14.3 Group work

14.3: Functions of multiple variables

1. Find $f_{xx}(x,y)$, $f_{xy}(x,y)$, and $f_{yy}(x,y)$

   (a) $f(x,y) = \dfrac{2x+x^2}{y}$

   Solution:

   $$f_x = \frac{2+2x}{y} \qquad f_y = -\frac{2x+x^2}{y^2} \qquad f_{xy} = -\frac{2+2x}{y^2}$$
   $$f_{xx} = \frac{2}{y} \qquad f_{yy} = \frac{4x+2x^2}{y^3}$$

   (b) $f(x,y) = \ln(x)\sin(\pi y)$

   Solution:

   $$f_x = \frac{\sin(\pi y)}{x} f_{xx} \quad = -\frac{\sin(\pi y)}{x^2} \quad f_y = \pi \ln(x)\cos(\pi y) f_{yy} \quad = -\pi^2 \ln(x)\sin(\pi y) \quad f_{xy} = \frac{\pi\cos(\pi y)}{x}$$

2. Find all first order partial derivatives of the function $h(x,y,z,t) = x^2 y\cos(z/t)$

   Solution:

   $$h_x = 2xy\cos(z/t)$$
   $$h_y = x^2\cos(z/t)$$
   $$h_z = -\frac{x^2 y\sin(z/t)}{t}$$
   $$h_t = \frac{x^2 yz\sin(z/t)}{t^2}$$

3. Verify Clairaut's theorem for $f(x, y) = (2x + 3y^2)^{10}$

Solution:

$$f_x = 20(2x+3y^2)^9 \qquad f_y = 60y(2x+3y^2)^9 f_{yx} = 1080y(2x+3y^2)^8 \tag{1}$$
$$f_{xy} = 20(9)(6y)(2x+3y^2)^8 = 1080y(2x+3y^2)^8$$

4. Consider the surface

$$\frac{x^2}{4} + 2y^2 - z^2 = 1$$

Use implicit differentiation to express both $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ as functions of x, y, and z.

Solution:

$$\frac{\partial}{\partial x}\left(\frac{x^2}{4} + 2y^2 - z^2\right) = \frac{\partial}{\partial x}(1)$$
$$\frac{1}{2}x - 2z\frac{\partial z}{\partial x} = 0$$
$$\frac{\partial z}{\partial x} = \frac{x}{4z}.$$

Similiarly,

$$\frac{\partial z}{\partial y} = \frac{2y}{z}.$$