Detrended Fluctuation Analysis

Introduction:

- Used to determine the self-affinity of a signal
- Self-affinity - property of fractal type time-series. It describes the way in which a smaller segment of a fractal structure is similar to the whole structure.
- Used mostly in time-series analysis
- Can also be applied in cases where statistical functions such mean and variance are non-stationary (i.e., they change with time).
- Related to measures based on spectral techniques like auto-correlations and fourier transforms.

1/F Structure (aka scale-free network or fractal):

- Before beginning Detrending Fluctuation Analysis (DFA), it is important to understand the 1/f concept.
- ‘1/f’ refers to the structure of the power spectrum of the brain dynamics (and other physiological functions too).
- 1/f is a kind of pink noise.
- As power increases, the frequency decreases and vice-versa.
- Sometimes, the mean of the data is not an accurate measure of the data.
- There may be a large number of small-sized entities or vice-versa that impact the mean of the data.

1/F Structure (contd.):

- Size of the entities of the data is related to their frequency of occurrence. This is a **scale-free network**.
- Most biological signals possess this 1/f noise.
- It poses a problem when we are trying to do time-series analysis.
- It does not mean that the 1/f noise is always bad for analysis.
- But, in order to observe the brain's response to certain external stimuli, we can remove the 1/f noise from the data to focus on fast time-scale dynamics.

Steps to perform DFA on signals:

1. Convert to mean-centered cumulative sum, i.e., subtract each value from the overall mean and take the cumulative sum of the entire set of obtained values.
2. Define log-spaced scales.
3. Choose a scale, split the signal into epochs based on the chosen scale, detrend it, and compute its individual RMS values.
4. Repeat step 3 for all the other scales, and find overall average of each scale's RMS.
5. Compute linear fit between log-scales and log-RMS.

Precursor step:

In electrophysiology, it is better to perform DFA on the power spectrum of the signal or on its amplitude spectrum and not on the original time-domain signal. So, we can first compute its amplitude time-series using wavelet convolution (or any other method), and then proceed with DFA.

Points to remember:

1. Slope of the previously mentioned final linear-fit graph gives us the Hurst Exponent.
2. Systems that have a long-range memory, show strong positive auto-correlations in time and have a Hurst exponent (DFA value) generally between 0.5 and 1.
3. Hurst exponent is indicative of a system in critical state.
4. Since DFA is something that analyses fluctuations over slower time periods like minutes/hours/decades, it cannot be applied to data that is only a few seconds long. Has to be long data.
5. No. of RMS values corresponds to no. of scales.

Points to remember: (contd.)

