Multivariable Functions

MTH 234 - Summer 2021

Learning Objectives

- Define and graph multivariable functions via level curves.

Functions of Two Variables

Definition

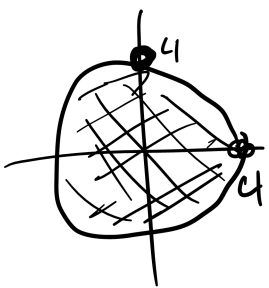
- A func. of two variables assigns a pair of real numbers (x,y) to each point in a set D. We denote this by $f(x,y)$ or $z = f(x,y)$.
- The graph of f is all points $(x,y,z) \in \mathbb{R}^3$ with $z = f(x,y)$ and $(x,y) \in D$.

* Domain and range as expected. Domain = D, range = $\{f(x,y) \mid (x,y) \in D\}$
* (x,y) independent, z dependent

Graphing Functions of Two Variables

Consider $f(x,y) = \sqrt{16 - x^2 - y^2}$. First, sketch the domain of f. Second, graph $z = f(x,y)$ using traces of $x = 0$, $y = 0$, and $z = 0$.

Just need $16 - x^2 - y^2 \geq 0$ or $16 \geq x^2 + y^2$. Circle of radius ≤ 4:



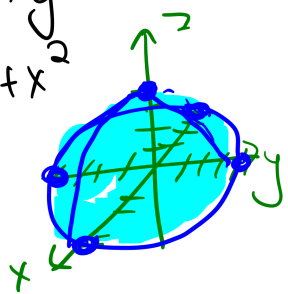
$z = f(x,y) \Leftrightarrow z^2 = 16 - x^2 - y^2$, $z \geq 0$

$z = 0 \Leftrightarrow 16 = x^2 + y^2$

$x = 0 \Leftrightarrow 16 = z^2 + y^2$

$y = 0 \Leftrightarrow 16 = z^2 + x^2$

* Mathematica if time.



Level Curves

Definition

- The level curve of a function are curves given by $f(x,y)=k$

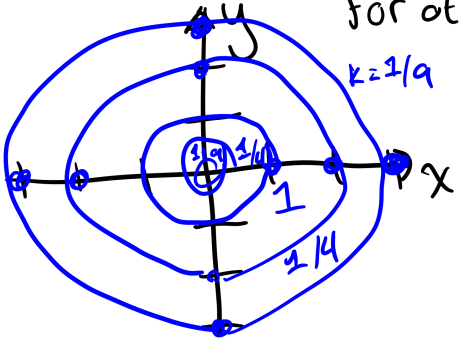
*Mathematica

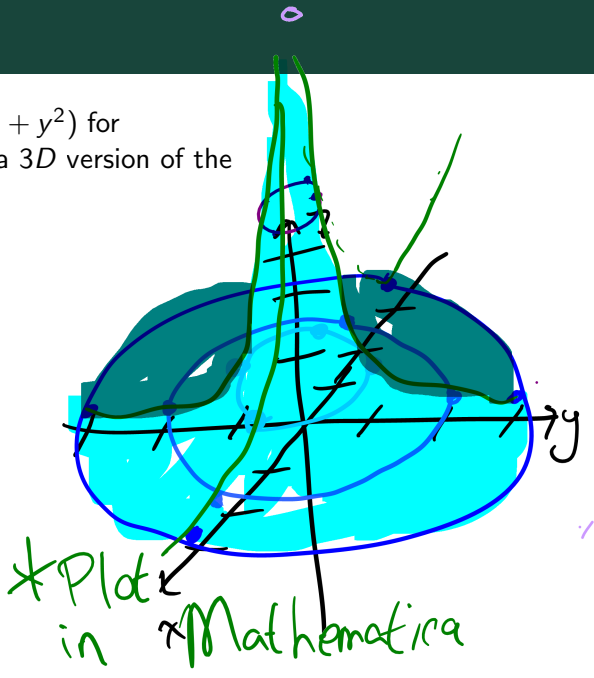
Level Curves Example

Sketch the level curves of $f(x, y) = 1/(x^2 + y^2)$ for $k = 1/9, 1/4, 1, 4, 9$. Use these to sketch a $3D$ version of the graph.

$1/9 = \frac{1}{(x^2+y^2)} \Leftrightarrow 9 = x^2 + y^2$

similar for others





Questions

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