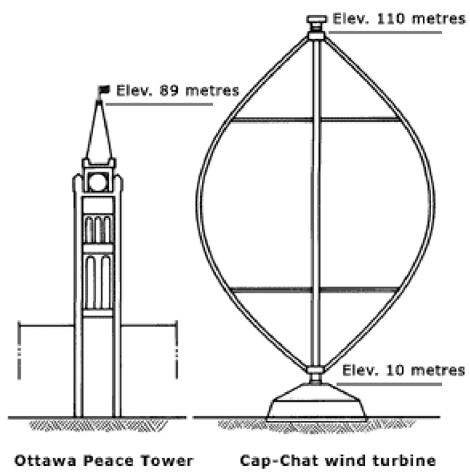
Vertical Axis Wind Turbine VAWT

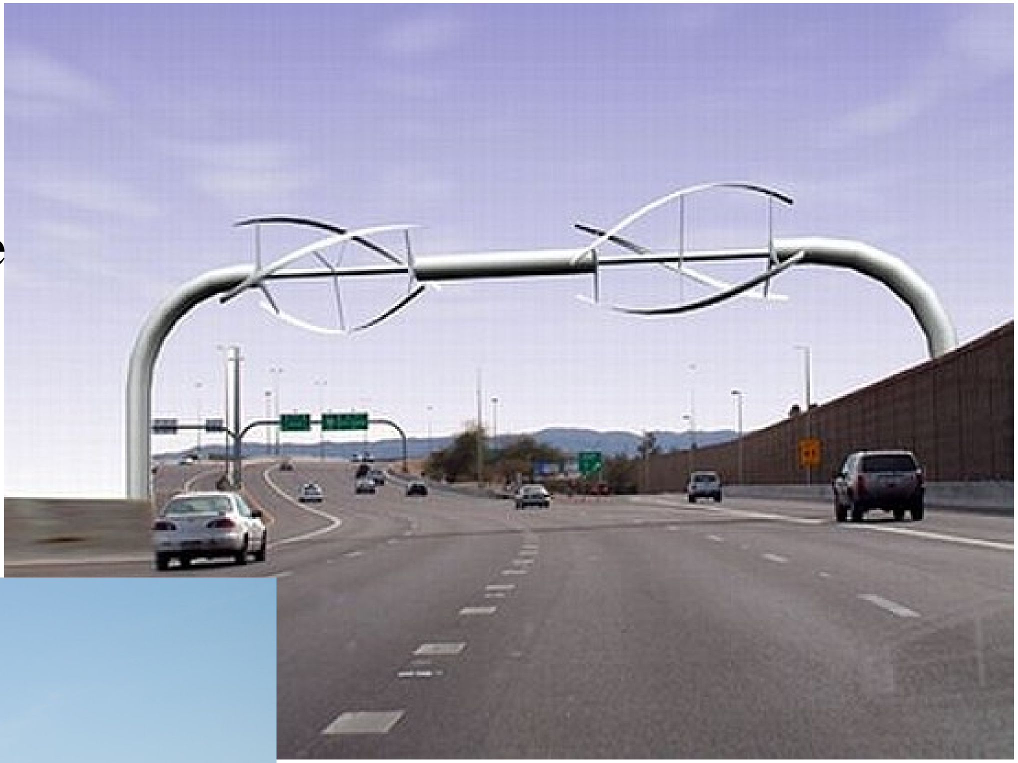


After today:

- You understand how a VAWT works
- Can model a VAWT using a simple BEM type model

