Pattern Recognition

Programming Assignment #1

Professor :

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Task 1: Eigen analysis

Perform eigen analysis on the covariance matrix for pixel representations of images in Dataset 1. Use eigenvectors corresponding to significant eigen values for reconstruction of images .

In this task we are reconstructing the images (64x64 gray scale images) using covariance matrix and eigen values.

1.Theory:

- Calculate the Covarience Matrix as **Cov(*X*, *Y*) = Σ (X_i - *X*) (Y_i - *Y*) / *N***;
  where N is number of elements in the dataset.
- From this matrix calculate the eigen values and eigen vectors.
- Sort these eigen values in descending order as well as the vectors in the matrix.Take some of the most significant eigen values.
- Reconstruct the image .

2.Comparison of the original images and reconstructed images

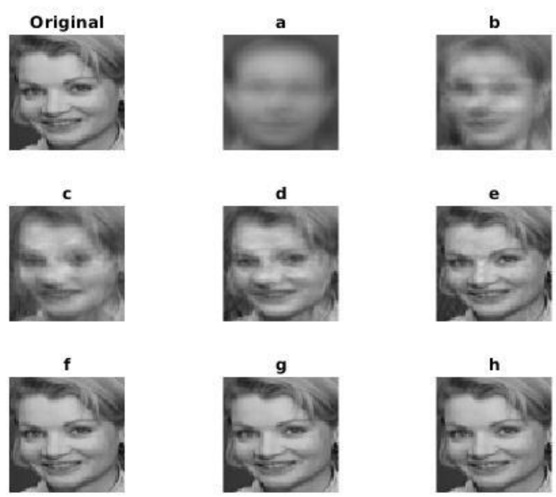

Figure 1 Original and Reconstructed image with Eigen values 1, 10, 20, 40, 80, 160, 320, 640 respectively

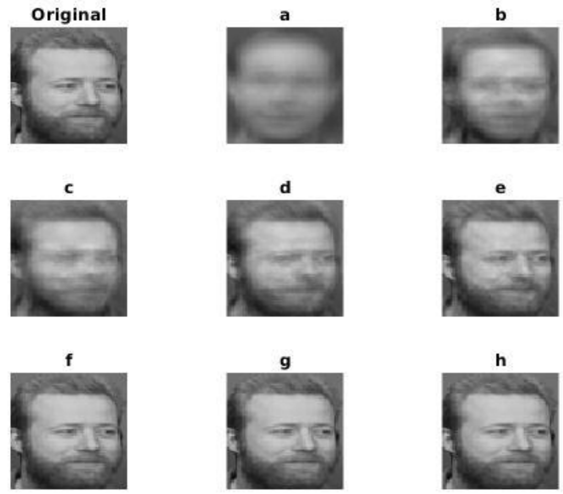

Figure 2 Original and Reconstructed image with Eigen values 1, 10, 20, 40, 80, 160, 320, 640 respectively

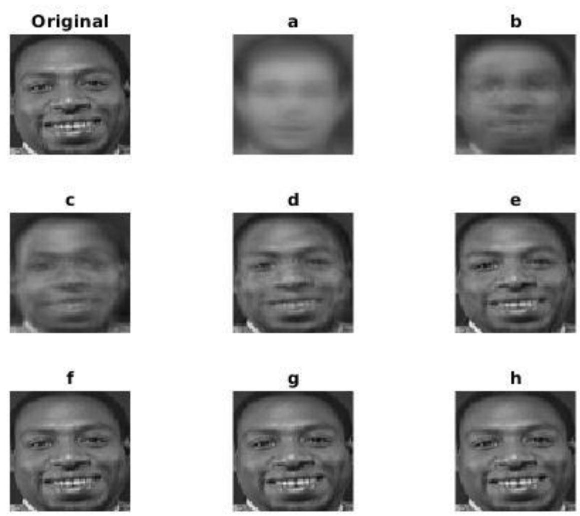

Figure 3 Original and Reconstructed image with Eigen values 1, 10, 20, 40, 80, 160, 320, 640 respectively

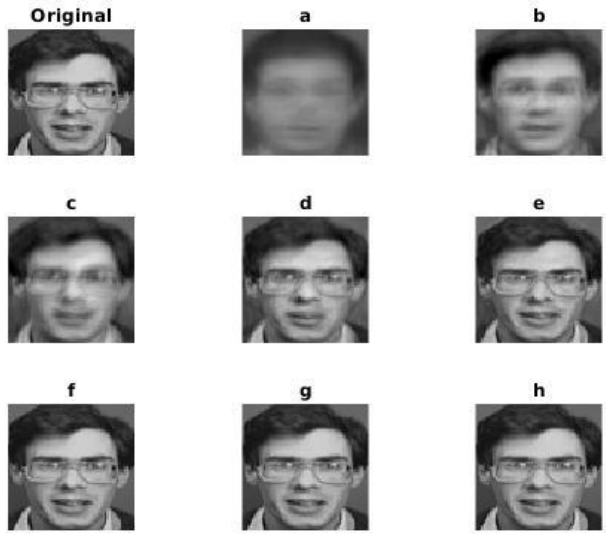

Figure 4 Original and Reconstructed image with Eigen values 1, 10, 20, 40, 80, 160, 320, 640 respectively

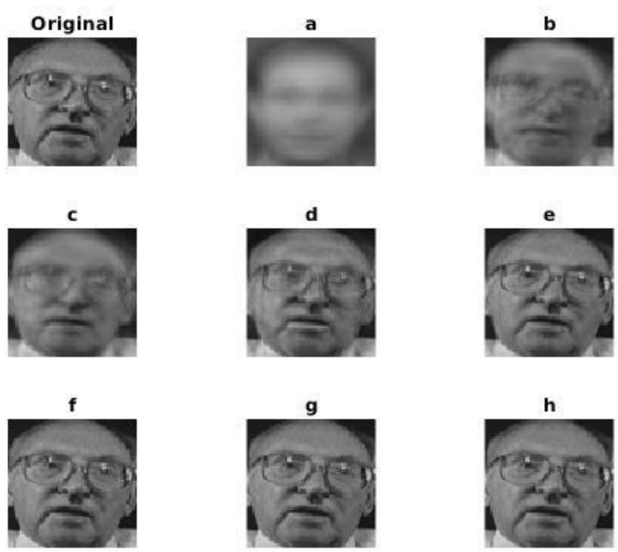

Figure 5 Original and Reconstructed image with Eigen values 1, 10, 20, 40, 80, 160, 320, 640 respectively

Observation :

Last three reconstructed images are same for significant values 160,320,640 and there is no change because for these values the eigen vectors are dependent.

Task 2: Regression Models

- Model 1. Polynomial curve fitting for Dataset 2

- In this task we are using cos(2∏x) + tanh(2∏x) this function by generating 100 random values for X and calculating values of Y by adding Gaussian noise with zero mean.
  So, Y = cos(2∏x) + tanh(2∏x) + Gaussian Noise.
- We have divided the dataset into test , train and validation in the ratio (70:20:10) .

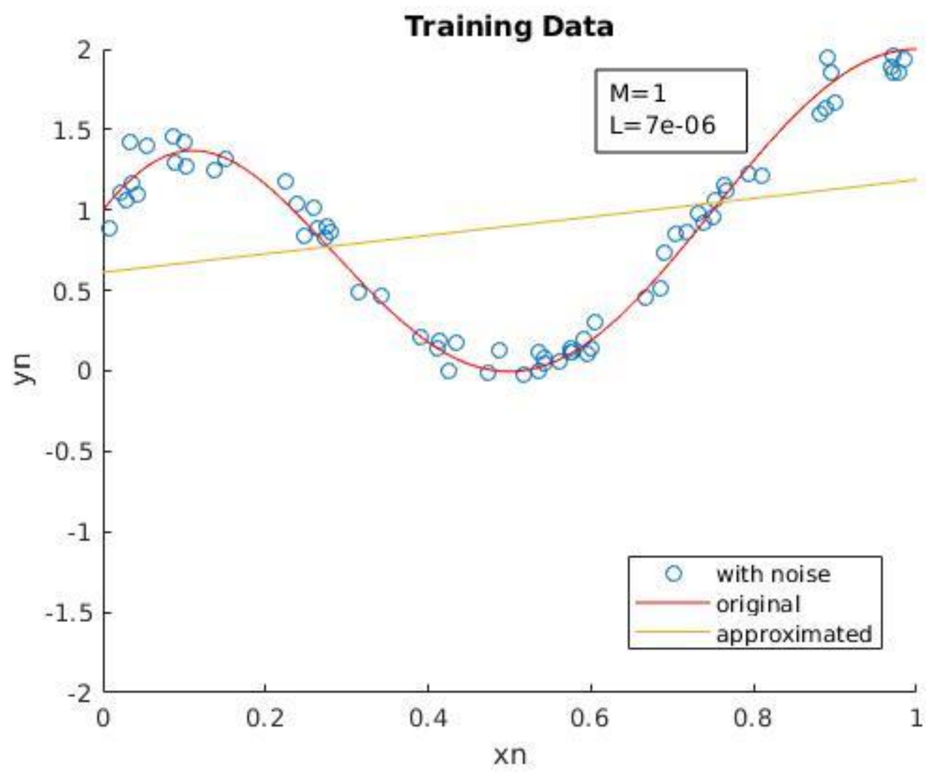

fig. 2.1.1 Curve Fitting of xn vs yn with M=1 ,L=0.000007 for training data set

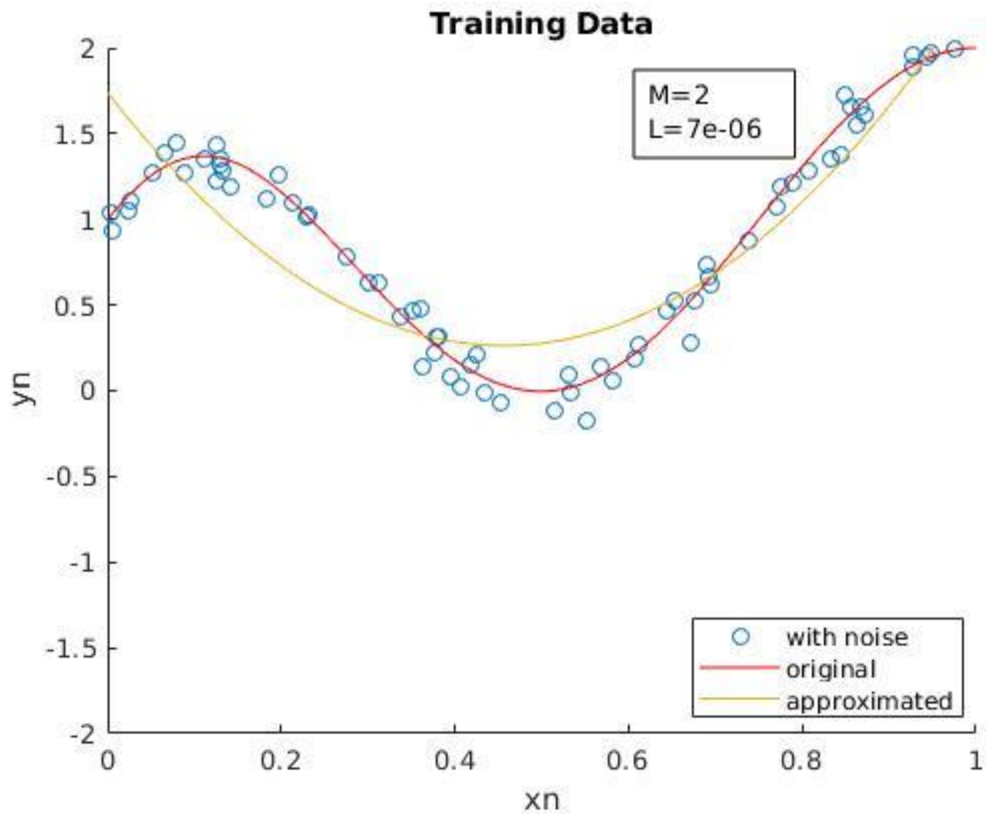

fig. 2.1.2 Curve Fitting of xn vs yn with M= 2, L=0.000007 for training data set

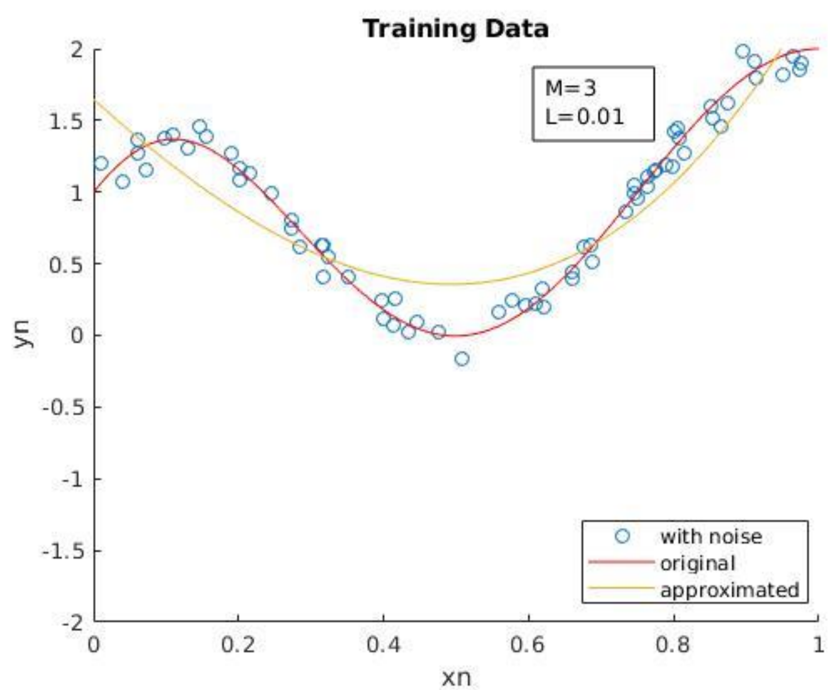

fig. 2.1.3 Curve Fitting of xn vs yn with M=3 , l=0.01 for training data set

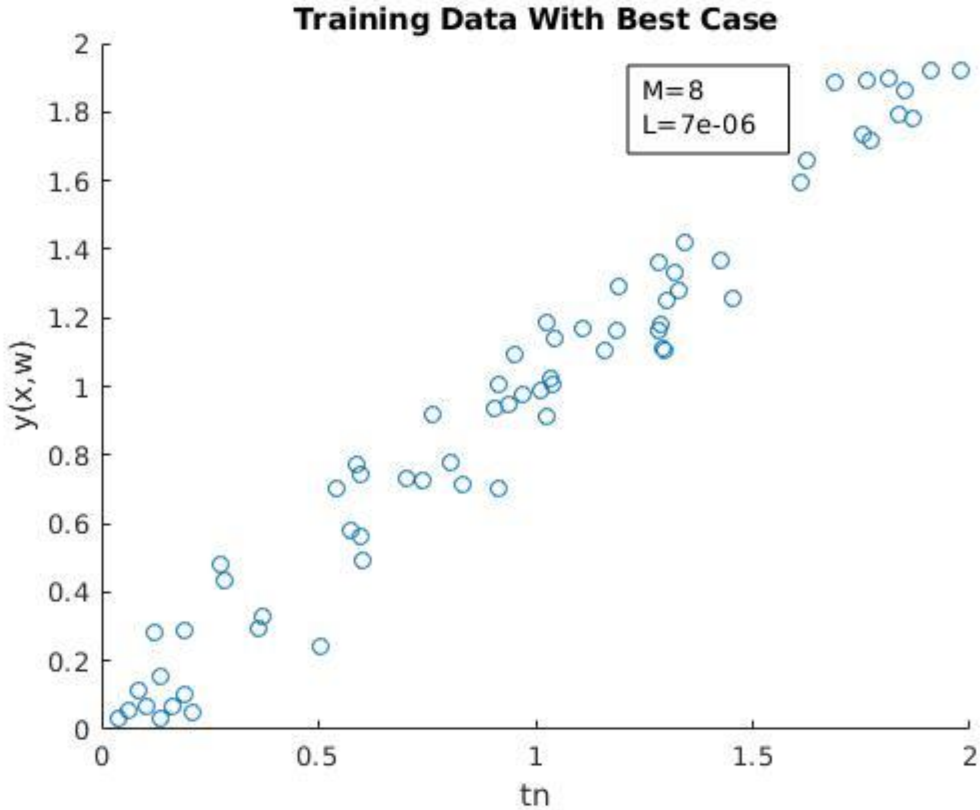

fig. 2.1.4 Curve Fitting of xn vs y(x,w) with M=8 , L=0.000007 for training data set

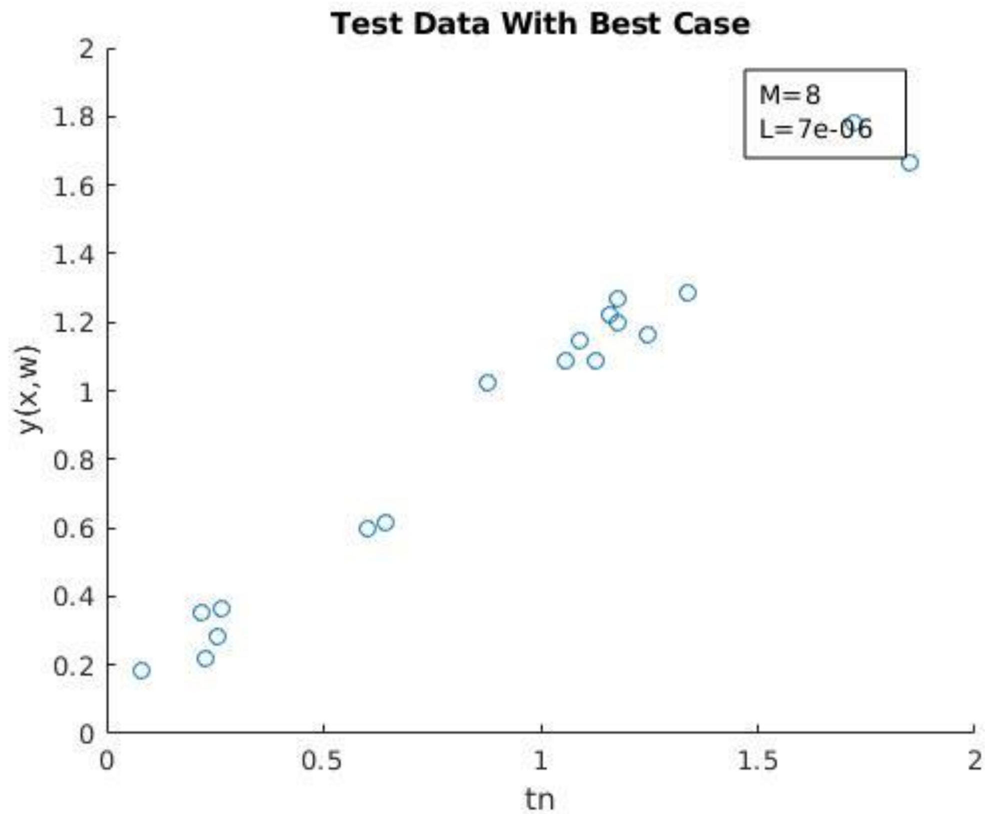

fig. 2.1.5 Scatter plot of tn vs y(x,w) for test data set with M=8, L=0.000007

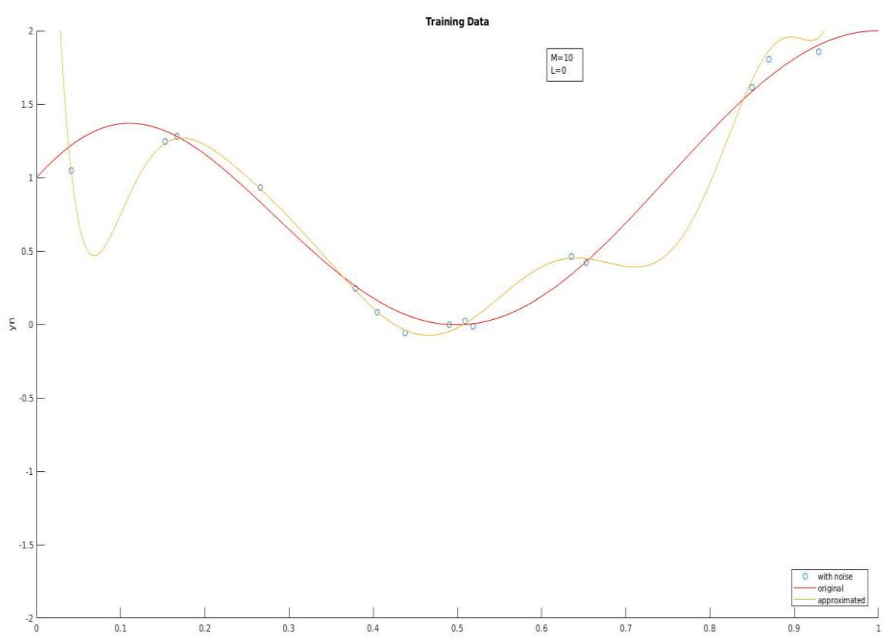

fig 2.1.8: Scatter plot with overfitting for training data with M=10 , L=0

Observation from fig.2.1.8 :

