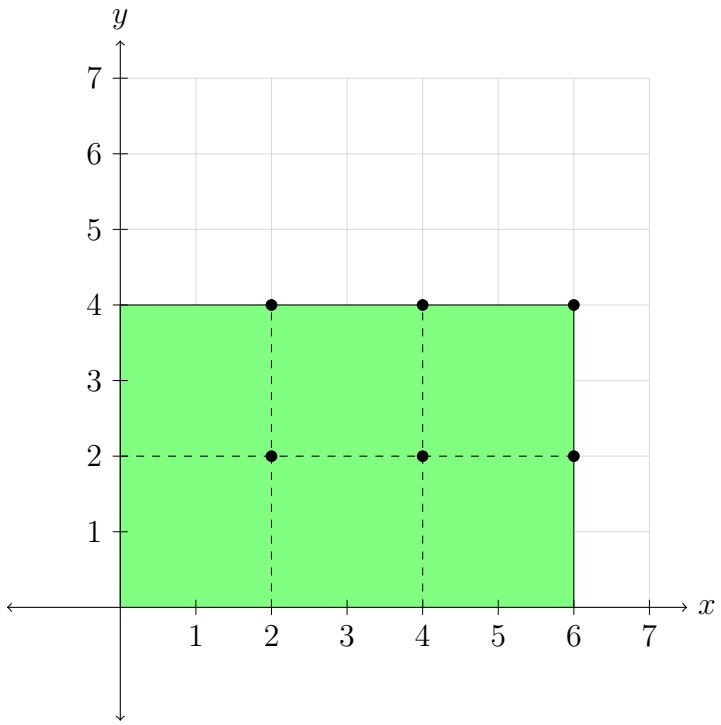
15.1 - Double Integrals over Rectangles

1. Extimate the volume of the solid that lies below the surface $z = xy$ and above the rectangle contained in the xy-plane $R = [0, 6] \times [0, 4]$. Use $\Delta x = 2$ and $\Delta y = 2$ and each sample point (x_k, y_k) is the upper right corner of the kth rectangle.

   R is drawn to the right with dashed lines representing the appropriate rectangles and the sample points are marked.



2. Estimate $\int\int_R \sin(x + y)\, dA$ where $R = [0, \pi] \times [0, \pi]$ and $\Delta x = \Delta y = \pi/2$ with sample points at the lower left corners of each rectangle.

3. Evaluate each double integral by identifying it as the volume of a solid and using a known formula

   (a) $\int\int_R 3\ dA$ where $R = \{(x, y) \mid -2 \leq x \leq 2, 1 \leq y \leq 6\}$

   (b) $\int\int_R (4 - 2y)\ dA$, $R = [0, 1] \times [0, 1]$

   Hint: First draw $z = 4 - 2y$ in the yz-plane and then use that to visualize what the portion of the plane $z = 4 - 2y$ looks like over the given region.