B be the mean and standard deviation of these daily returns and let the corresponding annualized values be obtained as $\mu = 252\,A$ and $\sigma = \sqrt{252}\,B$. You now have all the data $(T, S(0), \mu, \sigma)$ to simulate a GBM. Try to do an exercise similar to Q.4 of Lab 09, with these values. Compare your estimated stock price value with the actual value.

2. Simulate a jump diffusion process with the following discretization:

$$X(t_{i+1}) = X(t_i) + \left(\mu - \frac{1}{2}\sigma^2\right)(t_{i+1} - t_i) + \sigma[W(t_{i+1}) - W(t_i)] + \sum_{j=N(t_i)+1}^{N(t_{i+1})} \log(Y_j),$$

   in the time interval $[0, 1]$. The parameter values to be used are $X(0) = 5$, $\mu = 0.06$ and $\sigma = 0.3$. Divide the interval $[0, 1]$ into 1000 sub-intervals.

   (a) In the first step generate the process X on $[0, 1]$ considering only the Wiener process W but not the jump process.

   (b) Now do the simulation of X including the jumps (that follow a Poisson process) with the following 5 different values of $\lambda = 0.01, 0.05, 0.1, 0.5, 1$. The Y_j values have be taken from $Y_j = 0.1Z_j + 1$, where $Z_j \sim \mathcal{N}(0, 1)$.

   (c) Plot the sample paths generated in Part (a) and Part (b) in the same graph.

   (d) Repeat the above for 4 more sets of sample paths.

Put all your observations in the report.

Submission Deadline: October 25, 2024, 22:00 hrs

For the background material, refer to Glasserman (Chapter 3, Sections 3.1 & 3.2)

1. Generate 10 sample paths for the standard Brownian Motion in the time interval $[0, 5]$ using the recursion
$$W(t_{i+1}) = W(t_i) + \sqrt{t_{i+1} - t_i} \cdot Z_{i+1},$$
with 5000 generated values for each of the paths. Plot all the sample paths in a single figure. Also estimate $E[W(2.5)]$ and $E[W(5)]$ from the 10 paths that you have generated.

2. Repeat the above exercise with the following Brownian motions ($BM(\mu, \sigma^2)$) discretization
$$X(t_{i+1}) = X(t_i) + \mu(t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i} \cdot Z_{i+1}.$$
Take $X(0) = 5$ and the following sets of values: (a) $\mu = 0.09$ and $\sigma = 0.5$; (b) $\mu = 0.9$ and $\sigma = 0.25$; and (c) $\mu = 0.9$ and $\sigma = 0.05$.

3. The Euler approximated recursion with time dependent μ and σ is given by
$$Y(t_{i+1}) = Y(t_i) + \mu(t_i)(t_{i+1} - t_i) + \sigma(t_i)\sqrt{t_{i+1} - t_i} \cdot Z_{i+1}.$$
Repeat the above exercise by taking
$$Y(0) = 5,\ \mu(t) = 0.0325 - 0.05t,\ \sigma(t) = 0.012 + 0.0138t - 0.00125t^2.$$

4. Let us simulate a geometric Brownian motion (GBM), which is the most fundamental model for the value of a financial asset. As with the case of a BM, we have a simple recursive procedure to simulate a GBM at $0 = t_0 < t_1 < \cdots < t_n = T$ as:
$$S(t_{i+1}) = S(t_i)\exp\left(\left[\mu - \frac{1}{2}\sigma^2\right](t_{i+1} - t_i) + \sigma\sqrt{t_{i+1} - t_i} \cdot Z_{i+1}\right),\ i = 0, 1, \ldots, n-1,$$
where $Z_1, Z_2, \ldots, Z_n$ are independent $\mathcal{N}(0, 1)$ variates. In the interval $[0, 5]$, taking both positive and negative values for μ and for at least two different values of σ^2, simulate and plot at least 10 sample paths of the GBM (taking sufficiently large number of sample points, say 5000 values, for each path). Also, by generating a large number of sample paths, compare the actual and simulated distributions of $S(5)$.

   Now, taking $S(0) = 5, \mu = 0.06, \sigma = 0.3$, first simulate $S(1)$ by taking $n = 50$ steps to reach time 1 (i.e., with time intervals of length 0.02). Then, compute the expectation of $S(1)$ (i.e., $E(S(1))$) by simulating N values of $S(1)$. By varying values for N from 1 to 1000, plot the values of $E(S(1))$ as a curve. Simulate and plot 5 such curves (N in the x-axis and $E(S(1))$ in the y-axis).

