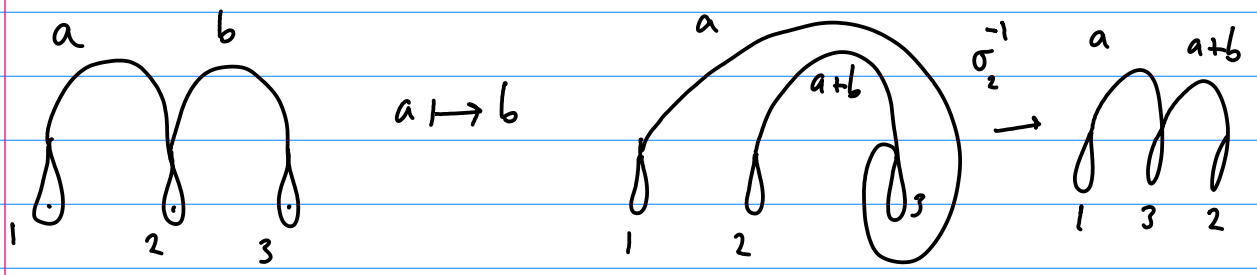
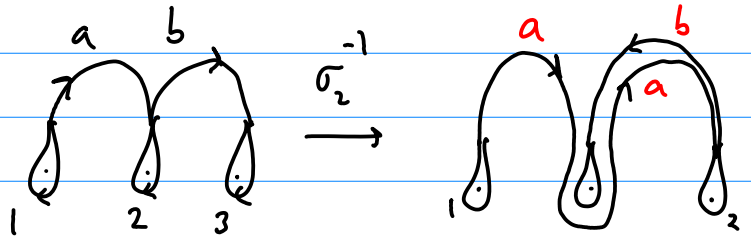
Train Track Maps from Automata

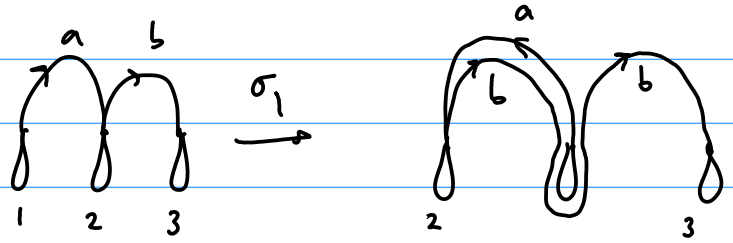


TT map:



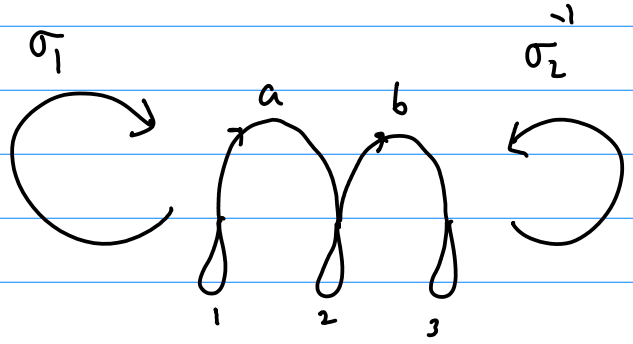
$a \mapsto a\bar{2}b$
$b \mapsto \bar{b}$

$1 \mapsto 1, 2 \mapsto 3, 3 \mapsto 2$



$a \mapsto \bar{a}$
$b \mapsto a\bar{2}b$
$1 \mapsto 2, 2 \mapsto 1, 3 \mapsto 3$

$a \mapsto \bar{a}$
$b \mapsto a\bar{2}b$
$123 \mapsto 213$



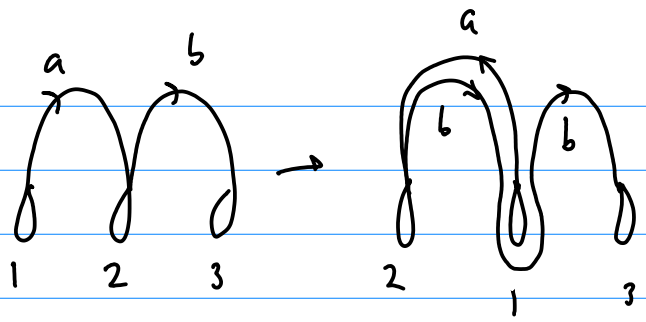
$a \mapsto a\bar{2}b$
$b \mapsto \bar{b}$
$123 \mapsto 132$

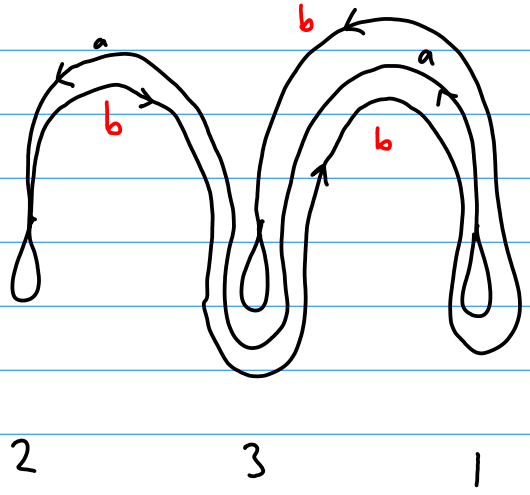
$\sigma_1\sigma_2^{-1}$:

$a \mapsto \bar{a} \mapsto \bar{b}2\bar{a}$

$b \mapsto a\bar{2}b \mapsto a\bar{2}b3\bar{b}$

Check:





$a \mapsto \bar{b}\,2\,\bar{a}$

$b \mapsto a\,\bar{2}\,b\,\bar{3}\,\bar{b}$

Works!

But is it in some sense as hard as finding the braid?
In a way (or exactly?) it keeps track of the embedding.