MTH 234 – Chapter 13

Vector Functions

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Contents

1.2 Parametrizing Curves – During Class

Objective(s):

- Parametrize curves.
- Understand why not all parametrizations are created equal.

In the book I feel like they skip quite quickly past how to parametrize curves but this because extremely important in CH16 so we need to spend some real time looking at the techniques and subtleties of parametrizing curves.

Example 1.9. Find a vector function that represents the curve of intersection of the surfaces $z = x^2y$ and $x^2 + y^2 = 4$

$t \in [0, 2\pi]$

$x = 2\cos t$
$y = 2\sin t$
$z = (2\cos t)^2 2\sin t$
$= 8\cos^2 t \sin t$

$r(t) = \langle 2\cos t, 2\sin t, 8\cos^2 t \sin t \rangle$

Example 1.10. Suppose I want to find vector function that represents the curve $y = x^2 + 2x + 1$, $x \in [0, 2]$ (note we are in $\mathbb{R}^2$). Which parametrization is best to start with and why?

(i) $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$

(ii) $\mathbf{r}(t) = \langle t, \star \rangle$

(iii) $\mathbf{r}(t) = \langle \star, t \rangle$

$t \in [0, 2]$

$x = t$
$y = t^2 + 2t + 1$

Then find a parametrization.

$r(t) = \langle t, t^2 + 2t + 1 \rangle$

Example 1.11. Suppose I want to find vector function that represents the curve $x = y^2 + 2y + 1$, $y \in [0, 2]$ (note we are in $\mathbb{R}^2$). Which parametrization is best to start with and why?

$y = t$ $\quad t \in [0,2]$

$x = t^2 + 2t + 1$

(i) $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$

(ii) $\mathbf{r}(t) = \langle t, \star \rangle$

(iii) $\mathbf{r}(t) = \langle \star, t \rangle$

$x = t$

$t = y^2 + 2y + 1$

$t = (y+1)^2$

$\sqrt{t} = y + 1$

$\pm\sqrt{t} - 1 = y$

Then find a parametrization.

Example 1.12. Suppose I want to find vector function that represents the curve $1 = x^2 + y^2$ (note we are in $\mathbb{R}^2$). Which parametrization is best to start with and why?

$\cos^2 t + \sin^2 t = 1$ $\quad x = 1\cos t$, $y = 1\sin t$ $\quad t \in [0, 2\pi]$

(i) $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$

(ii) $\mathbf{r}(t) = \langle t, \star \rangle$

$r(t) = \langle \cos t, \sin t \rangle$

(iii) $\mathbf{r}(t) = \langle \star, t \rangle$

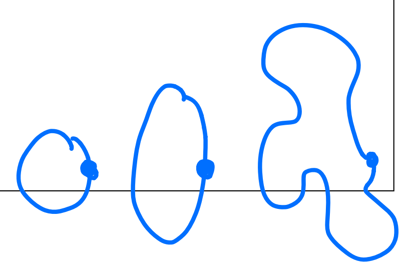
Then find a parametrization.

$y = \sin(2t)$ $\quad t \in [0, \pi]$

$x = \cos(2t)$

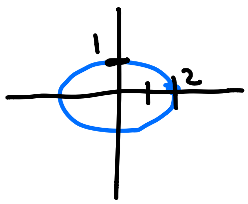
Tips to parametrizing in 2D

1. If $y = f(x)$, that is the curve passes the vertical line test then start with the parametrization $\langle t, \star \rangle$.
2. If $x = g(y)$, that is the curve passes the horizontal line test then start with the parametrization $\langle \star, t \rangle$.
3. If the curve is closed (starts and stops at same point) then start with the parametrization $\langle \cos t, \sin t \rangle$.



Note, sometimes there are still difficulties.

Example 1.13 (Skip if need time)**.** Parametrize the ellipse $\frac{x^2}{4} + y^2 = 1$ (in $\mathbb{R}^2$).



