First, I get the three equations for VB, as seen from the three perspectives of the turbines and grid respectively.

Turbine 1

```
clear

fprintf("Turbine 1 equations")
```

```
Turbine 1 equations
```

```
syms VPOC I11 I12 I21 I22 I31 I32 P1 P2 VA1 VA2 VB VS1 VS2 Z11 Z12 Z21 Z22 Z31 Z32 Z41 Z42 Z5

eq1 = VB == VA1 - I21 * Z21
```

eq1 = $\mathrm{VB} = \mathrm{VA}_1 - I_{21}\, Z_{21}$

```
% substitute I21
I21 = I11 - VA1 / Z31;
eq1 = subs(eq1, 'I21', I21)
```

eq1 =

$$\mathrm{VB} = \mathrm{VA}_1 - Z_{21}\left(I_{11} - \frac{\mathrm{VA}_1}{Z_{31}}\right)$$

```
% substitute VA1
VA1 = VS1 - I11 * Z11;
eq1 = subs(eq1, 'VA1', VA1)
```

eq1 =

$$\mathrm{VB} = \mathrm{VS}_1 - I_{11}\, Z_{11} - Z_{21}\left(I_{11} - \frac{\mathrm{VS}_1 - I_{11}\, Z_{11}}{Z_{31}}\right)$$

```
%substitute I11
I11 = P1 / VS1;
eq1 = subs(eq1, 'I11', I11)
```

eq1 =

$$\mathrm{VB} = \mathrm{VS}_1 - Z_{21}\left(\frac{P_1}{\mathrm{VS}_1} - \frac{\mathrm{VS}_1 - \frac{P_1\, Z_{11}}{\mathrm{VS}_1}}{Z_{31}}\right) - \frac{P_1\, Z_{11}}{\mathrm{VS}_1}$$

```
fprintf("%s", eq1)
```

```
VB == VS1 - Z21*(P1/VS1 - (VS1 - (P1*Z11)/VS1)/Z31) - (P1*Z11)/VS1
```

Turbine 2

...equation is the same as turbine 1, only a few values are different.

```
fprintf("Turbine 2 equations")
```

```
Turbine 2 equations
```

```
eq2 = subs(eq1, {VS1, Z31, P1, VS1, Z11, Z21}, {VS2, Z32, P2, VS2, Z12, Z22})
```

```
eq2 =
```

$$\mathrm{VB} = \mathrm{VS}_2 - Z_{22}\left(\frac{P_2}{\mathrm{VS}_2} - \frac{\mathrm{VS}_2 - \dfrac{P_2 Z_{12}}{\mathrm{VS}_2}}{Z_{32}}\right) - \frac{P_2 Z_{12}}{\mathrm{VS}_2}$$

```
fprintf("%s", eq2)
```

```
VB == VS2 - Z22*(P2/VS2 - (VS2 - (P2*Z12)/VS2)/Z32) - (P2*Z12)/VS2
```

Grid / Point of Connection

```
%Step 1
% substitute I31 and I32
eq3 = VPOC == VB - (I31 + I32) * Z5
```

```
eq3 =
```

$$\mathrm{VPOC} = \mathrm{VB} - Z_5\ (I_{31} + I_{32})$$

```

I31 = I21 - VB / Z41;
I32 = I22 - VB / Z42;

eq3 = subs(eq3, {'I31', 'I32'}, {I31, I32})
```

```
eq3 =
```

$$\mathrm{VPOC} = \mathrm{VB} + Z_5\left(\frac{\mathrm{VA}_1}{Z_{31}} - I_{22} - I_{11} + \frac{\mathrm{VB}}{Z_{41}} + \frac{\mathrm{VB}}{Z_{42}}\right)$$

```
%Step 2
% substitute I21 and I22
I21 = I11 - VA1 / Z21;
I22 = I12 - VA2 / Z22;
eq3 = subs(eq3, {'I21', 'I22'}, {I21, I22})
```

```
eq3 =
```