1. For test data set , error = 1.1682
   For training data set , error = 0.0397

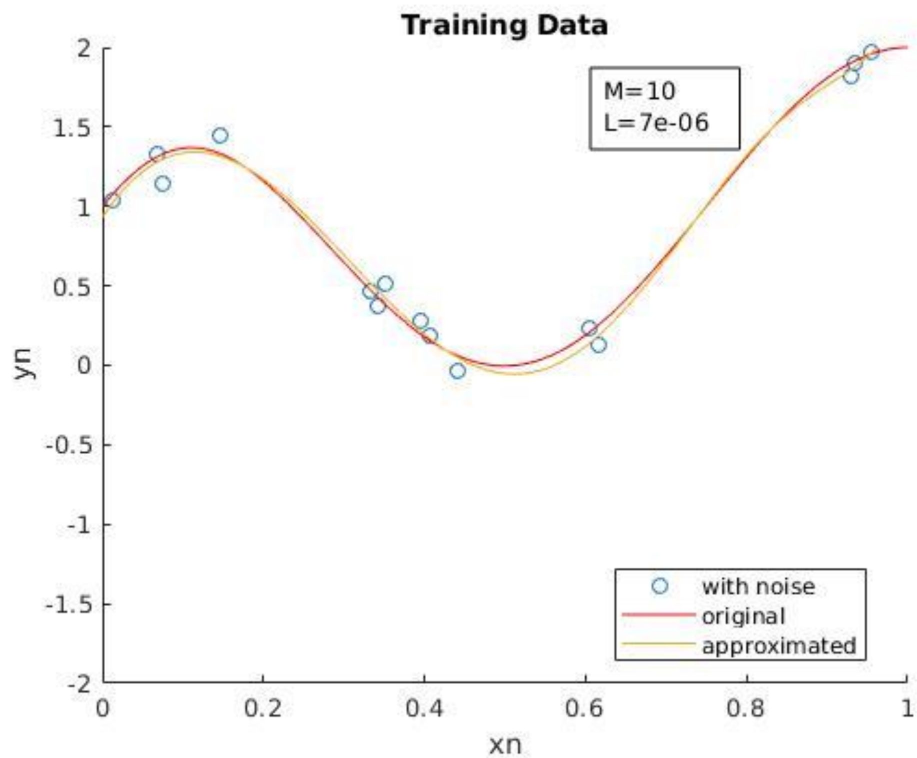

fig 2.1.9: Scatter plot with overfitting reduced for training data with M=10,L=0.000007

Observation from fig.2.1.9 :

1. For test data set , error = 0.1521
   For training data set , error = 0.0924

		Train			
M\L	0	0.000007	0.007	0.7	1
2	0.2466	0.247	0.6108	2.1117	1.9322
3	0.2441	0.2443	0.26971	0.3999	2.6062
5	0.0986	0.1261	0.5734	2.2768	2.4737
8	0.0981	0.1196	0.5123	1.8703	2.0185

Table 2.1.1 : Observations of errors with given degree of polynomial with given lambda for traning data

M\L	0	0.000007	0.007	0.7	1
2	0.2358	0.2362	0.6046	2.1125	1.9314
3	0.2468	0.247	0.4113	2.6879	2.593
5	0.1033	0.1344	0.5641	2.2704	2.4591
8	0.1039	**0.124**	0.4824	1.873	2.0147

Table 2.1.2 Observations of errors with given degree of polynomial with given lambda for validation data

M\L	0	0.000007	0.007	0.7	1
2	0.2672	0.2676	0.6321	2.0963	1.9131
3	0.2596	0.2598	0.4134	2.6926	2.5926
5	0.124	0.1542	0.5904	2.2892	2.4792
8	0.1253	0.1441	0.5368	1.8876	2.0319

Table 2.1.3 Observations of error with given degree of polynomial with given lambda for test data

- Model 2 : Linear model for regression using polynomial basis functions for Dataset 3.

Ans :

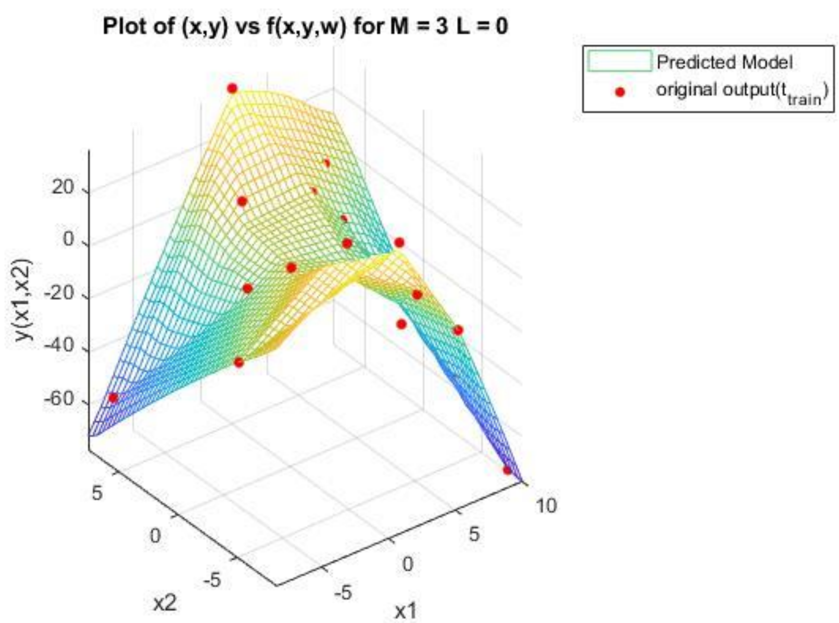

fig.2.2.1 : 3-D plot of Bivariate data using Polynomial basis functions for train20