$x = 2\cos t$

$y = \sin t$

$t \in [0, 2\pi]$

$\frac{4\cos^2 t}{4} + \sin^2 t = 1$

Example 1.14 (Skip if need time)**.** Parametrize the circle $x^2 + 2x + y^2 = 8$ (in $\mathbb{R}^2$).

$x^2 + 2x + 1 + y^2 = 9$

$(x+1)^2 + y^2 = 9$

$x = 3\cos t - 1$

$y = 3\sin t$

$r(t) = \langle 3\cos t - 1, 3\sin t \rangle$

$t \in [0, 2\pi]$

Example 1.15. Just to assure you there is harder math out there the hyperbola $x^2 - y^2 = 1$ (in $\mathbb{R}^2$) is best parametrized by:

$$\mathbf{r}(t) = \langle \cosh t, \sinh t \rangle$$

$y = 4z^2 + z^4$

Example 1.16. Find a vector function that represents the curve of intersection of the two surfaces $y = 4z^2 + x^2$ and $x = z^2$

$t = z^2$

$\pm\sqrt{t} = z$

$\times \quad x = t$

$\times \quad y = t \qquad t = 4z^2 + x^2$

$t = 4z^2 + z^4$

$z = t \qquad x = t^2$

$y = 4t^2 + t^4$

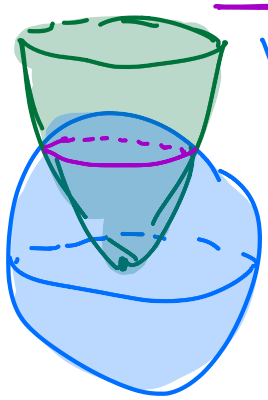
$r(t) = \langle t^2, 4t^2 + t^4, t \rangle$

$t \in (-\infty, \infty)$

Tips to parametrizing in 3D

1. If $y = f(x)$ and $z = g(x)$, that is the curve passes the $x = k$ vertical plane test then start with the parametrization $\mathbf{r}(t) = \langle t, \star, \star \rangle$.
2. If $x = f(y)$ and $z = g(y)$, that is the curve passes the $y = k$ vertical plane test then start with the parametrization $\mathbf{r}(t) = \langle \star, t, \star \rangle$.
3. If $x = f(z)$ and $y = g(z)$, that is the curve passes the $z = k$ horizontal plane test then start with the parametrization $\mathbf{r}(t) = \langle \star, \star, t \rangle$.
4. If the curve is closed (starts and stops at same point) then consider a parametrization with $\sin(t)$ and $\cos(t)$ in it.

Example 1.17. Find a vector function that represents the curve of intersection of the sphere $x^2 + y^2 + z^2 = 6$ and the paraboloid $z = x^2 + y^2$.



$\cancel{-3 = x^2 + y^2}$

$x^2 + y^2 + 4 = 6$

$2 = x^2 + y^2$

$x = \sqrt{2}\cos t$

$y = \sqrt{2}\sin t$

$z + z^2 = 6$

$z^2 + z - 6 = 0$

$(z+3)(z-2) = 0$

$\cancel{z = -3}$, $z = 2$

$r(t) = \langle \sqrt{2}\cos t, \sqrt{2}\sin t, 2 \rangle$

$t \in [0, 2\pi]$

$0 = \sin t$

$1 = 1 + \sin t$

Example 1.18. Find a vector function that represents the curve of intersection of the surfaces $z = \sqrt{x^2 + y^2}$ and $z = 1 + y$.

$1 + y = \sqrt{x^2 + y^2}$

$(1+y)^2 = x^2 + y^2$

$1 + 2y + \cancel{y^2} = x^2 + \cancel{y^2}$

$1 + 2y = x^2$

$x = \cos t$

$y = \sin t$

$z = \sqrt{\cos^2 t + \sin^2 t}$

$= \sqrt{1} = 1$

$x = t$

$1 + 2y = t^2$

$2y = t^2 - 1$

$y = \dfrac{t^2 - 1}{2}$

$z = 1 + \dfrac{t^2 - 1}{2}$

$r(t) = \left\langle t, \dfrac{t^2-1}{2}, \dfrac{t^2+1}{2} \right\rangle$ $t \in (-\infty, \infty)$

2.2 Tangent Lines and Unit Tangent Vectors – During Class

Objective(s):

- Parametrize tangent lines to vector functions.
- Define unit tangent vectors and be able to calculate it.

Calc 1 Theorem

$y - y_0 = m(x - x_0)$

$y = m(x - x_0) + y_0$

(a) $f'(a)$ is the slope of $f(x)$ at a.

(b) $L(x) = f'(a)(x - a) + f(a)$ is the linearization of $f(x)$ at a

(aka it is the tangent line to $f(x)$ through $x = a$).

Theorem 2.6.

(a) $\vec{r}\,'(a)$ is the direction (or tangent vector) of $\mathbf{r}(t)$ at a.

(b) $\mathbf{L}(t) = \vec{r}\,'(a)(t - a) + \vec{r}(a)$ is the linearization of $\mathbf{r}(t)$ at a

(aka it is the tangent line to $\mathbf{r}(t)$ through $t = a$).

$2t - t^2 = 0$

$t(2 - t) = 0$

Example 2.7. Find parametric equations for the tangent line to the curve $\mathbf{r}(t) = \langle t, e^{-t}, 2t - t^2\rangle$ through the point $(0, 1, 0)$.

$x \; y \; z$

$t = 0$

$L(t) = \vec{v}t + \vec{r}_0$

$= \vec{r}\,'(a)(t - a) + \vec{r}(a)$

$= r'(0)(t - 0) + \vec{r}(0)$

$\langle 1, -1, 2\rangle t + \langle 0, 1, 0\rangle$

$r'(t) = \langle 1, -e^{-t}, 2 - 2t\rangle$

$r'(0) = \langle 1, -1, 2\rangle$

$x = t$

$y = -t + 1$

$z = 2t$

Definition(s) 2.8. The Unit tangent vector is defined to be

$$\mathbf{T}(t) = \frac{r'(t)}{|r'(t)|}$$

Example 2.9. If $\mathbf{r}'(5) = \langle 2, 1, 2\rangle$ find $\mathbf{T}(5)$

$$\vec{T}(5) = \frac{r'(5)}{|r'(5)|} = \frac{\langle 2,1,2\rangle}{\sqrt{4+1+4}} = \left\langle \frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right\rangle \leftarrow$$

$$= \frac{1}{3}\langle 2,1,2\rangle \leftarrow$$

Example 2.10. Find the unit tangent vector of $\mathbf{r}(t) = te^{-t}\mathbf{i} + \arctan t\mathbf{j} + 2e^t\mathbf{k}$

$$T(t) = \frac{r'(t)}{|r'(t)|} = \frac{\left\langle e^{-t} - te^{-t}, \frac{1}{1+t^2}, 2e^t\right\rangle}{\sqrt{(e^{-t}-te^{-t})^2 + \left(\frac{1}{1+t^2}\right)^2 + 4e^{2t}}}$$

Example 2.11. Find the unit tangent vector of $\mathbf{r}(t) = te^{-t}\mathbf{i} + \arctan t\mathbf{j} + 2e^t\mathbf{k}$ when $t = 0$

$$T(0) = \frac{\langle 1,1,2\rangle}{\sqrt{1+1+4}} = \frac{1}{\sqrt{6}}\langle 1,1,2\rangle$$

4.2 Additional Theorems and Practice – During Class

Objective(s):

- Go over some theorems that are covered in the book and refrain from memorizing them. Instead think where they come from!
- Practice more problems covering physical applications of vector calculus.

