CS4090 Programming Project

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1 Abstract

The successful purification of imperfect entangled states is a critical step in establishing reliable quantum communication schemes under real-world conditions. In this paper, we compare the performance of three purification protocols under different noise scenarios, namely the Extreme Photon Loss (EPL) protocol [1][2], the DEJMPS protocol [3] and the Three-to-one protocol [4], using the quantum network simulator NetQASM. We find that all protocols need high gate fidelities to perform successful purification, with the EPL and Three-to-one protocols being more suited to high noise scenarios. The DEJMPS protocol provides a high success probability but requires even higher gate fidelity.

2 Introduction

The reliable distribution of quantum states is a requirement for practical quantum communications such as superdense coding [5], quantum key distribution [6], and many other related quantum communication processes. However, current real-world channels are subjected to varying degrees of noise. Thus, it is necessary to have a method for generating highly entangled pairs out of imperfect shared quantum states. This process is called quantum entanglement distillation or purification. Two key aspects of such a purification protocol are its throughput, the number of entangled pairs generated per second, and its quality, the fidelity of the post-distillation state. Both aspects are investigated in this paper.

2.1 Noise sources

In this paper, we chose to model the noise inherent to real-world systems via two different parameters. The **Entangled State Noise** represents the imperfect generation of entangled pairs between Alice and Bob, so that their initial pairs are in a Werner state of

$$\rho_{AB} = p\left|\Phi_{00}\right\rangle\left\langle\Phi_{00}\right| + \frac{1-p}{4}\mathbb{1}_4.$$

That means that every noise in the entanglement generation process is modeled via the depolarization parameter p, corresponding to a depolarizing channel acting on a perfect Bell state.

Similarly, gates will work imperfectly in the real world. It is again treated by introducing a depolarization channel after the applied (perfect) gate with a depolarization parameter p_U. With this, the state ρ_{noisy}, after apply the gate, is given by

$$\rho_{noisy}(p_u) = (1-p_U)\left|\Psi_{per}\right\rangle\left\langle\Psi_{per}\right| + \frac{p_u}{2}\mathbb{1}_4,$$

Ψ_{per} being the state after application of the ideal gate. To further understand the different protocols discussed in this paper here are brief summaries of the intricacies of each of these:

2.2 EPL protocol

The EPL protocol is a simple two-to-one distillation protocol, with the setup shown in Figure 1.

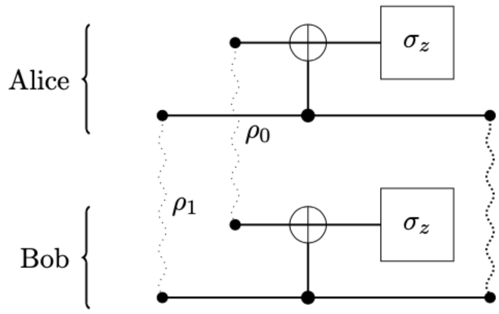


Figure 1: Quantum circuit implementing the EPL quantum distillation protocol.

It encompasses the following steps: 1) Generate two identical shared pairs. 2) Alice and Bob both apply a CNOT-operation as shown. 3) Alice and Bob both measure the second Qubit in the computational basis. If both measure one, they declare success and keep pair one else, they declare failure.

2.3 DEJMPS protocol

Like the EPL protocol, the DEJMPS quantum protocol performs a two to one copy distillation. The quantum circuit responsible for this protocol is described in Figure 2. Where the two quantum gates U_A and U_B are given by:

$$U_A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}, U_B = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \tag{1}$$

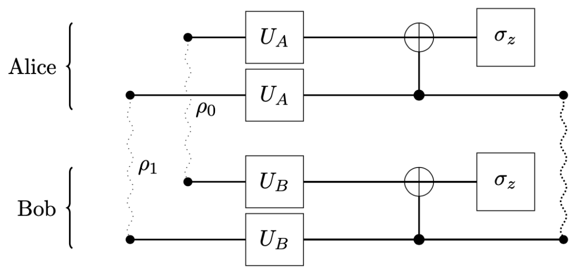


Figure 2: Quantum circuit implementing the DEJMPS quantum distillation protocol.

