

# OBJECTIVES

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*Olga Diamanti*

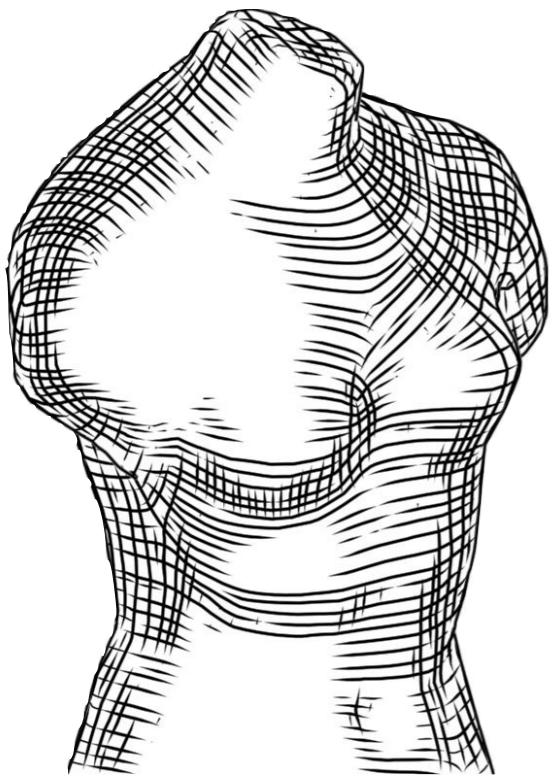
*Geometric Computation Group*

*Stanford University*

# OBJECTIVES - FIELD FAIRNESS

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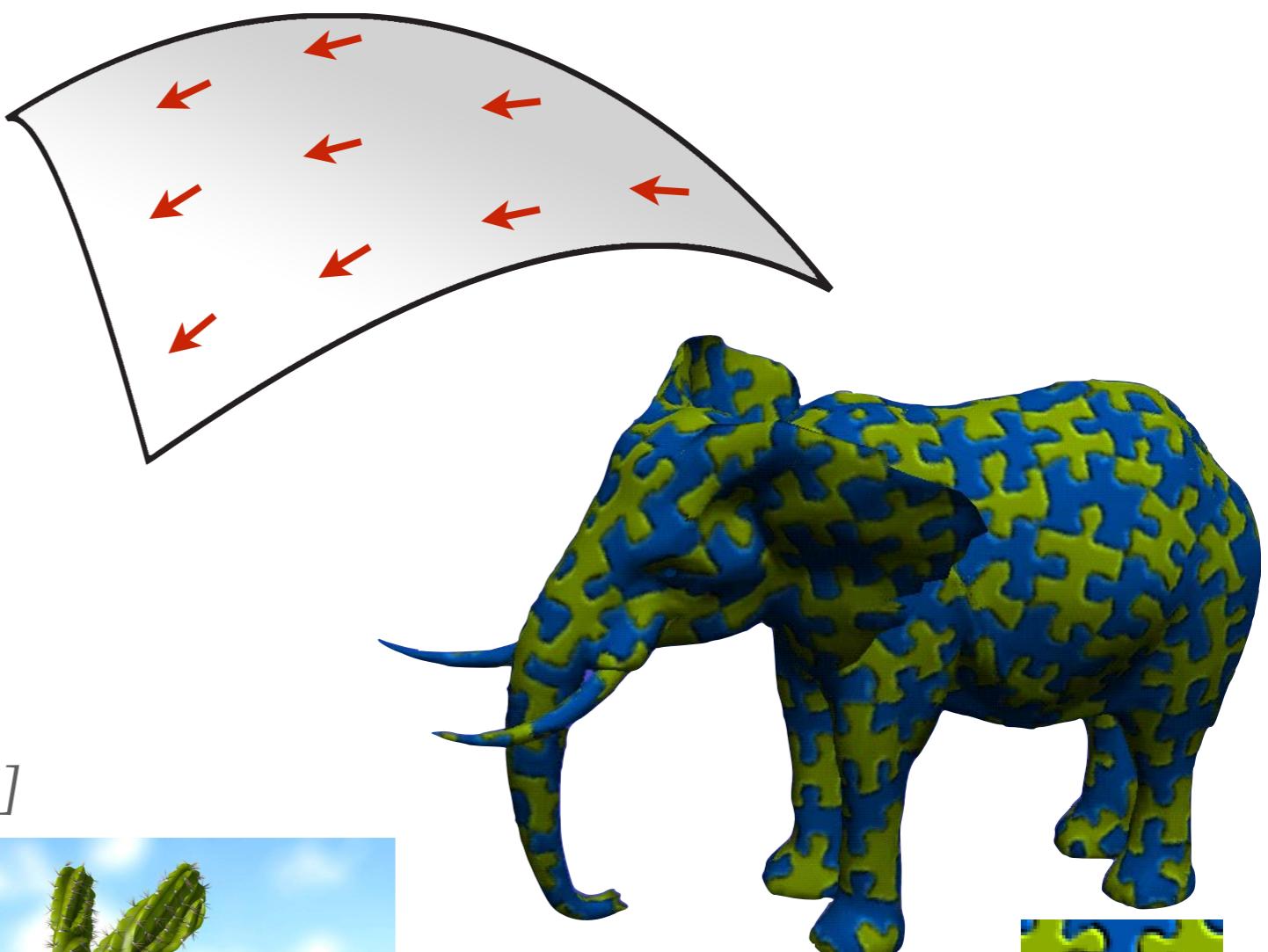
- “As-parallel-as-possible”



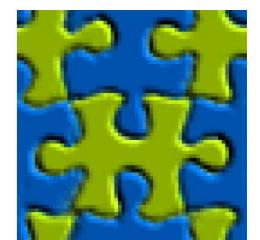
[Palacios et al. 2007]



[Ray et al. 2006]  
[Knöppel et al. 2015]

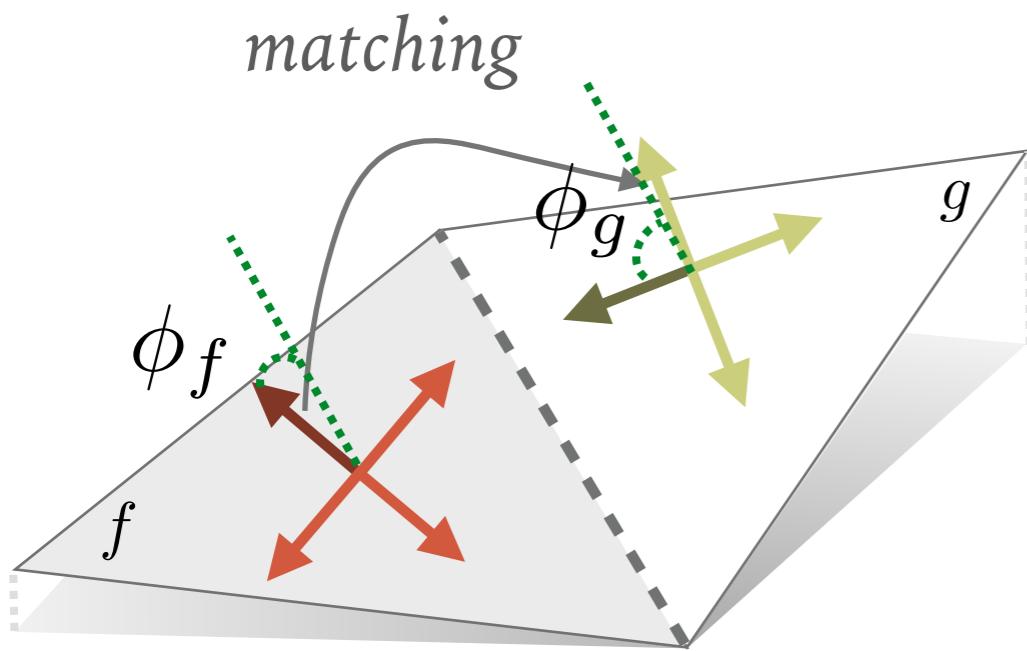


[Turk et al. 2001]



# OBJECTIVES - FIELD FAIRNESS

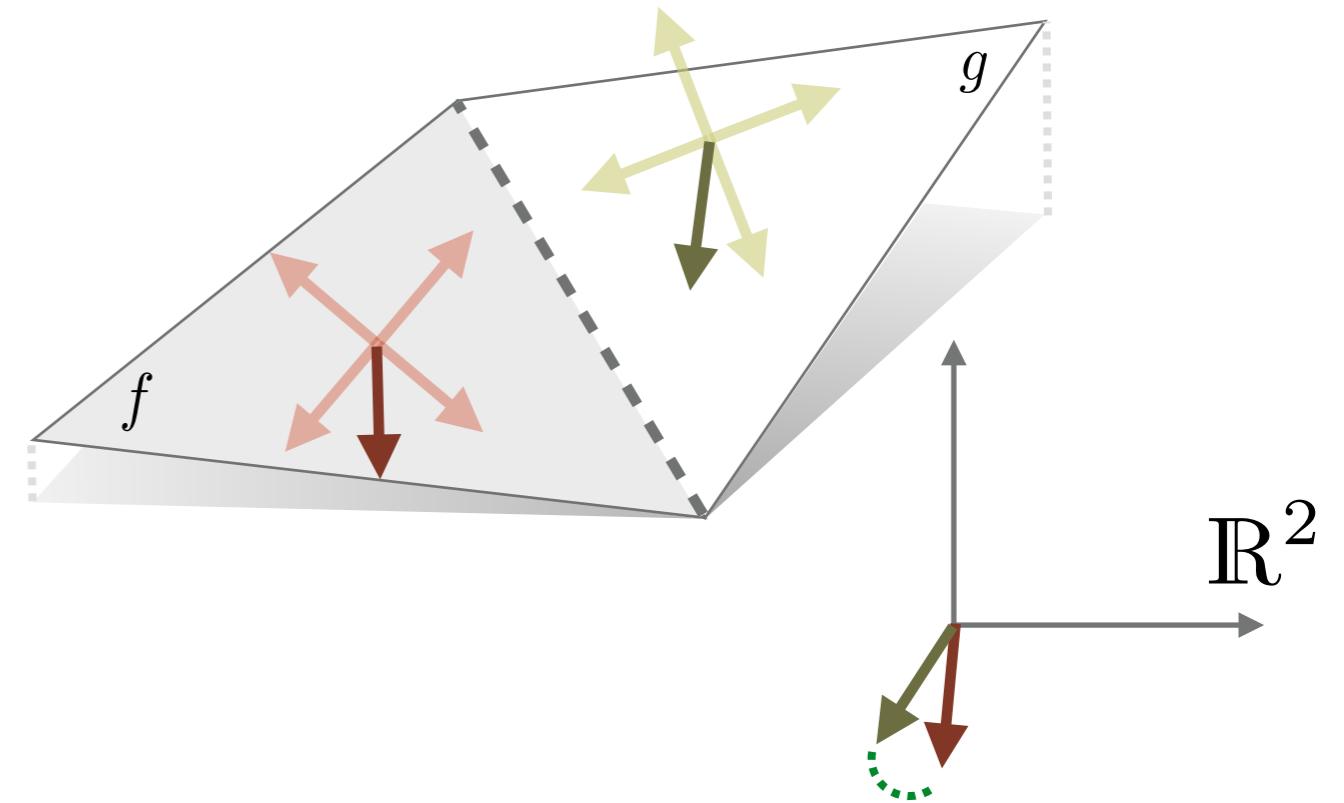
- “As-parallel-as-possible”



angle-based approaches

compare matching angles

[Hertzmann et al. 2000] [Crane et al. 2010]  
[Ray et al. 2008] [Ray et al. 2009]  
[Bommes et al. 2009] [Jakob et al. 2015]



cartesian/complex approaches

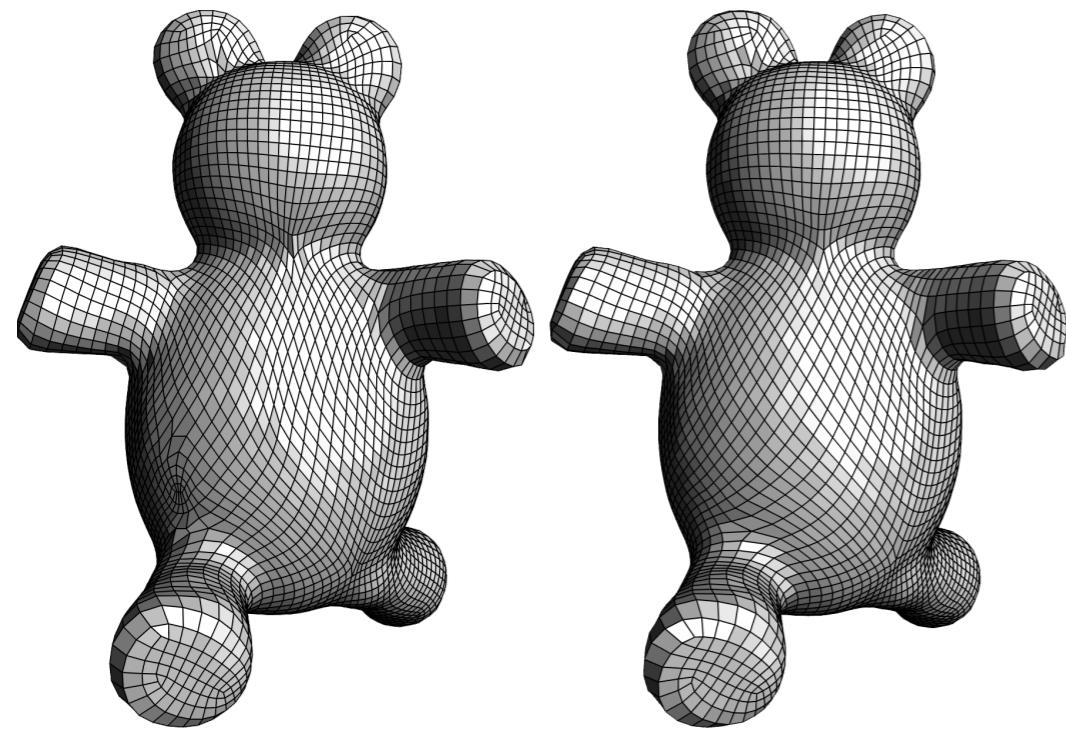
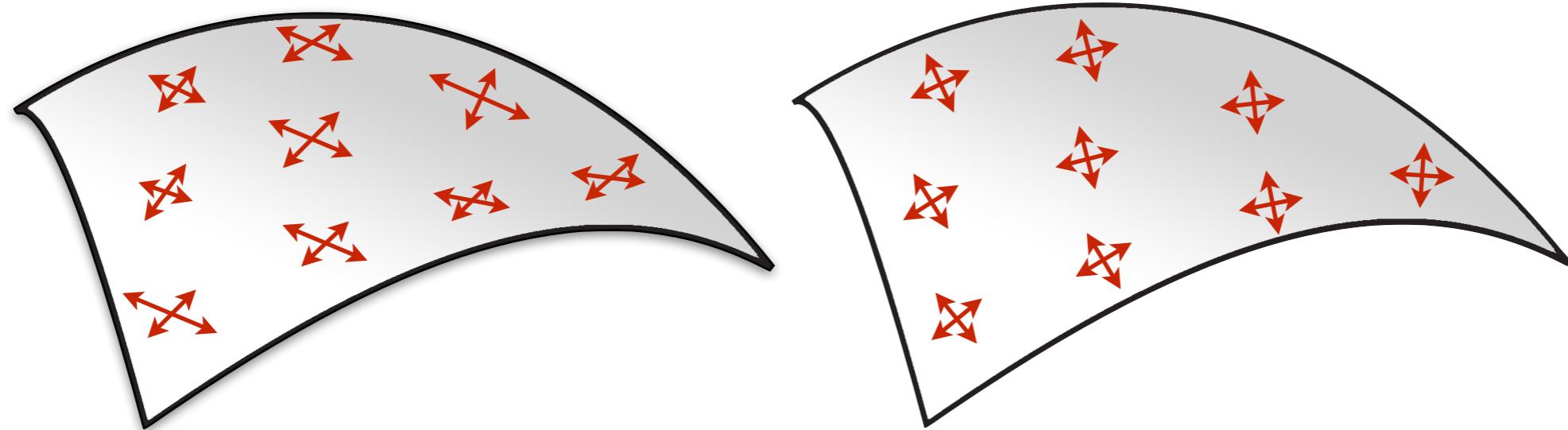
compare representative vectors

[Ray et al. 2006] [Knöppel et al. 2013]  
[Palacios et al. 2007] [Diamanti et al. 2014]

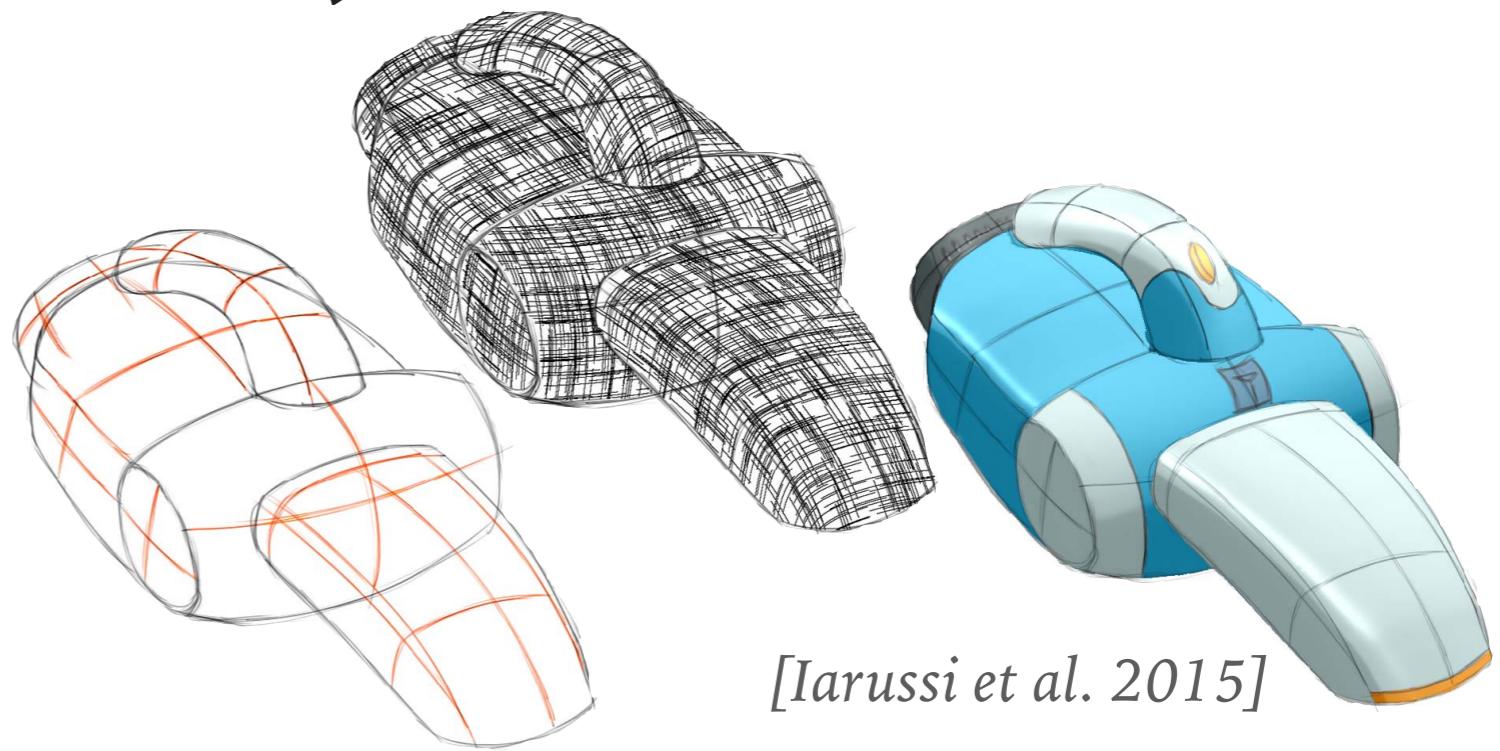
# OBJECTIVES - FIELD FAIRNESS

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- Orthogonality



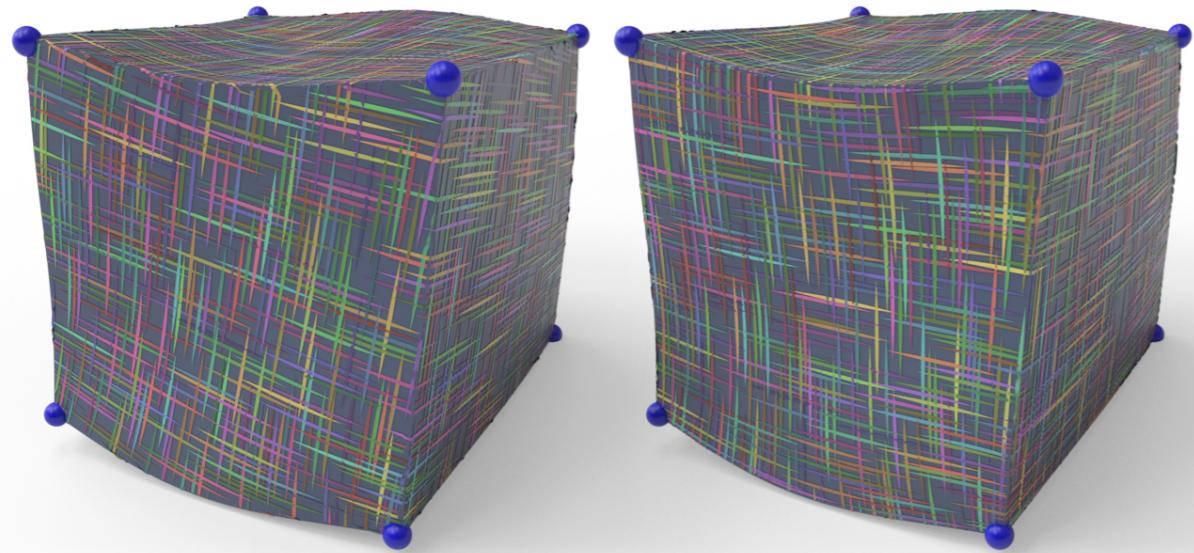
[Diamanti et al. 2014]



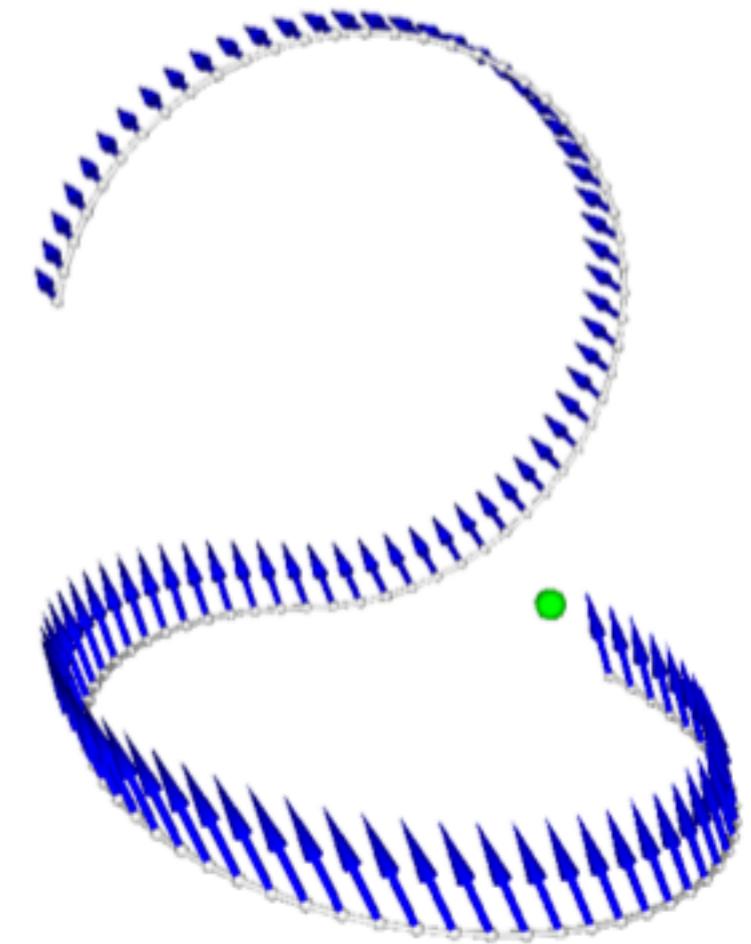
[Iarussi et al. 2015]

