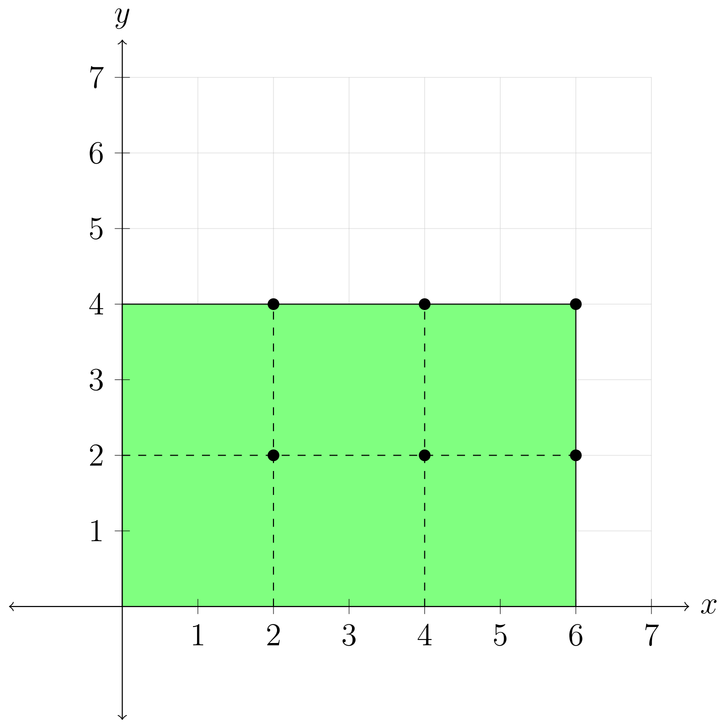
15.1 - Double Integrals over Rectangles

1. Extimate the volume of the solid that lies below the surface $z = xy$ and above the rectangle contained in the xy-plane $R = [0, 6] \times [0, 4]$. Use $\Delta x = 3$ and $\Delta y = 2$ and each sample point (x_k, y_k) is the upper right corner of the kth rectangle.

R is drawn to the right with dashed lines representing the appropriate rectangles and the sample points are marked.



Solution: Let $f(x, y) = xy$. The sample points are

$$(2, 2), (2, 4), (4, 2), (4, 4), (6, 2), \quad \text{and} \quad (6, 4).$$

Therefore

$$\begin{aligned}\iint_R xy\ dA &= f(2,2)\Delta A + f(2,4)\Delta A + f(4,2)\Delta A + f(4,4)\Delta A + f(6,2)\Delta A + f(6,4)\Delta A \\ &= 4(4) + 8(4) + 8(4) + 16(4) + 12(4) + 24(4) \\ &= 288.\end{aligned}$$

2. Estimate $\int\int_R \sin(x + y)\ dA$ where $R = [0, \pi] \times [0, \pi]$ and $\Delta x = \Delta y = 2$ with sample points at the lower left corners of each rectangle.

Solution: Again, let $f(x, y) = \sin(x + y)$. The sample points are

$$(0, 0), (\pi/2, 0), (0, \pi/2), (\pi/2, \pi/2)$$

and $\Delta A = (\pi/2)(\pi/2) = \pi^2/4$.

Then

$$\begin{aligned}\iint_R f(x,y)\ dA &= f(0,0)\Delta A + f(\pi/2,0)\Delta A + f(0,\pi/2)\Delta A + f(\pi/2,\pi/2) \\ &= 0 + \pi^2/4 + \pi^2/4 + 0 \\ &= \pi^2/2.\end{aligned}$$

3. Evaluate each double integral by identifying it as the volume of a solid and using a known formula

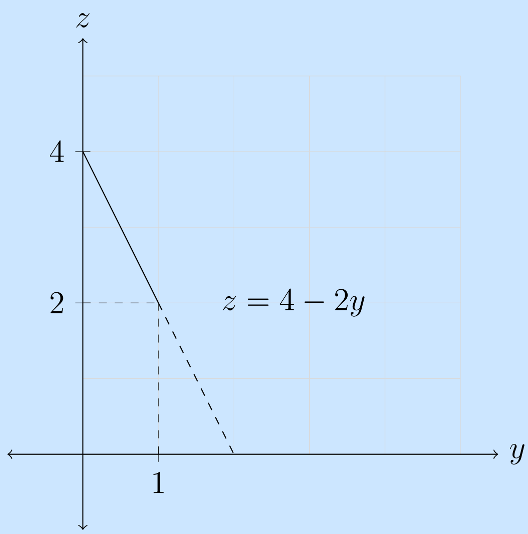
   (a) $\int\int_R 3\ dA$ where $R = \{(x, y) \mid -2 \leq x \leq 2, 1 \leq y \leq 6\}$

   Solution: This is a "box" with dimensions $4 \times 5 \times 3$ so it's volume is 60.

   (b) $\int\int_R (4 - 2y)\ dA$, $R = [0, 1] \times [0, 1]$

   Hint: First draw $z = 4 - 2y$ in the yz-plane and then use that to visualize what the portion of the plane $z = 4 - 2y$ looks like over the given region.

   The solid is the shape below 1 unit thick, which means the volume is simply the area of the shaded box and triangle.



Solution: