DIRECTIONAL FIELDS

SYNTHESIS, DESIGN, AND PROCESSING

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Developed with:

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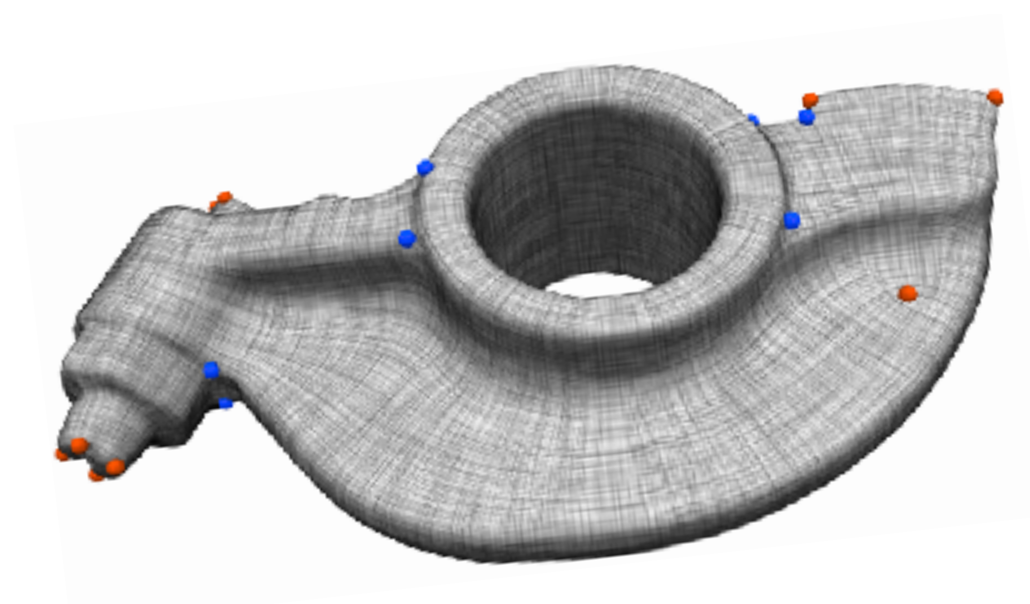
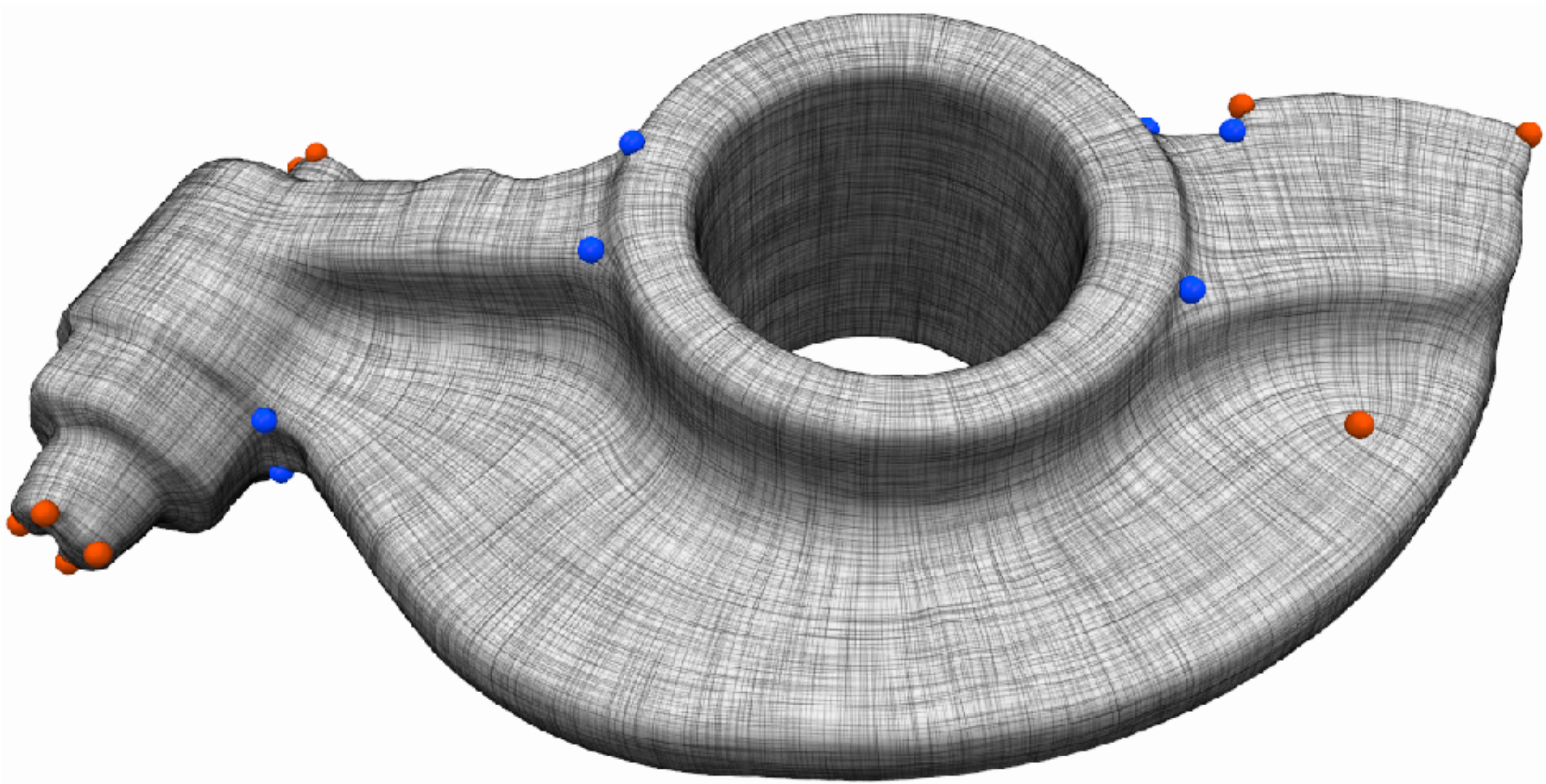
Daniele Panozzo	*New York University*
Olga Diamanti	*Stanford University*
David Bommes	*RWTH Aachen University*
Klaus Hildebrandt	*TU Delft*
Mirela Ben-Chen	*Technion*

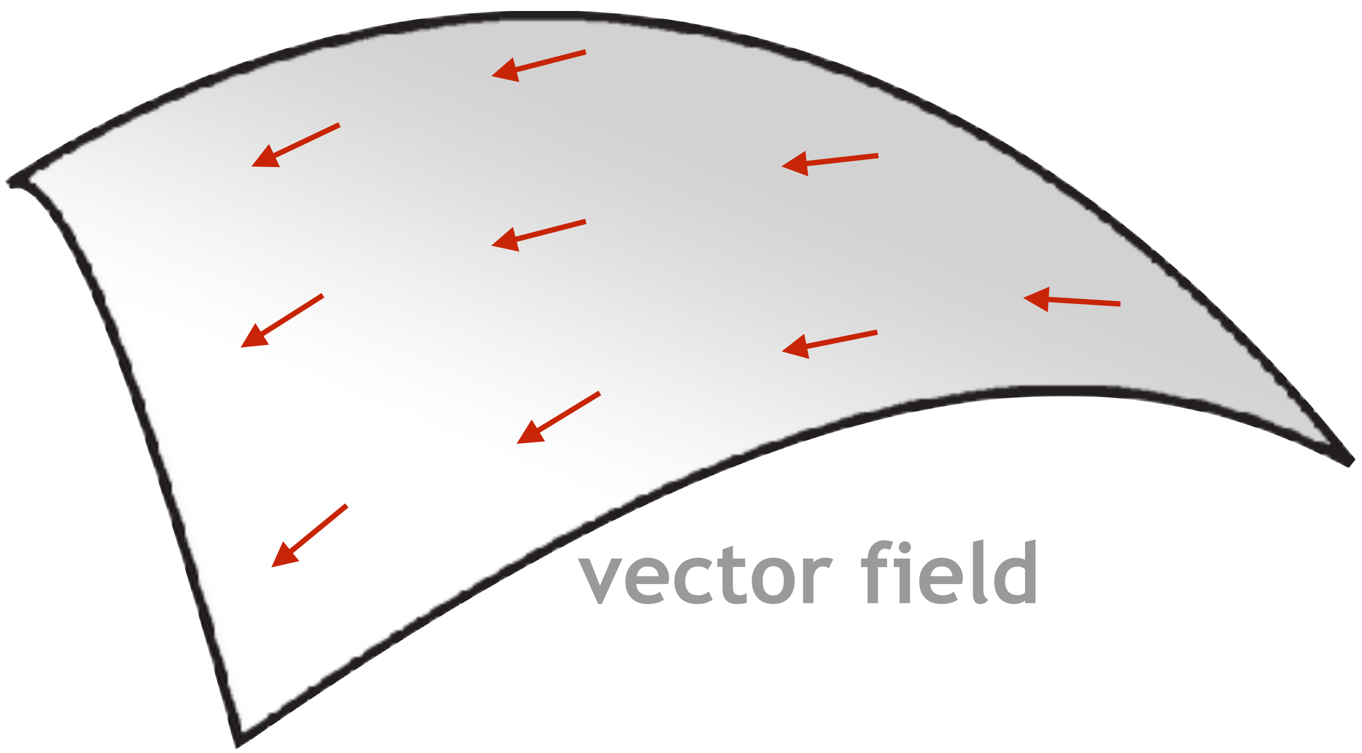


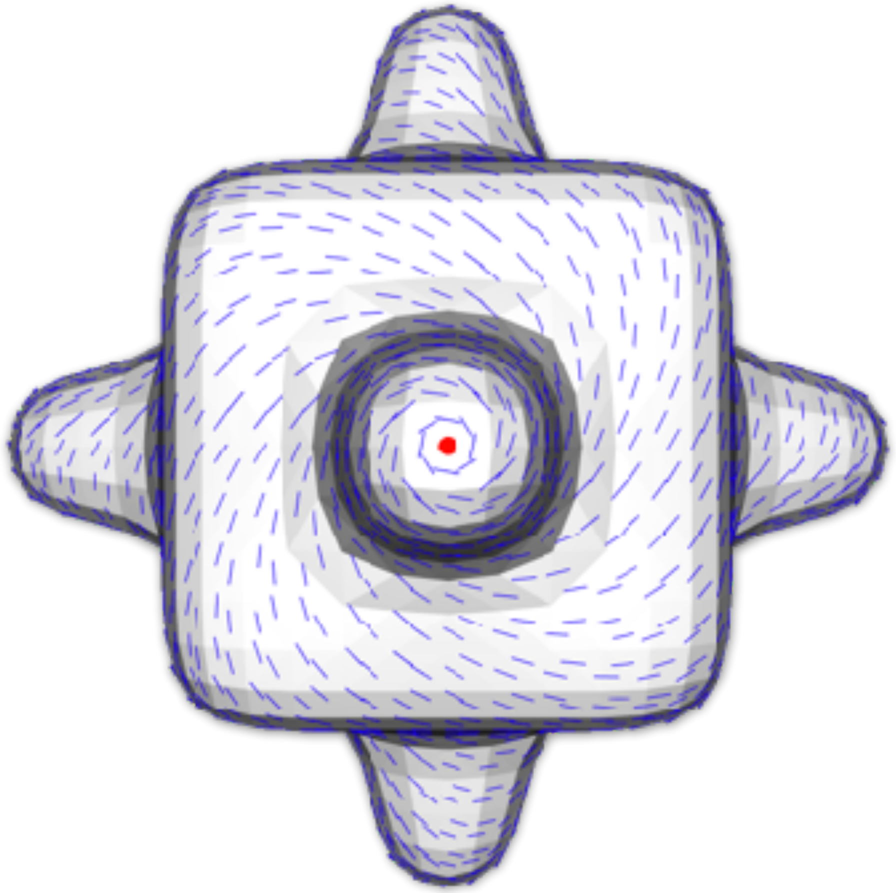
github.com/avaxman/DirectionalFieldSynthesis

DIRECTIONAL FIELDS



FIELDS ON SURFACES





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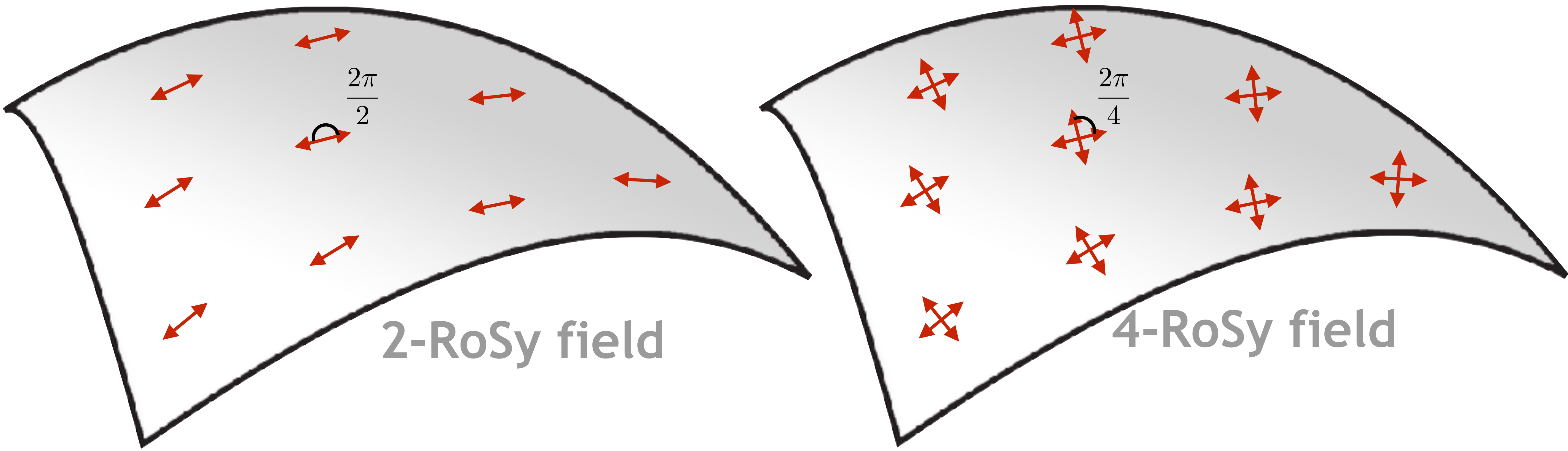
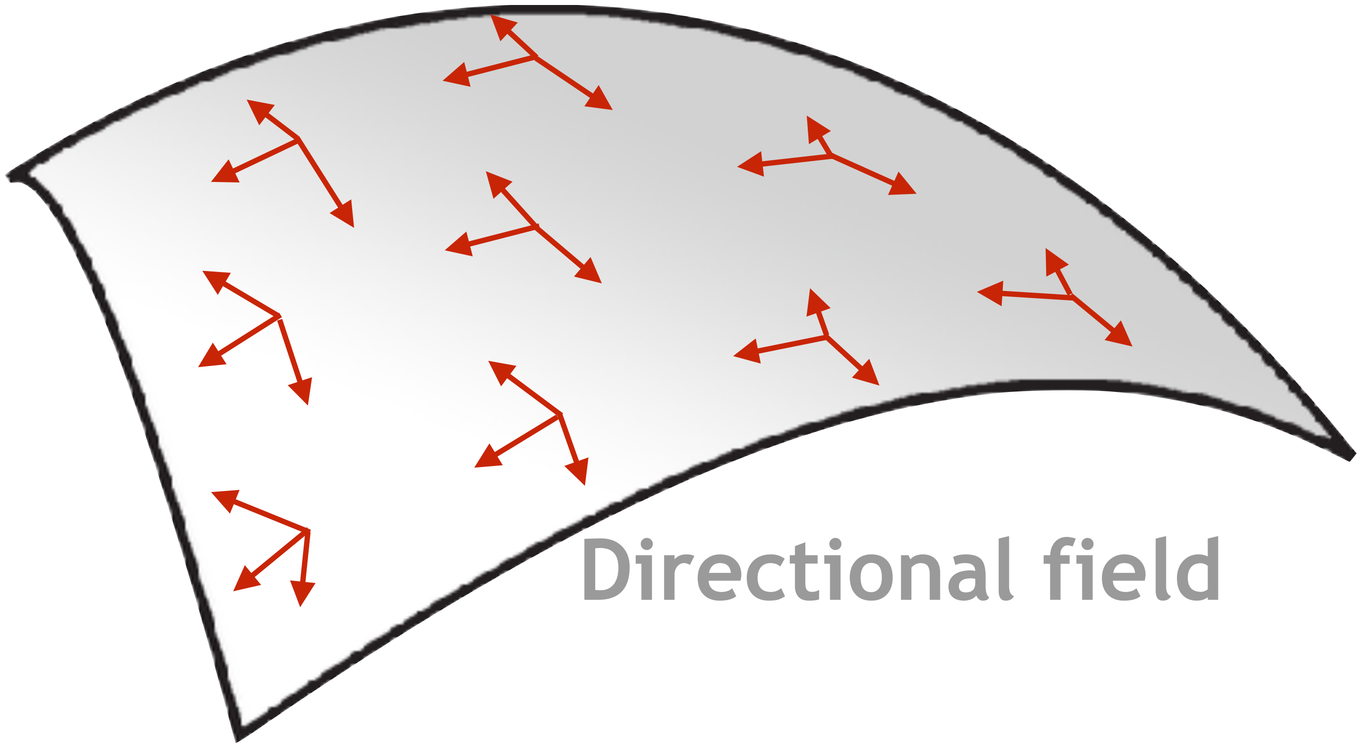
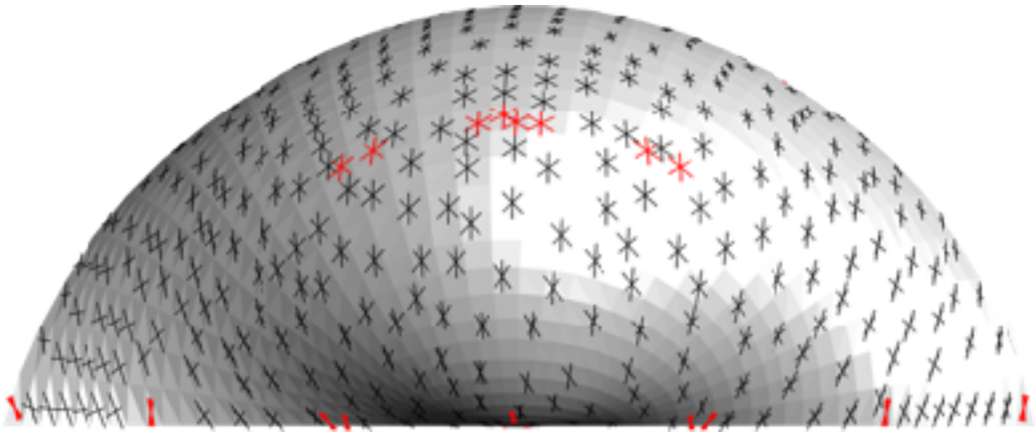


GENERAL, NON-SYMMETRIC FIELDS

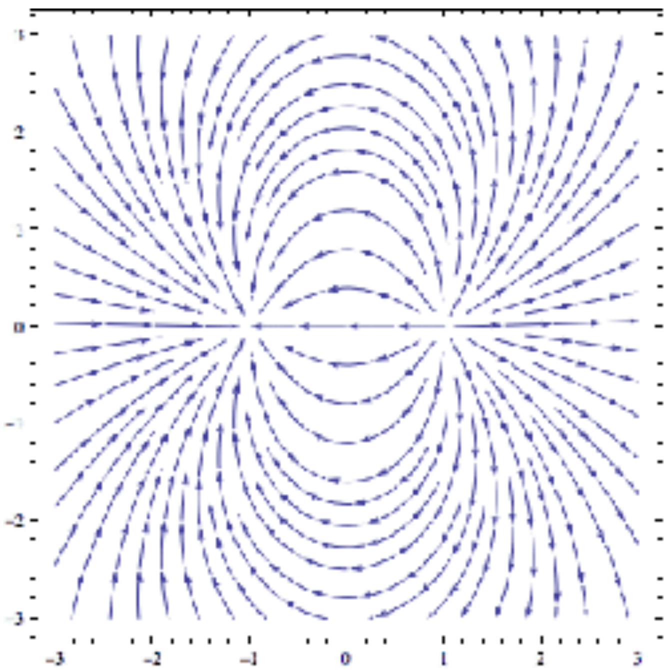


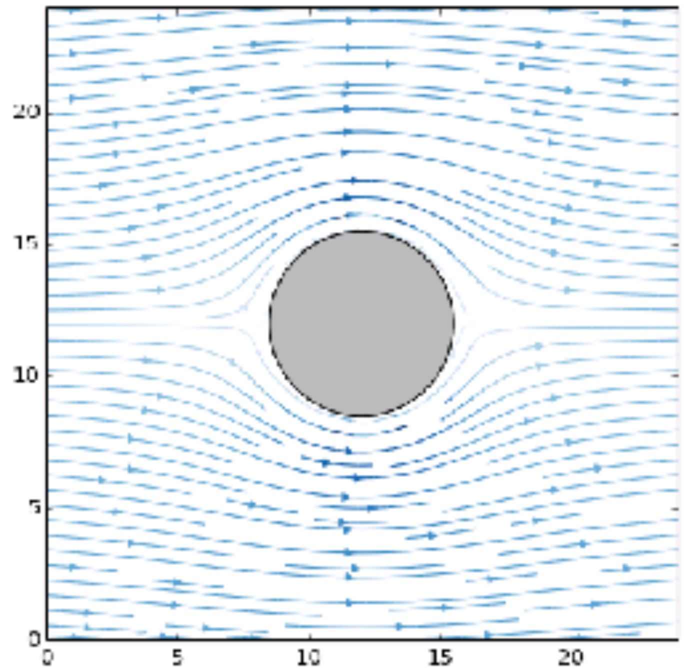


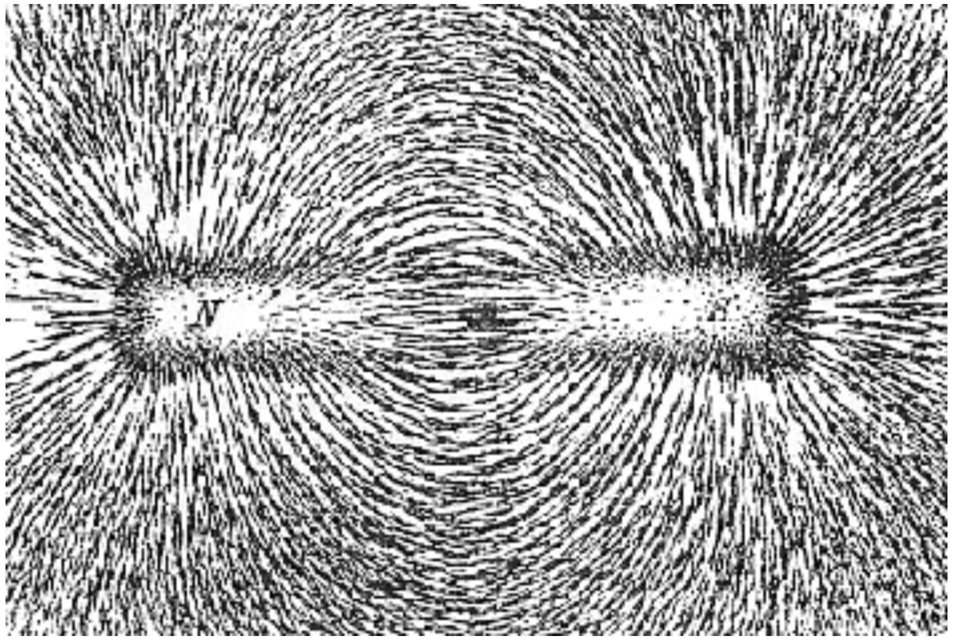
How do we create and analyze such fields?

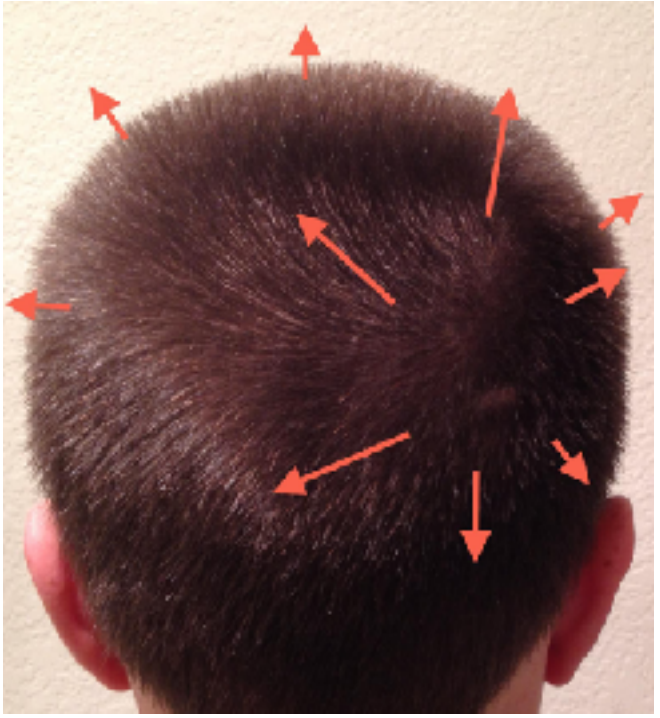
DIRECTIONAL FIELDS

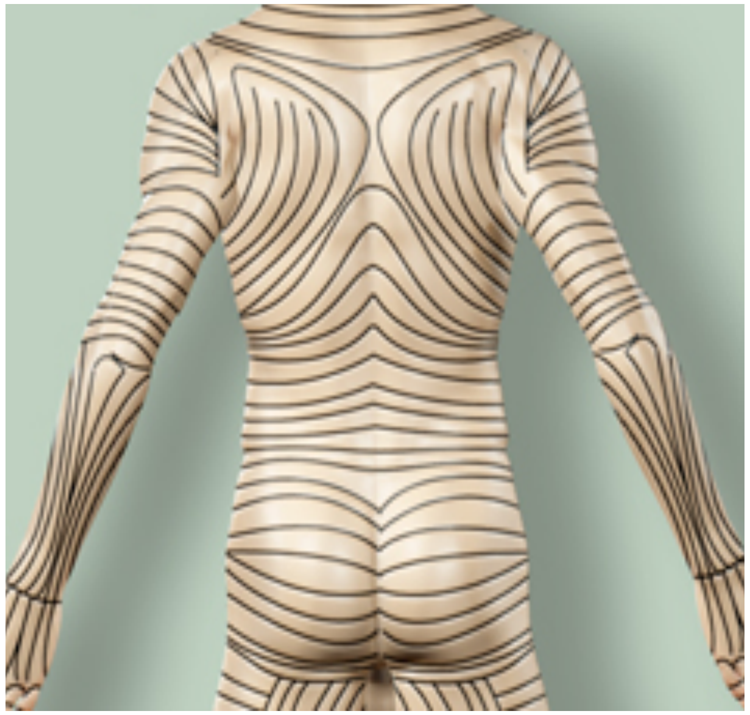
- Appearance in nature

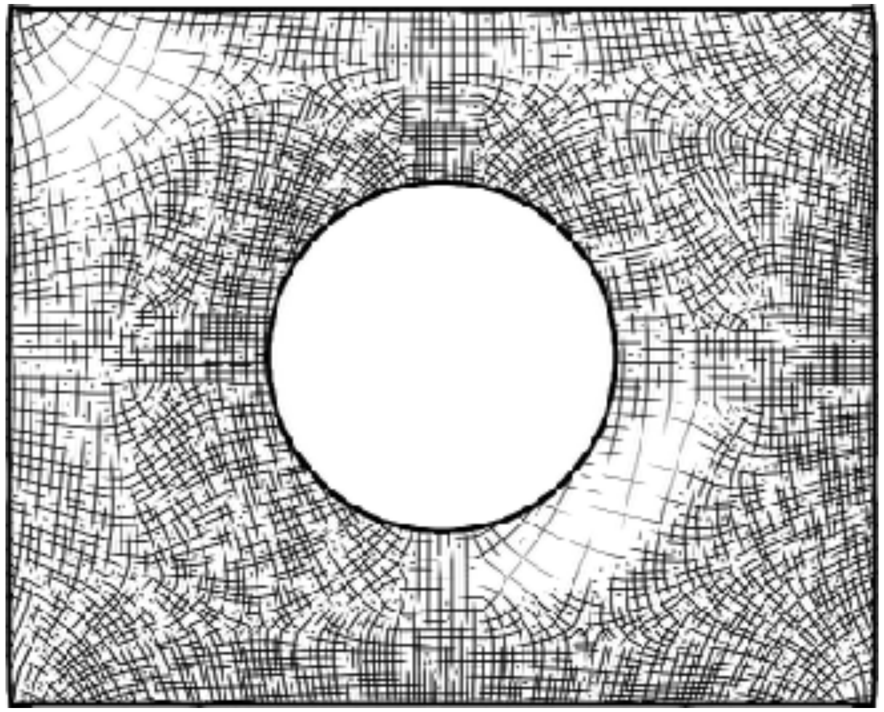








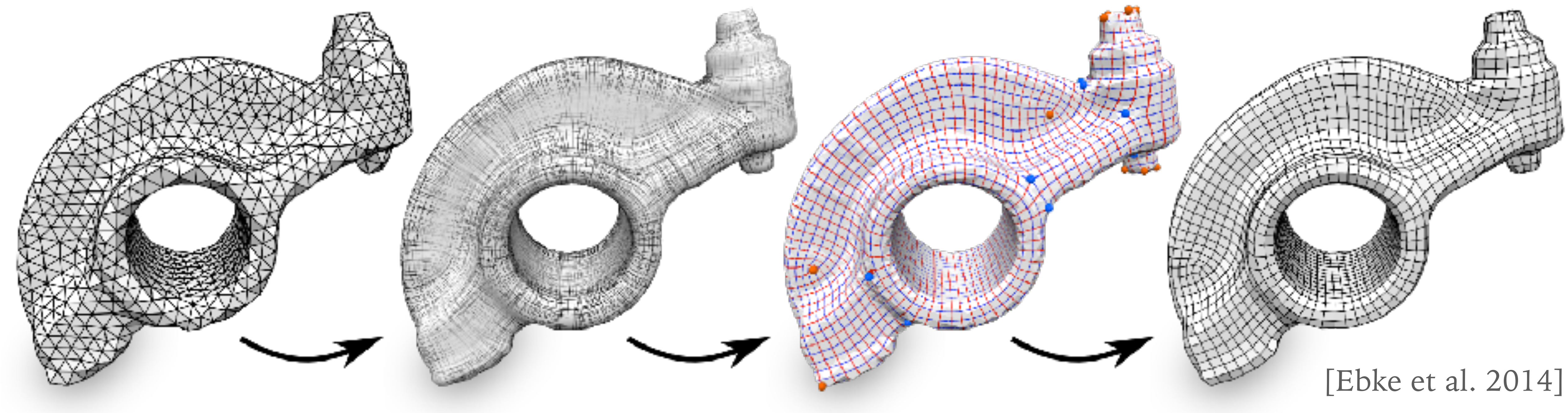




[Pietroni et al. 2015]

APPLICATION EXAMPLES

- Meshing

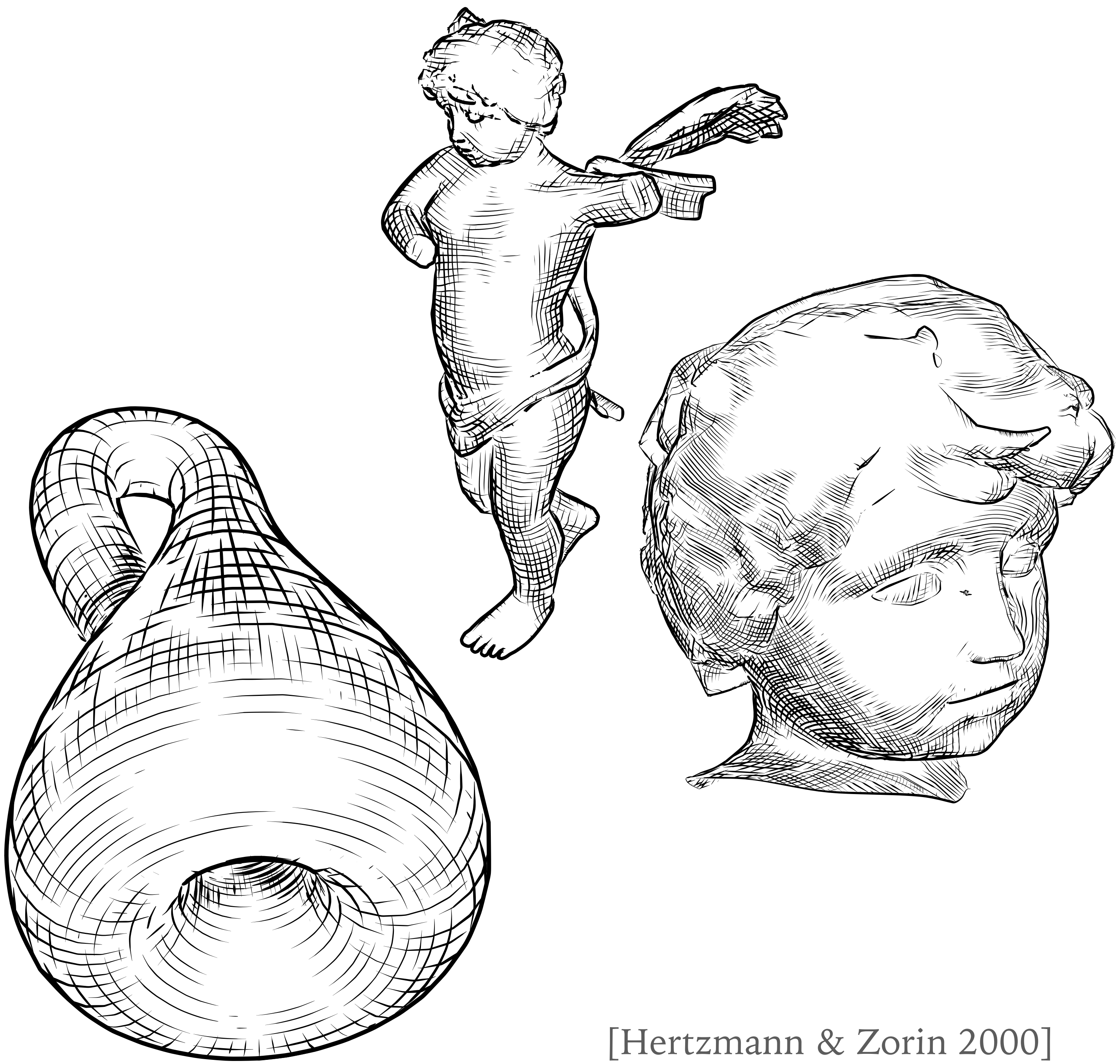


[Ebke et al. 2014]

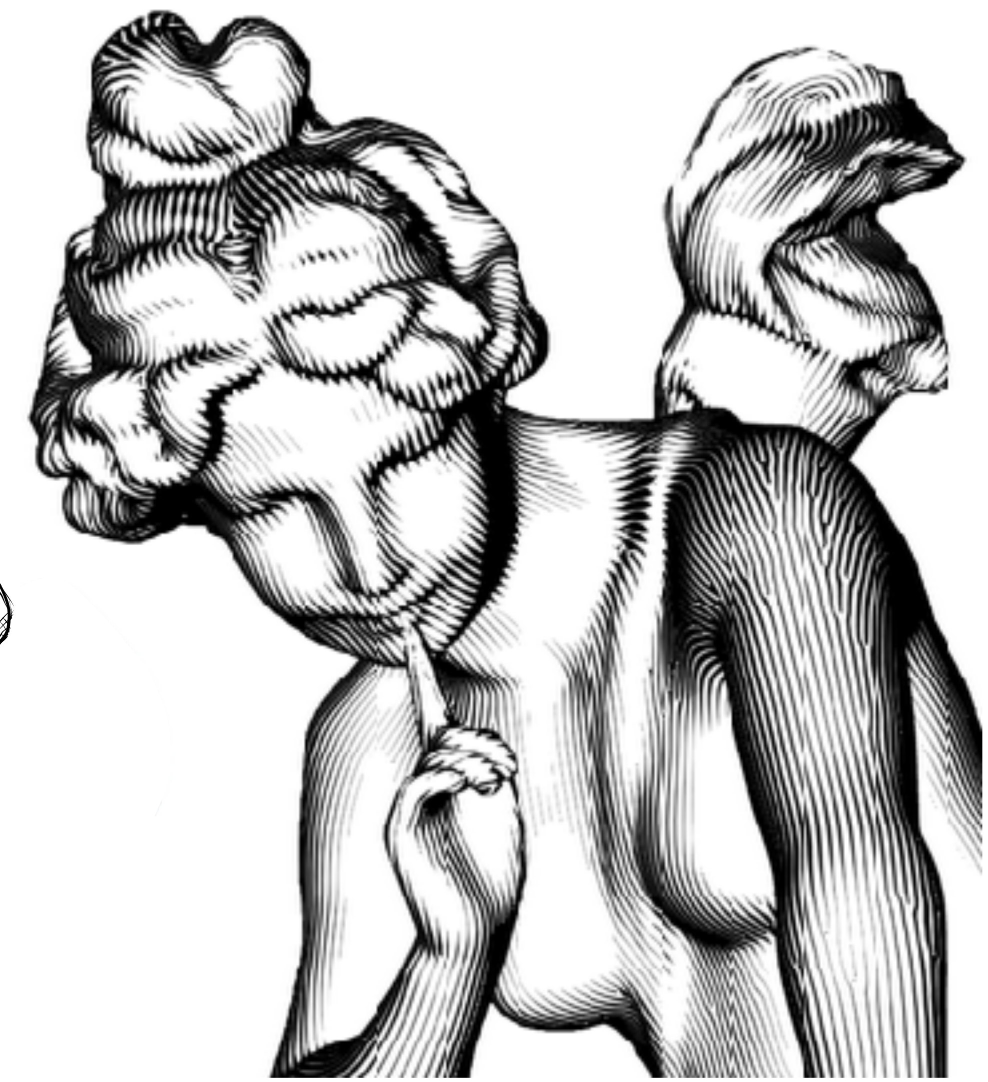
[Jakob et al. 2015]

APPLICATION EXAMPLES

- Illustration



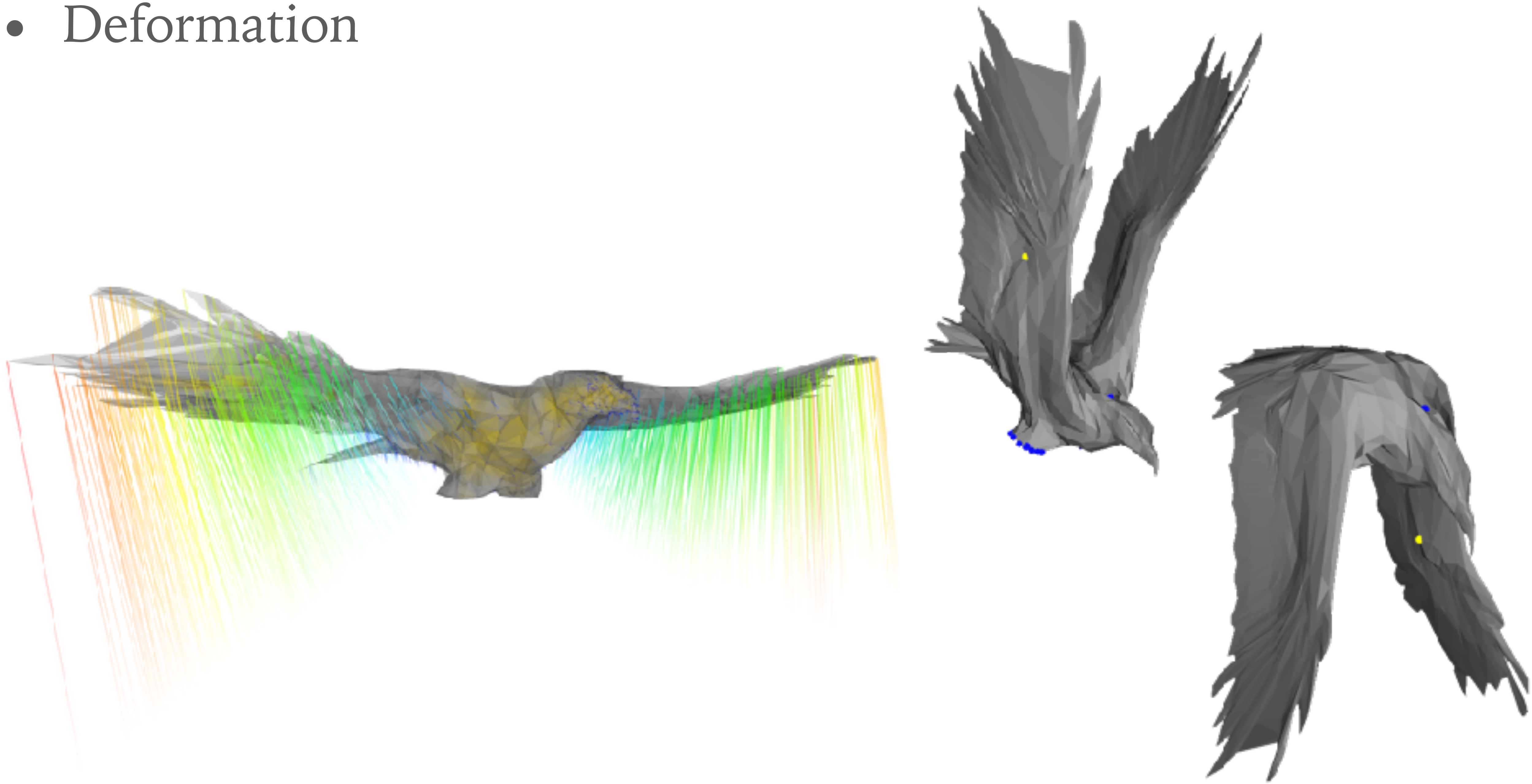
[Hertzmann & Zorin 2000]



[Knöppel et al. 2015]

APPLICATION EXAMPLES

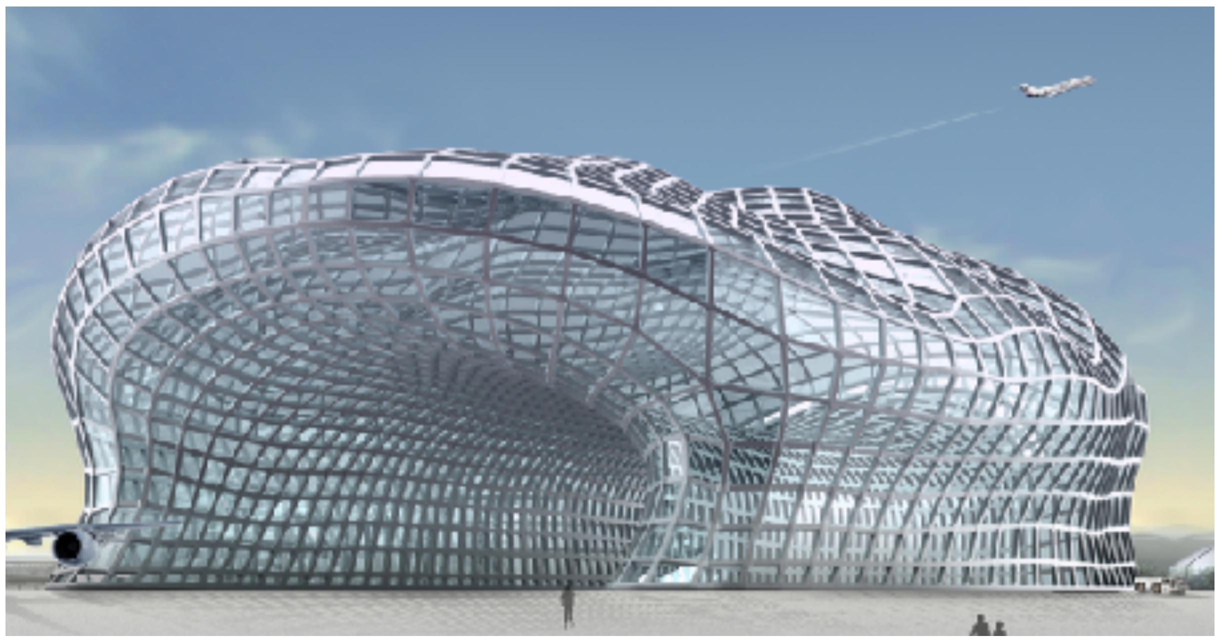
- Deformation



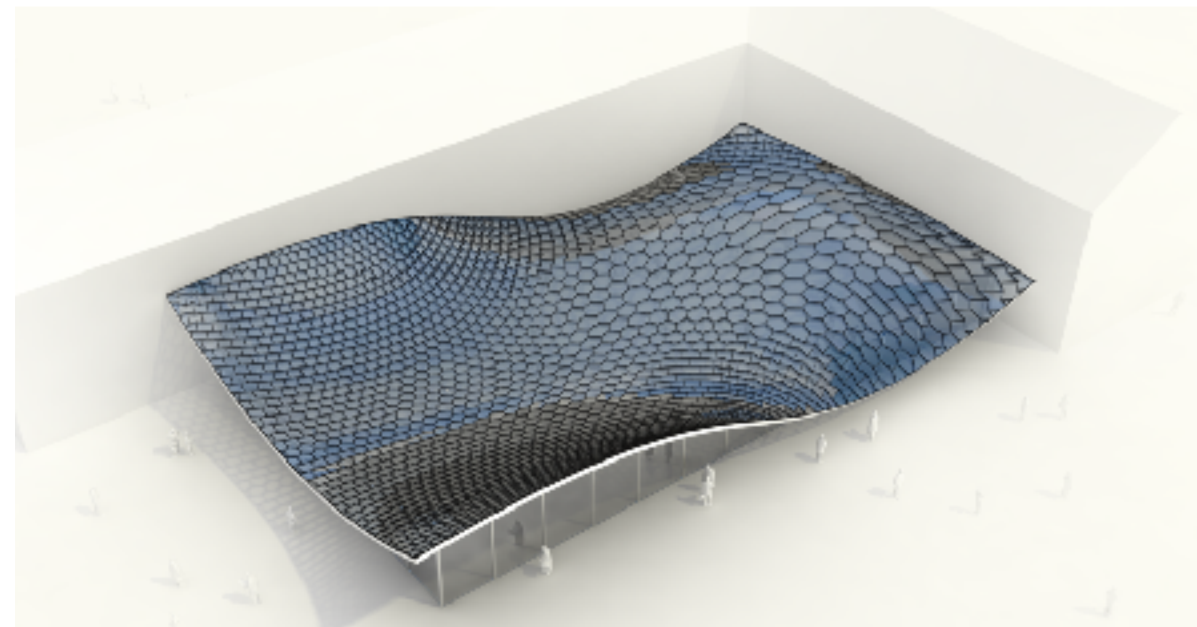
[Martinez Esturo et al. 2014]

APPLICATION EXAMPLES

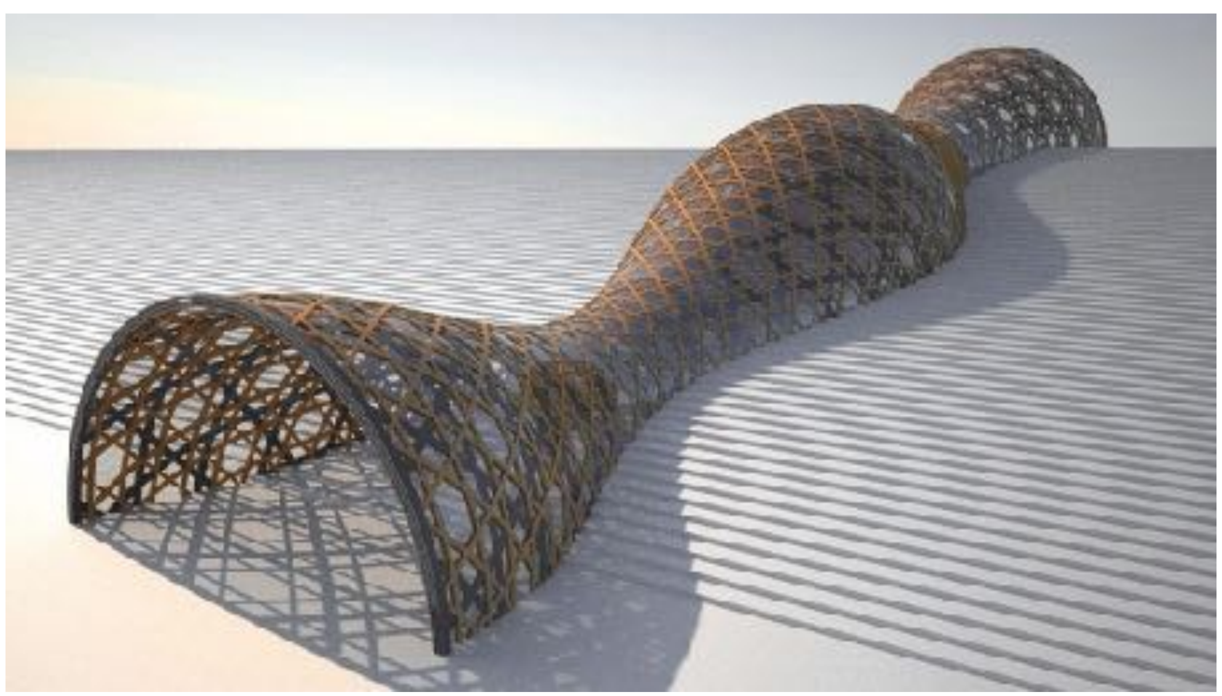
- Architecture



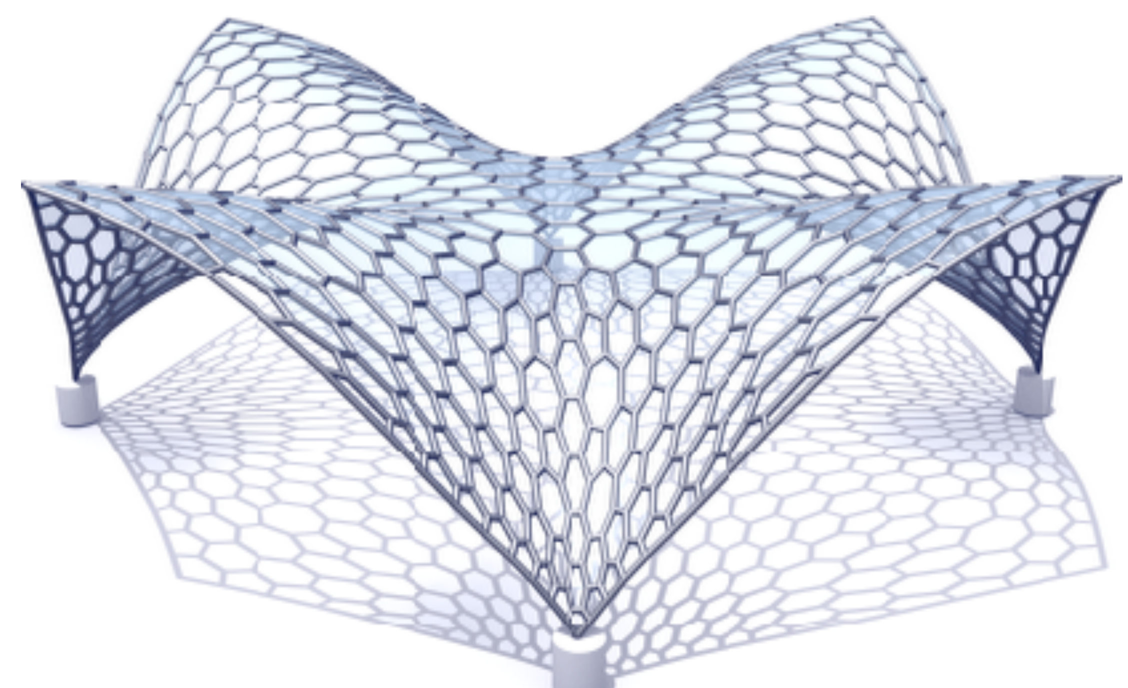
[Liu et al. 2011]



[Vaxman et al. 2015]



[Pottmann et al. 2010]



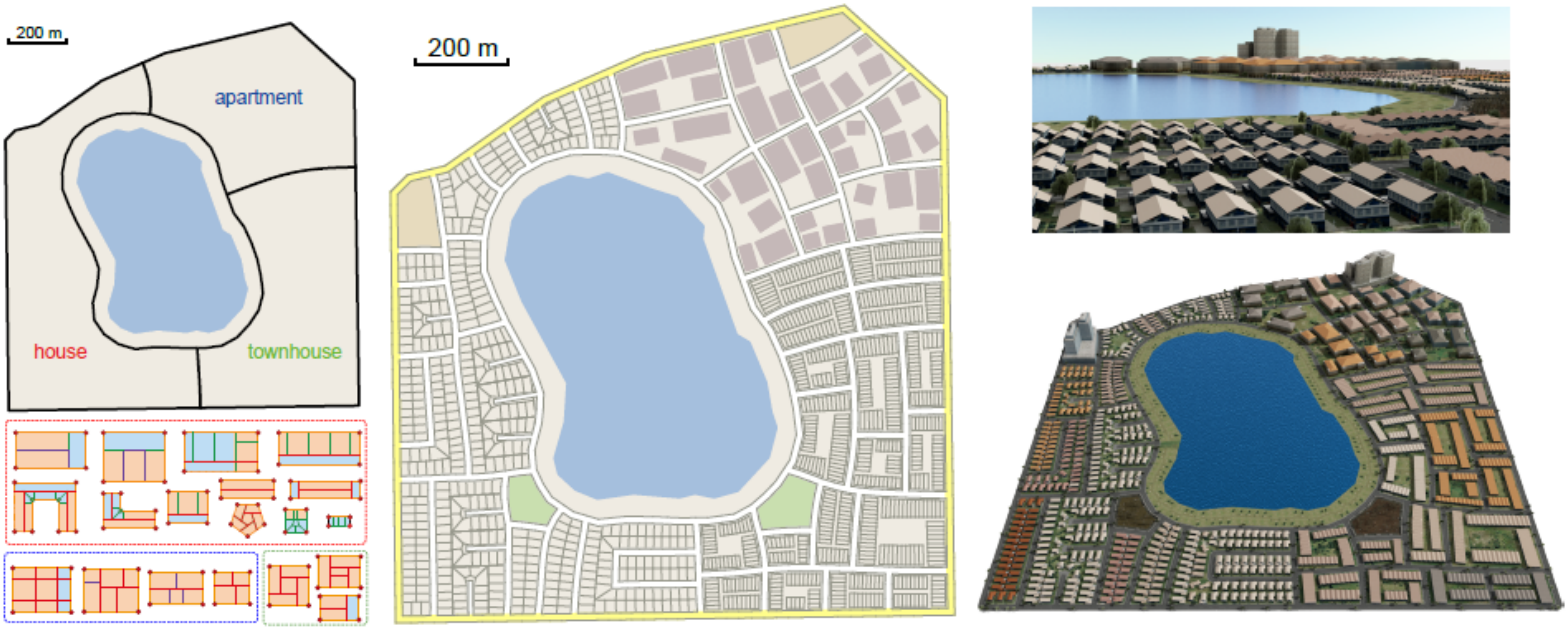
[Pietroni et al. 2015]

APPLICATION EXAMPLES

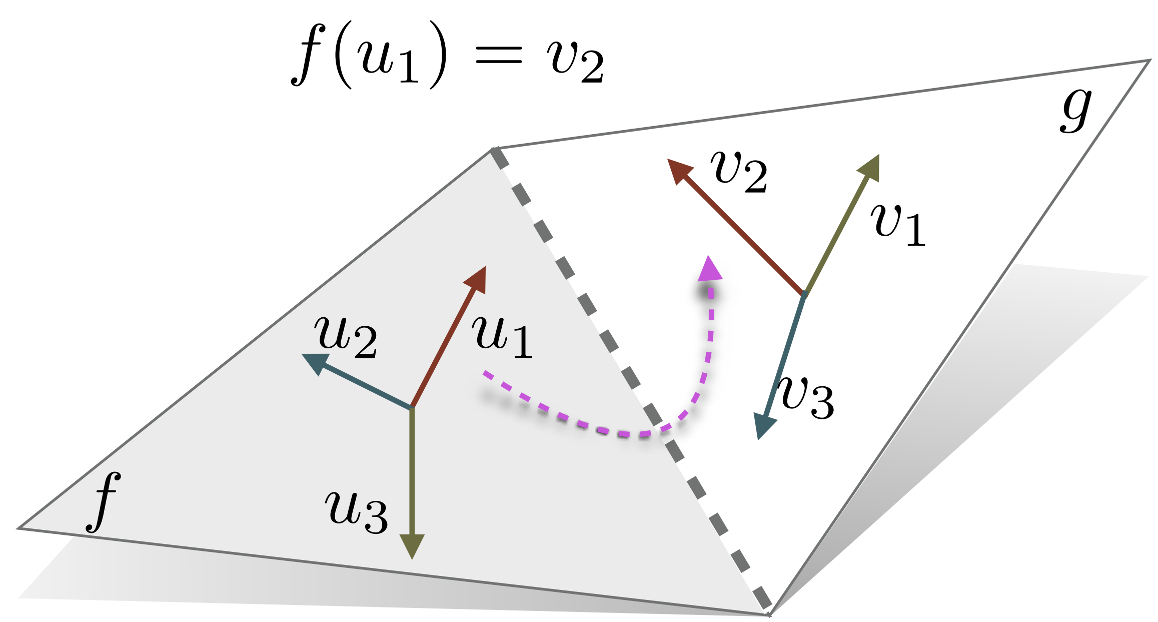
- Data Analysis

[Ferreira et al. 2013]

URBAN PLANNING

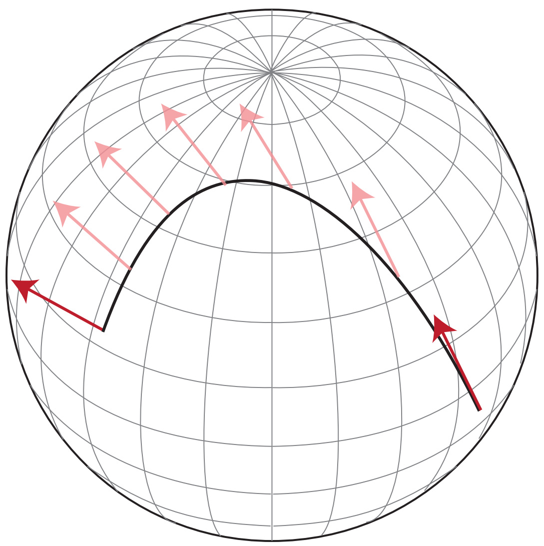


[Yang et al. 2013]



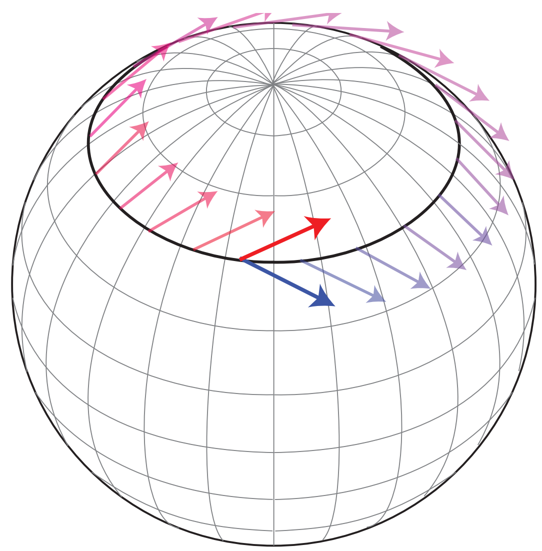
DISCRETIZATION

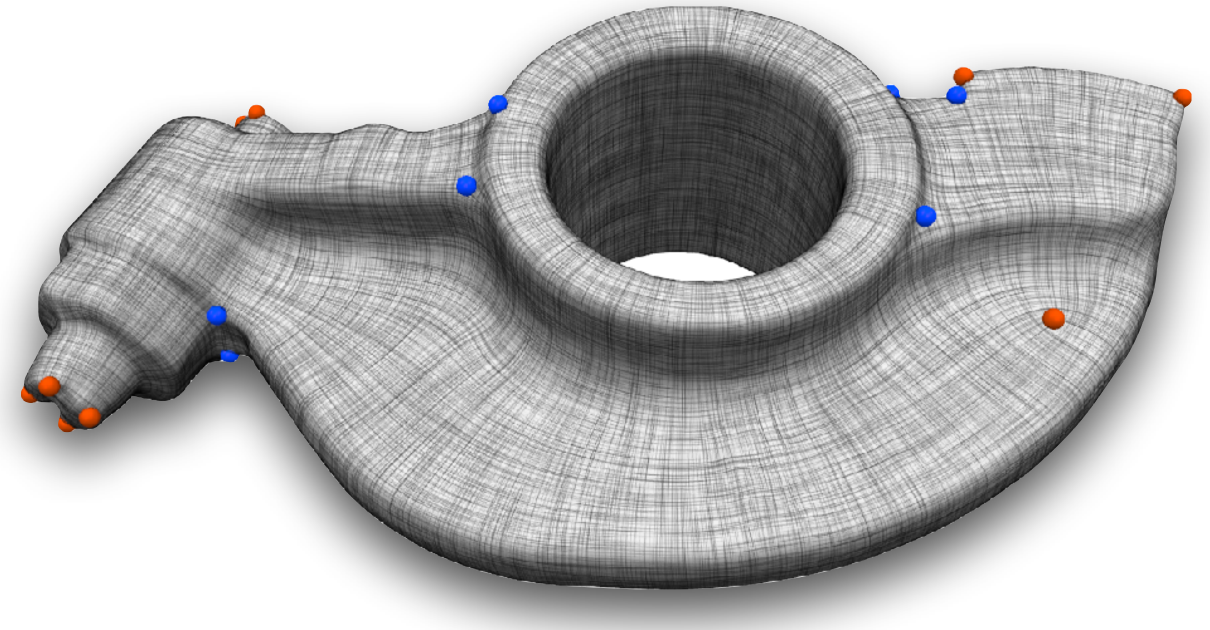
ALL IS WELL KNOWN IN THE CONTINUUM



Parallel Transport

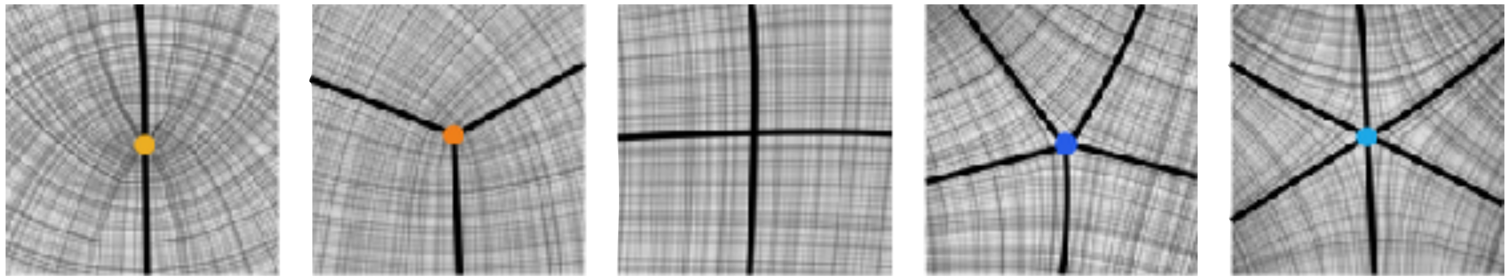
Holonomy





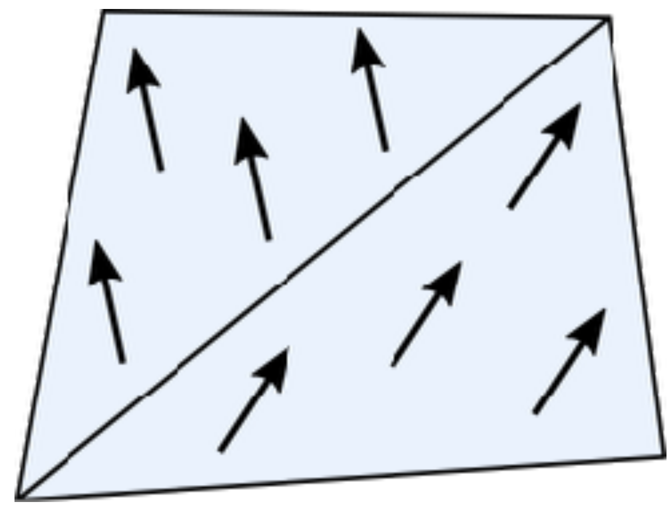
Poincare-Hopf Theorem

Singularities

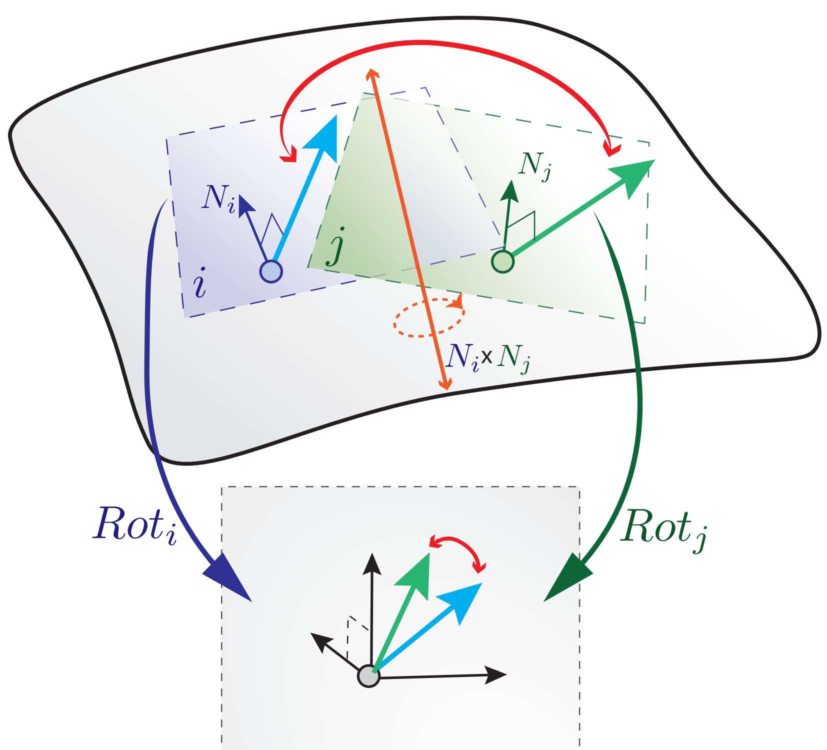


CHALLENGES IN THE DISCRETE SETTING

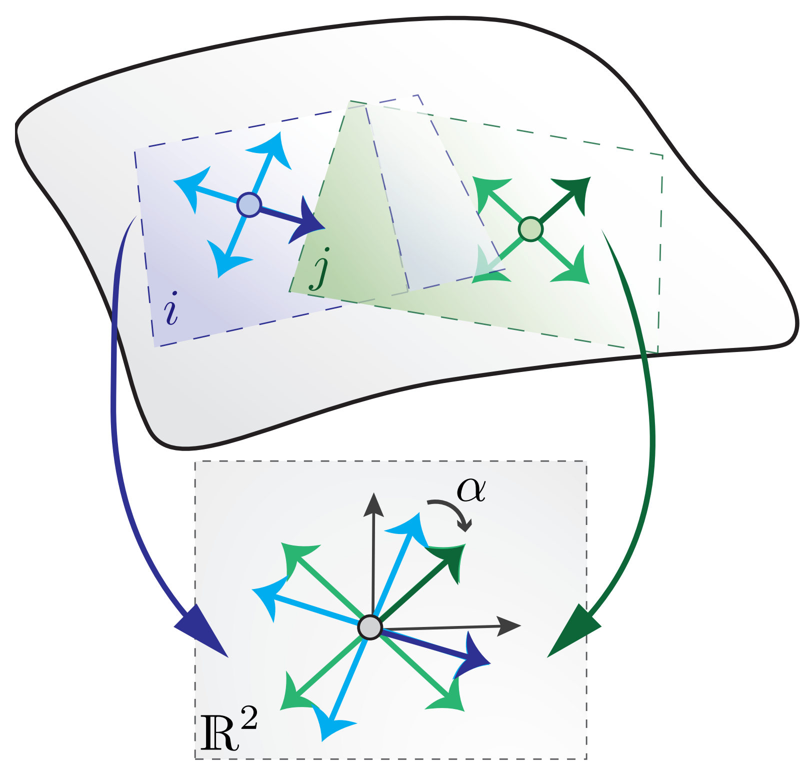
Discontinuity



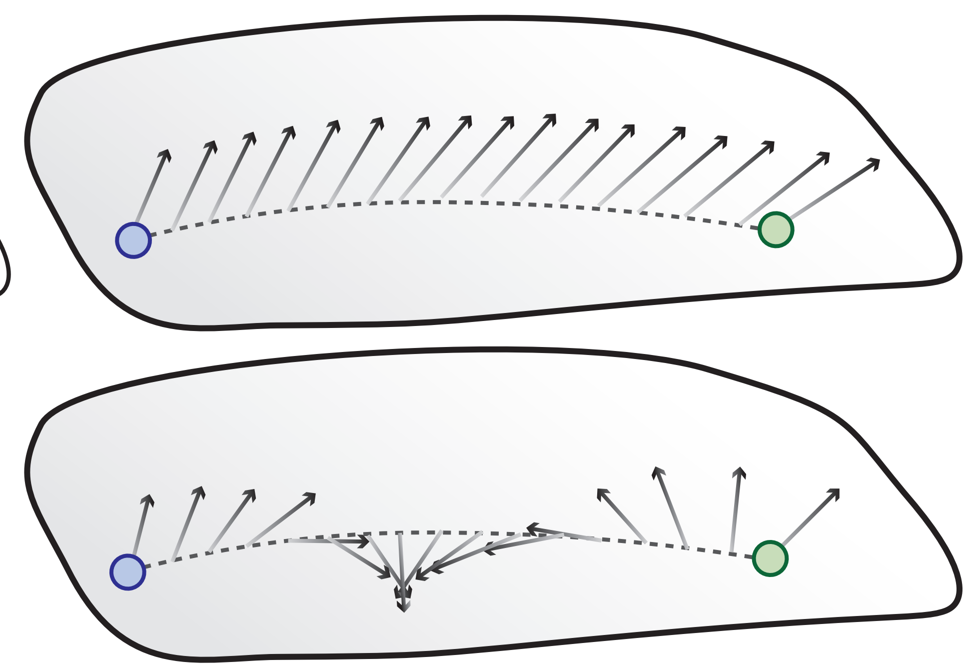
Connection



Matching

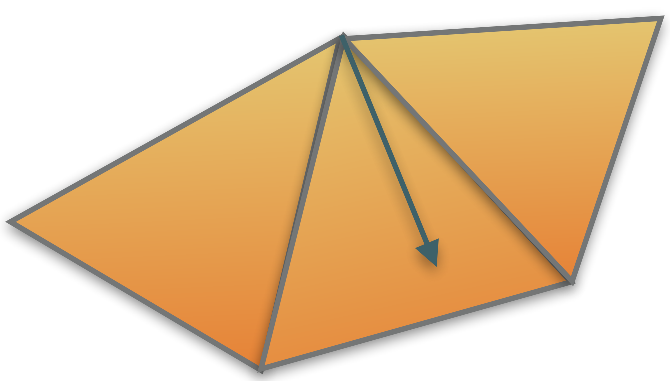


Interpolation



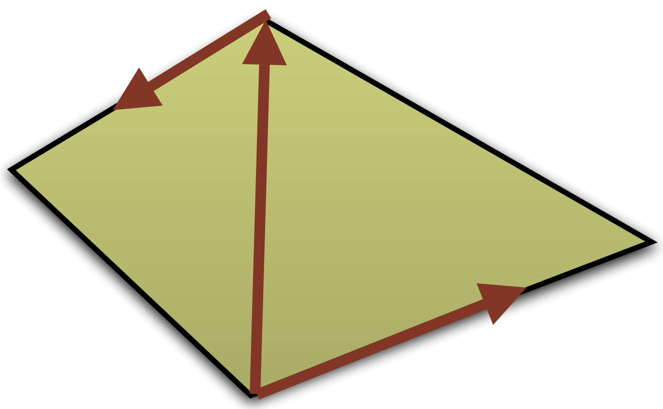
TANGENT SPACES

Vertex based



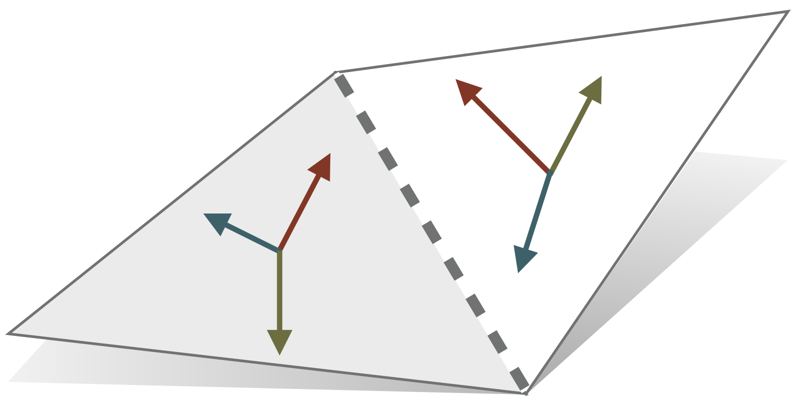
[Polthier and Schmies 98]
[Zhang *et al.* 2006]
[Knöppel *et al.* 2013]

Edge based



[Desbrun *et al.* 2005]
[Fisher *et al.* 2007]
[Ben-Chen *et al.* 2010]

Face based



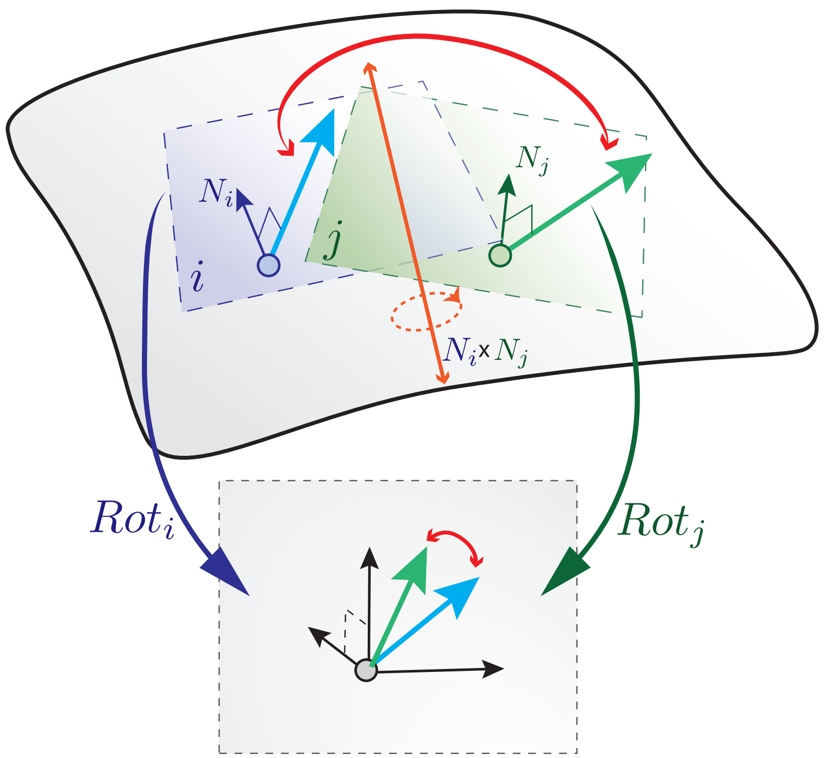
[Bommes *et al.* 2009]
[Crane *et al.* 2010]
[Diamanti *et al.* 2014]

Choice of tangent space and differential implications: see [deGoes *et al.* 2016].

DISCRETE CONNECTION

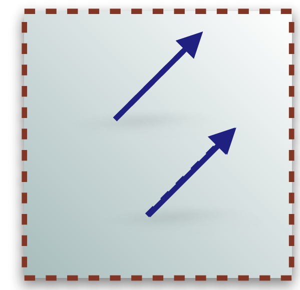
- Bijective linear map between adjacent tangent spaces.
- Popular choice: flattening + single axis system.

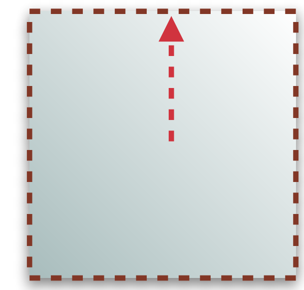
[Ray *et al.* 2008]
[Crane *et al.* 2010]
[Knöppel and Pinkall 2015]

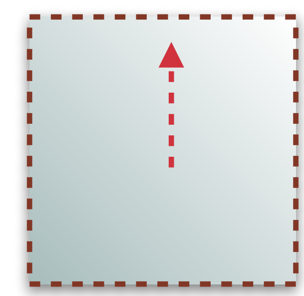


DISCRETE TOPOLOGY: ROTATION

- What happens in between?
- Valid rotation choices:







$$\delta_{ij} = \frac{\pi}{4} + 2\pi k, k \in \mathbb{Z}$$

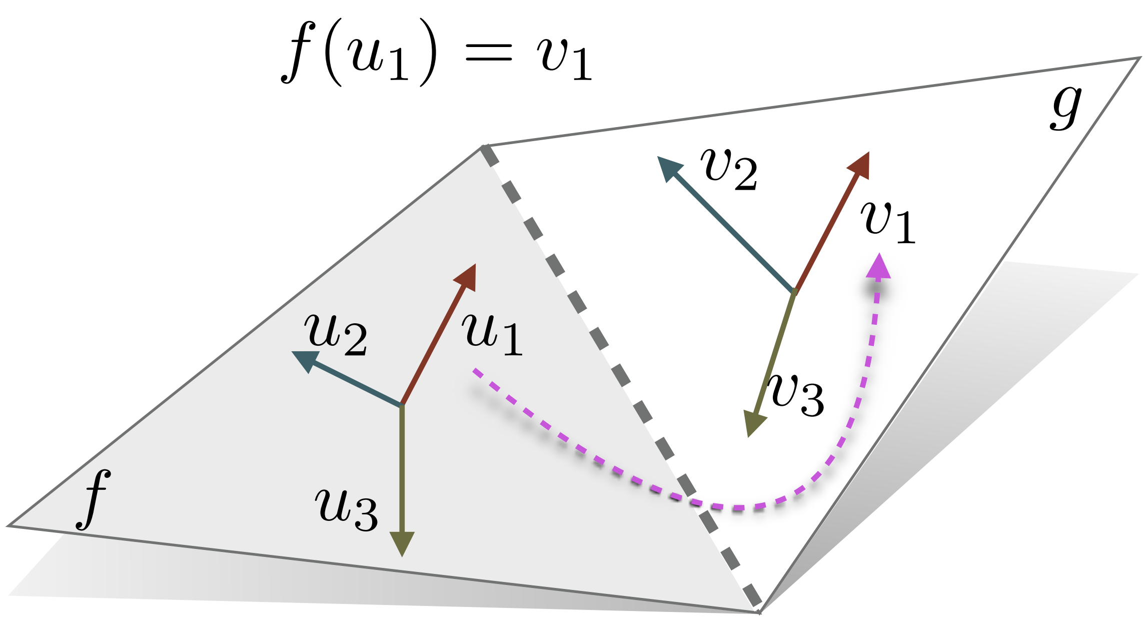
Principal Rotation *Period Jump*

[Li *et al.* 2006]

- Implicit/Cartesian: can only assume principal.
- Explicit/polar/angle-based: period given.

MATCHING

- Which direction to which other?
- Reduction: order-preserving. [Diamanti *et al.* 2014]
- *N*-directional: *N* possible choices.
    - How best to choose?



MATCHING

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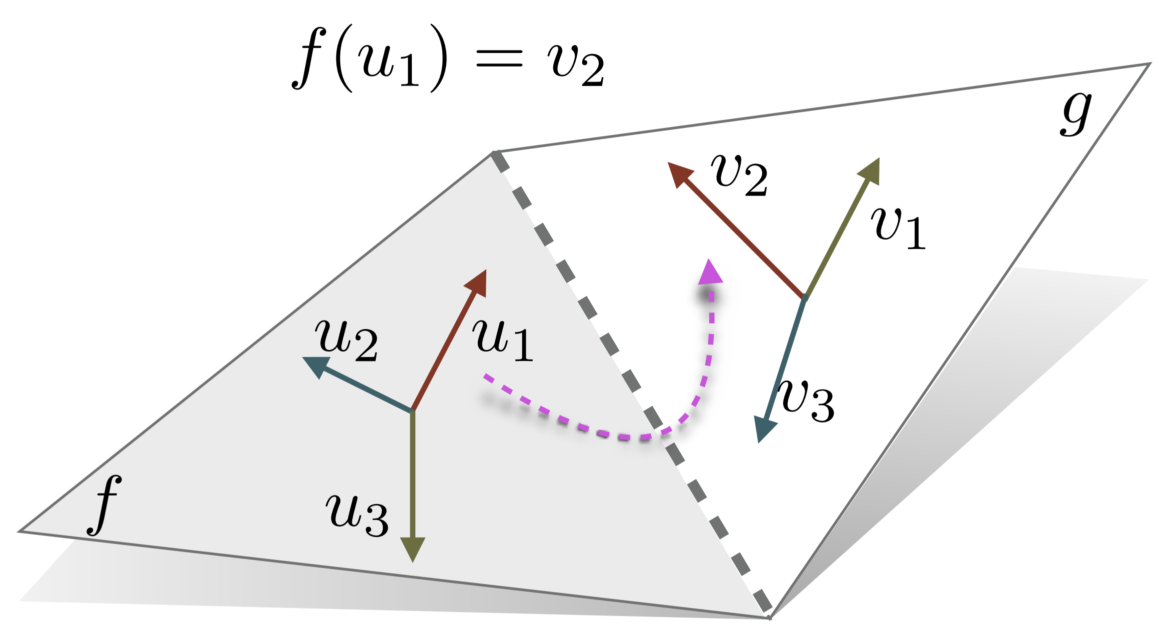
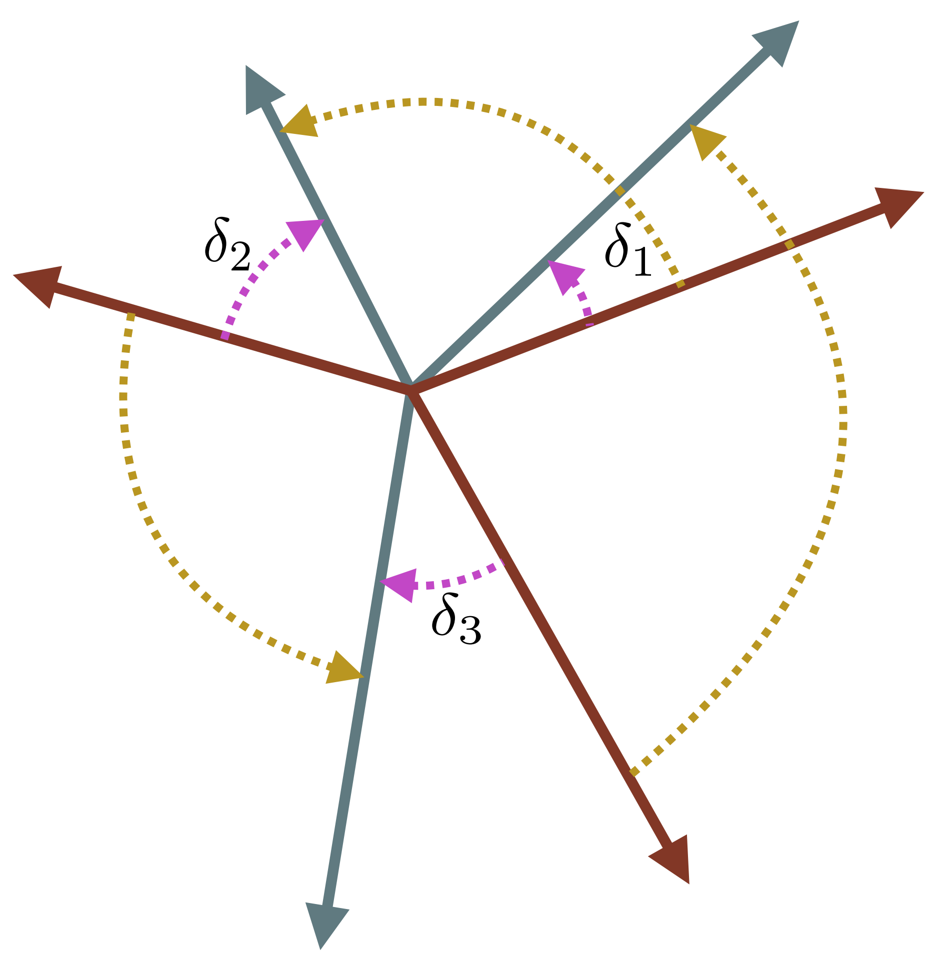


EFFORT

- Sum of matching rotations.
    - Intuition: generalize "closest angle" to "minimum effort".
- All order-preserving matchings differ by $2\pi k, k \in \mathbb{Z}$
- Principal matching: the matching with $Y = [-\pi, \pi)$

$$Y_1 = \delta_1 + \delta_2 + \delta_3$$

$$Y_2 = \delta_1 + \delta_2 + \delta_3 + 2\pi$$

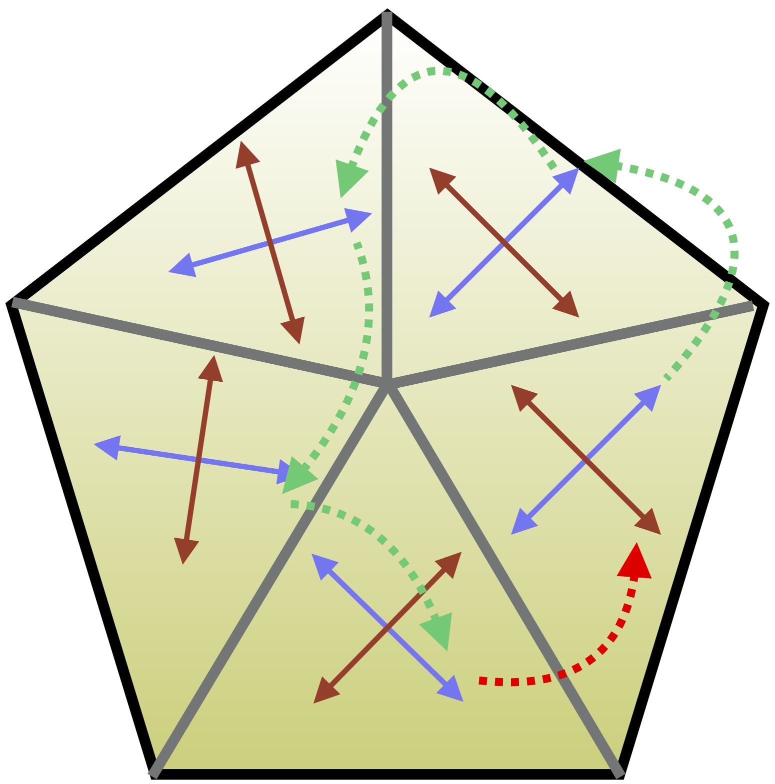


SINGULARITIES

- Around a matching cycle, directional returns to itself.
- Up to a different matching!

- Directional field as trivial connections. [Crane *et al.* 2010]
  - Induced Curvature: $\frac{2\pi}{k}$

- Regular cycles: index 0
- Sum of indices: $2 - 2g$

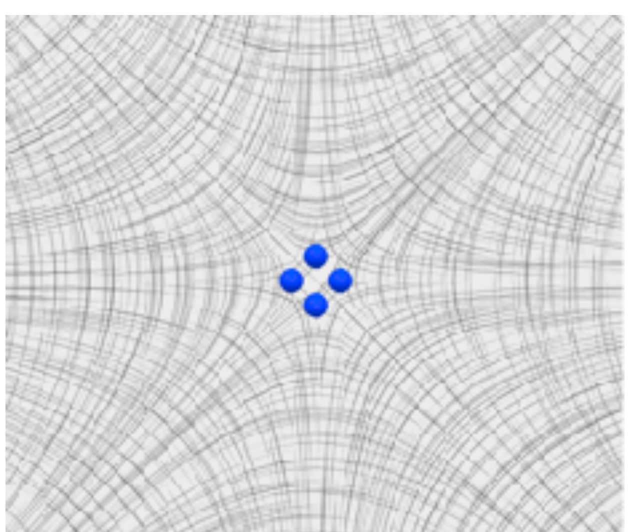


Index of singularity: $\frac{1}{k}$

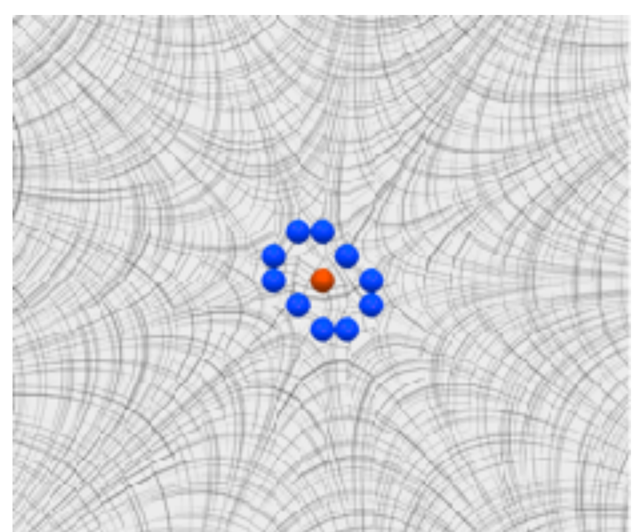
SAMPLING PROBLEM

- Implicit field: principal matching assumed.
- Low valence cycles: limited rotation sums.
- Higher order singularities cannot be represented!
- In practice: promoting low-degree singularity cycles ("singularity party").

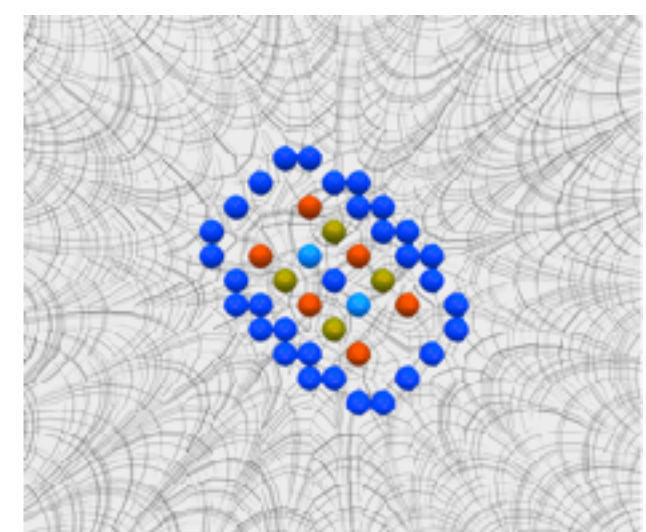
$$-\frac{4}{4}$$

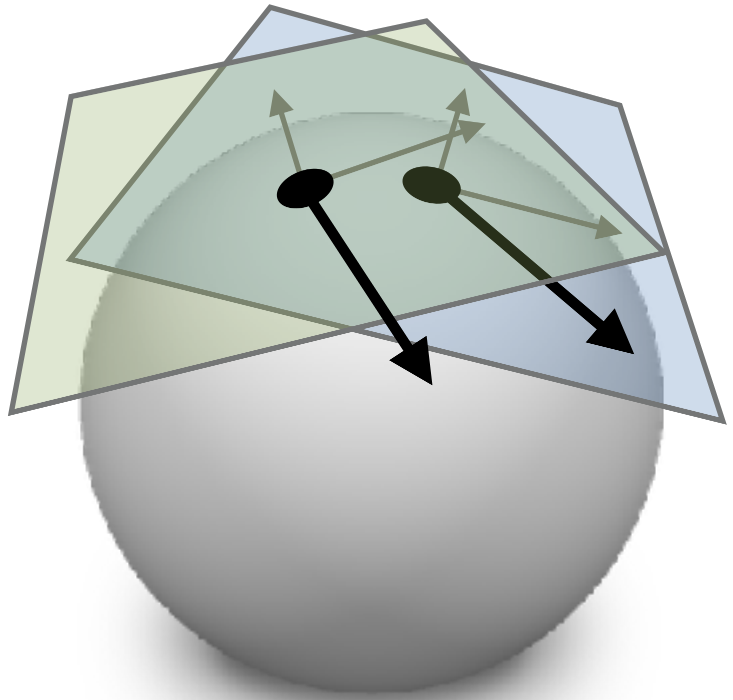


$$-\frac{9}{4}$$



$$-\frac{21}{4}$$





REPRESENTATION

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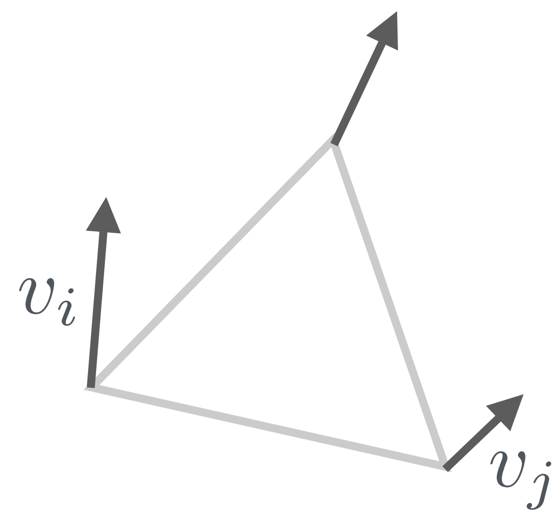
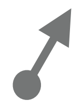
Model problem:

$|\nabla v|^2 \to \min$



v_i "$-$" v_j

REPRESENTATION

- 1 directional 

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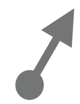
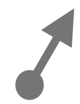
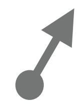
- 1 directional 
    - Cartesian

REPRESENTATION

- 1 directional 
  - Cartesian

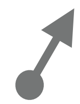
$$\begin{pmatrix} x \\ y \end{pmatrix}$$

REPRESENTATION

- 1 directional 
  - Cartesian

$$\begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

REPRESENTATION

- 1 directional 
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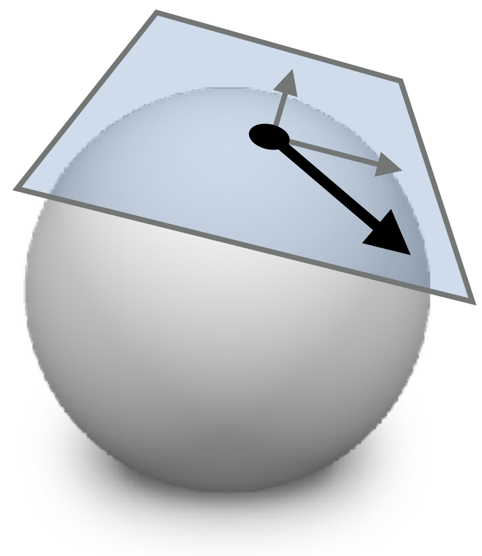
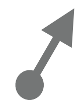
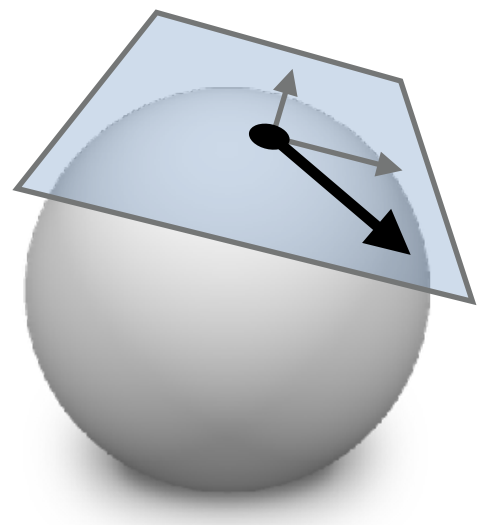


REPRESENTATION

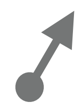
- 1 directional 
  - Cartesian

$$\begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

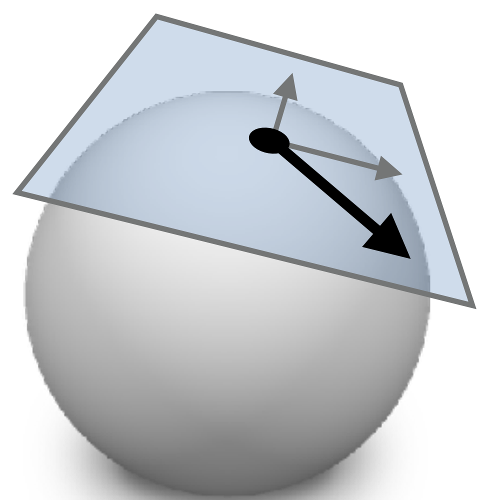


$$\begin{pmatrix} b_{1,p} & b_{2,p} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

REPRESENTATION

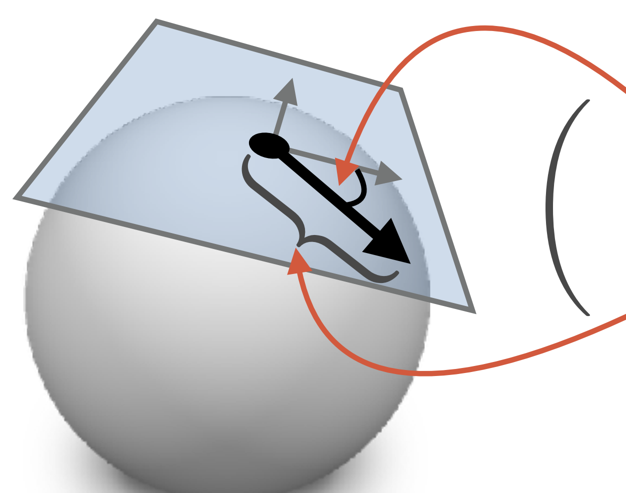
- 1 directional 
  - Cartesian

$$\begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$



$$\begin{pmatrix} b_{1,p} & b_{2,p} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

  - Polar



$$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

REPRESENTATION

- 1 directional

  direction (unit length) field

  - Cartesian

    $$\begin{pmatrix} x \\ y \end{pmatrix}$$

    $$x^2 + y^2 = 1$$

  - Polar

    $$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

    $$\begin{pmatrix} \theta \end{pmatrix}$$

REPRESENTATION

- 1 directional

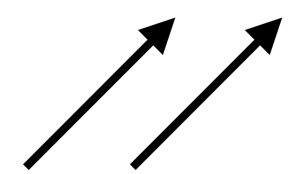
  direction (unit length) field

  - Cartesian

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

$$x^2 + y^2 = 1$$

$$\begin{pmatrix} 0.7 \\ 0.7 \end{pmatrix} - \begin{pmatrix} 0.7 \\ 0.7 \end{pmatrix} = 0$$



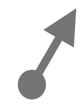
  - Polar

$$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

$$\begin{pmatrix} \theta \end{pmatrix}$$

$$45^\circ - 405^\circ \neq 0$$

REPRESENTATION

- 1 directional 
  - Cartesian

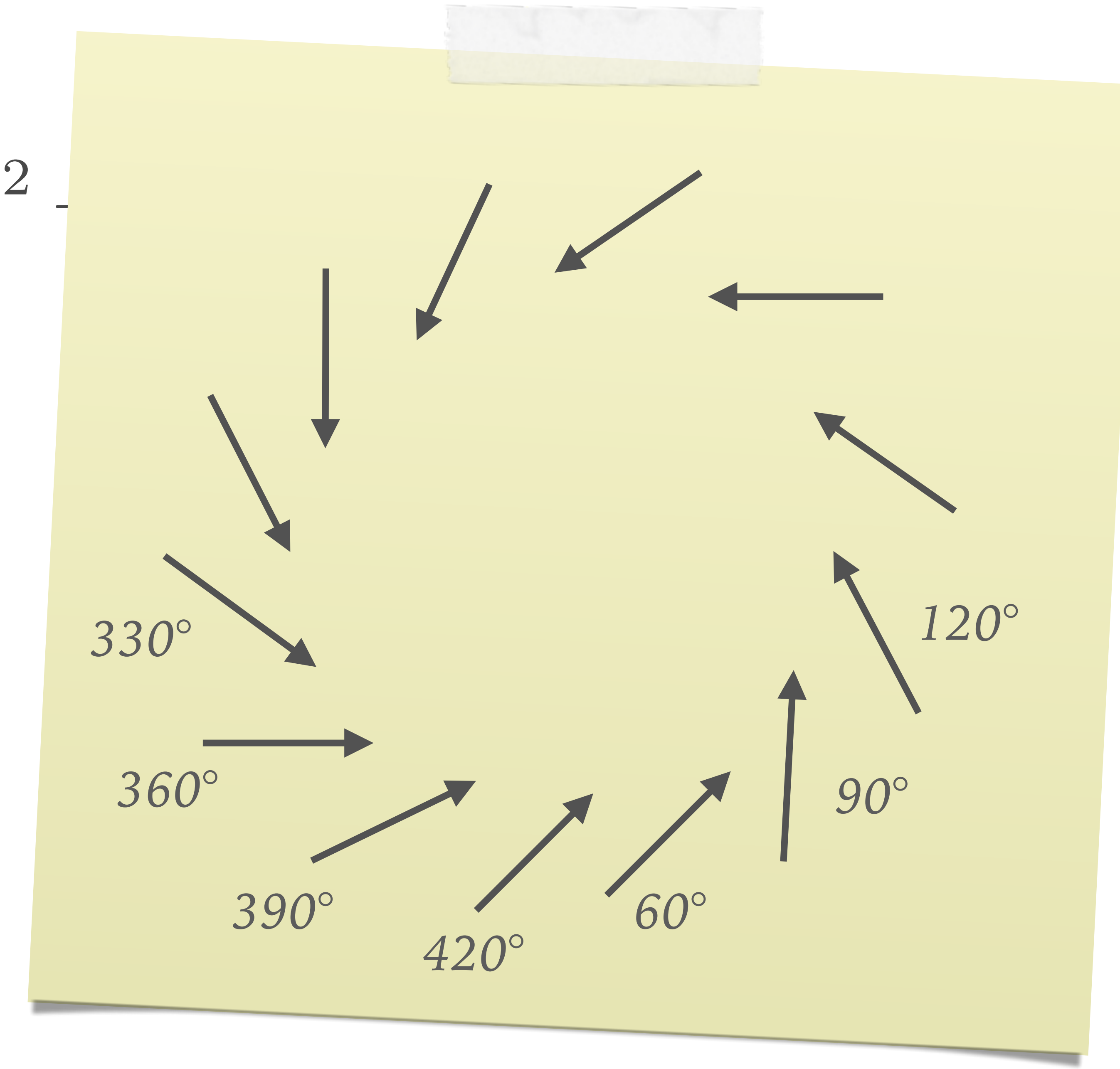
    $$\begin{pmatrix} x \\ y \end{pmatrix}$$

  - Polar

    $$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

direction field

x^2 - 0



REPRESENTATION

- 1 directional
  - Cartesian

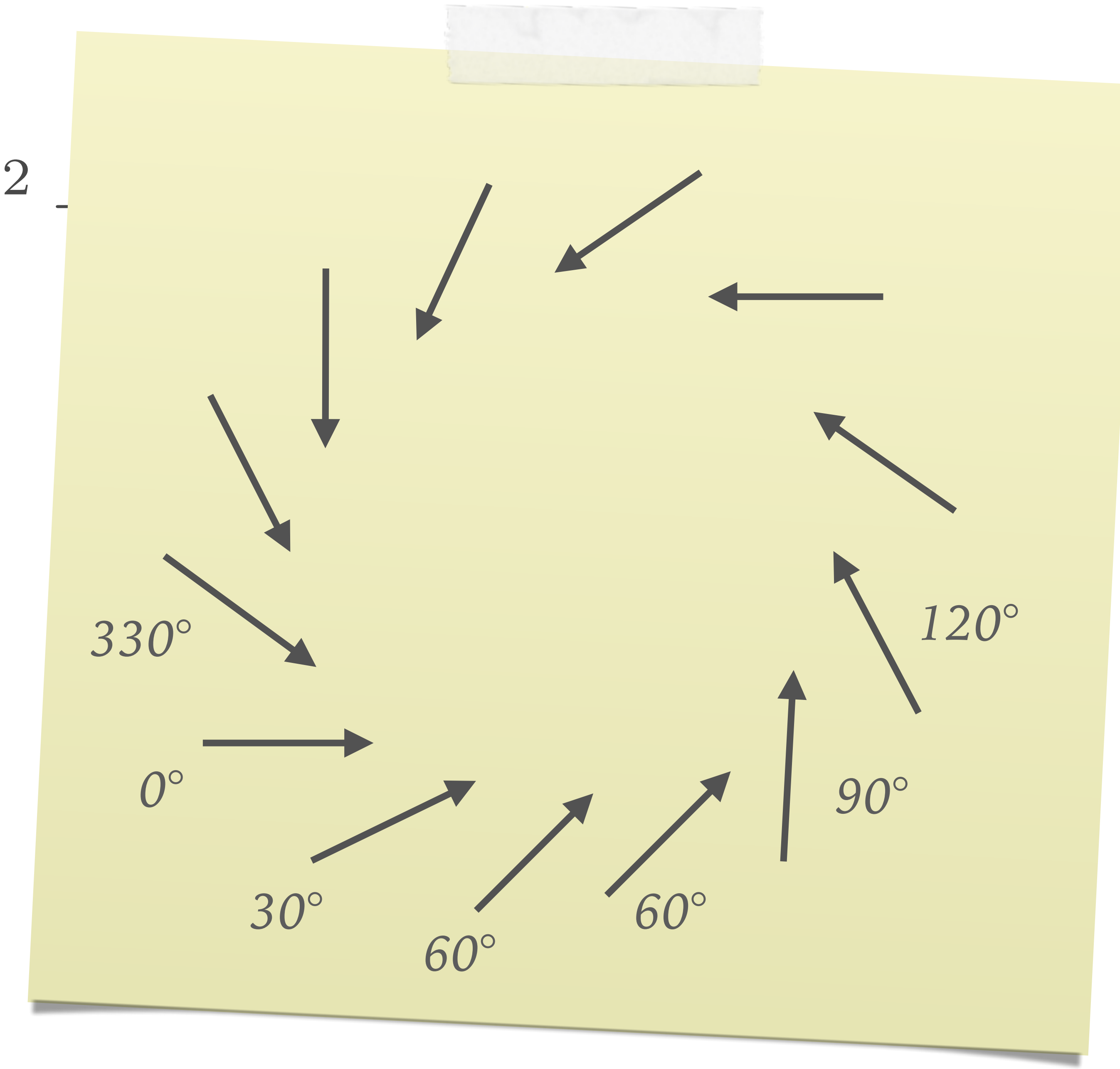
    $$\begin{pmatrix} x \\ y \end{pmatrix}$$

  - Polar

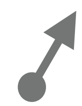
    $$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

direction field

x^2 - ... 0

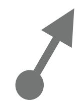


REPRESENTATION

- 1 directional 
  - Cartesian
  - Polar

	direction field	2π-invariance
$\begin{pmatrix} x \\ y \end{pmatrix}$	$x^2 + y^2 = 1$	built-in
$\begin{pmatrix} \theta \\ r \end{pmatrix}$	$\begin{pmatrix} \theta \end{pmatrix}$	$\theta_i - \theta_j \mod 2\pi$

REPRESENTATION

- 1 directional 

	direction field	2π-invariance
• Cartesian $\begin{pmatrix} x \\ y \end{pmatrix}$	$x^2 + y^2 = 1$	built-in
• Polar $\begin{pmatrix} \theta \\ r \end{pmatrix}$	$\begin{pmatrix} \theta \end{pmatrix}$	$\min\limits_{k \in \mathbb{Z}} \ \theta_i - \theta_j + k2\pi$

REPRESENTATION

- 1 directional

direction field | 2π-invariance

  - Cartesian

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

$$x^2 + y^2 = 1$$

built-in

  - Polar

$$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

$$\begin{pmatrix} \theta \end{pmatrix}$$

$$\theta_i - \theta_j + k2\pi$$

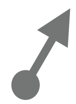
k const.

explicit choice of period

$\Rightarrow$ control over topology

[Li et al. 2006]

REPRESENTATION

- 1 directional 
  - Cartesian

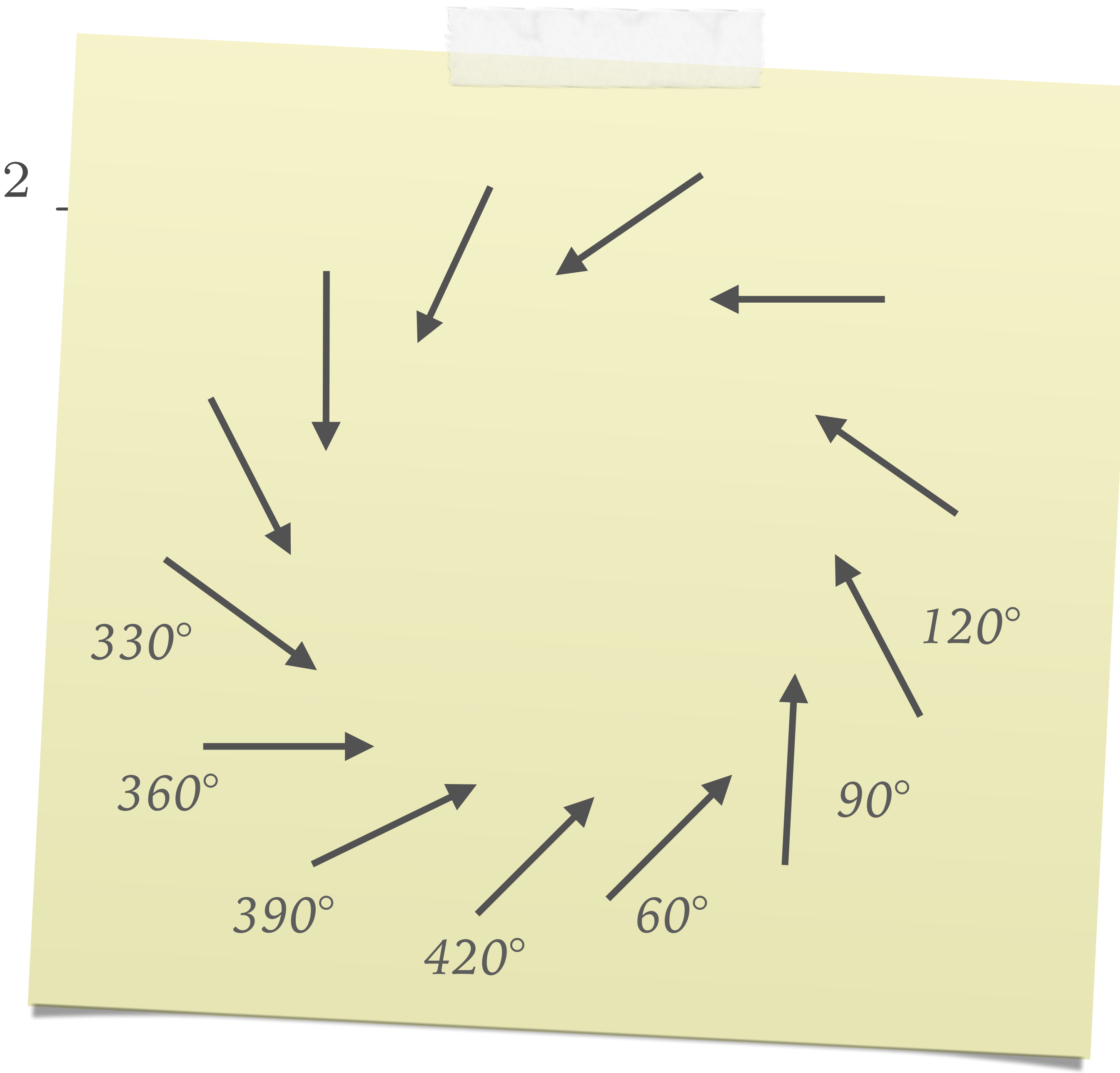
    $$\begin{pmatrix} x \\ y \end{pmatrix}$$

