DIRECTIONAL FIELDS

SYNTHESIS, DESIGN, AND PROCESSING

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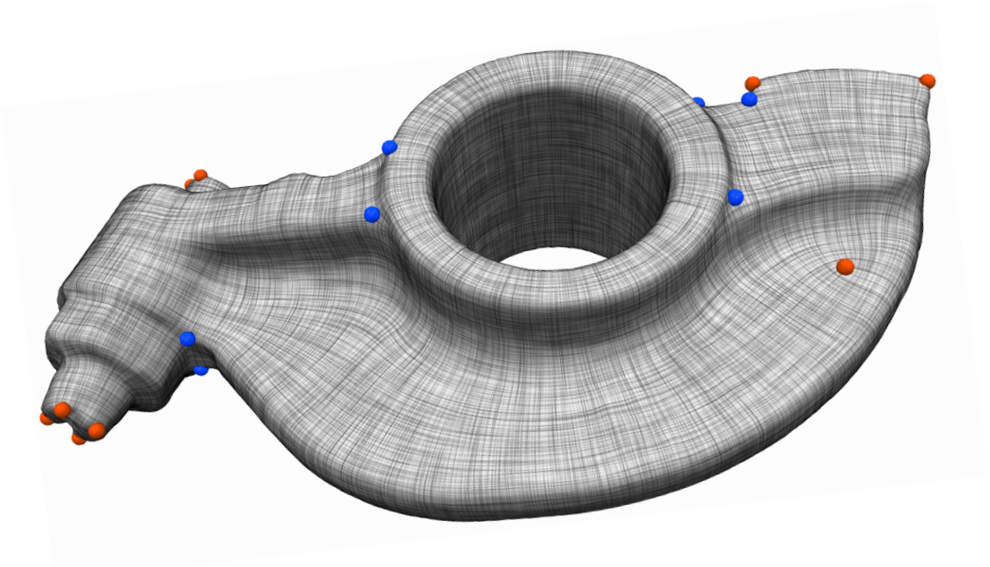
Olga Diamanti — *Stanford University*

Daniele Panozzo — *New York University*
David Bommes — *RWTH Aachen University*
Klaus Hildebrandt — *TU Delft*
Mirela Ben-Chen — *Technion*



OBJECTIVES

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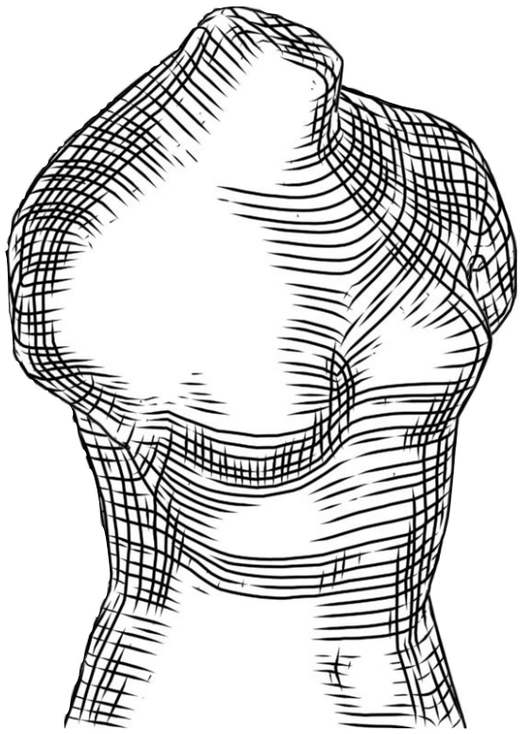
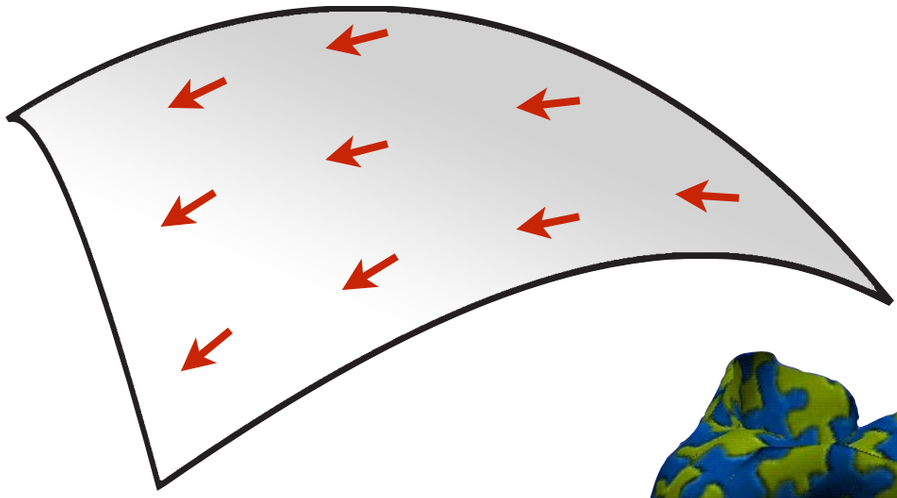
OBJECTIVES – FIELD FAIRNESS

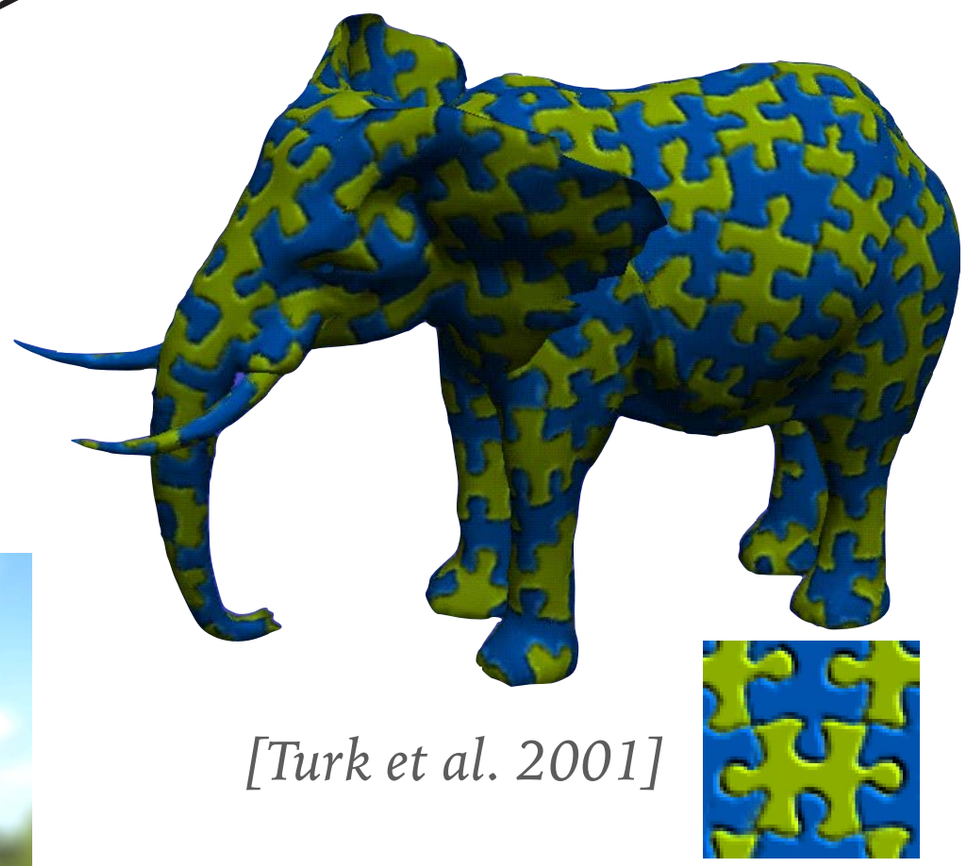
- “As-parallel-as-possible”



[Palacios et al. 2007]



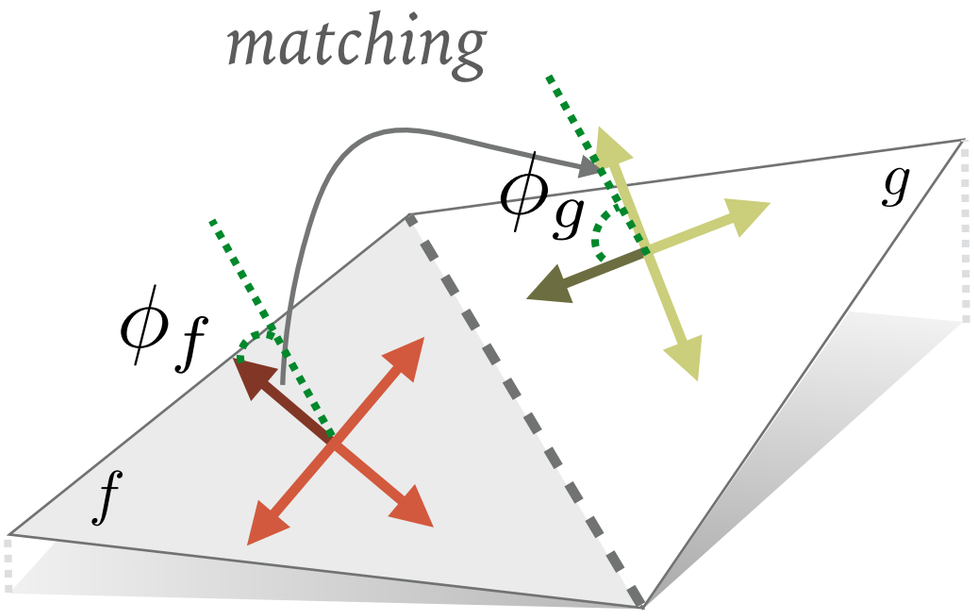
[Ray et al. 2006]
[Knöppel et al. 2015]

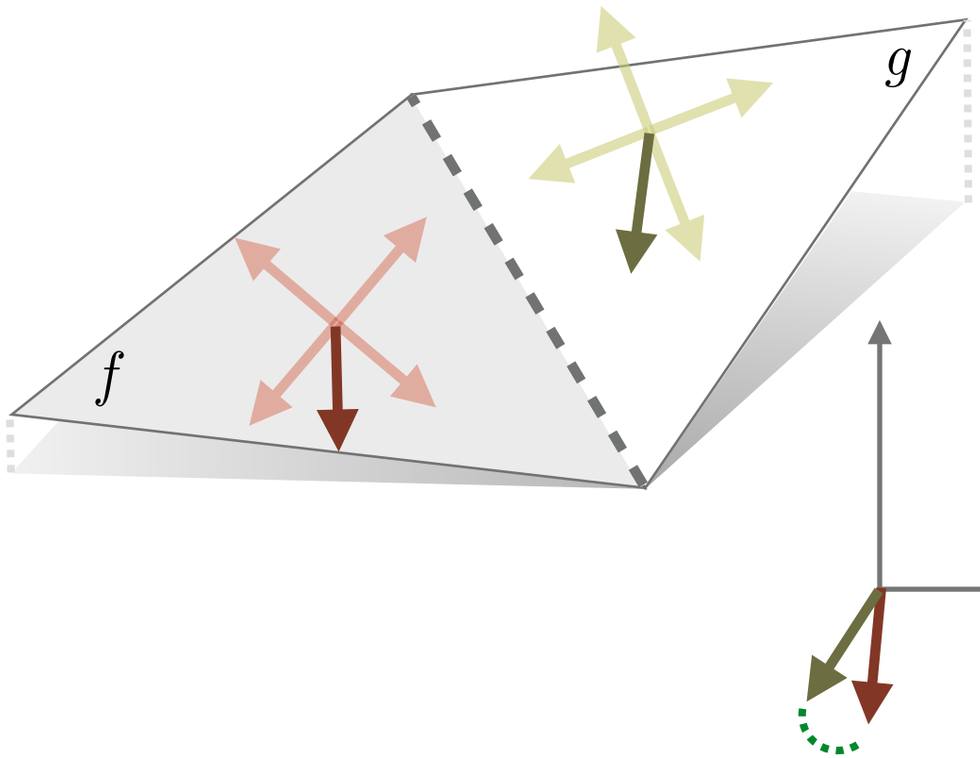


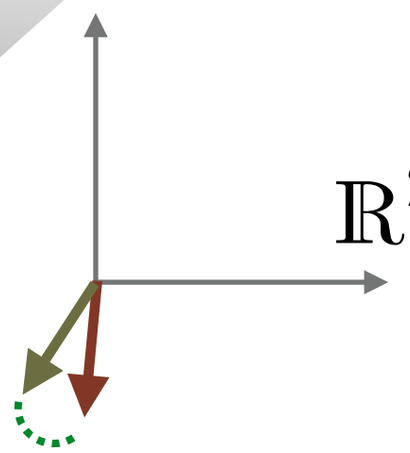
[Turk et al. 2001]

OBJECTIVES – FIELD FAIRNESS

- "As-parallel-as-possible"







angle-based approaches

compare matching angles

[Hertzmann et al. 2000] *[Crane et al. 2010]*
[Ray et al. 2008] *[Ray et al. 2009]*
[Bommes et al. 2009] *[Jakob et al. 2015]*

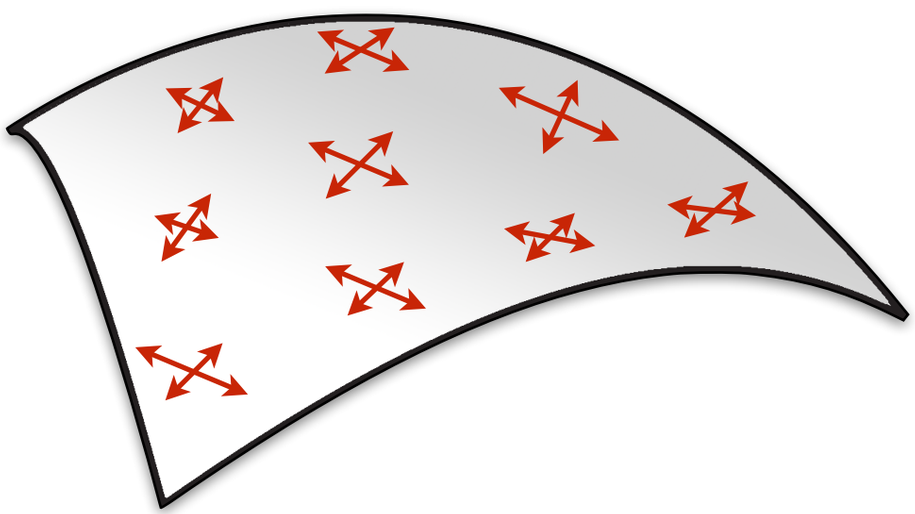
cartesian/complex approaches

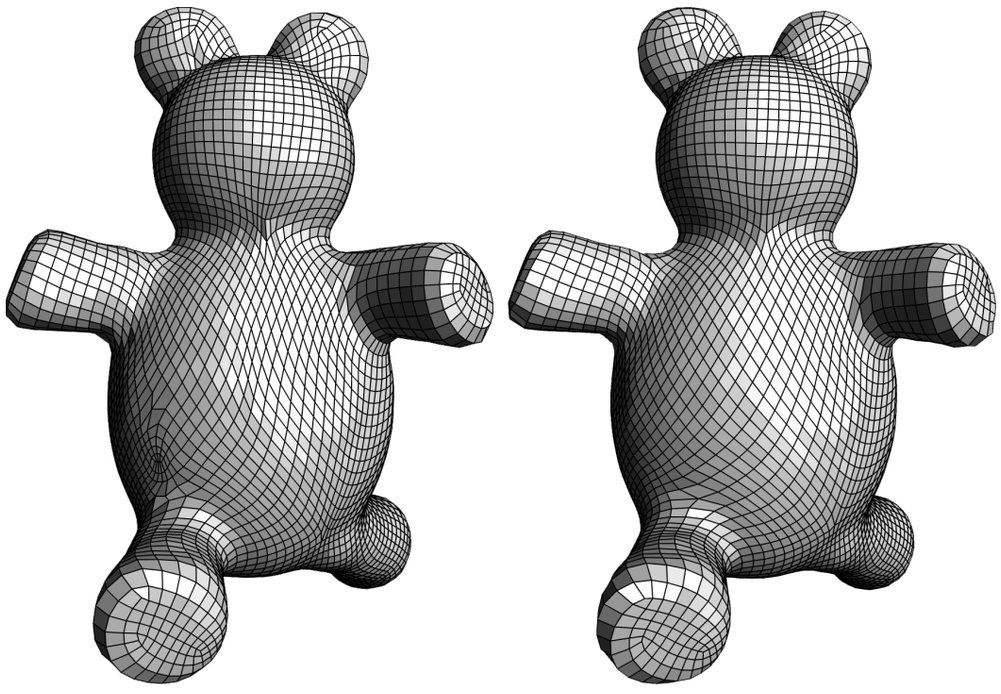
compare representative vectors

[Ray et al. 2006] *[Knöppel et al. 2013]*
[Palacios et al. 2007] *[Diamanti et al. 2014]*

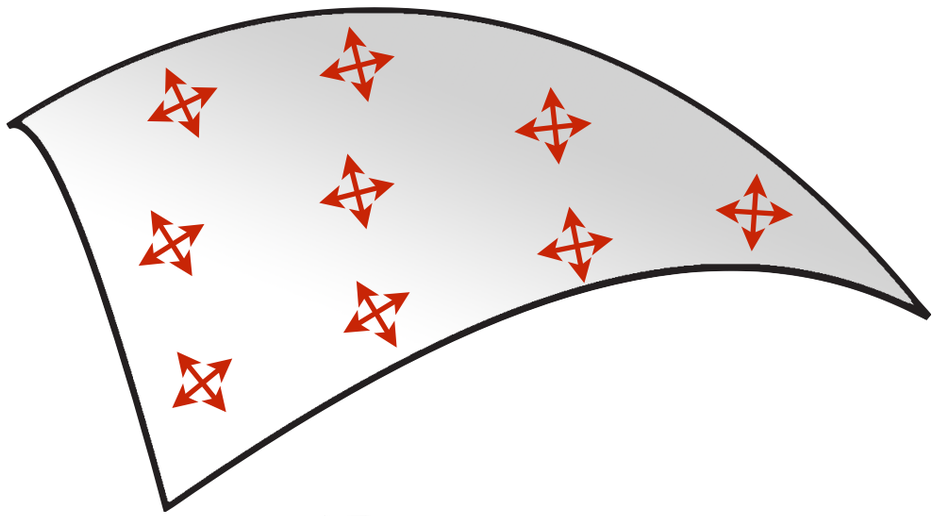
OBJECTIVES – FIELD FAIRNESS

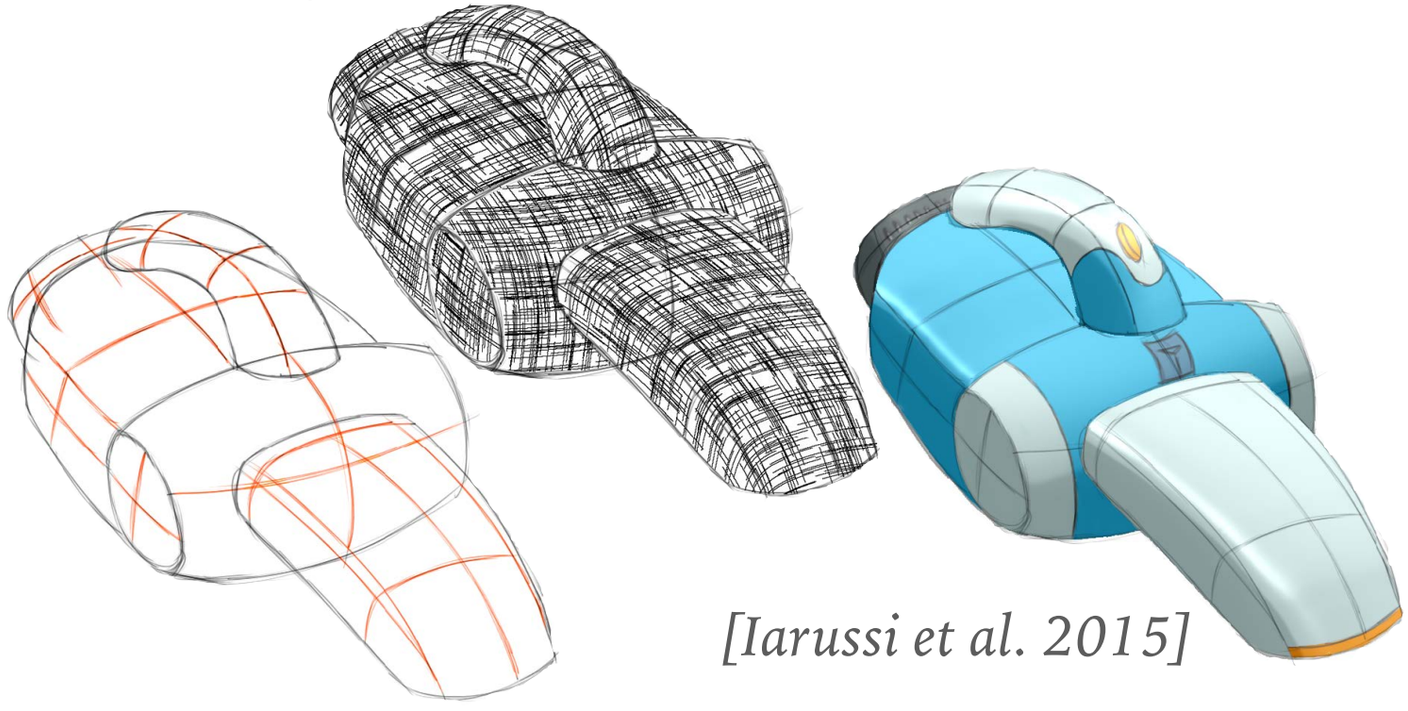
- Orthogonality





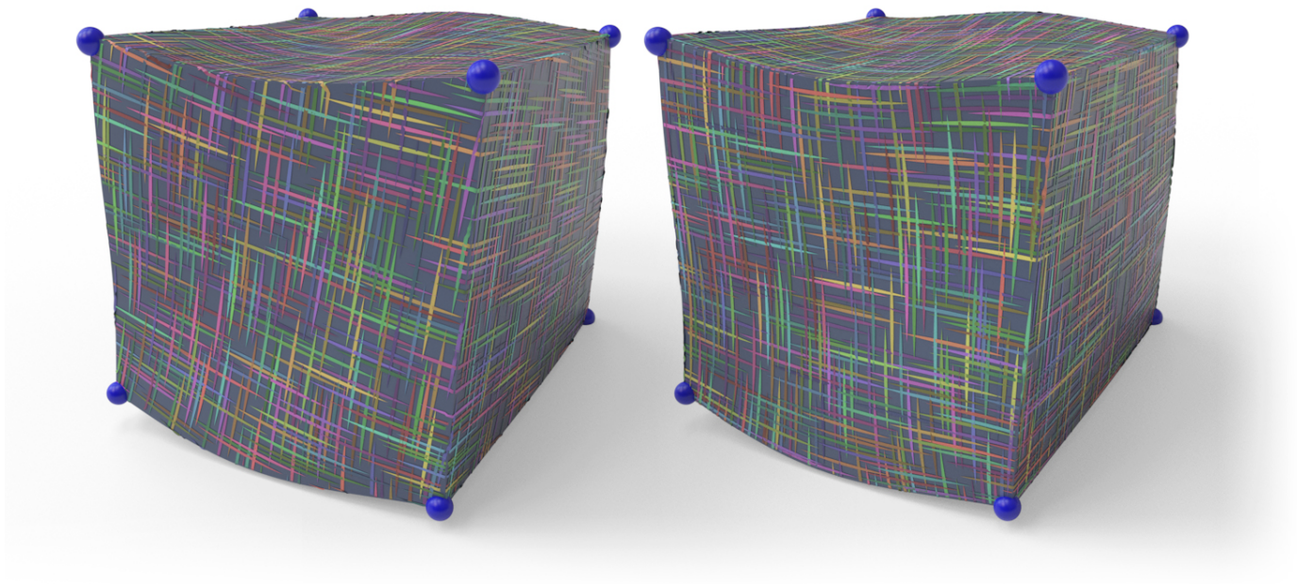
[Diamanti et al. 2014]