A VAWT is not always vertical

Gyro turbine more general phrase

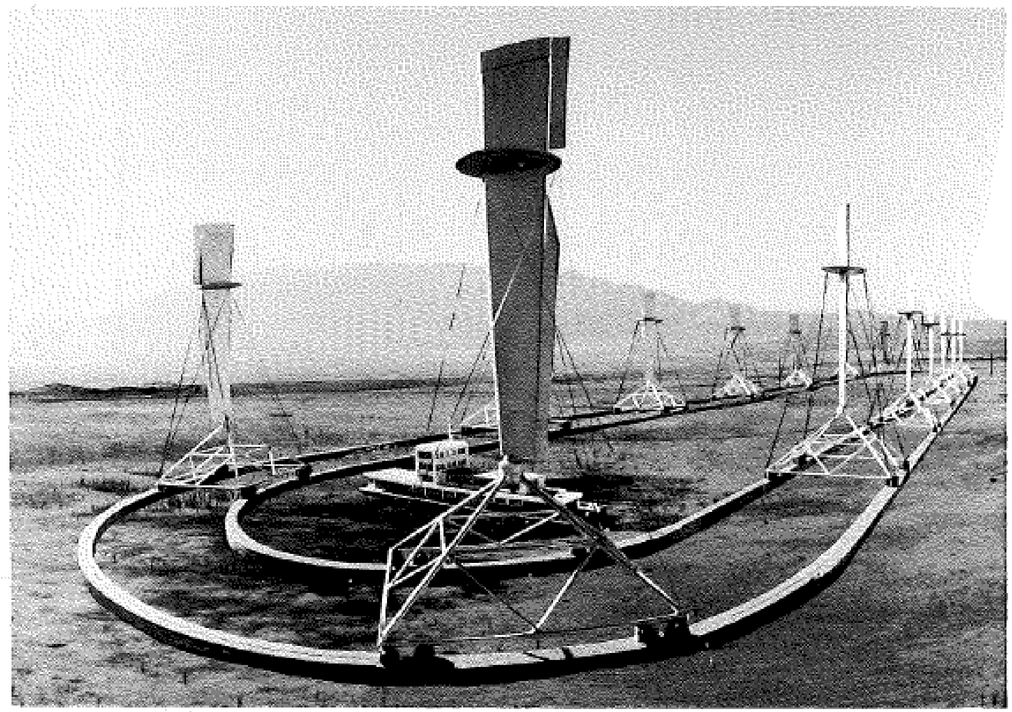


”Science fiction” never realized

80-MEGAWATT FREE-WING TURBINE

The Western Area Power Administration of the U S Department of Energy has contracted to buy the power from eleven of these installations. For more information contact:

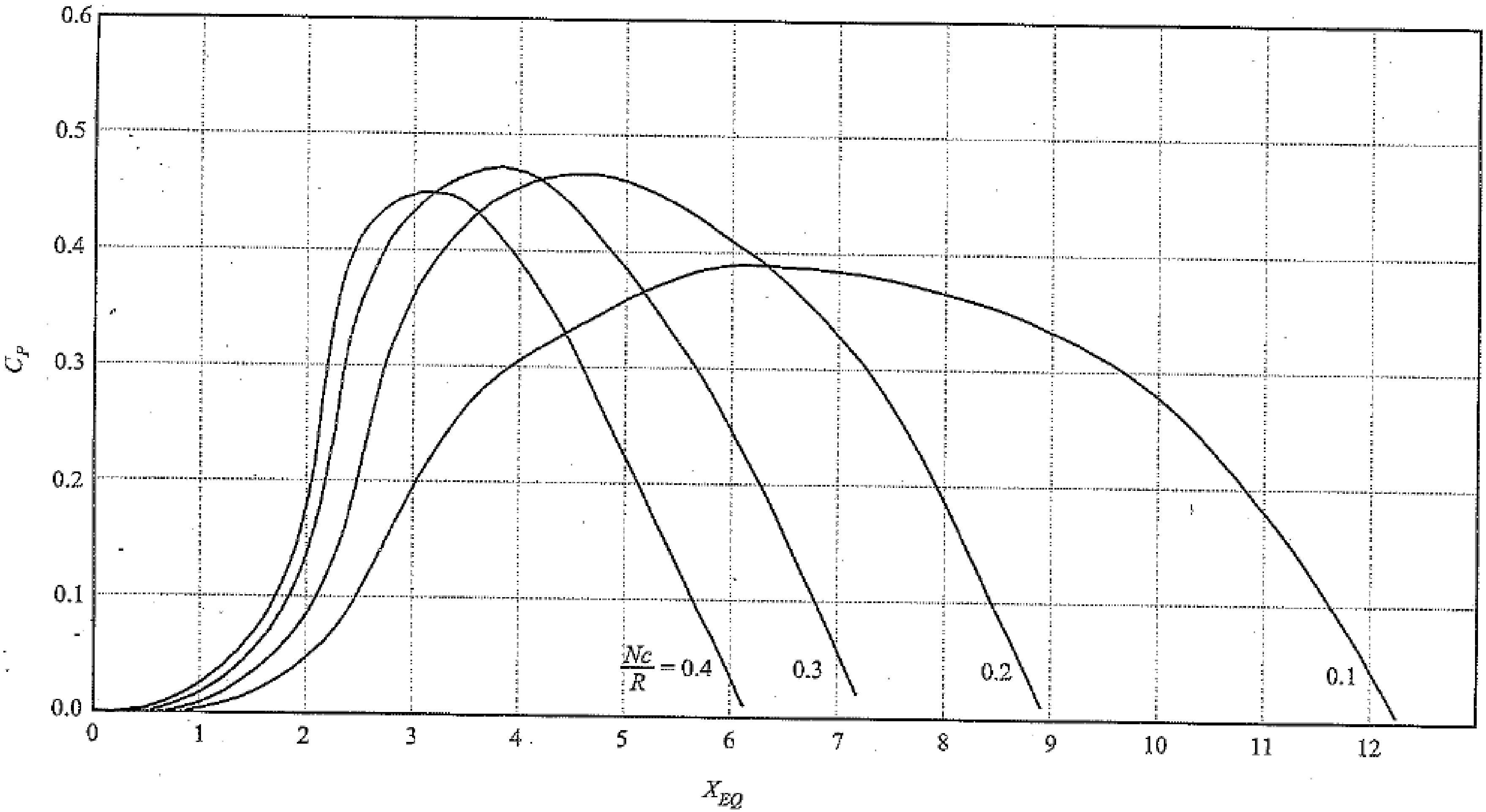
Free-Wing Turbine Corporation, 5278 Pinemont Drive, Suite A-120, Murray, Utah 84107, (801) 263-3481.



A big French project called Nenuphar was proposed
Ran out of money 2018 ☹

Picture of the planned project (two counter rotating VAWTs per floater)

Prototype built and tested in Marseille harbor



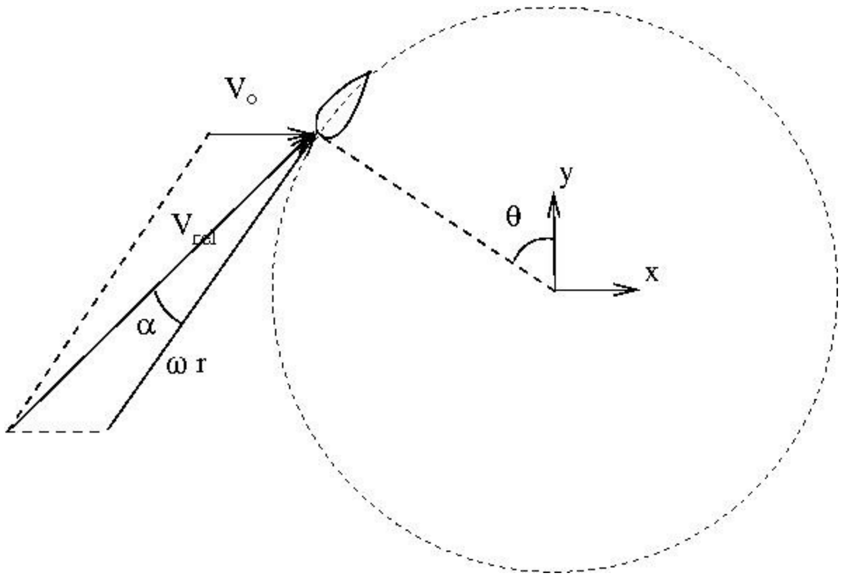
Typical variation between tip speed ratio $X_{EQ}=\omega R/V_o$ and solidity $S=Bc/R$ for a VAWT

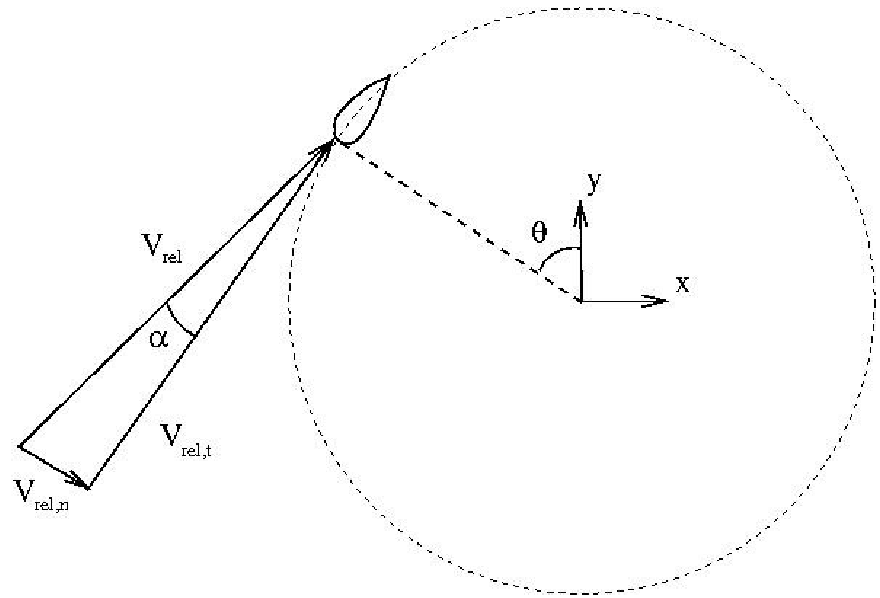
From: Wind Turbine Design with emphasis on Darrieus concept

