15.4 Double Integrals in Polar Coordinates

Useful Information.

$$\iint_R f(x,y)\ dA = \int_\alpha^\beta \int_a^b \ f(r\cos\theta, r\sin\theta)\ r\ dr\ d\theta$$

1. Find $\iint_R \ \sin(x^2+y^2)\ dA$ where R is the region in the first quadrant between the circles centered at the origin and radii 1 and 3.

2. Find the area inside one loop of the rose $r = \cos 3\theta$

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3. Find the volume above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2 = 1$.

4. Evaluate $\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}} \sin\left(x^2+y^2\right)\ dy\ dx$ by converting to polor coordinates.

5. Use polar coordinates to combine the sum

$$\int_{1/\sqrt{2}}^{1}\int_{\sqrt{1-x^2}}^{x} xy\ dy\ dx + \int_{1}^{\sqrt{2}}\int_{0}^{x} xy\ dy\ dx + \int_{\sqrt{2}}^{2}\int_{0}^{\sqrt{4-x^2}} xy\ dy\ dx$$

into a single double integral. Then evaluate the integral.