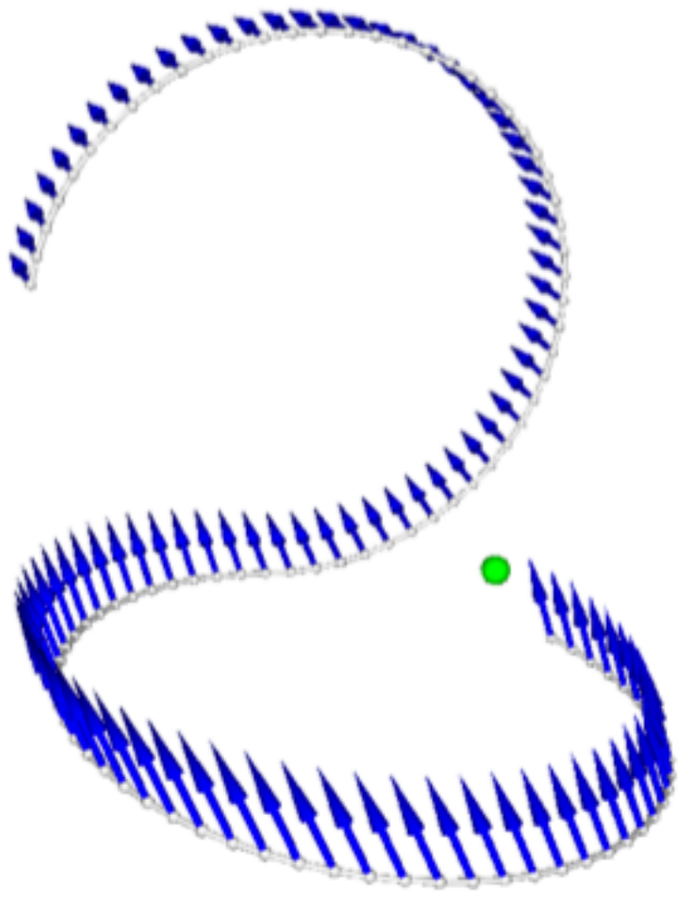


[Iarussi et al. 2015]

OBJECTIVES – ALIGNMENT TO FEATURES



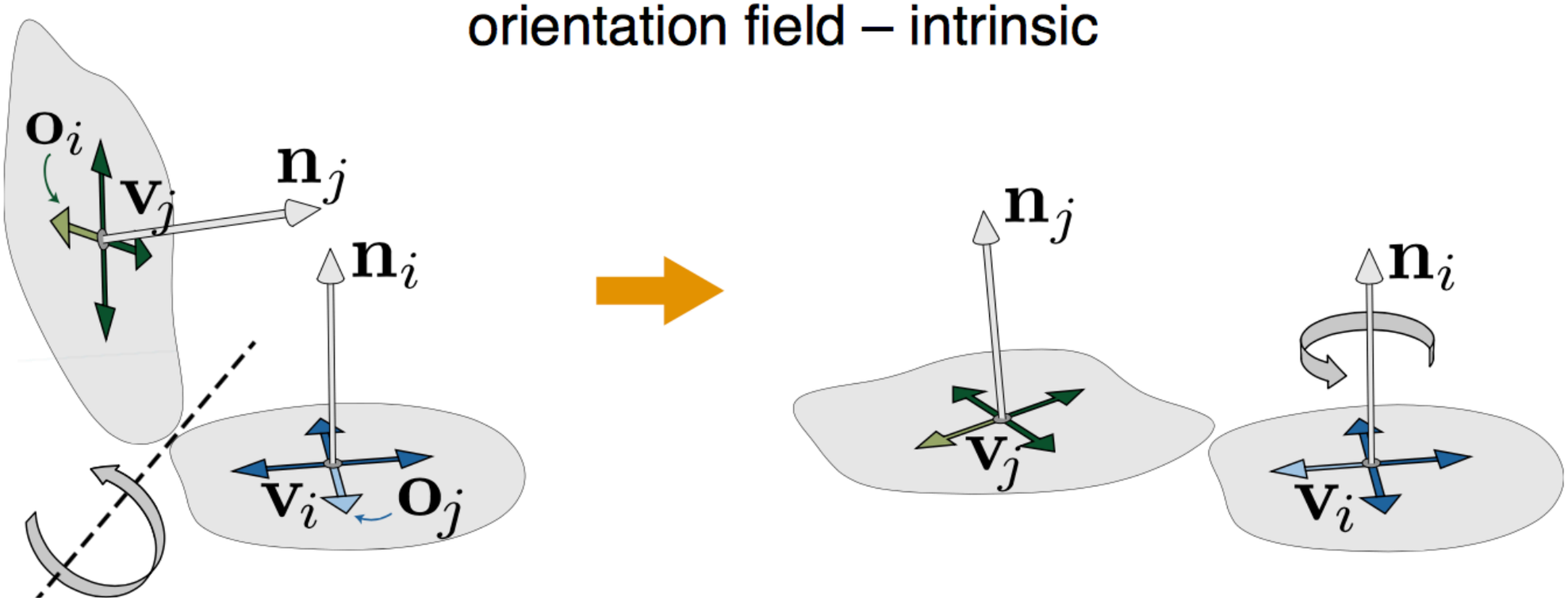
[Jakob et al. 2015]



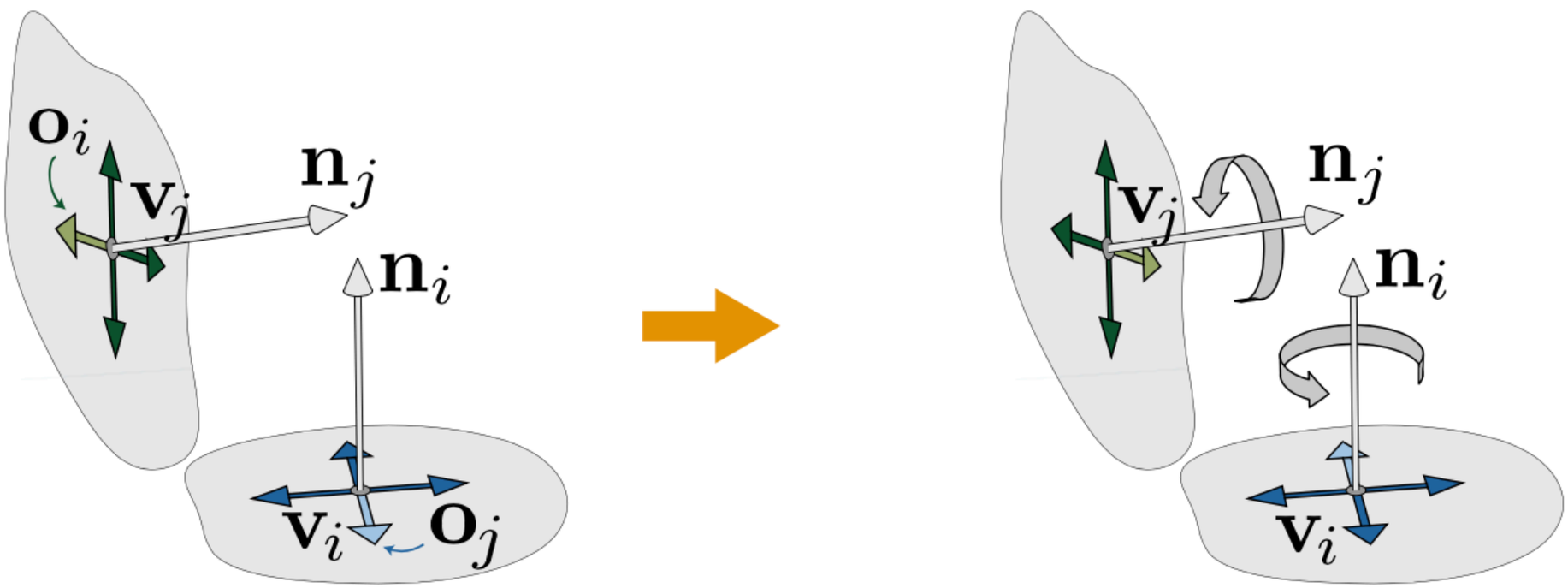
[Huang et al. 2016]

OBJECTIVES – ALIGNMENT TO FEATURES

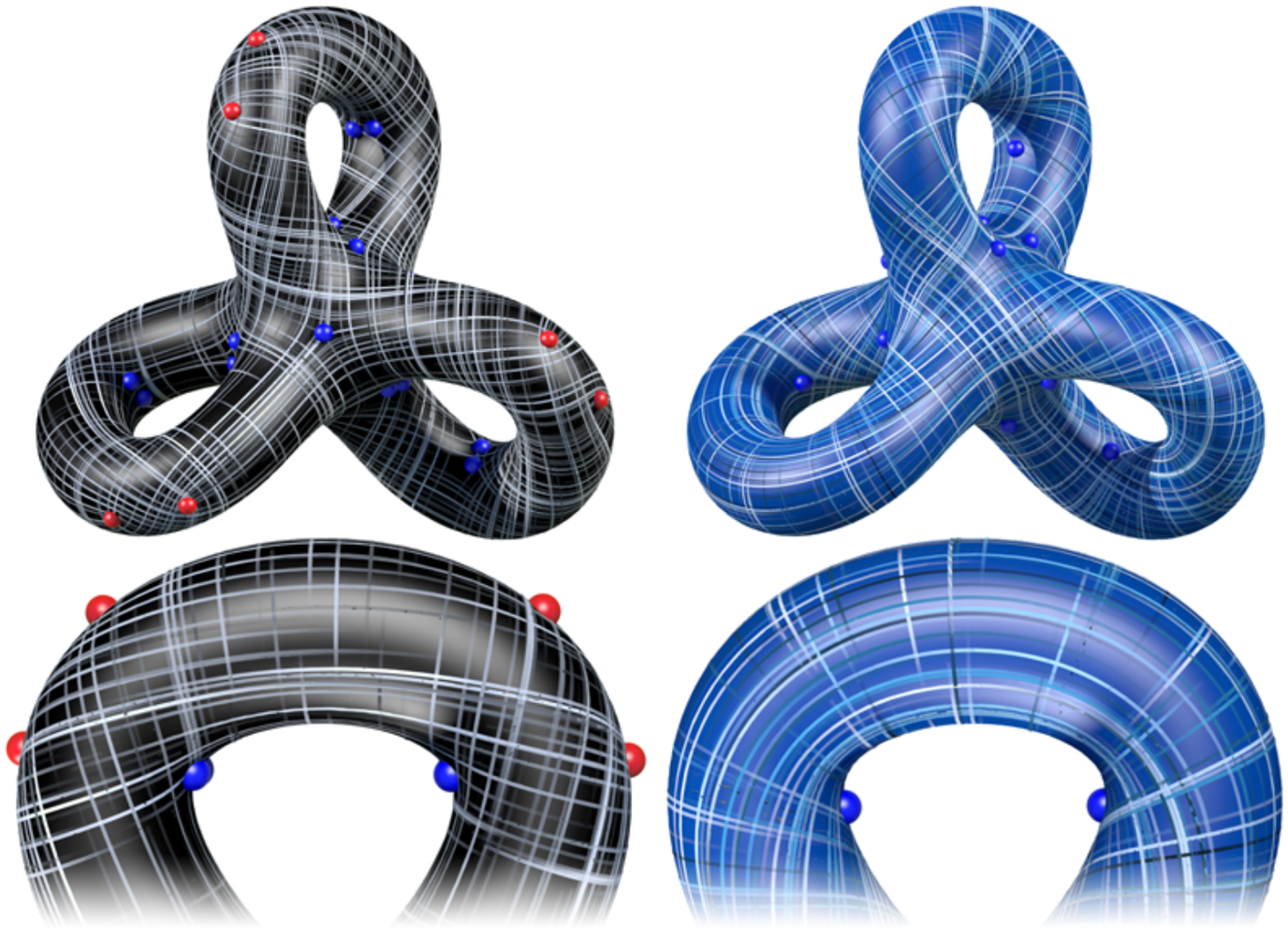
orientation field – intrinsic



orientation field – extrinsic

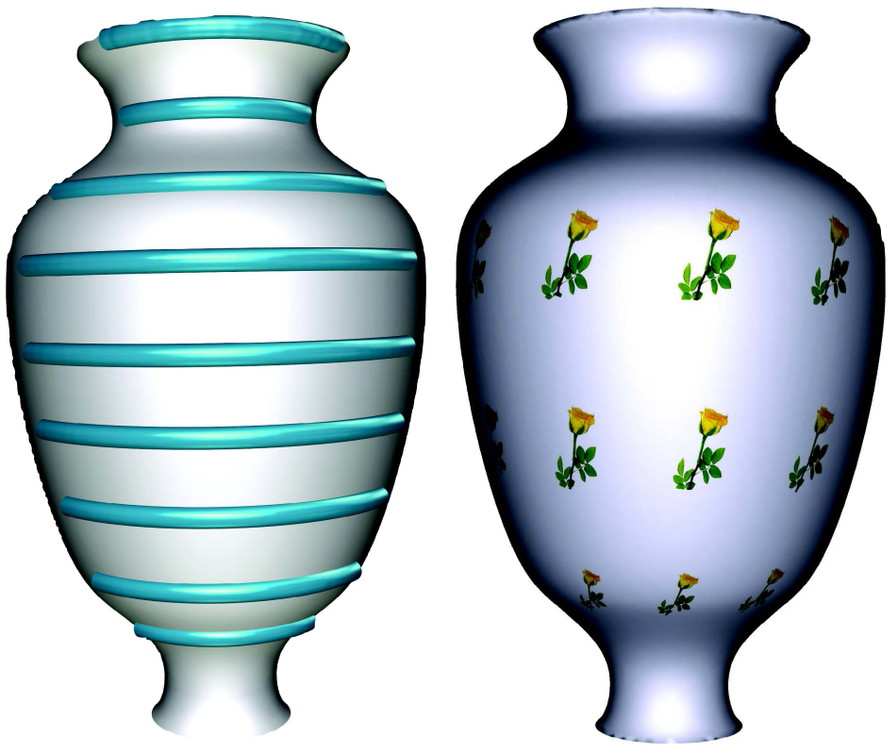


OBJECTIVES – SINGULARITY CONTROL

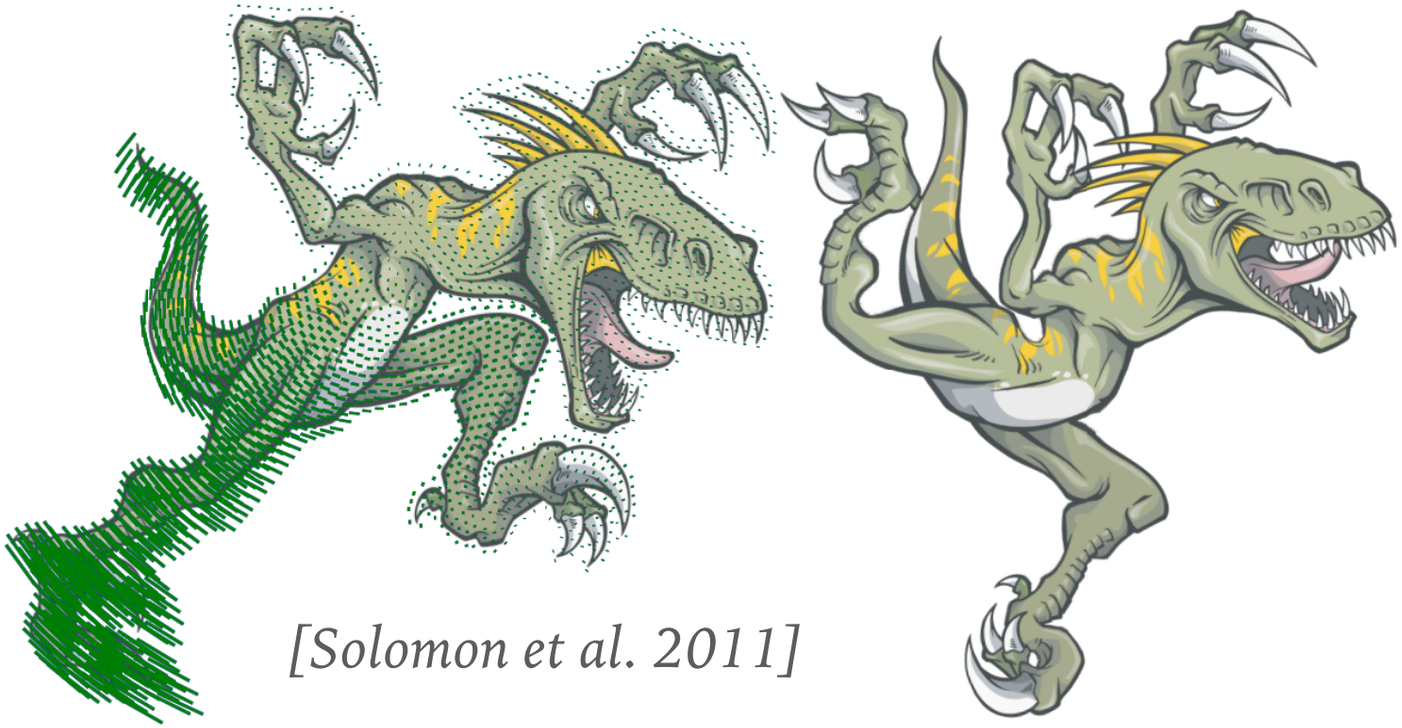


[Knöppel et al. 2013]

OBJECTIVES – ISOMETRY INDUCING

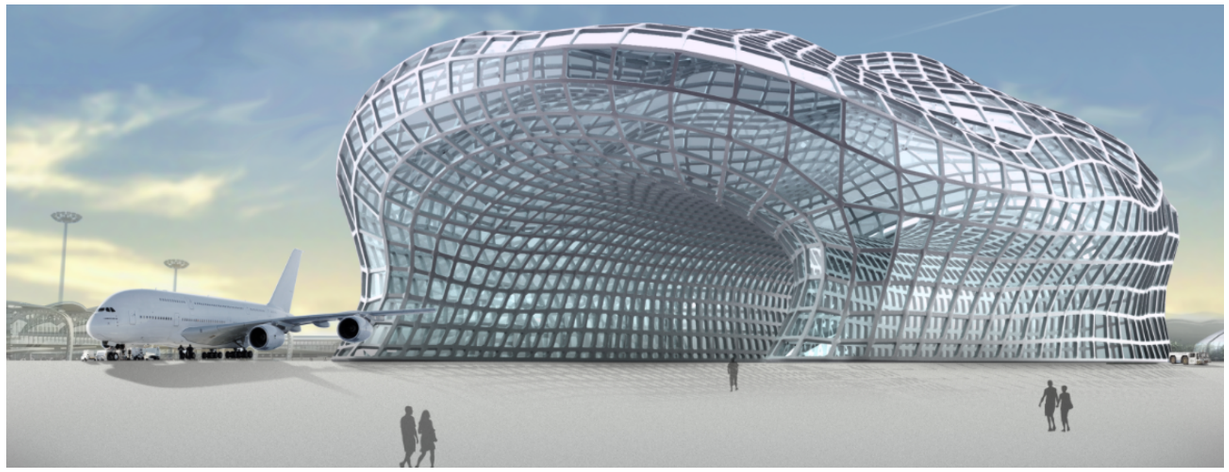


[Ben-Chen et al. 2010]

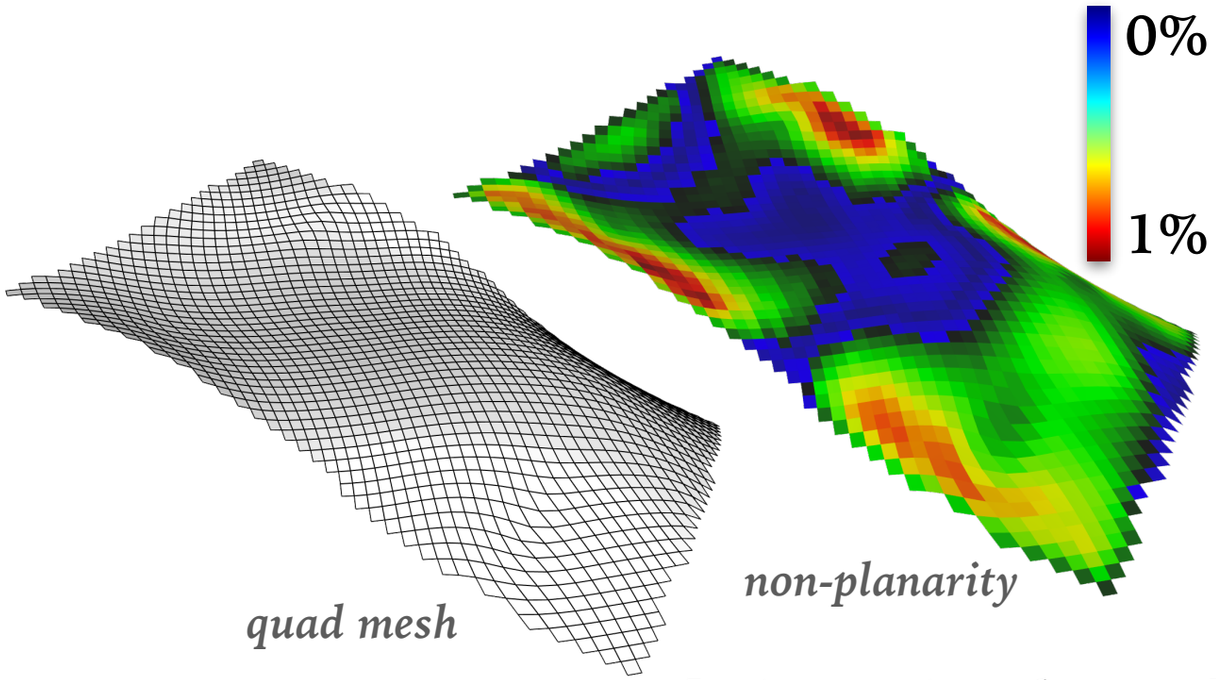


[Solomon et al. 2011]

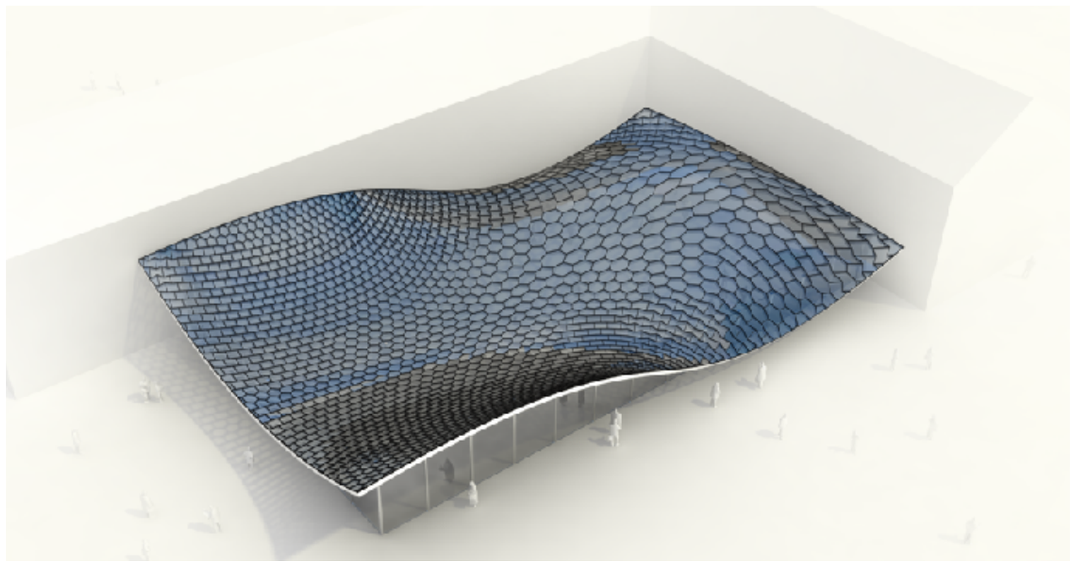
OBJECTIVES – CONJUGACY (PLANARITY)



[Liu et al. 2011]



[Diamanti et al. 2014]



[Vaxman et al. 2015]