  - Polar

    $$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

direction field

x^2 - 0



REPRESENTATION

- 1 directional
  - Cartesian

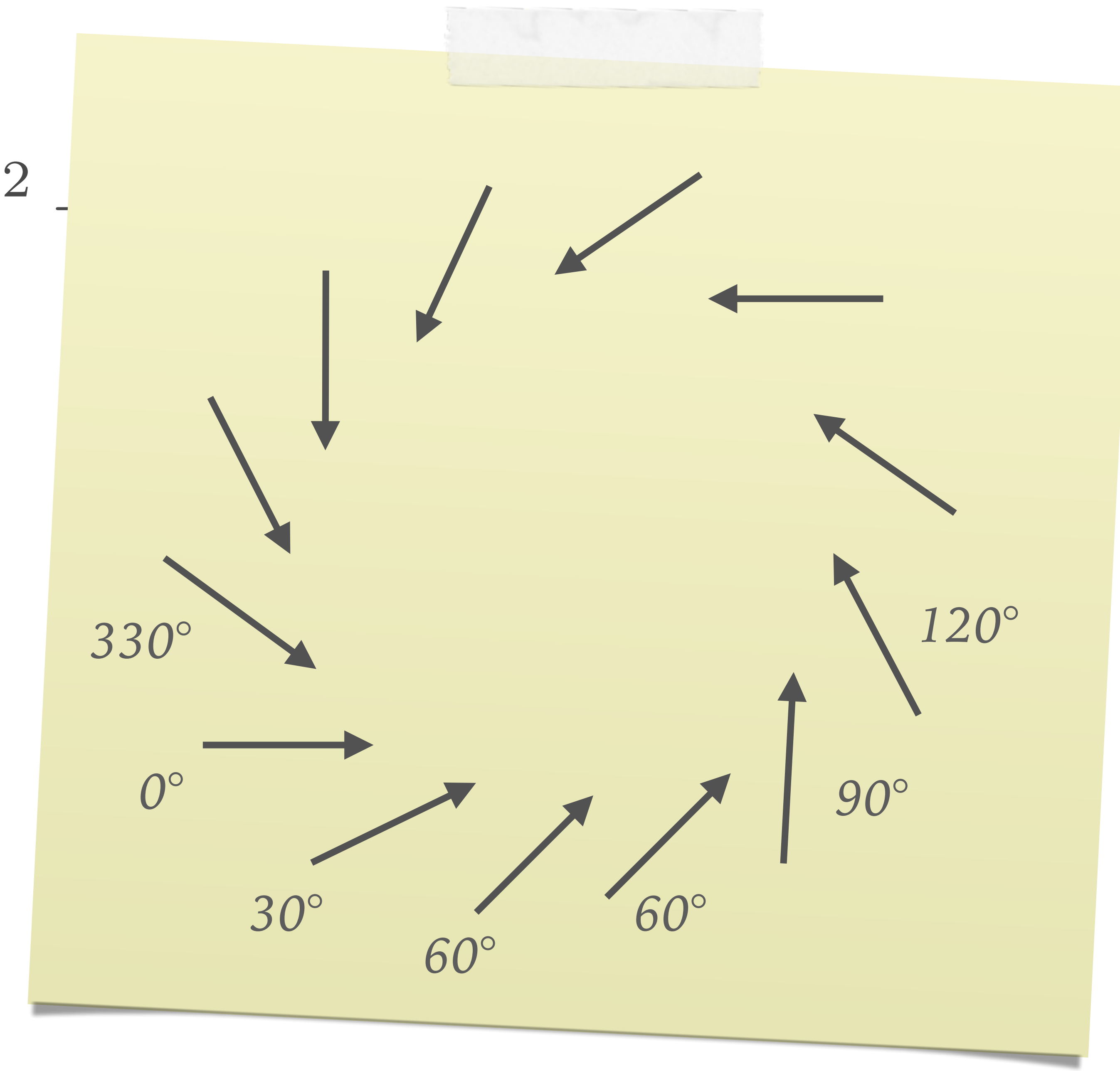
$$\begin{pmatrix} x \\ y \end{pmatrix}$$

  - Polar

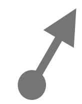
$$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

direction field

x^2 - ... 0



REPRESENTATION

- 1 directional 
    - Cartesian

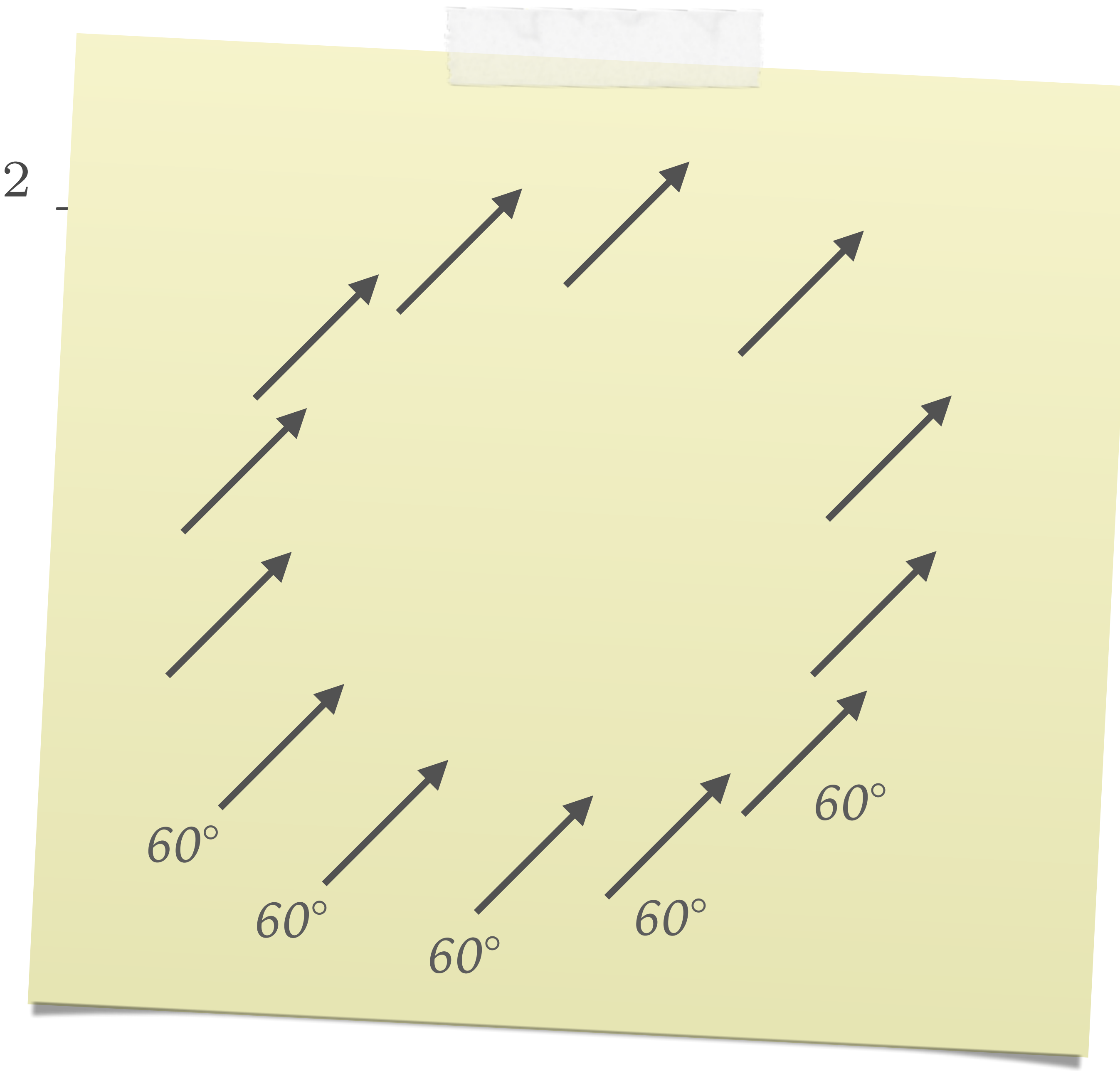
      $$\begin{pmatrix} x \\ y \end{pmatrix}$$

    - Polar

      $$\begin{pmatrix} \theta \\ r \end{pmatrix}$$

direction field

x^2 - 0



REPRESENTATION

- 1 directional

direction field | 2π-invariance

  - Cartesian

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

$$x^2 + y^2 = 1$$

built-in

  - Polar

$$\begin{pmatrix} \theta \\ r \end{pmatrix} \quad r\mathrm{e}^{\mathrm{i}\theta}$$

$$\begin{pmatrix} \theta \end{pmatrix}$$

$$\theta_i - \theta_j + k2\pi$$

k integer

explicit choice of period

$\Rightarrow$ control over topology

[Li et al. 2006]

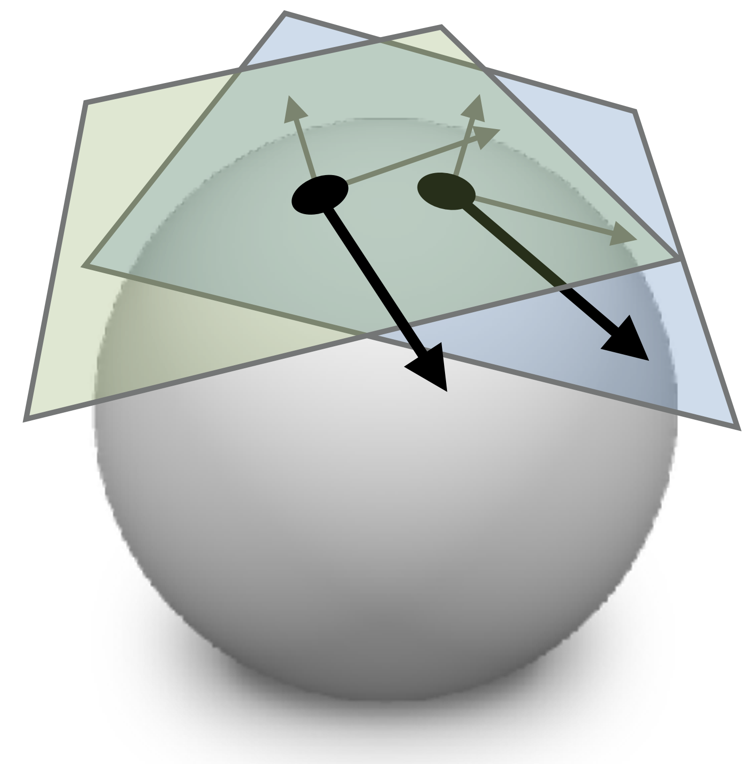
REPRESENTATION

- Differences between tangent vectors?
    - Cartesian

      $$\begin{pmatrix} x \\ y \end{pmatrix}$$

    - Angle

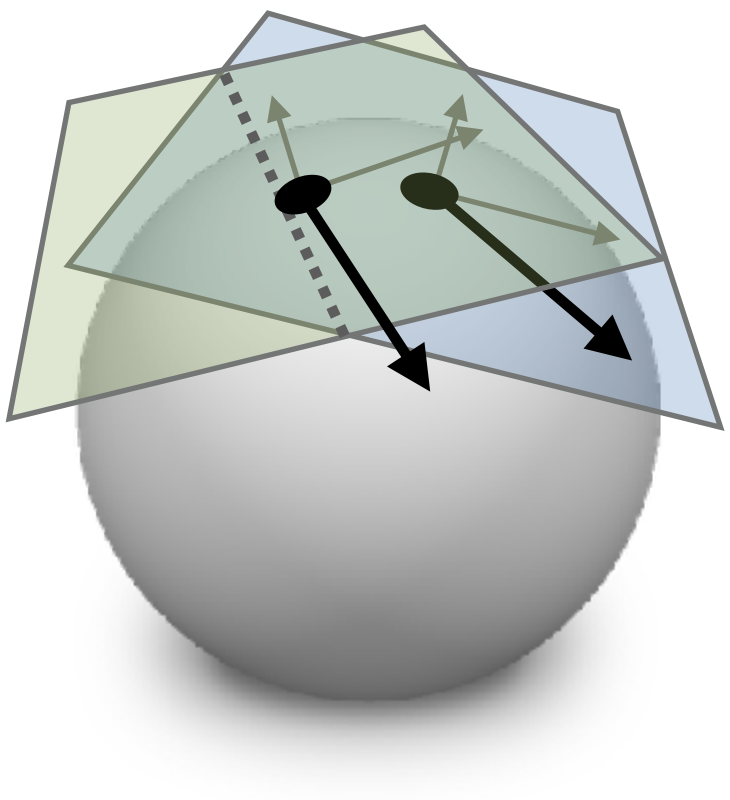
      $$\theta$$



REPRESENTATION

- Differences between tangent vectors?
  - Cartesian

$$\begin{pmatrix} x \\ y \end{pmatrix}_j - \begin{pmatrix} \cos X_{ij} & -\sin X_{ij} \\ \sin X_{ij} & \cos X_{ij} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}_i$$



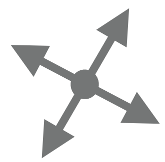
  - Angle

Period Jump

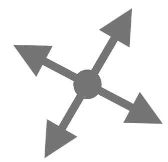
$$\theta_j - \theta_i + X_{ij} + p_{ij} 2\pi$$

Transition angle

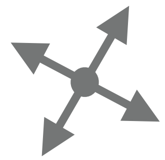
REPRESENTATION

- N directionals 

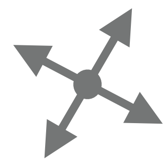
REPRESENTATION

- N directionals 
  - simply use multiple $\begin{pmatrix} x \\ y \end{pmatrix}$ or $\begin{pmatrix} \theta \\ r \end{pmatrix}$ per location?

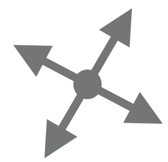
REPRESENTATION

- *N* directionals 
  - simply use multiple $\begin{pmatrix} x \\ y \end{pmatrix}$ or $\begin{pmatrix} \theta \\ r \end{pmatrix}$ per location?
    - perhaps okay for mere representation,
      but problematic for synthesis, optimization, …

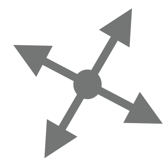
REPRESENTATION

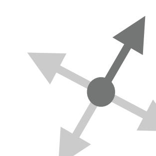
- N directionals 
  - simply use multiple $\begin{pmatrix} x \\ y \end{pmatrix}$ or $\begin{pmatrix} \theta \\ r \end{pmatrix}$ per location?
    - perhaps okay for mere representation,
      but problematic for synthesis, optimization, …
      - symmetries $\Rightarrow$ additional constraints

REPRESENTATION

- *N* directionals 
    - symmetric
        - just use one representative

REPRESENTATION

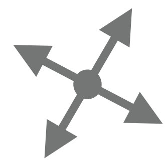
- N directionals 
  - symmetric
    - just use one representative
      - others implied by rotation by $k2\pi/N$



[Palacios & Zhang 2007]

[Ray et al. 2008]

REPRESENTATION

- N directionals 
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    - just use one representative
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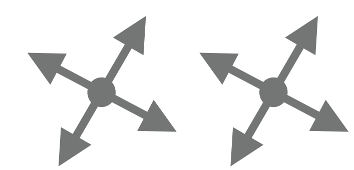
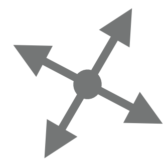


REPRESENTATION

- N directionals 
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    - just use one representative
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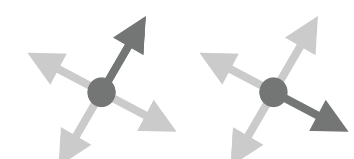
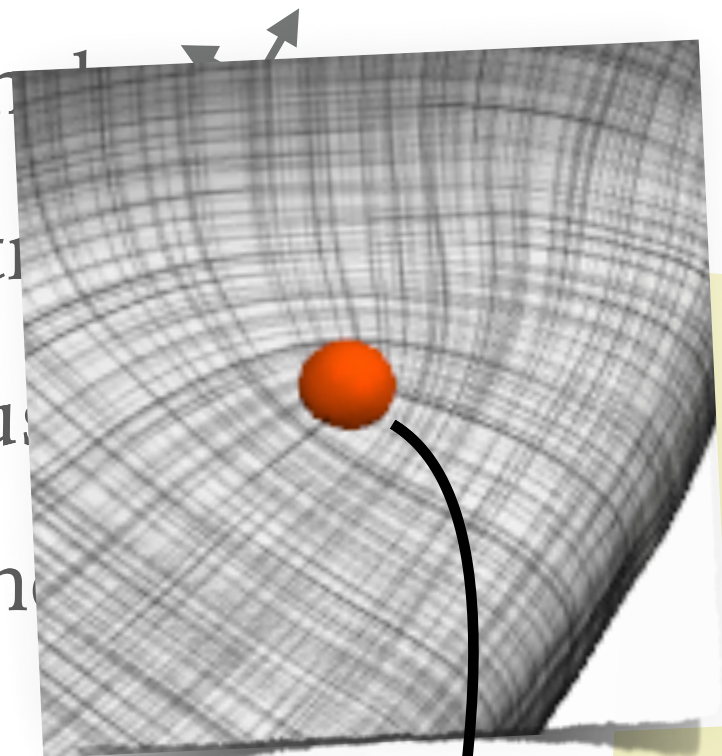
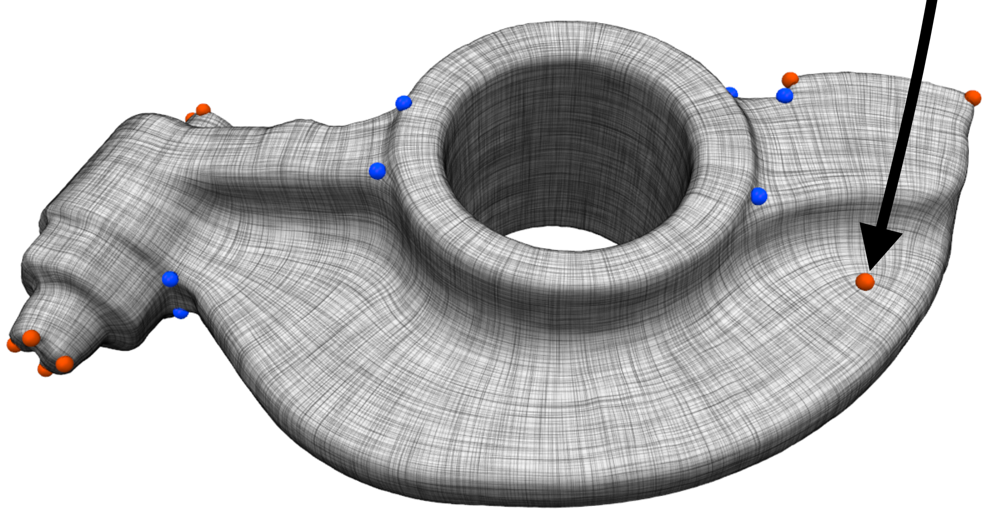
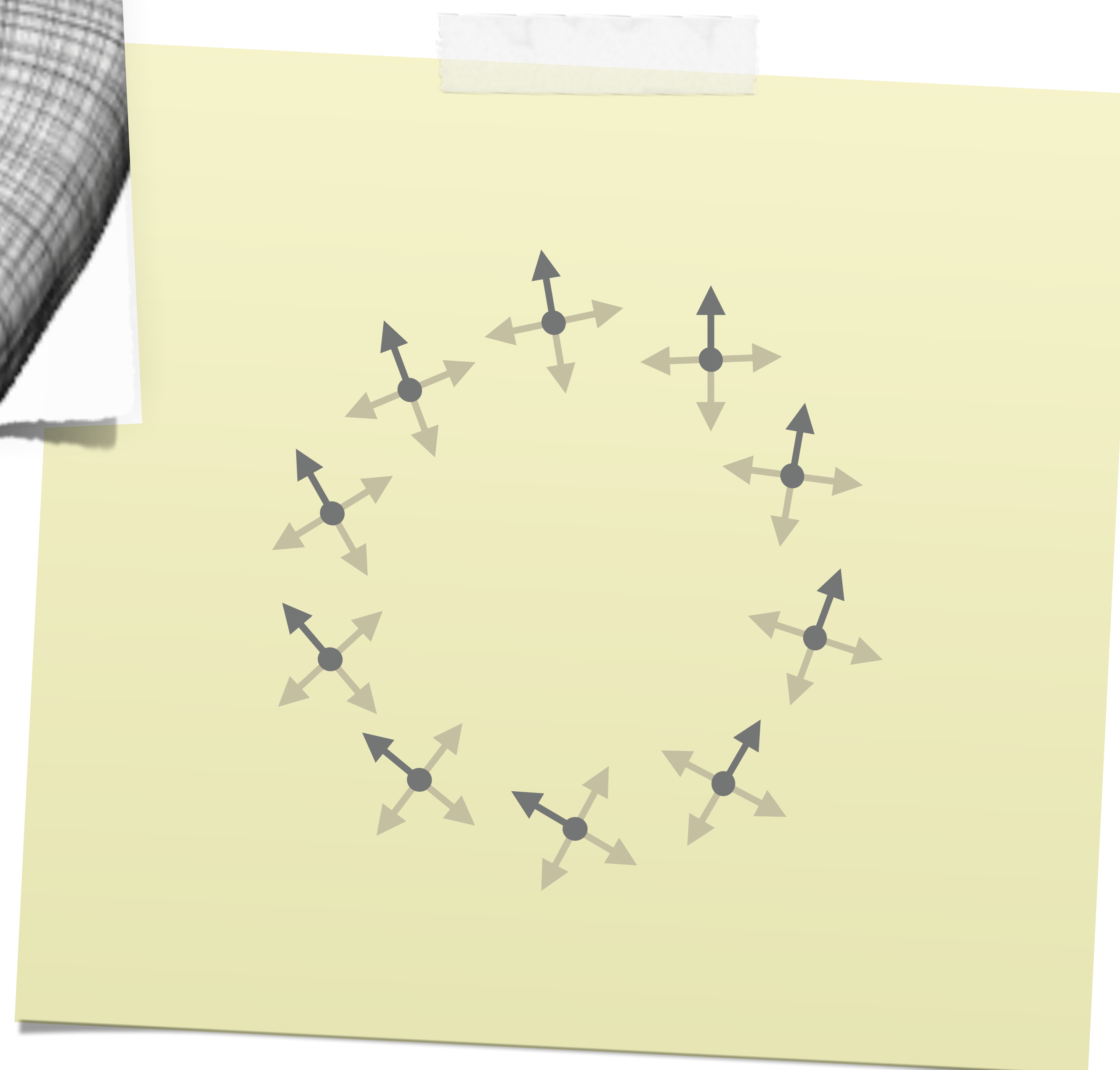


REPRESENTATION

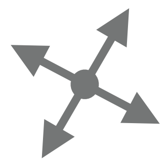
- *N* direction
  - symmetr
    - just us
      - oth







REPRESENTATION

- N directionals 
    - symmetric
        - just use one representative
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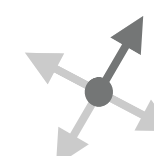
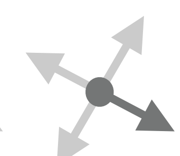
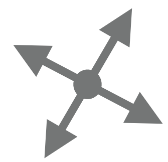
$$\theta_j - \theta_i + X_{ij} + p_{ij}2\pi/N$$

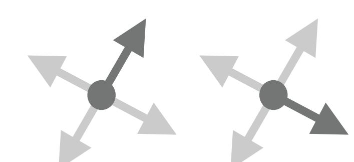




REPRESENTATION

- N directionals 
  - symmetric
    - just use one representative
      - others implied by rotation by $k2\pi/N$

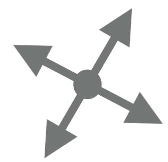
$$\theta_j - \theta_i + X_{ij} + p_{ij} 2\pi/N$$



      - Cartesian:

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

REPRESENTATION

- N directionals 
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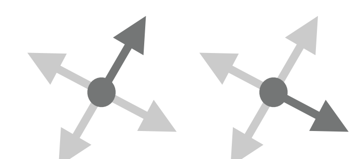
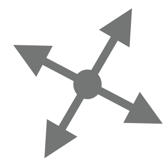
$$\theta_j - \theta_i + X_{ij} + p_{ij} 2\pi/N$$



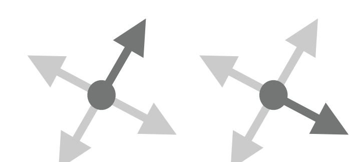
- Cartesian:

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \theta = \mathrm{atan2}(y, x)$$

REPRESENTATION

- N directionals 
    - symmetric
        - just use one representative
            - others implied by rotation by $k2\pi/N$

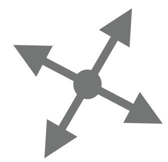
$$\theta_j - \theta_i + X_{ij} + p_{ij} 2\pi/N$$



- Cartesian:

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \theta = \operatorname{atan2}(y, x) \to \begin{pmatrix} \sin N\theta \\ \cos N\theta \end{pmatrix}$$

REPRESENTATION

- N directionals 
    - symmetric
        - just use one representative
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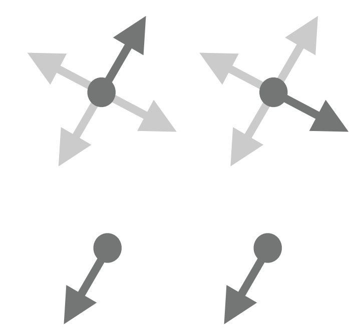
$$\theta_j - \theta_i + X_{ij} + p_{ij} 2\pi/N$$



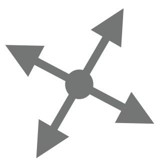
- Cartesian:

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \theta = \mathrm{atan2}(y, x) \rightarrow \begin{pmatrix} \sin N\theta \\ \cos N\theta \end{pmatrix}$$

"representation vector"

[Hertzmann & Zorin 2000]

REPRESENTATION

- N directionals 
  - symmetric
    - just use one representative
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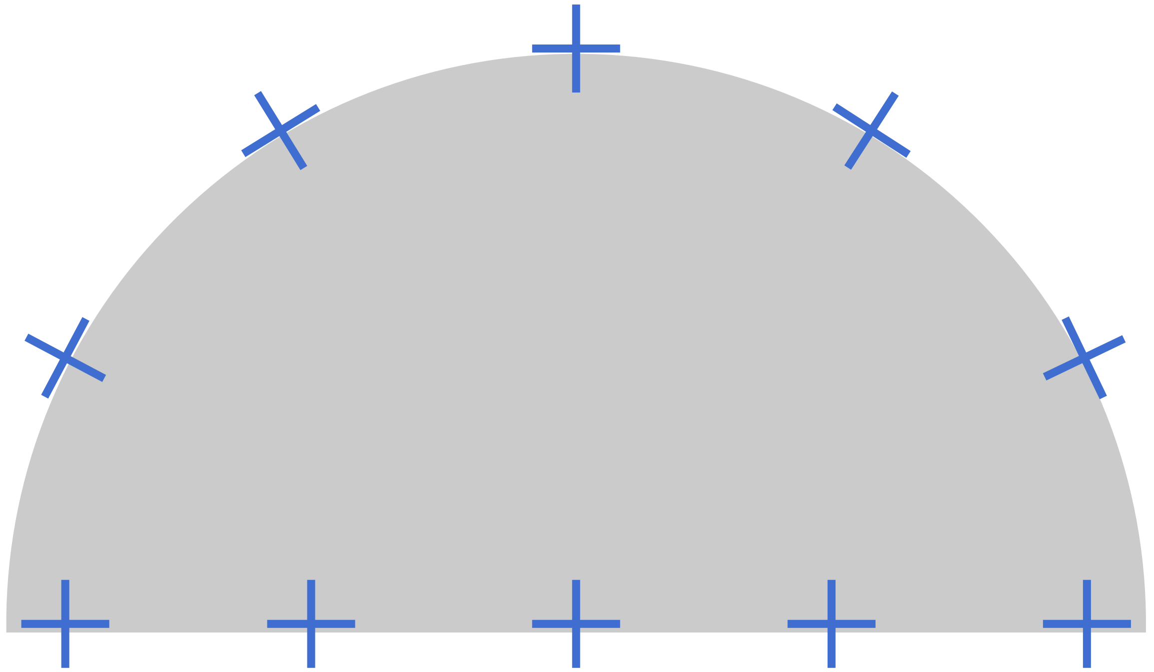
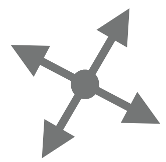


REPRESENTATION

- N directionals 
    - symmetric
        - just use one representative
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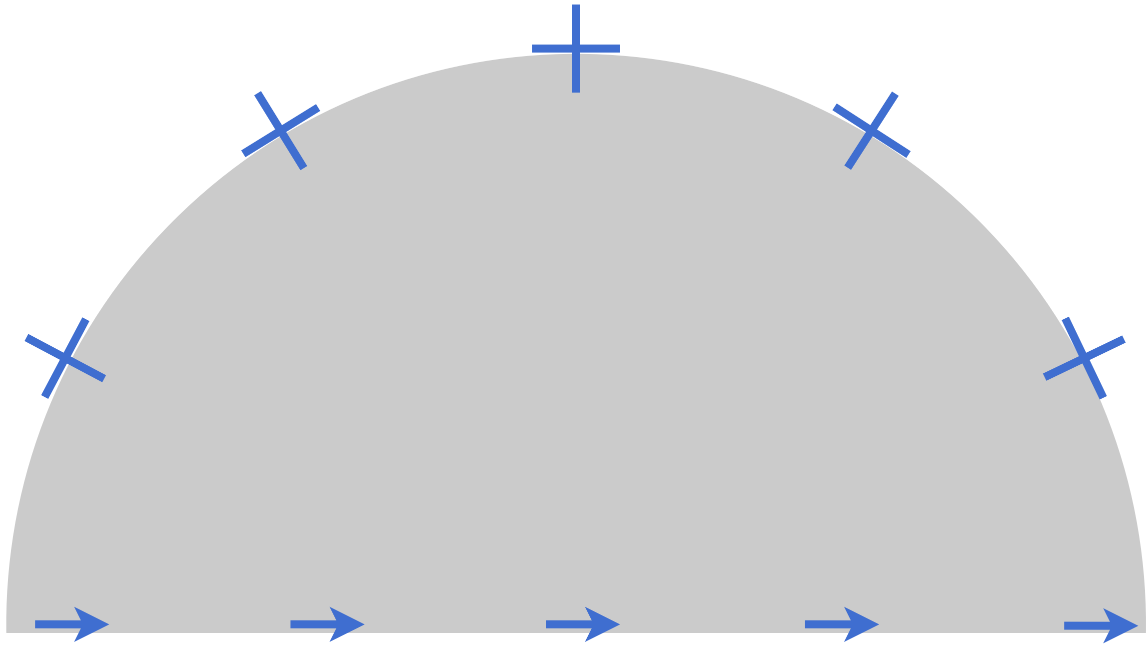
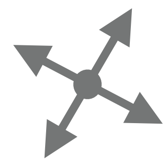


REPRESENTATION

- N directionals 
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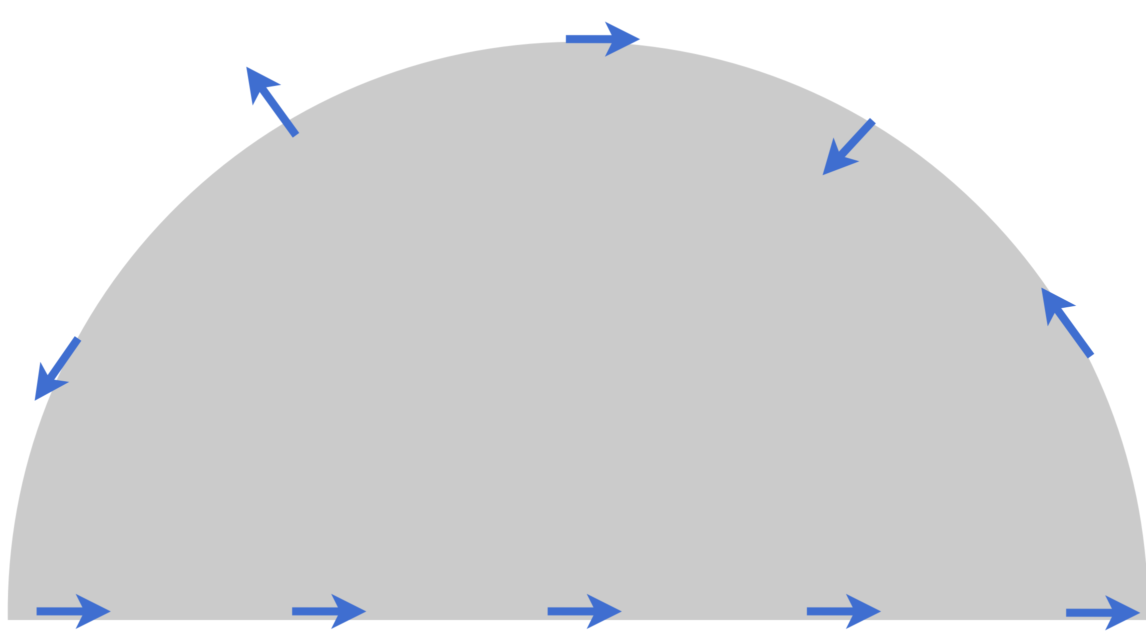
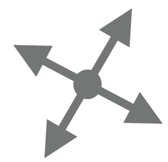


REPRESENTATION

- *N* directionals 
    - symmetric
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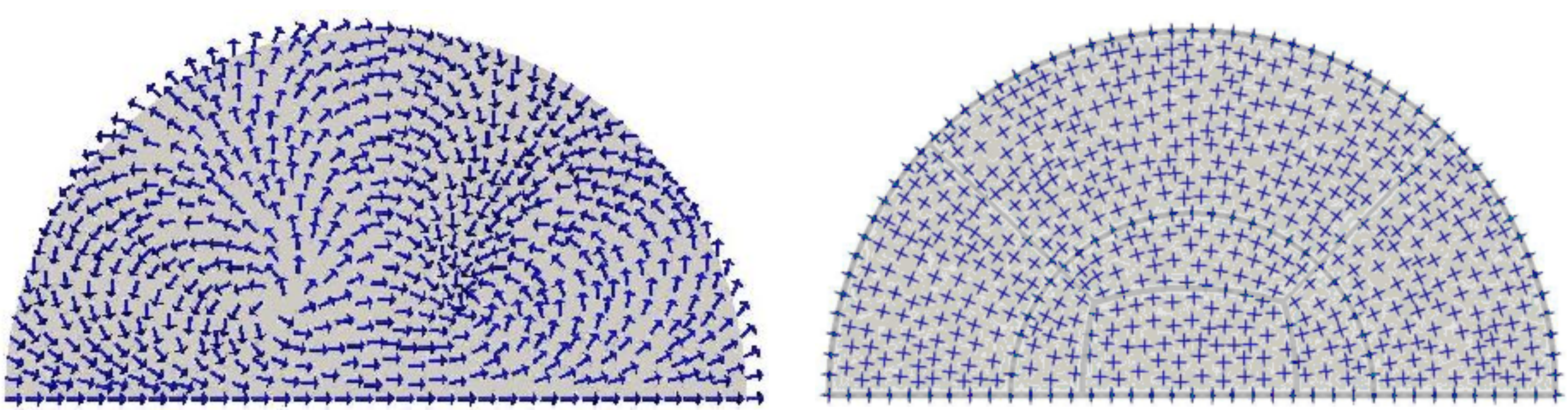
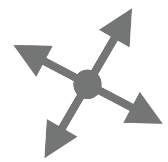


[Kowalski et al. 2013]

REPRESENTATION

- N directionals 
  - symmetric
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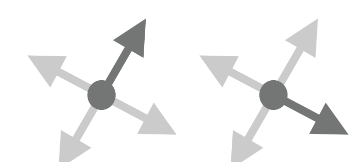
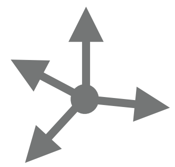


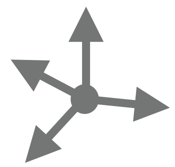
- Cartesian:

$$\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \theta = \mathrm{atan2}(y, x) \rightarrow \begin{pmatrix} \sin N\theta \\ \cos N\theta \end{pmatrix}$$

REPRESENTATION

- N directionals 
  - non-symmetric

REPRESENTATION

- N directionals 
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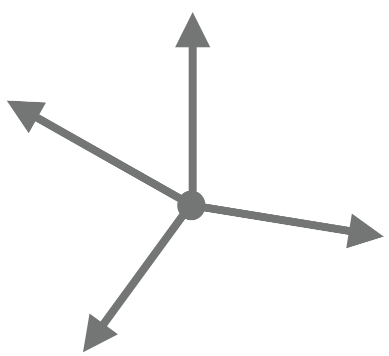
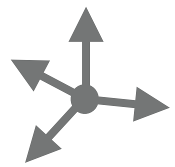


REPRESENTATION

- N directionals 
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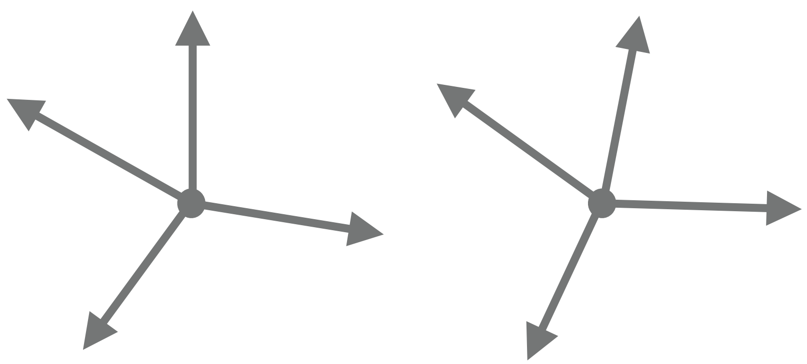