# OBJECTIVES – ALIGNMENT TO FEATURES

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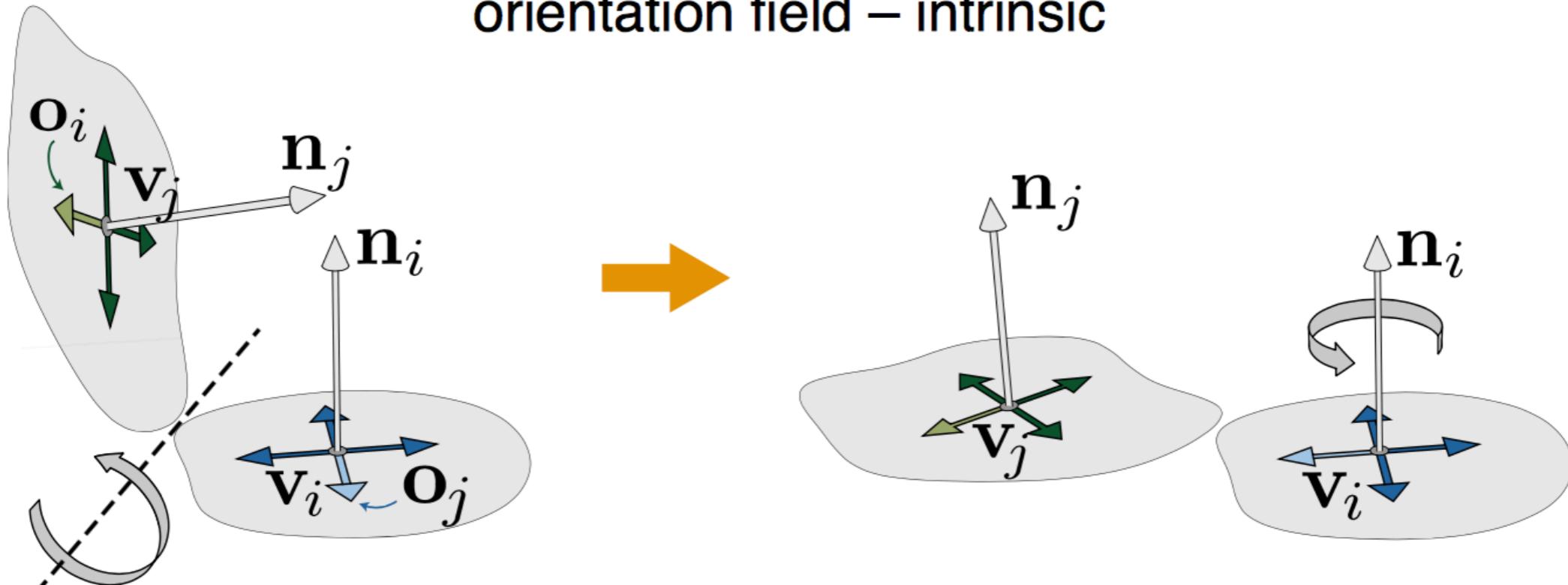
[*Jakob et al. 2015*]



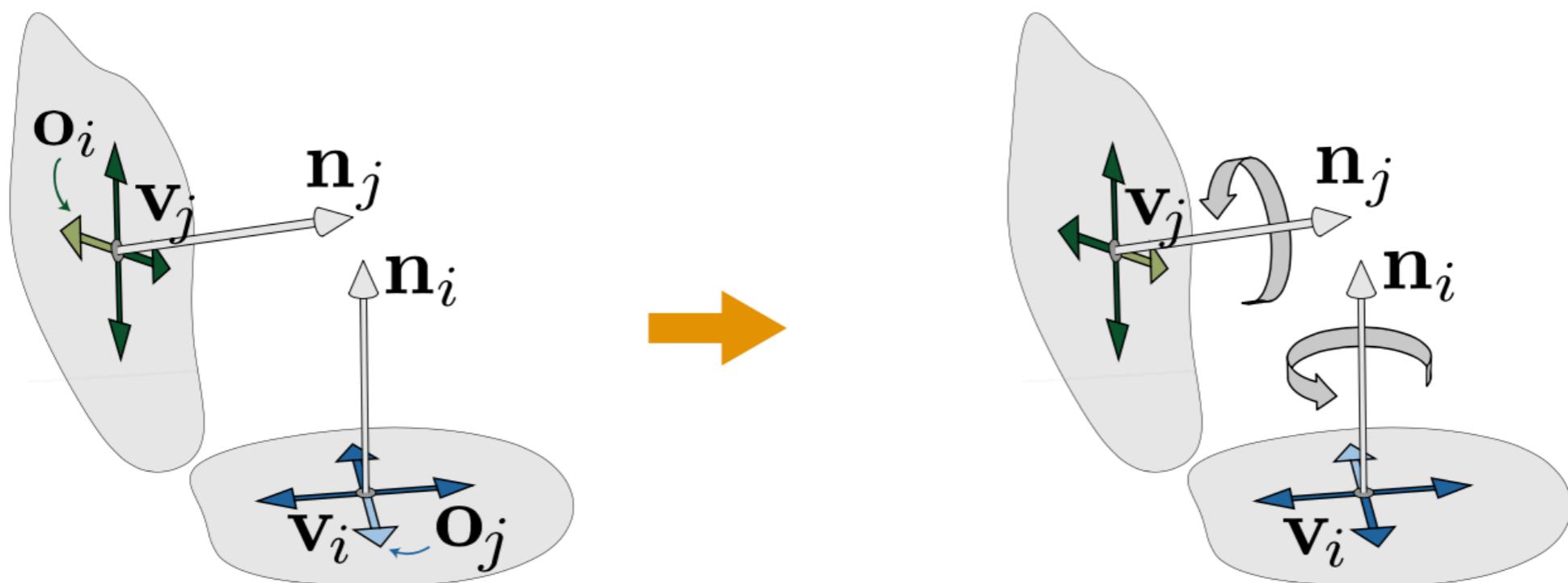
[*Huang et al. 2016*]

# OBJECTIVES – ALIGNMENT TO FEATURES

orientation field – intrinsic

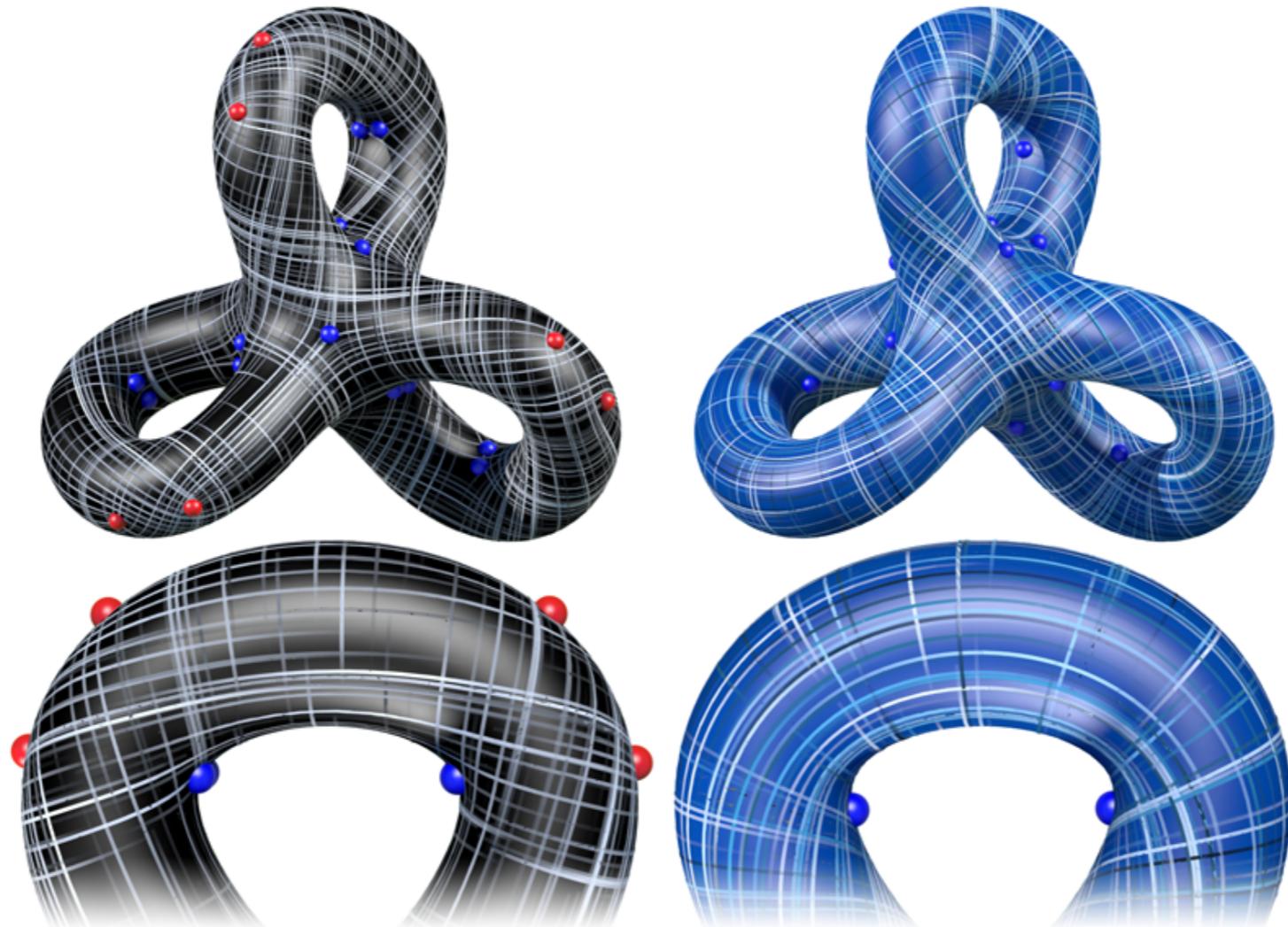


orientation field – extrinsic



# OBJECTIVES – SINGULARITY CONTROL

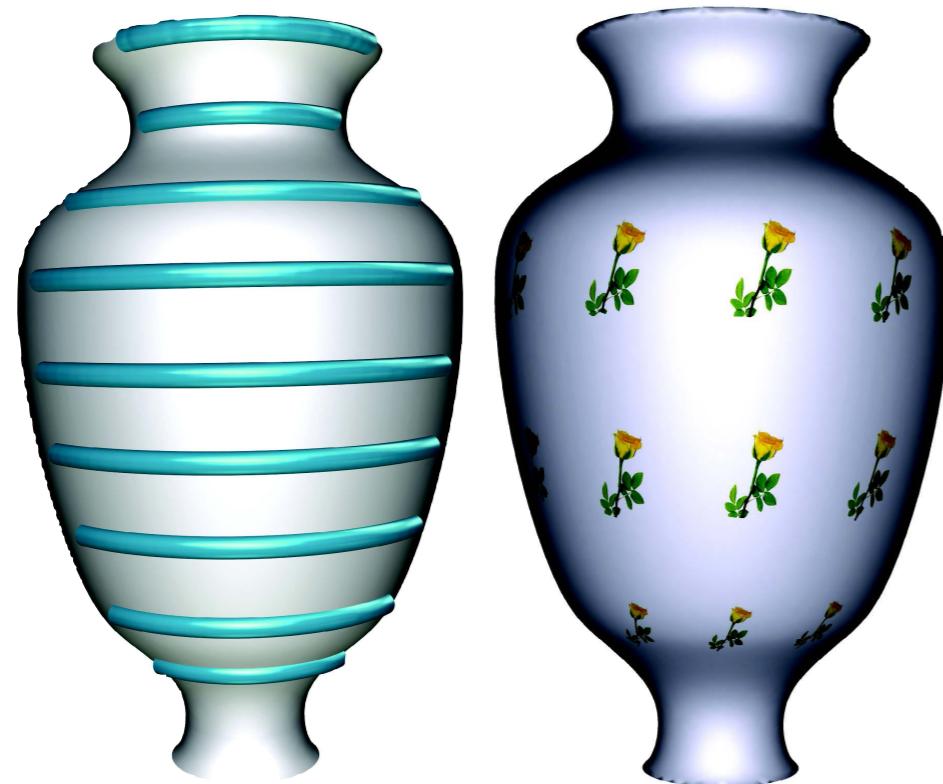
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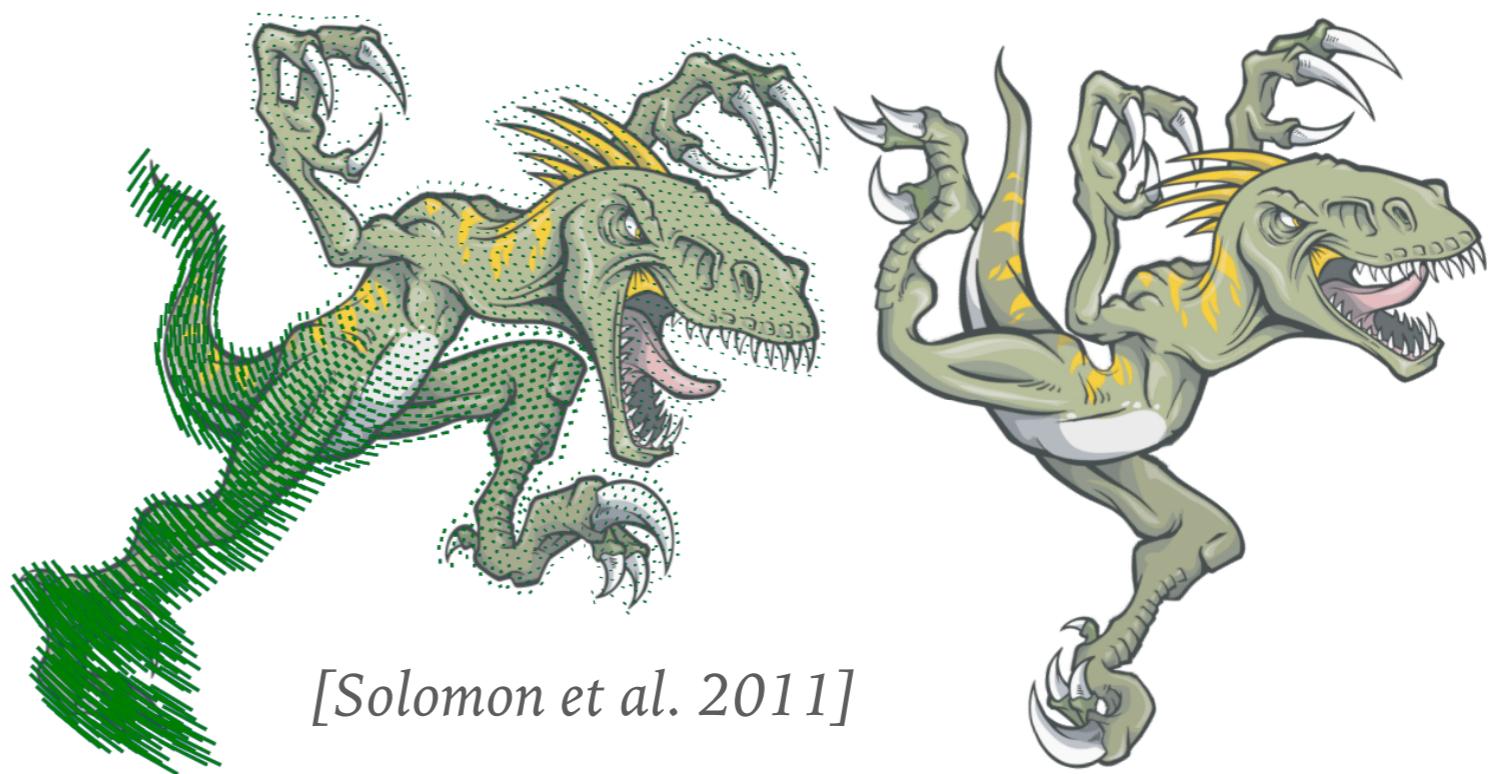
[Knöppel et al. 2013]

# OBJECTIVES – ISOMETRY INDUCING

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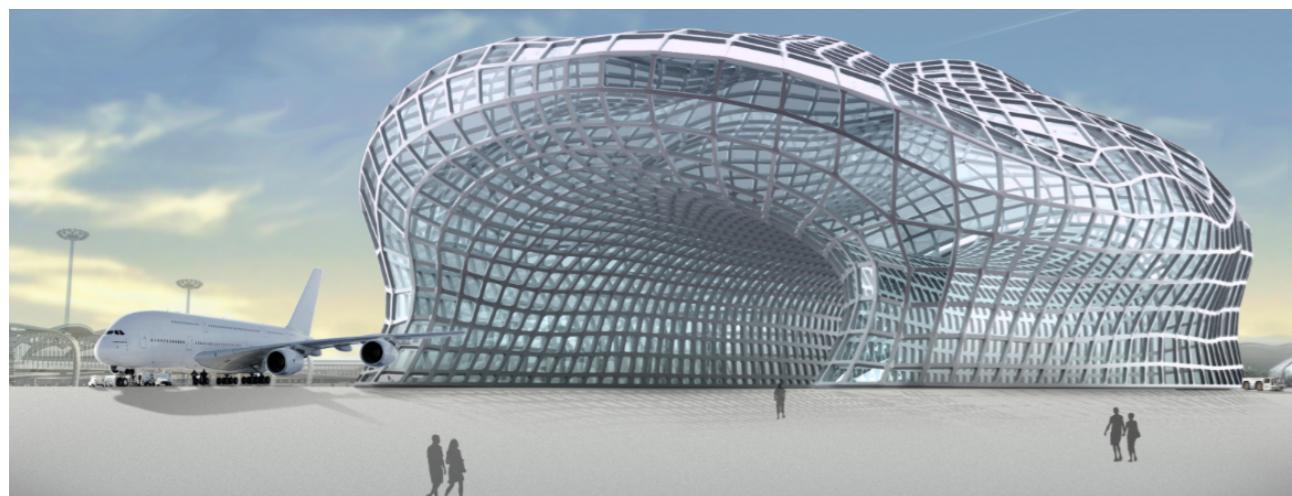
[ Ben-Chen et al. 2010]



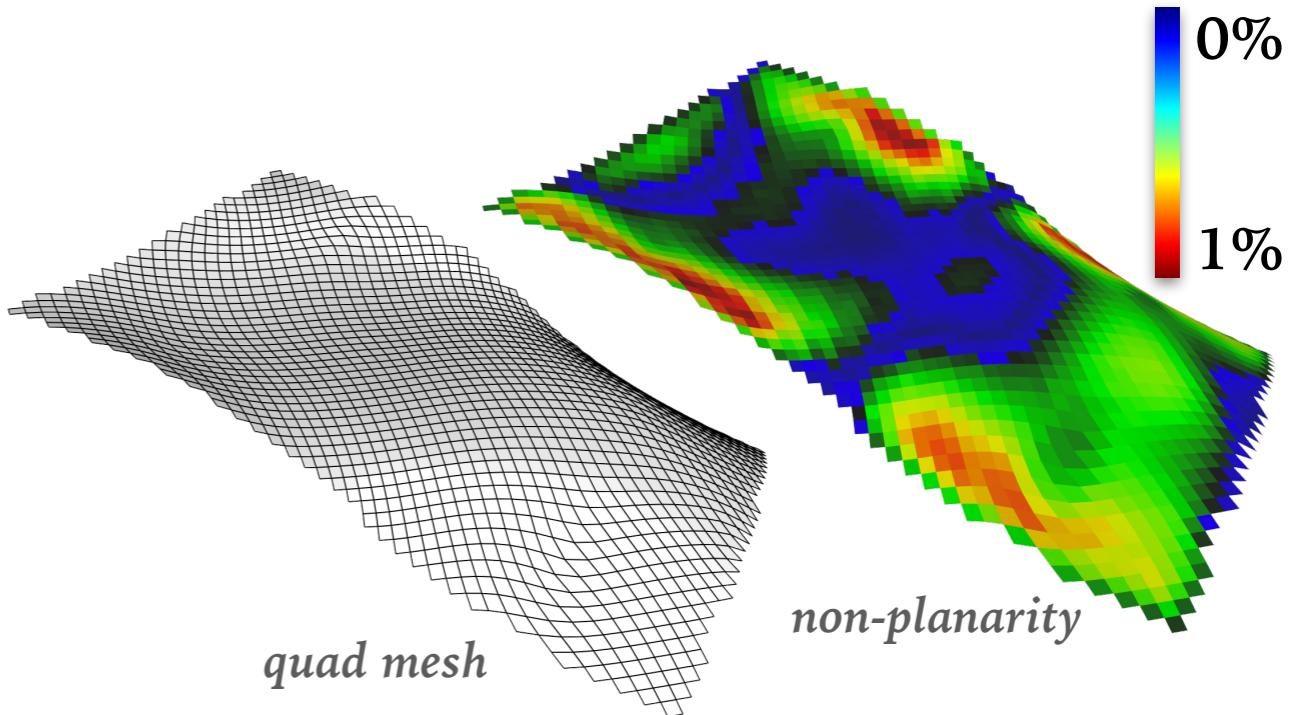
[Solomon et al. 2011]

# OBJECTIVES - CONJUGACY (PLANARITY)

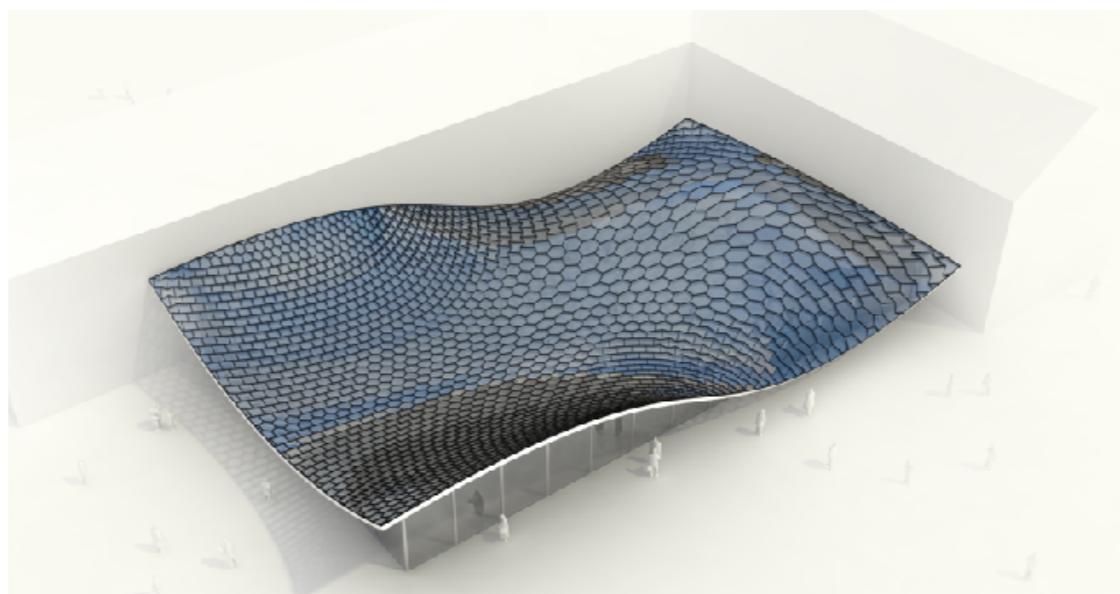
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[Liu et al. 2011]