CONSTRAINTS

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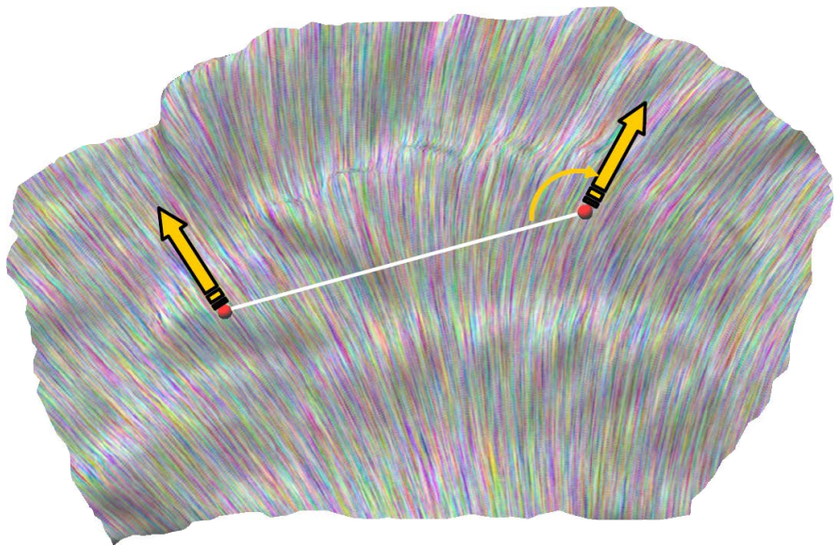
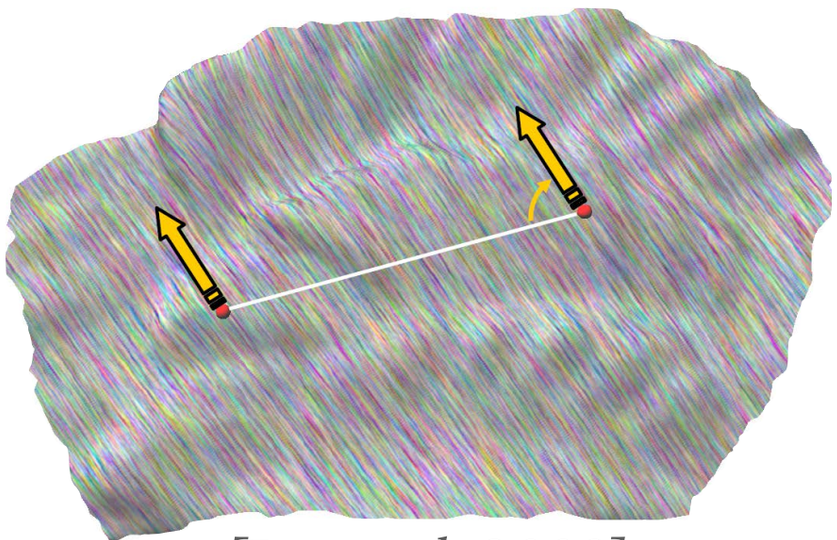
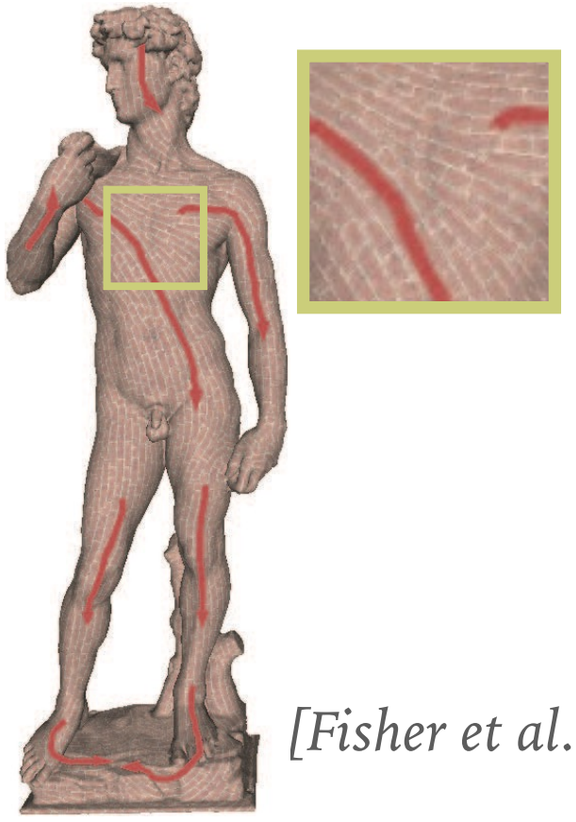
CONSTRAINTS – ALIGNMENT





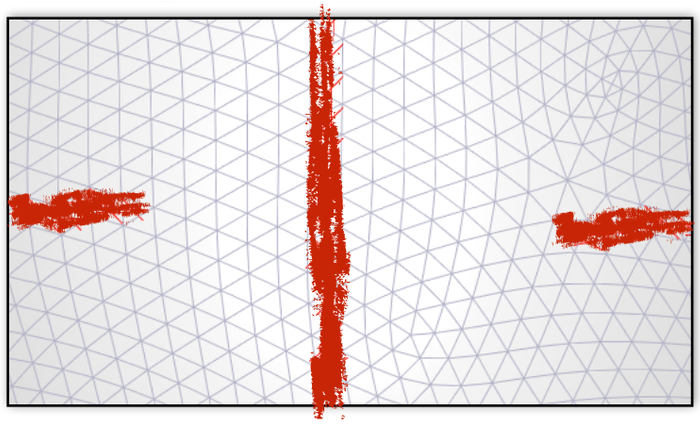
[Ray et al. 2008]

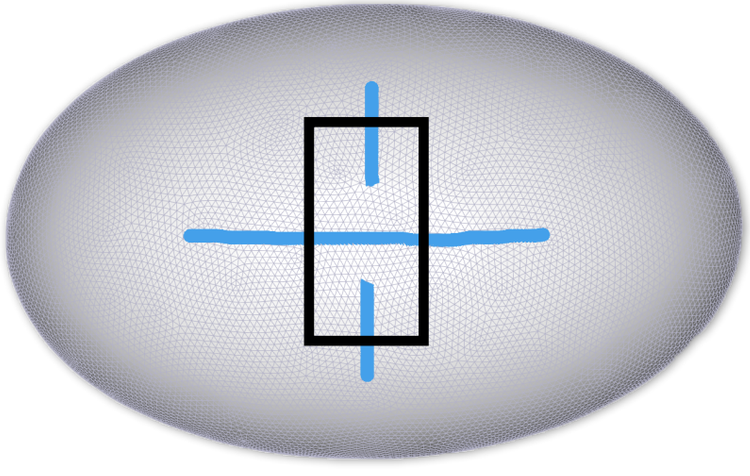


[Fisher et al. 2007]

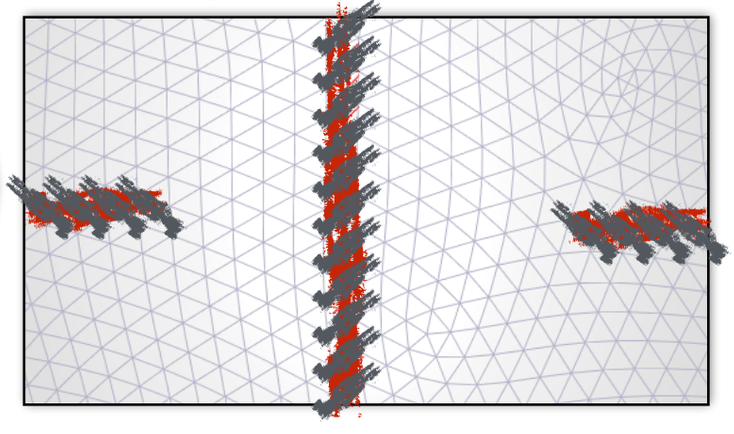
- Complete or Partial

partial constraints

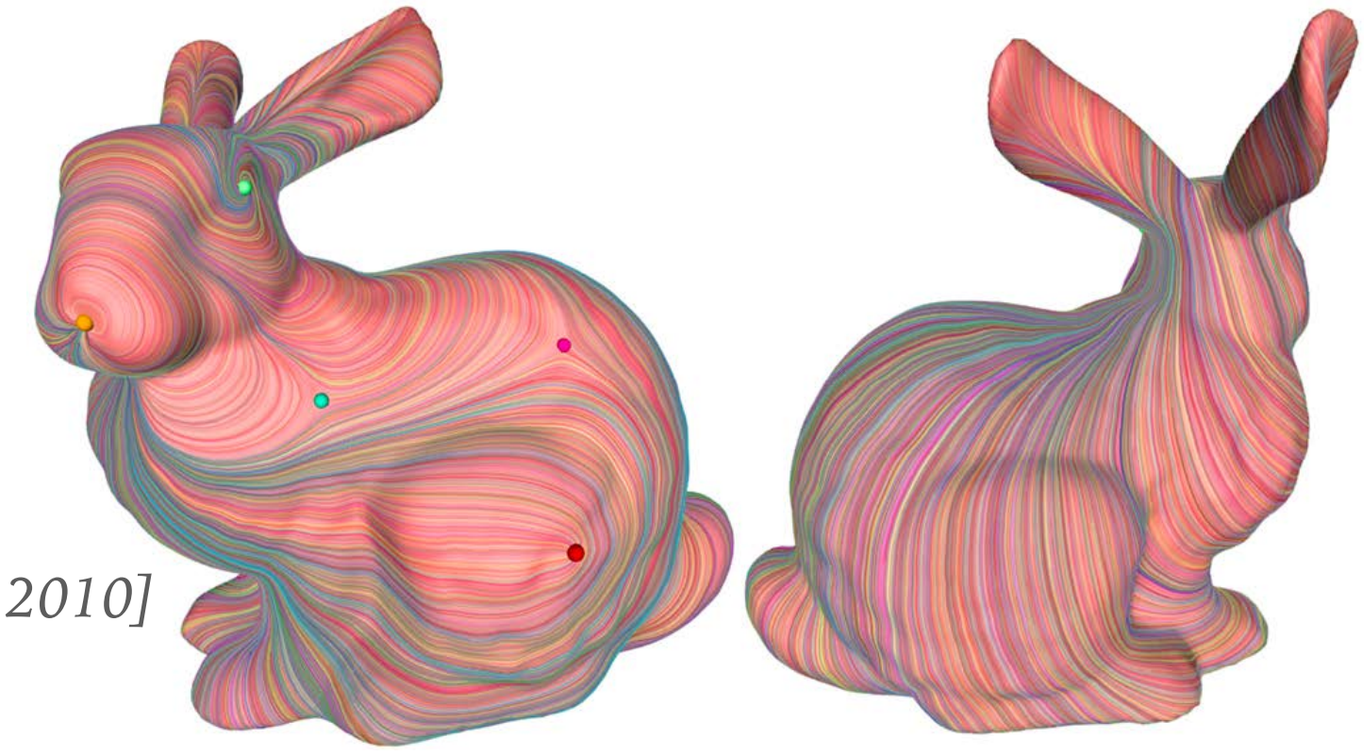




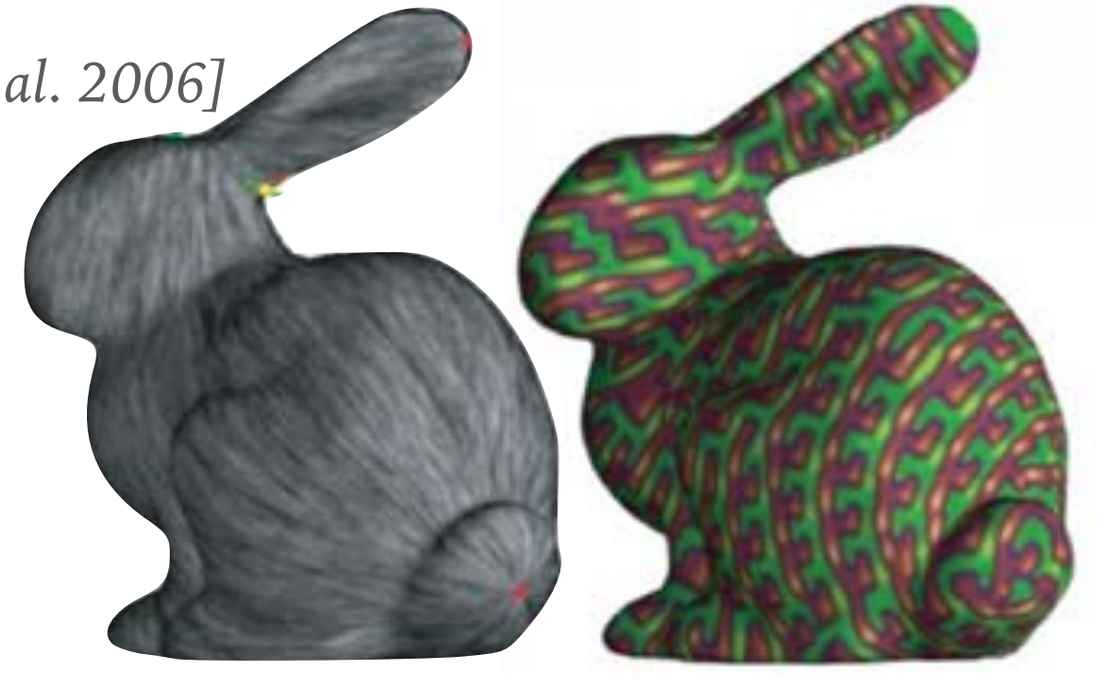
full constraints



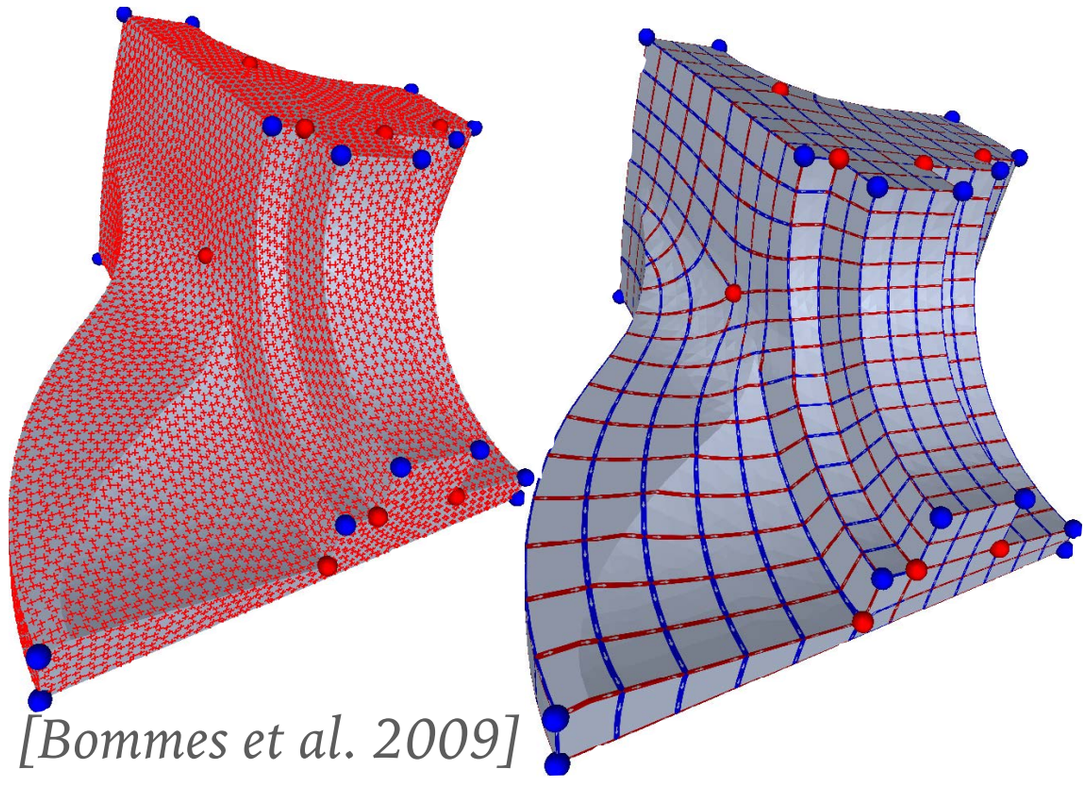
CONSTRAINTS – TOPOLOGY



[Crane et al. 2010]