Ion Paraschivoiu

Polytechnic International Press

Estimation of the angle of attack **neglecting** the induced velocities





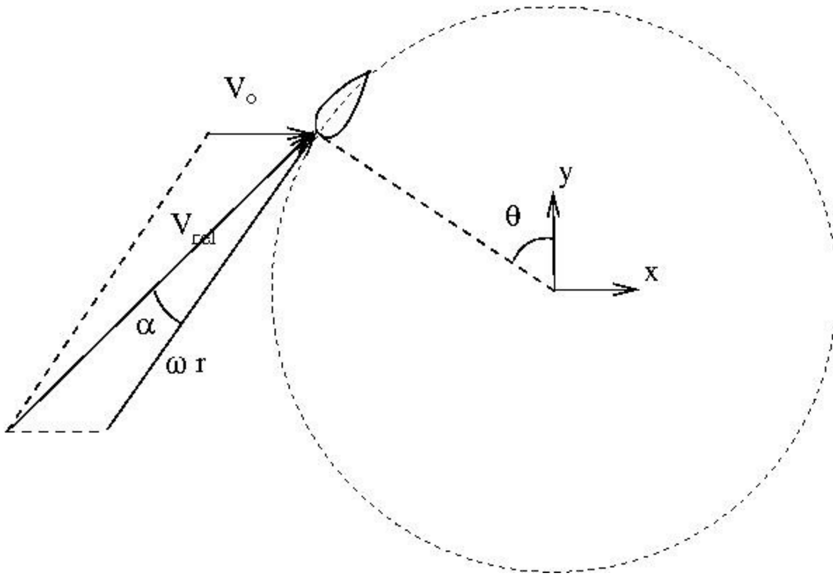
$$\mathbf{V_{rel}} = -\mathbf{V_{rot}} + \mathbf{V}_o$$

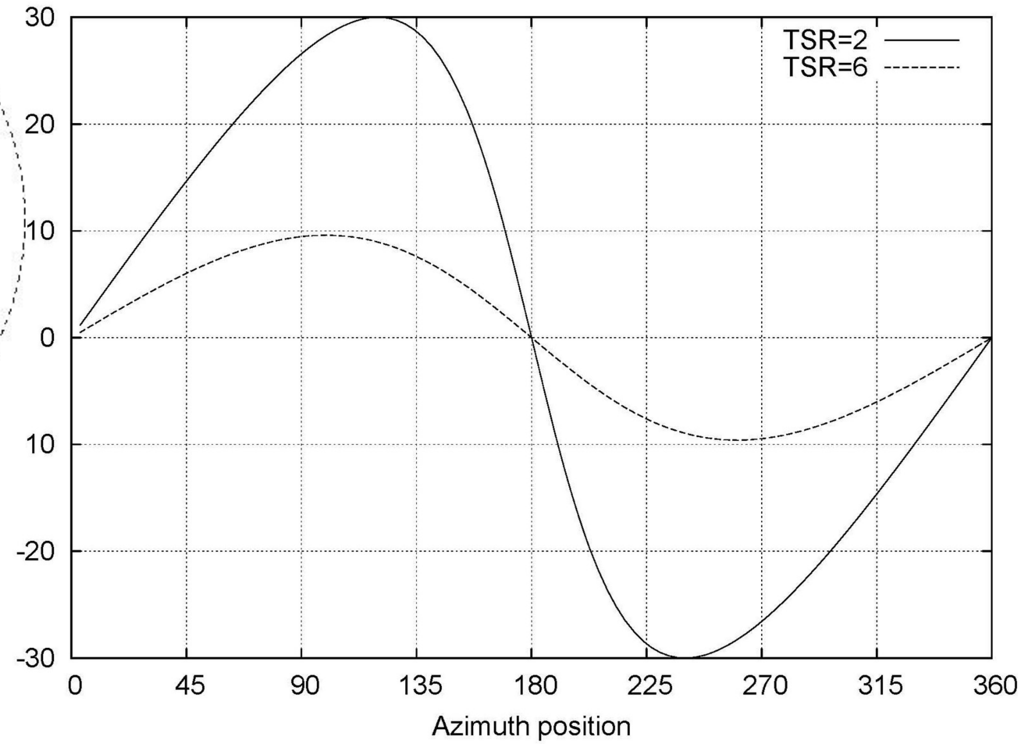
$$V_{rel,t} = \omega R + V_o \cos\theta$$

$$V_{rel,n} = V_o \sin\theta$$

$$\tan\alpha = \frac{V_{rel,n}}{V_{rel,t}} = \frac{V_o \sin\theta}{\omega R + V_o \cos\theta} = \frac{\sin\theta}{\lambda + \cos\theta}$$