[Diamanti et al. 2014]



[Vaxman et al. 2015]

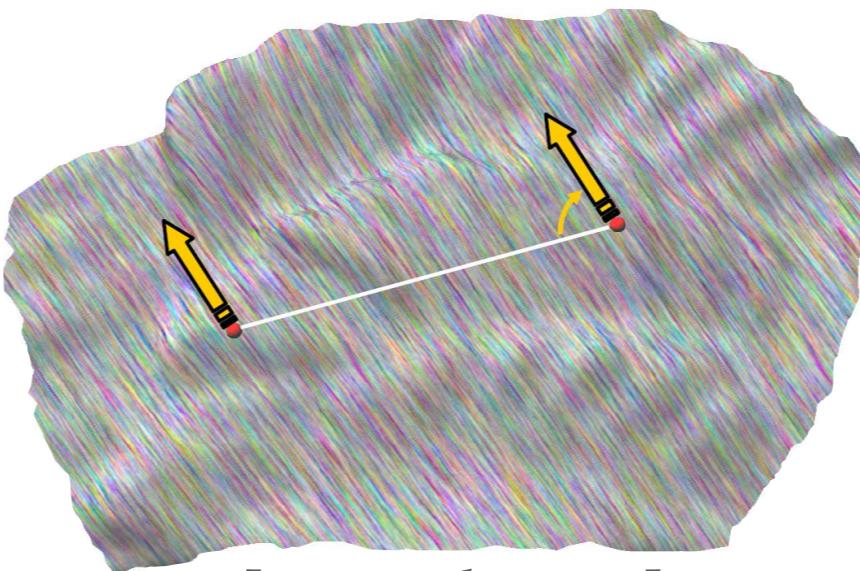
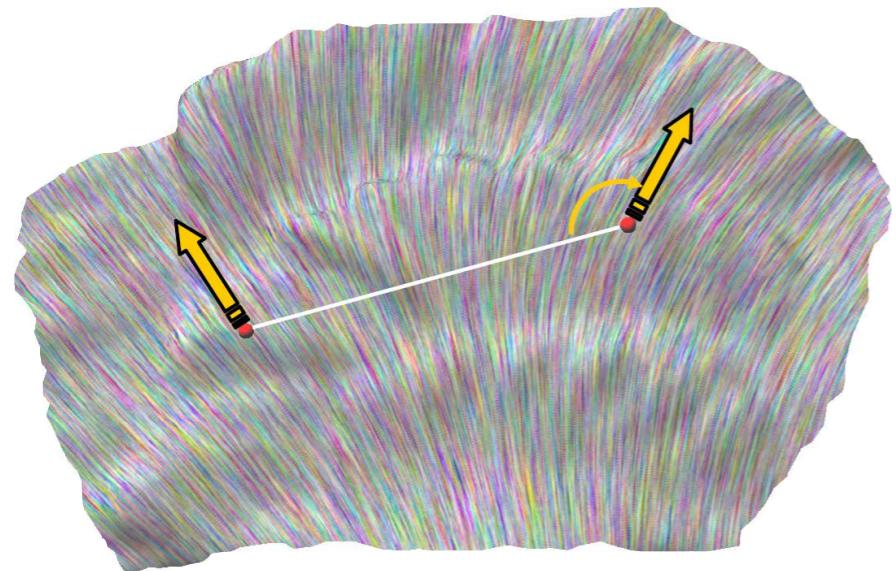
# CONSTRAINTS

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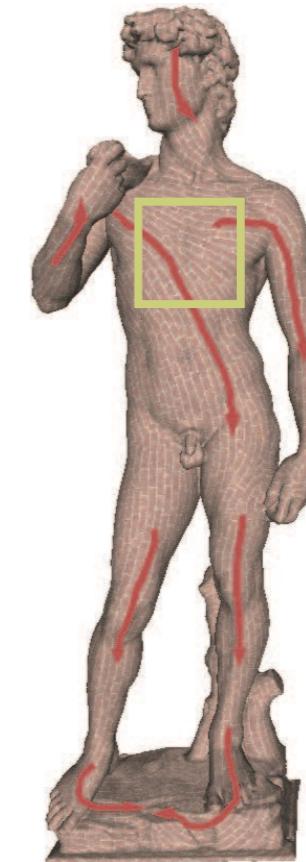
*Olga Diamanti*

*Geometric Computation Group  
Stanford University*

# CONSTRAINTS - ALIGNMENT



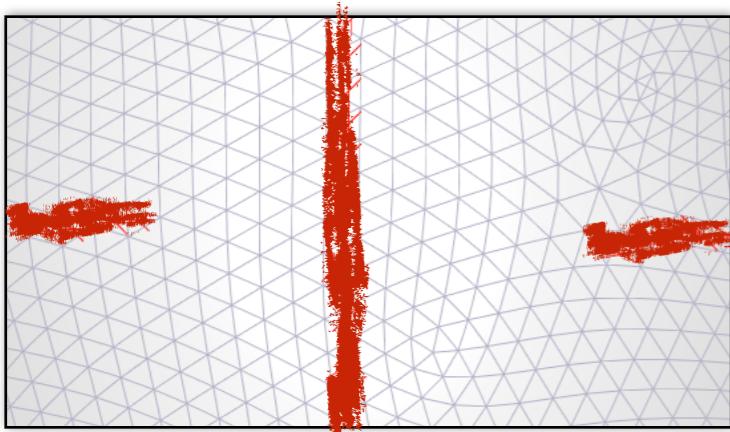
[Ray et al. 2008]



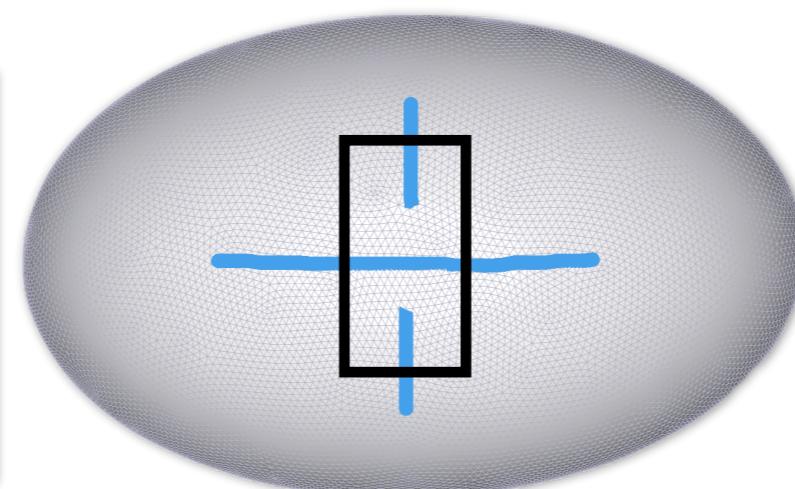
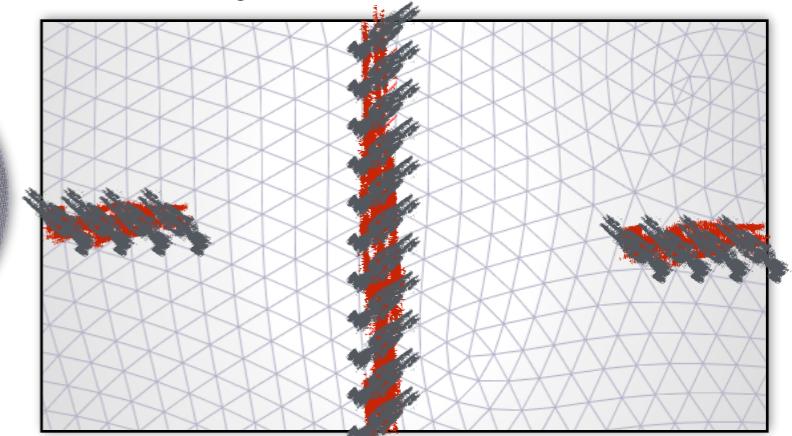
[Fisher et al. 2007]

- Complete or Partial

*partial constraints*

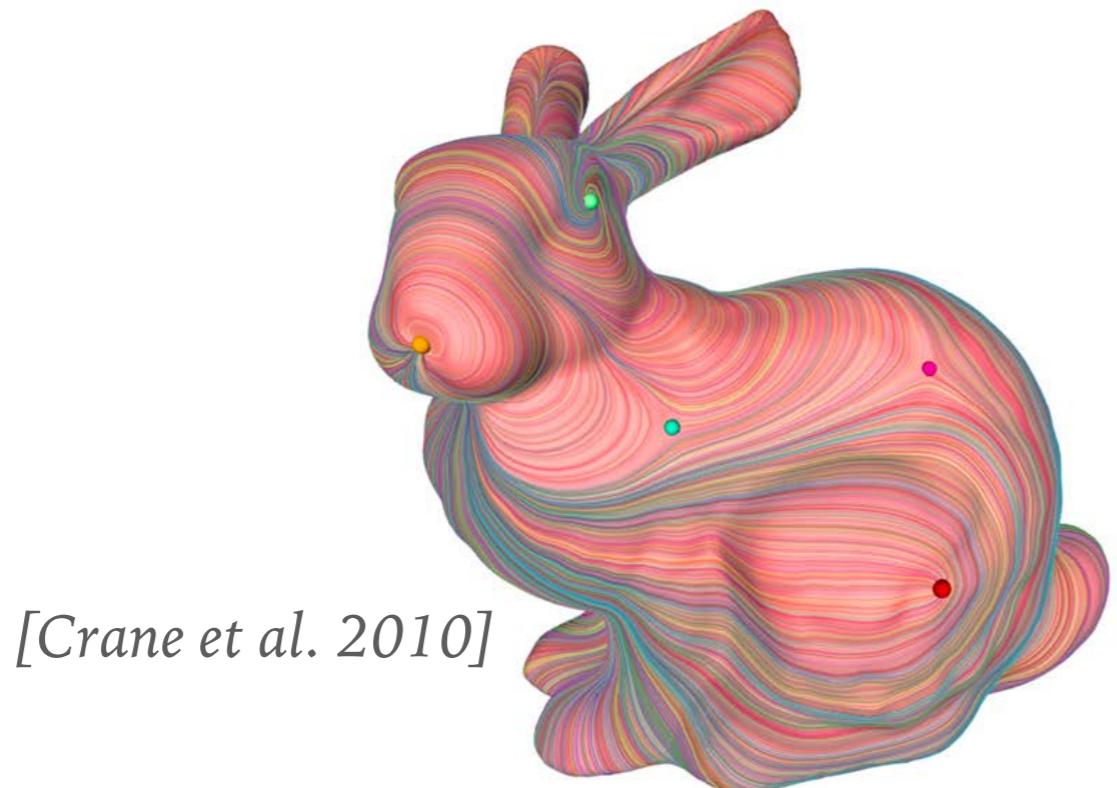


*full constraints*



# CONSTRAINTS - TOPOLOGY

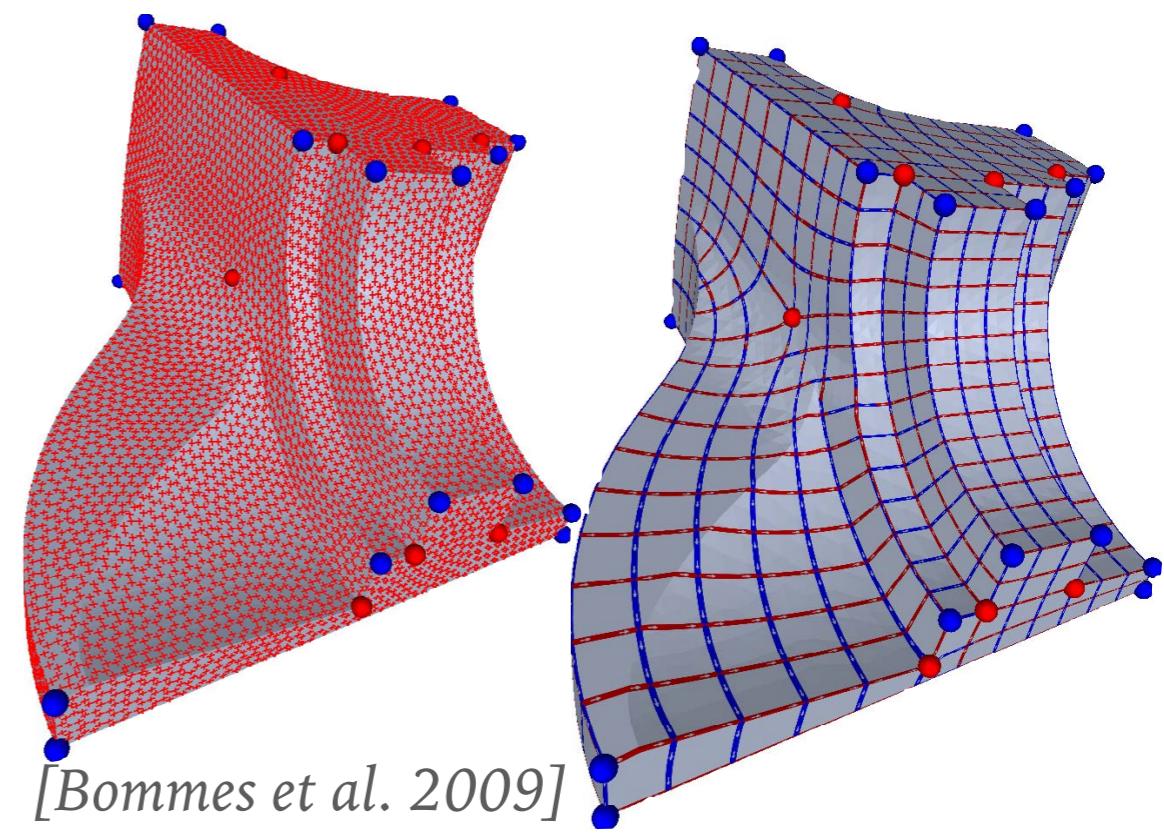
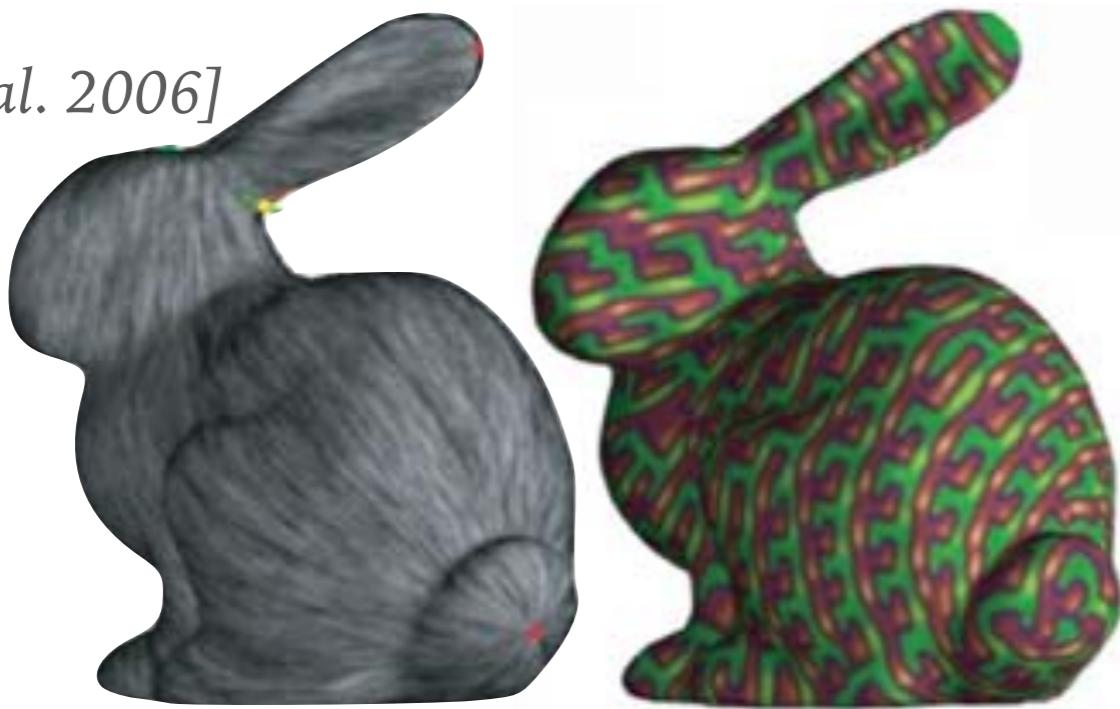
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[Crane et al. 2010]



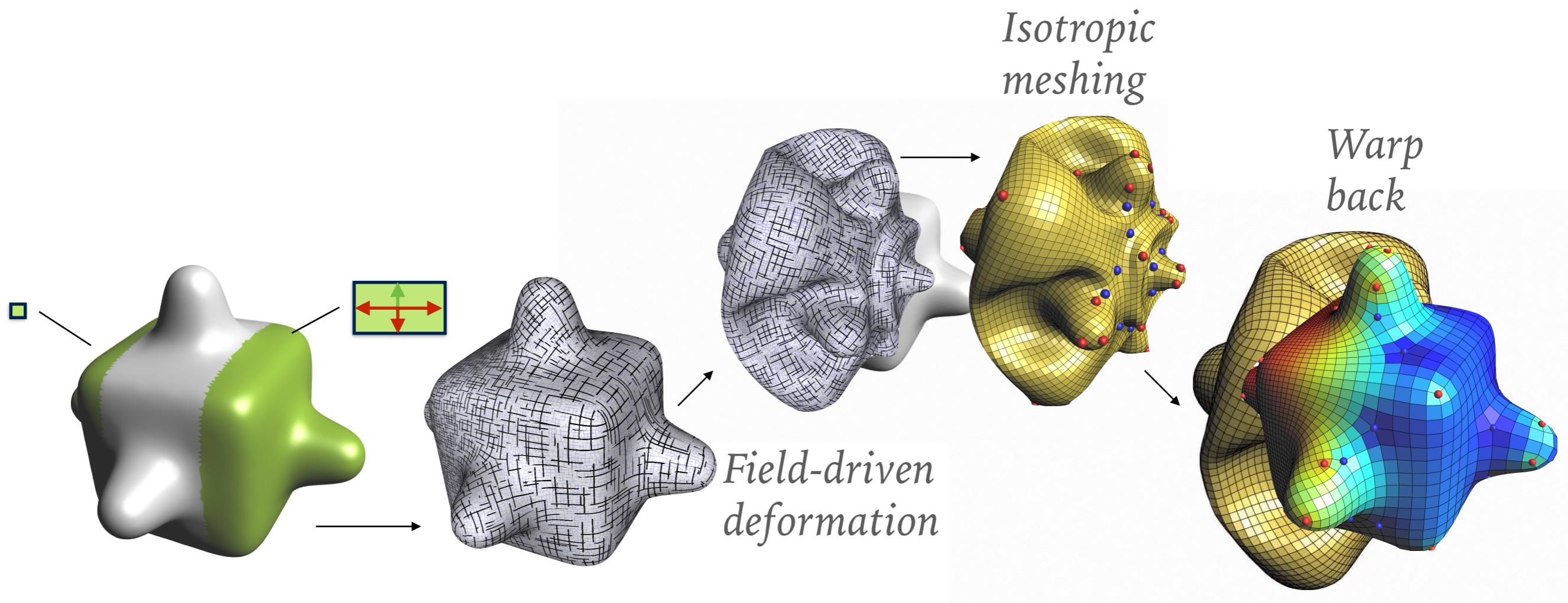
[Zhang et al. 2006]



[Bommes et al. 2009]

# CONSTRAINTS - SCALE

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*Frame field interpolation*

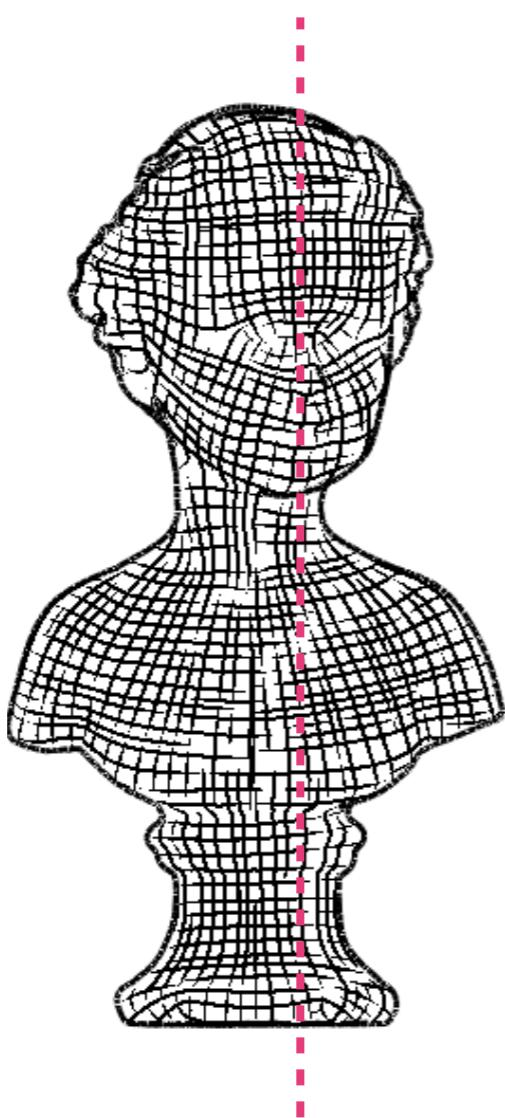
*Isotropic  
meshing*

*Warp  
back*

[Panizzo et al. 2014]

