# R401: Statistical and Mathematical Foundations

Lecture 15: Convex Optimization

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# Lecture Contents

# Basic formulation with inequality constraints

We now look at a problem which is very similar to the case of optimization with equality constraints:

$$f(x_1, \dots, x_n) \to \max \tag{1}$$

s.t.

$$\begin{array}{l} g_1(x_1, \dots, x_n) \le b_1 \\ g_2(x_1, \dots, x_n) \le b_2 \\ \dots \\ g_m(x_1, \dots, x_n) \le b_m \end{array} . \tag{2}$$

In vector notation:

$$f(\mathbf{x}) \to \max$$

s.t.

$$\mathbf{g}(\mathbf{x}) \le \mathbf{b}.$$

# Basic formulation with inequality constraints

- A vector $\mathbf{x}$ satisfying the constraints (2) is called *admissible* or *feasible*.
- In some alternative (but essentially equivalent) formulations the constraints take the form $g_i(x_1, \ldots, x_n) \leq 0$ or $g_i(x_1, \ldots, x_n) \geq 0$ for $i = 1, \ldots, m$.
- The set of admissible vectors is called the *admissible (feasible) set*.
- Note that with inequality constraints the requirement $m < n$ is not necessary. Intuitively, this is because an inequality constraint is much more forgiving: think of a line vs. a half-plane.
- We focus on maximization problems here. Notice that minimizing a function $f(x)$ is equivalent to maximizing $-f(x)$, so there is no loss of generality in our choice.

# Readings

Main references:
Sydsæter et al. *Further mathematics for economic analysis*. Chapter 3.

Additional readings: