R401: Statistical and Mathematical Foundations

Lecture 14: Unconstrained Optimization. Static Optimization with Equality Constraints. Lagrange Multipliers

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General Principles and Caveats for the Optimization Module

- Emphasis on practicality over rigour
- Consequently, algorithmic approach and “recipes” rather than proofs
- Also, existence and relevant properties of various objects are often implicitly assumed
- Pathologies and mathematical peculiarities discussed only in special cases

Lecture Contents

1. Warm-up: Basic Optimization in $\mathbb{R}^1$

Warm-up: Basic Optimization in $\mathbb{R}^1$

Fact

For a differentiable function $f : \mathbb{R} \to \mathbb{R}$, a necessary condition for a local extremum is

$$f'(x) = 0.$$

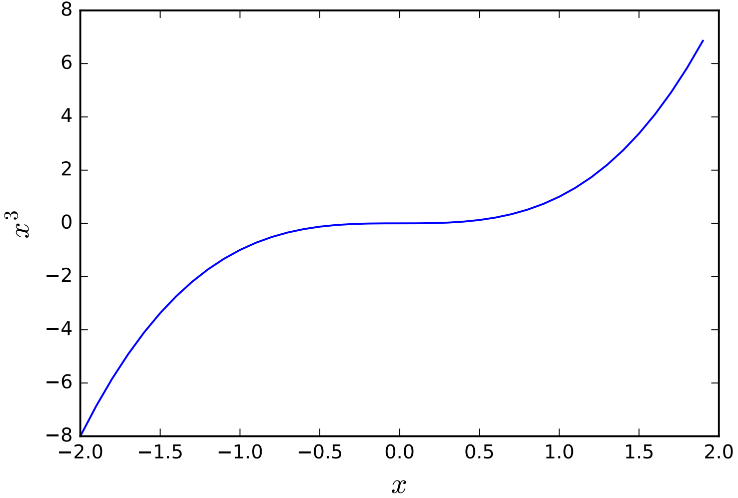
Example

If $f(x) = ax^2 + bx + c$, then $f'(x) = 2ax + b$ and the condition $f'(x) = 0$ yields the familiar $x = -\frac{b}{2a}$ (recall your high-school days). Depending on the sign of a, this is a maximum or a minimum (What is the relationship?).

Example

If $f(x) = x^3$, then $f'(x) = 3x^2$ and $f'(x) = 0 \Rightarrow x = 0$.
Does the function attain a maximum or a minimum at $x = 0$?

Warm-up: Basic Optimization in $\mathbb{R}^1$



Warm-up: Basic Optimization in $\mathbb{R}^1$

The answer is “neither”! The point $x = 0$ is not a local extremum of $f(x) = x^3$. This illustrates the pitfalls of using necessary conditions to find extrema – they supply only candidates that need to be checked further.

Recipe

1.
2.
3.