# CONSTRAINTS - SYMMETRY

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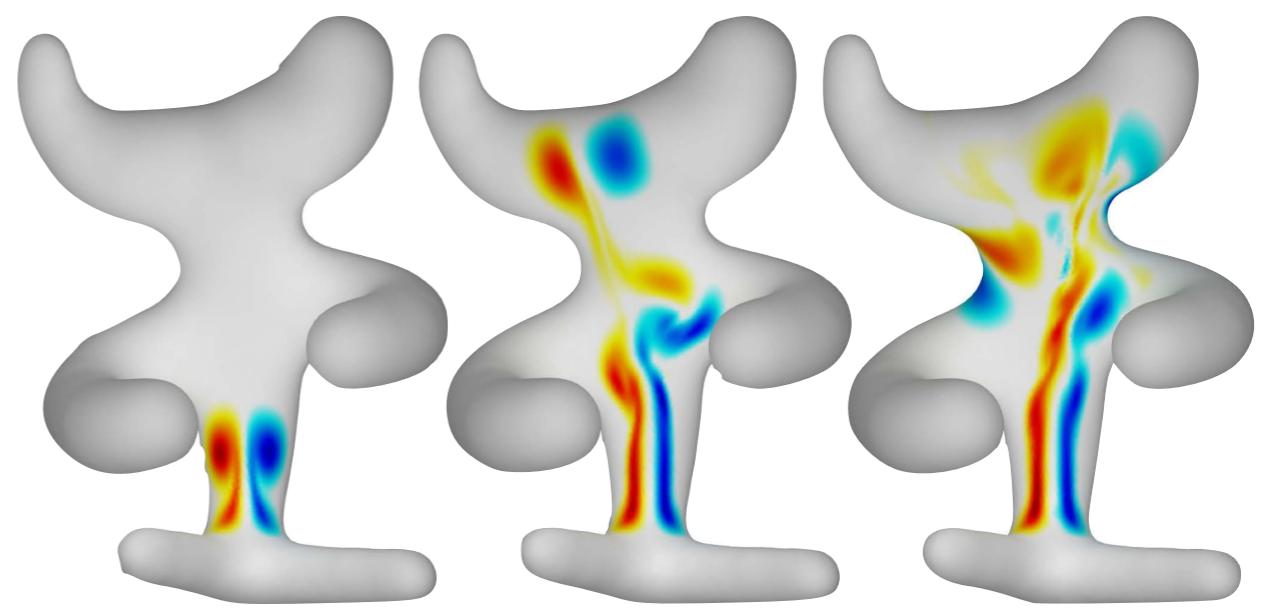
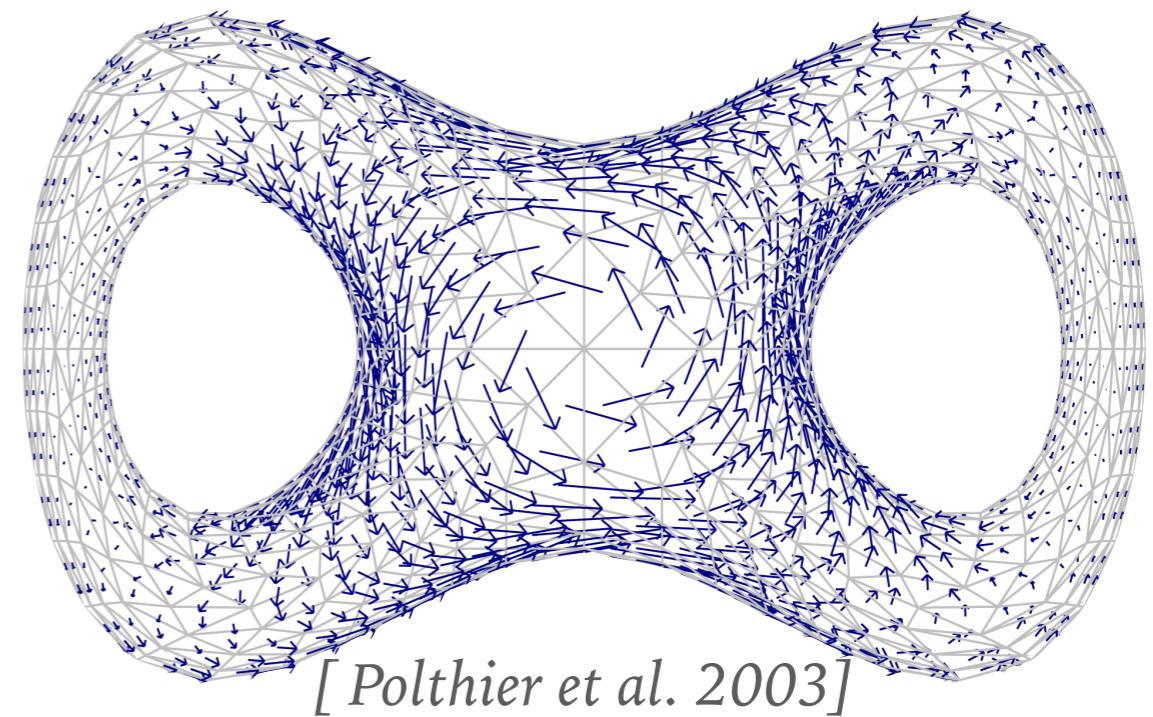
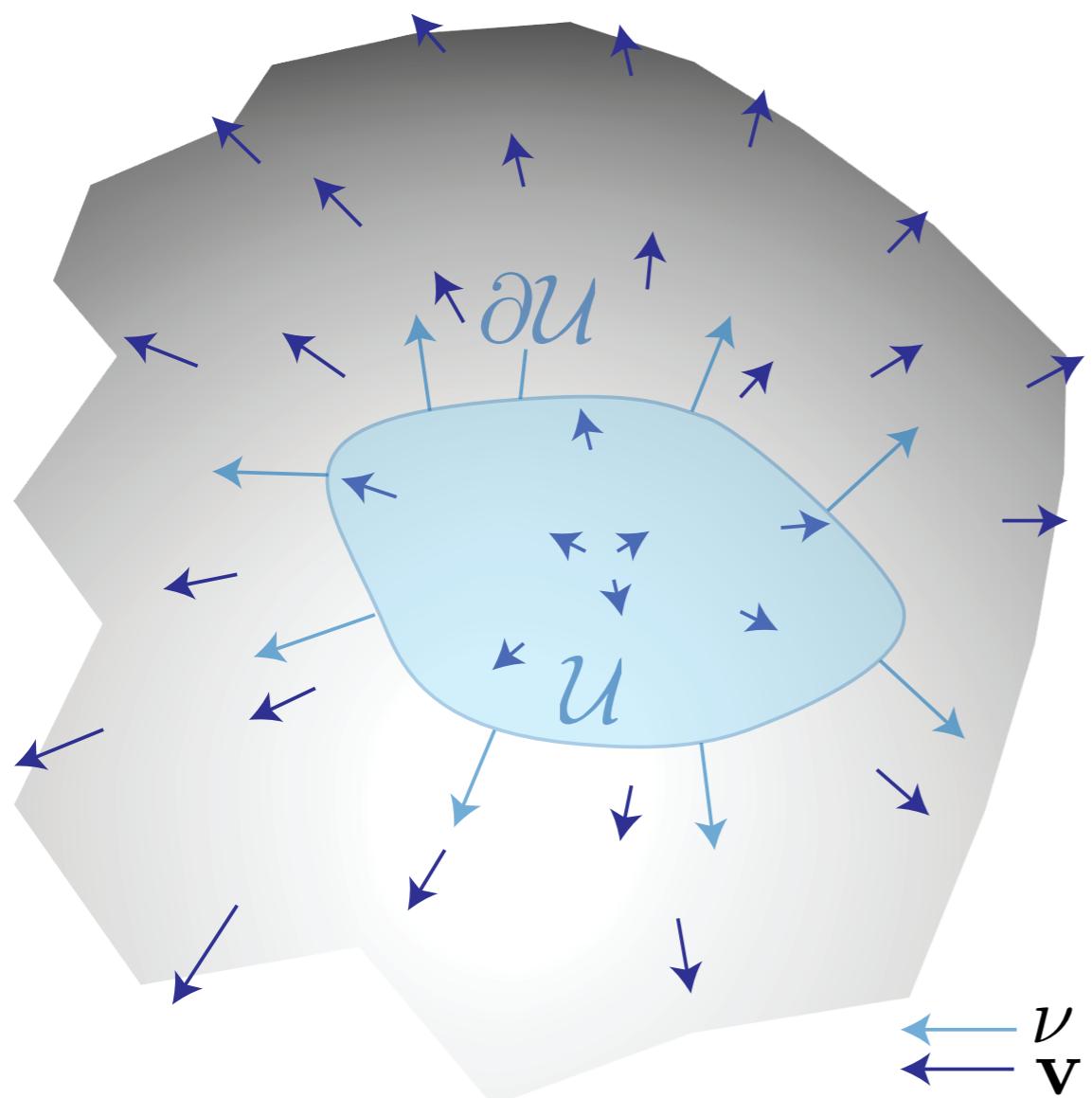
[ Panozzo et al. 2012]



[ Azencot et al. 2013]

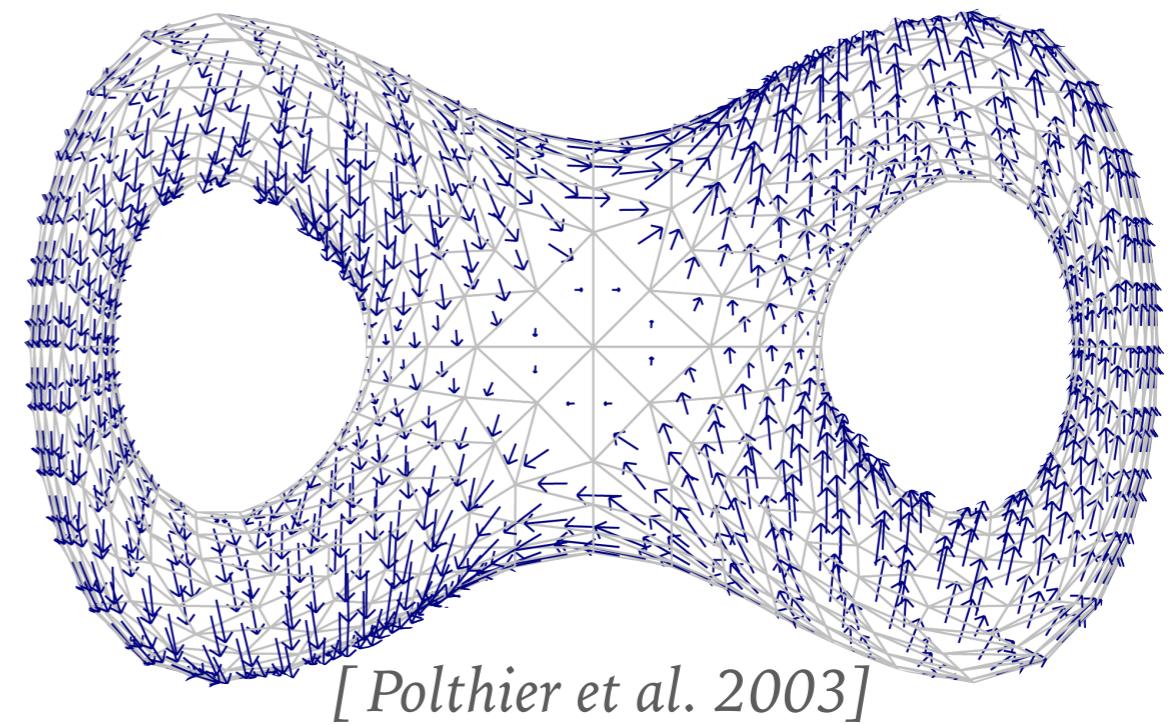
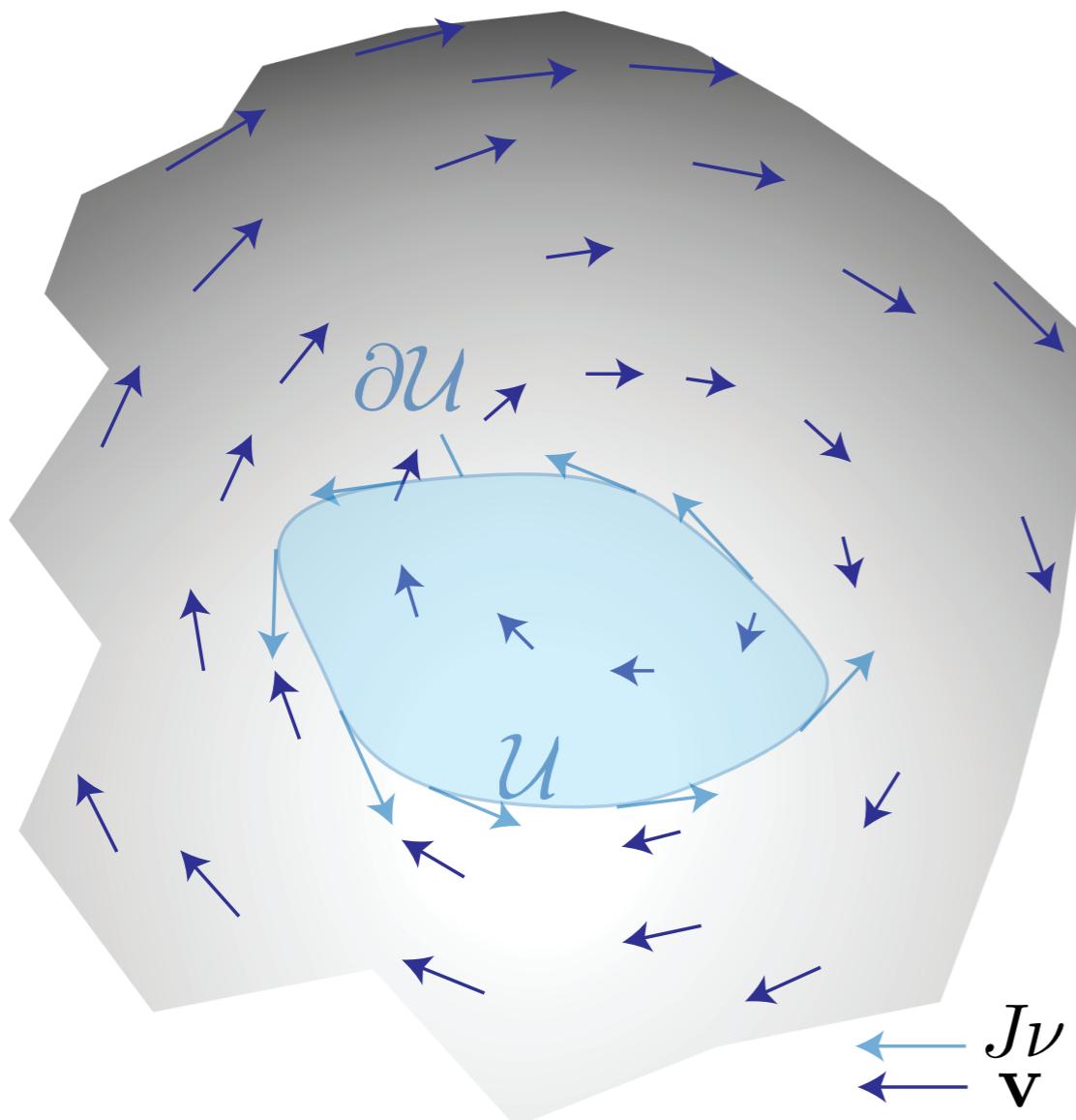
# CONSTRAINTS - DIFFERENTIAL - DIVERGENCE

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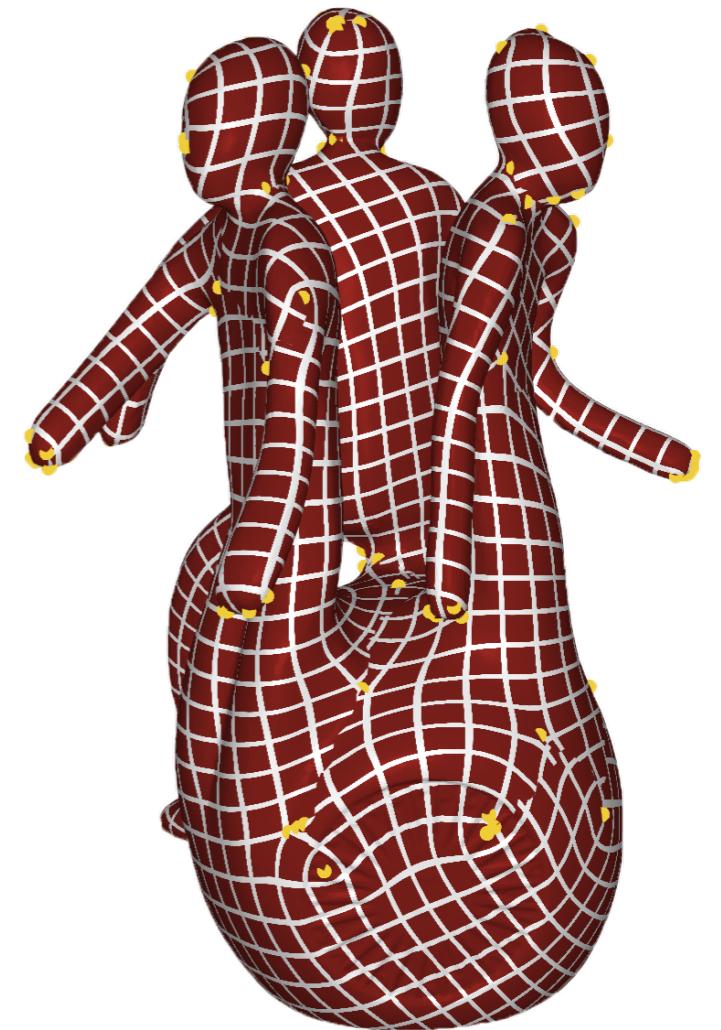
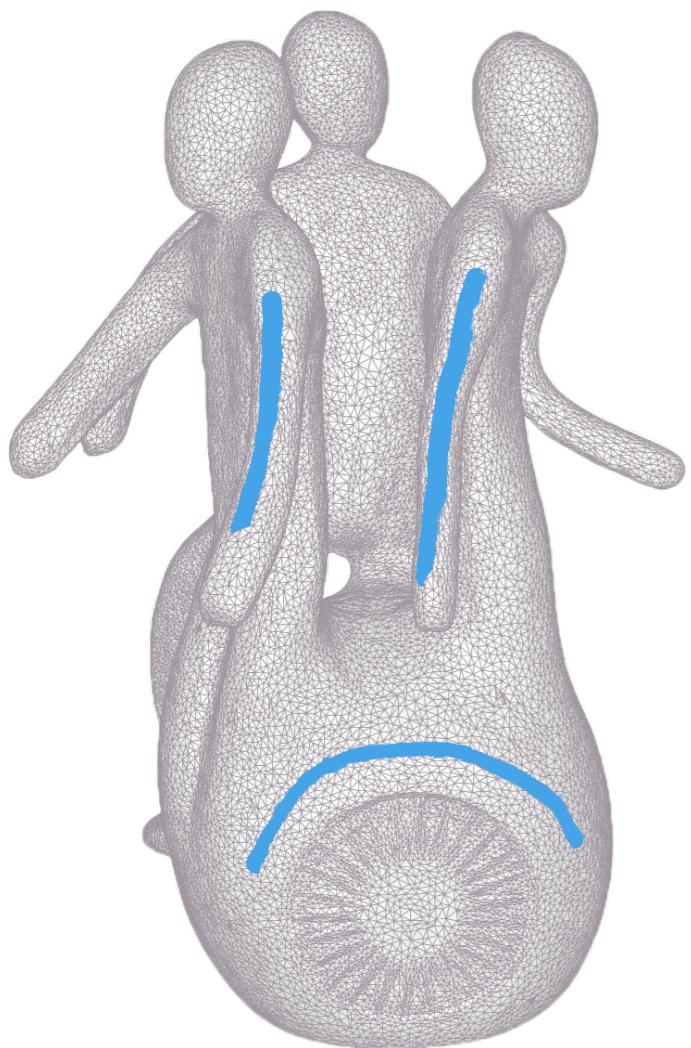
# CONSTRAINTS - DIFFERENTIAL - CURL

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# CONSTRAINTS - DIFFERENTIAL - CURL

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[Diamanti et al. 2015]

# HOW TO CHOOSE THE RIGHT METHOD?

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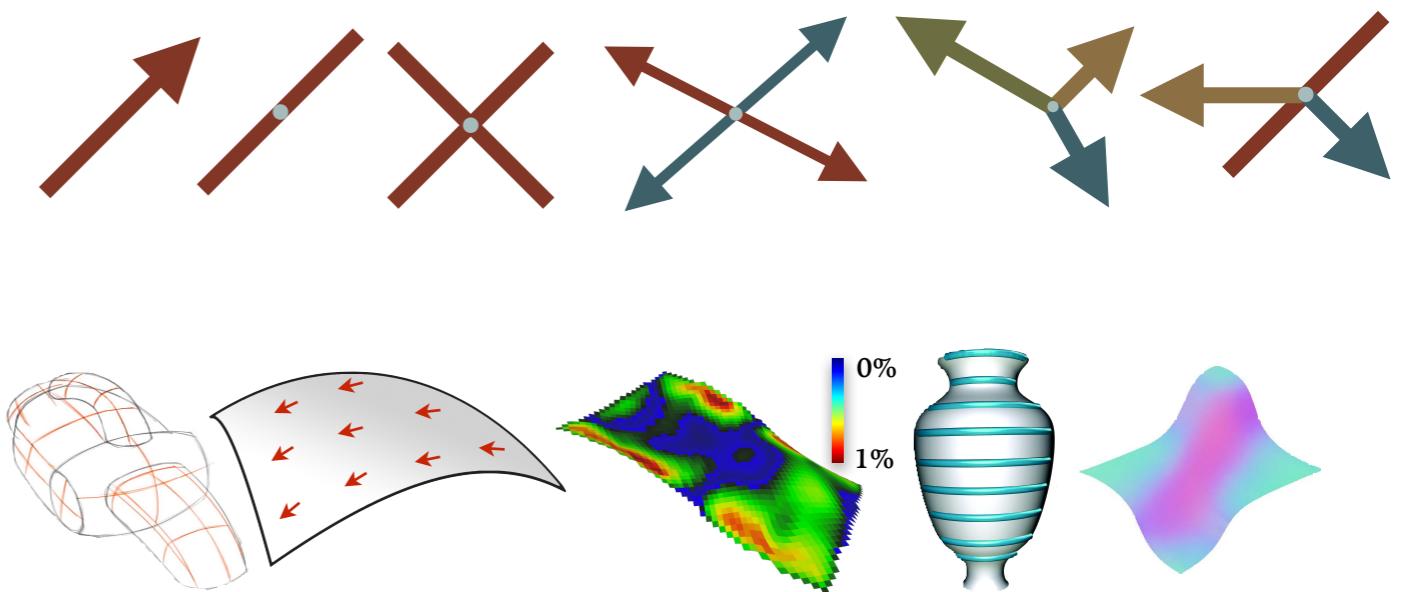
*Olga Diamanti*

*Geometric Computation Group  
Stanford University*

# CHOOSING THE RIGHT METHOD

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- Choose the type of object
- Determine what the ideal field is (objective)
- Consider the types of guarantees required
- Consider design strategy in terms of efficiency, convergence,...
  - this affects the choice of representation
- Consider discretization preference

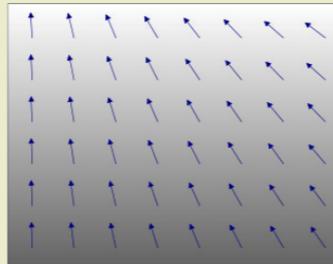
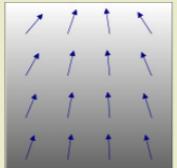


# CASE STUDY: VECTOR FIELDS



## Objective: Fairness

[Pedersen et al. 1995]



[Turk et al. 2001]



[Praun et al. 2000]

## Objective: Other

Isometries - Killing

[Ben-Chen et al. 2010] [Azencot et al. 2013]