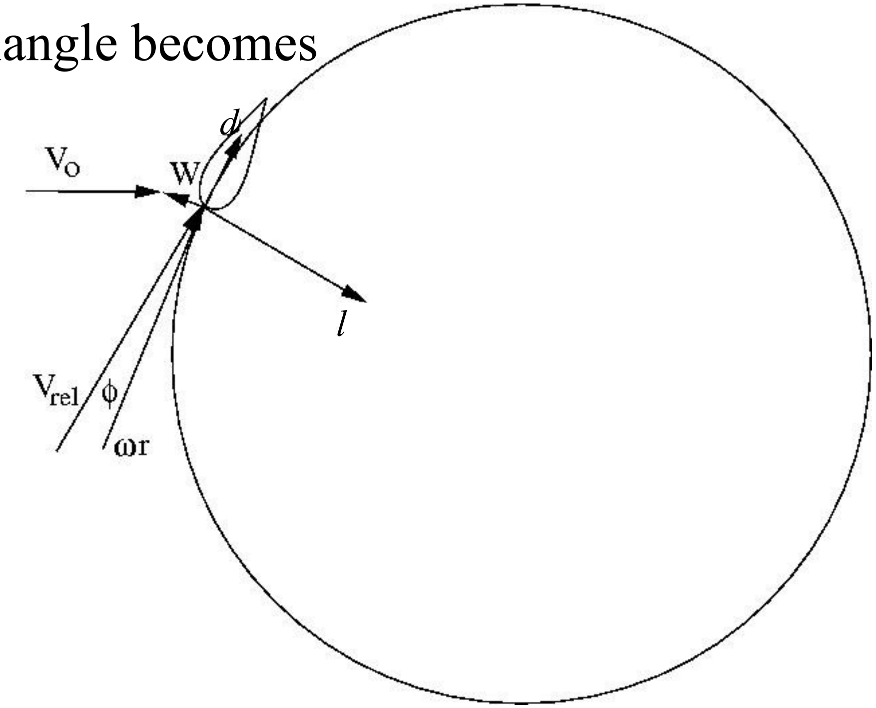
One of the big challenges of VAWTs
AoA and thus loads will change in time





$$\tan\alpha = \frac{\sin\theta}{\lambda + \cos\theta}$$

Including the induced wind the velocity triangle becomes



$$\mathbf{V_{rel}} = -\mathbf{V_{rot}} + \mathbf{W} + \mathbf{V_o}$$

$$\alpha = \phi - \theta_p$$

$$l = \tfrac{1}{2}\rho V_{rel}^2 c C_l$$

$$d = \tfrac{1}{2}\rho V_{rel}^2 c C_d$$

Lift and drag projected normal to rotor (l is normal to V_{rel})

$$p_n = l\cos\phi + d\sin\phi$$

$$p_t = l\sin\phi - d\cos\phi$$

Again, just as a HAWT: If the induced velocity and the airfoil data are known then the loads are known

How to calculate the performance (Power, Thrust)

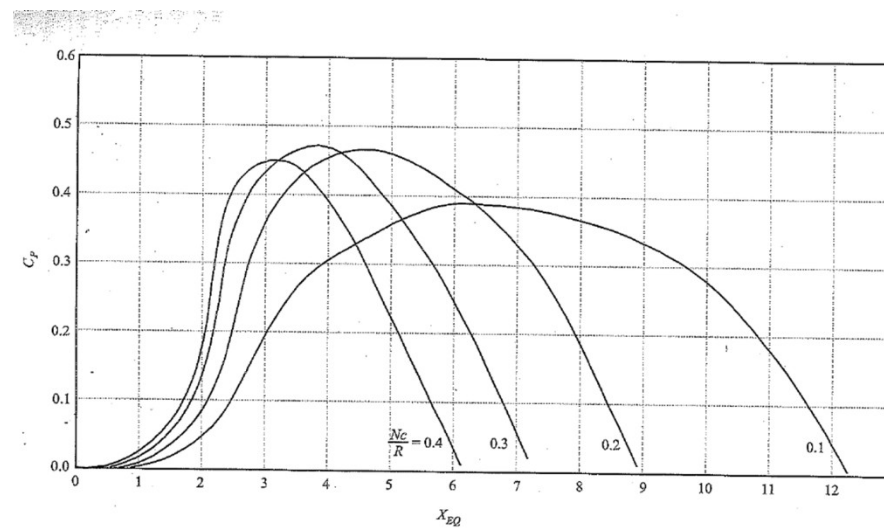
1) Strip theory (Single disc or double disc)

