15.8 Triple Integrals in Cylindrical Coordinates

1. Sketch or describe the surface given (all coordinates are cylindrical):
   (a) $r = 5$

   (b) $\theta = 0$

   (c) $\theta = \pi/4$

   (d) $z = 4 - r^2$

   (e) $0 \leq r \leq 2$, $-\pi/2 \leq \theta \leq \pi/2$, $0 \leq z \leq 1$.

2. Convert the following (given in rectangular coordinates) to cylindrical coordinates:
   (a) The point $(-1, 1, 1)$

   (b) The point $(2, -\pi/2, 1)$

   (c) $z = x^2 - y^2$

   (d) $x^2 - x + y^2 + z^2 = 1$

3. Sketch the surface whose volume is given by the integral

$$\int_{-\pi/2}^{\pi/2}\int_{0}^{2}\int_{0}^{r^2} r\ dz\ dr\ d\theta.$$

What is the volume of this surface?

4. Use cylindrical coordinates to evaluate the following:

(a) $\iiint_E (x+y+z)\ dV$ where E is the solid in the first octant that lies under the parabaloid $z = 4-x^2-y^2$.

(b) $\iiint_E \sqrt{x^2+y^2}\ dV$ where E is the region that lies inside the cylinder $x^2+y^2 = 16$ and between the planes $z = -5$ and $z = 4$.

5. Evaluate by changing to cylindrical coordinates:

$$\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{2} xz\ dz\ dx\ dy.$$