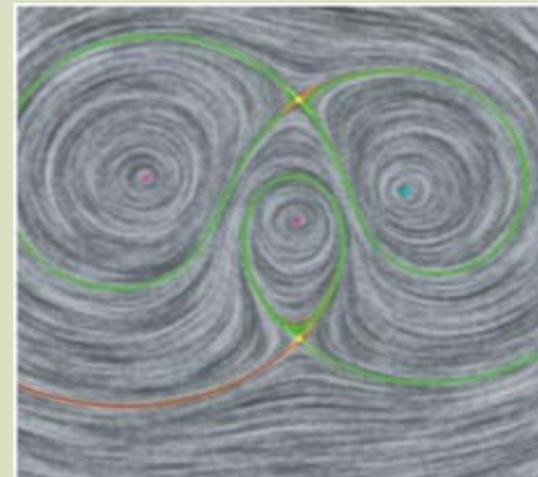
Curl / Divergence Control

[Fisher et al. 2007]

[Zhang et al. 2006]

Symmetries

[Azencot et al. 2013]



## Constraints: Directional

hard

[Pedersen et al. 1995]

[Praun et al. 2000]

[Azencot et al. 2013]

soft

[Turk et al. 2001]

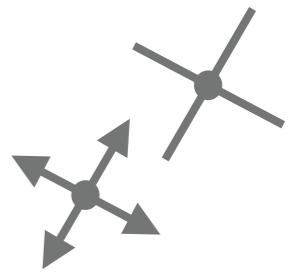
[Zhang et al. 2006]

soft + singularity control

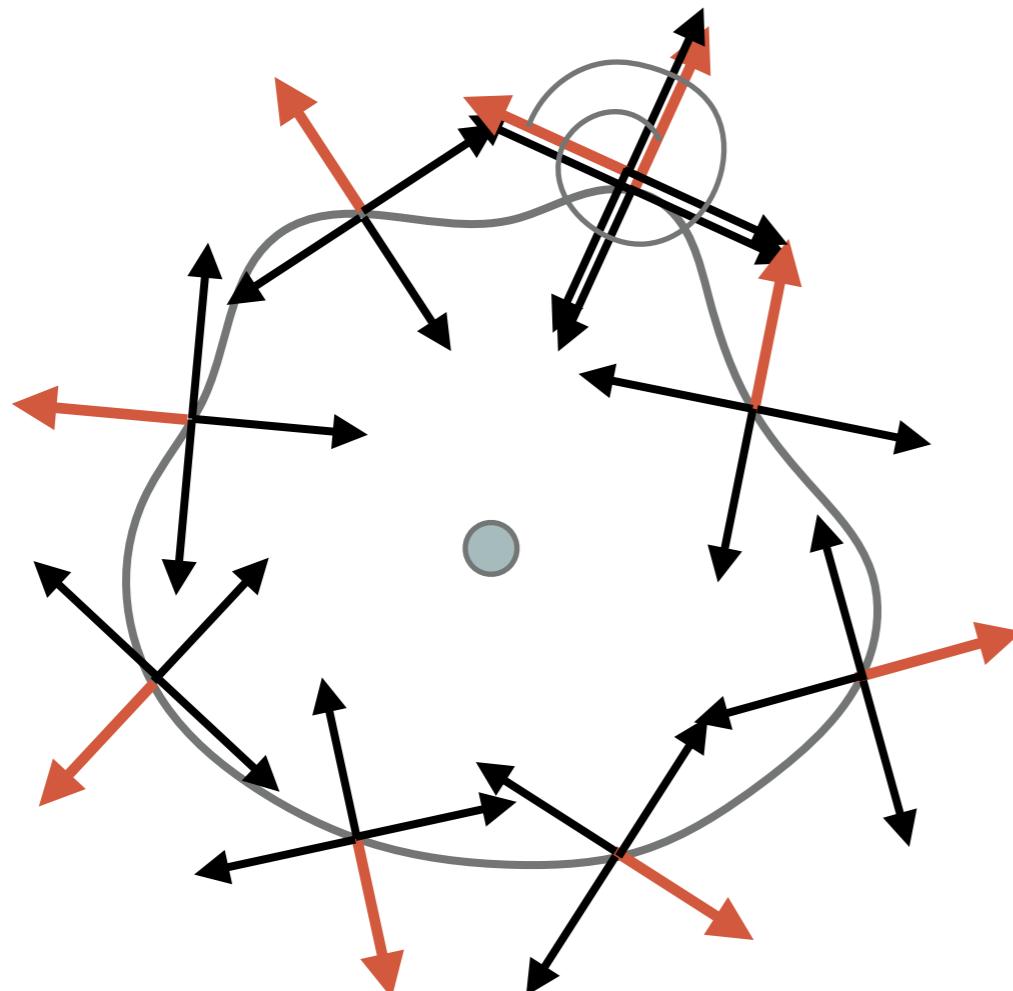
[Fisher et al. 2007]

[Zhang et al. 2006]

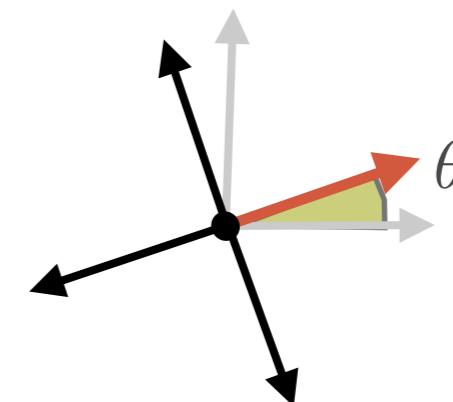
# CASE STUDY: N-DIRECTIONAL FIELD, FIXED TOPOLOGY



- Objective: “As-parallel-as-possible”
- Singularity prescription straightforward with angle based representations

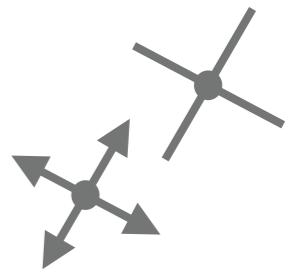


*desired singularity index = 5/4*

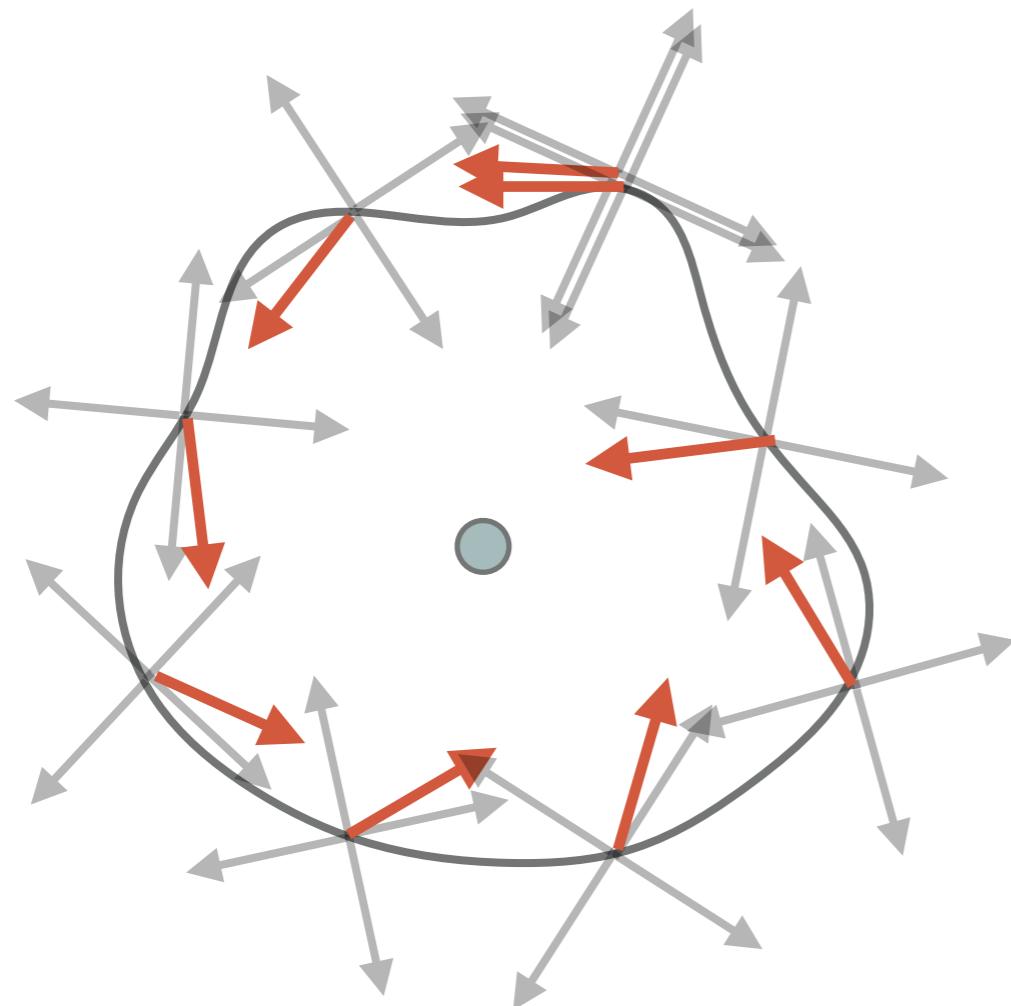


*angle-based representation  
of a 4-direction field*

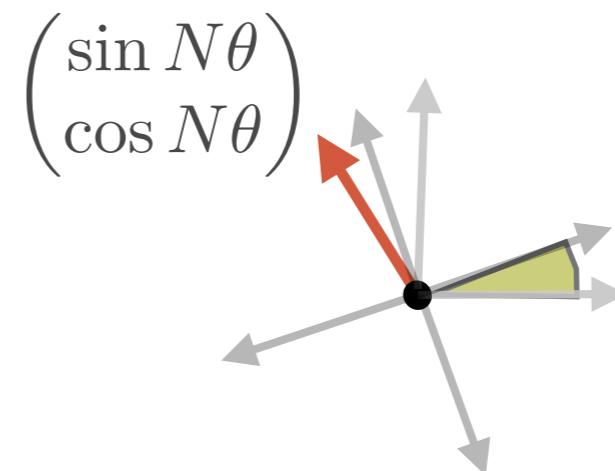
# CASE STUDY: N-DIRECTIONAL FIELD, FIXED TOPOLOGY



- Objective: “As-parallel-as-possible”
- Cartesian/Complex representative insensitive to  $2\pi$  rotations



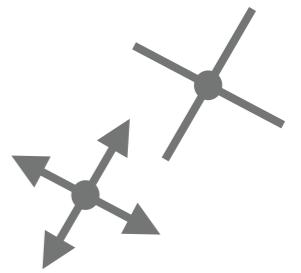
*desired singularity index = 5/4*



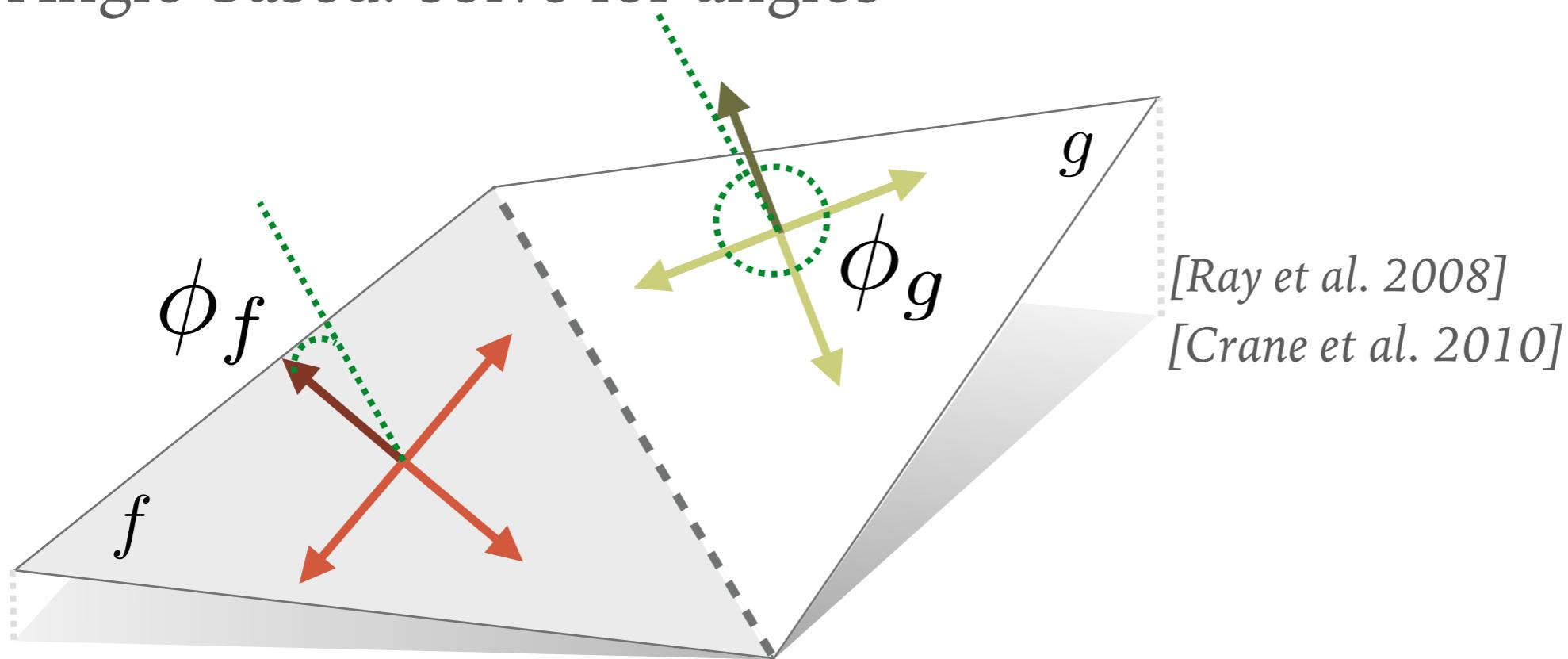
*cartesian representation of  
a 4-direction field*

*How to prescribe angle difference  
 $2\pi + \pi/4$  with principal rotation/  
matching?*

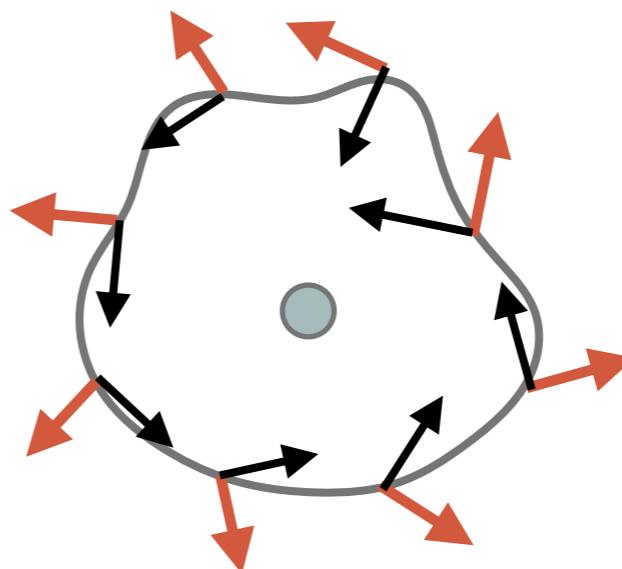
# CASE STUDY: N-DIRECTIONAL FIELD, FIXED TOPOLOGY



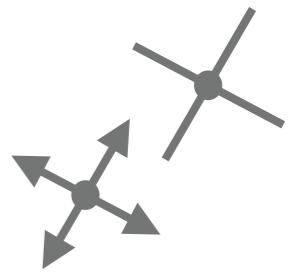
- Angle-based: solve for angles



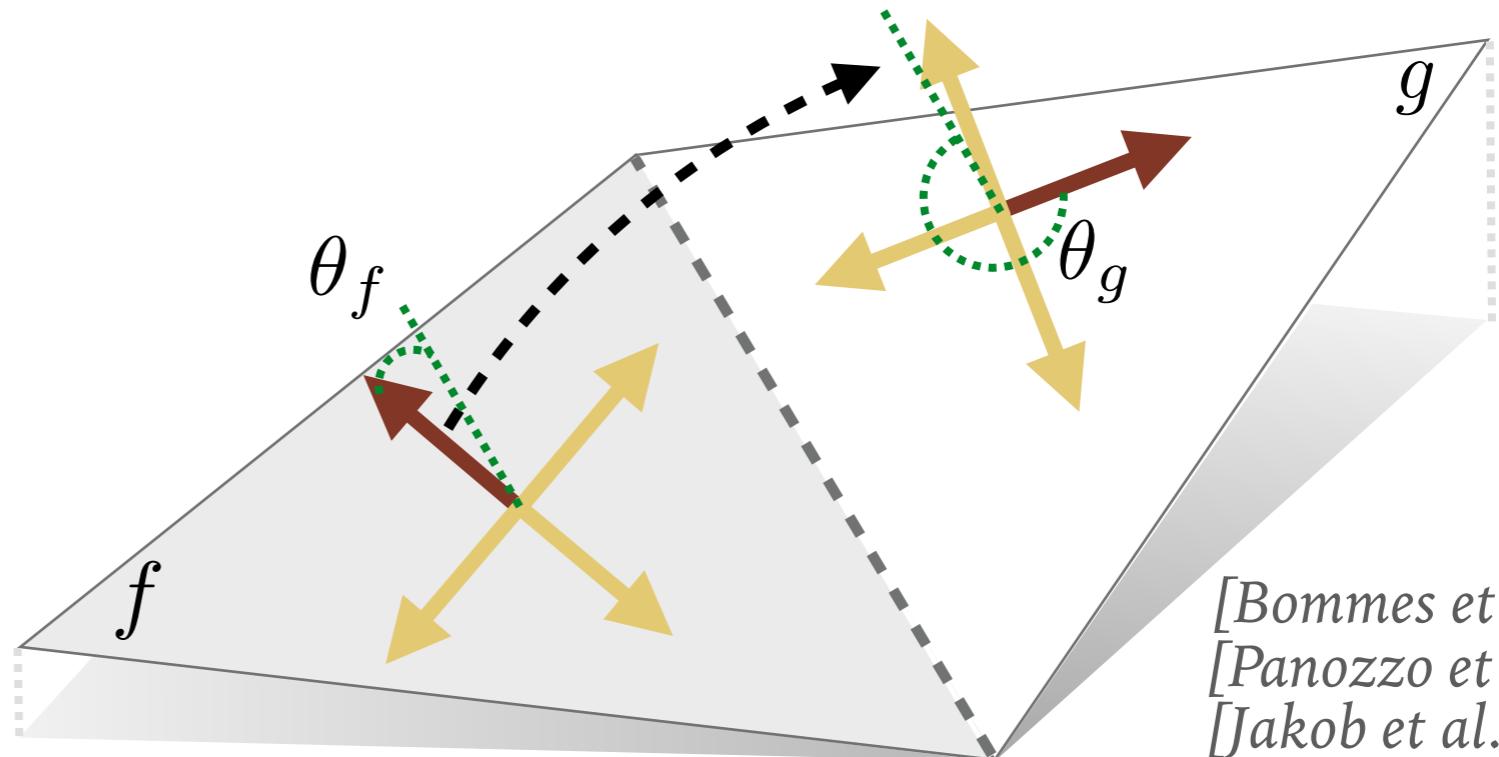
- linear system
- at least one constraint needed



# CASE STUDY: N-DIRECTIONAL FIELD, FREE TOPOLOGY



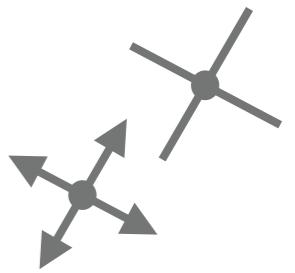
- Objective: “As-parallel-as-possible”
- Explicitly model topology (typically angle-based)
  - Matchings are explicitly modeled
  - Mixed Integer Optimization
  - Local minima



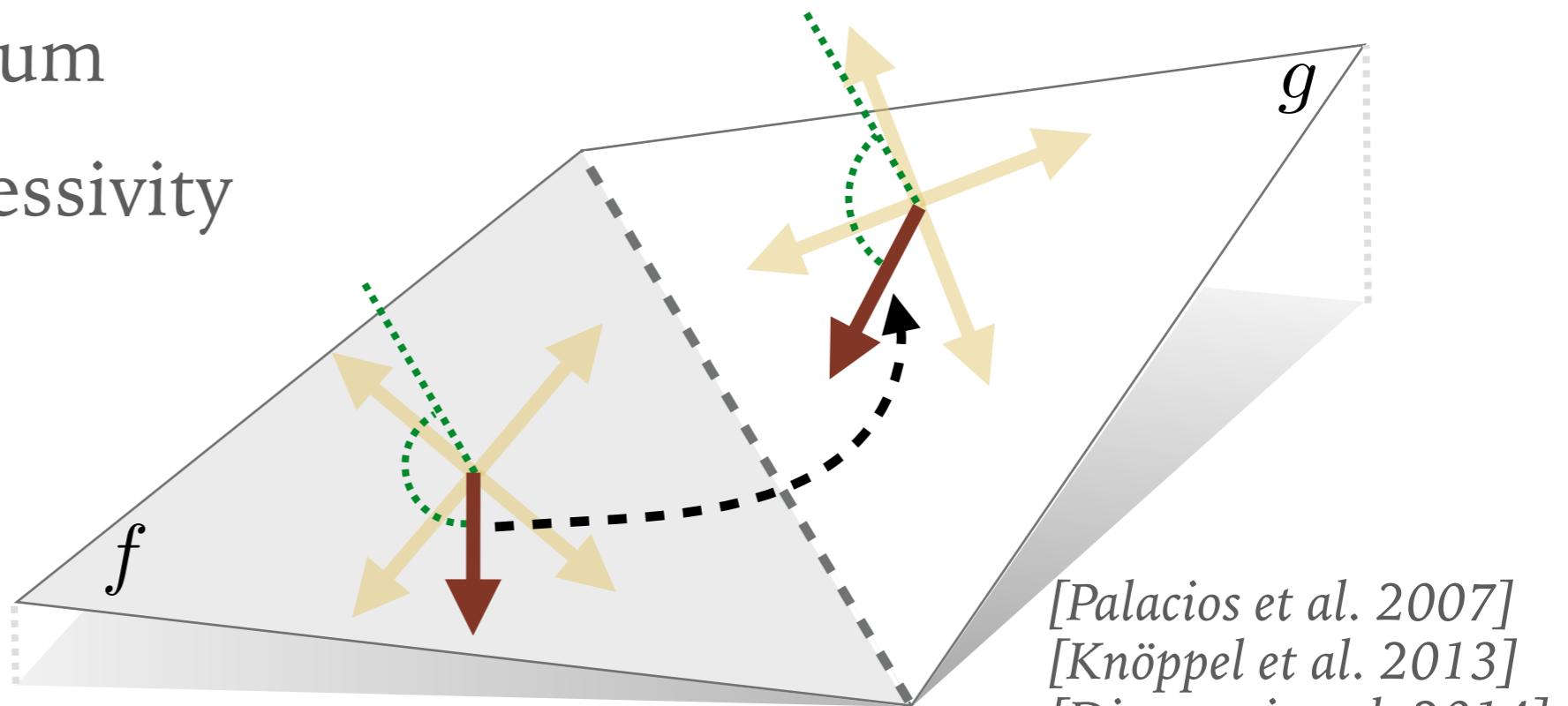
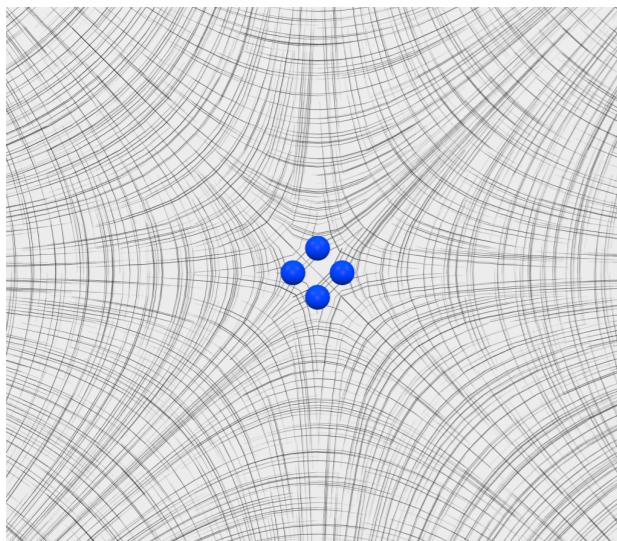
[Bommes et al. 2009]  
[Panozzo et al. 2012]  
[Jakob et al. 2015]

$$(\theta_f + \rho_{fg} \frac{\pi}{2} - \theta_g)^2 \quad \rho_{fg} \in \mathbb{I}$$

