R404: Advanced Estimation Techniques

Topic: Bayesian Methods in Econometrics

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Lecture Contents

Review: the philosophy of the Bayesian approach

A classical estimation example

- Consider a sample of iid observations $x_1, \ldots, x_n$ coming from a random variable $\xi \sim N(\mu, \sigma^2)$.
- We are interested in obtaining an estimate of the mean.
- A standard approach would be to use the method of maximum likelihood (MML).
- We construct the likelihood function:

$$\begin{aligned} L(x_1, \ldots, x_n | \mu, \sigma^2) &= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma^2}} \\ &= \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} e^{-\frac{1}{2}\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{\sigma^2}}. \end{aligned} \tag{1}$$

A classical estimation example

- According to the MML, we maximize L w.r.t. μ.
- It is well-known (or, if your recollections are hazy, you can derive it) that the solution is given by the estimator $\hat{\mu} = \sum_{i=1}^{n} x_i / n$, i.e. the sample mean.
- By definition, the statistic $\hat{\mu}$ is a random variable.
- Consequently, if we keep repeating the experiment and regenerating the n observations, we'll obtain new samples $\tilde{x}_1, \ldots, \tilde{x}_n, \tilde{\tilde{x}}_1, \ldots, \tilde{\tilde{x}}_n$ etc. and therefore new values of $\hat{\mu}$.
- For each of those samples the corresponding value $\hat{\mu}$ will be our estimate of the unknown parameter μ.

A classical estimation example

- When we speak of the statistical properties of $\hat{\mu}$ like unbiasedness or consistency, we are implicitly referring to an ability to repeat the experiment many times or to extend the sample size n within an experiment.
- In this context, any probabilistic reasoning about $\hat{\mu}$ is based on a *classical* notion of probability as the theoretical limit of the ratio of occurrences of an event to the total number of trials (i.e. the relative frequency).
- It can be argued that this notion of probability is "objective" – it derives from an experiment and reflects mechanisms external to an observer.
- At the same time it is operational only when repeatability is ensured.
  - A football player is allowed to shoot one penalty but misses. Does it matter that he is the best scorer in his team?

Subjective probability

- The idea of probability is often used in contexts where the classical interpretation as the limit of the relative frequency is not applicable.
- Consider a person making the following statement:

  The probability that there is life on Mars is 1/1000000.

- What is this person trying to say?
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Readings

Additional readings: