Jean-Luc Thiffeault <jeanlucth@gmail.com>

Trains

3 messages

Jean-Luc Thiffeault <jeanluc@math.wisc.edu> Thu, Apr 30, 2009 at 3:02 PM
To: Toby Hall <T.Hall@liverpool.ac.uk>

Hi Toby,

Erwan Lanneau and I are looking at some minimising braids on the disc with 6 punctures. We've got a problematic braid: our methods (automata) say it's pA, but your code says reducible (but with the same dilatation). Looking at the output of your code, I can build the transition matrix, and it appears to me to be irreducible. I thought that the transition matrix being irreducible (with positive entropy) was enough to be pA? Am I missing something?

Cheers,

Jean-Luc

> braid
Enter number of braid strings: 6
Enter braid generators separated by spaces, ending with 0:
1 2 3 -5 -4 -3 0
> train
Now have an efficient fibred surface: Growth 2.01536, Entropy 0.700796
Isotopy class is Reducible
> print

Graph on surface with 6 peripheral loops:
Vertex number 1 with image vertex 7:
Edges at vertex are: 1 -1 12
Region 1

Vertex number 2 with image vertex 2:
Edges at vertex are: -4 10 5
Region 4

Vertex number 3 with image vertex 5:
Edges at vertex are: 7 -7 -11
Region 5

Vertex number 4 with image vertex 1:
Edges at vertex are: -2 13 2
Region 1

Vertex number 5 with image vertex 4:
Edges at vertex are: -3 -8 9 3
Region 2

Vertex number 6 with image vertex 6:
Edges at vertex are: -9 4 -5
Region 3

Vertex number 7 with image vertex 3:

Edges at vertex are: -6 14 6
Region 4

Vertex number 8 with image vertex 8:
Edges at vertex are: -12 8 -13
Region 1

Vertex number 9 with image vertex 9:
Edges at vertex are: -14 -10 11
Region 4

Edge number 1 from vertex 1 to vertex 1:
Type: Peripheral about puncture number 1
Image is: 6
Path (1 -> 1): 1 -1

Edge number 2 from vertex 4 to vertex 4:
Type: Peripheral about puncture number 2
Image is: 1
Path (1 -> 1): -2 2

Edge number 3 from vertex 5 to vertex 5:
Type: Peripheral about puncture number 3
Image is: 2
Path (2 -> 2): -3 3

Edge number 4 from vertex 6 to vertex 2:
Type: Peripheral about puncture number 4
Image is: 4
Path (3 -> 4): -4

Edge number 5 from vertex 2 to vertex 6:
Type: Peripheral about puncture number 4
Image is: 5
Path (4 -> 3): 4

Edge number 6 from vertex 7 to vertex 7:
Type: Peripheral about puncture number 5
Image is: 7
Path (4 -> 4): -5 5

Edge number 7 from vertex 3 to vertex 3:
Type: Peripheral about puncture number 6
Image is: 3
Path (5 -> 5): -6 6

Edge number 8 from vertex 8 to vertex 5:
Type: Main
Image is: -13
Path (1 -> 2): -2

Edge number 9 from vertex 5 to vertex 6:
Type: Main
Image is: 13 8 -3 9
Path (2 -> 3): -3

Edge number 10 from vertex 2 to vertex 9:
Type: Main
Image is: 10 -14 -6 14
Path (4 -> 4):

Edge number 11 from vertex 9 to vertex 3:
Type: Main
Image is: -10 -4 -9
Path (4 -> 5): -5

Edge number 12 from vertex 1 to vertex 8:
Type: Main
Image is: 14 -10 -4 -9 3 -8
Path (1 -> 1):

Edge number 13 from vertex 4 to vertex 8:
Type: Main
Image is: 12
Path (1 -> 1):

Edge number 14 from vertex 7 to vertex 9:
Type: Main
Image is: -11
Path (4 -> 4):

Reducible Isotopy class
Vertex 1:
Gates are: {-1}, {12}, {1}
Infinitesimal edges join -1 to 12, 1 to 12
Vertex 2:
Gates are: {10}, {5}, {-4}
Infinitesimal edges join -4 to 10
Vertex 3:
Gates are: {-7}, {-11}, {7}
Infinitesimal edges join -11 to -7, -11 to 7
Vertex 4:
Gates are: {13}, {2}, {-2}
Infinitesimal edges join -2 to 13, 2 to 13
Vertex 5:
Gates are: {-8, 9}, {3}, {-3}
Infinitesimal edges join -8 to -3, -8 to 3
Vertex 6:
Gates are: {4}, {-5}, {-9}
Infinitesimal edges join -9 to 4
Vertex 7:
Gates are: {14}, {6}, {-6}
Infinitesimal edges join -6 to 14, 6 to 14
Vertex 8:
Gates are: {8}, {-13}, {-12}
Infinitesimal edges join -13 to 8, -13 to -12, -12 to 8
Vertex 9:
Gates are: {-10}, {11}, {-14}
Infinitesimal edges join -14 to -10, -14 to 11, -10 to 11

Hall, Toby <T.Hall@liverpool.ac.uk> Fri, May 1, 2009 at 4:01 AM
To: Jean-Luc Thiffeault <jeanluc@math.wisc.edu>

Dear Jean-Luc,

I think this braid is reducible. On drawing it, you can see that the 4th puncture is fixed, and the 4th string goes behind all of the other strings. Thus the curve which goes round all 6 punctures except is deformed to go in front of the 4th one is a reducing curve.

As regards the algorithm: if you have an efficient graph map with an irreducibile transition matrix, then the isotopy

class is pA _provided_ that all the gates at each vertex are connected by infinitesimal edges (see Proposition 3.3.2 of BH). In this example, the gates at vertices 2 and 6 are not all connected by infinitesimal edges.

If you delete the 4th string, you get the 5-braid 1 2 -4 -3, which is pA (and has the same growth rate 2.01536).

How are things?

Toby.

From: Jean-Luc Thiffeault [jeanluc@math.wisc.edu]
Sent: 30 April 2009 21:02
To: Hall, Toby
Subject: Trains
[Quoted text hidden]

Jean-Luc Thiffeault <jeanluc@math.wisc.edu> Fri, May 1, 2009 at 8:38 AM
To: "Hall, Toby" <T.Hall@liverpool.ac.uk>

Hi Toby,

Many thanks for that! It's clearly reducible, of course, if you know where to look! This means that our "automaton" approach should be viewed with a grain of salt: we are probably detecting spurious pA's (but at least I don't think we are leaving any out!). Of course, we might not be implementing it properly, so maybe it can be strengthened.

Erwan is here in Madison right now and we are working on a couple of papers. There is one that is almost set to go (missing a few things in appendices) -- I'd love to hear you comments. We also have a "general reader" summarized version at http://arxiv.org/abs/0904.0778/

Hope you're well,

Jean-Luc
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