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- :
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-
-

:

$$\max_{\{c_t\}} \sum_{t=0}^{\infty} \beta^t \ln c_t$$

$$k_{t+1} = (1-\delta)k_t + y_t - c_t, \quad k_0 > 0 -$$

$$y_t = Ak_t^{\alpha}, \quad 0 < \underline{\epsilon} \leq c_t \leq \bar{\epsilon}, \quad \alpha, \beta \in (0,1), \quad A > 0$$

:

$$\sum_{t=0}^{\infty} \beta^t \underbrace{\ln((1-\delta)k_t + Ak_t^{\alpha} - k_{t+1})}_{=F(x_t, x_{t+1})}.$$

:

$$F_y(x_t, x_{t+1}) + \beta F_x(x_{t+1}, x_{t+2}) = 0$$

:

$$\frac{-1}{(1-\delta)k_t + Ak_t^{\alpha} - k_{t+1}} + \beta\frac{(1-\delta) + \alpha Ak_{t+1}^{\alpha-1}}{(1-\delta)k_{t+1} + Ak_{t+1}^{\alpha} - k_{t+2}} = 0$$

$$G(x_{t+2}, x_{t+1}, x_t) = 0,$$

..   II .

:

$$\frac{-1}{(1-\delta)k_t + Ak_t^{\alpha} - k_{t+1}} + \beta \frac{(1-\delta) + \alpha A k_{t+1}^{\alpha-1}}{(1-\delta)k_{t+1} + Ak_{t+1}^{\alpha} - k_{t+2}} = 0$$

$$G(x_{t+2}, x_{t+1}, x_t) = 0,$$

.. II .

: $\beta = 0.95,\ A = 1,\ \delta = 0.05,\ \alpha = 0.5\ \ k_0 = 2?$

:

$$\frac{-1}{(1-\delta)k_t + Ak_t^{\alpha} - k_{t+1}} + \beta\frac{(1-\delta) + \alpha A k_{t+1}^{\alpha-1}}{(1-\delta)k_{t+1} + Ak_{t+1}^{\alpha} - k_{t+2}} = 0$$

$$G(x_{t+2}, x_{t+1}, x_t) = 0,$$

.. II .

: $\beta = 0.95,\ A = 1,\ \delta = 0.05,\ \alpha = 0.5$ $k_0 = 2$? , ?

$X \subset \mathbb{R}^n$ – , $x = (x^1, \dots, x^n)$.

, $\forall x \in X,\ \exists U(x) \subset \mathbb{R}^m, U(x) \neq \emptyset$. $u = (u^1, \dots, u^m)$.

(): $f^0(x, u)$ $x \in X$, $u \in U(x)$

:

(1) $$x_{t+1} = f(x_t, u_t), \quad x_0 -$$

$f(x, u)$, X, $x \in X$, $u \in U(x)$.

!

(2)

$$\mathbf{u} = \{u_t\},\ t = 0, 1, 2, \ldots, \quad \textbf{(??)} \qquad \{x_{t+1}\},\ t = 0, 1, 2, \ldots,$$

$$j(x_0, \mathbf{u}) = \sum_{t=0}^{\infty} \beta^t f^0(x_t, u_t) \tag{2}$$

$$v(x_0) = \sup_{\mathbf{u}} j(x_0, \mathbf{u}). \tag{3}$$

.

$\beta \in (0,1)$.

$\mathrm{FC}(x_0)$ $\{u_t\}_{t=0}^{\infty}$ $x_0 \in X$, .. x_{t+1} (**??**) $u_t \in U(x_t)$, $t = 0, 1, 2, \ldots,$ x_0.

$\{x_{t+1}^*, u_t^*\}$, $t = 0, 1, 2, \ldots$, .. $\{u_t^*\} \in \mathrm{FC}(x_0)$,

$$v(x_0) = j(x_0, \mathbf{u}^*), \ \mathbf{u}^* = \{u_t^*\}.$$

:

1.

$$\mathcal{L}(x_1, x_2, \ldots, u_0, u_1, \ldots) = \sum_{t=0}^{\infty} \beta^t \left[f^0(x_t, u_t) + \lambda_t' \cdot [f(x_t, u_t) - x_{t+1}] \right]$$

