15.10

Useful Information.

- The **Jacobian** for a tranformation T given by $x = x(u, v)$ and $y = y(u, v)$ is

$$\frac{(x, y)}{(x, y)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u} \cdot \frac{\partial y}{\partial v} - \frac{\partial x}{\partial v} \cdot \frac{\partial y}{\partial u}$$

- $$\iint_R f(x, y)\ dA = \iint_S f(x(u, v), y(u, v)) \left| \frac{(u, v)}{(u, v)} \right|\ du\ dv$$

1. Let S be the square $\{(u, v)\ :\ 0 \leq u \leq 1, 0 \leq v \leq 1\}$ and let T be the transformation

$$x = v,\ \ y = u(1 + v^2)$$

   (a) Let L_1,L_2,L_3, and L_4 denote the left, bottom, right, and top sides of S respectively. L_1 is the line $u = 0$ and $0 \leq v \leq 1$. So on L_1, $x = v$ and $y = 0$. So

$$L_1 \mapsto y = 1, \quad \text{with} 0 \leq x \leq 1$$

   Express the image of L_2, L_3, and L_4 similarly.

   (b) Sketch the image of S under the transformation given.

2. Repeat the above instructions for S the triangular region of the uv-plane with vertices $(0, 0)$, $(1, 1)$, and $(0, 1)$ with the transformation

$$x = u^2, y = v.$$

3. Let R be the parallelogram with vertices $(0,0),(4,3),(2,4),(-2,1)$. Let S be the square $[0,1]\times[0,1]$. Find a transformation that maps S onto R.

   Suggestion: Experiment and try stuff. What points in S are sent to the corners of R?

4. Evaluate the integral
$$\iint_R e^{(x+y)/(x-y)}\, dA$$
   where R is the trapezoidal region with vertices $(1,0),(2,0),(0,-2),(0,-1)$.

   (a) Since this is not easy as written, we want to do a change of variables. Based on the given function, we will try
$$u = x+y, \quad v = x-y.$$
   Then we want to use the transformation T given by $x = \frac{1}{2}(u+v)$ and $y =$?

   (b) Setting $u = x+y$ and $v = x-y$, what is the image of the trapezoidal region given?

   (c) Evaluate the integral.

5. Evaluate $\iint_X xy\ dA$ where R is the region in the first quadrant bounded by

$$y = x, \quad y = 3x, \quad xy = 1, \quad xy = 3.$$

using the transformation $x = u/v, y = v$.

(a) Complete the following, determining the image of each line or curve:

- $$y = x \quad \mapsto \quad v^2 = u$$
- $$y = 3x \quad \mapsto$$
- $$xy = 1 \quad \mapsto \quad u = 1$$
- $$xy = 3 \quad \mapsto$$

(b) Rewrite the original double integral using the given transformation in the formr

$$\int_a^b \int_c^d f(u, v)\ dv\ du$$

What are the values of a, b, c, d and $f(u, v)$?

(c) Evaluate the integral.