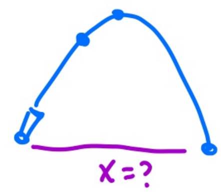
The book includes the following theorem:

Theorem 4.4. Assume that α is the angle of elevation and that $|\mathbf{v}(0)| = v_0$ is the initial speed. If we consider the initial position to be $\mathbf{r}(0) = \langle 0, 0\rangle$ and assume that gravity is the only force acting on the projectile then we have that:

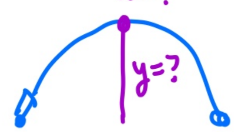
(a) Acceleration is $\mathbf{a}(t) = \langle 0, -g\rangle$

(b) Velocity is $\mathbf{v}(t) = \langle V_0 \cos\alpha, V_0 \sin\alpha - gt\rangle$

(c) Position is $\mathbf{r}(t) = \langle V_0 \cos\alpha\, t, V_0 \sin\alpha\, t - \frac{g}{2}t^2\rangle$



(d) Horizontal distance (range) is $\dfrac{V_0^2 \sin(2\alpha)}{g}$ which occurs at $t = \dfrac{2V_0 \sin\alpha}{g}$.

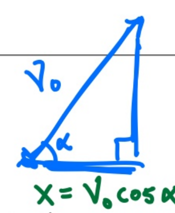


(e) Maximum height is $\dfrac{V_0^2 \sin^2\alpha}{2g}$ which occurs at $t = \dfrac{V_0 \sin\alpha}{g}$.

It may be very tempting to use these to solve WeBWorK problems. However they are very difficult to memorize and only apply is select situations. Instead you should know how these are derived (much easier and logical to memorize) and will help you do well on quizzes and exams. For the following please use the notation $\mathbf{r}(t) = \langle x(t), y(t)\rangle$.

$\sin\alpha = \dfrac{y}{V_0}$

$V_0 \sin\alpha = y$



Ideas of Proofs

(a) Given by statement "Only force acting on projectile is gravity"

(b) Integrate $\mathbf{a}(t)$, Use $\mathbf{v}(0) = \langle V_0 \cos\alpha, V_0 \sin\alpha\rangle$ to solve for constant of integration.

(c) Integrate $\mathbf{v}(t)$, Use $\mathbf{r}(0) = \langle 0, 0\rangle$ to solve for constant of integration.

(d) Solve $y(t) = 0$ for t, plug into $x(t)$.

(e) Solve $y'(t) = 0$ for t, plug into $y(t)$.

Example 4.5. Find the velocity, accelerations, and speed of the particle with position function $\mathbf{r}(t) = \langle t, t^2, 2 \rangle$.

velocity $\rightarrow v(t) = r'(t) = \langle 1, 2t, 0 \rangle$

acceleration $\rightarrow a(t) = v'(t) = \langle 0, 2, 0 \rangle$

speed $\rightarrow |v(t)| = \sqrt{1 + 4t^2}$

Example 4.6. Find the velocity and position vectors of a particle that has the given conditions

$$\mathbf{a}(t) = \langle 1, 2, 0 \rangle, \quad \mathbf{v}(0) = \langle 0, 0, 1 \rangle, \quad \mathbf{r}(1) = \langle 1, 2, 0 \rangle$$

$\vec{v}(t) = \langle 1,2,0 \rangle t + \vec{C}$

$\langle 0,0,1 \rangle = \langle 1,2,0 \rangle \cdot 0 + \vec{C}$

$\langle 0,0,1 \rangle = \vec{C}$

$\vec{v}(t) = \langle 1,2,0 \rangle t + \langle 0,0,1 \rangle$

$\vec{r}(t) = \langle 1,2,0 \rangle \frac{t^2}{2} + \langle 0,0,1 \rangle t + \vec{D}$

$\langle 1,2,0 \rangle = \langle 1,2,0 \rangle \frac{1}{2} + \langle 0,0,1 \rangle + \vec{D}$

$\langle 1,2,0 \rangle = \langle \frac{1}{2}, 1, 1 \rangle + \vec{D}$

$\langle \frac{1}{2}, 1, -1 \rangle = \vec{D}$

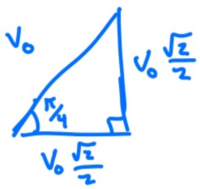
$\vec{r}(t) = \langle 1,2,0 \rangle \frac{t^2}{2} + \langle 0,0,1 \rangle t + \langle \frac{1}{2}, 1, -1 \rangle$

$=$

$\alpha = 45°$ $y=0$ $x=30$ V_0

Example 4.7. A ball is thrown at an angle of 45° to the ground. If the ball lands 30m away, what was the initial speed of the ball?

$g = 10$



$a(t) = \langle 0, -10\rangle$

$v(t) = \langle 0, -10t\rangle + \vec{C}$

$v(t) = \langle 0, -10t\rangle + \langle V_0\frac{\sqrt{2}}{2}, V_0\frac{\sqrt{2}}{2}\rangle$

$\vec{r}(t) = \langle 0, -5t^2\rangle + \langle V_0\frac{\sqrt{2}}{2}t, V_0\frac{\sqrt{2}}{2}t\rangle + \vec{C}$

$\vec{r}(t) = \langle 0, -5t^2\rangle + \langle V_0\frac{\sqrt{2}}{2}t, V_0\frac{\sqrt{2}}{2}t\rangle + \langle 0, 0\rangle$

$x = 30$ and $y = 0$ at some time (let's call it $t_1 > 0$)

$\langle 30, 0\rangle = \langle V_0\frac{\sqrt{2}}{2}t_1, -5t_1^2 + V_0\frac{\sqrt{2}}{2}t_1\rangle$

$30 = V_0\frac{\sqrt{2}}{2}t_1$

$\frac{60}{V_0\sqrt{2}} = t_1$

$0 = -5t_1^2 + V_0\frac{\sqrt{2}}{2}t_1$

$0 = -5t_1 + V_0\frac{\sqrt{2}}{2}$

$t_1 = V_0\frac{\sqrt{2}}{10}$

$\frac{60}{V_0\sqrt{2}} = V_0\frac{\sqrt{2}}{10}$

$\frac{600}{2} = V_0^2$

$300 = V_0^2$

$\sqrt{300} = V_0$

$\boxed{10\sqrt{3} = V_0}$

Example 4.8. A ball is thrown eastward into the air from the origin (in the direction of the positive x−axis). The initial velocity is $50\mathbf{i} + 80\mathbf{k}$ with speed measured in feet per second. The spin of the ball results in a southward acceleration of 4 ft/s^2, so the acceleration vector is $\mathbf{a} = -4\mathbf{j} - 32\mathbf{k}$. Where does the ball land and with what speed? (**NOTE:** We are in 3 dimensions!)

$r(0) = \langle 0, 0, 0\rangle$ $v(0)$ ← due to gravity $z = 0$

$a(t) = \langle 0, -4, -32\rangle$

$v(t) = \langle 0, -4t, -32t\rangle + C$

$\langle 50, 0, 80\rangle = \langle 0, 0, 0\rangle + \vec{C}$

$v(t) = \langle 50, -4t, 80 - 32t\rangle$

$r(t) = \langle 50t, -2t^2, 80t - 16t^2\rangle + C$

$\langle 0, 0, 0\rangle = \langle 0, 0, 0\rangle + \vec{C}$

$r(t) = \langle 50t, -2t^2, 80t - 16t^2\rangle$

$80t - 16t^2 = 0$

$80 - 16t = 0$

$80 = 16t$

$5 = t$

$\boxed{r(5) = \langle 250, -50, 0\rangle}$

$v(5) = \langle 50, -20, -80\rangle$

$= 10\langle 5, -2, -8\rangle$

$|v(5)| = 10\sqrt{25 + 4 + 64}$

$= \boxed{10\sqrt{93} \text{ ft/s}}$

3.2 Additional Practice with Arc Length – During Class

Objective(s):

- Calculate the length of more curves!
- Calculate the arc length function and use it to solve some problems!