$$\mathrm{VPOC} = \mathrm{VB} + Z_5\left(\frac{\mathrm{VA}_2}{Z_{22}} - I_{12} - I_{11} + \frac{\mathrm{VA}_1}{Z_{31}} + \frac{\mathrm{VB}}{Z_{41}} + \frac{\mathrm{VB}}{Z_{42}}\right)$$

```

% Step 3
% substitute VA1 and VA2
VA1 = VS1 - I11 * Z11;
VA2 = VS2 - I12 * Z12;
eq3 = subs(eq3, {'VA1', 'VA2'}, {VA1, VA2})
```

```
eq3 =
```

$$\mathrm{VPOC} = \mathrm{VB} + Z_5\left(\frac{\mathrm{VB}}{Z_{41}} - I_{12} - I_{11} + \frac{\mathrm{VB}}{Z_{42}} + \frac{\mathrm{VS}_1 - \dfrac{P_1 Z_{11}}{\mathrm{VS}_1}}{Z_{31}} + \frac{\mathrm{VS}_2 - I_{12} Z_{12}}{Z_{22}}\right)$$

```

% Step 4
```

```
% substitute I11 and I12
I11 = P1 / VS1;
I12 = P2 / VS2;
eq3 = subs(eq3, {'I11', 'I12'}, {I11, I12})
```

```
eq3 =
```

$$\mathrm{VPOC} = \mathrm{VB} + Z_5\left(\frac{\mathrm{VB}}{Z_{41}} - \frac{P_2}{\mathrm{VS}_2} - \frac{P_1}{\mathrm{VS}_1} + \frac{\mathrm{VB}}{Z_{42}} + \frac{\mathrm{VS}_2 - \frac{P_2 Z_{12}}{\mathrm{VS}_2}}{Z_{22}} + \frac{\mathrm{VS}_1 - \frac{P_1 Z_{11}}{\mathrm{VS}_1}}{Z_{31}}\right)$$

```
eq3 = VB == solve(eq3, VB)
```

```
eq3 =
```

$$\mathrm{VB} = \frac{\mathrm{VPOC} + Z_5\left(\frac{P_1}{\mathrm{VS}_1} + \frac{P_2}{\mathrm{VS}_2} - \frac{\mathrm{VS}_2 - \frac{P_2 Z_{12}}{\mathrm{VS}_2}}{Z_{22}} - \frac{\mathrm{VS}_1 - \frac{P_1 Z_{11}}{\mathrm{VS}_1}}{Z_{31}}\right)}{Z_5\left(\frac{1}{Z_{41}} + \frac{1}{Z_{42}}\right) + 1}$$

```
fprintf("%s", eq3)
```

```
VB == (VPOC + Z5*(P1/VS1 + P2/VS2 - (VS2 - (P2*Z12)/VS2)/Z22 - (VS1 - (P1*Z11)/VS1)/Z31))/(Z5*(1/Z41 + 1/Z42) + 1)
```

Criteria functions

The equations that the solver should solve

$$\mathrm{VB}_{t1}(\mathrm{VS1}) - \mathrm{VB}_{\mathrm{POC}}(\mathrm{VS1}, \mathrm{VS2}) = 0$$

$$\mathrm{VB}_{t2}(\mathrm{VS2}) - \mathrm{VB}_{\mathrm{POC}}(\mathrm{VS1}, \mathrm{VS2}) = 0$$

```
fprintf("Criteria functions:")
```

```
Criteria functions:
```

```
cc1 = rhs(eq1) - rhs(eq3);
cc2 = rhs(eq2) - rhs(eq3);
fprintf("%s", cc1)
```

```
VS1 - Z21*(P1/VS1 - (VS1 - (P1*Z11)/VS1)/Z31) - (VPOC + Z5*(P1/VS1 + P2/VS2 - (VS2 - (P2*Z12)/VS2)/Z22 - (VS1 - (P1*Z11)/VS1)/Z31))/(Z5*(1/Z41 + 1/Z42) + 1) - (P1*Z11)/VS1
```

```
fprintf("%s", cc2)
```

```
VS2 - Z22*(P2/VS2 - (VS2 - (P2*Z12)/VS2)/Z32) - (VPOC + Z5*(P1/VS1 + P2/VS2 - (VS2 - (P2*Z12)/VS2)/Z22 - (VS1 - (P1*Z11)/VS1)/Z31))/(Z5*(1/Z41 + 1/Z42) + 1) - (P2*Z12)/VS2
```

```
clear
f = 50;
w = 2 * pi * f;
```