2) "Unsteady BEM" method

3) 2-D Vortex methods

4) 2-D/3-D Actuator Line theory

5) 2-D/3-D Fully resolved CFD



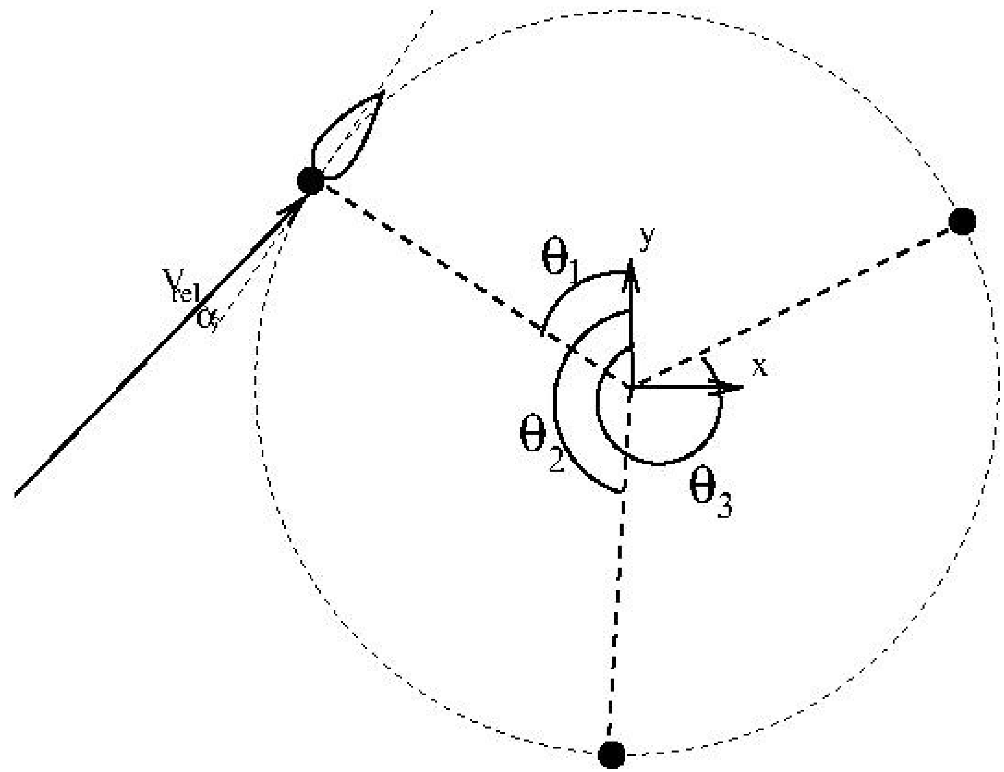
“Unsteady BEM” approach - runs in the time domain

The azimuthal positions of the blades are updated in every time step and the aerodynamic loads at the blade positions estimated from the velocity triangle

$$\theta_1(t+\Delta t) = \theta_1(t) + \omega \Delta t$$

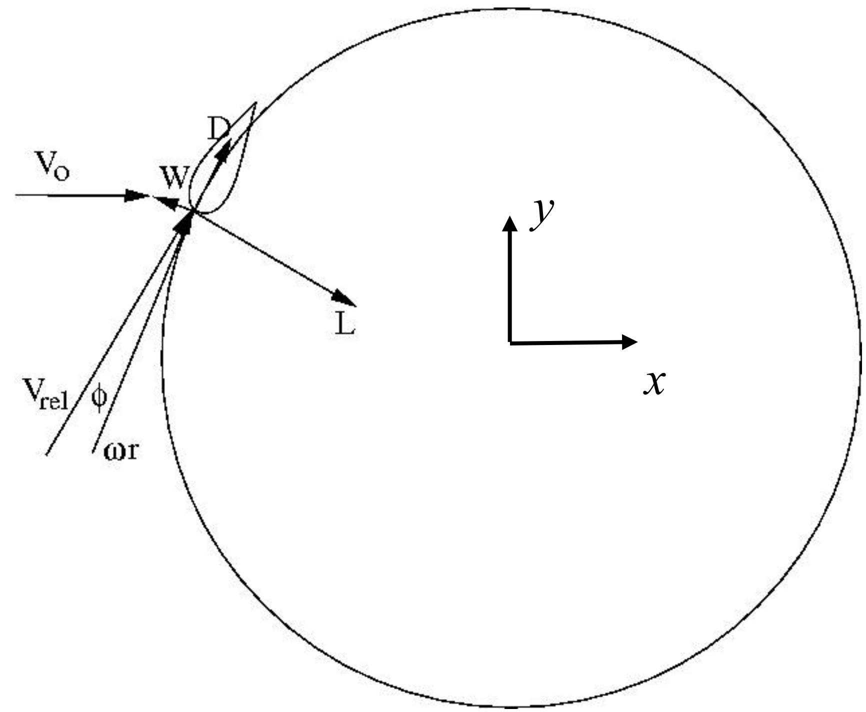
$$\theta_2(t+\Delta t) = \theta_1(t+\Delta t) + 2\pi/3$$