Example 3.6. Find the length of the curve: $\mathbf{r}(t) = \left\langle 2t, t^2, \frac{t^3}{3} \right\rangle, \quad t \in [0, 1]$

$$L(C) = \int_a^b |r'(t)|\, dt$$

$$r'(t) = \langle 2, 2t, t^2 \rangle$$

$$|r'(t)| = \sqrt{4 + 4t^2 + t^4}$$

$$L(c) = \int_0^1 \sqrt{4+4t^2+t^4}\, dt$$

$$\int_0^1 \sqrt{(2+t^2)^2}\, dt = \int_0^1 2+t^2\, dt$$

$$= \left[2t + \frac{t^3}{3}\right]_0^1 = \boxed{2 + \frac{1}{3}} = \boxed{7/3}$$

Example 3.7. Find the length of the curve: $\mathbf{r}(t) = \mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}, \quad 0 \le t \le 2$

$$r'(t) = \langle 0, 2t, 3t^2 \rangle$$

$$|r'(t)| = \sqrt{0 + 4t^2 + 9t^4}$$

$$\int_0^2 \sqrt{4t^2 + 9t^4}\, dt$$

$$\int_0^2 \sqrt{t^2(4+9t^2)}$$

$$\int_0^2 t\sqrt{\underline{4+9t^2}}\, dt$$

$u = 4 + 9t^2$

$du = 18t\, dt$

$\frac{1}{18} du = t\, dt$

$u = 0 \to 4$

$u = 2 \to 40$

$$\int_4^{40} \sqrt{u}\ \frac{1}{18}\, du$$

$$\frac{1}{18}\left[\frac{2}{3} u^{3/2}\right]_4^{40} = \frac{1}{27}\left(40^{3/2} - 4^{3/2}\right)$$

$$\frac{1}{27}\left(40^{3/2} - 8\right)$$

Question 1

Multiple choice

Suppose the velocity of a projectile is given by $\mathbf{v}(t) = \langle 1, 2t, -\sin t\rangle$ and the initial position is $\langle 1, 3, 0\rangle$. Find the position function

$r(0) = \langle 1, 3, 0\rangle$

A. $\mathbf{r}(t) = \langle 1, 2t, -\sin t\rangle$

B. $\mathbf{r}(t) = \langle t, t^2, \cos t\rangle$

C. $\mathbf{r}(t) = \langle t+1, t^2+3, \cos t\rangle$

D. $\mathbf{r}(t) = \langle t+1, t^2+3, -1+\cos t\rangle$

E. None of the above

$$\int \langle 1, 2t, -\sin t\rangle \, dt$$

$$r(t) = \langle t, t^2, +\cos t\rangle + \langle \ , \ , \ \rangle$$

$$r(0) = \langle 0, 0, 1\rangle + \langle 1, 3, -1\rangle$$

$$r(t) = \langle t, t^2, \cos t\rangle + \langle 1, 3, -1\rangle$$

$$= \langle t+1, t^2+3, \cos t - 1\rangle$$

Example 3.8. Find the length of the curve: $\mathbf{r}(t) = \langle \cos t, \sin t, \ln(\cos t)\rangle, \quad t \in [0, \pi/4]$

$$r'(t) = \left\langle -\sin t, \cos t, \frac{1}{\cos t} \cdot -\sin t \right\rangle$$

$$|r'(t)| = \sqrt{\sin^2 t + \cos^2 t + \tan^2 t}$$

$$\frac{\sin^2 t}{\cos^2 t} + \frac{\cos^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}$$

$$\tan^2 t + 1 = \sec^2 t$$

$$= \sqrt{1 + \tan^2 t} = \sqrt{\sec^2 t}$$

$$= \sec t$$

$$\int_0^{\pi/4} \sec t \, dt = \Big[\ln|\sec t + \tan t|\Big]_0^{\pi/4}$$

$$= \ln\left|\sec\tfrac{\pi}{4} + \tan\tfrac{\pi}{4}\right| - \ln|\sec 0 + \tan 0|$$

$$\sec\frac{\pi}{4} = \frac{1}{\cos\frac{\pi}{4}} = \frac{1}{\sqrt{2}/2} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

$$= \ln|\sqrt{2} + 1| - \ln|1 + 0|$$

$$= \ln(\sqrt{2} + 1)$$

Example 3.9. Find the arc length parametrization of the curve: $\mathbf{r}(t) = \langle 2\cos t, 2\sin t, 3t\rangle$ with base point π

