14.4 Group Work

14.4 - Tangent Planes and Linear Approximations

1. Find the equation to the tangent plane of $z = x^2 + xy + 3y^2$ at the point $1, 1, 5$.

Solution:

$$\frac{\partial z}{\partial x}(x, y) = 2x + y$$
$$\frac{\partial z}{\partial x}(1, 1) = 3$$
$$\frac{\partial z}{\partial y}(x, y) = x + 6y$$
$$\frac{\partial z}{\partial y}(1, 1) = 7$$

Which gives the equation

$$z - 5 = 3(x - 1) + 7(y - 1)$$

or equivalently

$$z = 3x + 7y - 5$$

2. Find the linearization to $f(x, y) = x^3y^4$ at the point $(1, 1)$ and use it to approximate the value of $(1.01)^3(.9)^4$.

Solution:

$$\frac{\partial f}{\partial x}(x, y) = 3x^2y^4$$
$$\frac{\partial f}{\partial x}(1, 1) = 3$$
$$\frac{\partial f}{\partial y}(x, y) = 4x^3y^3$$
$$\frac{\partial f}{\partial xy}(1, 1) = 4$$

So the linearization is given by

$$\begin{aligned} L(x, y) &= f(1, 1) + f_x(1, 1)(x - 1) + f_y(1, 1)(y - 1) \\ &= 1 + 3(x - 1) + 4(y - 1) \\ &= 3x + 4y - 6 \end{aligned}$$

Then we can approximate

$$\begin{aligned}(1.01)^3(0.9)^4 &\approxeq L(1.01, 0.9)\\ &= 3(1.01) + 4(.9) - 6\\ &= 3.03 + 3.6 - 6\\ &= 0.63\end{aligned}$$

If you use an expensive calculator to estimate the value you get $(1.01)^3(0.9)^4 \approxeq 0.6300000000000008$.

3. Find the linearization to

$$f(x, y) = \sqrt{x + y} + (y)^4$$

at the point $(3, 1)$ and use this to estimate the value of $\sqrt{4.25} + (.75)^4$.

Solution:

$$\begin{aligned}f(1, 1) &= \sqrt{3 + 1} + (1)^4 = 2 + 1 = 3\\ f_x(x, y) &= \frac{1}{2\sqrt{x + y}} \cdot \frac{\partial f}{\partial x}(x + y) = \frac{1}{2\sqrt{x + y}}\\ f_x(3, 1) &= \frac{1}{2\sqrt{3 + 1}} = \frac{1}{4}\\ f_y(x, y) &= \frac{1}{2\sqrt{x + y}}\\ f_y(3, 1) &= \frac{1}{2\sqrt{3 + 1}} = \frac{1}{4}\end{aligned}$$

$$\begin{aligned}L(x, y) &= f(1, 1) + f_x(3, 1)(x - 3) + f_y(3, 1)(y - 1)\\ &= 3 + \frac{1}{4}(x - 3) + \frac{1}{4}(y - 1)\end{aligned}$$

4. If $f(x, y)$ is differentiable with $f(2, 5) = 6$, $f_x(2, 5) = 1$, and $f_y(2, 5) = -1$, estimate the value of $f(2.2, 4.9)$.

Solution:

5.