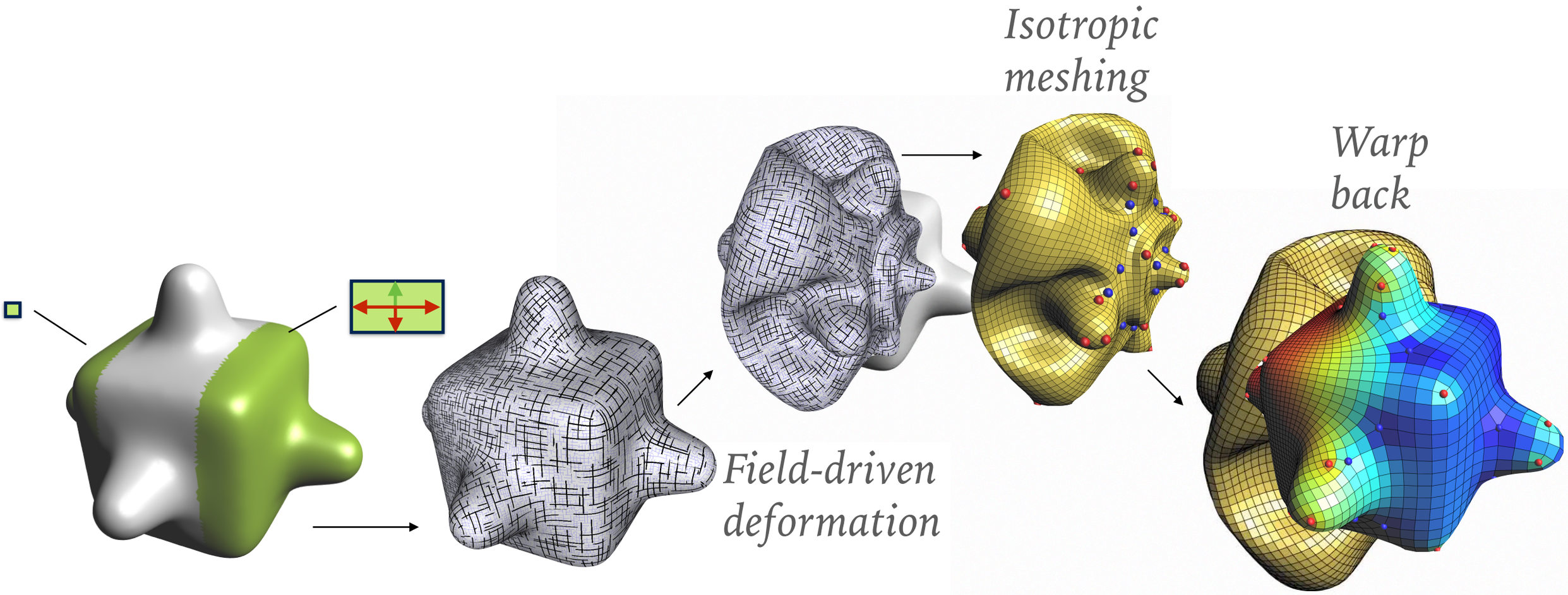


[Zhang et al. 2006]



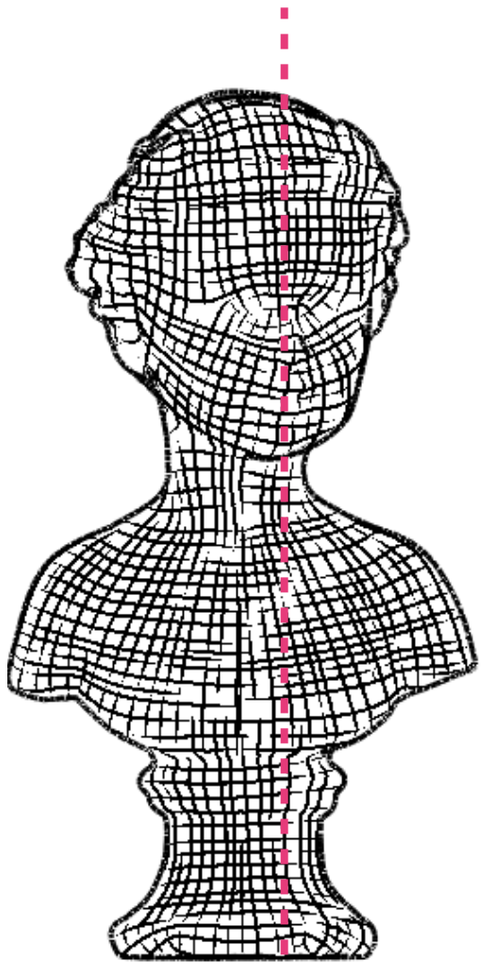
[Bommes et al. 2009]

CONSTRAINTS – SCALE

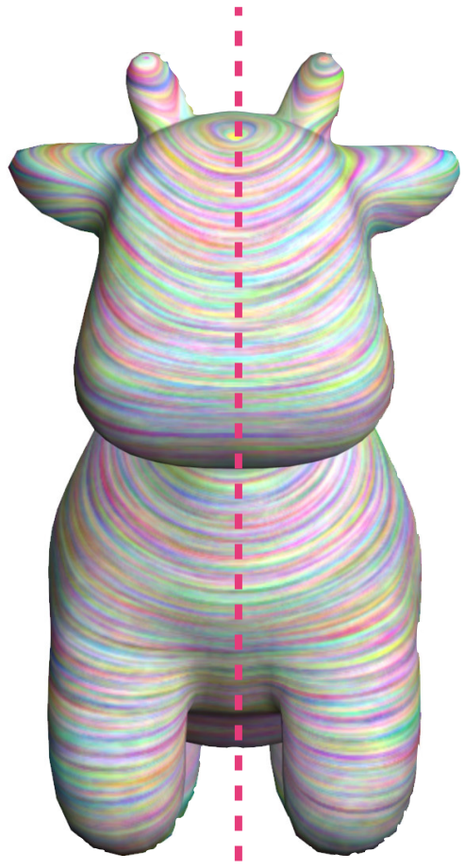


[Panozzo et al. 2014]

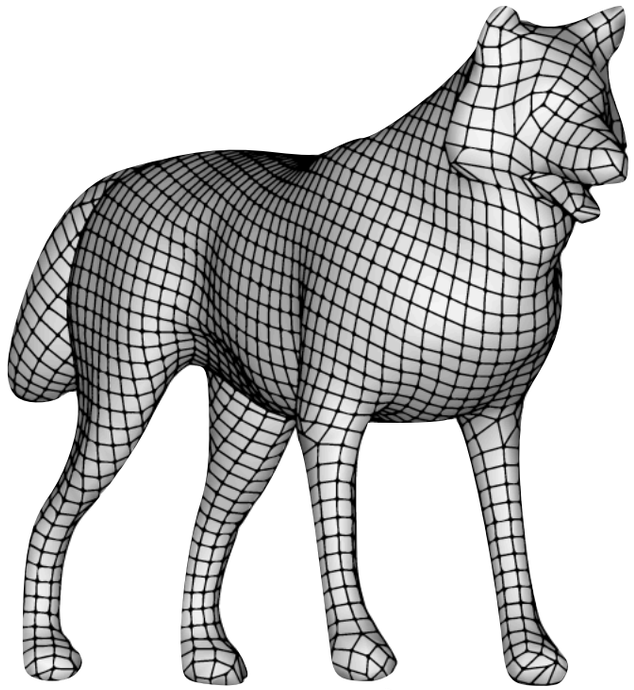
CONSTRAINTS – SYMMETRY

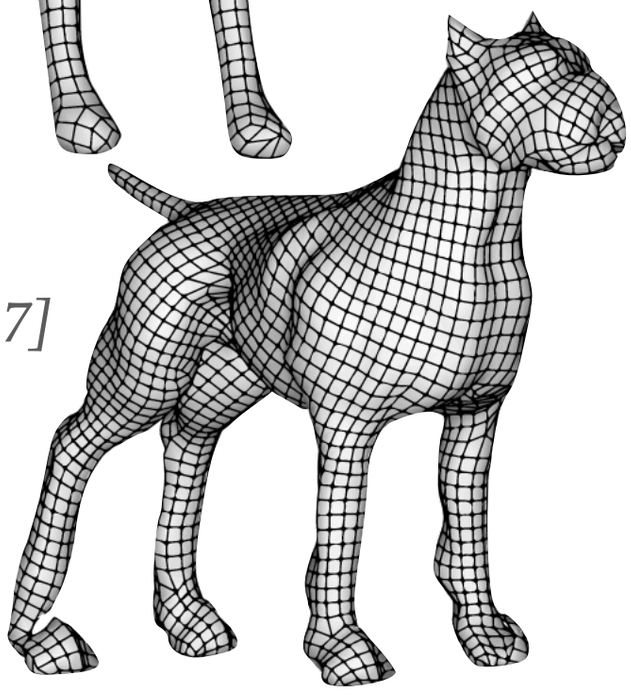


[Panozzo et al. 2012]



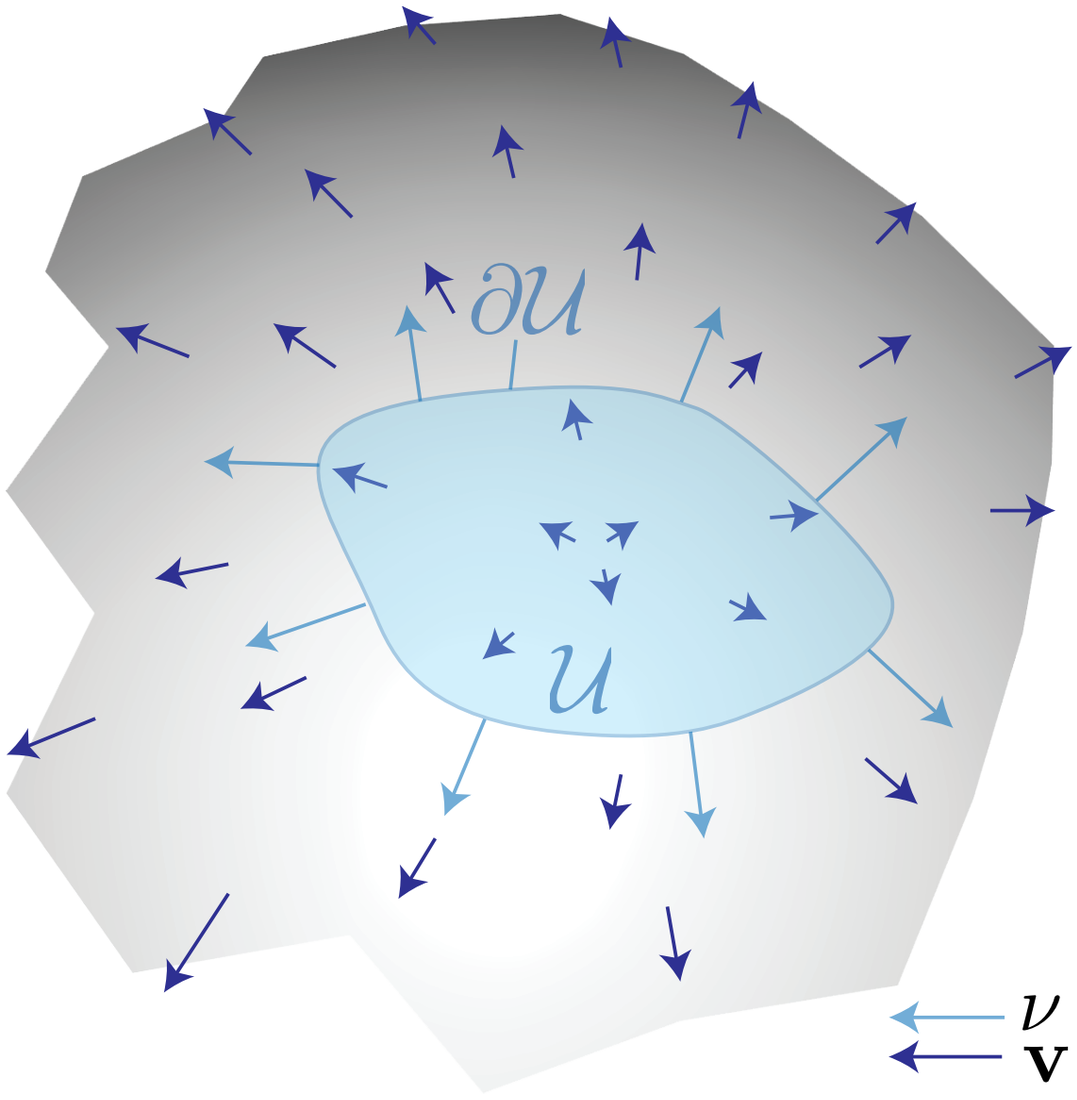
[Azencot et al. 2013]

