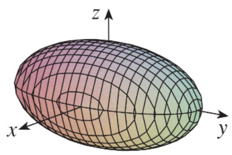
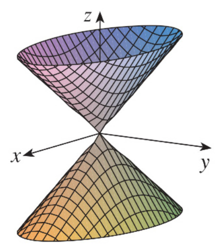
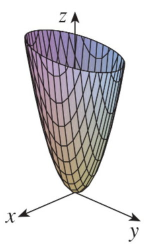
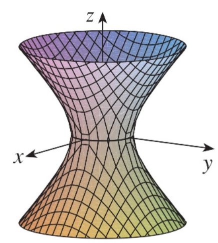
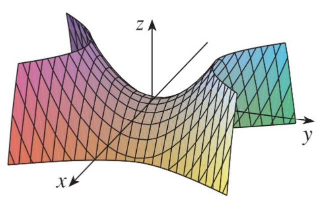
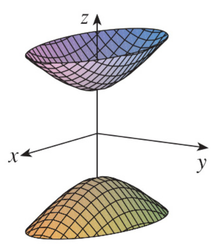
12-6: Cylinders and Quadric Surfaces

TABLE 1 Graphs of quadric surfaces

Surface	Equation	Surface	Equation
Ellipsoid 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ All traces are ellipses. If $a = b = c$, the ellipsoid is a sphere.	Cone 	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$.
Elliptic Paraboloid 	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.	Hyperboloid of One Sheet 	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.
Hyperbolic Paraboloid 	$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where $c < 0$ is illustrated.	Hyperboloid of Two Sheets 	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ Horizontal traces in $z = k$ are ellipses if $k > c$ or $k < -c$. Vertical traces are hyperbolas. The two minus signs indicate two sheets.

From Stewart's Calculus 7E

1. Classify each curve as a: Cylinder, Ellipsoid, Elliptical Paraboloid, Elliptical Cone, Hyperboloid of one sheet, Hyperboloid of two sheets, or a Hyperbolic paraboloid.

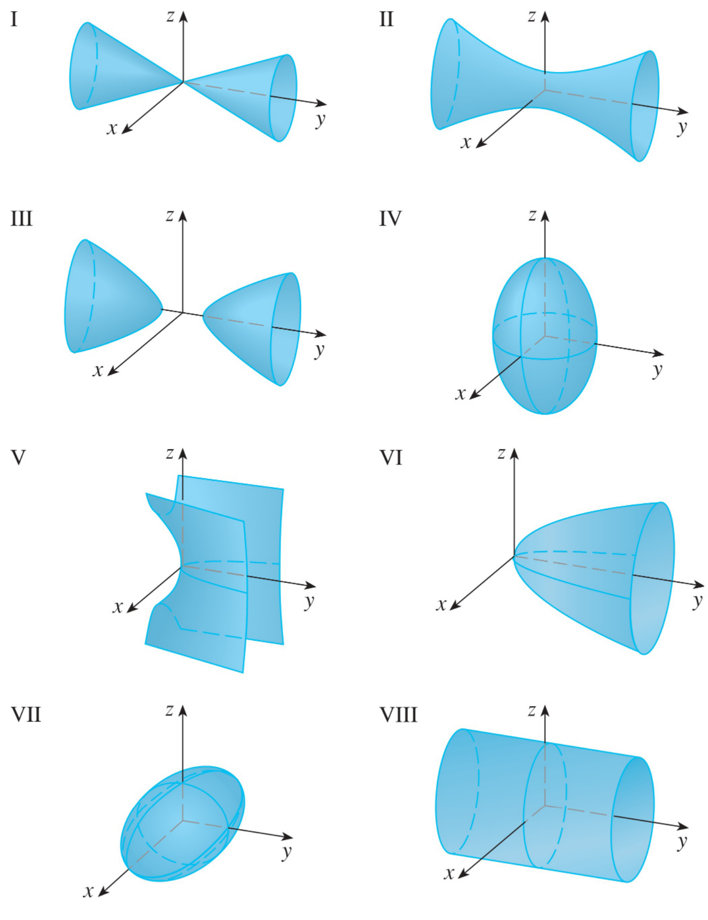
   Draw a sketch demonstrating how each surface is oriented in 3-space.

   (a) $x = y^2 + 4z^2$

(b) $x^2 = y^2 + 4z^2$

(c) $9x^2 - y^2 + z^2 = 0$

(d) $-x^2 + y^2 - z^2 = 1$



From Stewart's Calculus 7e

2. Idenfity the shapes above with the equation that represents them.

(a) $x^2 + 4y^2 + 9z^2 = 1$

(b) $9x^2 + 4y^2 + z^2 = 1$

(c) $x^2 - y^2 + z^2 = 1$

(d) $-x^2 + y^2 - z^2 = 1$

(e) $y = 2x^2 + z^2$

(f) $y^2 = x^2 + 2z^2$

(g) $x^2 + 2z^2 = 1$

(h) $y = x^2 - z^2$