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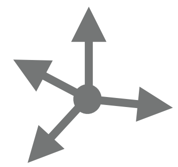
- N directionals 
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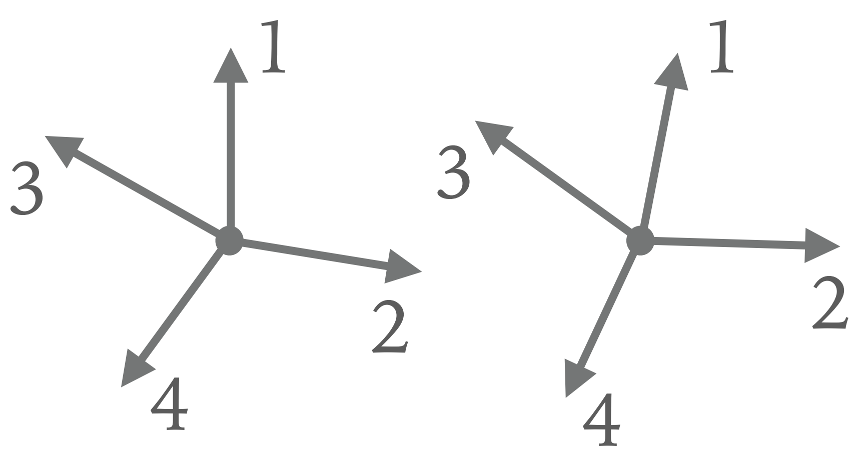
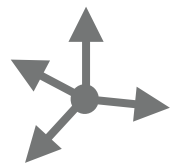
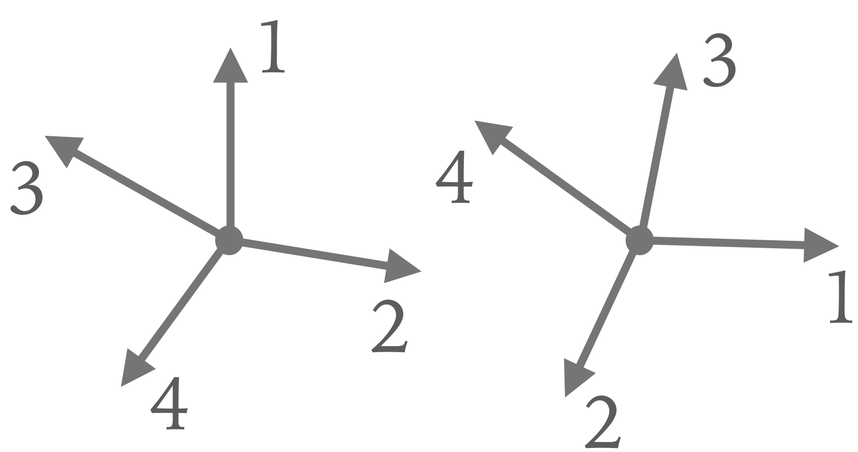
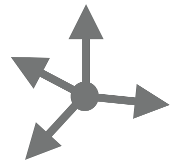


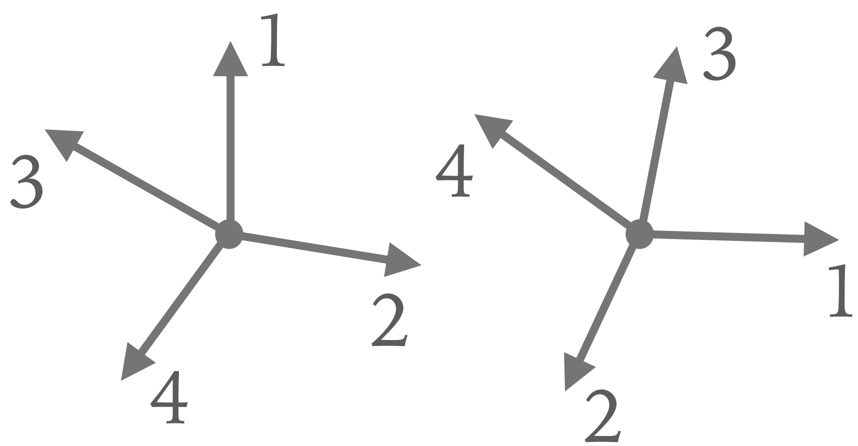
REPRESENTATION

- N directionals 
    - non-symmetric



REPRESENTATION

- N directionals 
  - non-symmetric

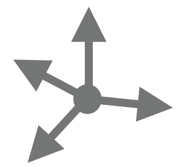


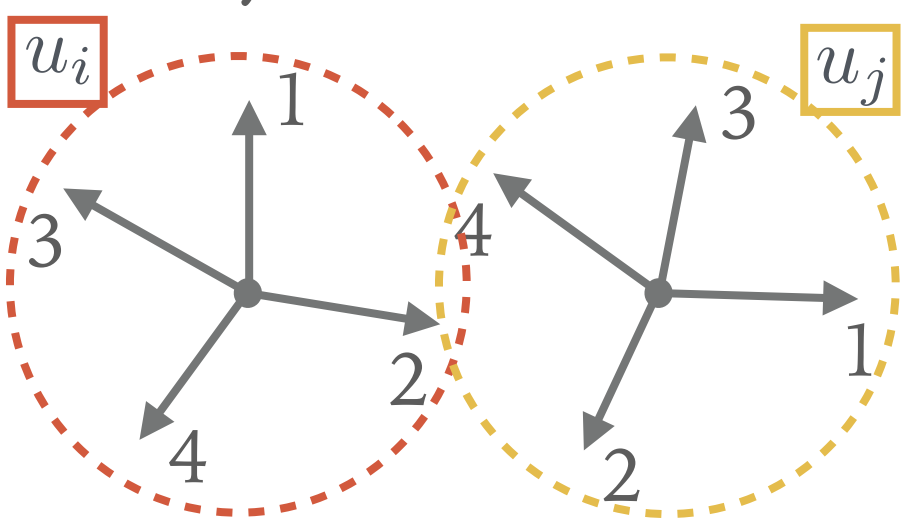
$F(v_1, v_2, v_3, v_4) = u$

F symmetric

$F(v_1, v_2, v_3, v_4) = F(v_3, v_2, v_4, v_1)$

REPRESENTATION

- N directionals 
    - non-symmetric



$F(v_1, v_2, v_3, v_4) = u$

F symmetric

F invertible (up to symmetry)

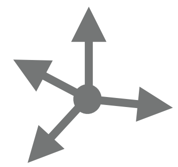
$$(x-a)(x-b)(x-c)$$
$$= x^3 + (a+b+c)x^2 + (ab+ac+bc)x + (abc)$$
$$\rightarrow (a+b+c \mid ab+ac+bc \mid abc)$$
$$(\quad u \quad , \quad v \quad , \quad w \quad) =: F(a,b,c)$$

N-PolyVector

[Diamanti et al. 2014]

$$(a+b+c+d \mid ab+ac+ad+bc+bd+cd \mid abc+abd+acd+bcd \mid abcd)$$

REPRESENTATION

- N directionals 
    - use complex numbers, complex polynomials

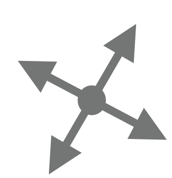
      $$a = x + iy$$
      $$a = re^{i\theta}$$

    - non-symmetric

$$(a+b+c+d \mid ab+ac+ad+bc+bd+cd \mid abc+abd+acd+bcd \mid abcd)$$

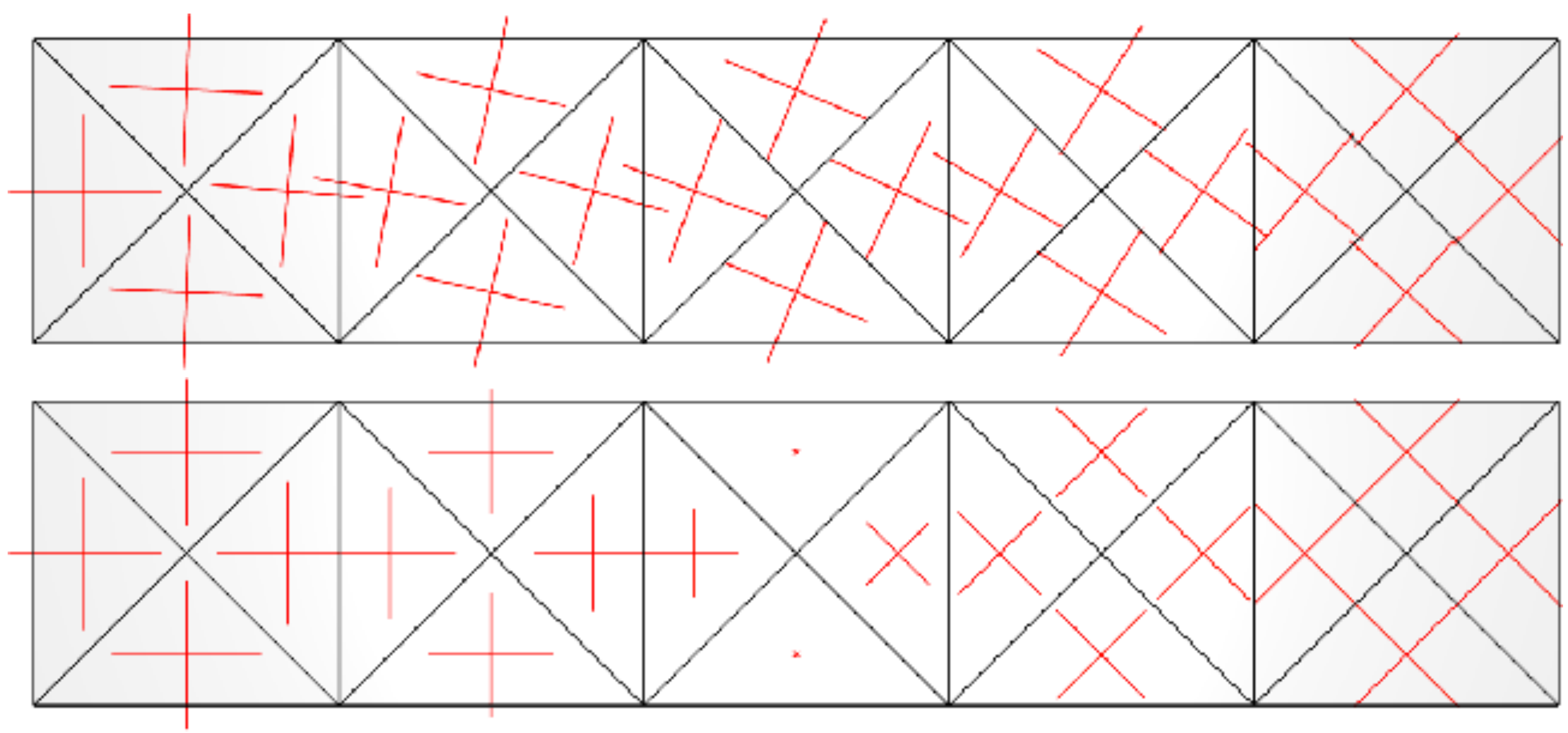
- symmetric

$$(\underbrace{a+b+c+d}_{0} \mid \underbrace{ab+ac+ad+bc+bd+cd}_{0} \mid \underbrace{abc+abd+acd+bcd}_{0} \mid \underbrace{abcd}_{\substack{-a^4 \\ =-re^{i4\theta}}})$$

 $\rightarrow \begin{pmatrix} \sin N\theta \\ \cos N\theta \end{pmatrix}$

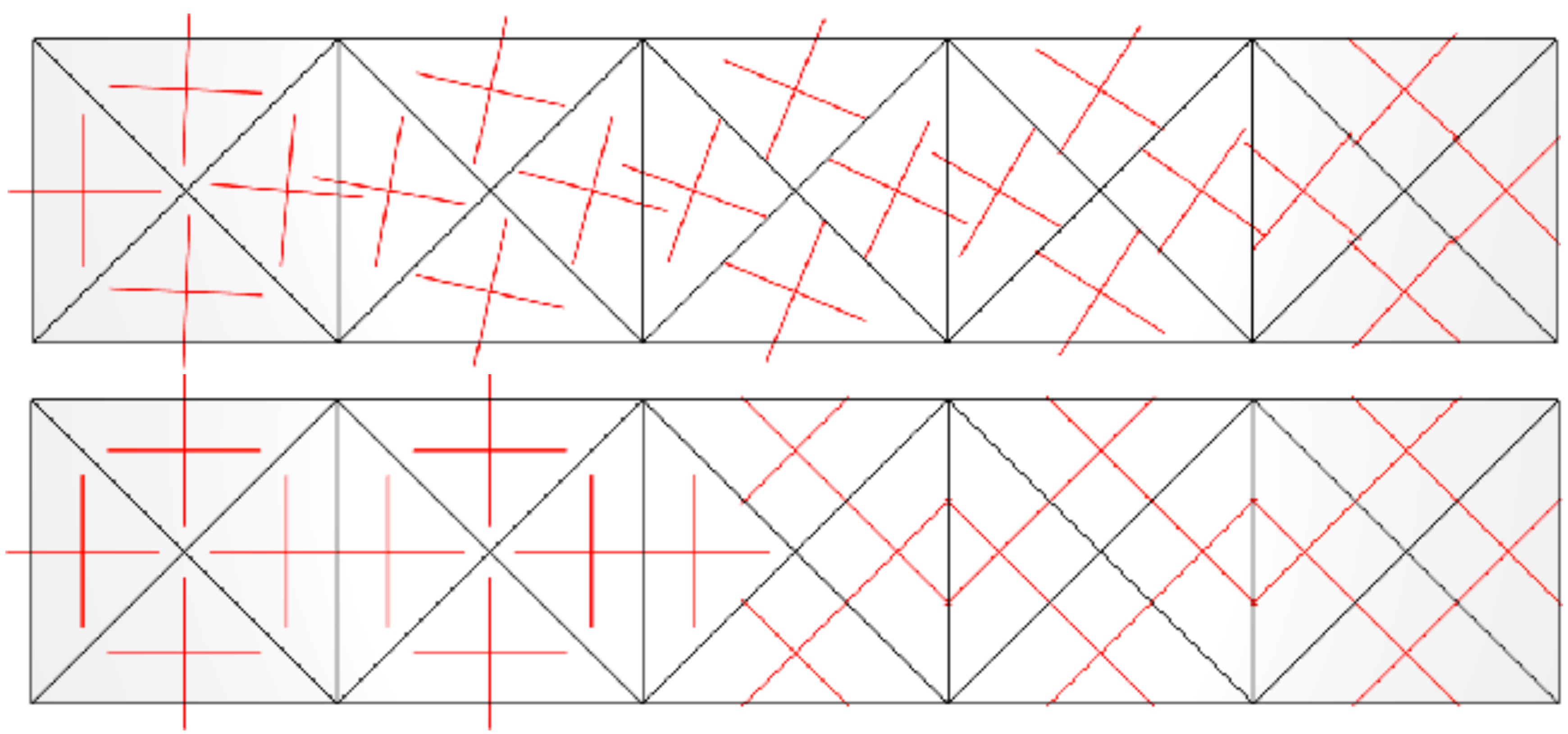
REPRESENTATION

	fixed topology	free topology
smoothest	**angle** linear	**angle** linear, mixed-integer **Cartesian** linear, non-linear constraints Eigenvalue problem



REPRESENTATION

	fixed topology	free topology
smoothest	angle linear	angle linear, mixed-integer Cartesian linear, non-linear constraints Eigenvalue problem

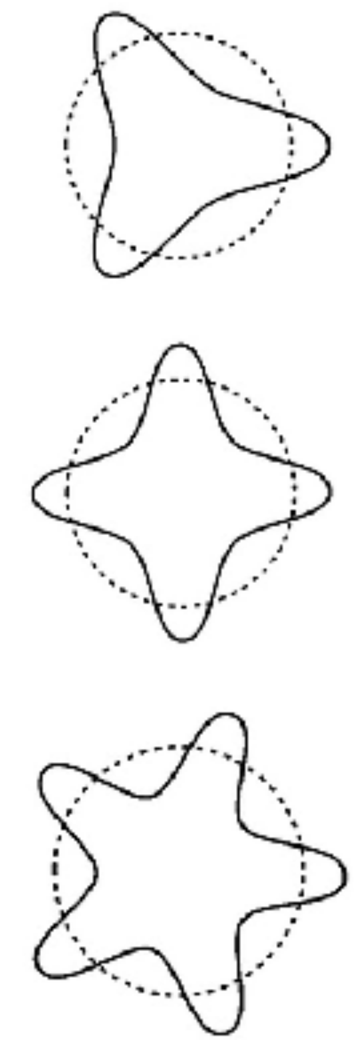


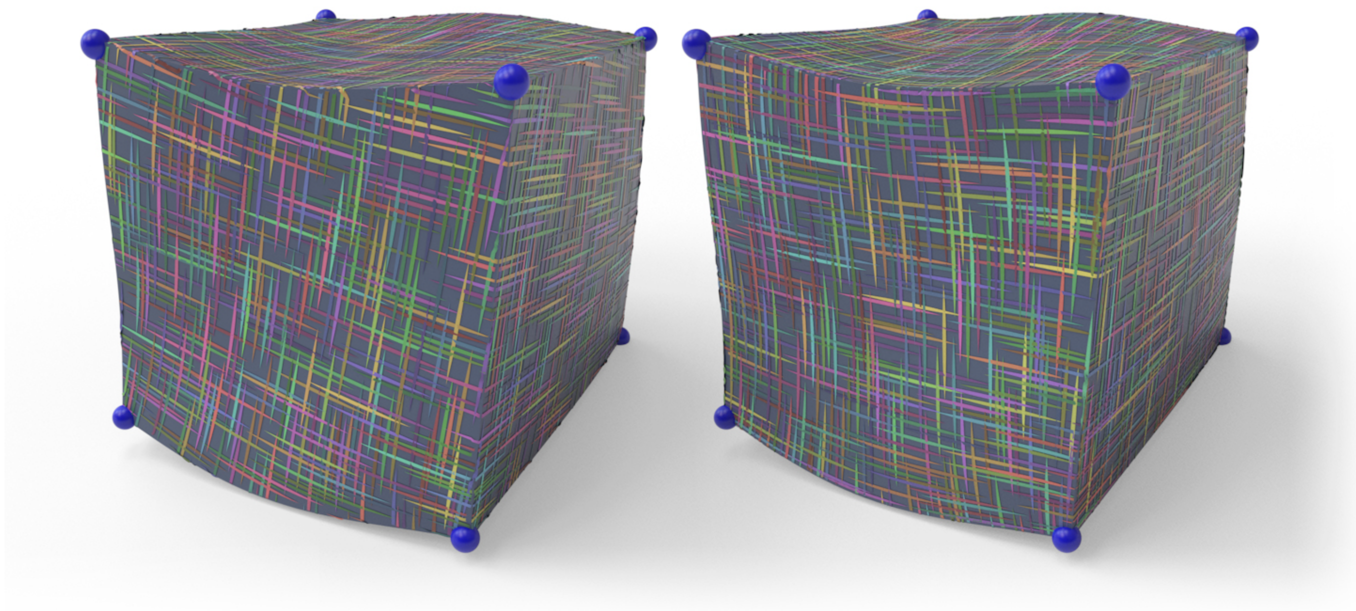
REPRESENTATION

	fixed topology	free topology
smoothest	**angle** linear	**angle** linear, mixed-integer **Cartesian** linear, non-linear constraints Eigenvalue problem
+ directional constraints	**angle** linear, (mixed integer) **1-form** linear, linear constraints	**angle** linear, mixed-integer **Cartesian** linear, non-linear constraints linear, linear constraints

REPRESENTATION

- Alternatives
  - Extrema of periodic circular functions (SH)
  - Eigenvectors of tensors (spd matrices)
  - Functional representation
  - …

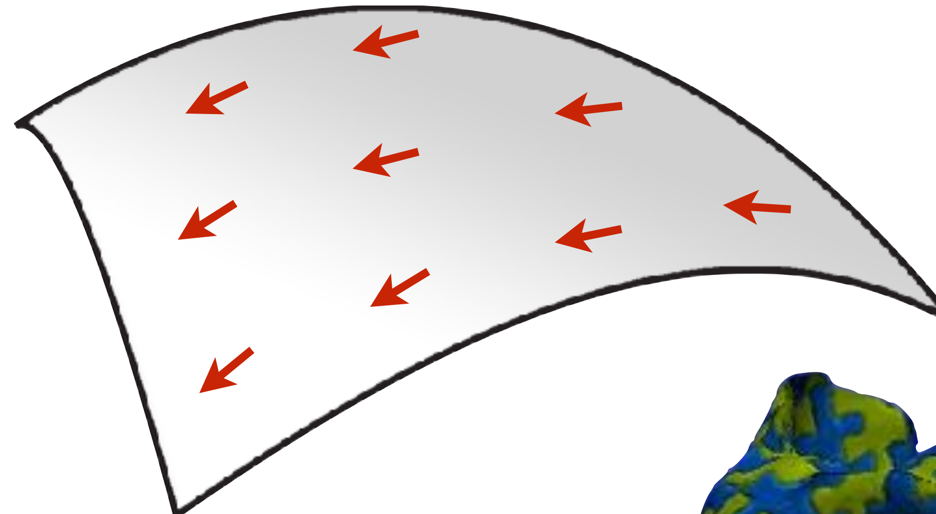


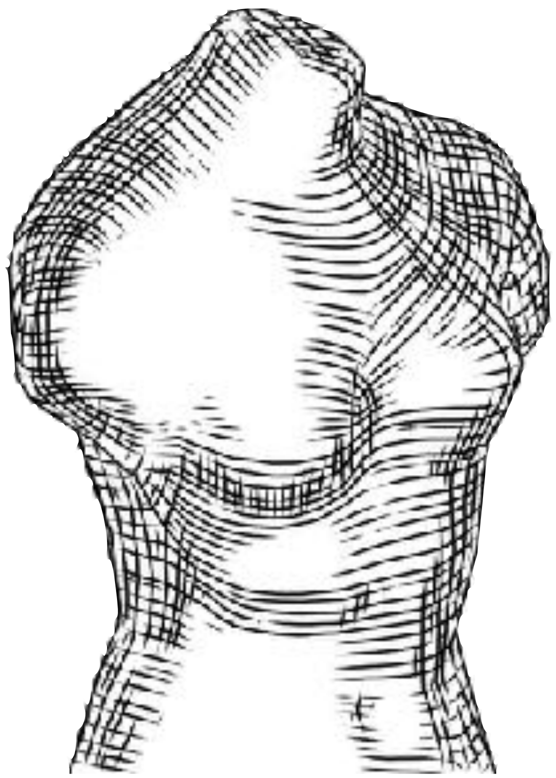


OBJECTIVES

OBJECTIVES - FIELD FAIRNESS

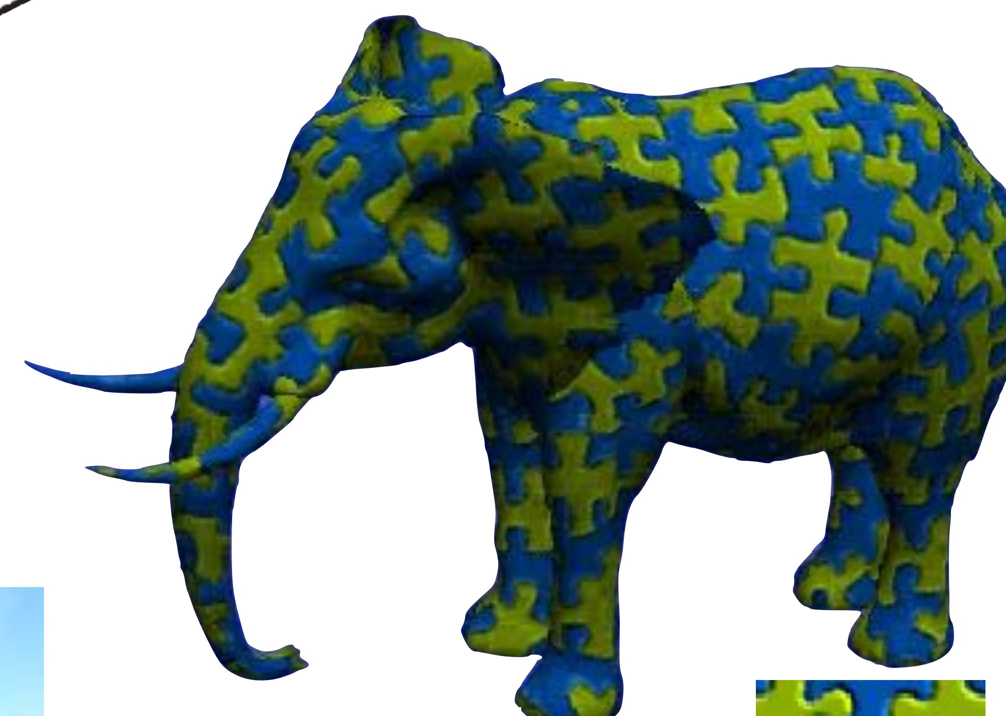
- “As-parallel-as-possible”





[Palacios et al. 2007]

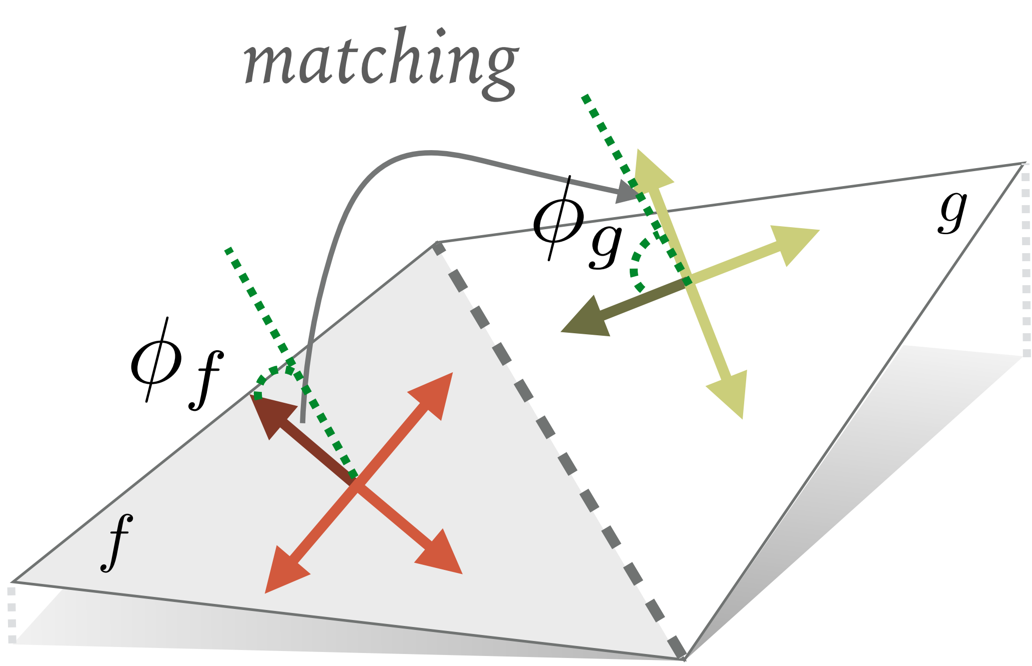
[Ray et al. 2006]
[Knöppel et al. 2015]

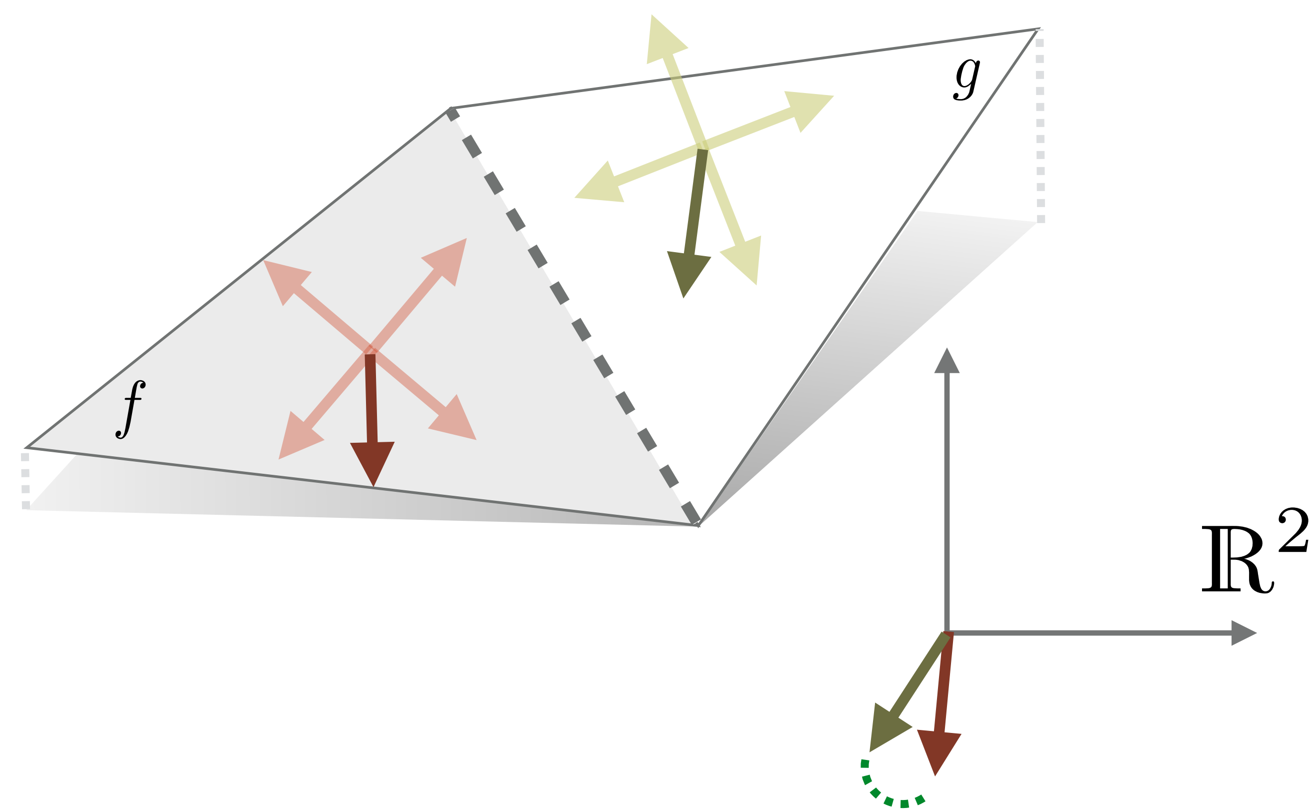


[Turk et al. 2001]

OBJECTIVES – FIELD FAIRNESS

- "As-parallel-as-possible"





<u>angle-based approaches</u>

compare matching angles

[Hertzmann et al. 2000] [Crane et al. 2010]
[Ray et al. 2008] [Ray et al. 2009]
[Bommes et al. 2009] [Jakob et al. 2015]

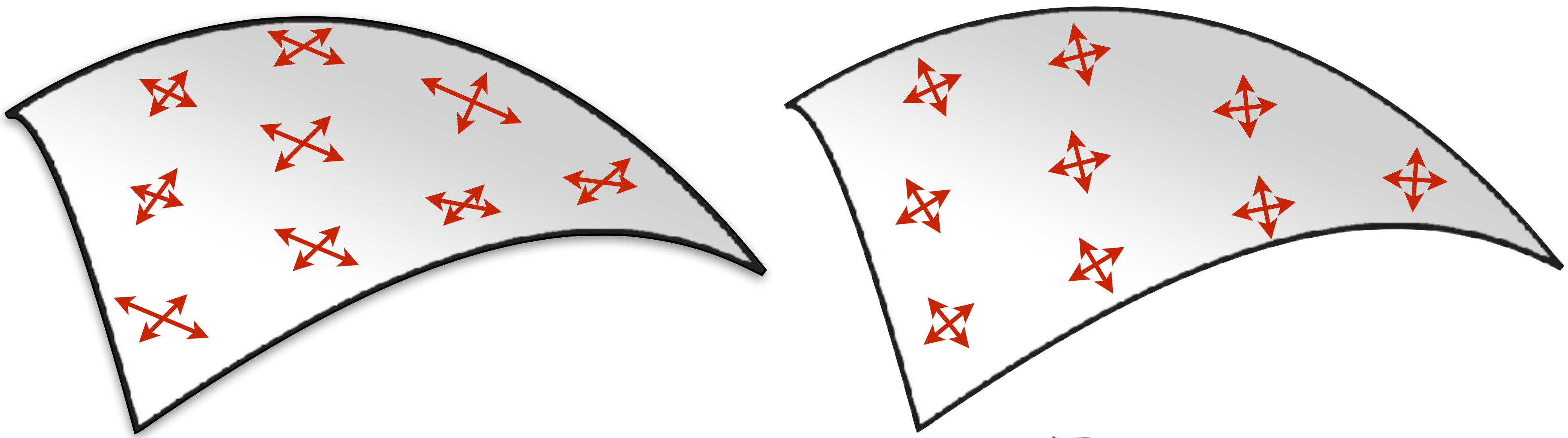
<u>cartesian/complex approaches</u>

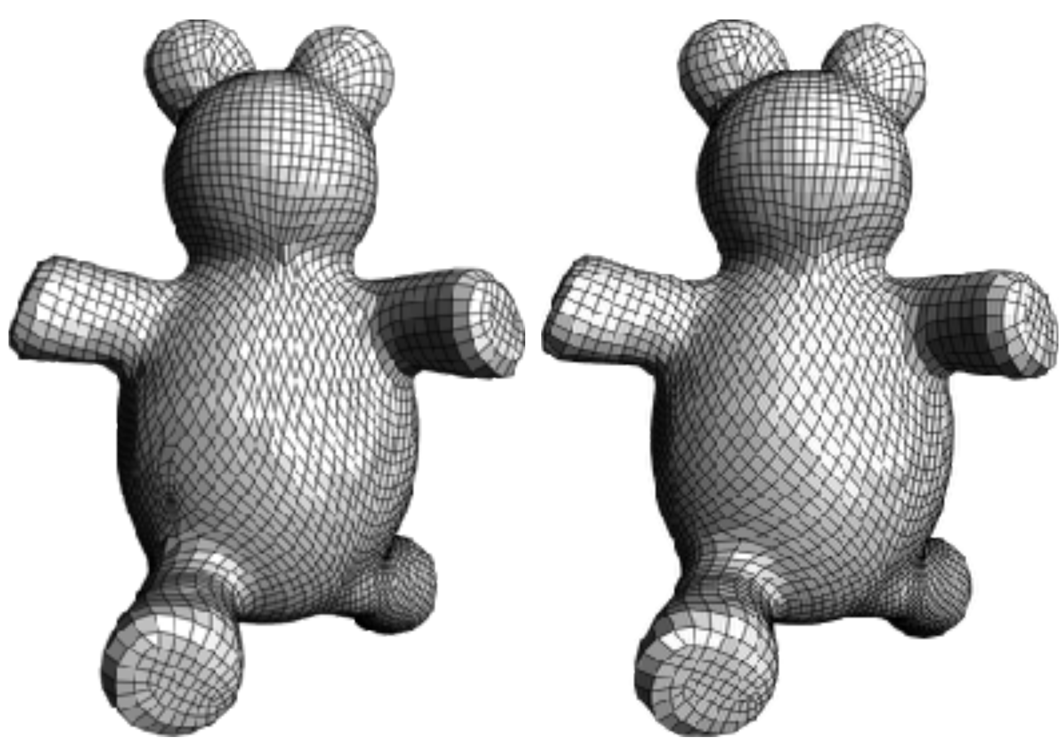
compare representative vectors

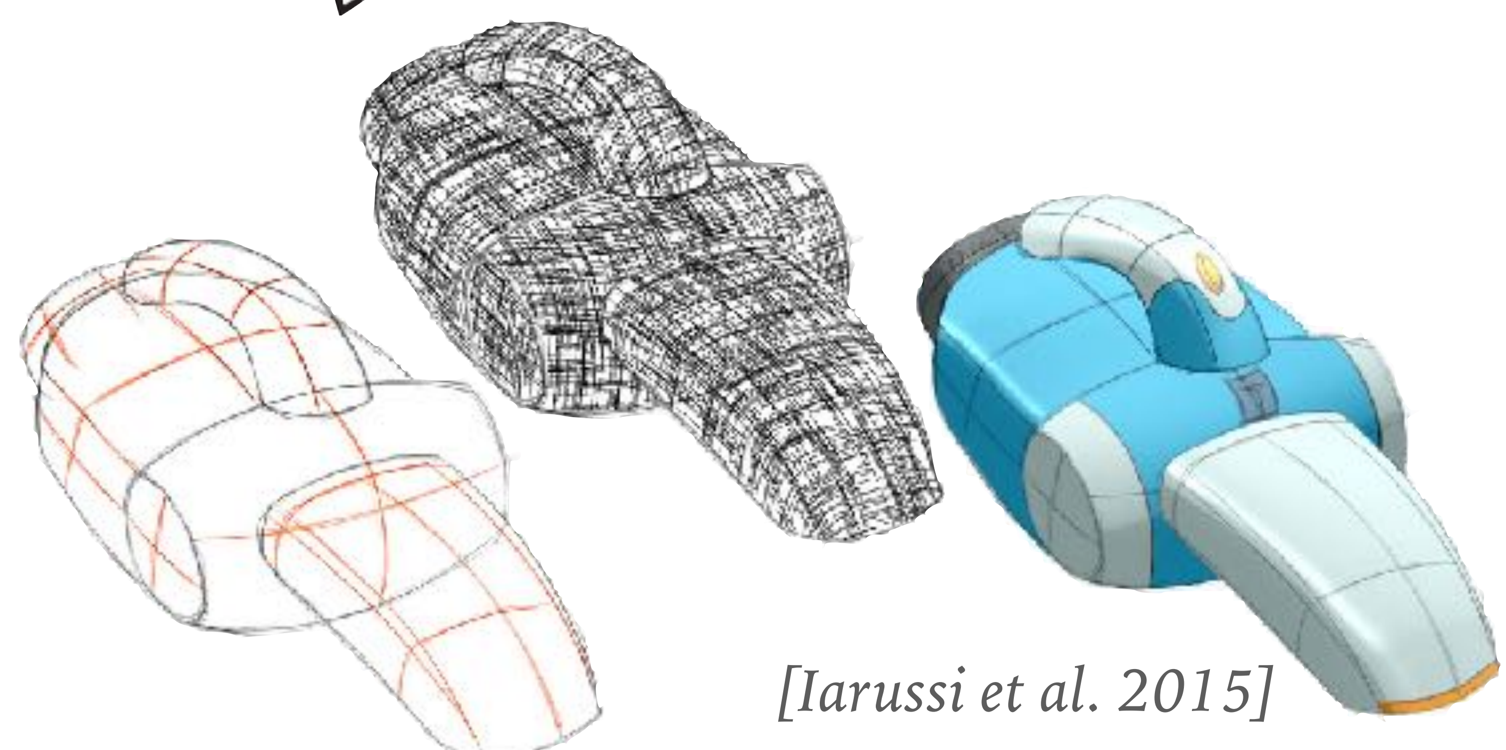
[Ray et al. 2006] [Knöppel et al. 2013]
[Palacios et al. 2007] [Diamanti et al. 2014]

