Partial Derivatives

MTH 234 - Summer 2021

Learning Objectives

- Calculate partial derivatives of multivariable functions.
- State and apply Clairaut's Theorem.

Partial Derivatives

Definition

- The partial derivative of f w.r.t. x at (a,b) is:

$$f_x(a,b) = \lim_{h \to 0} \frac{f(a+h,b) - f(a,b)}{h}$$

$$f_x = \frac{\partial f}{\partial x} = D_x f = \frac{\partial}{\partial x} f(x,y) \text{ etc.}$$

* We do y as expected

* Just pretend y is constant then take derivate w.r.t. x.

Partial Derivatives Example 1

Consider $f(x, y) = x^2y + \sin(x, y)$. *

Find $f_x(x, y)$.

$$f_x(x,y) = \frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial x}\sin(xy)$$

$$= 2xy + y\cos(xy)$$

Find $\frac{\partial f}{\partial \cancel{x}}$ y

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(x^2y) + \frac{\partial}{\partial y}\sin(xy)$$

$$= x^2 + x\cos(xy)$$

Not equal :o

Second Partial Derivatives

Definition

- $(f_x)_x = f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2}$
- $(f_x)_y = f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x}$

Similarly for y and f_{yx}.

*Notice order flips between subscript and partial "del."

Second Partial Derivatives Example

Consider $g(x,y) = xy + x\sin(y) + \frac{y}{x}$.

Find $g_{xy}(x,y)$.

$$= \frac{\partial}{\partial y}\frac{\partial}{\partial x}\left(xy + x\sin(y) + \frac{y}{x}\right)$$

$$= \frac{\partial}{\partial y}\left(y + \sin(y) - \frac{y}{x^2}\right)$$

$$= 1 + \cos(y) - \frac{1}{x^2}$$

Find $\frac{\partial^2 g}{\partial x \partial y}$.

$$\frac{\partial}{\partial x}\frac{\partial}{\partial y}\left(xy + x\sin(y) + \frac{y}{x}\right)$$

$$= \frac{\partial}{\partial x}\left(x + x\cos(y) + \frac{1}{x}\right)$$

$$= 1 + \cos(y) - \frac{1}{x^2}$$

EQUAL!

$=$

Clairaut's Theorem

Theorem

- If f_{xy} and f_{yx} are continuous and defined around a point (a,b) then $f_{xy}(a,b) = f_{yx}(a,b)$.

$* \dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial^2 f}{\partial y \partial x}$ for nice enough f.

$f(x,y) = \dfrac{xy(x^2-y^2)}{x^2+y^2}$ extend to $(0,0)$ fails

Questions

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