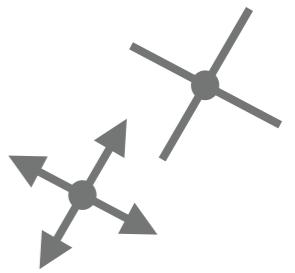
# CASE STUDY: N-DIRECTIONAL FIELD, FREE TOPOLOGY



- Objective: “As-parallel-as-possible”
- Implicit topology with principal matchings (typically cartesian/complex)
  - Linear problem
  - Global optimum
  - Limited expressivity

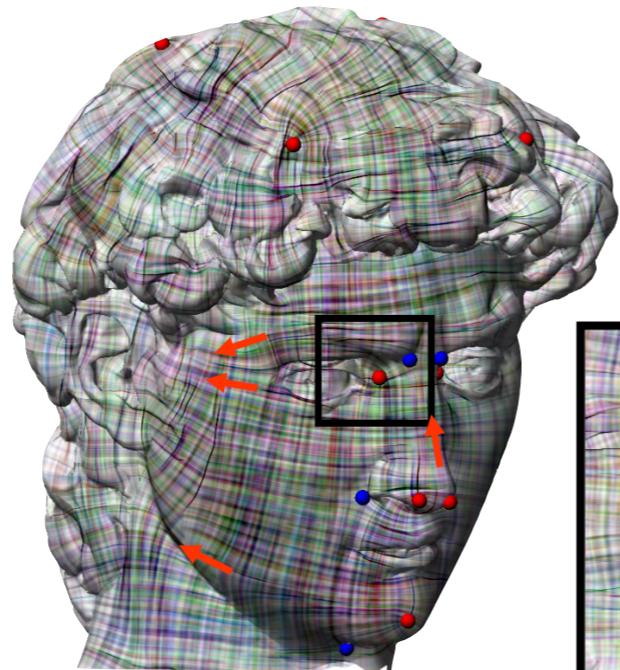


[Palacios et al. 2007]  
[Knöppel et al. 2013]  
[Diamanti et al. 2014]



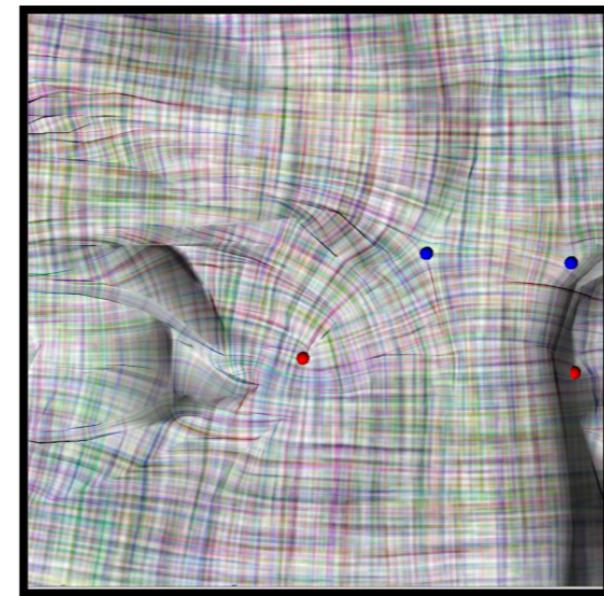
# HOW ABOUT CONSTRAINTS?

- Sparse “Hard”



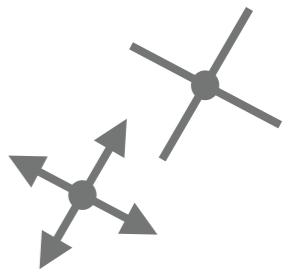
[Crane et al. 2010]  
[Bommes et al. 2009]  
[Hertzmann et al. 2000]  
[Diamanti et al. 2014]

[Palacios et al. 2007]  
[Ray et al. 2008]  
[Jakob et al. 2015]

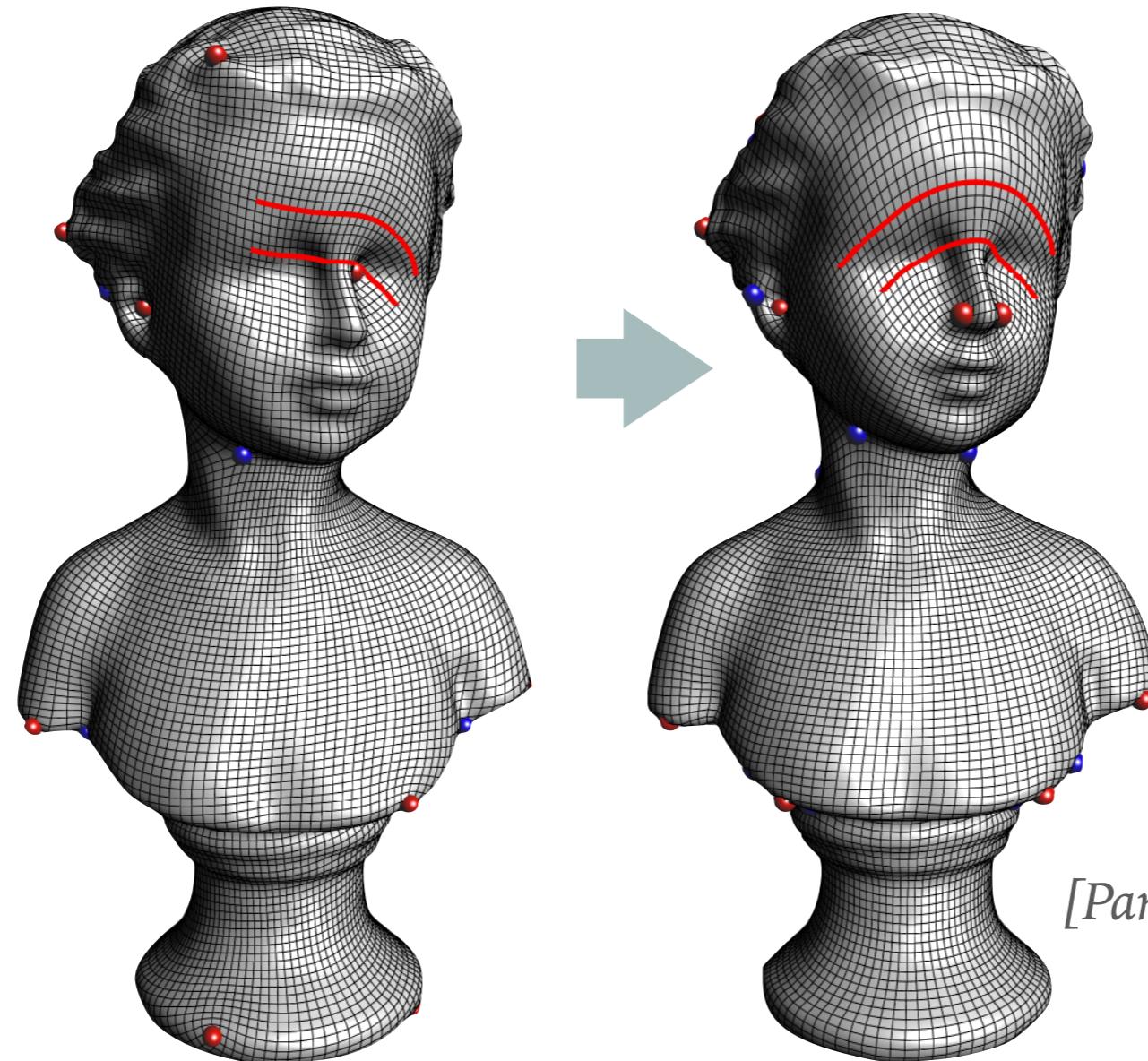


- Also partial constraints! [Iarussi et al. 2015] [Diamanti et al. 2015]

# HOW ABOUT CONSTRAINTS?



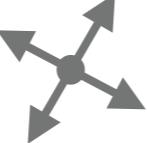
- Soft constraints

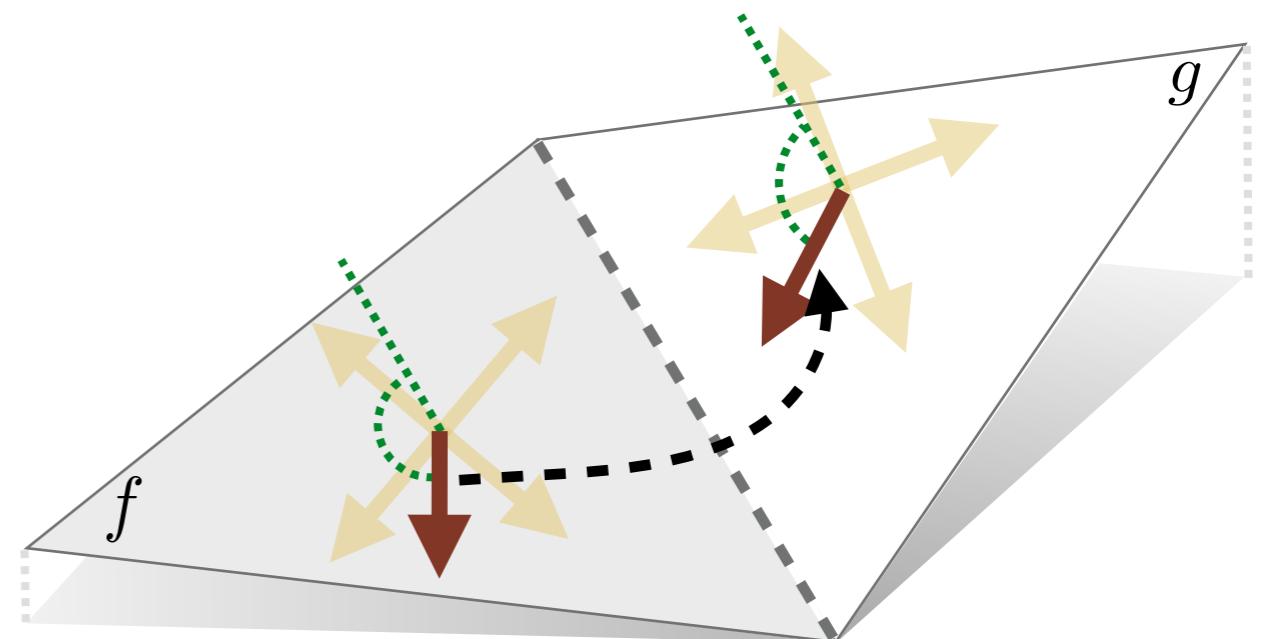


[Panozzo et al. 2012]

# AVOIDING THE TRIVIAL SOLUTION

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- Eg. N-Vector Fields  with a cartesian/complex representation
- Zero Field is perfectly smooth!
- Constraints are necessary
  - Per-vector unit-norm constraint [Palacios et al. 2007]
  - Integrated norm constraint [Knöppel et al. 2013]
  - Enough hard constraints [Diamanti et al. 2014]



# COMPARATIVE ANALYSIS

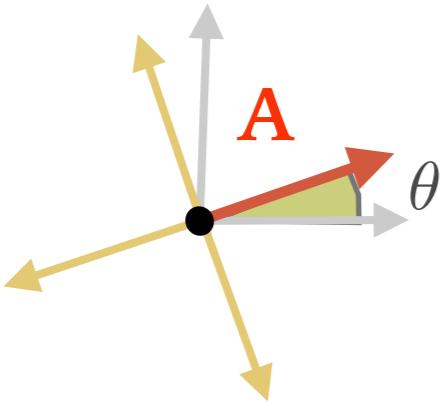
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*Olga Diamanti*

*Geometric Computation Group  
Stanford University*

# COMPARISONS AND DEMOS

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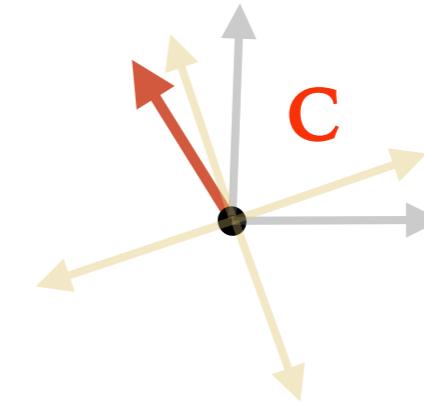


angle-based approaches

[Bommes et al. 2009]

[Panozzo et al. 2014]

[Jakob et al. 2015]



cartesian/complex approaches

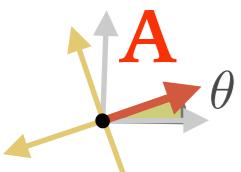
[Knöppel et al. 2013]

[Diamanti et al. 2014]

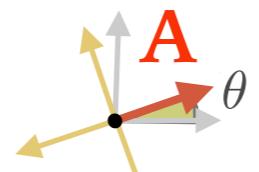
[Diamanti et al. 2015]

# SINGULARITIES

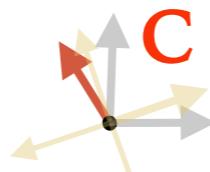
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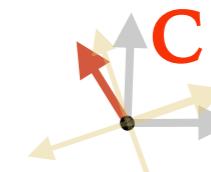
[Bommes et al. 2009]



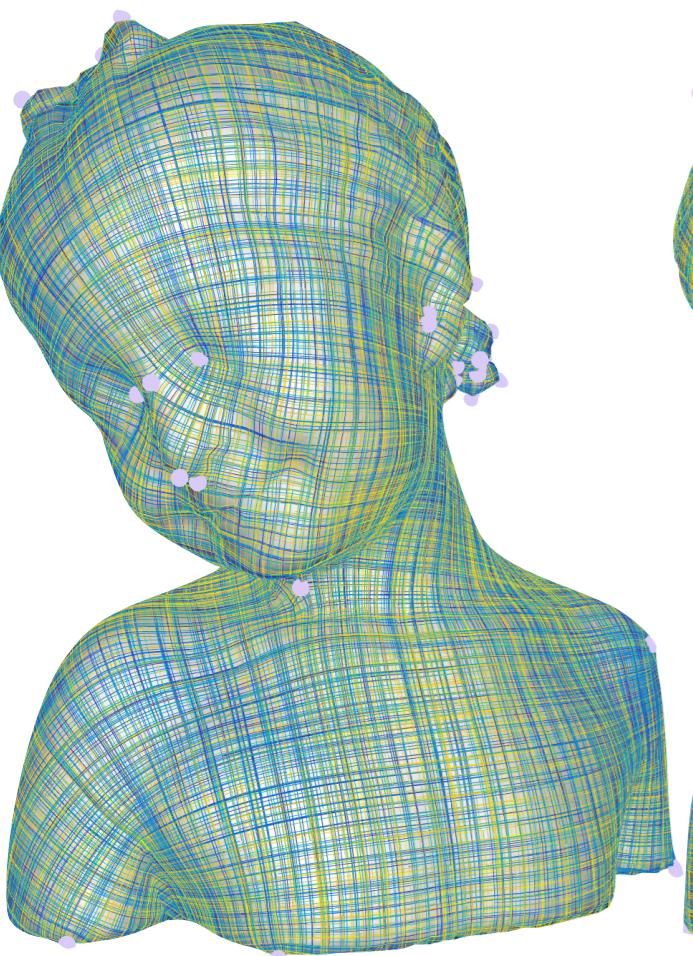
[Jakob et al. 2015]



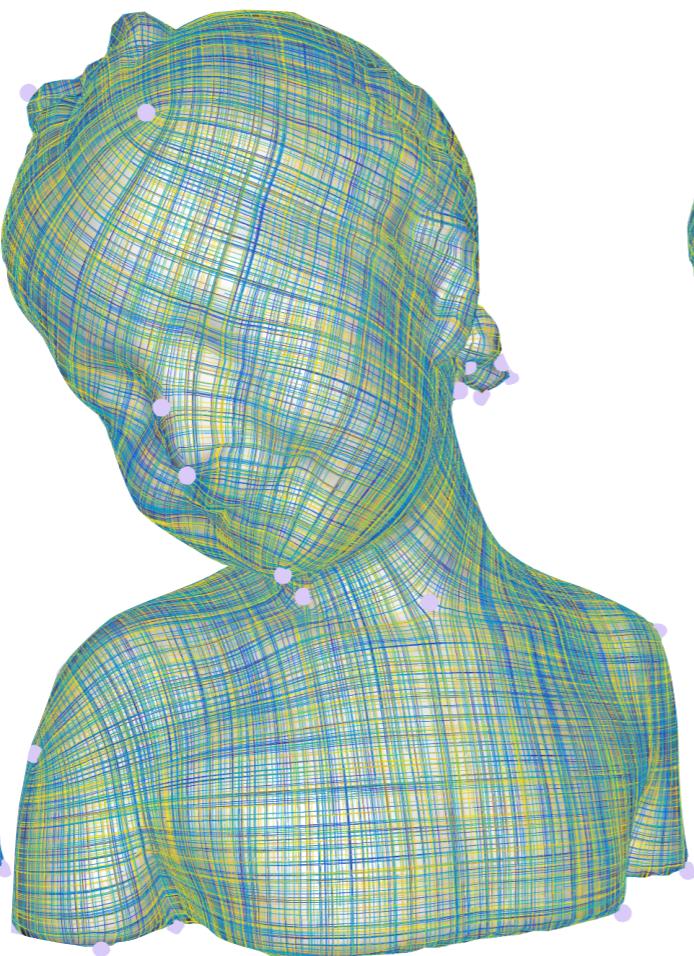
[Knöppel et al. 2013]



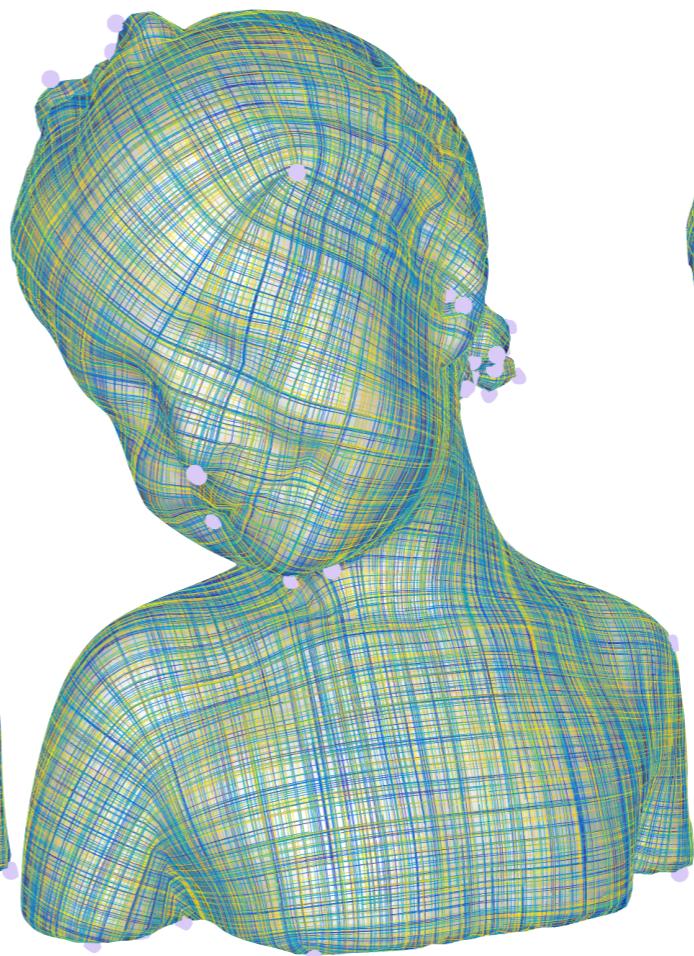
[Diamanti et al. 2015]



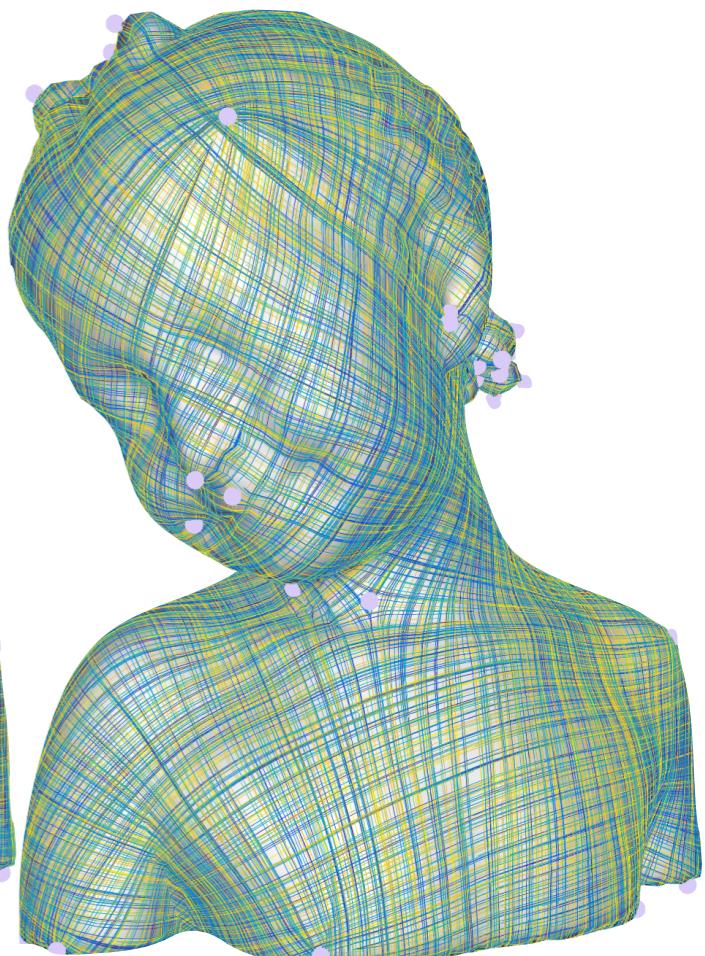
48 singularities



42 singularities



62 singularities

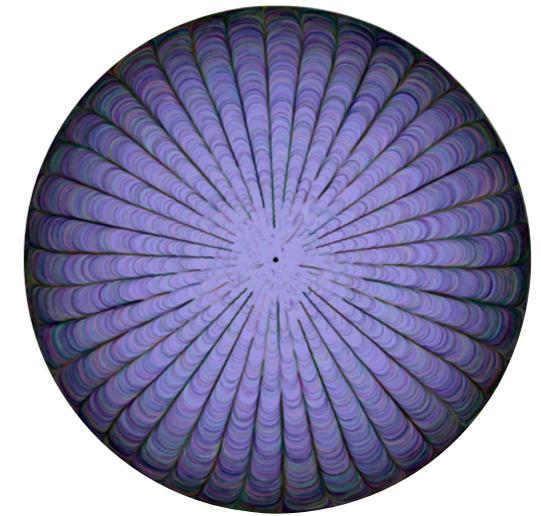


56 singularities

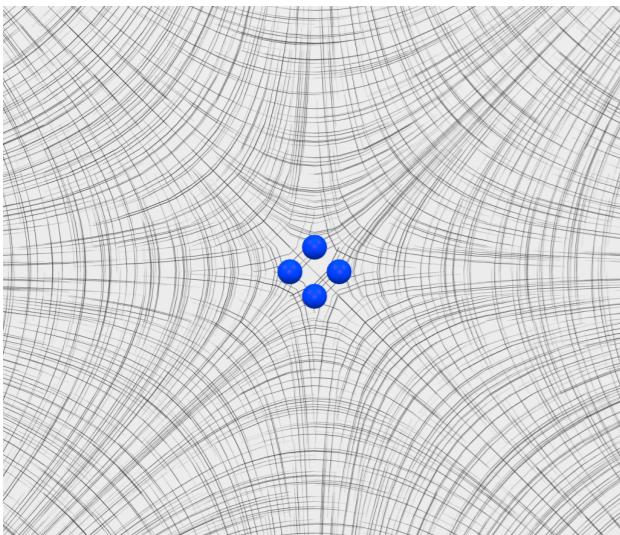
# SINGULARITIES – SAMPLING PROBLEM

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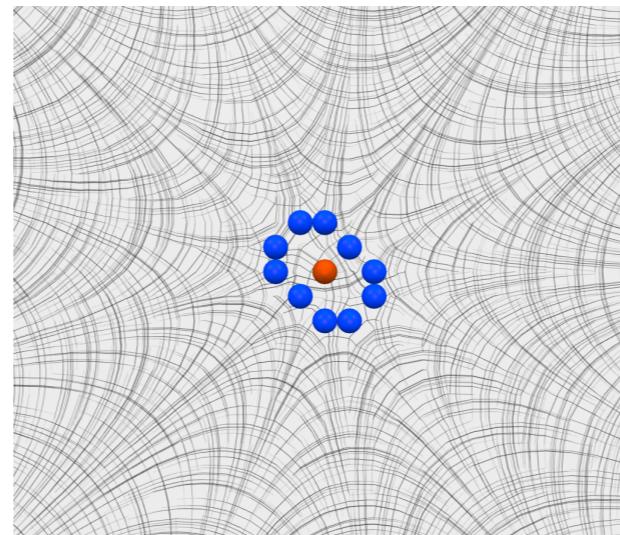
- Implicit field: principal matching assumed.
- Low valence cycles: limited rotation sums.
- Higher order singularities cannot be represented!
- In practice: promoting low-degree singularity cycles (“**singularity party**”).



