Chapter 01 - Mobile Robot Kinematics

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Adapted from “Principles of Robot Autonomy” by D. Gammelli, J. Lorenzetti, K. Luo, G. Zardini, and M. Pavone

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Without a model that links objectives to actuation, even simple point-to-point tasks stall.

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Both perspectives are necessary to describe what a robot can actually do.

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The right model balances complexity against accuracy for the objective at hand.

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Picking the right abstraction speeds up planning without losing the essential behavior.

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Build practical kinematic models for wheeled robots.

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Trajectories are mappings $\boldsymbol{\xi}(t) : \mathbb{R} \to \mathbb{R}^n$ that describe motion over time.

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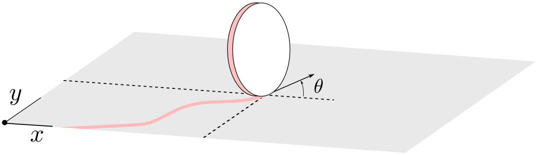

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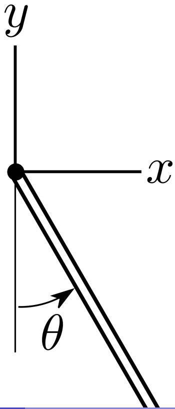

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$$\frac{dh_i(\boldsymbol{\xi})}{dt} = \frac{dh_i(\boldsymbol{\xi})}{d\boldsymbol{\xi}}\dot{\boldsymbol{\xi}} = 0$$

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The no-slip wheel constraint is not integrable, so it does not limit which (x, y) can be reached.

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Any system subject to at least one such constraint is nonholonomic.

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Example: For $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$, any scalar multiple of $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ lies in the null space because $A \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \mathbf{0}$.

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Any feasible velocity is a combination of those motions.

Building Kinematic Models

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Inputs $\boldsymbol{u}$ parameterize allowable motions of dimension $n - k$.

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Studying canonical cases makes it easier to adapt kinematic reasoning to new robots.

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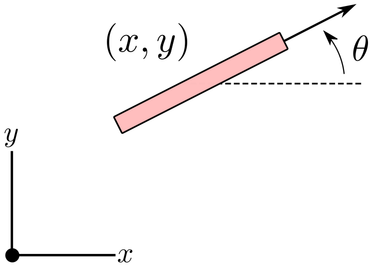

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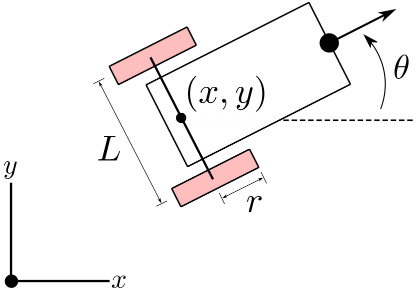

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Wheel positions: $p_l = [x - \frac{L}{2}\sin\theta,\ y + \frac{L}{2}\cos\theta]$ and $p_r = [x + \frac{L}{2}\sin\theta,\ y - \frac{L}{2}\cos\theta]$.

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The redundant constraints confirm that a single nonholonomic condition governs the chassis.

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$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{r}{2}\cos\theta & \frac{r}{2}\cos\theta \\ \frac{r}{2}\sin\theta & \frac{r}{2}\sin\theta \\ \frac{r}{L} & -\frac{r}{L} \end{bmatrix} \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix}.$$

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Some tasks therefore augment kinematics with lightweight dynamics.

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Revisit models as tasks evolve to ensure accuracy remains aligned with objectives.