$\lambda_t = (\lambda_t^1, \ldots, \lambda_t^n),\ t = 0, 1, 2, \ldots,$ $(\cdot)$ ( ):

$$\lambda_t' \cdot [f(x_t, u_t) - x_{t+1}] = \sum_{i=1}^{n} \lambda_t^i [f^i(x_t, u_t) - x_{t+1}^i].$$

(2)

2. , $\mathcal{L}$ x_t u_t, I :

$$
(4)\quad
\begin{aligned}
&\beta\left[f^0_{x^k_t}(x_t,u_t)+\sum_{i=1}^{n}\lambda^i_t f^i_{x^k_t}(x_t,u_t)\right]=\lambda^k_{t-1},\ k=1,\dots,n,\\
&f^0_{u^j_t}(x_t,u_t)+\sum_{i=1}^{n}\lambda^i_t f^i_{u^j_t}(x_t,u_t)=0,\ j=1,\dots,m.
\end{aligned}
$$

3. **(??)** **(??)** (-) $\{u_t\}_{t=0}^{\infty}$, -, $\{x_{t+1},u_t\}_{t=0}^{\infty}$. (**(??)** **(??)** .)

(3)

:

$$\mathcal{L}_x = \beta^t f_x^0(x_t, u_t) + \beta^t f_x'(x_t, u_t) \cdot \lambda_t - \beta^{t-1}\lambda_{t-1} = 0 \Rightarrow$$

(5) $$\beta(f_x^0(x_t, u_t) + f_x'(x_t, u_t) \cdot \lambda_t) = \lambda_{t-1}.$$

$$\mathcal{L}_u = \beta^t f_u^0(x_t, u_t) + \beta^t f_u'(x_t, u_t) \cdot \lambda_t = 0 \Rightarrow$$

(6) $$f_u^0(x_t, u_t) + f_u'(x_t, u_t) \cdot \lambda_t = 0$$

?

- :

$$(7) \qquad v(x) = \sup_{u \in U(x)} \left[f^0(x,u) + \beta v(f(x,u)) \right].$$

(??) $U(x)$. $u = \nu(x)$ - , .

$$(8) \qquad v(x) = f^0(x,\nu(x)) + \beta v(f(x,\nu(x))).$$

,

$$(9) \qquad f_u^0(x,\nu(x)) + \beta f_u'(x,\nu(x)) \cdot v_x(f(x,\nu(x))) = 0.$$

(**??**) x,

$$\begin{aligned} v_x(x) =& f_x^0(x,\nu(x)) + \nu_x'(x) \cdot f_u^0(x,\nu(x)) + \\ & \beta \left[f_x'(x,\nu(x)) + \nu_x'(x) \cdot f_u'(x,\nu(x)) \right] \cdot v_x(f(x,\nu(x))) \\ =& f_x^0(x,\nu(x)) + \beta f_x'(x,\nu(x)) \cdot v_x(f(x,\nu(x))) + \\ & \underbrace{\nu_x'(x) \cdot f_u^0(x,\nu(x)) + \beta \nu_x'(x) \cdot f_u'(x,\nu(x)) \cdot v_x(f(x,\nu(x)))}_{=0 \ \ (\textbf{??})}. \end{aligned}$$

(10) $$v_x(x) = f_x^0(x, \nu(x)) + \beta f_x'(x, \nu(x)) \cdot v_x(f(x, \nu(x))).$$

$x = x_t^*$ $u_t^* = \nu(x_t^*)$, **(??)** **(??)**

(11) $$f_u^0(x_t^*, u_t^*) + \beta f_u'(x_t^*, u_t^*) \cdot v_x(x_{t+1}^*) = 0,$$

(12) $$v_x(x_t^*) = f_x^0(x_t^*, u_t^*) + \beta f_x'(x_t^*, u_t^*) \cdot v_x(x_{t+1}^*).$$

$\lambda_t := \beta v_x(x_{t+1}^*)$ **(??)** **(??)**, **(??)** **(??)**.

$\{\lambda_t\}$ $\{x^*_{t+1}, u^*_t\}$, $t = 0, 1, 2, \ldots,$ (**??**) (**??**).

1. $f^0(x, u)$ $f(x, u)$ (x, u);
2. $\lambda^1_t, \ldots, \lambda^n_t$, $t = 0, 1, 2, \ldots$;
3. X $\mathbb{R}^n_+$

$$\lim_{T\to\infty} \beta^T \lambda'_T \cdot x^*_{T+1} = 0,$$

$\{x^*_{t+1}, u^*_t\}$ (x_0) .

