15.3 Double Integrals over General Regions

1. Evaulate the iterated and double integrals

   (a) $\int_0^1 \int_{x^3}^{x} (x + 2y)\ dy\ dx$

   (b) $\int_0^1 \int_0^{s^2} \cos(s^3)\ dt\ ds$

   (c) $\iint_D y^2\ dA$ with $D = \{(x, y) \mid -1 \leq y \leq 1, -y - 2 \leq x \leq y\}$

2. Evaulate $\iint_D xy \, dA$, where D is the region enclosed by $y = x^2, y = 3x$

3. Evaluate $\iint_D y^2 \, dA$ where D is the triangular region with vertices $(0, 1), (1, 2), (4, 1)$.

4. Find the volume of the solid under the plane $x - 2y + z = 1$ and above the region bounded by $x + y = 1$ and $x^2 + y = 1$.

5. Sketch the region of integration and rewrite the integral with the order of integration switched

   (a) $\int_0^1 \int_0^y f(x, y)\, dx\, dy$

   (b) $\int_1^2 \int_0^{\ln(x)} f(x, y)\, dy\, dx$

6. Evaluate each integral by changing the order of integration

   (a) $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dy\, dx$

   (b) $\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy$

7. Find the average value of $f(x, y) = x \sin(y)$ over the region bounded by the curves $y = 0$, $y = x^2$, and $x = 1$.