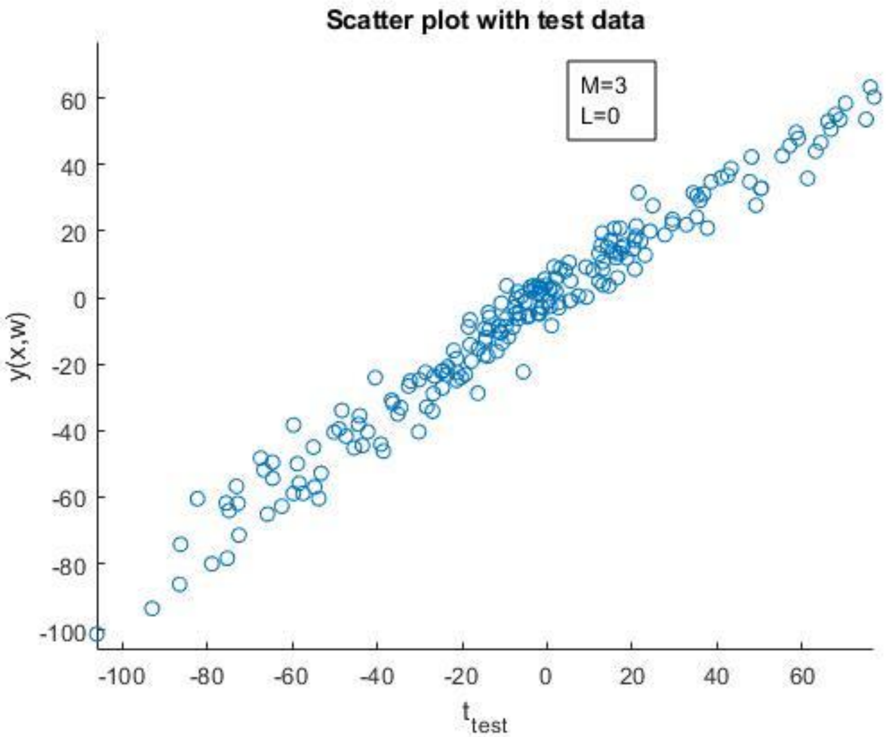

fig.2.2.2 : Scatter plot of tn vs y(x,w) for dataset train20

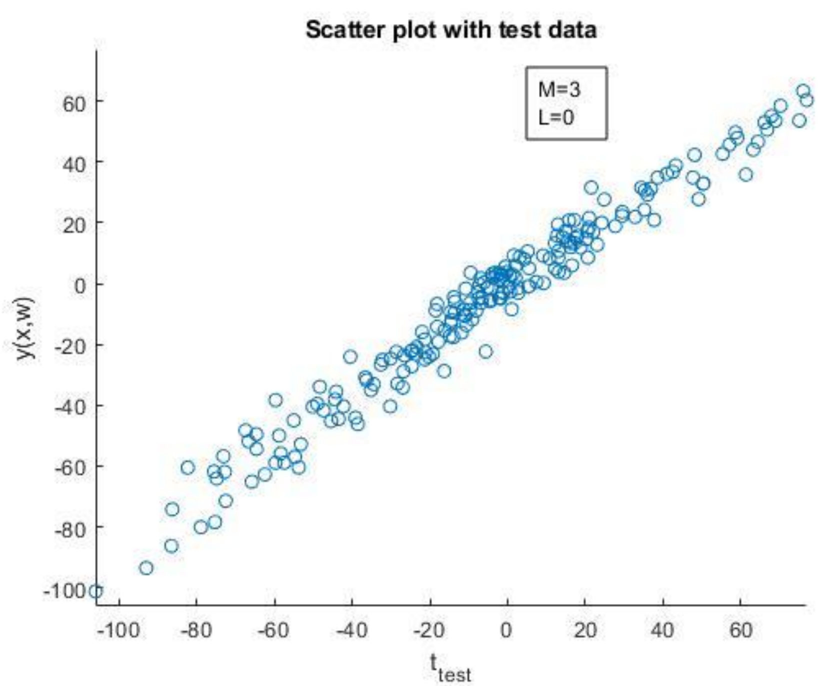

fig.2.2.3 : Scatter plot of tn vs y(x,w) for test data

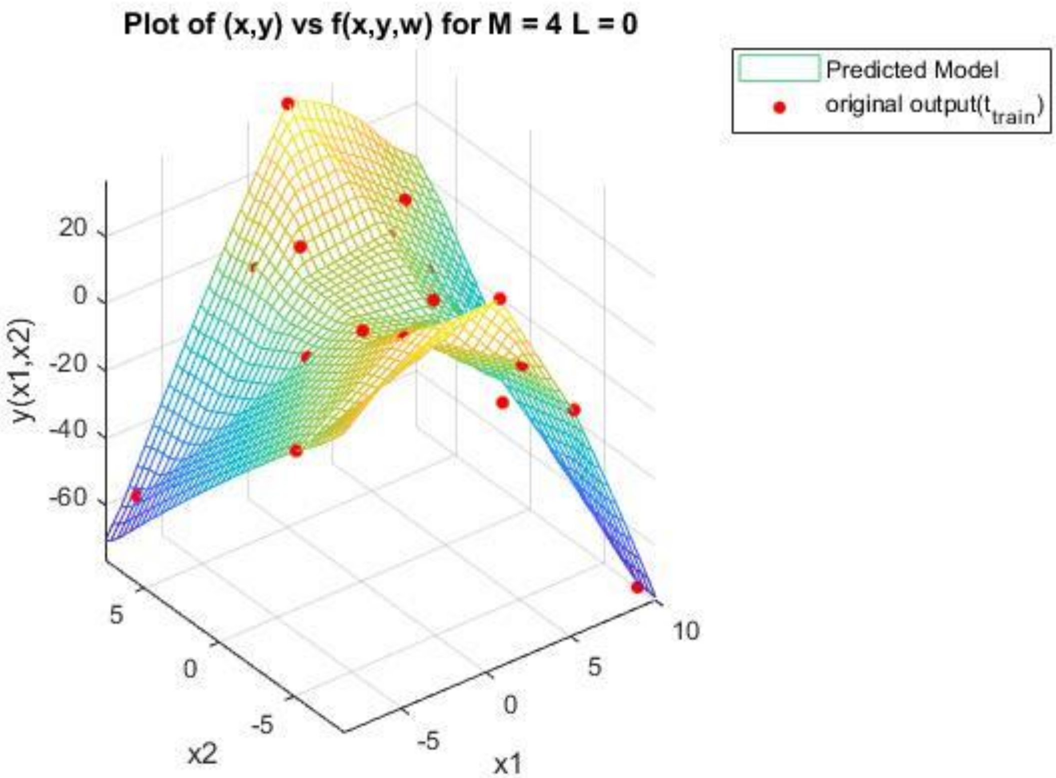

fig: 3-D plot of Bivariate data using Polynomial basis functions for train20 , Overfitting case of data with M=4 , L=0 for train 20

M\L	0	0.0001	0.1
2	4.2199	4.2203	4.5589
3	3.9648	3.9652	4.3299
4	3.664	3.6647	4.2912
5	10.671	0.0024	2.1711

Table 2.2.1 : observations of errors with different M and L for dataset train20

M\L	0	0.0001	0.1
2	8.4236	8.4239	8.7508
3	8.2581	8.2585	8.6124
4	12.0389	12.0396	12.748
5	640.5423	502.2222	514.0345

Table 2.2.2 : observations of errors with different M and L for test data

M\L	0	0.0001	0.1
2	8.2724	8.2727	8.6024
3	7.8719	7.8724	8.2301
4	11.7436	11.7443	12.4447
5	612.2612	478.5347	489.749

Table 2.2.3 : observations of errors for validation with different M and L for train 20

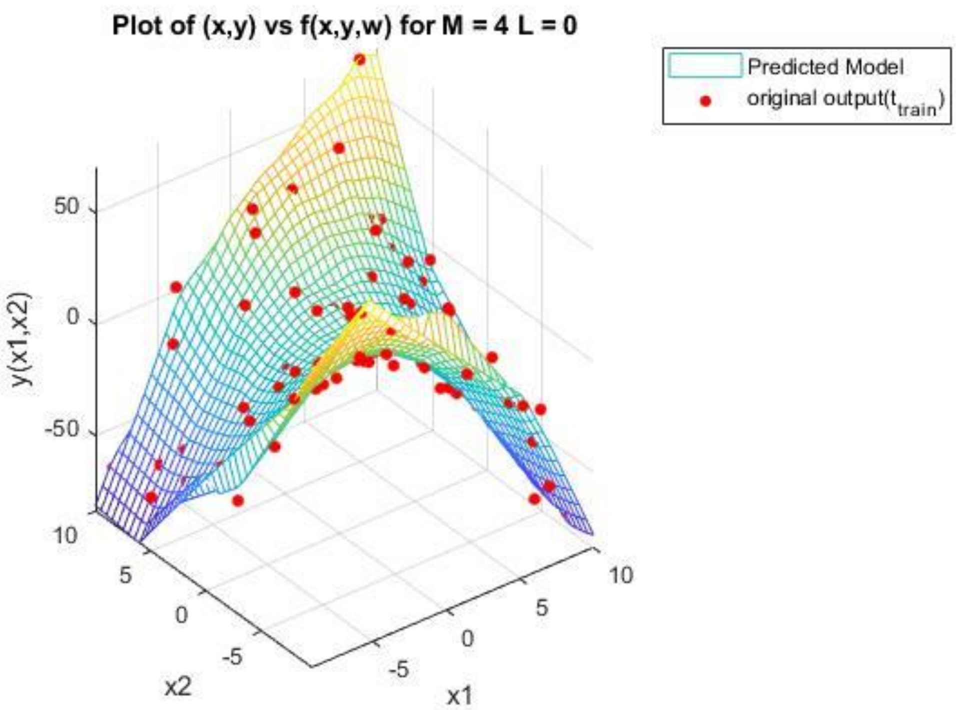

fig.2.2.4 : 3-D plot of Bivariate data using Polynomial basis functions for train100

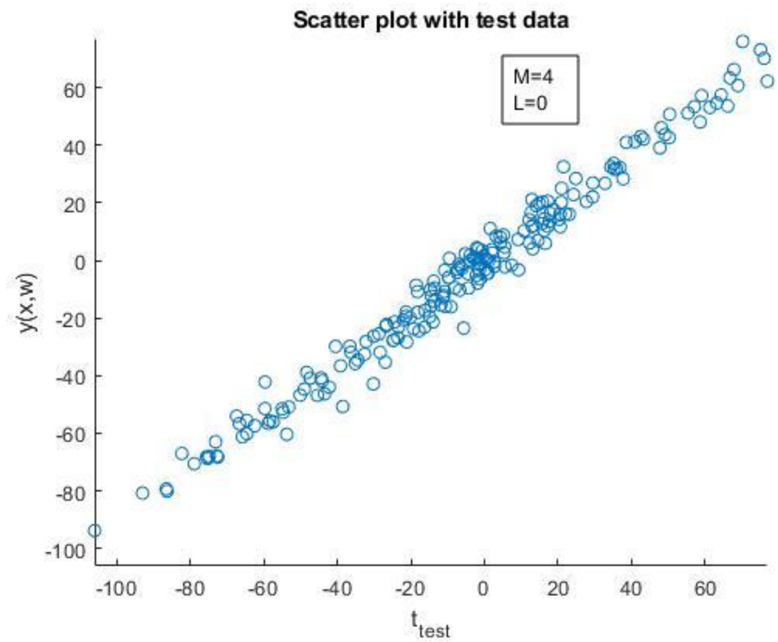

fig.2.2.5 : Scatter plot of tn vs y(x,w) for train100 test data

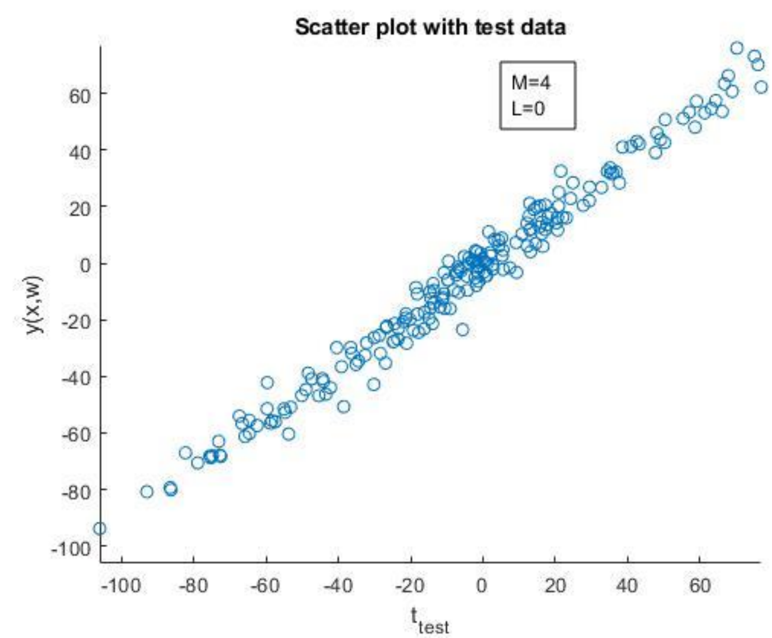

fig.2.2.6 : Scatter plot of tn vs y(x,w) for test data for dataset train100

M\L	0	0.0001	10
2	5.4076	5.4077	15.1802
3	5.3348	5.3349	16.5195
4	5.2559	5.2561	16.6752
5	5.1178	5.1181	8.837
10	3.106	3.1069	34.363

Table 2.2.4 : observations of errors with different M and L for dataset train100 data

M\L	0	0.0001	10
2	5.6291	5.6292	15.4005
3	5.7515	5.7516	16.9226
4	5.988	5.9881	17.3892
5	5.9548	5.955	10.0226
10	600.8491	600.8485	579.6039

Table 2.2.5: observations of errors for test data with different M and L for dataset train100 data

M\L	0	0.0001	10
2	5.784	5.7841	15.5603
3	5.9066	5.9069	17.09
4	6.1079	6.108	17.5151
5	6.4661	6.4663	15.6972
10	664.26	664.26	646.9021

Table 2.2.6: observations of errors for validation data with different M and L for dataset train100 data