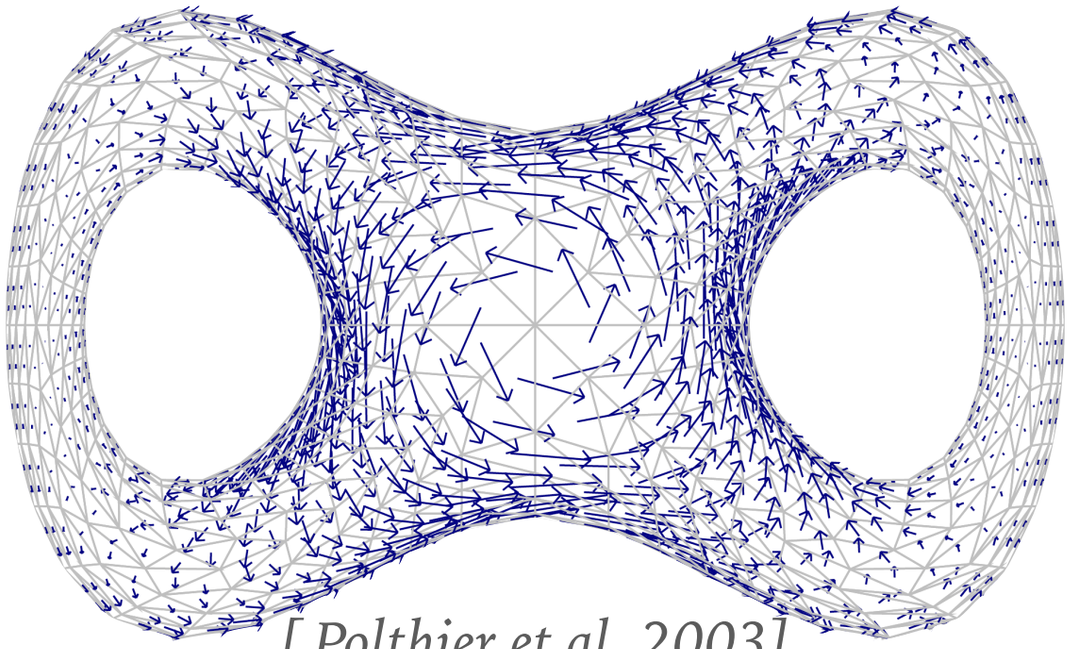




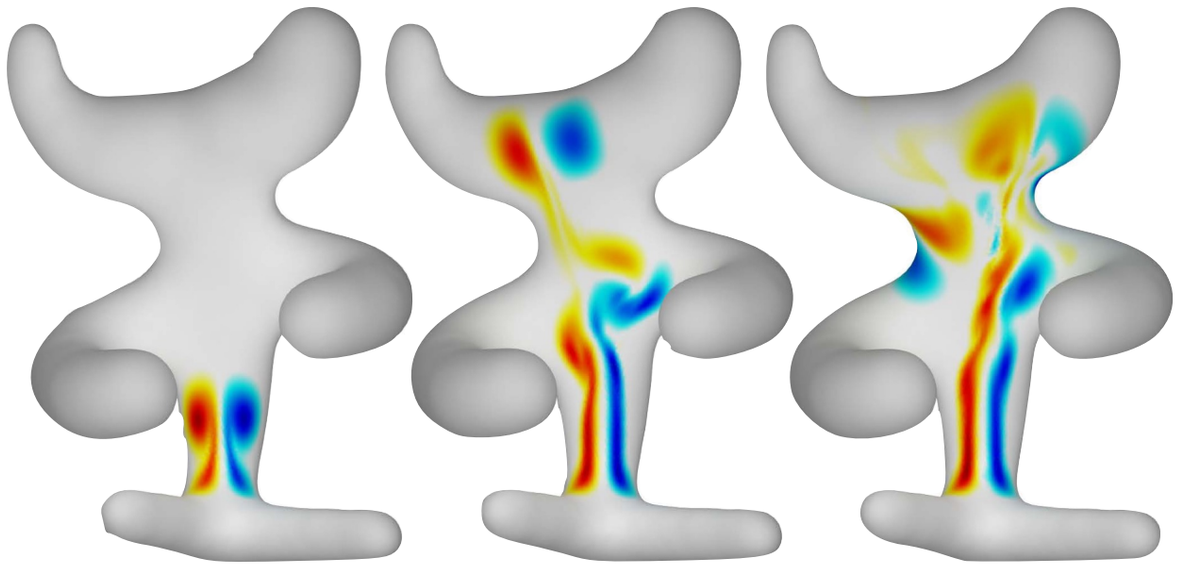
[Azencot et al. 2017]

CONSTRAINTS – DIFFERENTIAL – DIVERGENCE



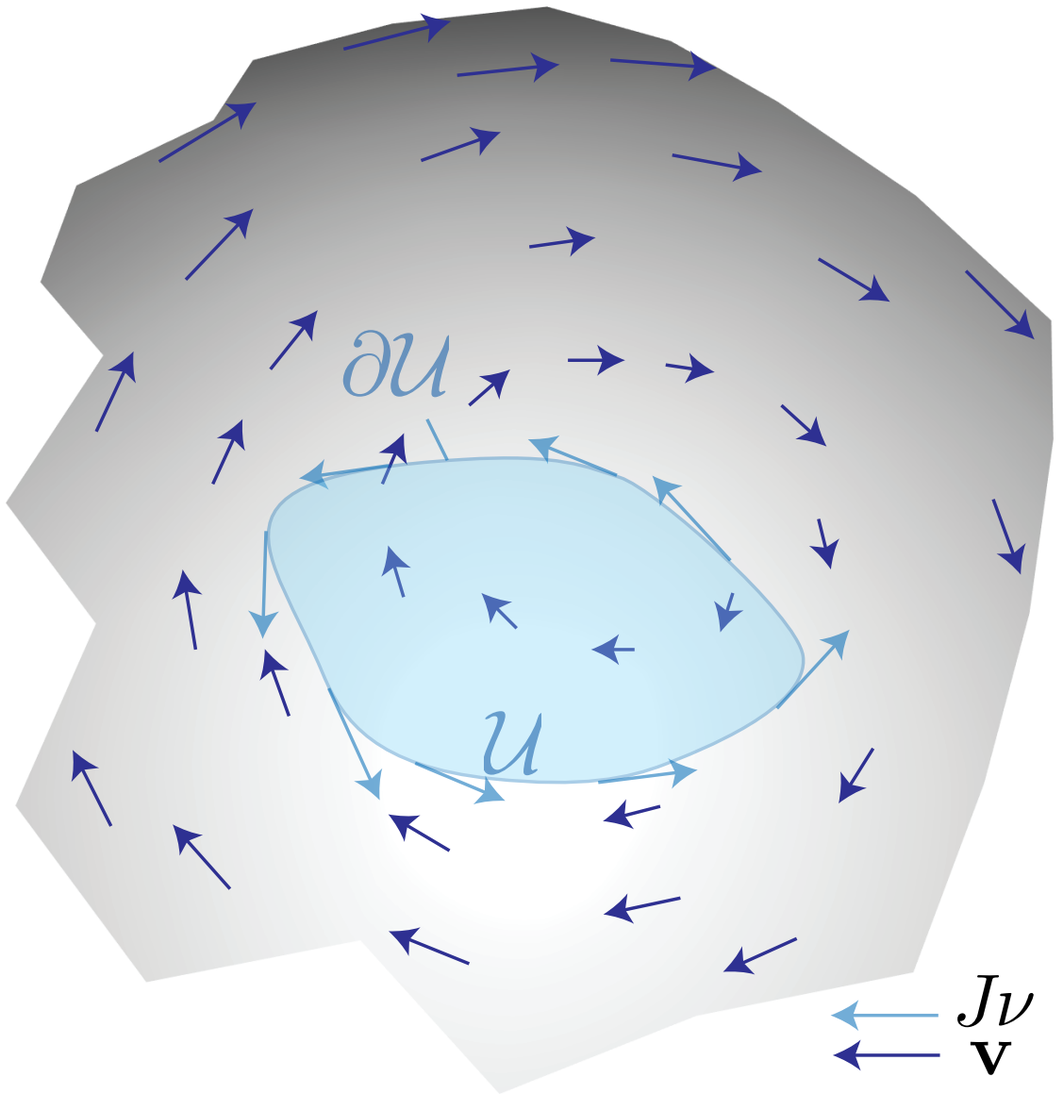


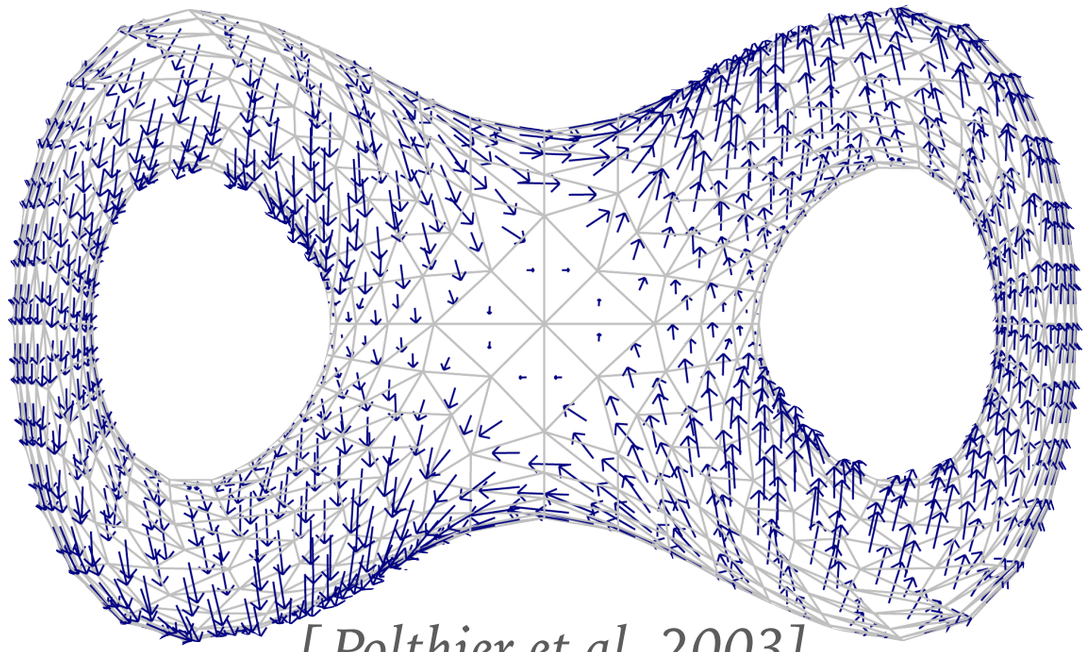
[Polthier et al. 2003]



[Azencot et al. 2014]

