15.10

Useful Information.

- The **Jacobian** for a tranformation T given by $x = x(u, v)$ and $y = y(u, v)$ is

$$\frac{(x, y)}{(x, y)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u} \cdot \frac{\partial y}{\partial v} - \frac{\partial x}{\partial v} \cdot \frac{\partial y}{\partial u}$$

-
$$\iint_R f(x, y)\ dA = \iint_S f(x(u, v), y(u, v)) \left| \frac{(u, v)}{(u, v)} \right|\ du\ dv$$

1. Show that the Jacobian of the transformation $x = 5u - v, y = u + 3v$ is 16.

2. Show that the Jacobian of the transformation $x = e^{-r} \sin\theta$, $y = e^{r} \cos\theta$ is $\sin^2\theta - \cos^2\theta$.

3. Evaluate $\iint_R (x - 3y)dA$ where R is the triangular region with vertices $(0, 0)$,$(2, 1)$, and $(1, 2)$ using the transformation $x = 2u + v$, $y = u + 2v$. Make sure you evaluate it in such a way that the answer you get is -3.

4. Use the transformation $x = 2u$, $y = 3v$ to evaluate $\iint_R x^2 dA$ where R is the region bounded by the ellipse $9x^2 + 4y^2 + 36$