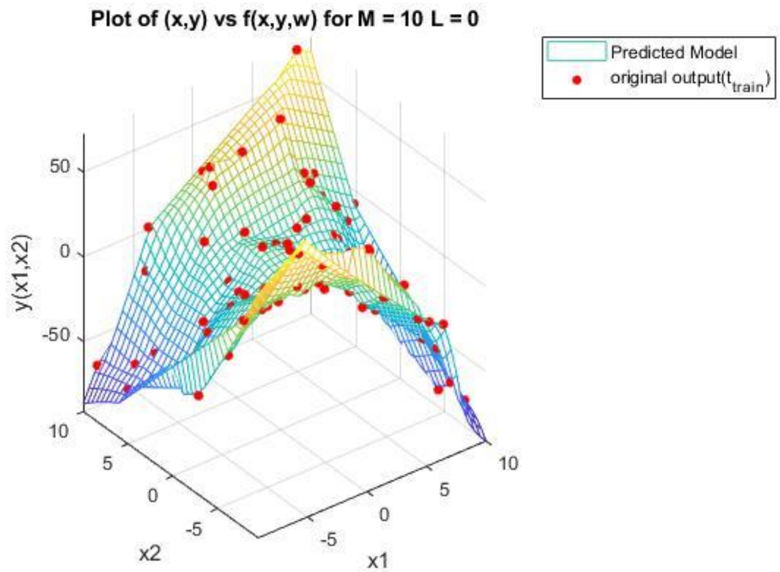

fig: 3-D plot of Bivariate data using Polynomial basis functions , Overfitting case of data with M=10 , L=0 for train 100

M\L	0	0.0001	10
2	5.1693	5.1694	16.7894
3	5.1617	5.1618	15.8475
4	5.1542	5.4544	15.2847
5	5.1509	5.151	15.5057
20	4.4621	4.4621	11.8413

Table 2.2.7: observations of errors with different M and L for dataset train1000

M\L	0	0.0001	10
2	5.0802	5.0803	16.6996
3	5.0791	5.0792	15.7642
4	5.11	5.1101	15.2399
5	5.0954	5.0955	15.45
20	7.3267	7.3267	14.6853

Table 2.2.8 : observations of errors for test data with different M and L for dataset train1000

M\L	0	0.0001	10
2	5.3769	5.377	16.9979
3	5.3892	5.3894	16.0761
4	5.3997	5.3998	15.5304
5	5.4101	5.4102	15.7647
20	10.7516	10.7516	18.1861

Table 2.2.9 : observations of errors for validation data with different M and L for dataset train1000

Observations :

1 . For training with 20 datasets , overfitting is observed at M=5 , and there is no significant difference in error values for test data when regularization was applied.

2 . When the dataset size is increased from 20 to 100 , overfitting at M=5 was removed ,but later on at M=10 , overfitting is observed , with no significant change in errors after regularization is applied.

3 . Again when dataset size is increased from 100 to 1000 , overfitting at M=10 is removed.

4.Overfitting gets removed when the dataset size is increased for that value of M for which it was observed before.

➢ Model 3 : Linear model for regression using Gaussian basis functions for Datasets 3 and 4

Case 1: Regression of Bivariate data using quadratic regularization

- Performed for different values of k , where k is number of clusters .
- Performed for different values of sigma , where sigma is variance .
- Performed for different values of lambda , where lambda is regularization factor.

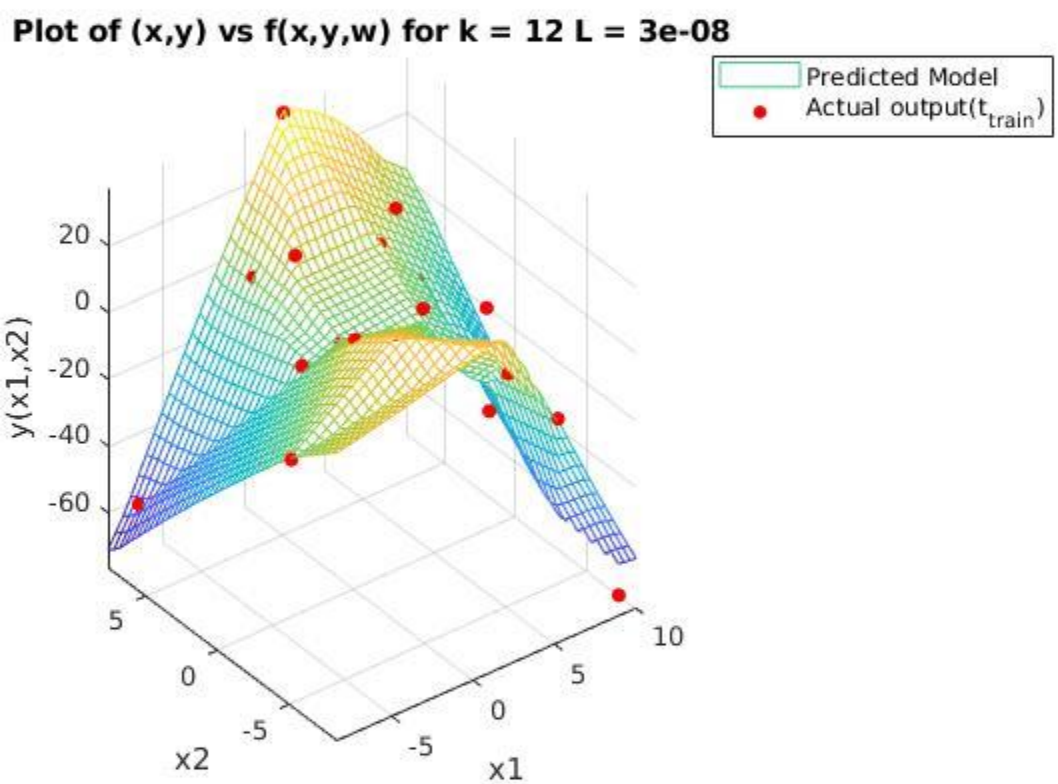

fig.2.3.1.1: 3D plot for bivariate data using quadratic regularization for 20 training dataset

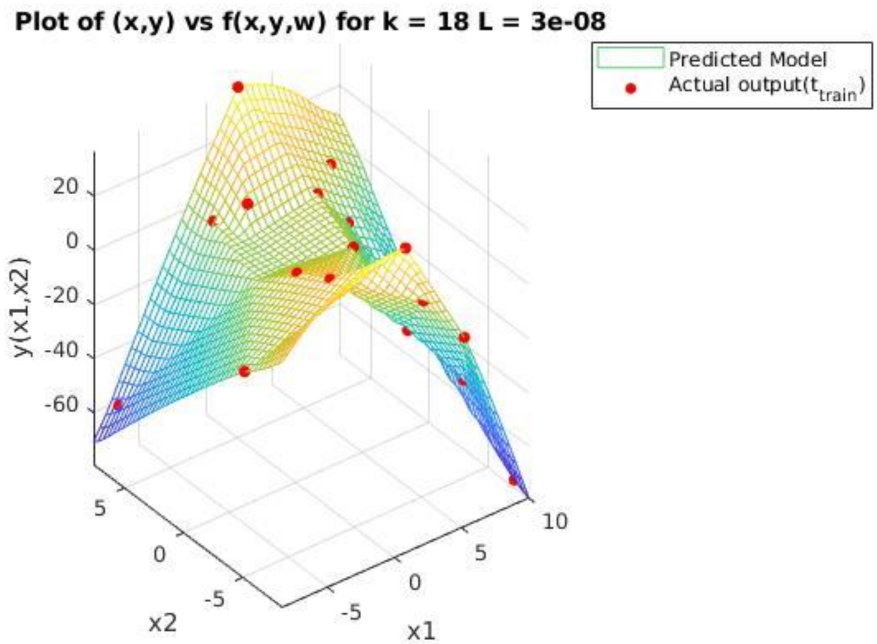

fig .2.3.1.2: 3D plot for overfitting case for bivariate data using quadratic regularization for 20 training dataset

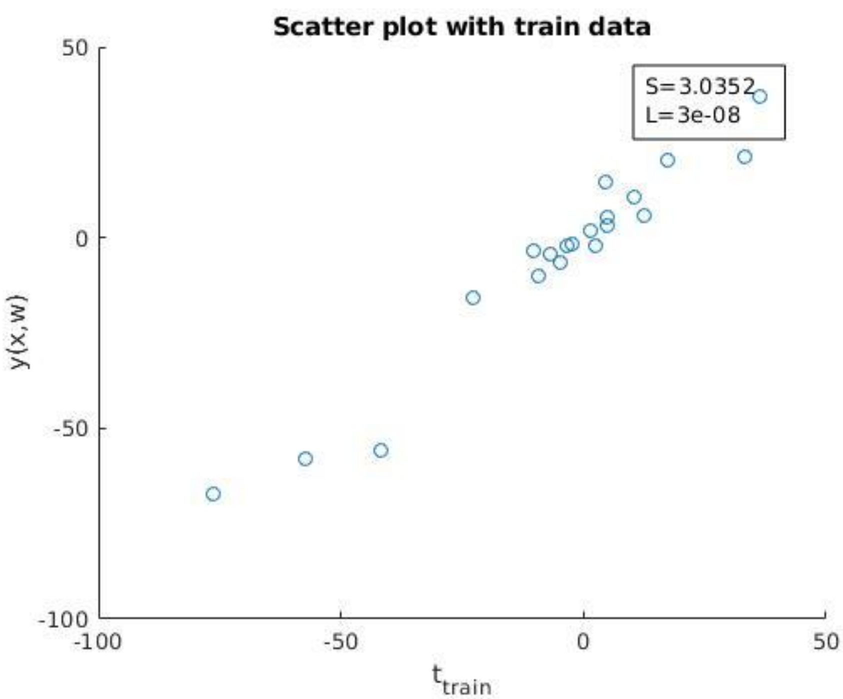

fig.2.3.1.3 : scatter plot for bivariate data using quadratic regularization for 20 training dataset

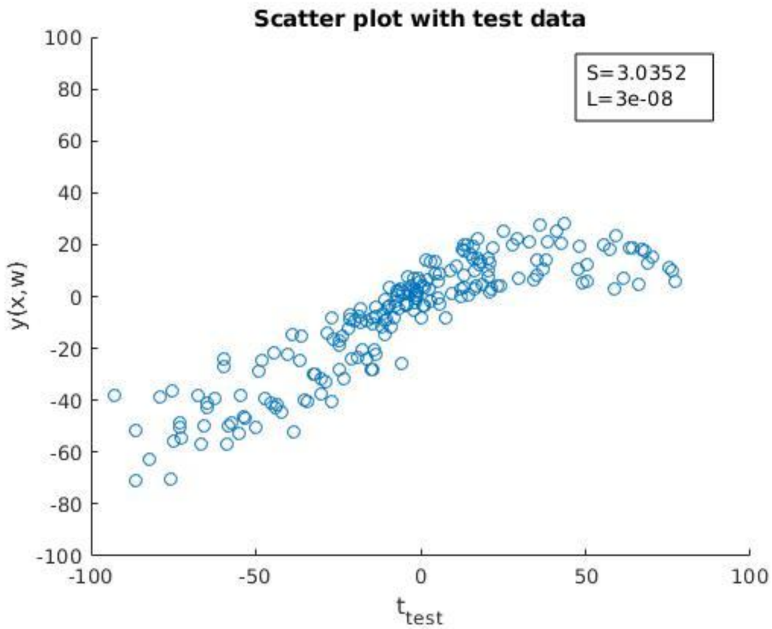

fig.2.3.1.4 : scatter plot for bivariate test data using quadratic regularization for model train using 20 training dataset

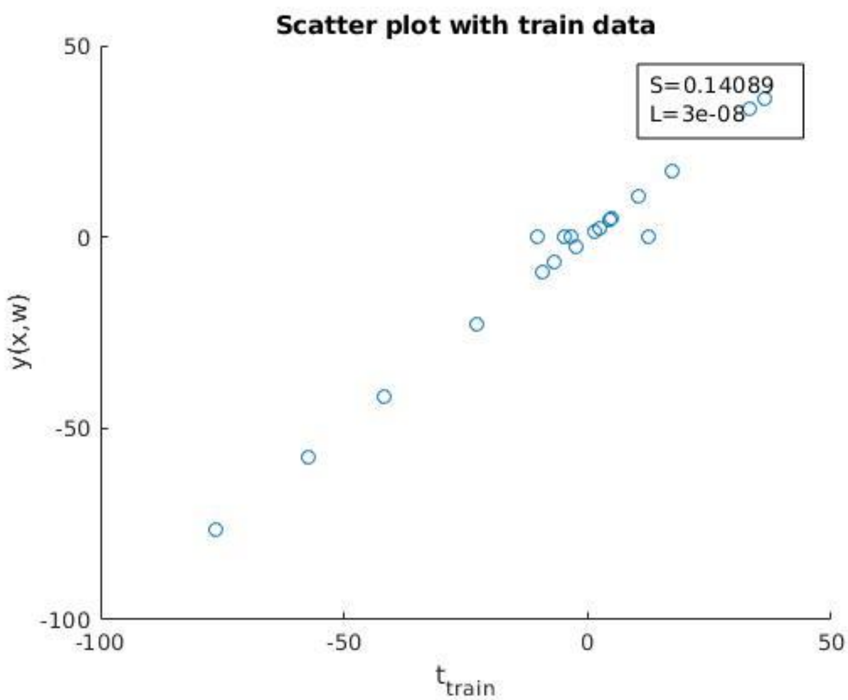

fig.2.3.1.5 : Scatter plot for bivariate data using quadratic regularization for 20 training dataset - Overfitting case

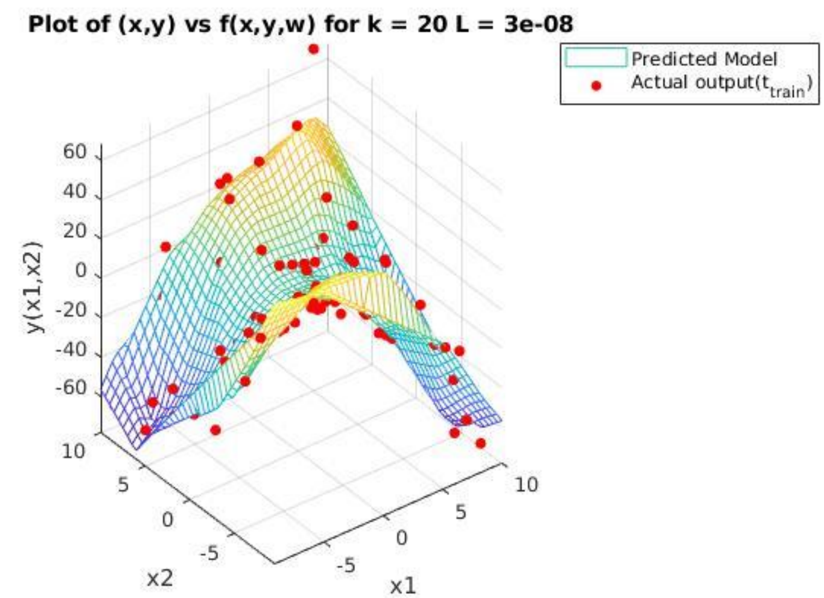

fig.2.3.1.6 : 3D plot for bivariate data using quadratic regularization for 100 training dataset

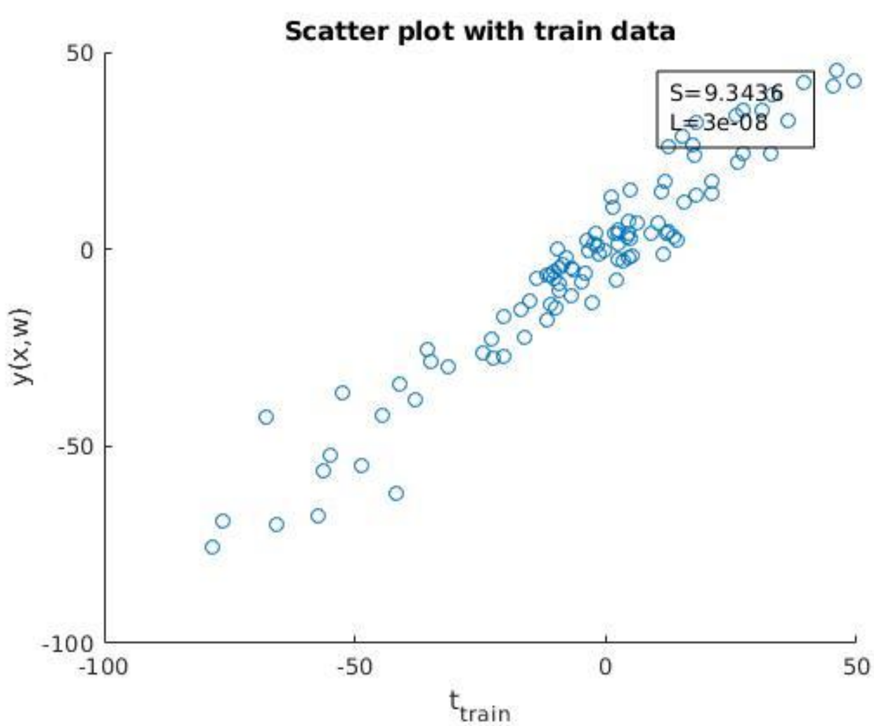

fig.2.3.1.7 : scatter plot for bivariate data using quadratic regularization for 100 training dataset

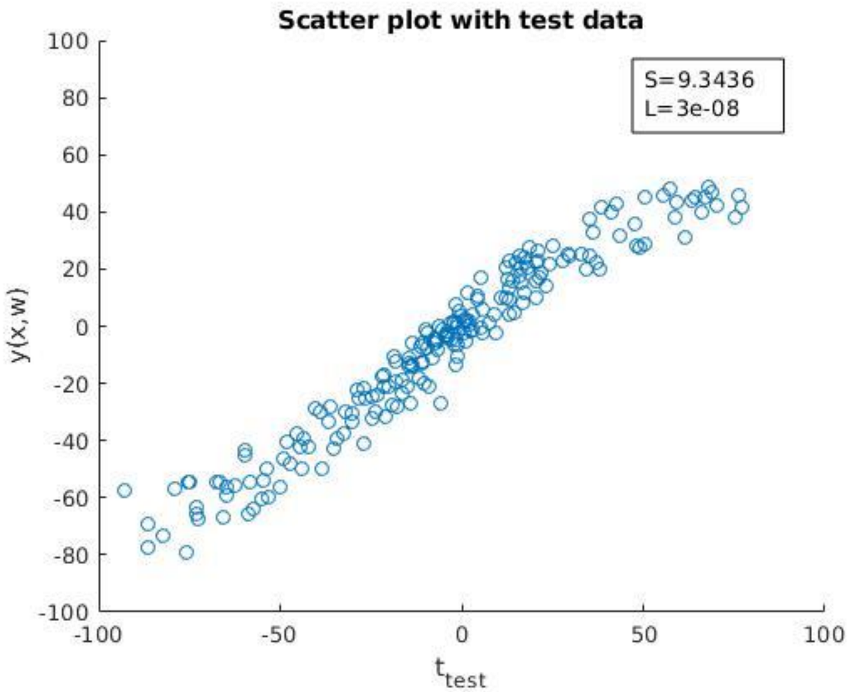

fig.2.3.1.8: scatter plot for bivariate test data using quadratic regularization for model train using 100 training dataset

Observation:

for 20 training dataset we see that when no of cluster becomes closer to 20 there is significant rise in test error . At k =18 overfitting occurs, and on adding L , we see error in test data gets reduced.

K\L	0	9.00E-05	0.1
2	26.3388	26.6197	27.32801
5	20.0441	29.6881	26.9052
12	8.4264	10.2653	1454.27
18	5.9099	8.44307	608.27
20	0	0.6656	611.63

Table 2.3.1: observations of errors with different M and L for dataset train20 for quadratic regularization for bivariate data

K\L	0	9E-05	0.1
2	37.7792	37.5693	38.27774
5	31.7458	40.64454	37.8547
12	22.489	25.1967	1469.51
18	169.3804	41.0609	642.933
20	38.0033	38.666	647.155

Table 2.3.2: observations of errors for test data with different M and L for dataset train20 for quadratic regularization for bivariate data

K\L	0	9E-05	0.1
2	38.5592	38.3303	39.03762
5	32.9926	41.4047	38.6152
12	23.6175	25.895	1470.51
18	194.3411	41.3887	643.244
20	38.3142	38.9977	647.466

Table 2.3.3: observations of errors for validation data with different M and L for dataset train20 for quadratic regularization for bivariate data

K\L	0	9.00E-05	0.1
2	27.7621	27.9501	28.17755
5	24.8436	32.4705	28.041
12	12.0511	6801.7	56.617
18	8.226	2588.78	1182.62
20	7.79	478.341	2733.92

Table 2.3.4 : observations of errors with different M and L for dataset train100 for quadratic regularization for bivariate data

K\L	0	9E-05	0.1
2	37.639	37.6664	37.892
5	33.537	42.187	37.757
12	17.662	6810.5	66.335
18	11.165	2593.79	1192.29
20	10.69	481.28	2743.42

Table 2.3.5 : observations of errors for test data with different M and L for dataset train100for quadratic regularization for bivariate data

K\L	0	9E-05	0.1
2	38.1521	38.2315	38.4569
5	34.441	42.7525	38.3228
12	18.218	6811.3	66.9
18	11.58	2594.21	1192.92
20	11.124	481.73	2744.125