Components

POC and 220kV export cable has two transformers between them and turbines, therefore they must be referred twice.

Wind farm transformer components are already referred to primary side, and must therefore be referred just once to the primary side of turbine transformer

Turbine 33kV cable lengths are not equal, as one turbine sits longer away in the park. These must be referred once to the primary side of turbine transformer.

```
% Point of connection, referred twice
VPOC_rr = 220e3 * 33/220 * 4/33;

% 220kV export cable, referred twice
r = 0.25; % Ohm pr. km
l = 2.5e-3; % Henry pr. km
c = 0.05e-6 % Farad pr. km
```

```
c = 5.0000e-08
```

```
len = 30; % km

Z_ex_shunt = 1 / (j * w * c * len)
```

```
Z_ex_shunt = 0.0000e+00 - 2.1221e+03i
```

```
Z_ex_ser = r * l + j * w * l * len
```

```
Z_ex_ser = 0.0006 +23.5619i
```

```

% wind farm transformer
% secondary side has already been referred to primary, no need to do more
l1 = 2154e-6; % Henry
l2 = l1;
r1 = 834e-3; % Ohm
r2 = r1;

Z_wft = r1 + r2 + j * w * (l1 + l2)
```

```
Z_wft = 1.6680 + 1.3534i
```

```

% Turbine cables, 33kV
r = 0.1; % Ohm pr. km
l = 0.6e-3; % H pr. km
c = 0.3e-6; % F pr. km
len1 = 6;
len2 = 16;

Z_30k_shunt_1 = 1 / (j * w * c * len1)
```

```
Z_30k_shunt_1 = 0.0000e+00 - 1.7684e+03i
```

```
Z_30k_shunt_2 = 1 / (j * w * c * len2)
```

```
Z_30k_shunt_2 = 0.0000e+00 - 6.6315e+02i
```

```

Z_30k_ser_1 = len1 * (r + j * w * l)
```

```
Z_30k_ser_1 = 0.6000 + 1.1310i
```

```
Z_30k_ser_2 = len2 * (r + j * w * l)
```

```
Z_30k_ser_2 = 1.6000 + 3.0159i
```

```
% Turbine transformer
% secondary side has already been referred to primary, no need to do more
l1 = 10e-6;
l2 = l1;
r1 = 42e-3;
r2 = r1;
% core and magnetization
Rc = 55;
Lm = 55e-3
```

```
Lm = 0.0550
```

```
Z_wtt_sec = r2 + j * w * l2
```

```
Z_wtt_sec = 0.0420 + 0.0031i
```

```
Z_wtt_prim = r1 + j * w * l1
```

```
Z_wtt_prim = 0.0420 + 0.0031i
```

```
Z_wtt_shunt = (Rc^-1 + (j * w * Lm)^-1)^-1
```

```
Z_wtt_shunt = 4.9407 +15.7266i
```

```
% Referring
fprintf("Referred values:")
```

```
Referred values:
```

```
Z_ex_shunt = Z_ex_shunt * (33/220)^2 * (4 / 33)^2
```

```
Z_ex_shunt = 0.0000 - 0.7015i
```

```
Z_ex_ser = Z_ex_ser * (33/220)^2 * (4 / 33)^2
```

```
Z_ex_ser = 0.0000 + 0.0078i
```

```
Z_wft = Z_wft * (4 / 33)^2;
Z_30k_ser_1 = Z_30k_ser_1 * (4/33)^2
```

```
Z_30k_ser_1 = 0.0088 + 0.0166i
```

```
Z_30k_ser_2 = Z_30k_ser_2 * (4/33)^2
```

```
Z_30k_ser_2 = 0.0235 + 0.0443i
```

```
Z_30k_shunt_1 = Z_30k_shunt_1 * (4/33)^2
```

```
Z_30k_shunt_1 = 0.0000 -25.9818i
```

```
Z_30k_shunt_2 = Z_30k_shunt_2 * (4/33)^2
```

```
Z_30k_shunt_2 = 0.0000 - 9.7432i
```

```
% Equivalent impedances
Z11 = Z_wtt_prim
```

```
Z11 = 0.0420 + 0.0031i
```

```
Z12 = Z_wtt_prim
```

```
Z12 = 0.0420 + 0.0031i
```

```
Z21 = Z_wtt_sec + Z_30k_ser_1
```

```
Z21 = 0.0508 + 0.0198i
```

```
Z22 = Z_wtt_sec + Z_30k_ser_2
```

```
Z22 = 0.0655 + 0.0475i
```

```
Z31 = Z_wtt_shunt
```

```
Z31 = 4.9407 +15.7266i
```

```
Z32 = Z_wtt_shunt
```

```
Z32 = 4.9407 +15.7266i
```

```
Z41 = Z_30k_shunt_1
```

```
Z41 = 0.0000 -25.9818i
```

```
Z42 = Z_30k_shunt_2
```

```
Z42 = 0.0000 - 9.7432i
```

```
Z5 = Z_wft + Z_ex_ser
```

```
Z5 = 0.0245 + 0.0277i
```

```
Z6 = Z_ex_shunt
```

```
Z6 = 0.0000 - 0.7015i
```

```
% Power Values for testing
P1 = 9775e3;
P2 = 9775e3;

% Solver
c = [Z11, Z12, Z21, Z22, Z31, Z32, Z41, Z42, Z5, P1, P2]
```

```
c = 1×11 complex
10^6 ×
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i ···
```

```
fun = @(x)equations(x, c);
x0 = [10, 10]
```

```
x0 = 1×2
    10    10
```

```
x = fsolve(fun, x0)
```

```
Equation solved.

fsolve completed because the vector of function values is near zero
as measured by the value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>
x = 1×2 complex
10^3 ×
   1.0615 + 0.1887i   1.1525 + 0.3060i
```