6. HURST EXPONENT (H)

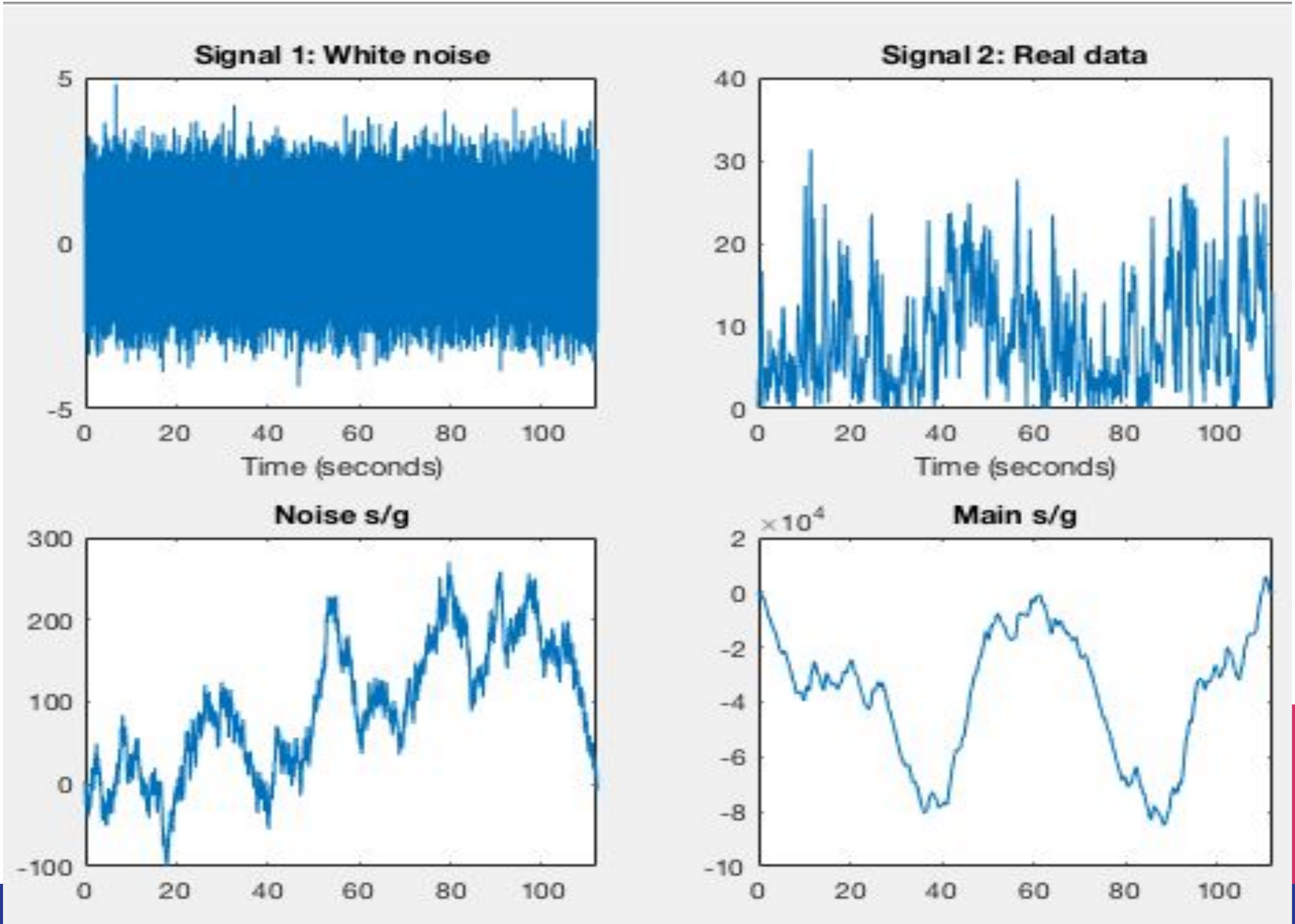
H~0.5 - Brownian time series: Indicates high degree of randomness, negligible correlation between observations and future values, predictability of such signals is very less.