$s(t)$ $\qquad a$

$$s(t) = \int_a^t |r'(u)| \, du$$

$$\vec{r}(u) = \langle 2\cos u, 2\sin u, 3u \rangle$$

$$\vec{r}'(u) = \langle -2\sin u, 2\cos u, 3 \rangle$$

$$|r'(u)| = \sqrt{4\sin^2 u + 4\cos^2 u + 9}$$

$$= \sqrt{4(1) + 9} = \sqrt{13}$$

$$s(t) = \int_\pi^t \sqrt{13} \, du = \Big[\sqrt{13}\, u\Big]_\pi^t$$

$$= \left(\sqrt{13}\, t - \sqrt{13}\,\pi\right)$$

Example 3.10. Consider the vector function $\mathbf{r}(t) = \langle \cos(-t), \sin(-t), t+1 \rangle$.

(a) Find the arc length function with the initial point $\langle 1, 0, 1 \rangle$.

$s(t)$ $\quad t = 0$

$$r(u) = \langle \cos(-u), \sin(-u), u+1 \rangle$$

$$r'(u) = \langle +\sin(-u), -\cos(-u), 1 \rangle$$

$$|r'(u)| = \sqrt{\sin^2(-u) + \cos^2(-u) + 1}$$

$$= \sqrt{1+1} = \sqrt{2}$$

$$s(t) = \int_0^t \sqrt{2}\, du = \sqrt{2}\, u \Big]_0^t = \sqrt{2}\, t$$

(b) Find the time in which a particle has traveled 7 units along the curve from $\langle 1, 0, 1 \rangle$ in the positive direction.

$$7 = \sqrt{2}\, t$$

$$7/\sqrt{2} = t$$

(c) Find the location of the particle in (b).

$$r\left(\frac{7}{\sqrt{2}}\right) = \left\langle \cos\left(-\frac{7}{\sqrt{2}}\right), \sin\left(\frac{-7}{\sqrt{2}}\right), \frac{7}{\sqrt{2}} + 1 \right\rangle$$

Example 3.11. Consider the surfaces $z = x^2 + y^2$ and $x^2 + y^2 = 9$.

(a) Parametrize the curve of intersection between the surfaces.

13.1

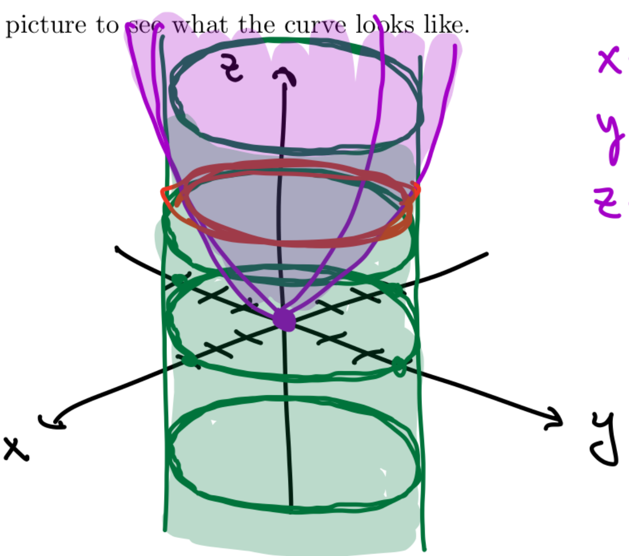
$x = 3\cos t$

$t \in [0, 2\pi]$ $\quad y = 3\sin t$

$z = 9$

(b) Sketch a picture to see what the curve looks like.

12.6



$x = 0 \rightarrow z = y^2$

$y = 0 \rightarrow z = x^2$

$z = 0 \rightarrow 0 = x^2 + y^2$

(c) Guess the length of the curve in problem (a). Use the arc length formula to check.

13.3

$C = \pi d$

$= \pi\, 2r$

$= \pi\, 2 \cdot 3$

$= 6\pi$

$r'(t) = \langle -3\sin t, 3\cos t, 0 \rangle$

$|r'(t)| = \sqrt{9\sin^2 t + 9\cos^2 t}$

$= \sqrt{9(1)} = 3$

$\int_0^{2\pi} 3\,dt = 3t \Big|_0^{2\pi} = 6\pi$