13.2 Derivatives and Integrals of Vector Functions

1. For each of the following

   (*i*) Sketch the following curves in the xy-plane

   (*ii*) Find $\boldsymbol{r}'(t)$

   (*iii*) Sketch the tangent vector $\boldsymbol{r}'(t)$ for the given value of t

   (a) $\boldsymbol{r}(t) = \langle t^2, t^3 \rangle$ with $t = -1$

   (b) $\boldsymbol{r}(t) = \langle t - 2, t^2 + 1 \rangle$ with $t = 1$

2. Find $\boldsymbol{r}'(t)$ given $\boldsymbol{r}(t) = \langle t\sin(t), t^2, \cos(2t)\rangle$

3. Find parametric equations for the tangent line to the curve $\boldsymbol{v}(t) = \left\langle 1 + 2\sqrt{t}, t^3 - t, t^3 + t\right\rangle$ at the point $(3, 0, 2)$

4. Evaluate the given integrals

(a)

$$\int_0^2 \left(t\boldsymbol{i} - t^2\boldsymbol{j} + 3t^5\boldsymbol{k}\right)\ dt$$

(b)

$$\int_0^1 \left(\frac{4}{1+t^2}\boldsymbol{j} + \frac{2t}{1+t^2}\boldsymbol{k}\right)\ dt$$

13.3 Arc Length and Curvature

$$\begin{aligned}
\text{Unit tangent vector} \quad & \mathbf{T}(t) = \frac{\boldsymbol{r}'(t)}{|\boldsymbol{r}'(t)|} \\
\text{Unit normal vector} \quad & \mathbf{N}(t) = \frac{\mathbf{T}(t)}{|\mathbf{T}'(t)|} \\
\text{Binormal vector} \quad & \mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t) \\
\text{Curvature} \quad & \kappa(t) = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\boldsymbol{r}'(t)|} = \frac{|\boldsymbol{r}'(t) \times \boldsymbol{r}''(t)|}{|\boldsymbol{r}'(t)|^3}
\end{aligned}$$

5. Find the unit tangent vector $\boldsymbol{T}(\pi/4)$ to the curve $\big\langle \sin^2(t), \cos^2(t), \tan^2(t) \big\rangle$.

6. Suppose you are at the point $(0, 0, 0)$ and move 2 units in the positive direction along the curve $\boldsymbol{r}(t) = \langle t^2, t^3, t^4 \rangle$. What point did you end up at? (Your answer may be messy, if so do not try to simplify.)

7. Find the length of the curve $\boldsymbol{r}(t) = \boldsymbol{i} + t^2\boldsymbol{j} + t^3\boldsymbol{k}$ with $0 \leq t \leq 1$. Then find the unit tangent vector $\boldsymbol{T}(t)$ and unit normal vector $\boldsymbol{N}(t)$.