15.9 Spherical Coord. & 16.5 Curl/Divergence

Spherical Coordinates

$$\boxed{x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad z = \rho\cos\phi}$$

$$\boxed{\rho^2 = x^2 + y^2 + z^2}$$

1. Convert the following, given in rectangular coordinates, to spherical coordinates
   (a) The point $(0, -2, 0)$

   (b) $z^2 = x^2 + y^2$

   (c) $x^2 + z^2 = 9$

2. Describe, sketch, or identify the surface given in spherical coordinates
   (a) $\phi = \pi/3$

   (b) $\rho = 2$

   (c) $\theta = \pi/2$

   (d) $\rho = \sin\theta \sin\phi$ (hint: try converting to rectangular coordinates using ρ^2)

3. Evaluate $\iiint_B (x^2 + y^2 + z^2)^2 \, dV$ where B is a ball centered at the origin with radius 5.

4. Find the volume of the solid that lies above the cone $\phi = \pi/3$ and below the sphere $\rho = 4\cos\phi$.

Curl and Divergence

If $\boldsymbol{F} = \langle P, Q, R \rangle$,

$$\boxed{\text{curl}\boldsymbol{F} = \nabla \times \boldsymbol{F} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}}$$

$$\boxed{\text{div}\boldsymbol{F} = \nabla \cdot \boldsymbol{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}}$$

5. Find the curl and divergence of each vector field
   (a) $\boldsymbol{F}(x, y, z) = (x + yz)\boldsymbol{i} + (y + xz)\boldsymbol{j} + (z + xy)\boldsymbol{k}$

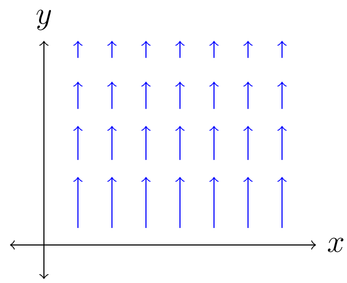
   (b) $\boldsymbol{F}(x, y, z) = \langle e^x \sin y, e^y \sin z, e^z \sin x \rangle$

6. Determine if the given vector field is conservative. If it is, find a function f so that $\boldsymbol{F} = \nabla f$.
   (a) $\boldsymbol{F}(x, y, z) = \langle y^2 z^3, 2xyz^3, 3xy^2 z^2 \rangle$

   (b) $\boldsymbol{F}(x, y, z) = \langle 3xy^2 z^2, 2x^2 yz^3, 3x^2 y^2 z^2 \rangle$

   (c) $\boldsymbol{F}(x, y, z) = \boldsymbol{i} + \sin z\boldsymbol{j} + y \cos z\boldsymbol{k}$

7. The vector field below is shown in the xy-plane and looks the same in every parallel plane, i.e. every plane of the form $z = k$.



(a) Is div$\boldsymbol{F}$ positive, negative, or zero?

(b) Explain why curl$\boldsymbol{F} = 0$.