$$\theta_3(t+\Delta t) = \theta_1(t+\Delta t) + 4\pi/3$$

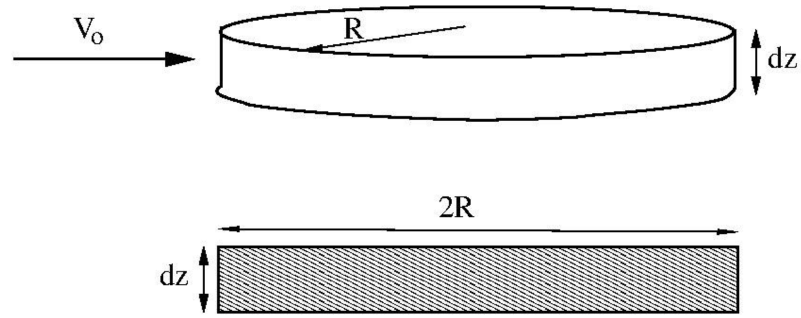


For each time step for each blade and for a **known** induced wind, W_i
i denotes blade nr. i

$$x_i = -R\sin\theta_i$$
$$y_i = R\cos\theta_i$$
$$V_{rel,x,i} = \omega y_i + V_o - W_{x,i}$$
$$V_{rel,y,i} = -\omega x_i + W_{y,i}$$
$$V_{norm,i} = (V_o - W_{x,i})\sin\theta_i - W_{y,i}\cos\theta_i$$
$$V_{\tan,i} = (V_o - W_{x,i})\cos\theta_i + W_{y,i}\sin\theta_i + \omega R$$
$$V_{rel,i}^2 = V_{norm,i}^2 + V_{\tan,i}^2$$
$$\phi_i = \mathrm{Arctan}(V_{norm,i} / V_{\tan,i})$$
$$\alpha_i = \phi_i - \theta_p$$
$$l_i = \tfrac{1}{2}\rho V_{rel,i}^2 c C_l(\alpha_i)$$
$$d_i = \tfrac{1}{2}\rho V_{rel,i}^2 c C_d(\alpha_i)$$
$$p_{n,i} = l_i\cos\phi_i + d_i\sin\phi_i$$
$$p_{t,i} = l_i\sin\phi_i - d_i\cos\phi_i$$
$$p_{x,i} = p_{n,i}\sin\theta_i - p_{t,i}\cos\theta_i$$
$$p_{y,i} = -p_{n,i}\cos\theta_i - p_{t,i}\sin\theta_i$$



The induced wind can be estimated from the loads.
First solve for the axial induction, a^{qs}, that is **in equilibrium** with the loads



Blade element and definition of C_T

$$C_T = \frac{\left(\frac{dT}{dz}\right) dz}{\frac{1}{2}\rho V_o^2 2R dz} = \frac{\sum_1^{NB} p_{x,i}}{\rho V_o^2 R}$$

Madsen approximation of steady momentum equation

$$a^{qs} = 0.286 \cdot C_T + 0.0586 \cdot C_T^2 + 0.0883 \cdot C_T^3$$

Time delay

The induced wind does not respond infinitely fast to a new thrust force **due to inertia,** and this **physical** phenomena can be **modelled** as a first order time filter

$$\frac{da(t)}{dt} \approx \frac{a^{qs} - a(t)}{\tau}$$ $a(t)$ is lagging the quasi steady value with a time constant τ

$$\tau \approx \frac{2R}{V_o}$$

$$\frac{1}{(a - a^{qs})} da = -\frac{dt}{\tau}$$

$$\int_{a(t-\Delta t)}^{a(t)} \frac{1}{(a(t) - a^{qs})} da = \frac{-1}{\tau} \int_{t-\Delta t}^{t} dt$$