Helper function

```
function F = equations(x, c)
    VS1 = x(1);
    VS2 = x(2);

    Z11 = c(1);
    Z12 = c(2);
    Z21 = c(3);
    Z22 = c(4);
    Z31 = c(5);
    Z32 = c(6);
    Z41 = c(7);
    Z42 = c(8);
    Z5 = c(9);
    P1 = c(10);
    P2 = c(11);

    VPOC = 10;

    F(1) = VS1 - Z21*(P1/VS1 - (VS1 - (P1*Z11)/VS1)/Z31) - (VPOC + Z5*(P1/VS1 + P2/VS2 - (VS2 - (P2*Z12)/VS2)/Z22 - (VS1 - (P1*Z11)/VS1)/Z31))/(Z5*(1/Z41 + 1/Z42) + 1) -
(P1*Z11)/VS1;
    F(2) = VS2 - Z22*(P2/VS2 - (VS2 - (P2*Z12)/VS2)/Z32) - (VPOC + Z5*(P1/VS1 + P2/VS2 - (VS2 - (P2*Z12)/VS2)/Z22 - (VS1 - (P1*Z11)/VS1)/Z31))/(Z5*(1/Z41 + 1/Z42) + 1) -
(P2*Z12)/VS2;
    %F(3) = (VPOC + Z5*(P1/VS1 + P2/VS2 - VA1/Z31 - VA2/Z32))/(Z5*(1/Z41 + 1/Z42) + 1);
end
```