The DEJMPS protocol encompasses the same steps as the EPL protocol with the exception of the second step 2), where there is the addition of the U_A and U_B gates and step three 3) where the success condition is that the outputs are equal.

2.4 Three-to-one protocol

The difference between this purification protocol and the two previously discussed protocols, is the use of three, instead of two, copies of a state to generate one coherent EPR pair. The circuit used for this protocol is illustrated in Figure 3.

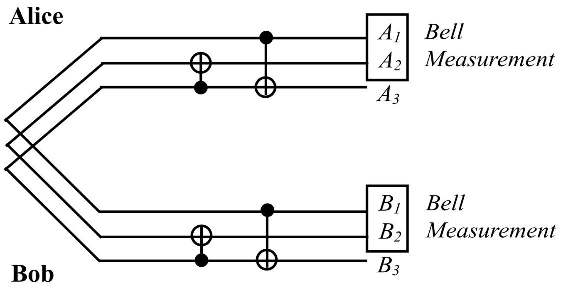


Figure 3: Quantum circuit implementing the three-to-on quantum distillation protocol [4].

The protocol itself, as described in [4], goes as follows: 1) Generate three identical copies of a given state. 2) Alice and Bob both apply two CNOT operations as depicted in Figure 3. 3) Perform a bell state measurement on the two-qubit states A_1A_2 and B_1B_2. If the measurement outcomes coincide, then keep the state of the pair A_3B_3. Otherwise, discard it. In practice, the bell state measurement was achieved by applying a Hadamard gate and a CNOT gate before measuring in the computational basis.

3 Research Questions

In this work, the following questions will be investigated on the aspects of success probability and the fidelity of successfully distilled states with the ideal EPR pair:

1. Given ideal gates, but imperfect initial entangled states, how do the protocols compare?
2. Given ideal initial states, but imperfect gates, how do the protocols compare?
3. With both errors present, how do the protocols compare?

The aspect of success probability is directly linked to the throughput of a system implementing this protocol, while the fidelity of distilled states is precisely our measure of quality. However, a protocol with low quality of output states would necessitate the iteration of the protocol to arrive at the desired actual fidelity, at the cost of throughput and memory. In this study, we decided to analyse these questions for a gate and depolarising fidelity between 0.7 and 1.0, as this is the scale at which most current Noisy Intermediate Scale Quantum (NISQ) devices function [7].

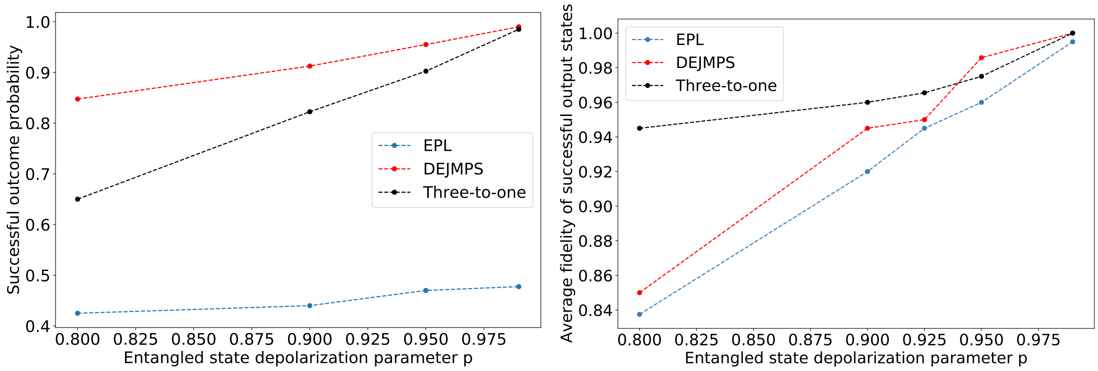
4 Results and Discussion

For each question, two measures were calculated and plotted against the noise level: the rate of success calculated from 400 runs (a) and average distilled state fidelity from 200 successful runs (b).