Table 2.3.6 : observations of errors for validation data with different M and L for dataset train100 for quadratic regularization for bivariate data

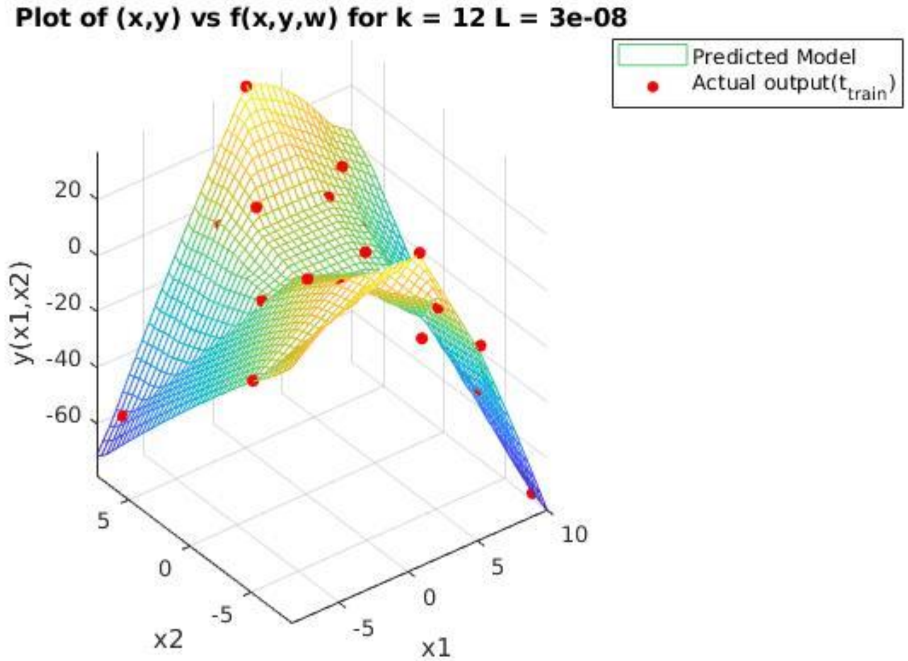

fig.2.3.2.1 : 3D plot for bivariate data using Tikhonov regularization for 20 training dataset

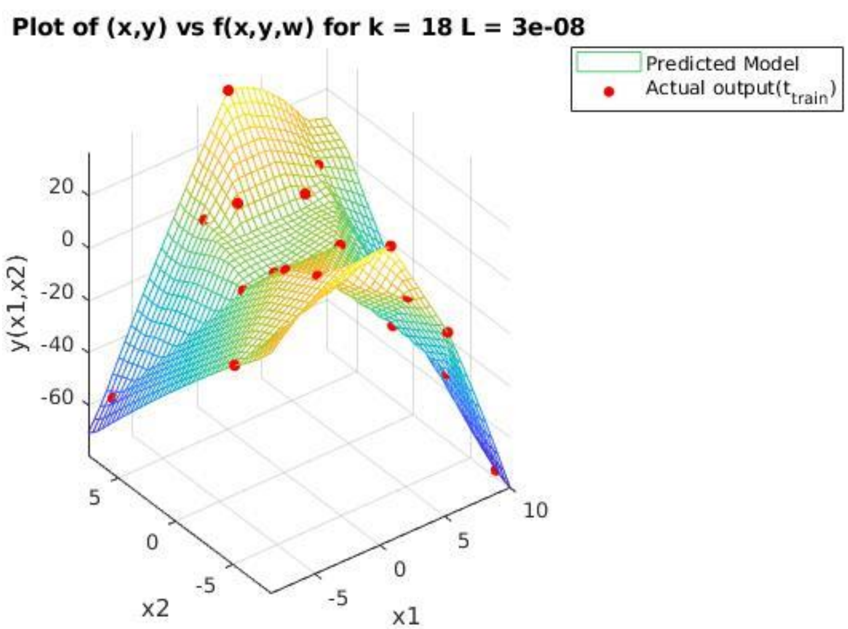

fig.2.3.2.2 : 3D plot for bivariate data using Tikhonov regularization for20 training dataset - Overfitting case

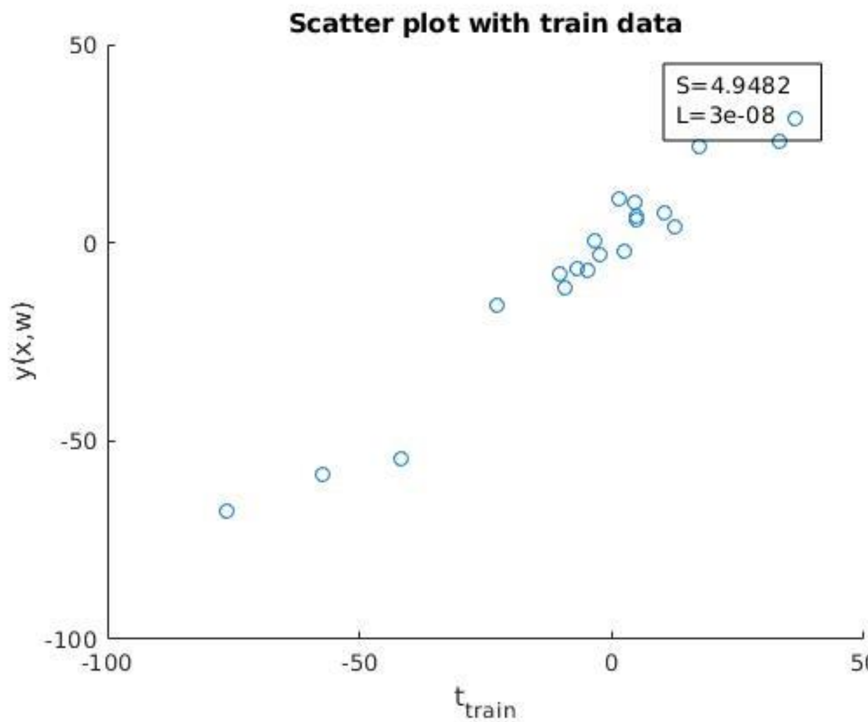

fig.2.3.2.3 : scatter plot for bivariate data using Tikhonov regularization for 20 training dataset

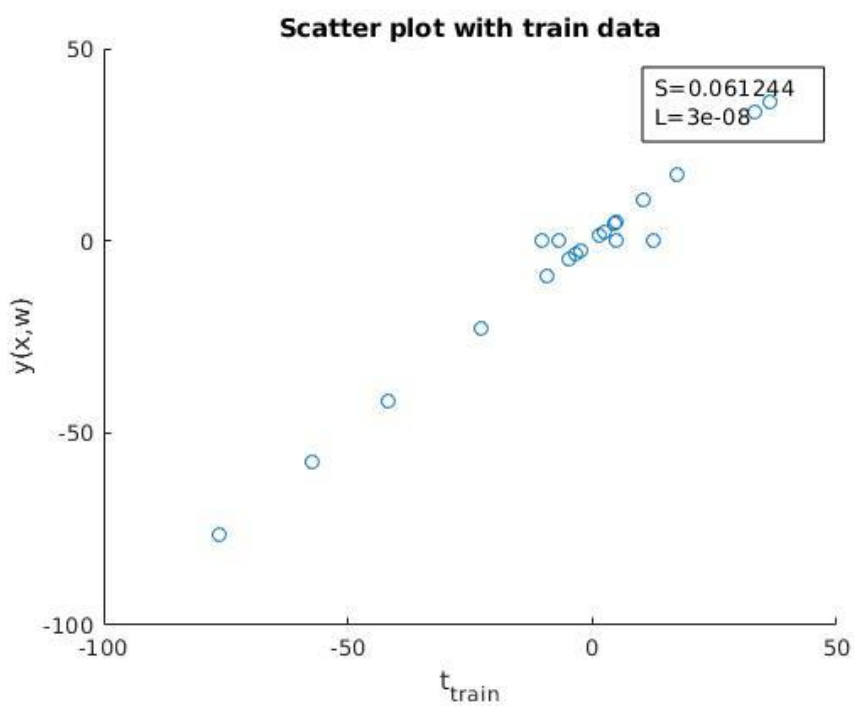

fig.2.3.2.4: scatter plot for bivariate data using Tikhonov regularization for 20 training dataset : Overfitting case

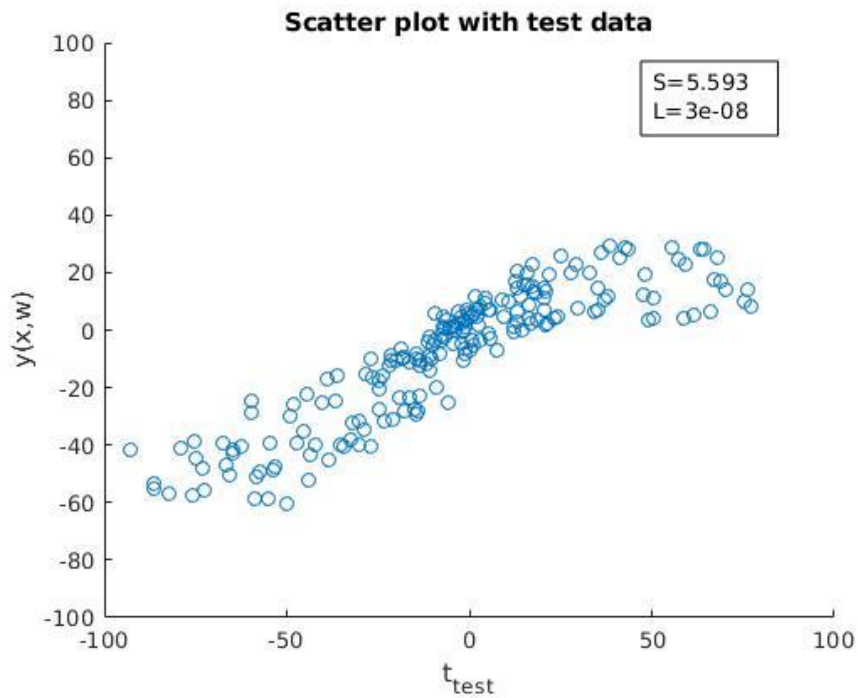

fig.2.3.2.5 : scatter plot for bivariate test data using Tikhonov regularization for model 20 training dataset

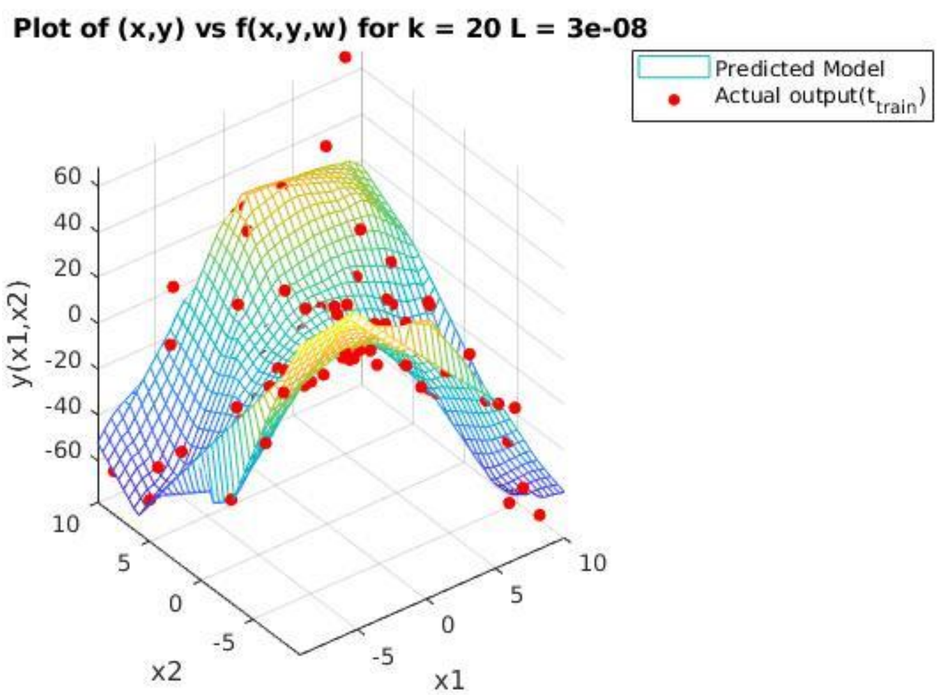

fig : fig.2.3.2.6 : 3D plot for bivariate data using Tikhonov regularization for 100 training dataset

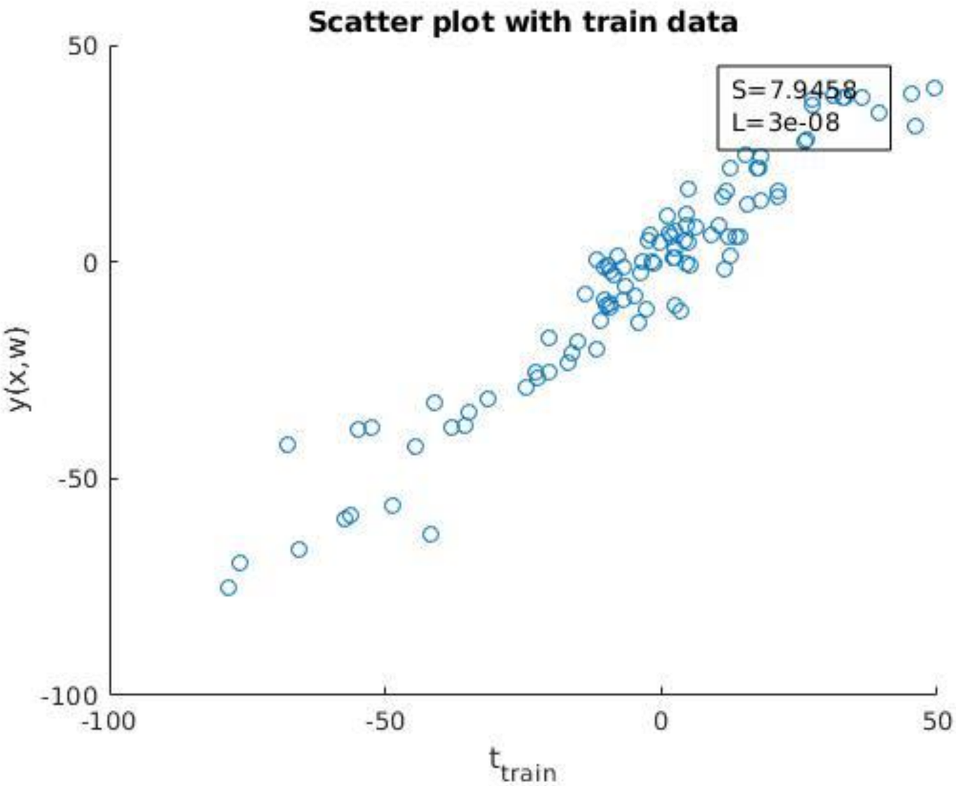

fig.2.3.2.7 : scatter plot for bivariate data using Tikhonov regularization for 100 training dataset

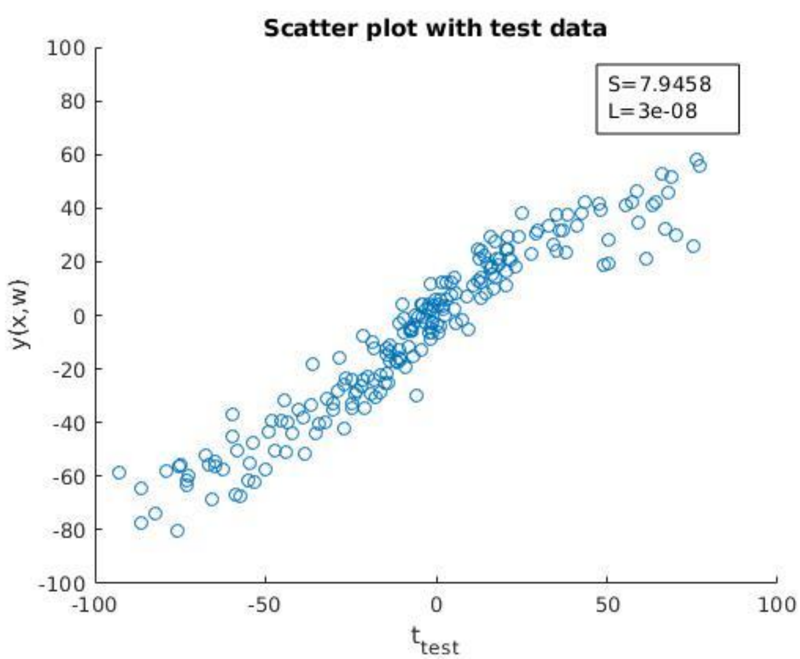

fig.2.3.2.8 : Scatter plot for bivariate test data using Tikhonov regularization for model 20 training dataset

Case 2 : Guassian Regression of multivariate data with Quadratic and Tikhonov regularization

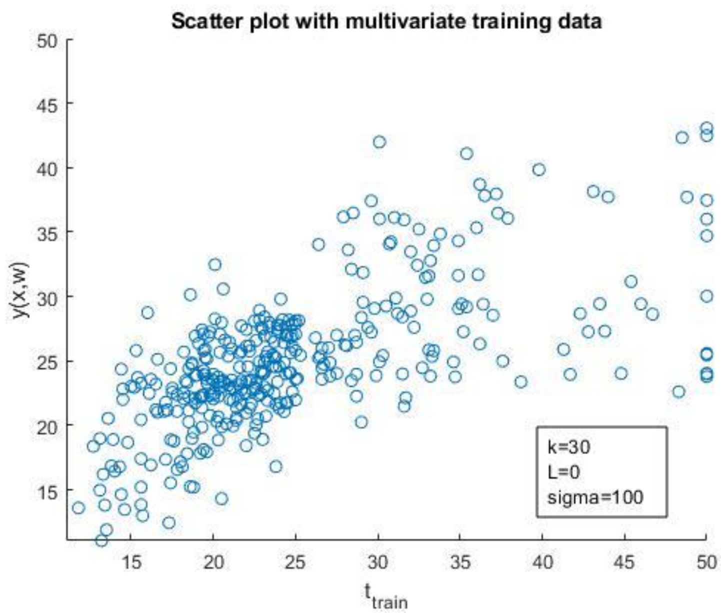

fig 2.3.3.1 : scatter plot for multivariate data without regularization for training dataset

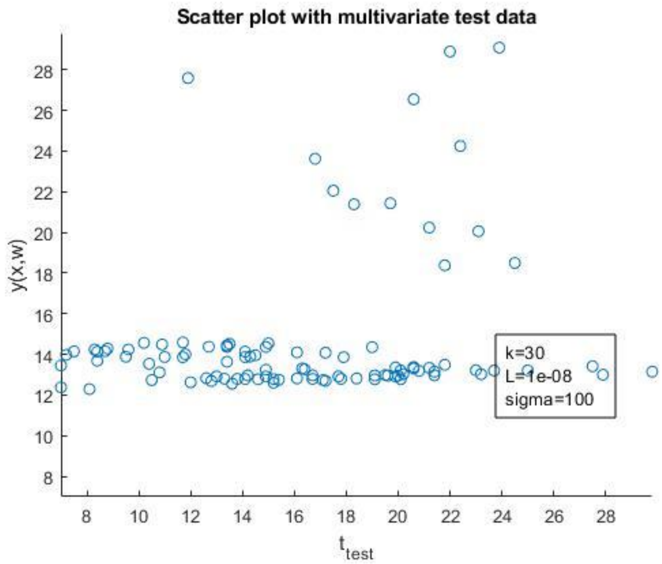

fig 2.3.3.2: scatter plot for multivariate data without regularization for test model

Observation :

Even if size of cluster is changed , there is no significant change in error .

			Quadratic	Tikhonov
K	**S**	**L=0**	**L=0.000001**	**L = 0.00000001**
7	15	7.6628	7.6067	7.6836
9	80	7.5413	9.7971	7.6156
17	60	7.1983	14.7864	7.4095
30	100	6.3456	188.2932	12.2614

Table 2.3.7 : observations of errors with different M and L for dataset train100 with quadratic regularization for multivariate data

			Quadratic	Tikhonov
K	**S**	**L=0**	**L=0.000001**	**L = 0.00000001**
7	15	6.2204	5.8633	5.7878
9	80	5.1845	7.2492	4.9942
17	60	4.6751	12.4396	6.346
30	100	5.5521	187.0699	10.8156

Table 2.3.8 : observations of errors for test data with different M and L for dataset train100 with quadratic regularization for multivariate data

			Quadratic	Tikhonov
K	**S**	**L=0**	**L=0.000001**	**L = 0.00000001**
7	15	13.0742	12.8628	12.816
9	80	12.6596	14.6814	12.3963
17	60	12.027	19.6268	13.0986
30	100	12.2153	194.0197	17.9307

Table 2.3.9: observations of errors for validation data with different M and L for dataset train100 with quadratic regularization for multivariate data