OBJECTIVES - FIELD FAIRNESS

- Orthogonality



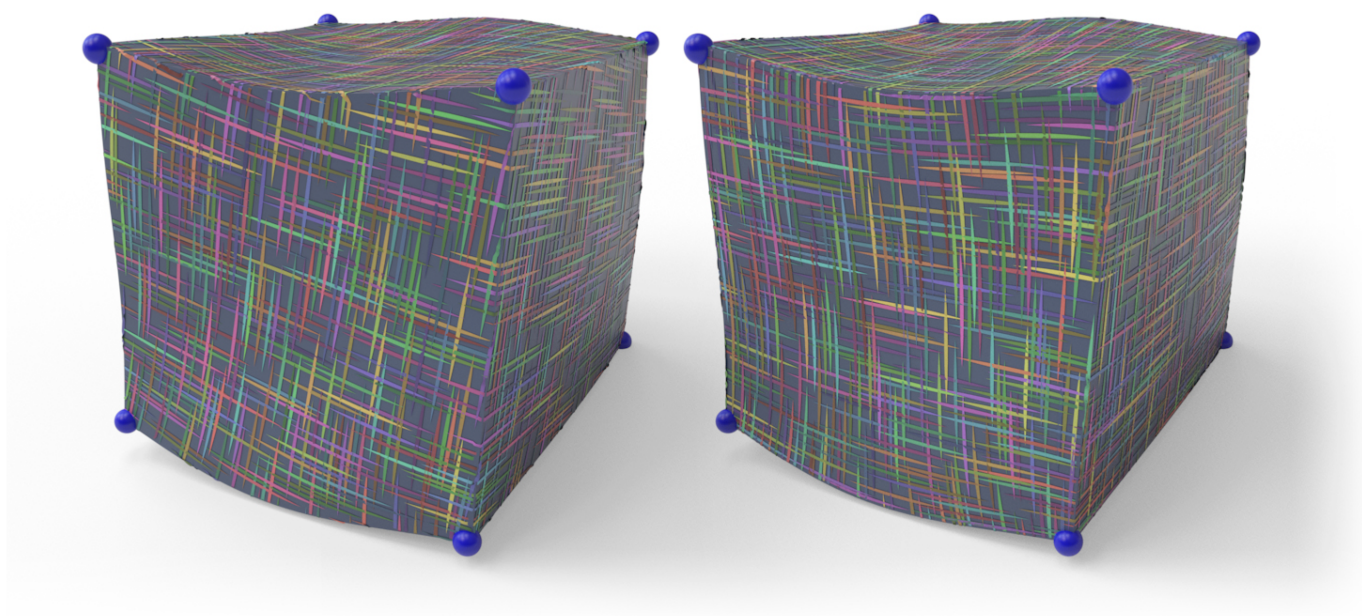




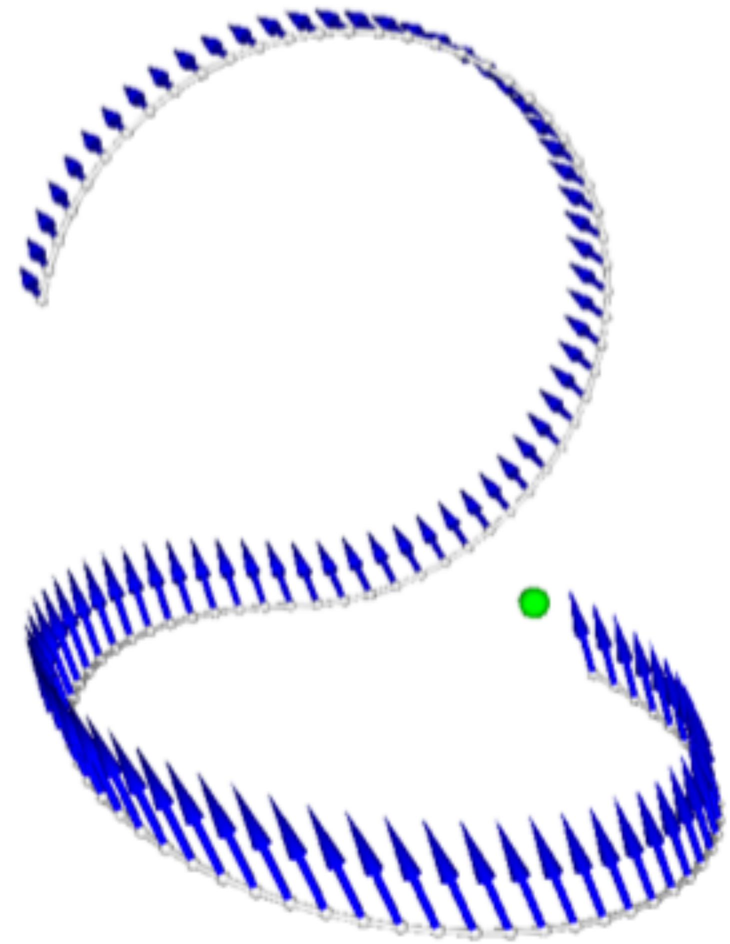
[Iarussi et al. 2015]

[Diamanti et al. 2014]

OBJECTIVES - ALIGNMENT TO FEATURES

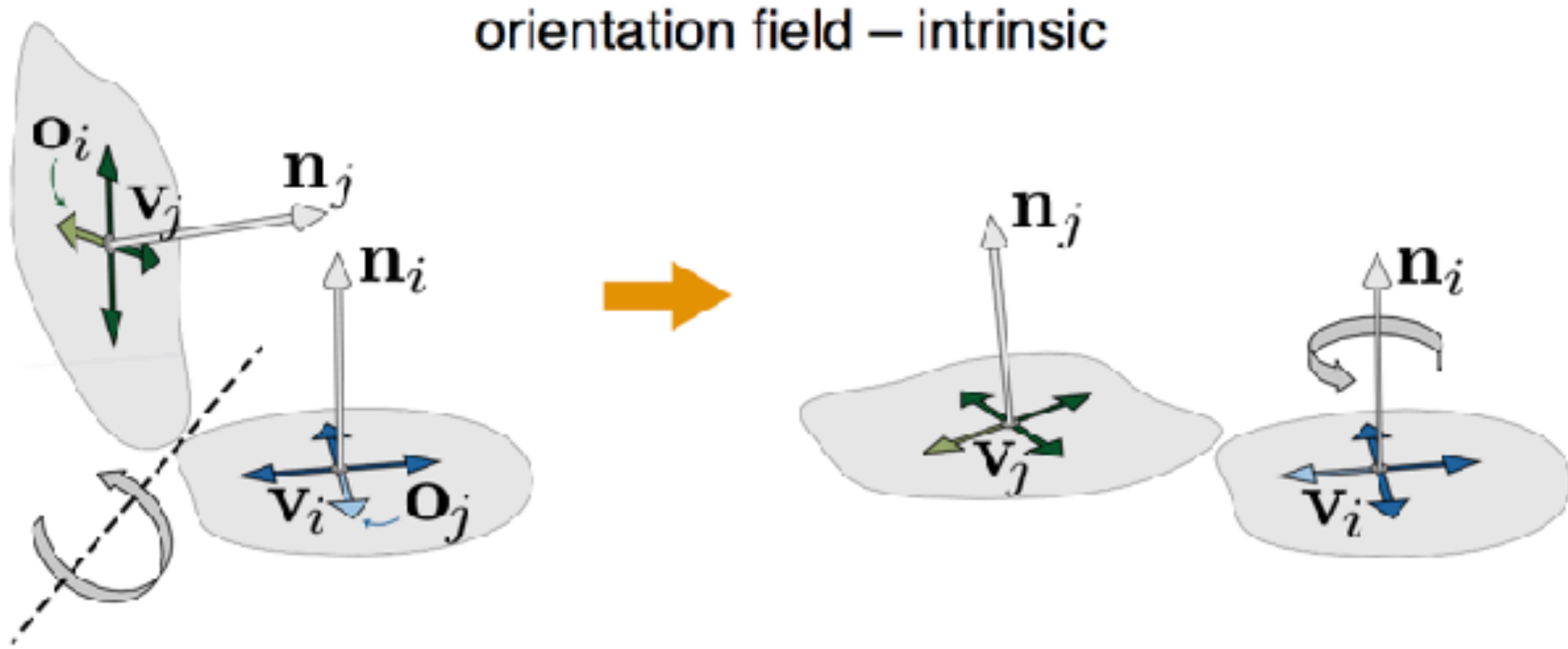


[Jakob et al. 2015]

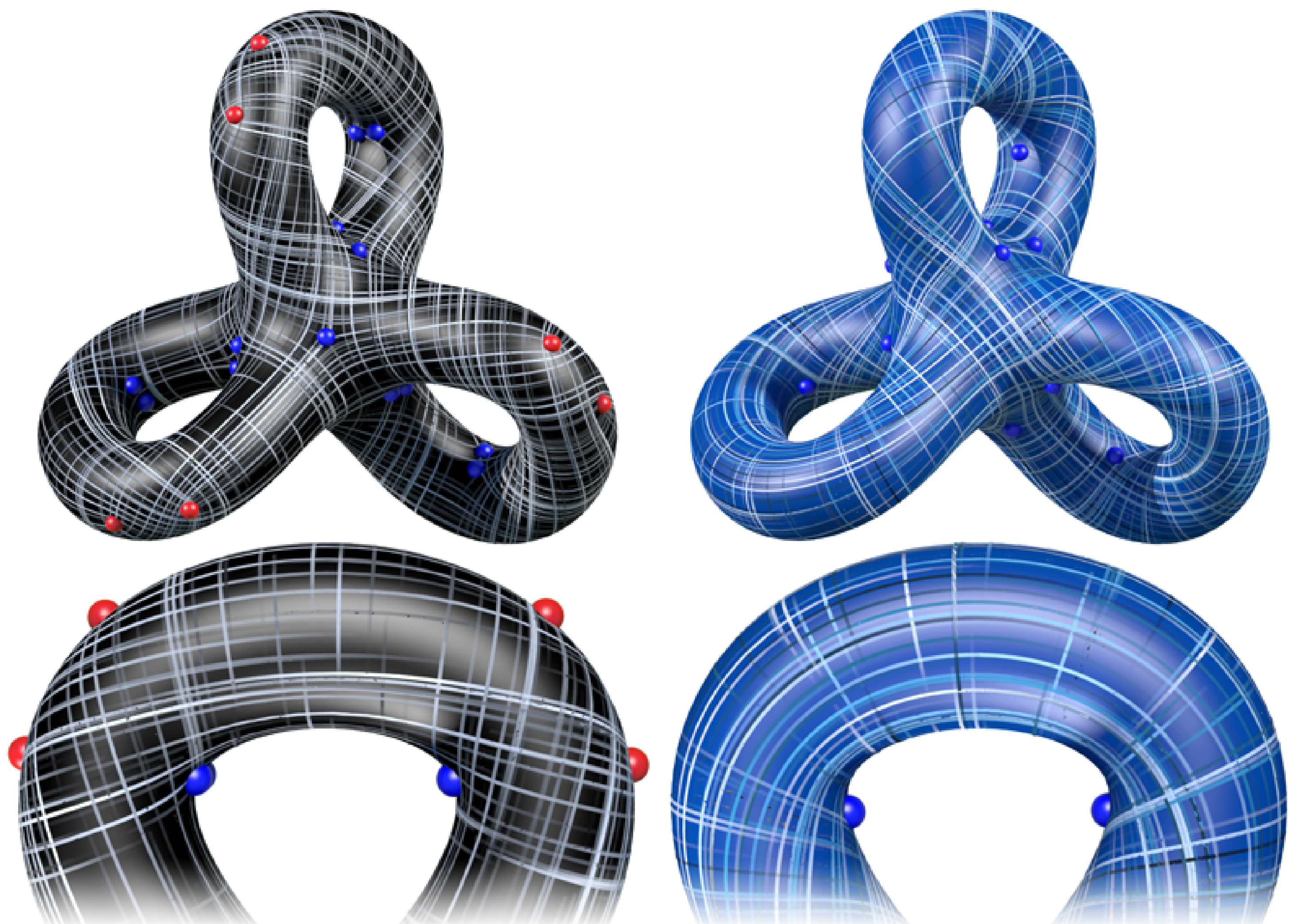


[Huang et al. 2016]

OBJECTIVES - ALIGNMENT TO FEATURES

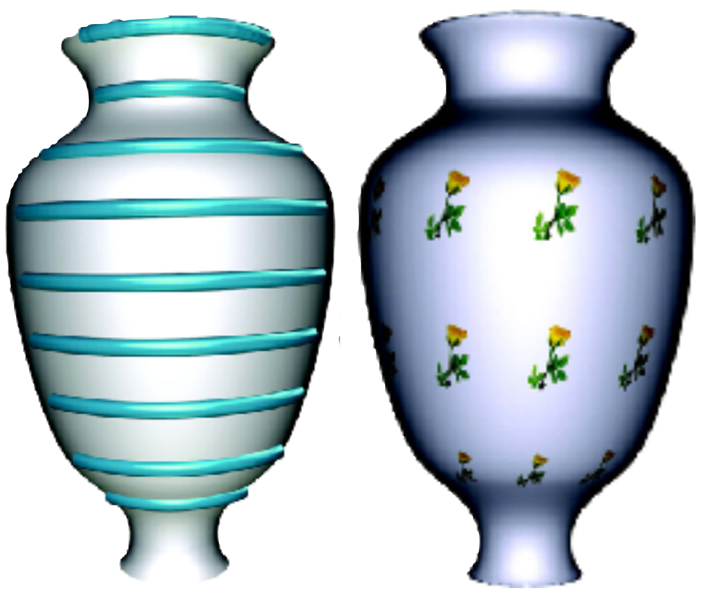


OBJECTIVES – SINGULARITY CONTROL



[Knöppel et al. 2013]

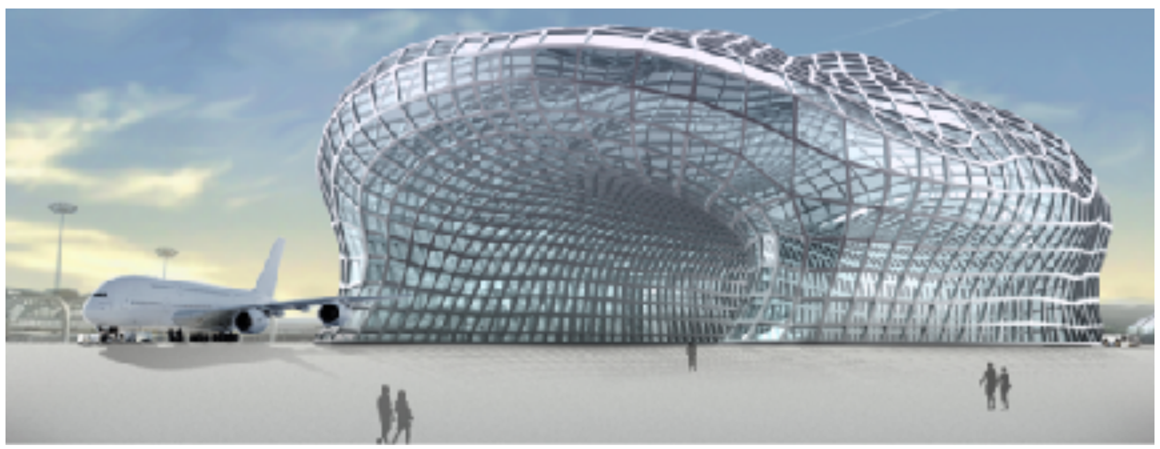
OBJECTIVES - ISOMETRY INDUCING



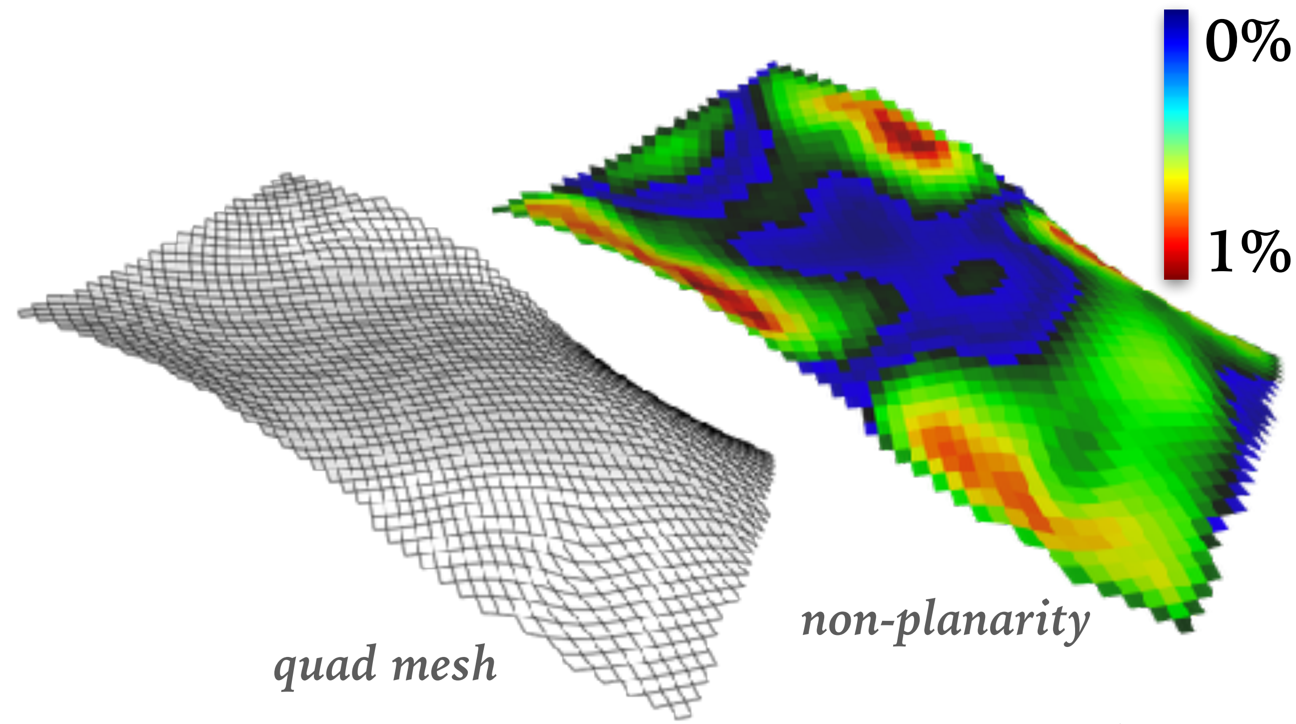
[Ben-Chen et al. 2010]

[Solomon et al. 2011]

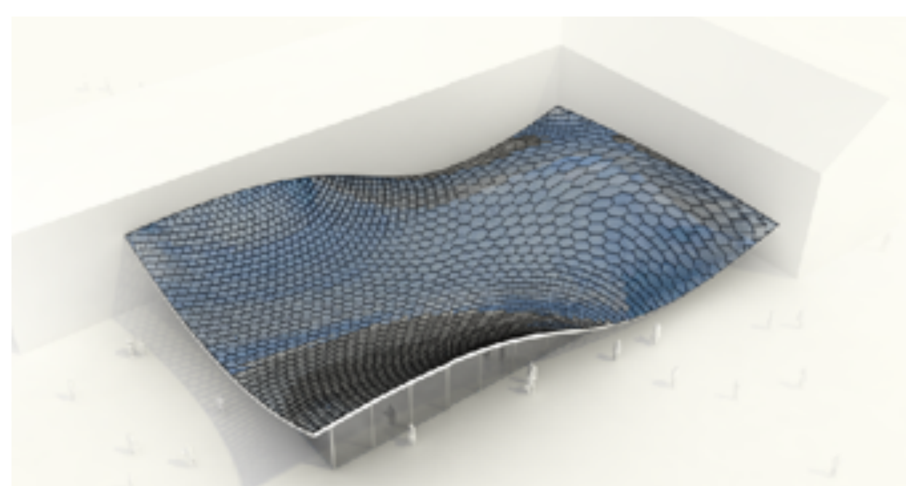
OBJECTIVES – CONJUGACY (PLANARITY)



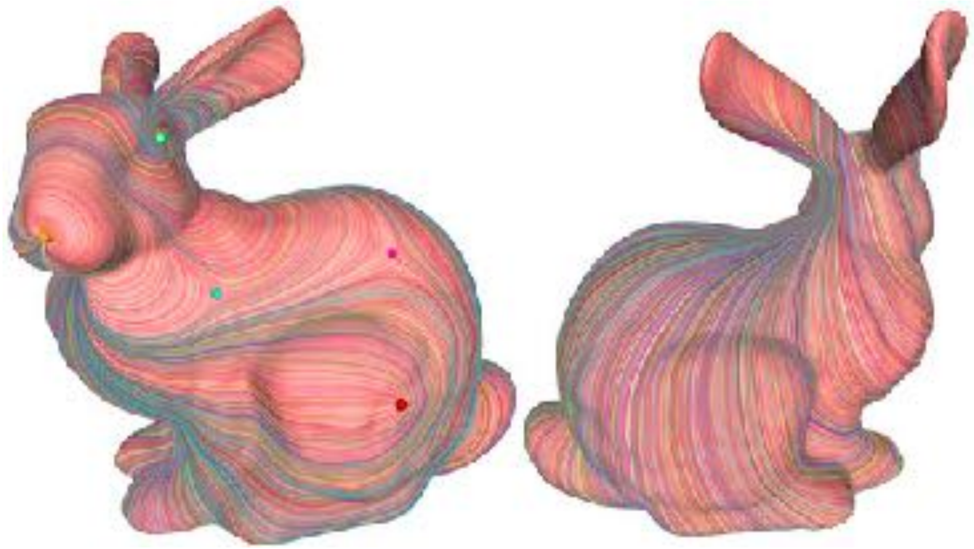
[Liu et al. 2011]



[Diamanti et al. 2014]

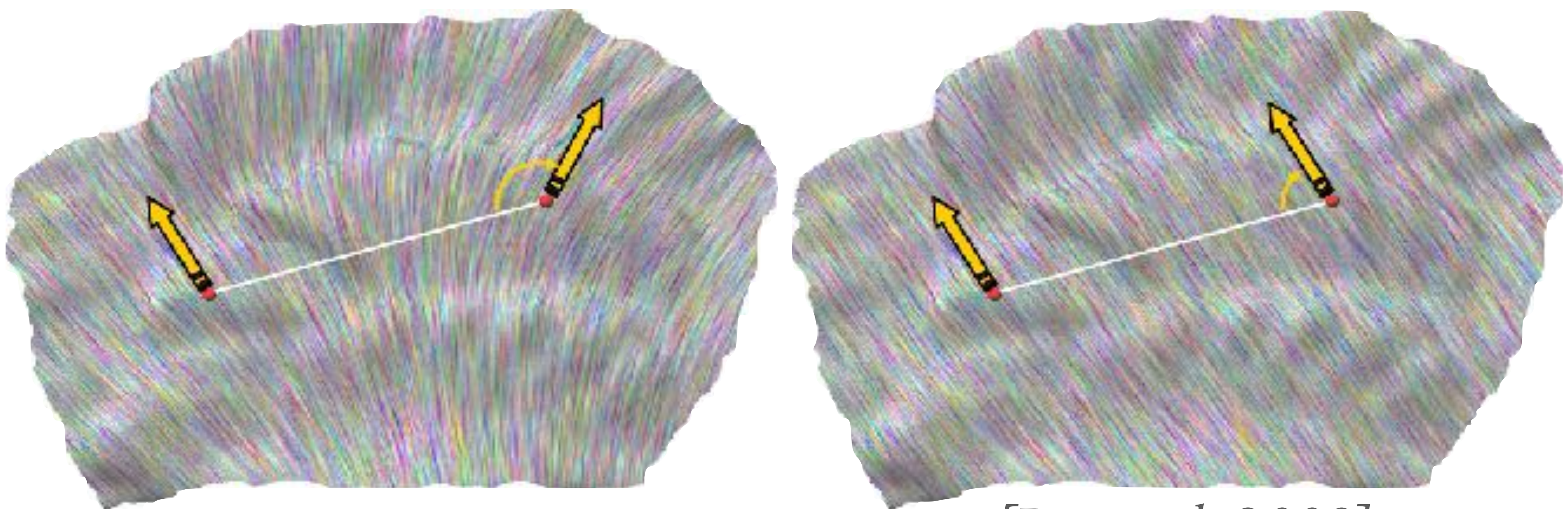


[Vaxman et al. 2015]

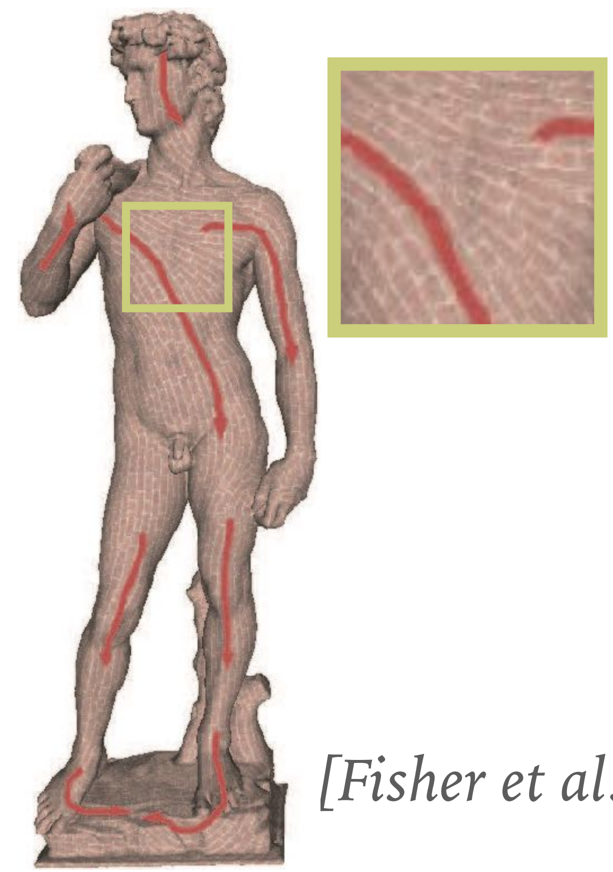


CONSTRAINTS

CONSTRAINTS - ALIGNMENT



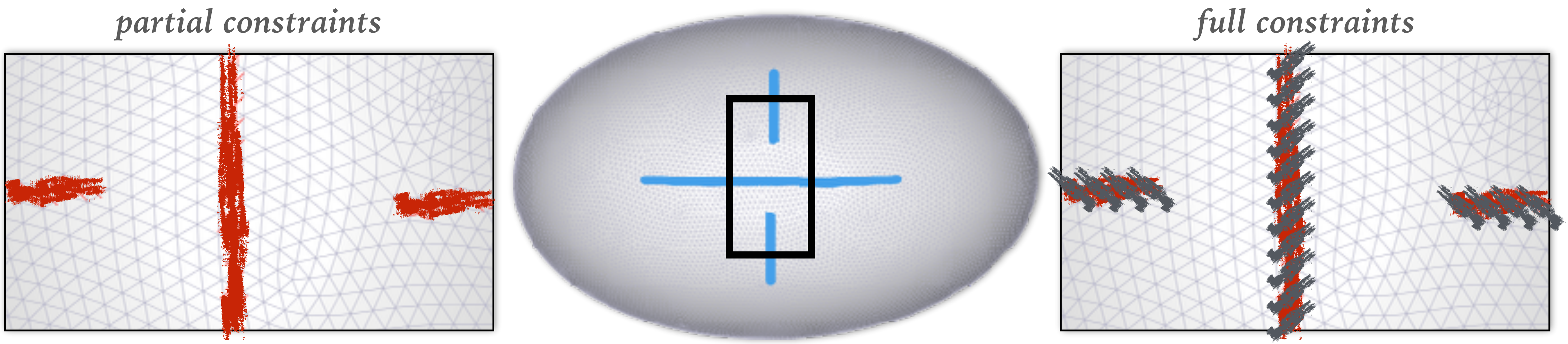
[Ray et al. 2008]



[Fisher et al. 2007]

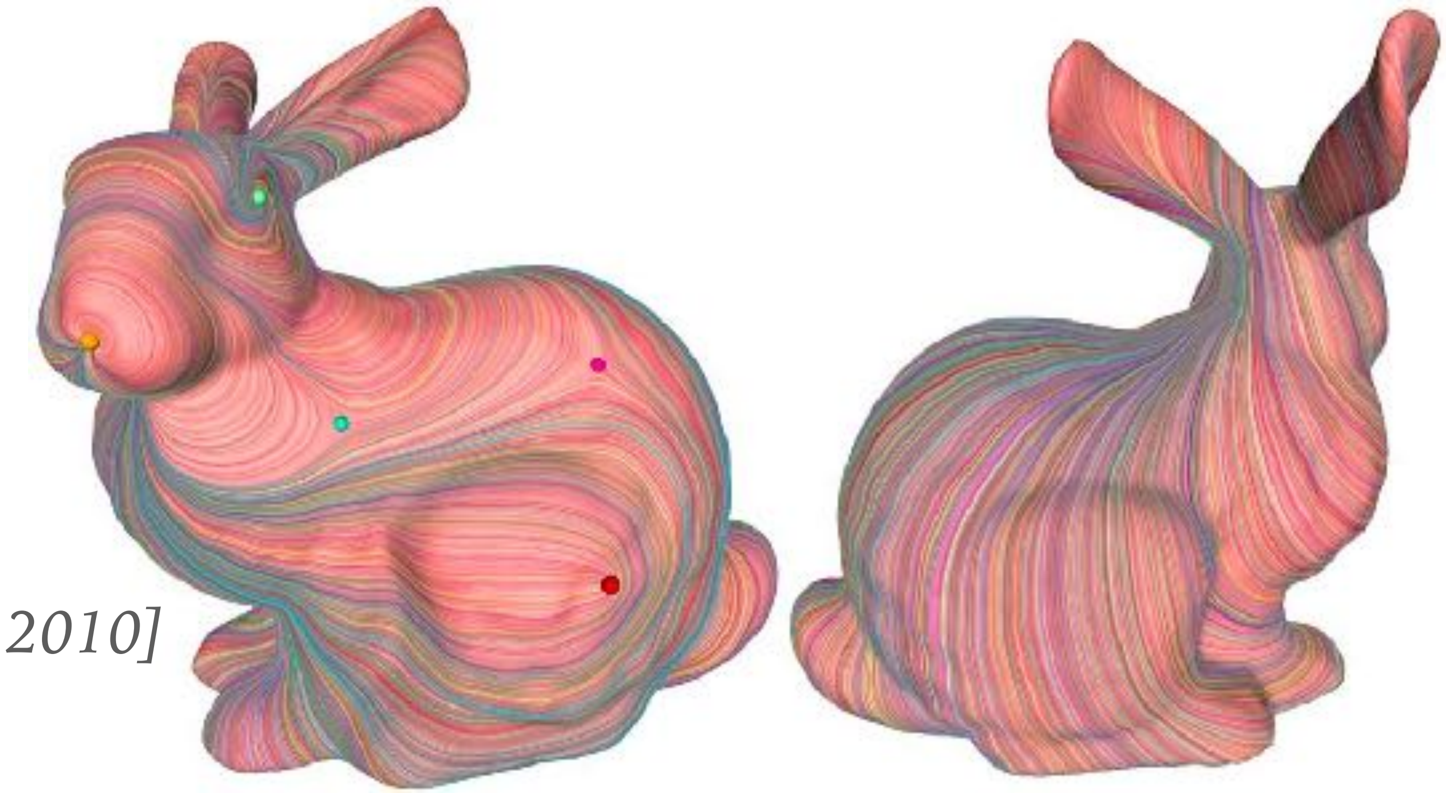
- Complete or Partial

partial constraints



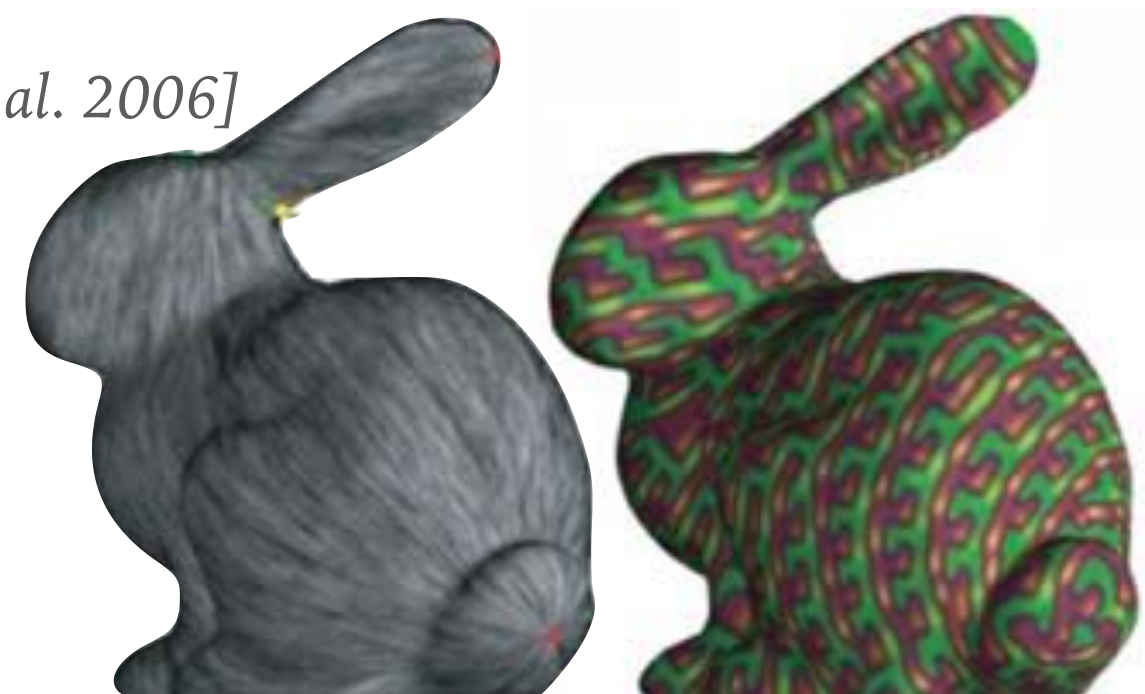
full constraints

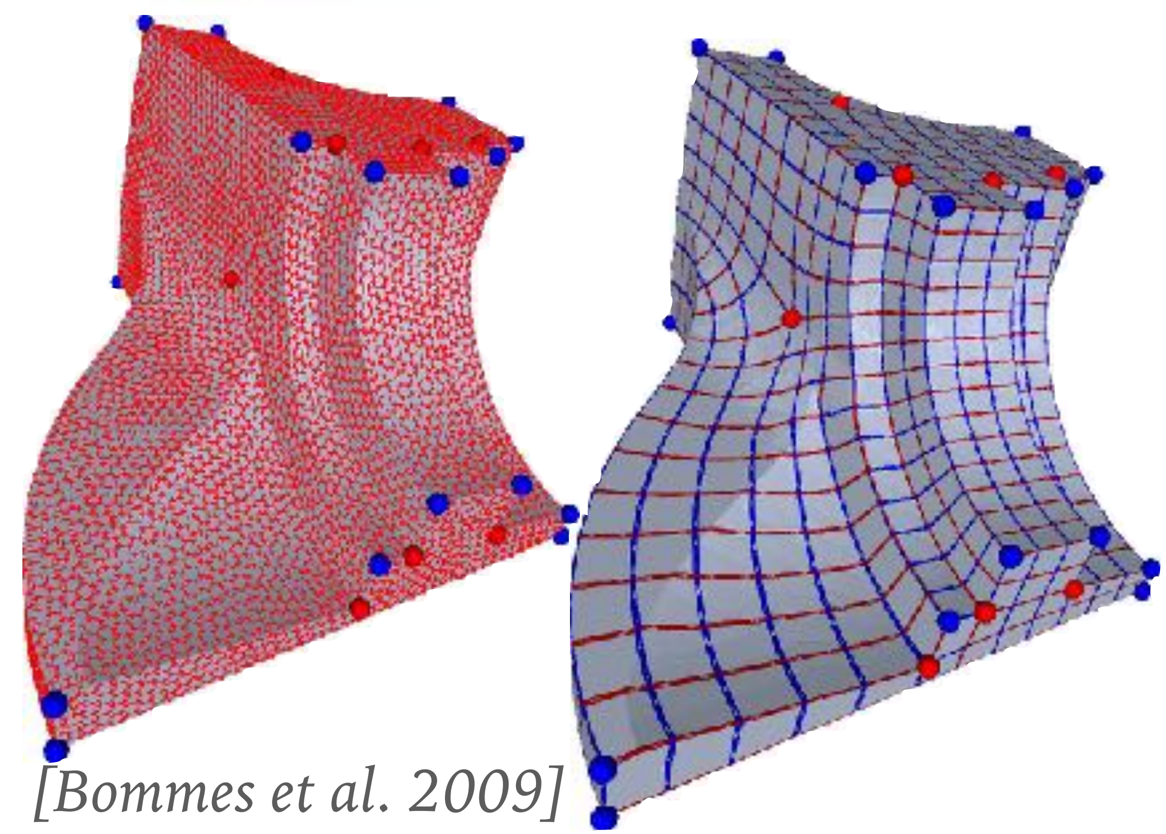
CONSTRAINTS - TOPOLOGY



[Crane et al. 2010]

