# 13.1 Vector Functions and Curves

## Parameterizing curves in $\mathbb{R}^2$

1. Sketch $\boldsymbol{r}(t) = \langle t, t^2 \rangle$ in $\mathbb{R}^2$ and express the curve as a function of $x$ and $y$.

2. Sketch $\boldsymbol{r}(t) = \langle 4, \cos(t), \sin(t) \rangle$ in $\mathbb{R}^3$ and express the curve as a function of $x$, $y$, and $z$.

**Definition.** A vector function $\boldsymbol{r}(t)$ is *continuous* at $t = a$ if

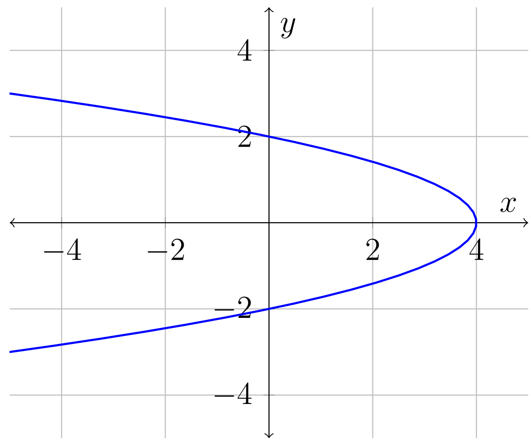
$$\lim_{t \to a} \boldsymbol{r}(t) = \boldsymbol{r}(a)$$

Note that by the above definition to check $\boldsymbol{r}(t)$ is continuous at $t = a$ you need to verify that $\boldsymbol{r}(a)$ and $\lim_{t \to a} \boldsymbol{r}(t)$ are both defined and equal to each other.

3. Show that $\boldsymbol{r}(t) = \langle \sin(\pi t) + 1, t^2 + t, 1 \rangle$ is continuous at $t = -1$.

## Parameterizing curves in $\mathbb{R}^2$

4. Find a vector function $\boldsymbol{r}(t)$, $t \in \mathbb{R}$ that represents the curve $x + y^2 = 4$ shown below.



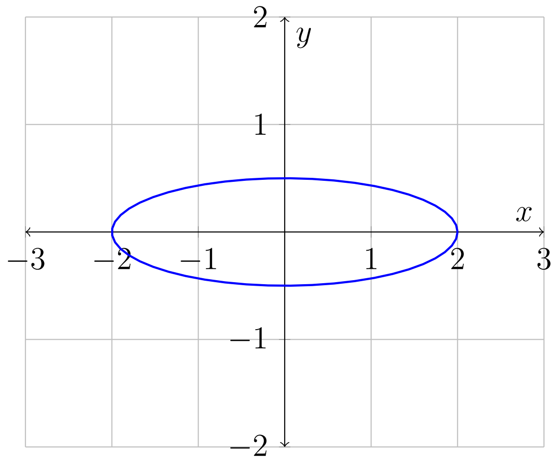

(a) Try parametezing the curve by setting $x = t$ and explain what makes that approach difficult (there is not one correct answer)..

(b) Now try parameterizing by setting $y = t$.

(c) Verify the vector function you found satisfies $x + y^2 = 4$ for any arbitrary choice of $t$.

(d) What would you do differently for the curve $x^2 + y = 4$?

5. Find a vector function $\boldsymbol{r}(t)$ that represents the curve $x^2 + 16y^2 = 4$




(a) Try parameterizing by setting $x = t$ and then try setting $y = t$. What makes these so difficult to work with?

(b) Rewrite the above equation so that it has the form

$$(f(x))^2 + (g(y))^2 = 1$$

(c) Find $\boldsymbol{r}(t)$ by setting $f(x) = \cos t$ and $g(y) = \sin t$. Be sure to include the domain for $t$.

(d) What do you need to change to represent the portion of $x^2 + 36y^2 = 4$ where $y \geq 0$? What about $x > 0$?

## Parameterizing curves in $\mathbb{R}^3$

6. Find a vector function that represents the curve of intersection between the two surfaces

$$y = 4z^2 + x^2 \quad \text{and} \quad x = z^2$$

Hint: Try $x = t$, $y = t$, and $z = t$ and see which gives you something you can work with.