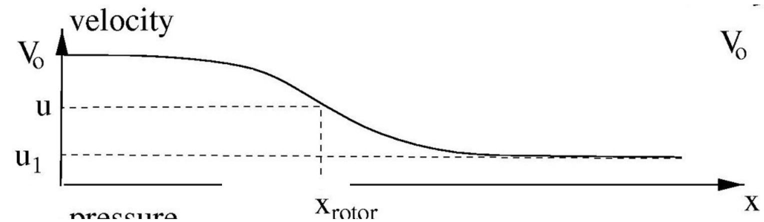
$$\boxed{a(t) = a^{qs} + (a(t - \Delta t) - a^{qs}) \cdot \exp\left(\frac{-\Delta t}{\tau}\right)}$$

Time relaxation with a time constant τ

The induced wind at the rotor mid plane becomes $W_{x=0} = a(t) V_o$

Opposite to a HAWT a VAWT blade also moves in the flow direction, x, and where the induced wind changes. A streamwise slope is assumed around midplane x=0

$$\frac{dW_x(x)}{dx} \approx \frac{W_{x=0}}{L} = \frac{a(t)V_o}{L}$$



By comparing with a vortex based code it appears that $L \approx 2.5R$ is a good choice (*i* denotes the blade number)

$$W_{x,i} = W_{x=0} + \frac{dW_x}{dx}\Delta x = W_{x=0} + \frac{W_{x=0}}{L}(-R\sin\theta_i) \approx W_{x=0}(1-0.4\sin\theta_i)$$

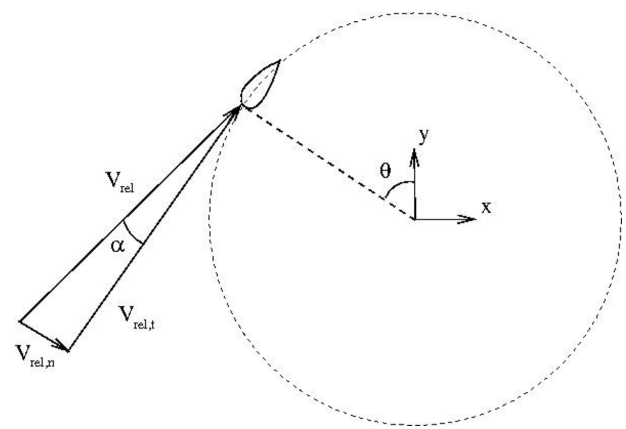
$$\boxed{W_{x,i} \approx a(t)V_o(1-0.4\sin\theta_i)}$$

Continuity equation used to estimate lateral induced wind speed

$$\frac{\Delta W_{y,i}}{\Delta y} \approx \frac{W_{y,i}}{R\cos\theta_i} = -\frac{\Delta W_{x,i}}{\Delta x} \approx \frac{W_{x,i}}{L}$$

$$\Downarrow$$

$$\boxed{W_{y,i} = \frac{R}{L}W_{x,i}\cos\theta_i = 0.4\cdot W_{x,i}\cos\theta_i}$$



So, a code simulating a VAWT in time domain “**unsteady BEM**” can look like this, where n denotes time:

One can use the old axial induction factor when estimating the induced wind at rotor midplane since the time constant in the relaxation is very large compared to the time increment

Initialize $a^{n=1}$ and $\theta_1^{n=1}$ at $t^{n=1}=0$

for $n=2:ntime$ % for each time

$t^n=(n-1)\Delta t$ % denotes time

$\theta_1^n=\theta_1^{n-1}+\omega\cdot\Delta t$ % azimuth position of blade#1

calculate induced wind at mid plane as $W_{x=0}^n=a^{n-1}V_o$

for $i=1:B$ %for each blade, i denotes blade nr.

$\theta_i^n=\theta_1^n+2\pi(i-1)/B$ %azimuth angle phase shifted for the other blades

calculate $W_{x,i}^n$ and $W_{y,i}^n$ (slide#15)

calculate $p_{x,i}^n$ and $p_{y,i}^n$ (slide#12)

end %blade loop ends

Compute C_T and C_p (slide#17)

update a^n from the quasi steady value and using the time relaxation (slides#13-14)

end % time loop ends

At each time step the thrust and torque can be calculated as

After the loads have been calculated the power per blade length can be calculated as

$$\frac{dT}{dz}(t) = \sum_1^B p_{x,i}$$

$$\frac{dP}{dz}(t) = \omega R \sum_1^B p_{t,i}$$

$$C_T(t) = \frac{\frac{dT}{dz}dz}{\frac{1}{2}\rho V_o^2 2Rdz} = \frac{\sum_1^B p_{x,i}}{\rho V_o^2 R}$$

$$C_p(t) = \frac{\frac{dP}{dz}dz}{\frac{1}{2}\rho V_o^3 2Rdz} = \frac{\omega \sum_1^B p_{t,i}}{\rho V_o^3}$$