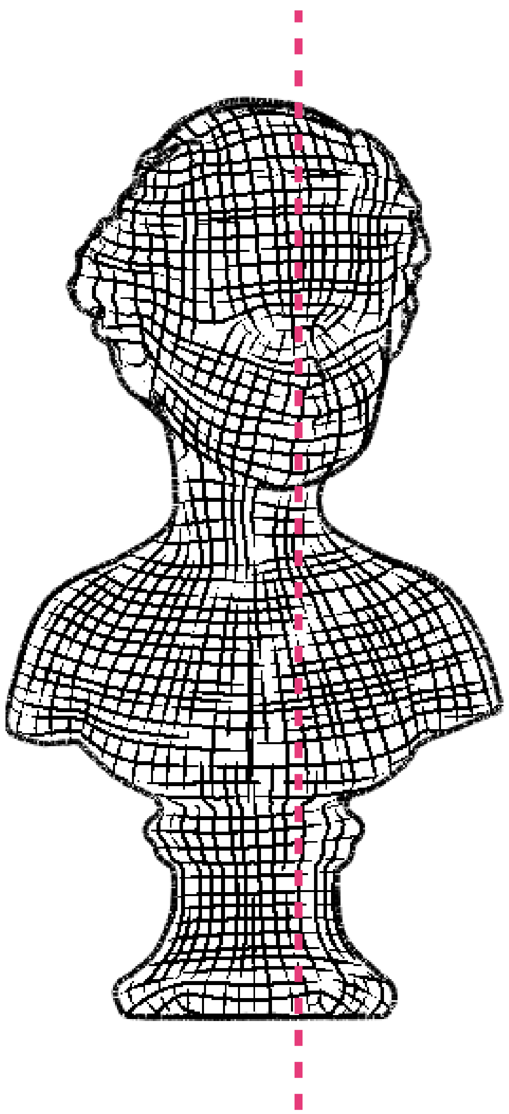
[Zhang et al. 2006]



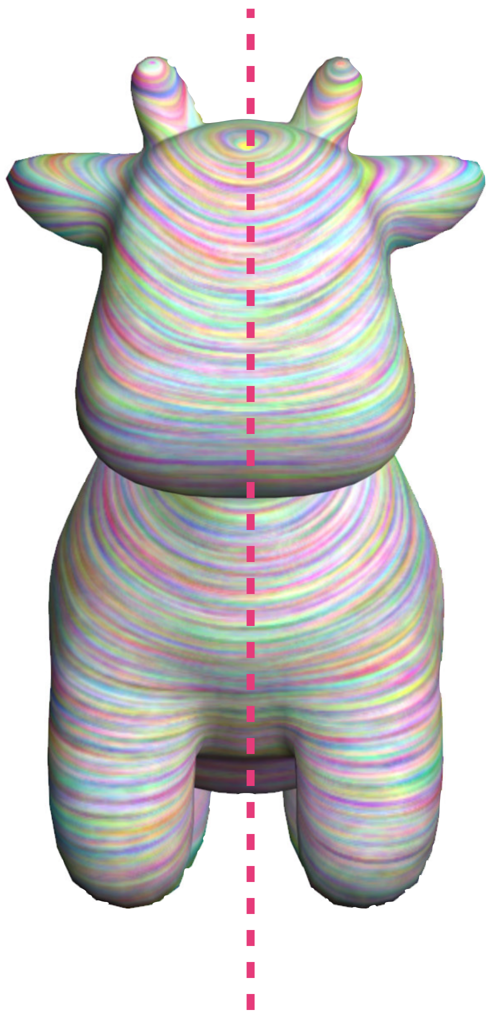


[Bommes et al. 2009]

CONSTRAINTS - SYMMETRY

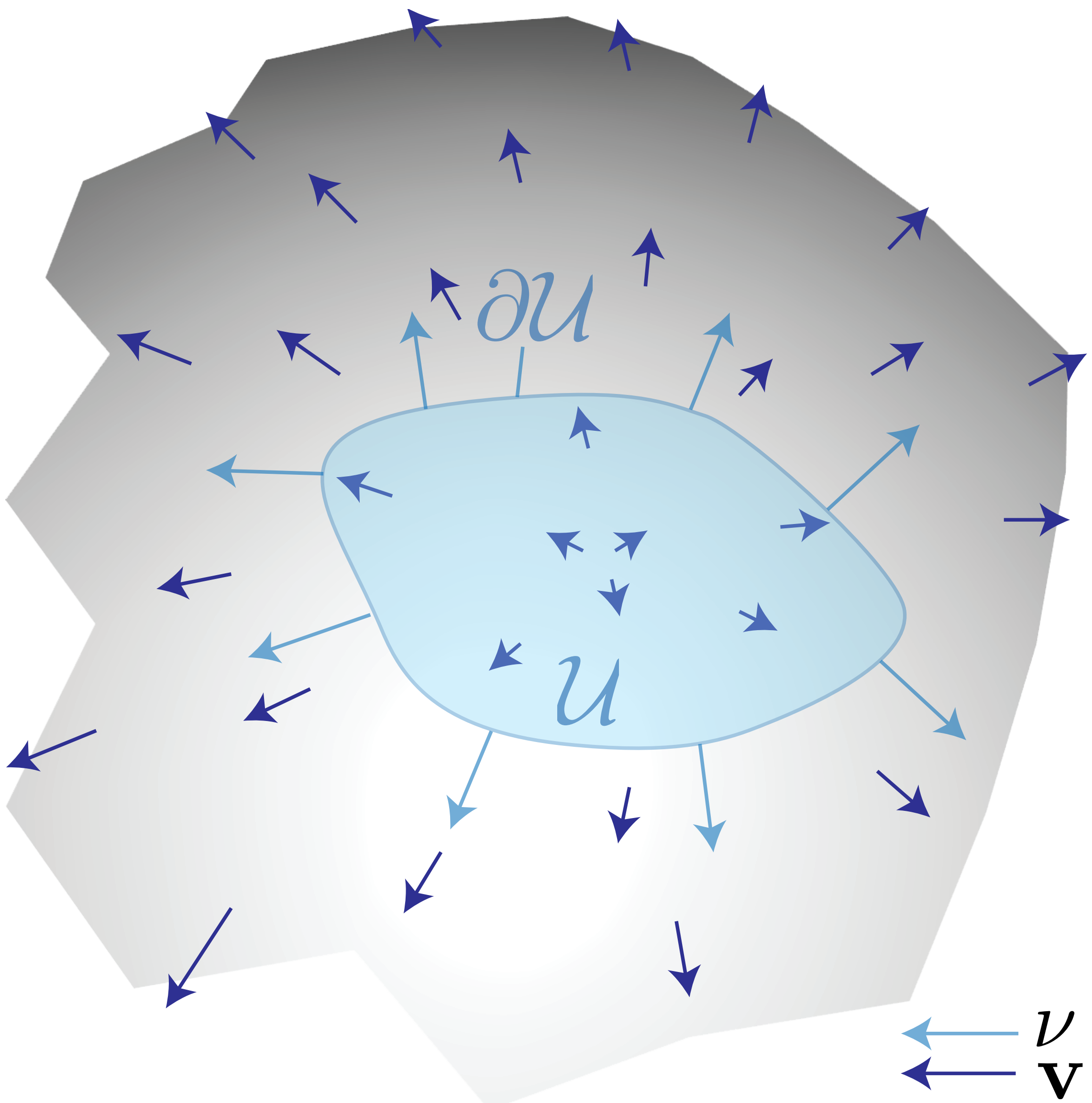


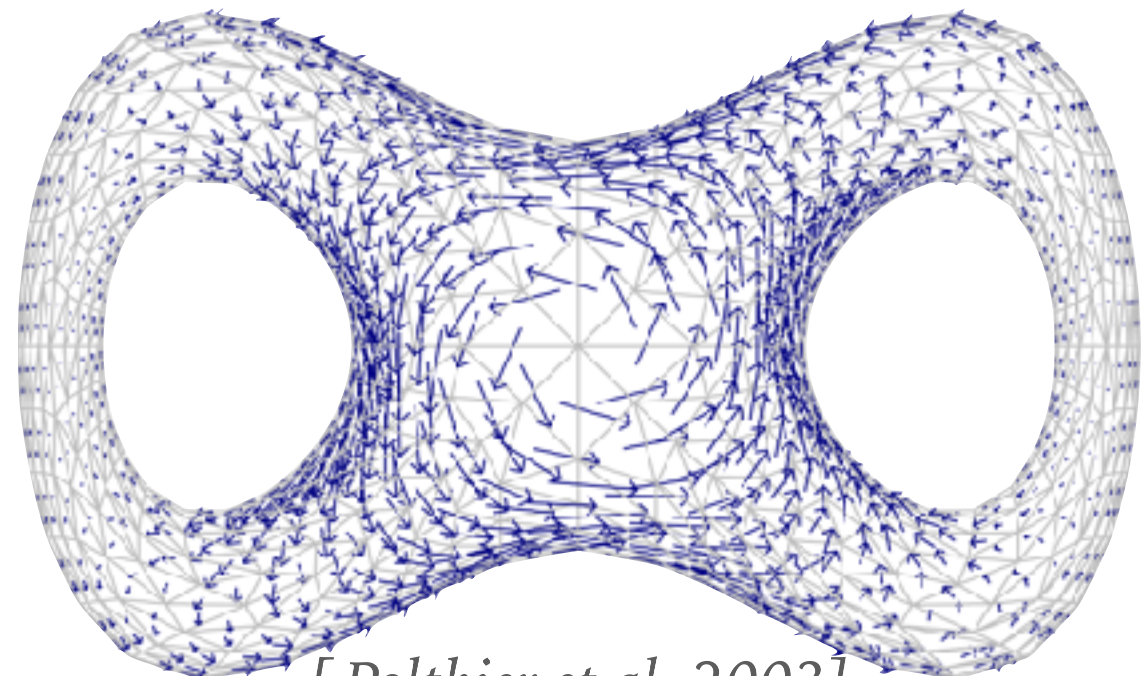
[Panozzo et al. 2012]



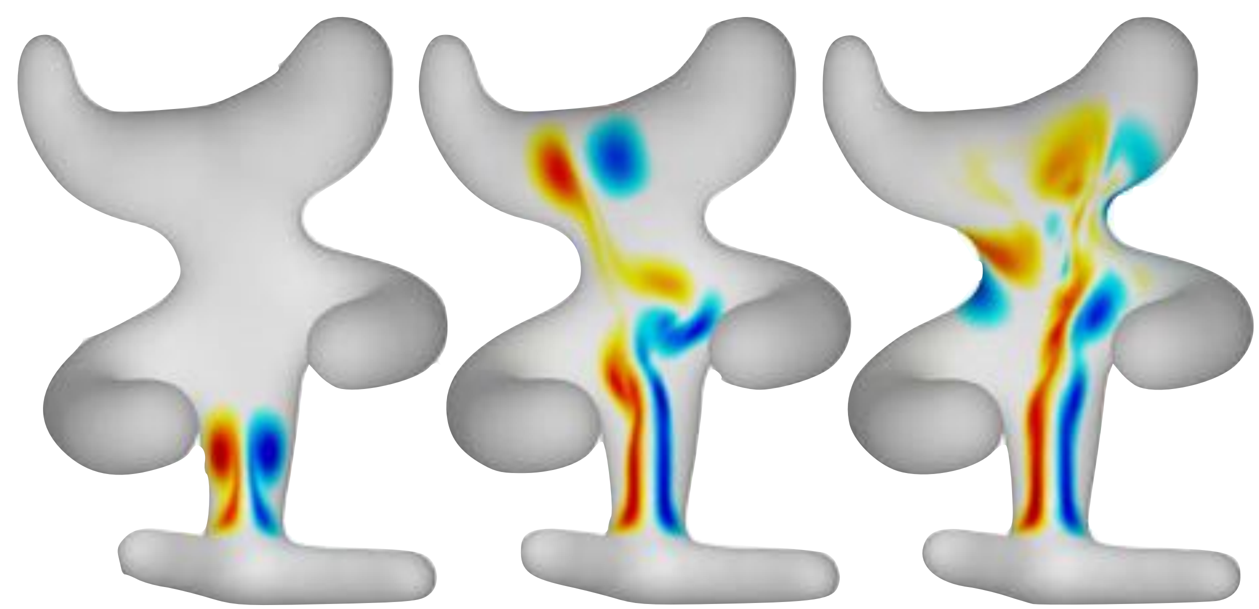
[Azencot et al. 2013]

CONSTRAINTS – DIFFERENTIAL – DIVERGENCE



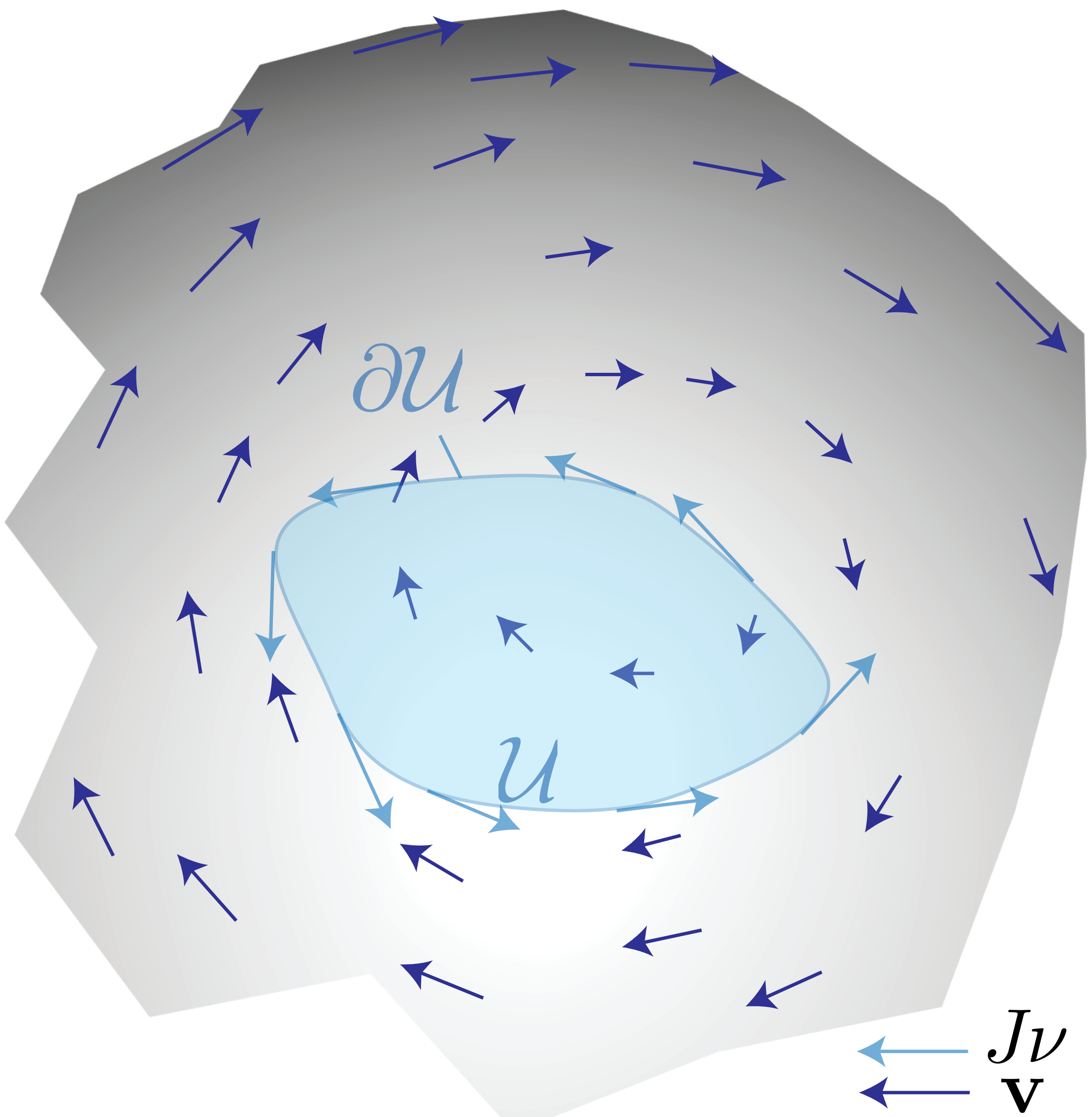


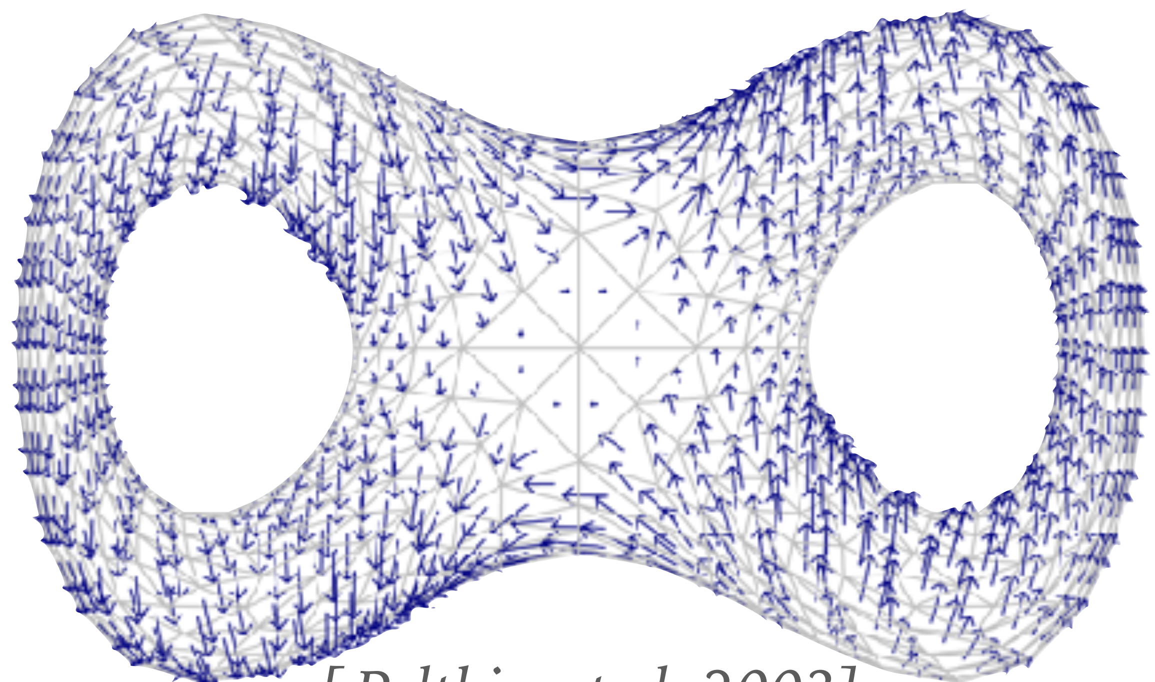
[Polthier et al. 2003]



[Azencot et al. 2014]

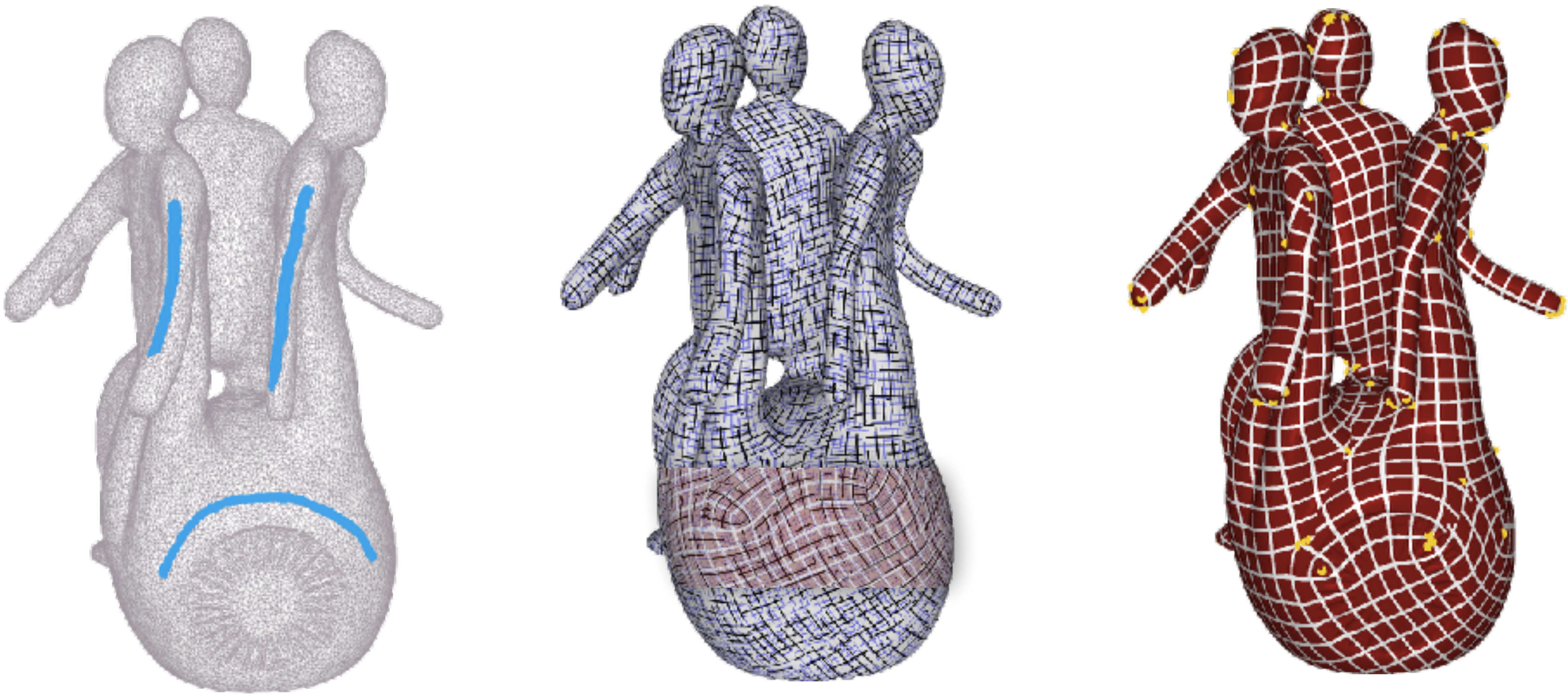
CONSTRAINTS – DIFFERENTIAL – CURL





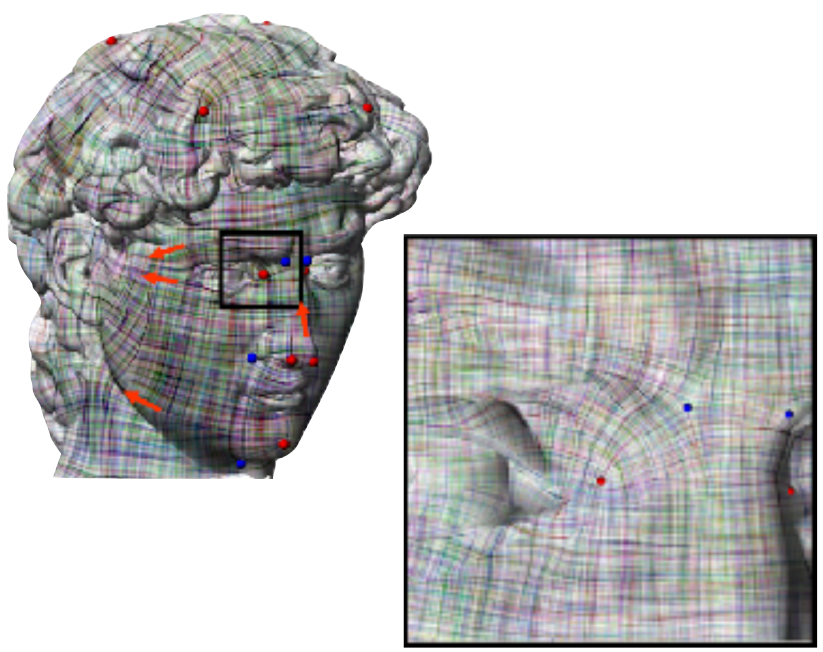
[Polthier et al. 2003]

CONSTRAINTS - DIFFERENTIAL - CURL



[Diamanti et al. 2015]

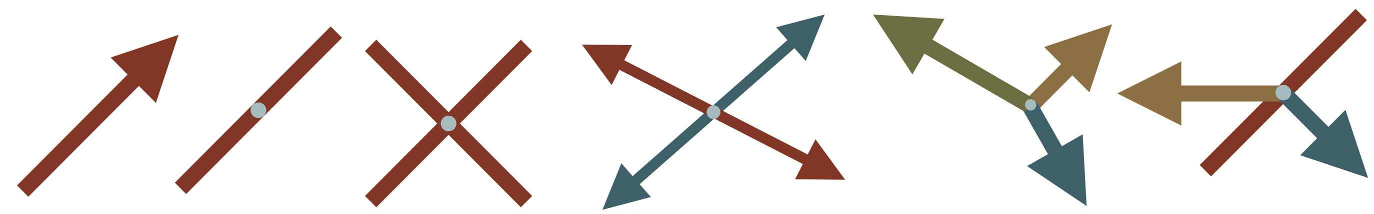
HOW TO CHOOSE THE RIGHT METHOD?



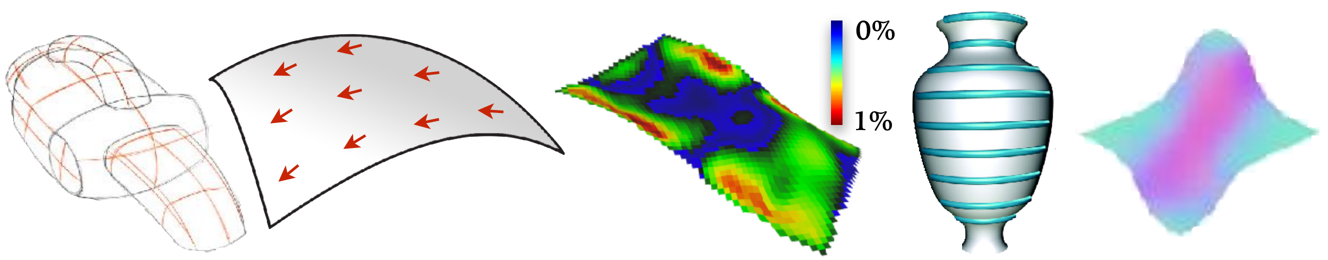
Demos: ./libdirectional/examples

CHOOSING THE RIGHT METHOD

- Choose the type of object



- Determine what the ideal field is (objective)

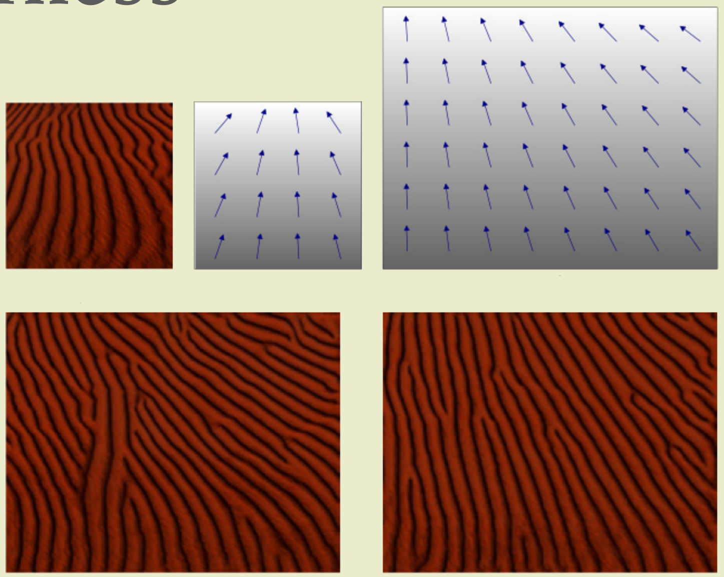


- Consider the types of guarantees required
- Consider design strategy in terms of efficiency, convergence,…
    - this affects the choice of representation
- Consider discretization preference

CASE STUDY: VECTOR FIELDS

Objective: Fairness

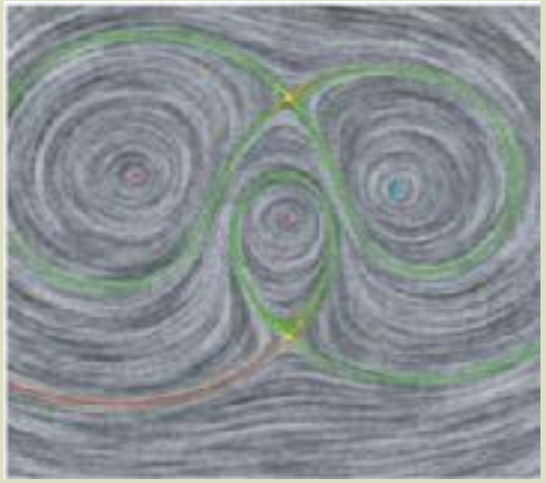
[Pedersen et al. 1995]
[Turk et al. 2001]
[Praun et al. 2000]



Objective: Other

Isometries - Killing
[Ben-Chen et al. 2010] [Azencot et al. 2013]

Curl / Divergence Control
[Fisher et al. 2007]
[Zhang et al. 2006]



Symmetries
[Azencot et al. 2013]

Constraints: Directional

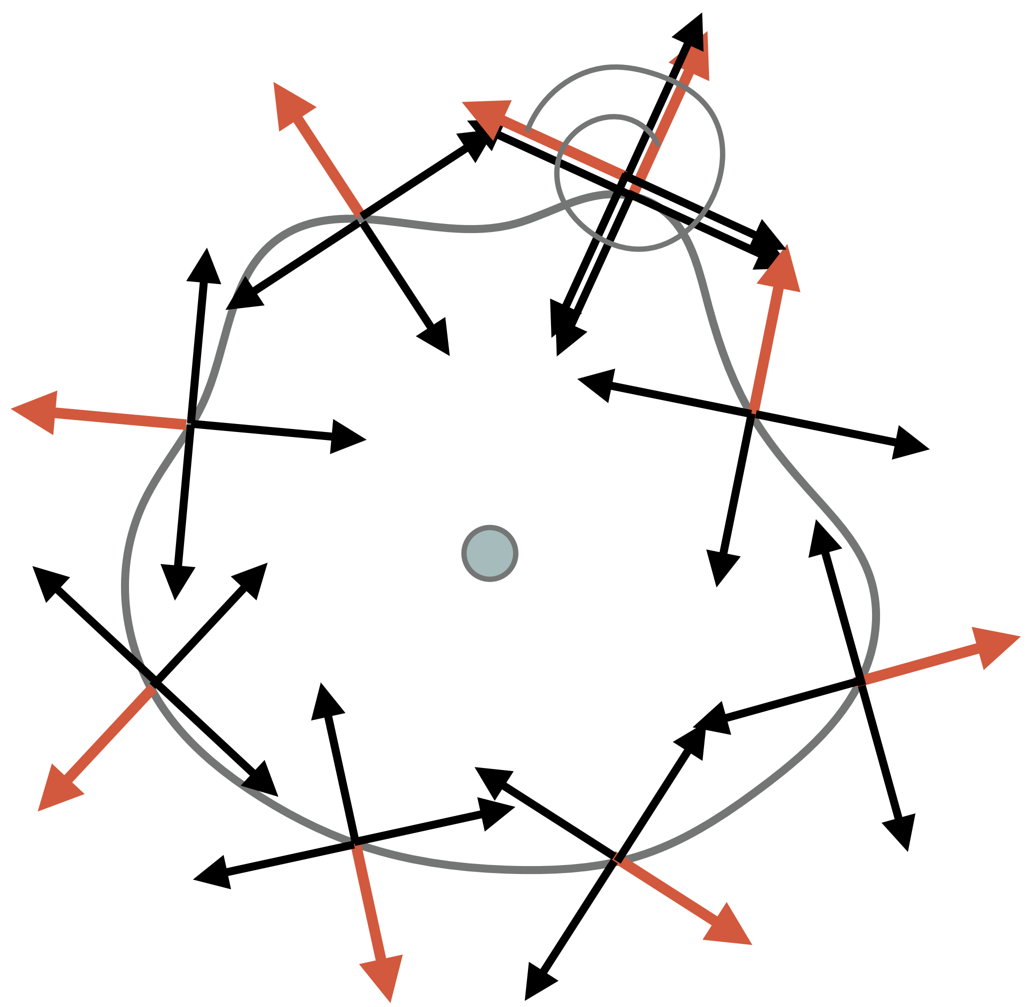
hard
[Pedersen et al. 1995]
[Praun et al. 2000]
[Azencot et al. 2013]

soft
[Turk et al. 2001]
[Zhang et al. 2006]

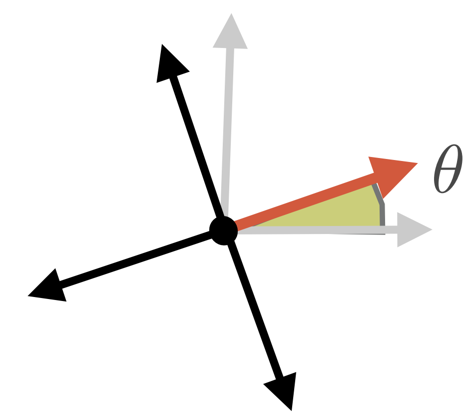
soft +singularity control
[Fisher et al. 2007]
[Zhang et al. 2006]

CASE STUDY: N-DIRECTIONAL FIELD, FIXED TOPOLOGY

- Objective: “As-parallel-as-possible”
- Singularity prescription straightforward with angle based representations



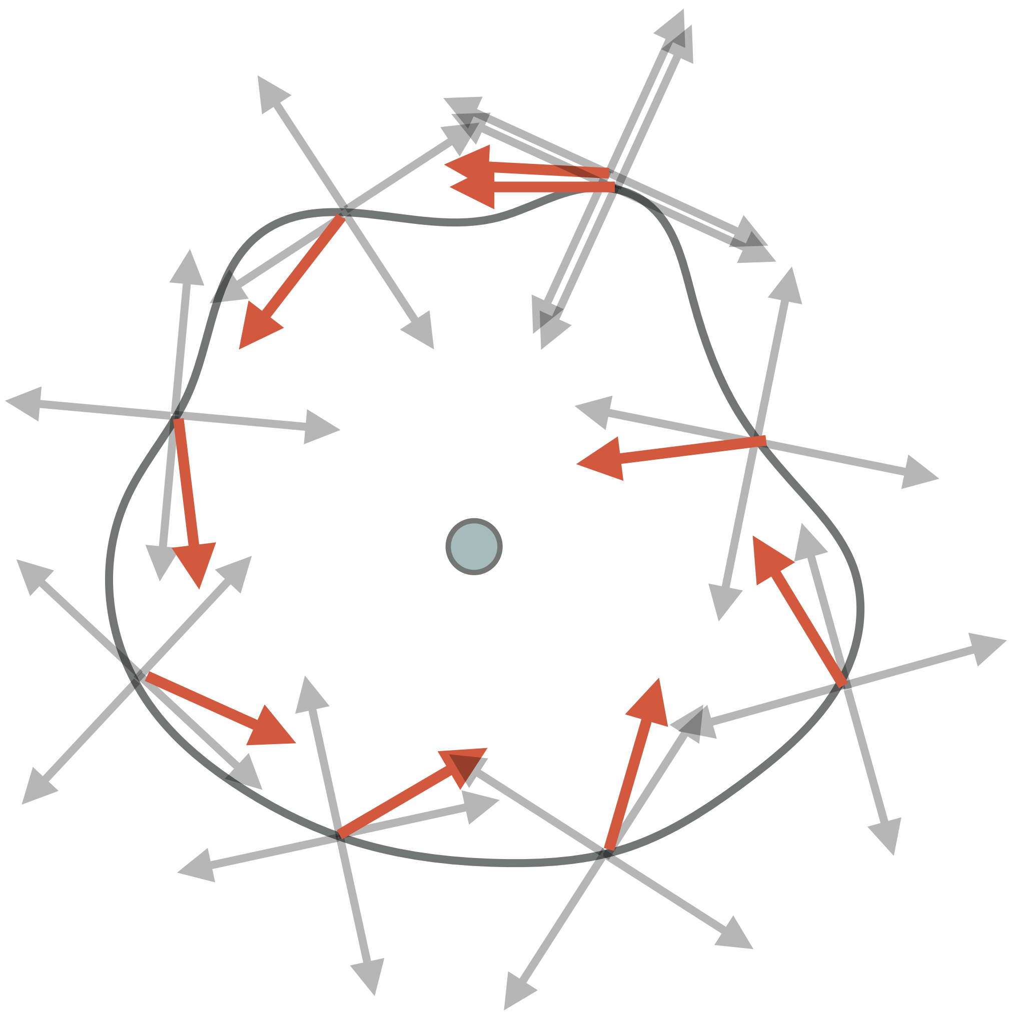
desired singularity index = 5/4



angle-based representation of a 4-direction field

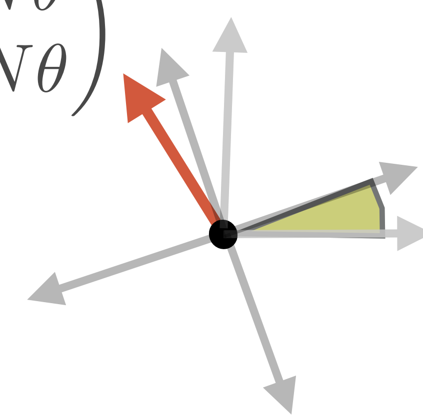
CASE STUDY: N-DIRECTIONAL FIELD, FIXED TOPOLOGY

- Objective: “As-parallel-as-possible”
- Cartesian/Complex representative insensitive to 2π rotations



desired singularity index = 5/4

$$\begin{pmatrix} \sin N\theta \\ \cos N\theta \end{pmatrix}$$

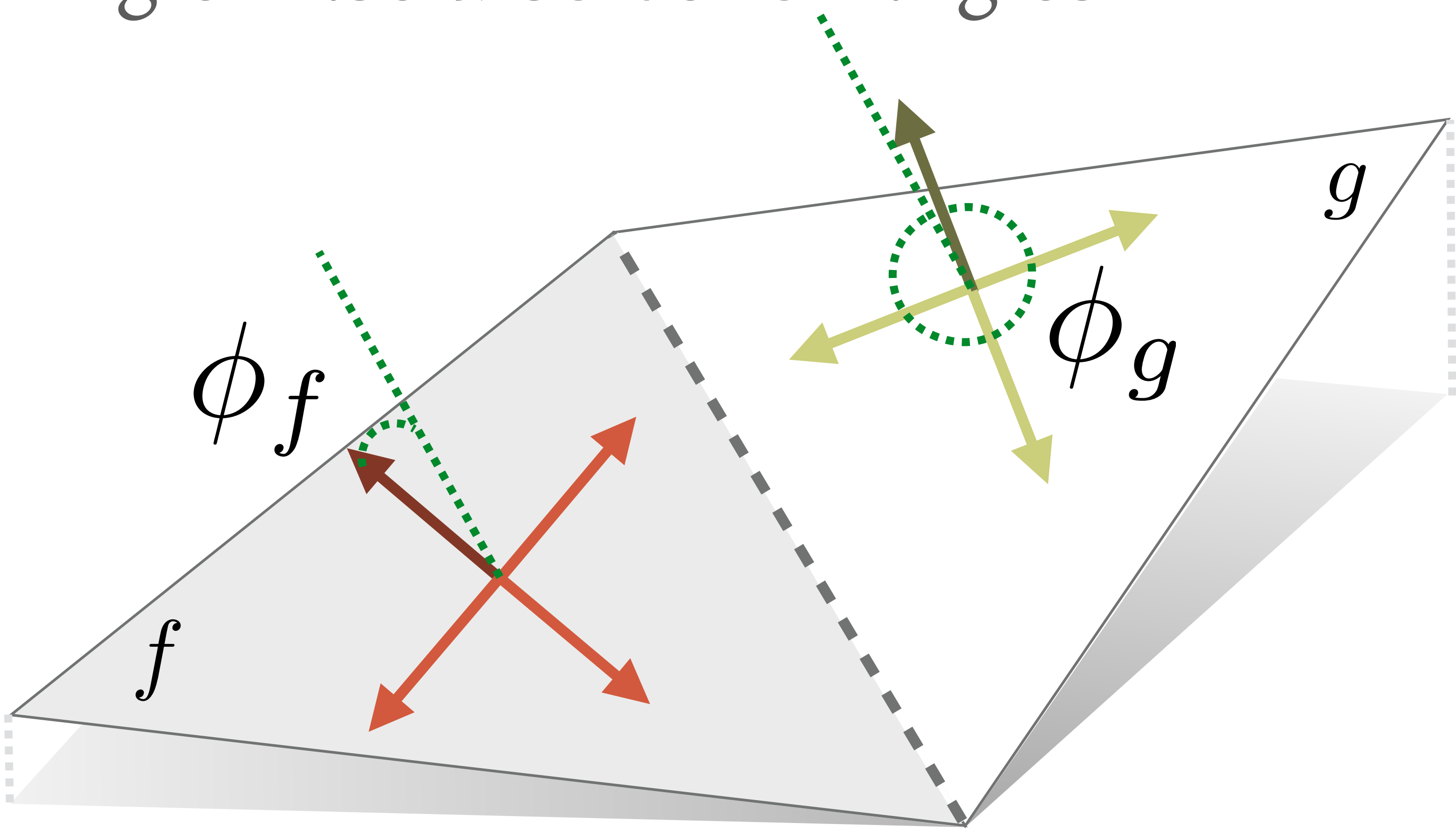


cartesian representation of a 4-direction field

How to prescribe angle difference $2\pi + \pi/4$ with principal rotation/matching ?