4.1 Ideal gates, noisy states

The first question concerns scenarios with noisy initial states but perfect gates. Figure 4 summarises the obtained results.

From the theory [8], it is known that the probability of success of the EPL protocol does not surpass $\frac{1}{2}$. Part (a) of Figure 4 confirms this. It further shows that for the studied range of depolarising errors, 0.80 to 1.00, the DEJMPS protocol will declare ”success” the most frequently out of the three protocols. Lastly, given the slopes of the three obtained lines, it would suggest that the EPL protocol will vary the least from its original success probability given worse entanglement generation.



(a) (b)

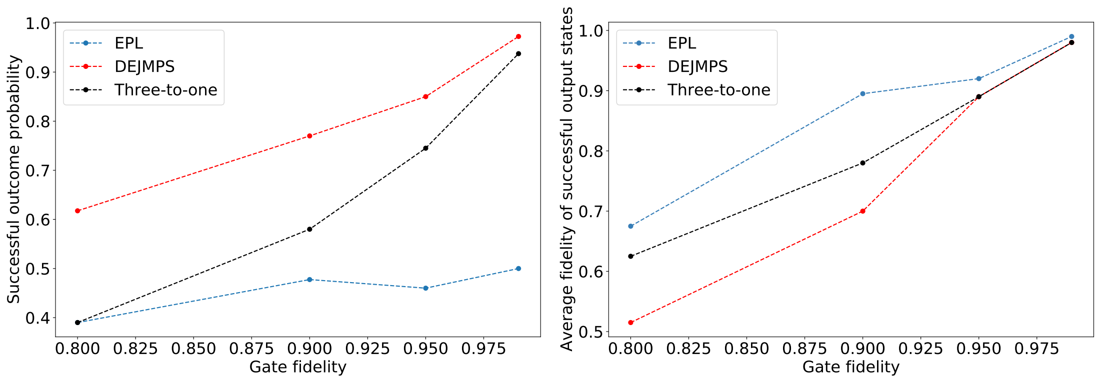
Figure 4: (a) Average success probability with perfect gate fidelity and varying entangled state noise. (b) Average generated state fidelity with perfect gates but varying entangled state noise.

Part (b) of Figure 4 illustrates the average fidelity of the successful runs. It is apparent that EPL performs consistently worse on this regard. On the other hand, the three-to-one protocol appears to generate the EPR states with the highest degree of fidelity. In their paper, [4] claim and show that their protocol is more efficient at producing accurate distillation than most known two to one protocols. In such, this result confirms their findings. The unusual behavior of the DEJMPS protocol at $p = 0.925$ is likely to be statistical noise, as time and computation power constraints limited the number of runs.

In this scenario, EPL is outperformed for the entire noise range, while a evaluation between DEJMPS and Three-to-one would likely depend on the exact scenario; for instance, memory limitations would favor Three-to-one as it would need fewer iterations. For low entangled state noise, their performance both in quality and throughput would be expected to be very close. However, all of these protocols always increase the output fidelity, so they all perform successful entanglement distillation in the considered noise regime.

4.2 Ideal states, noisy gates

Similarly to section 4.1 the success probability and output fidelity were evaluated, but this time for noisy quantum gates and perfect initial states. This test the damage of the protocols to existing high-fidelity states, which is relevant when considering iterating the protocols. Figure 5 illustrates the obtained results.



(a) (b)

Figure 5: (a) Average success probability with perfect gate fidelity and varying entangled state noise. (b) Average output state fidelity with perfect initial states but varying gate fidelity.

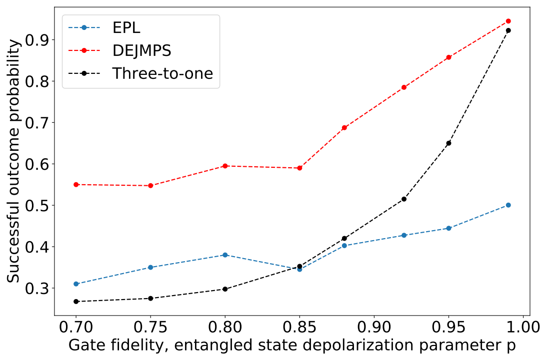
One of the main issues with current quantum devices is their limited size that prevents the allocations of qubits to error correction. As a result, the more noisy operations are performed, the less the results will be coherent. In the case of quantum distillation a similar pattern can be reasoned, namely that more operations will induce further sources of noise, but more EPR pairs to distill from could compensate for that noise.