4.15 Stokey-Lucas. , (**??**), , (**??**) (**??**).

$$\mathcal{L}_T(x_t, u_t) = \sum_{t=0}^{T} \beta^t \left\{ f^0(x_t, u_t) + \lambda_t' \cdot [f(x_t, u_t) - x_{t+1}] \right\}.$$

(13)

$$D := \mathcal{L}_T(x_t, u_t) - \mathcal{L}_T(x_t^*, u_t^*) = \sum_{t=0}^{T} \beta^t \lambda_t' \cdot (x_{t+1}^* - x_{t+1}) +$$

$$\sum_{t=0}^{T} \beta^t [f^0(x_t, u_t) + \lambda_t' \cdot f(x_t, u_t) - f^0(x_t^*, u_t^*) - \lambda_t' \cdot f(x_t^*, u_t^*)].$$

' '

$$\mathcal{L}_T(x_t,u_t)-\mathcal{L}_T(x_t^*,u_t^*)\ \ (=D)\ \ \le \sum_{t=0}^{T}\beta^t\lambda_t'\cdot(x_{t+1}^*-x_{t+1})+$$

$$\sum_{t=0}^{T}\beta^t[f_x^{0'}(x_t^*,u_t^*)\cdot(x_t-x_t^*)+f_u^{0'}(x_t^*,u_t^*)\cdot(u_t-u_t^*)+$$

$$\lambda_t'\cdot[f_x(x_t^*,u_t^*)\cdot(x_t-x_t^*)+f_u(x_t^*,u_t^*)\cdot(u_t-u_t^*)]]=$$

$$\sum_{t=0}^{T}\beta^t\lambda_t'\cdot(x_{t+1}^*-x_{t+1})+\sum_{t=0}^{T}\beta^t\underbrace{[f_x^{0'}(x_t^*,u_t^*)+\lambda_t'\cdot f_x(x_t^*,u_t^*)]}_{=\frac{\lambda_{t-1}'}{\beta}\ \ \textbf{(??)}}\cdot(x_t-x_t^*)$$

$$+\sum_{t=0}^{T}\beta^t\underbrace{[f_u^{0'}(x_t^*,u_t^*)+\lambda_t'\cdot f_u(x_t^*,u_t^*)]}_{=\mathbf{0}'\ \ \textbf{(??)}}\cdot(u_t-u_t^*).$$

..

$$D \le \sum_{t=0}^{T} \beta^t \lambda_t' \cdot (x_{t+1}^* - x_{t+1}) + \sum_{t=0}^{T} \beta^t \frac{\lambda_{t-1}'}{\beta} \cdot \underbrace{(x_t - x_t^*)}_{\text{N.B.: } x_0 = x_0^*} =$$

$$\sum_{t=0}^{T} \beta^t \lambda_t' \cdot (x_{t+1}^* - x_{t+1}) + \sum_{t=1}^{T} \beta^{t-1} \frac{\lambda_{t-1}'}{\beta} \cdot (x_t - x_t^*) =$$

$$\sum_{t=0}^{T} \beta^t \lambda_t' \cdot (x_{t+1}^* - x_{t+1}) + \sum_{t=0}^{T-1} \beta^t \frac{\lambda_t'}{\beta} \cdot (x_{t+1} - x_{t+1}^*) =$$

$$\underbrace{\sum_{t=0}^{T-1} \beta^t \lambda_t' \cdot (x_{t+1}^* - x_{t+1}) + \sum_{t=0}^{T-1} \beta^t \frac{\lambda_t'}{\beta} \cdot (x_{t+1} - x_{t+1}^*)}_{=0} +$$

$$\beta^T \lambda_T' \cdot (x_{T+1}^* - x_{T+1}) \underbrace{\le}_{\lambda_t, x_t \ge \mathbf{0}} \beta^T \lambda_T' \cdot x_{T+1}^* .$$

(5)

( ):

$$D \leq \beta^T \lambda_T' \cdot x_{T+1}^* \underset{T\to\infty}{\longrightarrow} 0,$$

..

$$\mathcal{L}_T(x_t^*, u_t^*) - \mathcal{L}_T(x_t, u_t) \geq 0,$$

$\{x_{t+1}^*, u_t^*\}$.