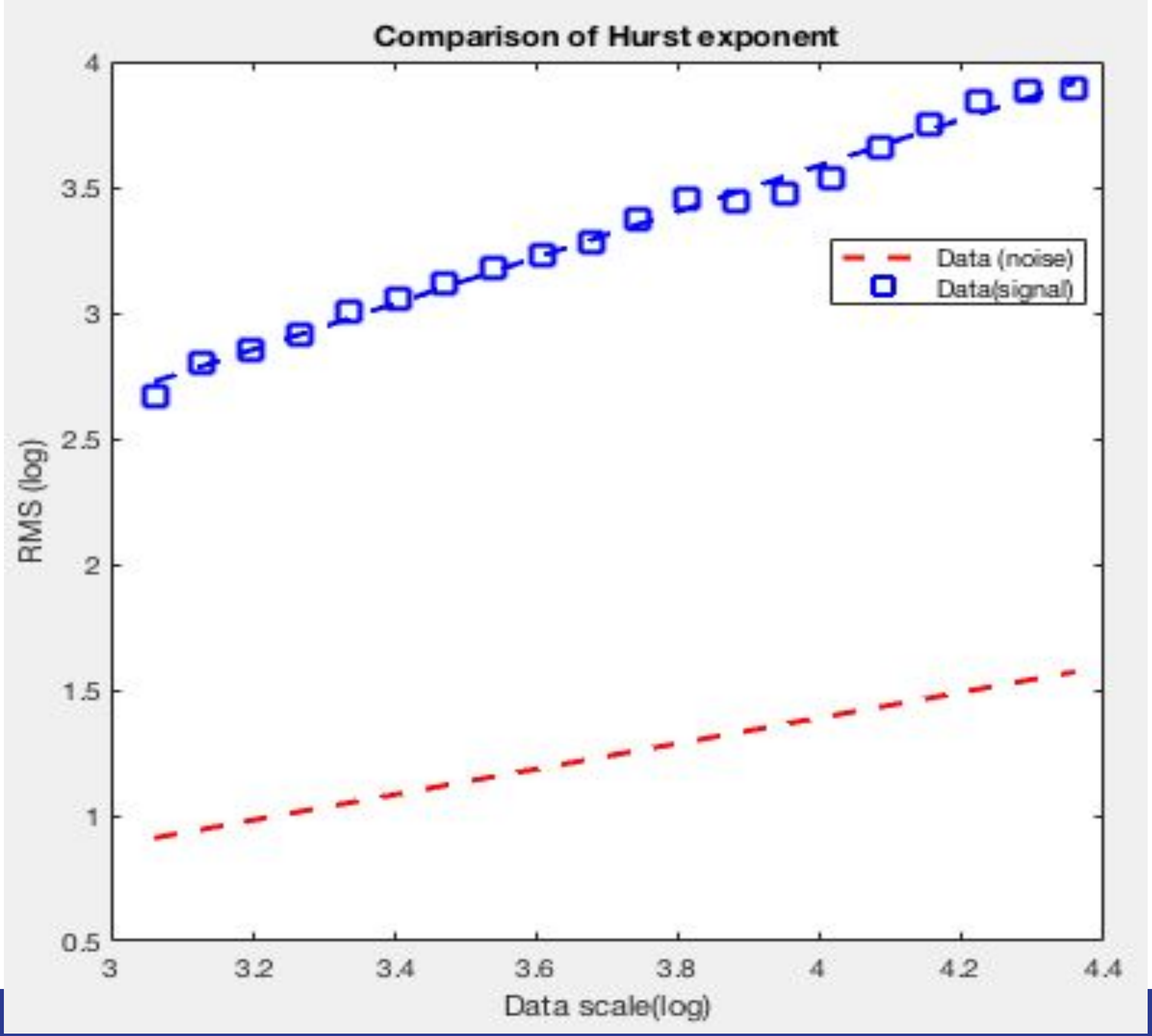
0<H<0.5 - Anti-persistent time series: Mean-reverting series, an increase will most likely be followed by a decrease or vice-versa (i.e., values will tend to revert to a mean). This means that future values have a tendency to return to a long-term mean.

Points to remember: (contd.)

<u>$0.5<H<1$ - Persistent time series:</u> In a persistent time series an increase in values will most likely be followed by an increase in the short term and a decrease in values will most likely be followed by another decrease in the short term. A Hurst exponent value between 0.5 and 1.0 indicates persistent behavior; the larger the H value the stronger the trend.

If $H>1$, it means that DFA is not an appropriate analysis technique for that time-series data.


Signal 1: White noise
Time (seconds)
Signal 2: Real data
Time (seconds)
Noise s/g
Main s/g
×10^4
5
0
-5
40
30
20
10
0
300
200
100
0
-100
2
0
-2
-4
-6
-8
-10
0
20
40
60
80
100



Comparison of Hurst exponent
Data (noise)
Data(signal)
RMS (log)
Data scale(log)
4
3.5
3
2.5
2
1.5
1
0.5
3
3.2
3.4
3.6
3.8
4
4.2
4.4

Hurst exponent:

Command Window

```
Hurst exponent of signal:
   -0.0716
    0.9145

Hurst exponent of noise:
   -0.5957
    0.4985
```

REFERENCES:

https://www.youtube.com/watch?v=-RmxLZF8adI

https://en.wikipedia.org/wiki/Detrended_fluctuation_analysis

https://www.frontiersin.org/articles/10.3389/fphys.2012.00450/full

https://pubsonline.informs.org/do/10.1287/LYTX.2012.04.05/full/