The results in part (a) demonstrate that the throughput of the DEJMPS and especially Three-to-one protocols is highly susceptible to gate noise, while EPL is less sensitive. Part (b) shows that the EPL protocol produces the highest quality states under increasing levels of gate noise, followed by the Three-to-one protocol.

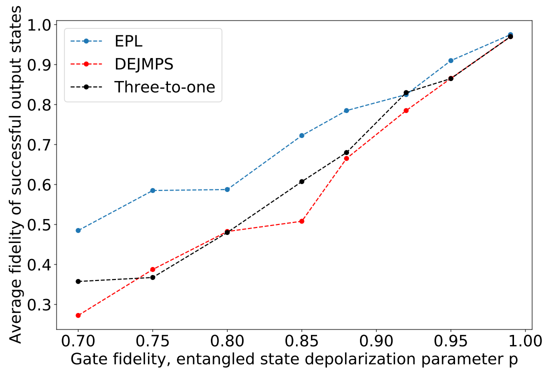
As previously addressed, EPL utilises the least amount of gates out of the studied protocols and is thus less subjected to noise. Conversely, the Three-to-one protocol has the highest amount of gates, and could be expected to perform worse in this scenario. The simulations confirm this, with the success probability already dropping to the level of the EPL protocol at a gate fidelity of 0.8. DEJMPS has again clearly the highest success probability, but lowest fidelity. Since the initial state was perfect, none of these processes can increase the output state fidelity. We can however see that the DEJMPS protocol will only be useful for a very high gate fidelity, as otherwise it will not be able to increase the output fidelity over the initial value.

4.3 Noisy states and noisy gates

Figure 6 shows the performance of the protocols for the case of both noise sources being relevant, modeled here by equating the gate fidelity $(1 - p_U)$ and depolarizing channel parameter p.



(a)



(b)

Figure 6: (a) Average success probability.(b) Average output state fidelity.

The main observation is that the quantum distillation protocols do not improve the the fidelity of the original EPR states under these noise levels. As discussed in section 4.2 this is due to the imperfect gate operations. In terms of success probability, the EPL protocol again is statistically limited to $\frac{1}{2}$, and the DEJMPS protocol is the most reliable. However, EPL generates the highest output state fidelities, especially for stronger noise, and hence is the closest to successful purification. DEJMPS has a slightly lower output fidelity, again due to the stronger fidelity dependence on gate noise. The Three-to-one protocol also falls short of successful entanglement distillation, with slightly higher output fidelities than DEJMPS, but considerably lower success probability. Considering section 4.2, it is to be expected that the Three-to-one and EPL protocols will be able to perform successful purification for a higher gate noise level than the DEJMPS protocol, which would need even higher fidelity gates but would have a considerably higher throughput.

5 Conclusion

We have investigated the influence of noise on three protocols. We have shown that the EPL protocol performs better than its competitors in scenarios with high noise from both sources, while being outperformed by the DEJMPS and Three-to-one protocols for lower gate noise levels. The Three-to-one protocol produces a very high fidelity in the case of ideal gates but high initial entangled state noise, while performing worse than the other protocols when both noise sources are strong. Paired with the additional time needed to generate two more qubits, this makes its application mainly suitable for scenarios with high gate fidelity and a priority on few iterations. The DEJMPS protocol always gives the highest success probability for every scenario, and thus has the highest throughput, but produces comparatively low-quality output states and would thus have to be iterated for most applications, making it undesirable for devices with limited memory and high fidelity requirement. In conclusion, the three protocols have all been shown to perform entanglement purification for ideal gates, but to fail for the considered case of $1 - p = p_u$. The development of high-fidelity gates thus is of the highest importance to successfully perform entanglement purification in the real world.

References

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