

15.2 Iterated Integrals

1. Evaluate the following

(a) $\int_1^4 \int_0^2 (6x^2y - 2x) \, dy \, dx$

(b) $\int_0^1 \int_0^1 v(u + v^2)^4 \, du \, dv$

(c) $\int_1^4 \int_1^2 \left(\frac{x}{y} + \frac{y}{x} \right) \, dy \, dx$

(d) $\iint_R ye^{-xy} \, dA$, where $R = [0, 2] \times [0, 3]$

2. If $\int_0^1 f(x, y) \, dy = 4x$ evaluate

$$\int_0^1 \int_2^3 x^2 f(x, y) \, dx \, dy$$

3. Find the volume of the solid that lies under the plane $4x + 6y - 2z + 15 = 0$ and above the rectangle $R = \{(x, y) \mid -1 \leq x \leq 2, -1 \leq y \leq 1\}$.

4. Find the volume of the solid in the first octant bounded by the cylinder $z = 16 - x^2$ and the plane $y = 5$.

5. Find the average value of $f(x, y) = x^2 y$ over the rectangle with vertices $(-1, 0), (-1, 5), (1, 5), (1, 0)$.