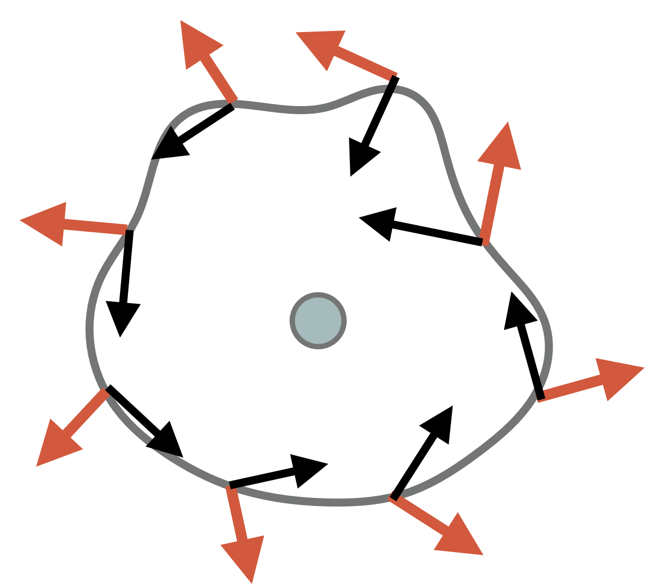
CASE STUDY: N-DIRECTIONAL FIELD, FIXED TOPOLOGY

- Angle-based: solve for angles



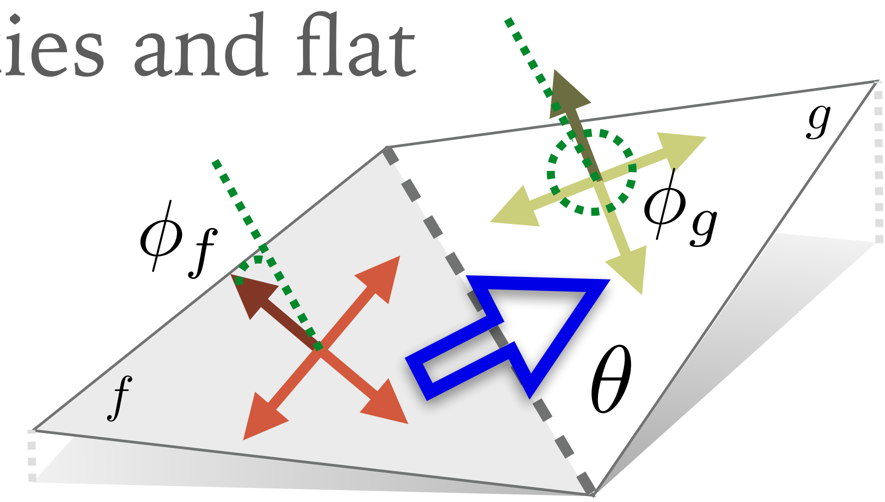
[Ray et al. 2008]
[Crane et al. 2010]

- ➤ *linear system*
- ➤ *at least one constraint needed*

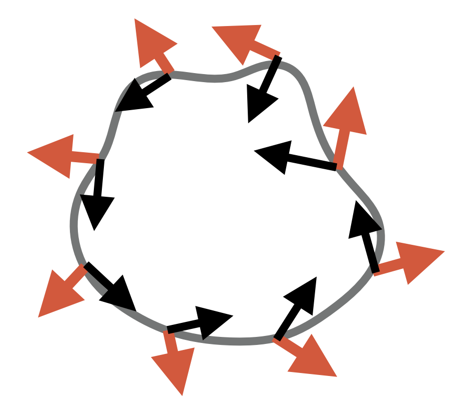


DEMO: TRIVIAL CONNECTIONS

- Input:
  - Original vertex curvatures k_0.
  - Prescribed curvatures k (cone singularities and flat vertices).



- Output:
  - Edge-based deviation from parallelity θ.
- Objective: as-parallel as possible.



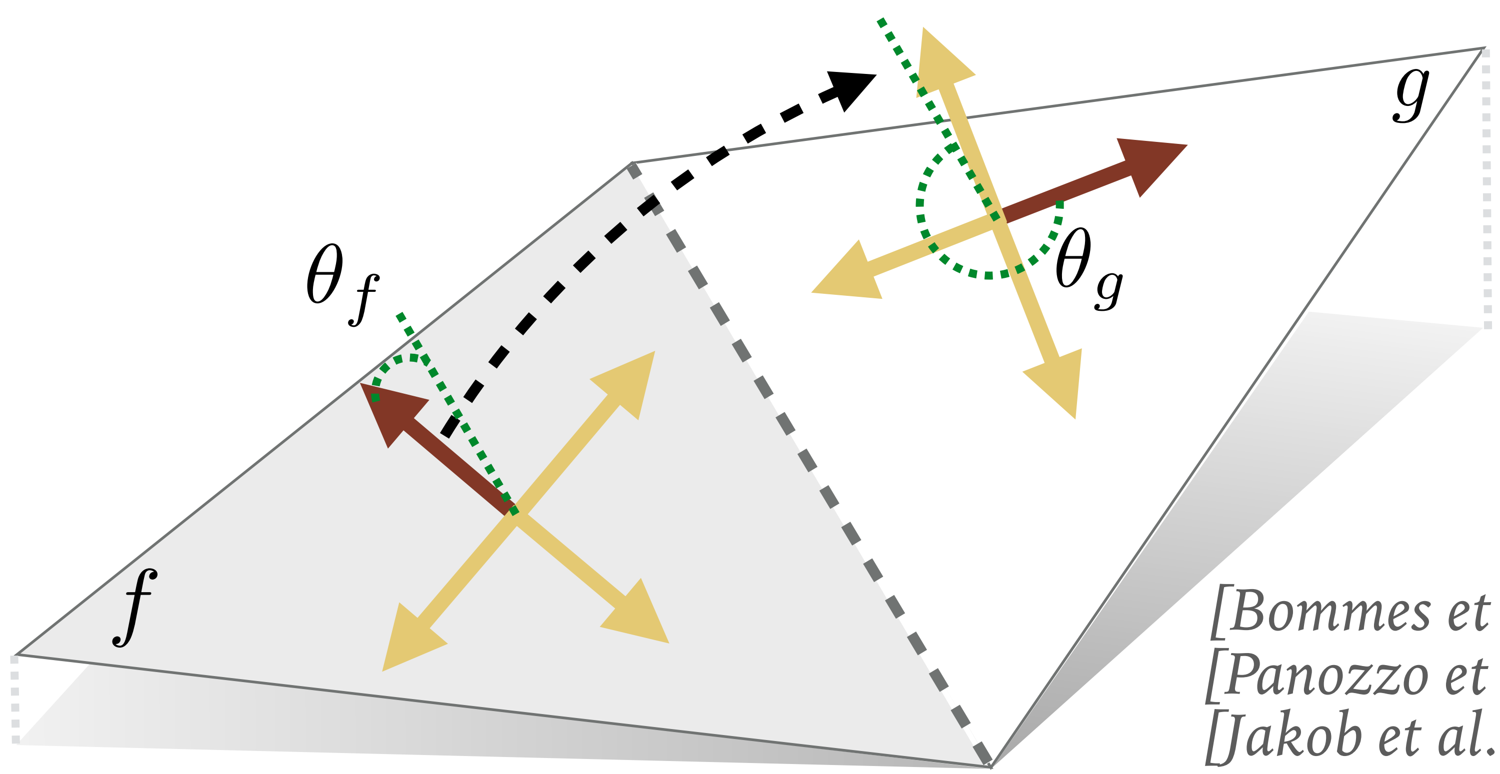
- Linearly-constrained quadratic minimization:

As parallel as possible —> $$\theta = argmin|\theta|^2 \ s.t.$$

Prescribed Singularities —> $$(d_0)^T\theta = k - k_0$$

CASE STUDY: N-DIRECTIONAL FIELD, FREE TOPOLOGY

- Objective: "As-parallel-as-possible"
- Explicitly model topology (typically angle-based)
    - Matchings are explicitly modeled
    - Mixed Integer Optimization
    - Local minima

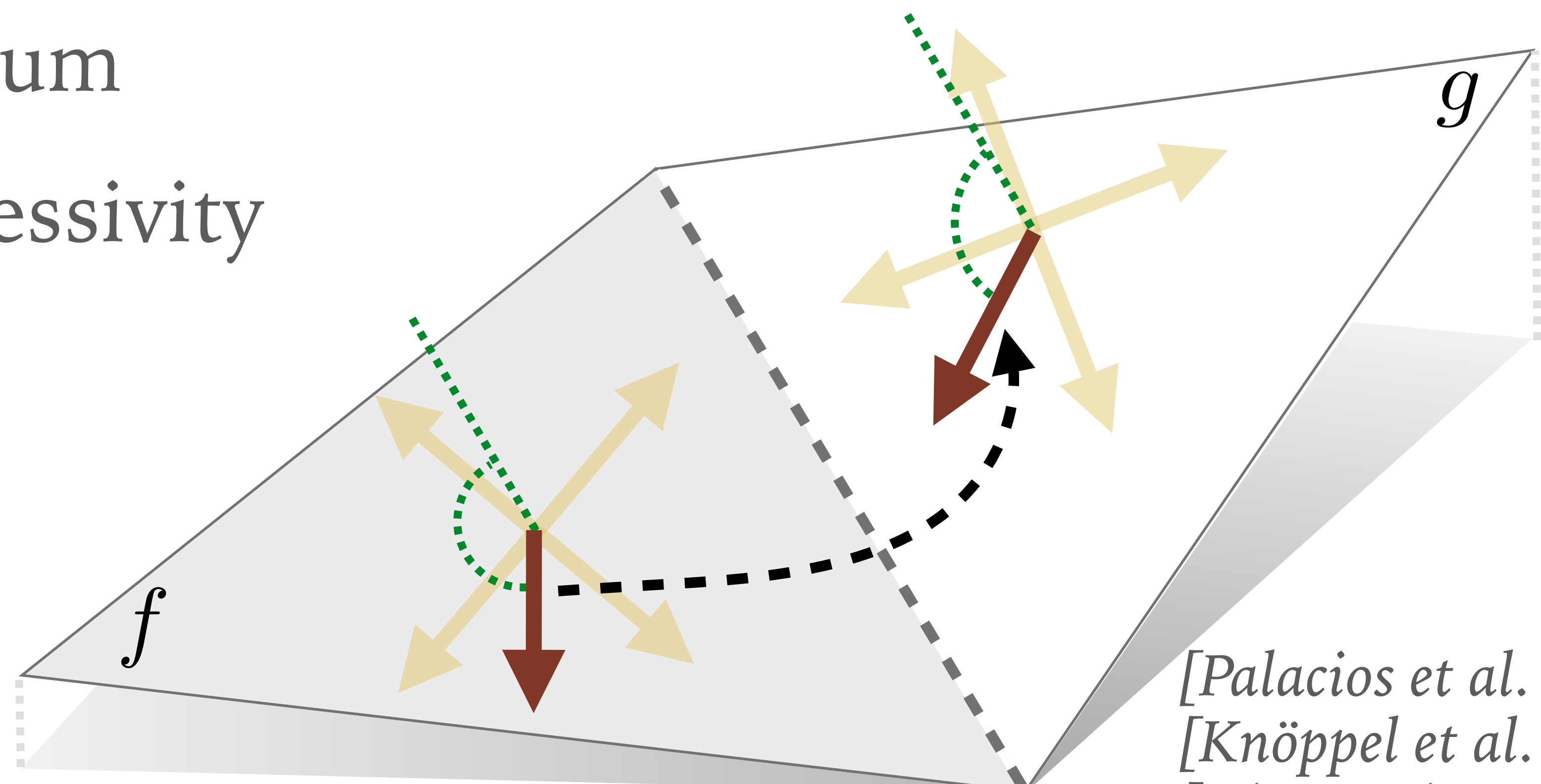


$$(\theta_f + \rho_{fg}\frac{\pi}{2} - \theta_g)^2 \quad \rho_{fg} \in \mathbb{I}$$

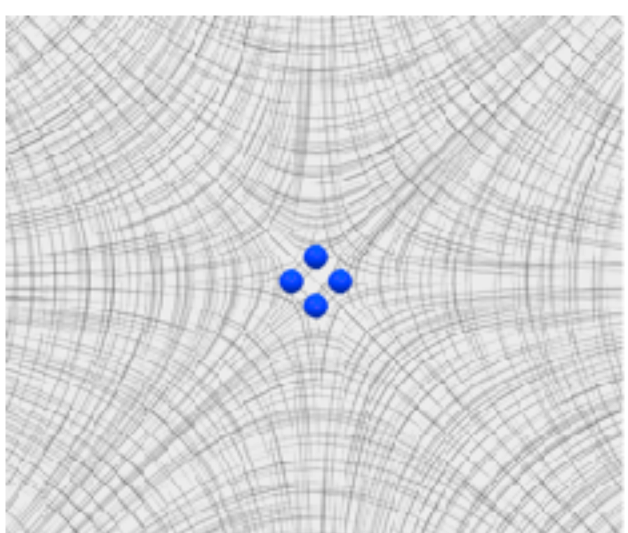
[Bommes et al. 2009]
[Panozzo et al. 2012]
[Jakob et al. 2015]

CASE STUDY: N-DIRECTIONAL FIELD, FREE TOPOLOGY

- Objective: “As-parallel-as-possible”
- Implicit topology with principal matchings (typically cartesian/complex)
    - Linear problem
    - Global optimum
    - Limited expressivity



[Palacios et al. 2007]
[Knöppel et al. 2013]
[Diamanti et al. 2014]



DEMO: GLOBALLY OPTIMAL

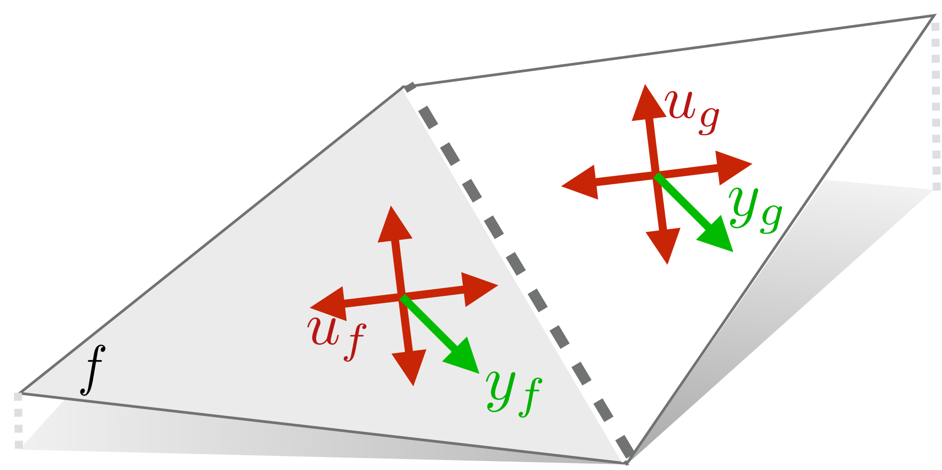
- Input: N-RoSy as single complex number per constrained face

$$y_f = (u_f)^N$$

- Output: interpolation to all free faces $y_{f_{known}} = (u_{f_{known}})^N$
- Minimizing energy: $\min_{\mathbf{y}} E_{smooth}(\mathbf{y})$

$$E_{smooth}(\mathbf{y}) = \sum_{(f,g)} \left|(\bar{e}_f)^N y_f - (\bar{e}_g)^N y_g\right|^2 = \mathbf{y}^T Q \mathbf{y}$$

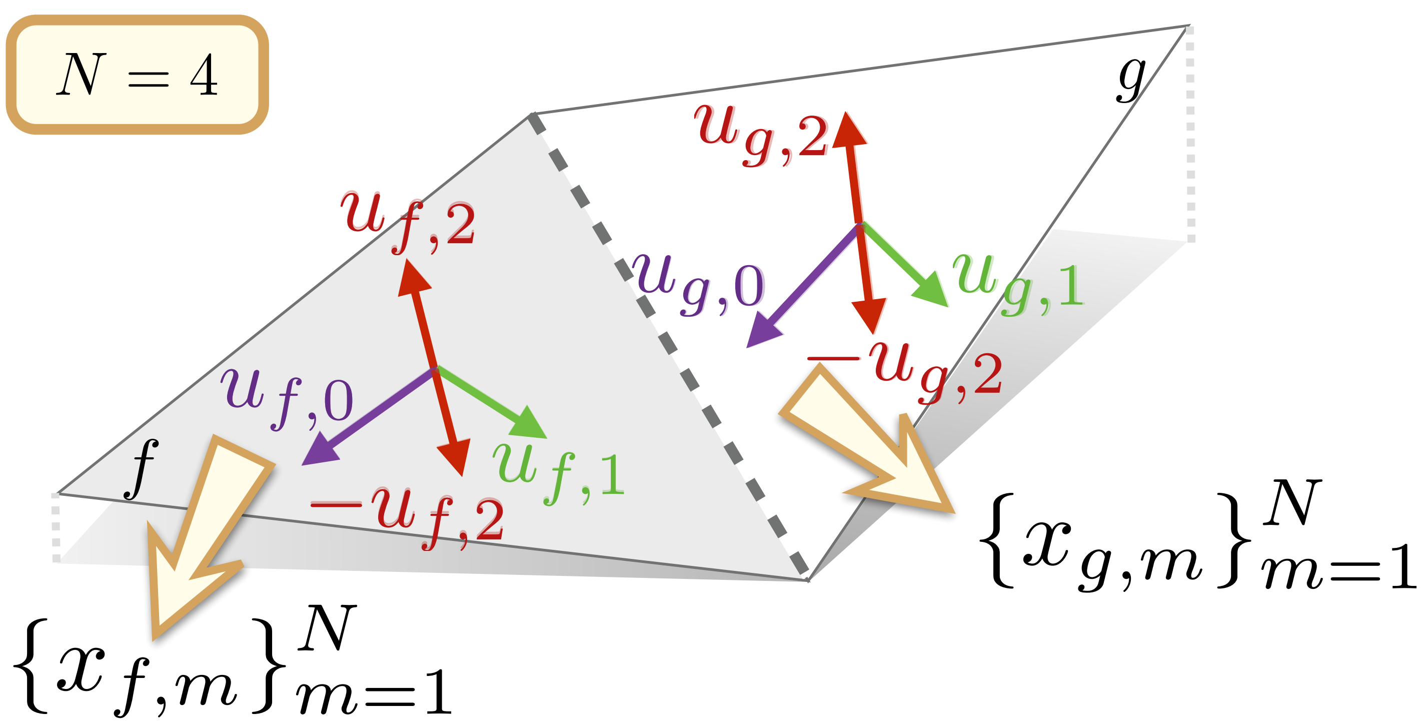
- Take roots of all y_f



[Knöppel 2013]

DEMO: POLYVECTORS

- Representing set of polynomial coefficients instead of single N-RoSy.
- Roots of polynomial = directional.
- Interpolating each like in **globally optimal.**



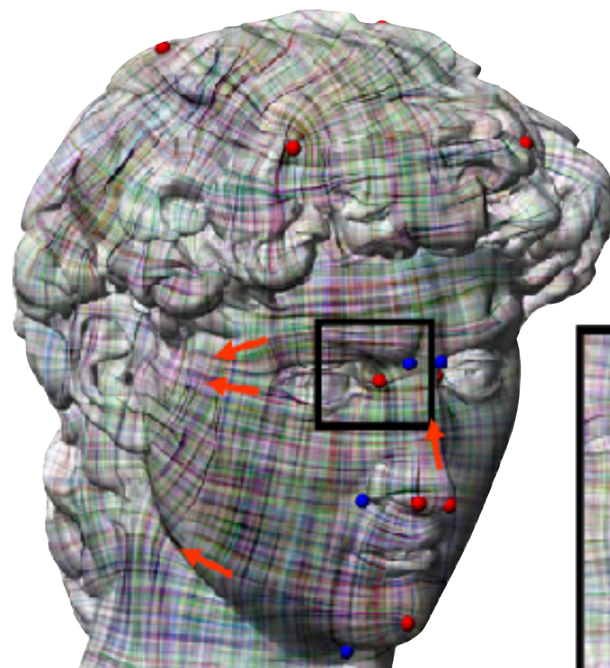
Ideally smooth field
of m-th coefficient

$$\boxed{x_{f,m}(\bar{e}_f)^m = x_{g,m}(\bar{e}_g)^m}$$

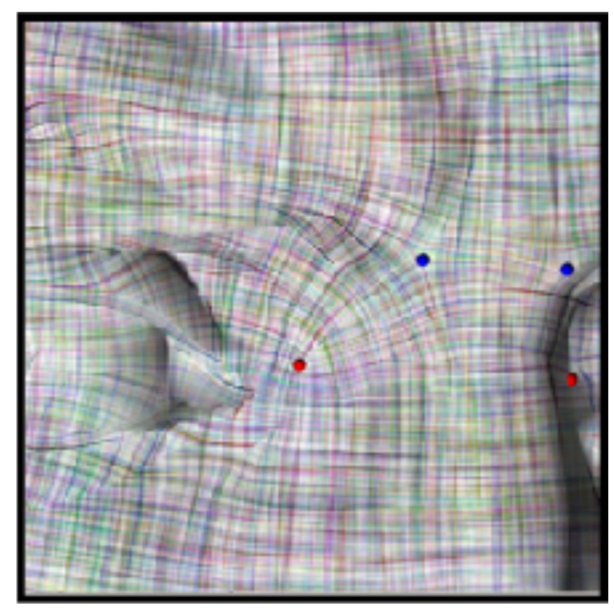
$$\forall m \in \{0, \ldots, N-1\}$$

HOW ABOUT CONSTRAINTS?

- Sparse "Hard"



[Palacios et al. 2007]
[Ray et al. 2008]
[Jakob et al. 2015]

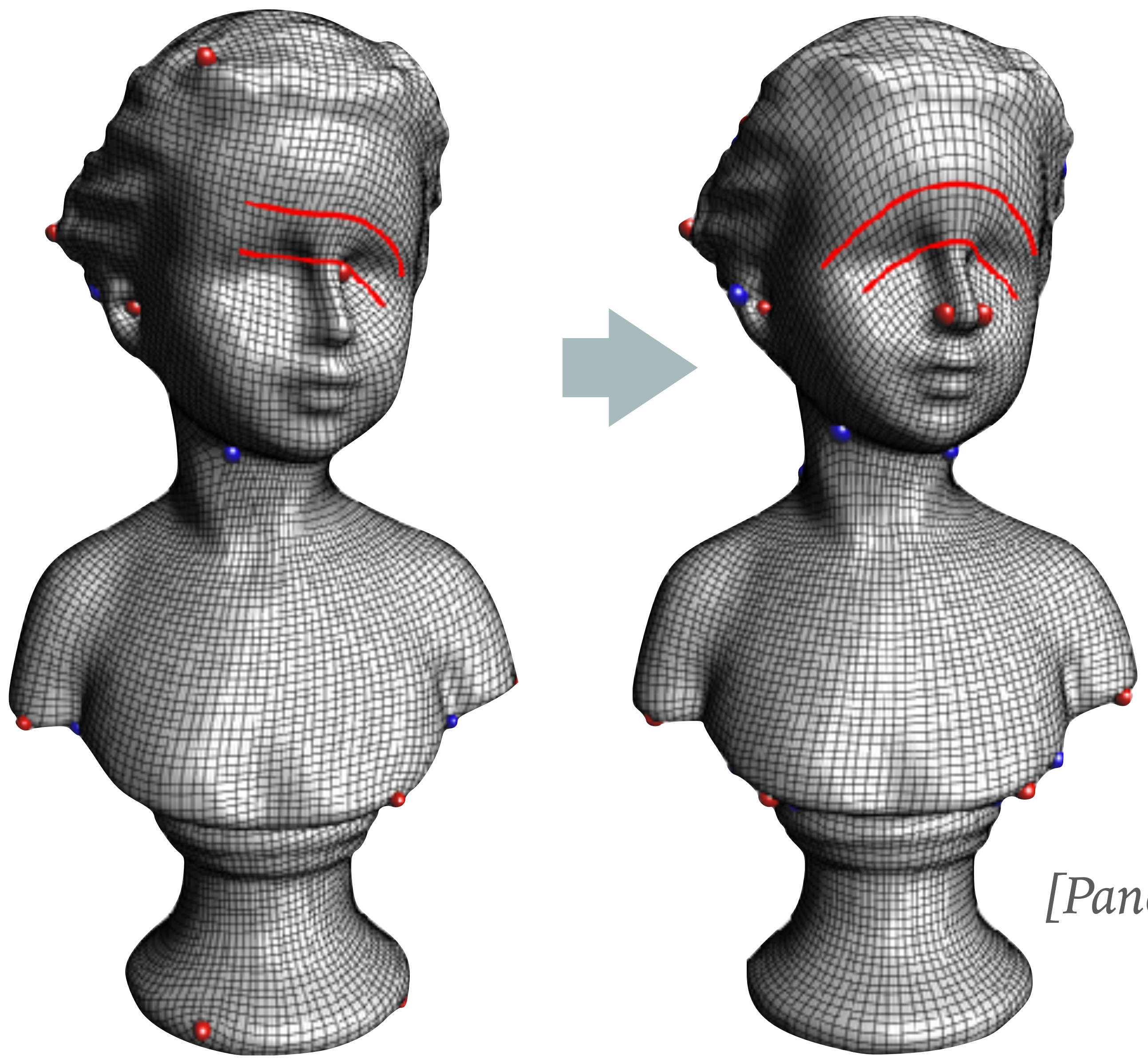


[Crane et al. 2010]
[Bommes et al. 2009]
[Hertzmann et al. 2000]
[Diamanti et al. 2014]

- Also partial constraints! [Iarussi et al. 2015] [Diamanti et al. 2015]

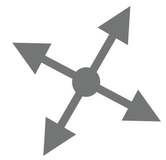
HOW ABOUT CONSTRAINTS?

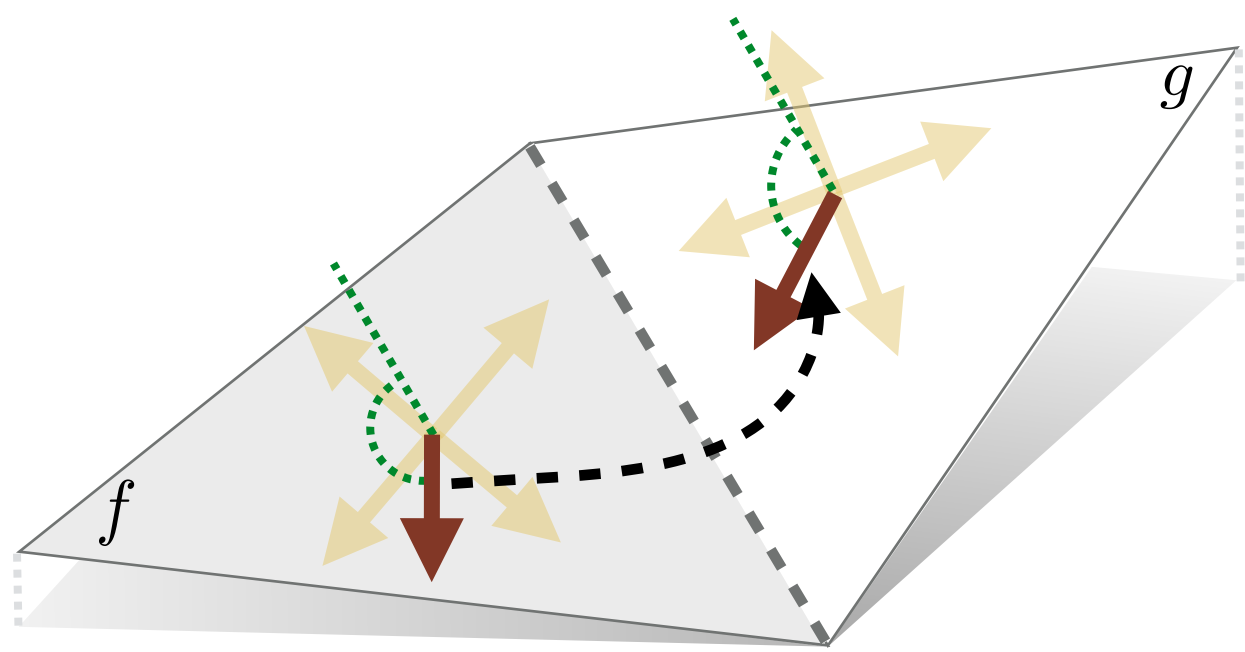
- Soft constraints

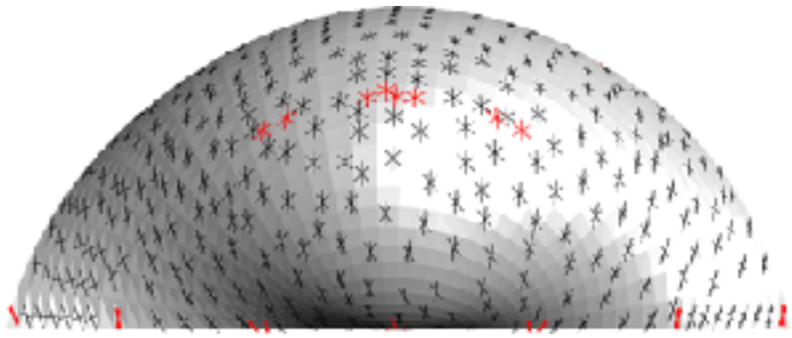


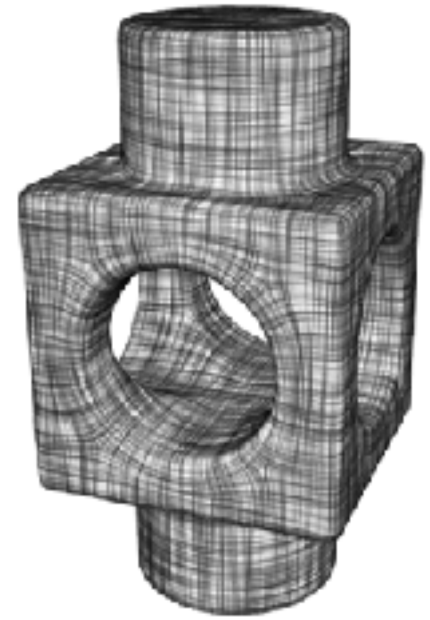
[Panozzo et al. 2012]

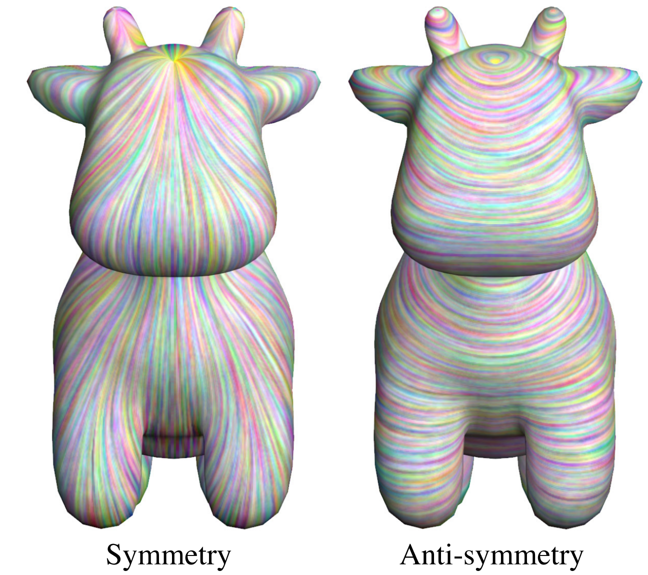
AVOIDING THE TRIVIAL SOLUTION

- Eg. N-Vector Fields  with a cartesian/complex representation
- Zero Field is perfectly smooth!
- Constraints are necessary
  - Per-vector unit-norm constraint [Palacios et al. 2007]
  - Integrated norm constraint [Knöppel et al. 2013]
  - Enough hard constraints [Diamanti et al. 2014]

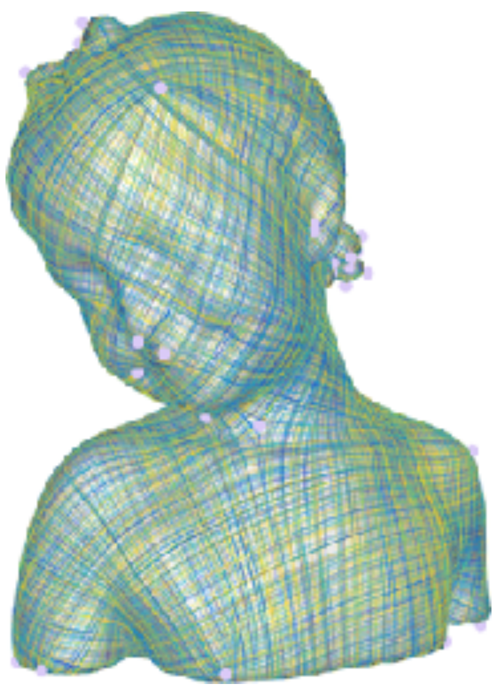


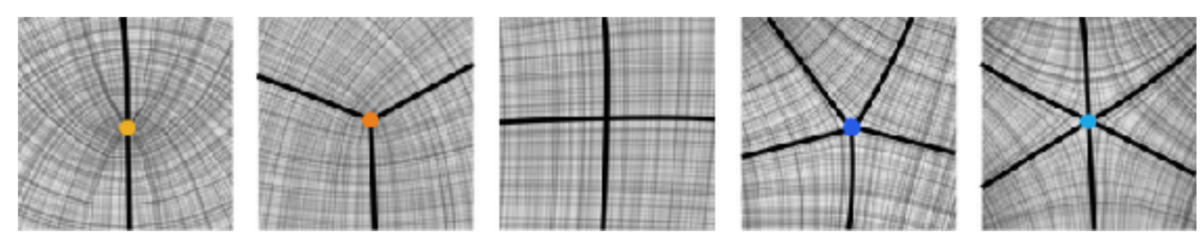






THANK YOU!





**Course repository: https://github.com/avaxman/DirectionalFieldSynthesis*