Put all your observations in the report.

Submission Deadline: October 18, 2024, 22:00 hrs

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1. (a) Use the following Monte Carlo estimator to approximate $I = E[e^U]$, where $U \sim U(0,1)$:

$$I_M = \frac{1}{M}\sum_{i=1}^{M} Y_i, \text{ where } Y_i = \exp(U_i) \text{ with } U_i \sim U(0,1).$$

Take the values of M to be $10, 10^2, 10^3, 10^4$ and 10^5. For each value of M, determine the sampling variance of the estimate. [You restart the random number generator from the same point (value) for each value of M.]

(b) Repeat the above exercise using antithetic variates and call the estimator $\widehat{I}_M$ (with comparable values of M).

(c) Repeat the above exercise using control variates taking U itself as the control variable, and call the estimator $\widetilde{I}_M$.

(d) Present the results that you have obtained in Parts (a), (b) and (c) in a tabular form. Your table must consist of the values of I_M, $\widehat{I}_M$ and $\widetilde{I}_M$, their sampling variances, and the percentage reduction in the variances of the improved estimates over the crude Monte carlo estimate. How do the values of the estimates compare with the actual value of I?

2. Suppose that the random variable Y is exponentially distributed with mean 1. Suppose further that, conditional on $Y = y$, the random variable X is Gaussian with mean y and variance 4. Taking the values of M to be $10, 10^2, 10^3, 10^4$ and 10^5, estimate by basic Monte Carlo the value of $P(X > 1)$. What would be the estimate if you use the variance reduction by conditioning idea, and how superior it is to the naive estimate? Can you improve it further by using either (a) antithetic variable technique, or (b) control variable technique? How much better these two new estimates are? Compare the estimates with sampling variances, and percentage reduction in variances.

3. Compound Poisson models are commonly used for rainfall and, here, we will look at stratifying such a model. In our simplified setting of such a model, we consider that the number of rainfall events (storms) in the coming month is given by the random variable N with $P\{N = 1\} = 0.19, P\{N = 2\} = 0.26, P\{N = 3\} = 0.24, P\{N = 4\} = 0.17, P\{N = 5\} = 0.14$. The depth of rainfall (in centimeters) in storm i is $D_i \sim Weib(k, \sigma)$ with shape $k = 0.8$ and scale $\sigma = 3$ (centimeters) and the storms are independent. The probability density function of $Weib(k, \sigma)$ distribution is given by

$$f(x) = \frac{k}{\sigma}\left(\frac{k}{\sigma}\right)^{k-1} e^{-(x/\sigma)^k} \quad \text{for } x > 0.$$

If the total rainfall is below 5 centimeters then an emergency water allocation will be imposed. Our goal is to approximate the probability of imposing the emergency water allocation in the coming month. Note that total rainfall is given by $\sum_{i=1}^{N} D_i$. Use simple Monte Carlo and stratification methods to approximate the probability based on $n = 10^2$ and 10^4. Also, provide the 99% confidence interval for the probability using both the methods.

Submission Deadline: October 04, 2024, 22:00 hrs

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1. Consider the expectation $I = E[\exp(\sqrt{U})]$ where, $U \sim U(0,1)$. Use the following antithetic method to approximate I:

$$\widehat{I}_M = \frac{1}{M}\sum_{i=1}^{M}\widehat{Y}_i, \text{ where } \widehat{Y}_i = \frac{\exp(\sqrt{U_i}) + \exp(\sqrt{1-U_i})}{2} \text{ with } U_i \sim U(0,1).$$

Take the values of M to be $\frac{10^2}{2}, 10^2, \frac{10^3}{2}, 10^3, \frac{10^4}{2}, 10^4, \frac{10^5}{2}$ and 10^5. Determine the 95% confidence interval for $\widehat{I}_M$ for all the values of M that you have taken.

2. Present the results that you have obtained in Question 1 of Lab 06 and Question 1 of Lab 07 in a tabular form. Your table must consist of the values of estimates (using the two methods), 95% confidence intervals for I (from two methods), and the ratio of widths of both the intervals. How do the estimates of I using the two methods compare with the actual value of I?