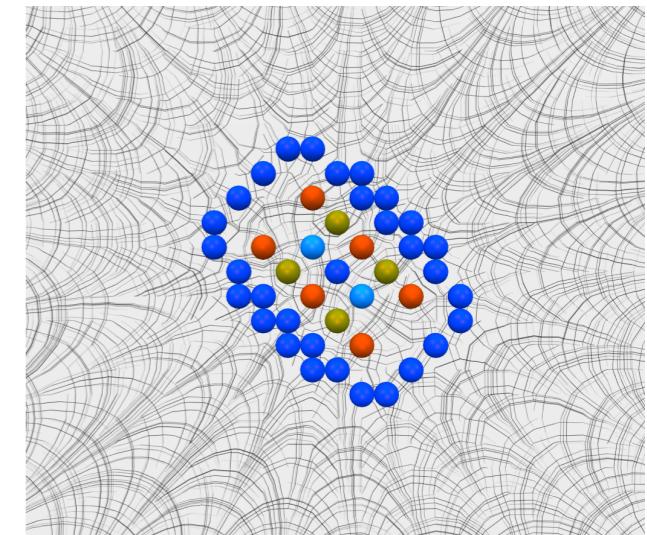
$$-\frac{4}{4}$$



$$-\frac{9}{4}$$

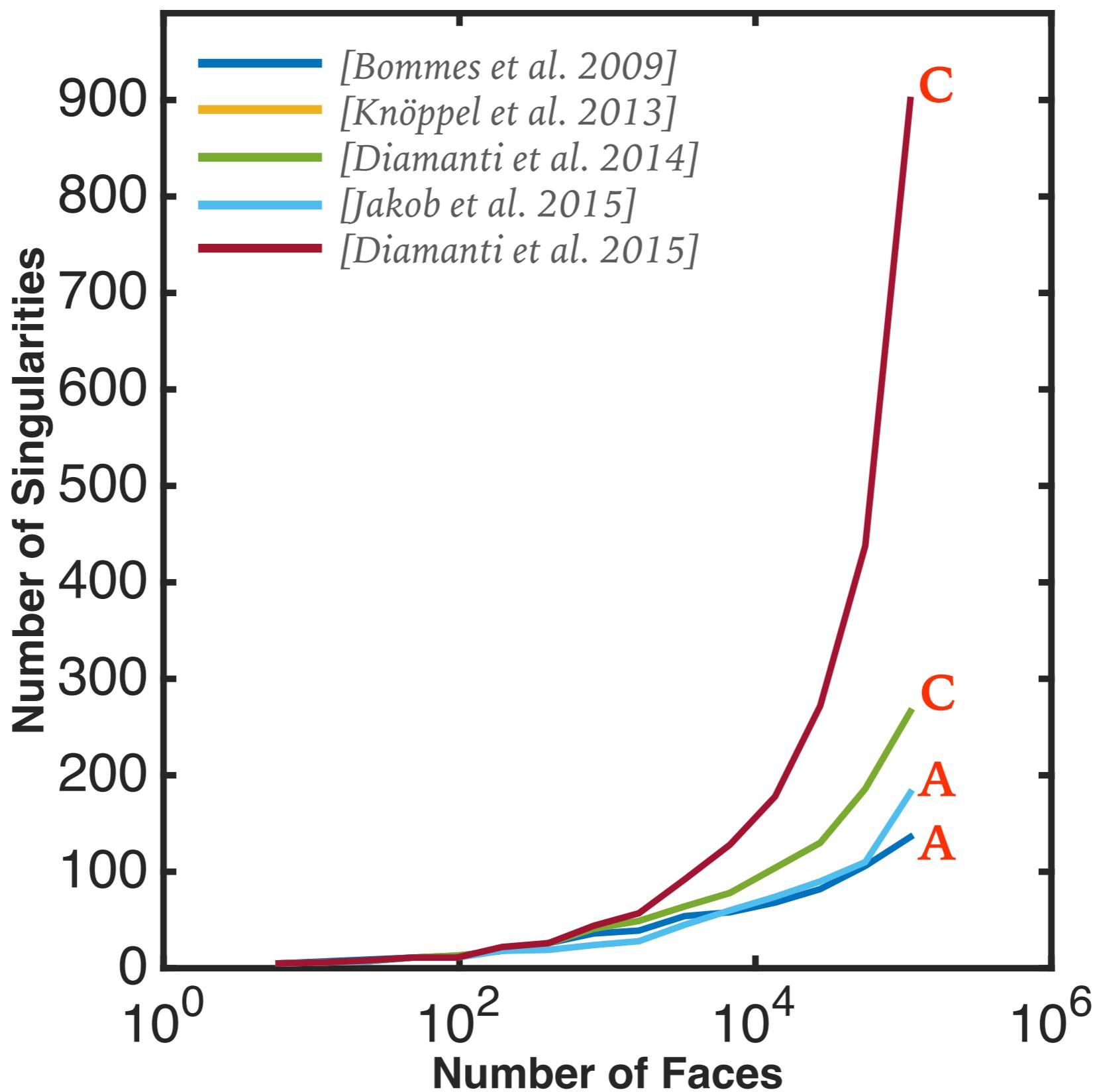


$$-\frac{21}{4}$$

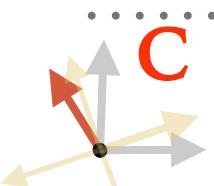


# SINGULARITIES

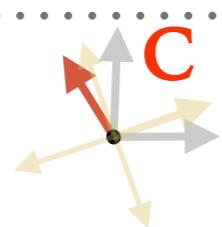
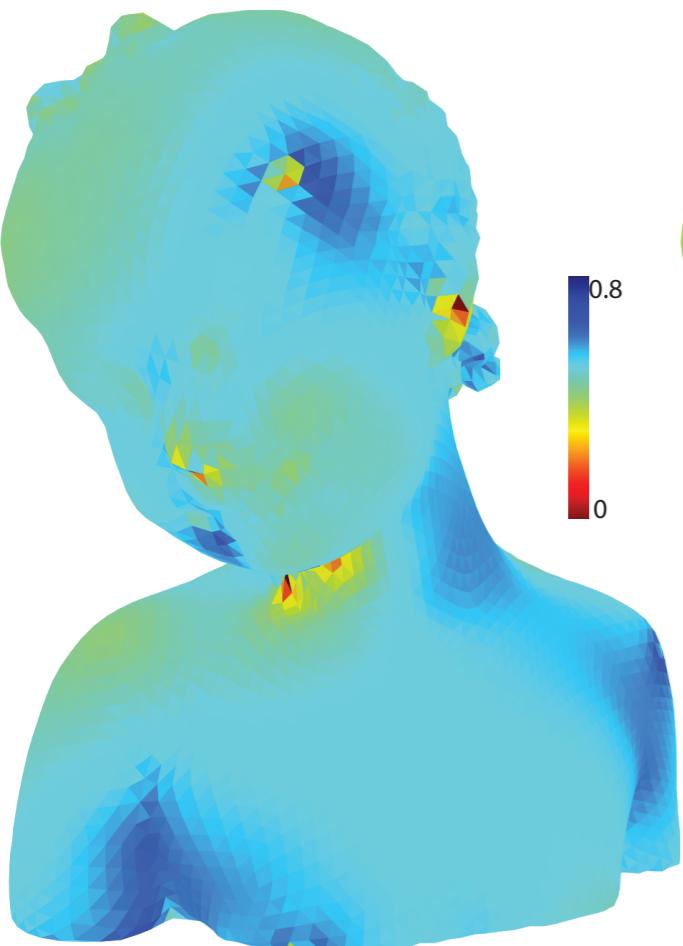
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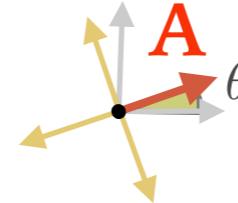
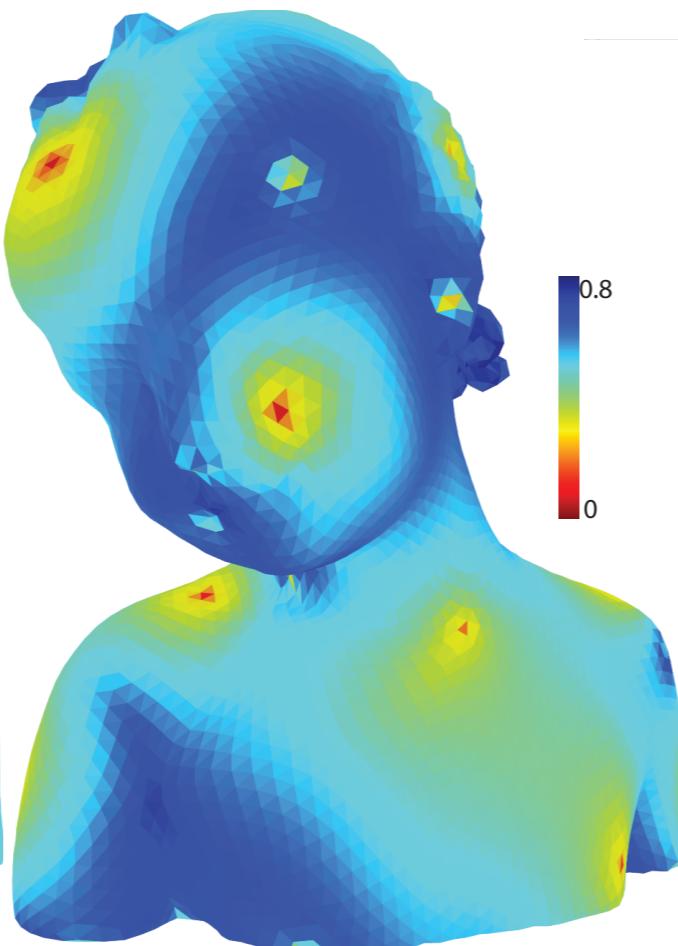
# INTEGRABILITY



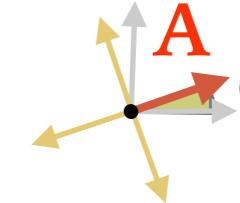
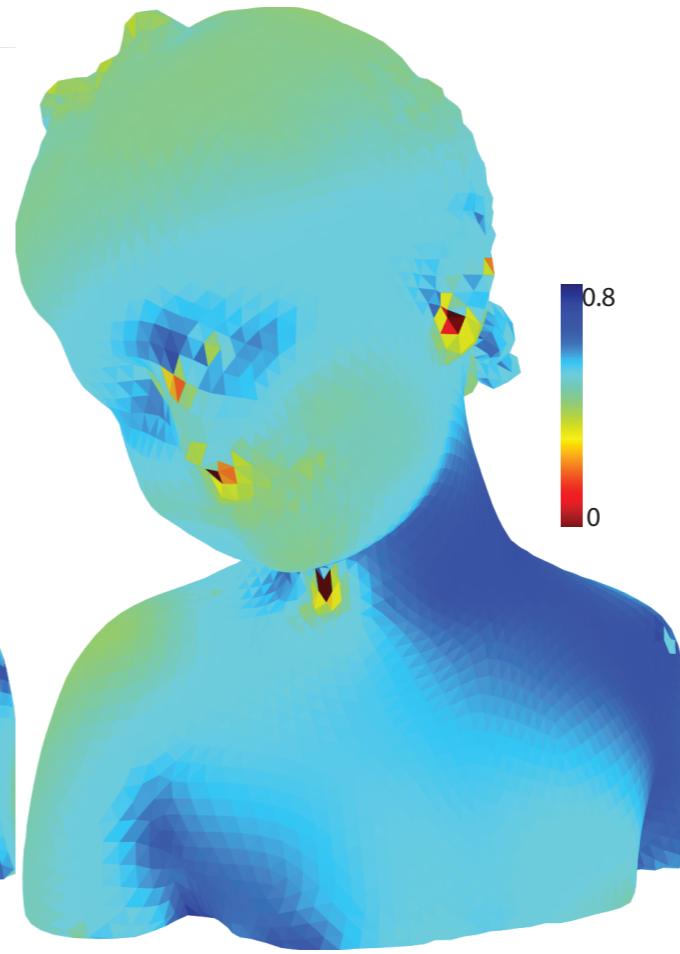
[Knöppel et al. 2013]



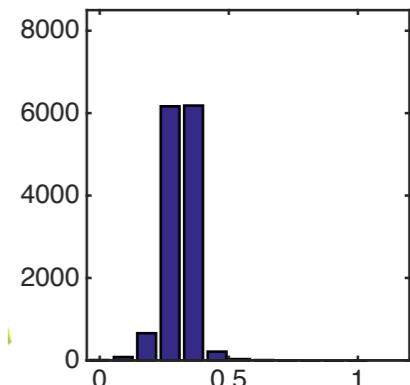
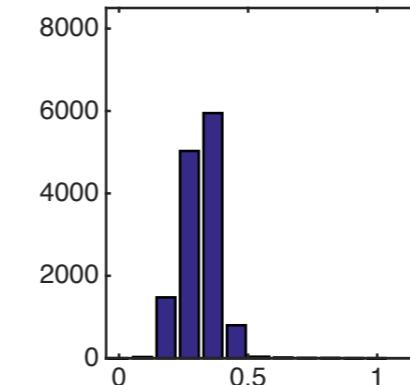
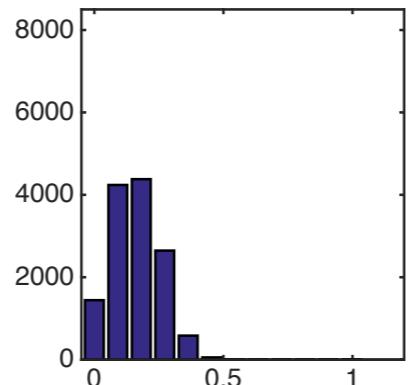
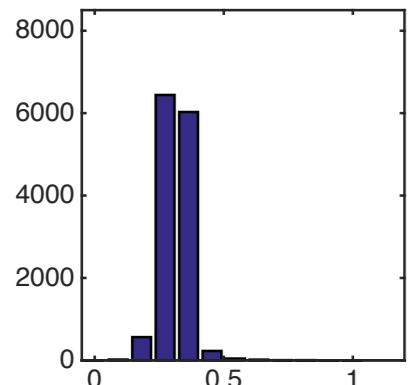
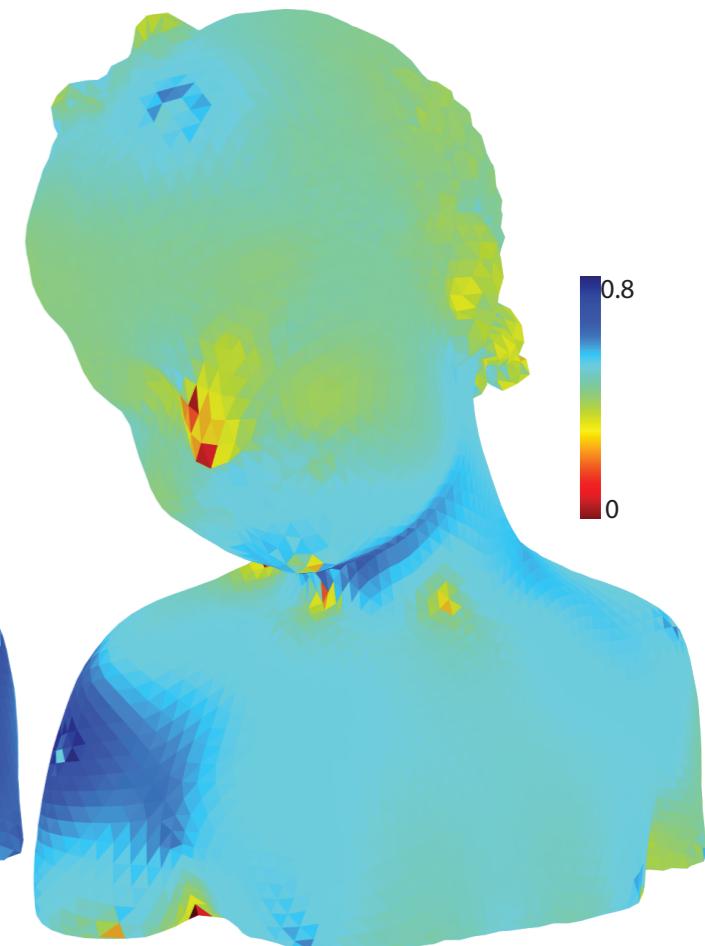
[Diamanti et al. 2014]



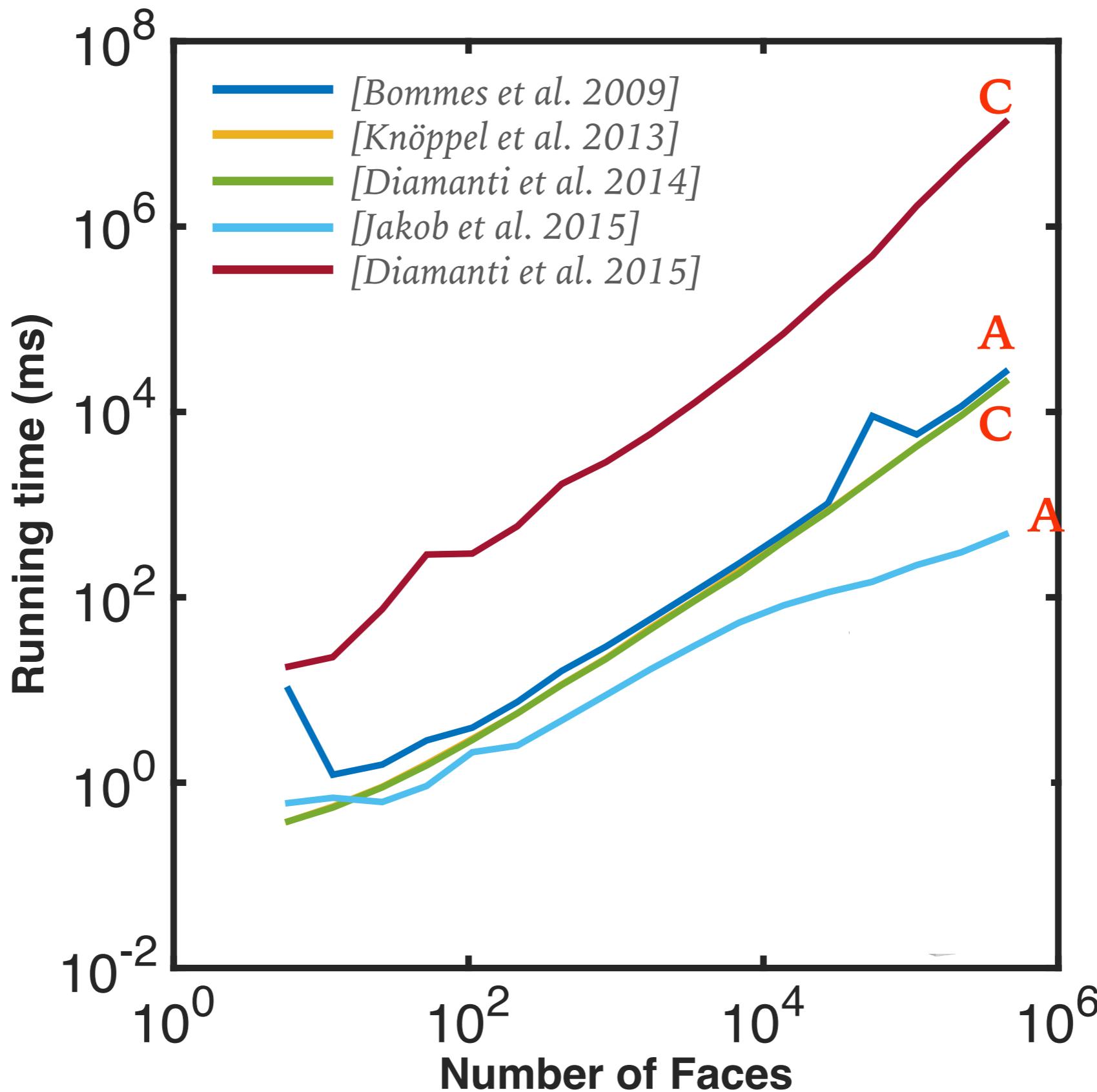
[Bommes et al. 2009]



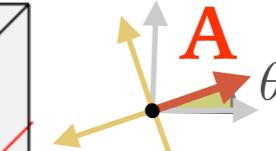
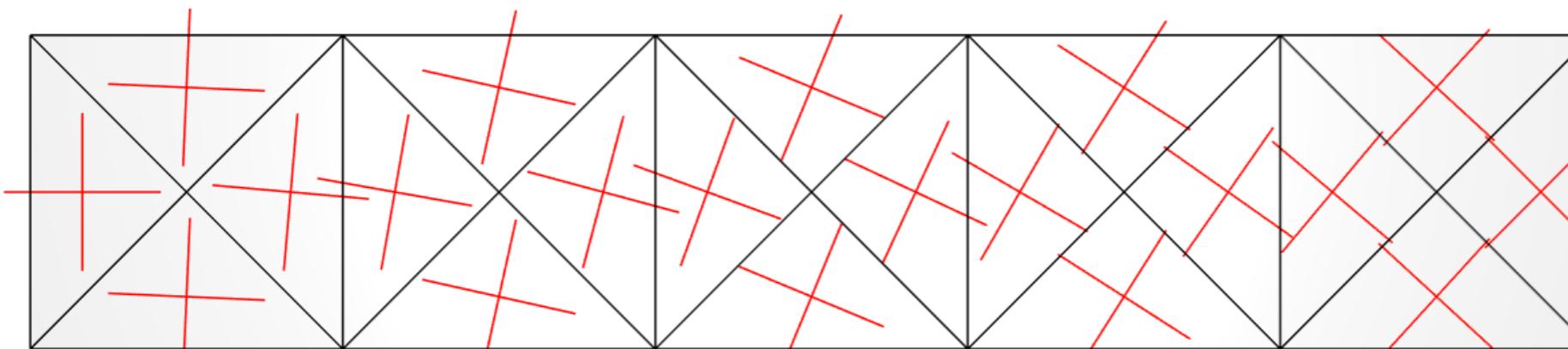
[Jakob et al. 2015]



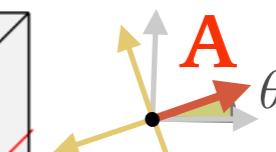
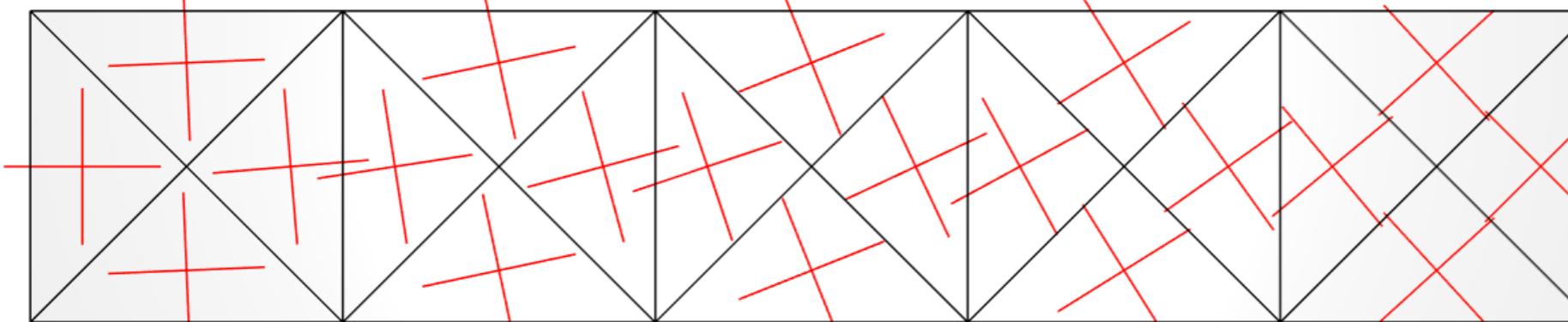
# SCALABILITY



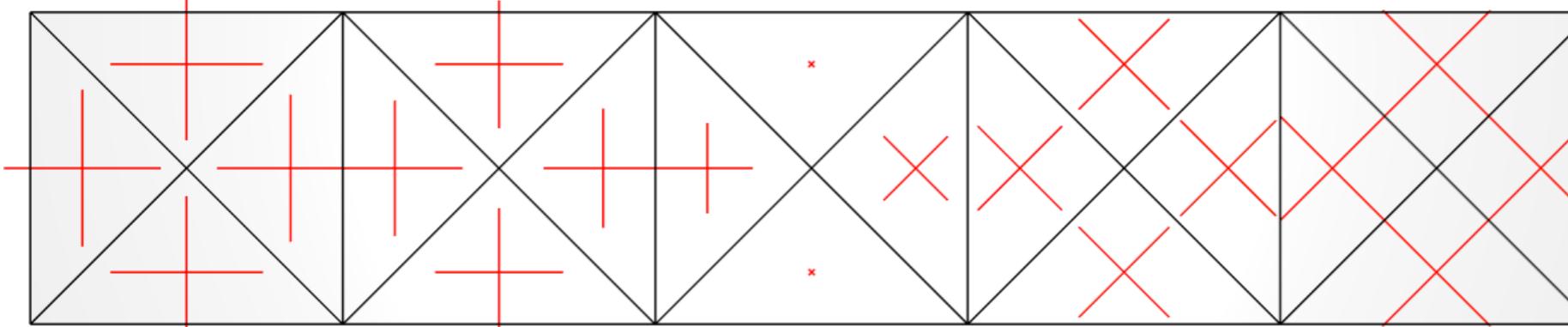
# 4-DIRECTIONAL FIELD



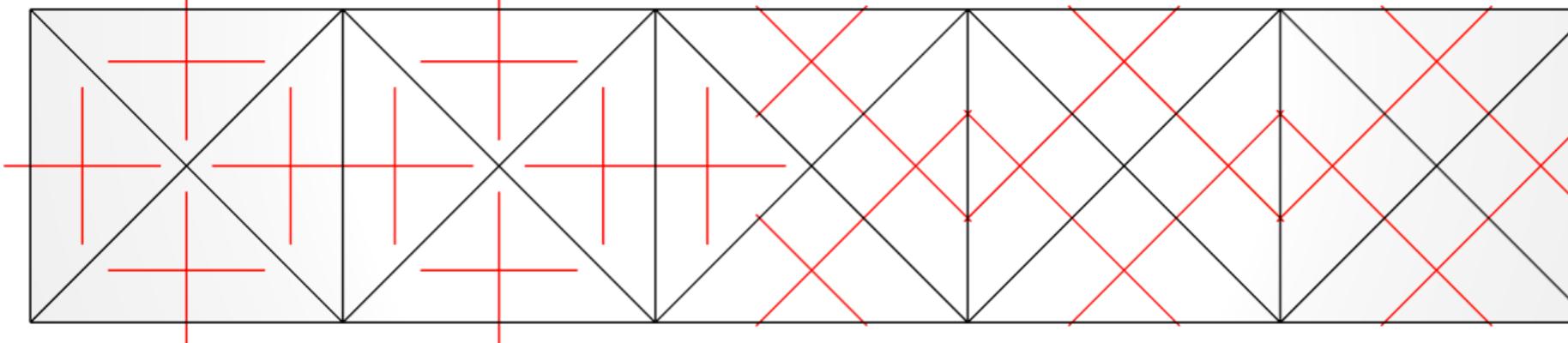
[Bommes et al. 2009]



[Jakob et al. 2015]



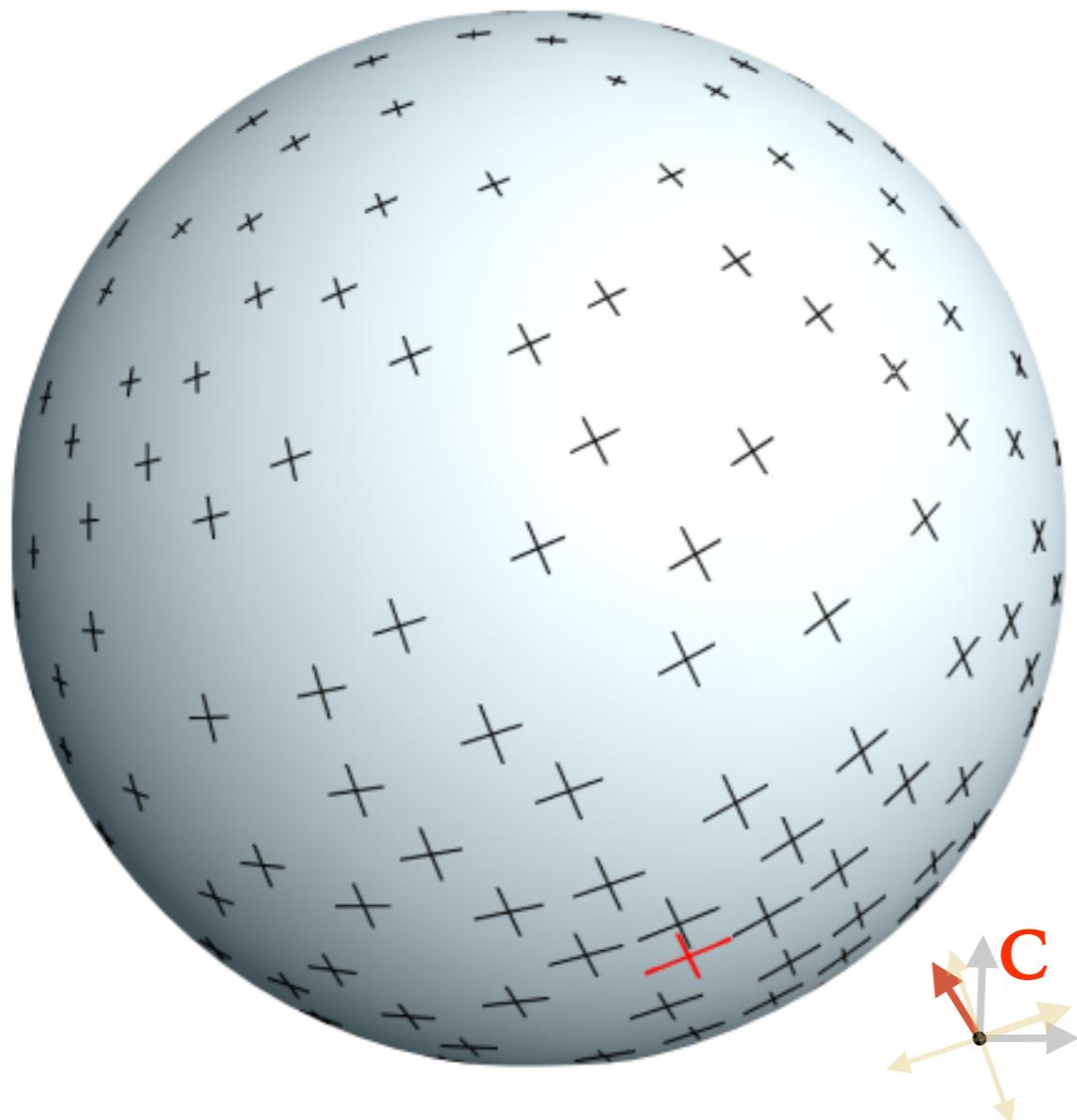
[Diamanti et al. 2014]



[Knöppel et al. 2013]

# MAGNITUDE IN CARTESIAN REPRESENTATIONS - CONSTRAINTS

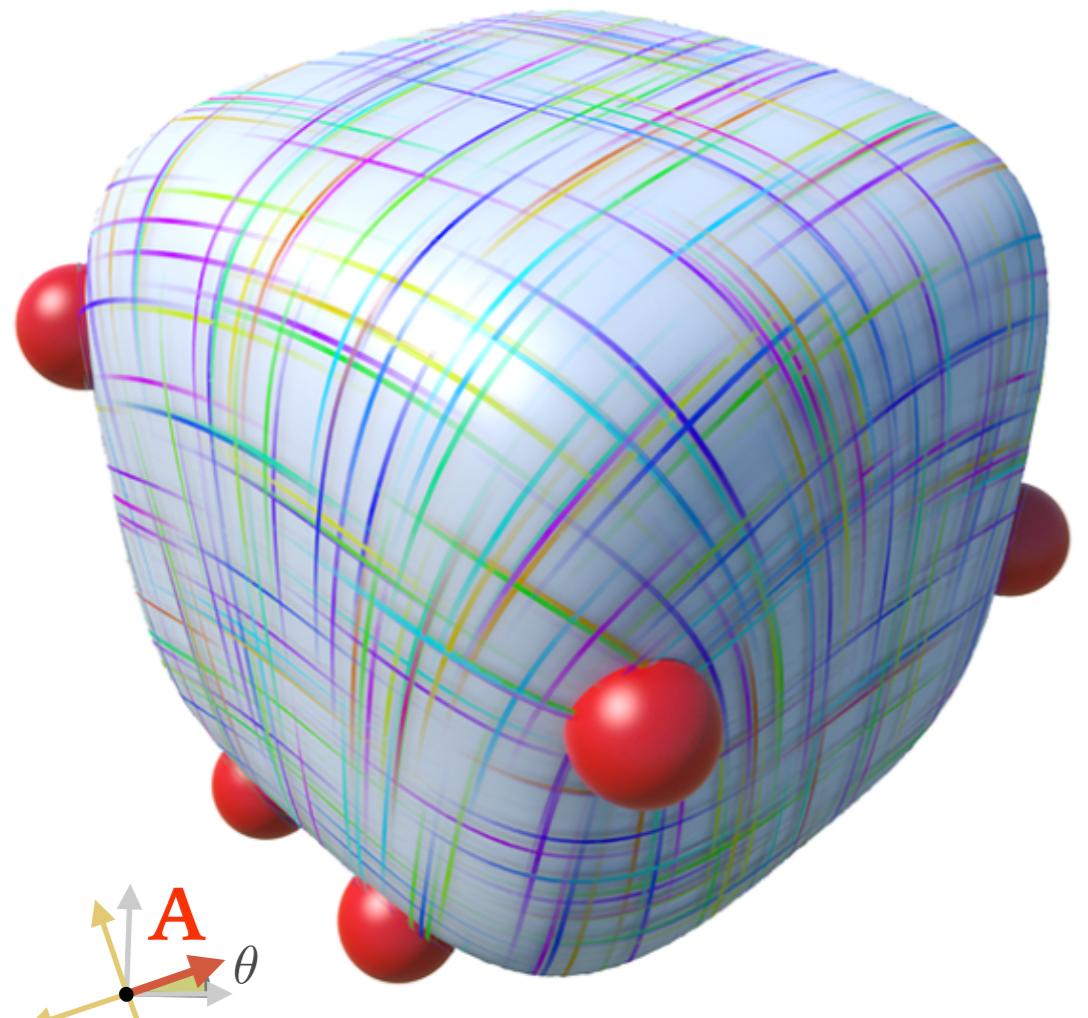
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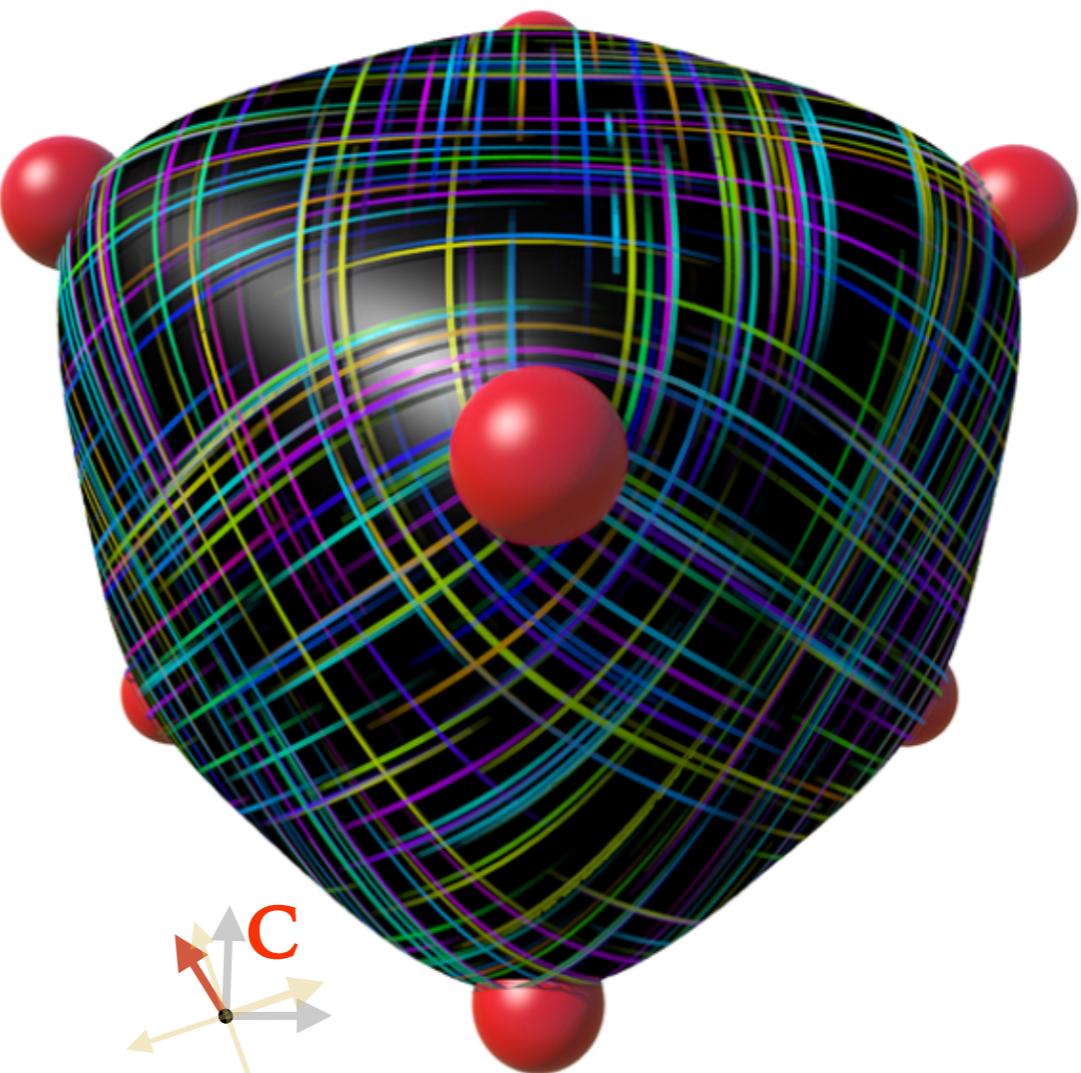
[Diamanti et al. 2014]

# EFFECT OF THE FORMULATION

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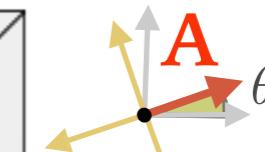
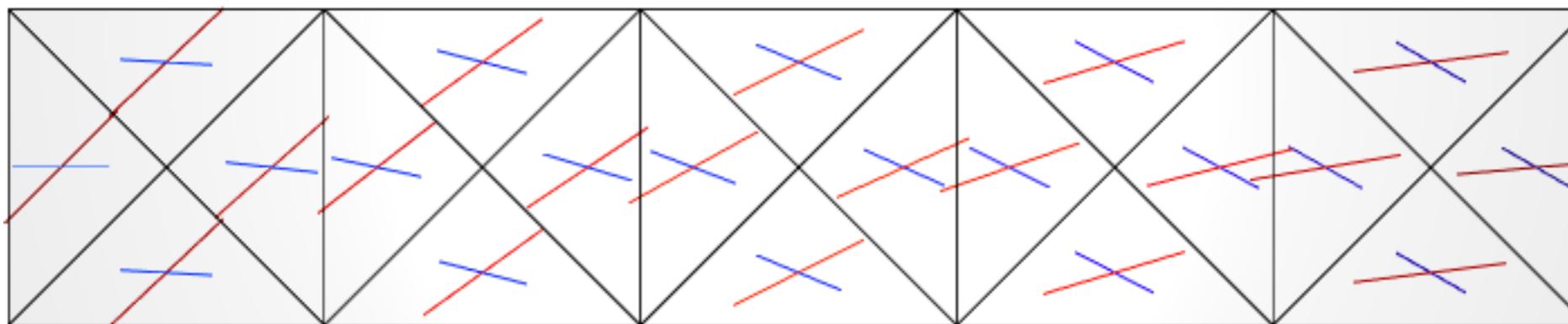


[Bommes et al. 2009]

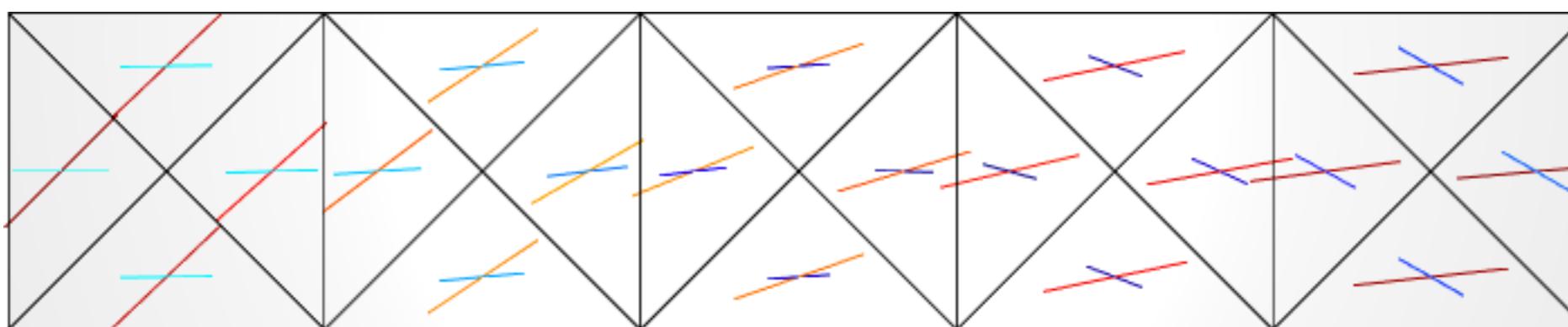


[Knöppel et al. 2013]

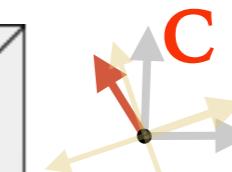
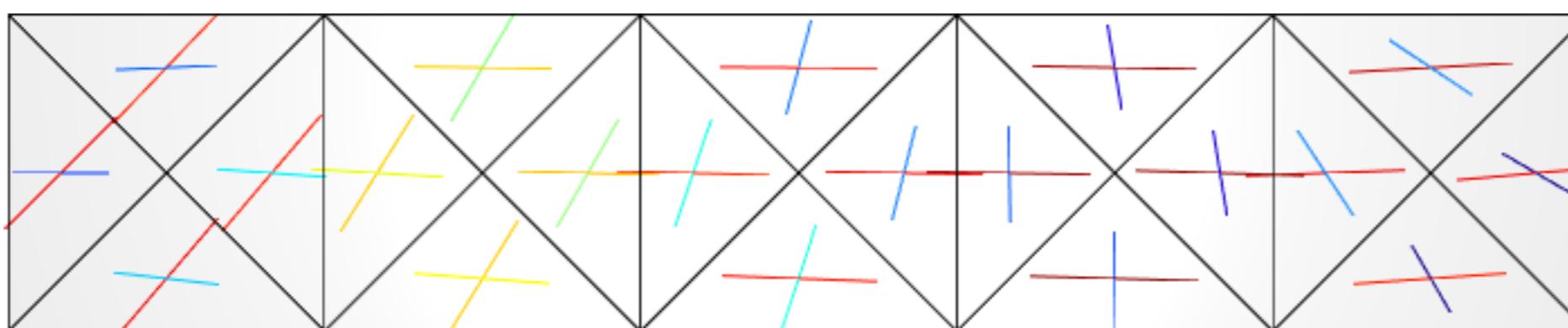
# $2^2$ -VECTOR FIELD



[Panozzo et al. 2014]



[Diamanti et al. 2014]



[Diamanti et al. 2015]