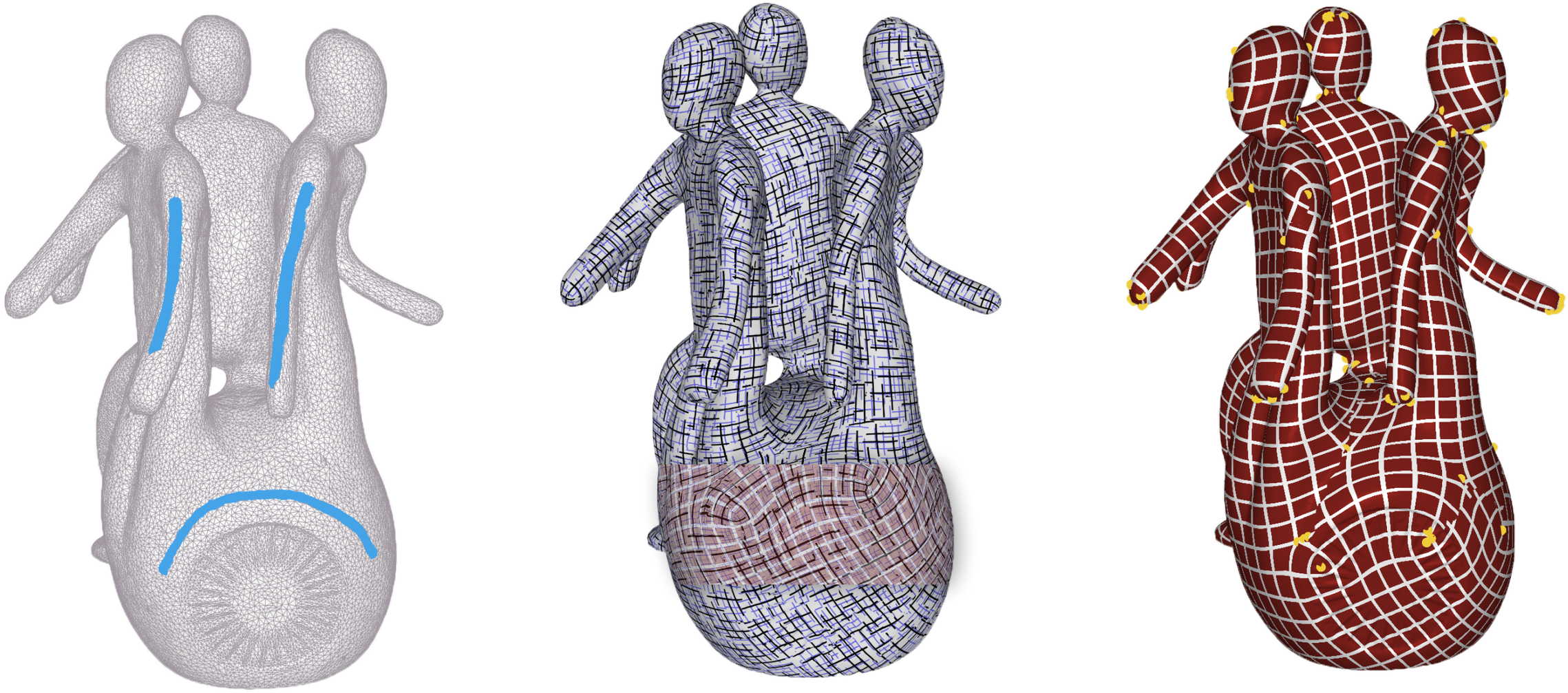
CONSTRAINTS – DIFFERENTIAL – CURL





[Polthier et al. 2003]

CONSTRAINTS – DIFFERENTIAL – CURL



[Diamanti et al. 2015]

HOW TO CHOOSE THE RIGHT METHOD?

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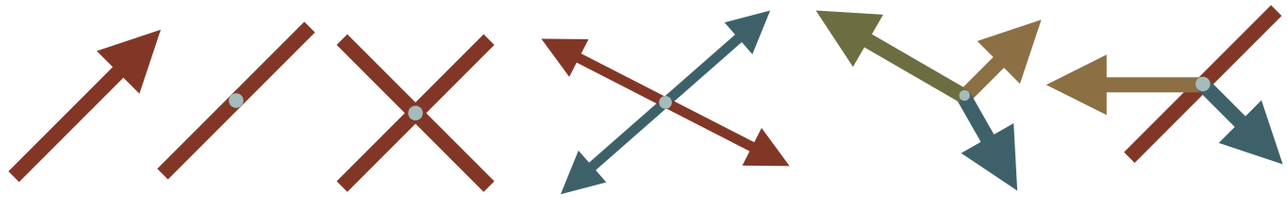
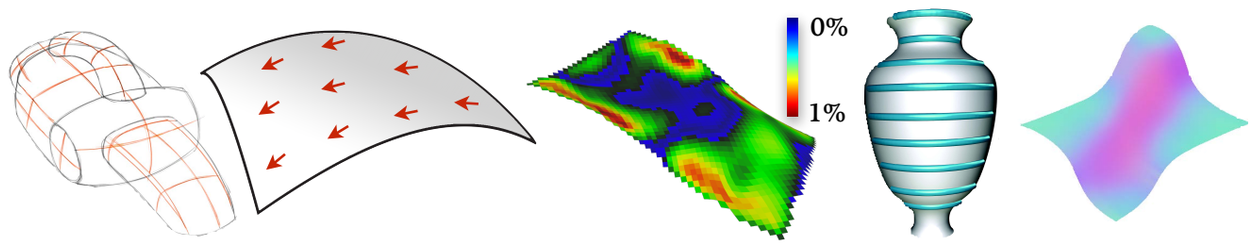
CHOOSING THE RIGHT METHOD

- Choose the type of object



- Determine what the ideal field is (objective)

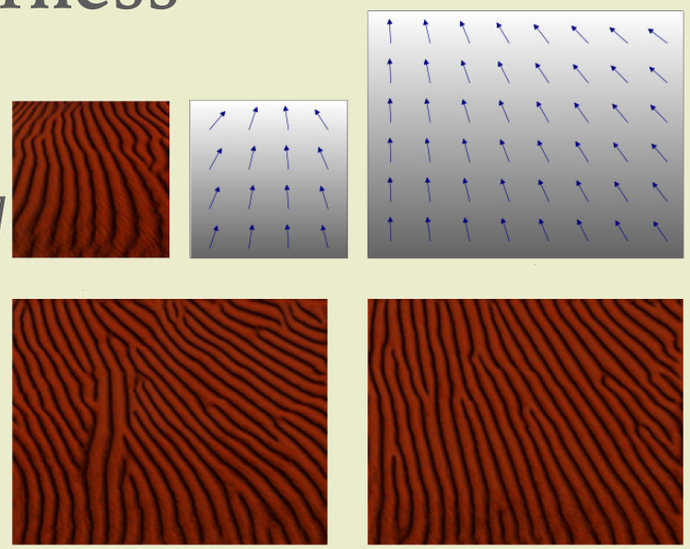


- Consider the types of guarantees required
- Consider design strategy in terms of efficiency, convergence,…
  - this affects the choice of representation
- Consider discretization preference

CASE STUDY: VECTOR FIELDS

Objective: Fairness

[Pedersen et al. 1995]
[Turk et al. 2001]
[Praun et al. 2000]

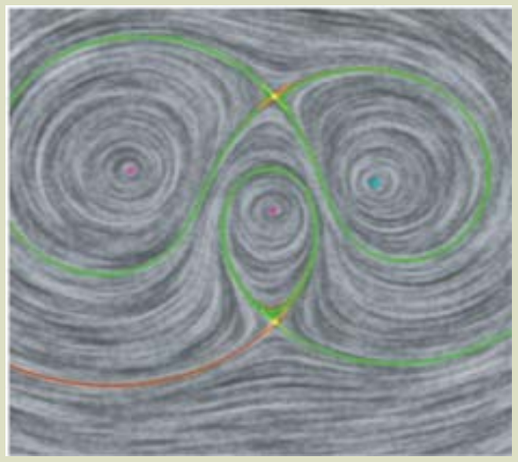


Objective: Other

Isometries - Killing
[Ben-Chen et al. 2010] [Azencot et al. 2013]

Curl / Divergence Control
[Fisher et al. 2007]
[Zhang et al. 2006]

Symmetries
[Azencot et al. 2013]



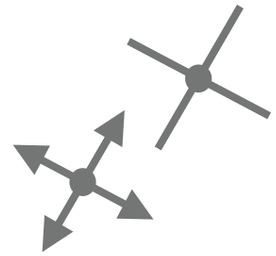
Constraints: Directional

hard
[Pedersen et al. 1995]
[Praun et al. 2000]
[Azencot et al. 2013]

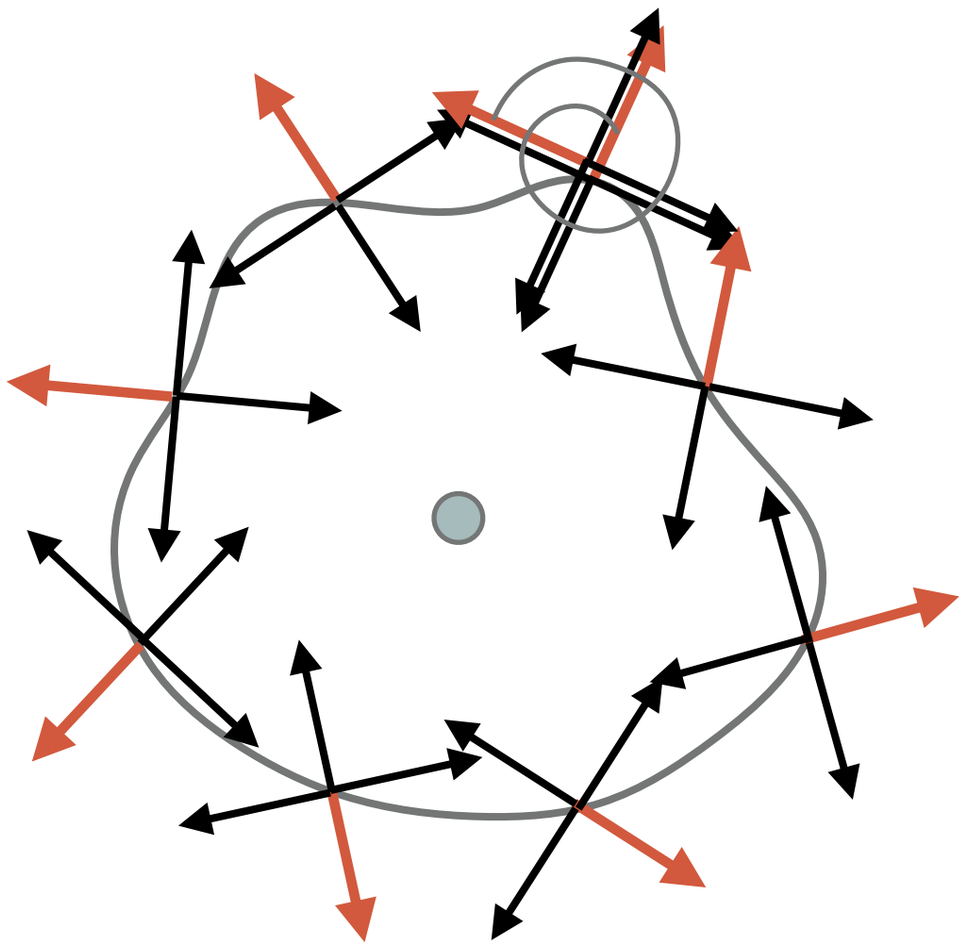
soft
[Turk et al. 2001]
[Zhang et al. 2006]

soft +singularity control
[Fisher et al. 2007]
[Zhang et al. 2006]

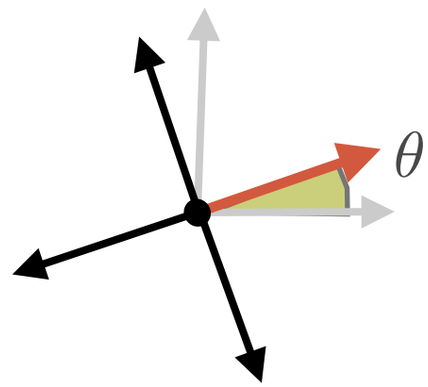
CASE STUDY: N-DIRECTIONAL FIELD, FIXED TOPOLOGY



- Objective: “As-parallel-as-possible”
- Singularity prescription straightforward with angle based representations

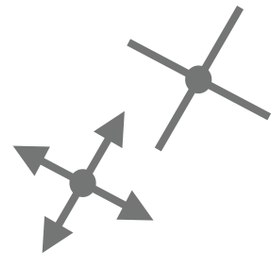


desired singularity index = 5/4

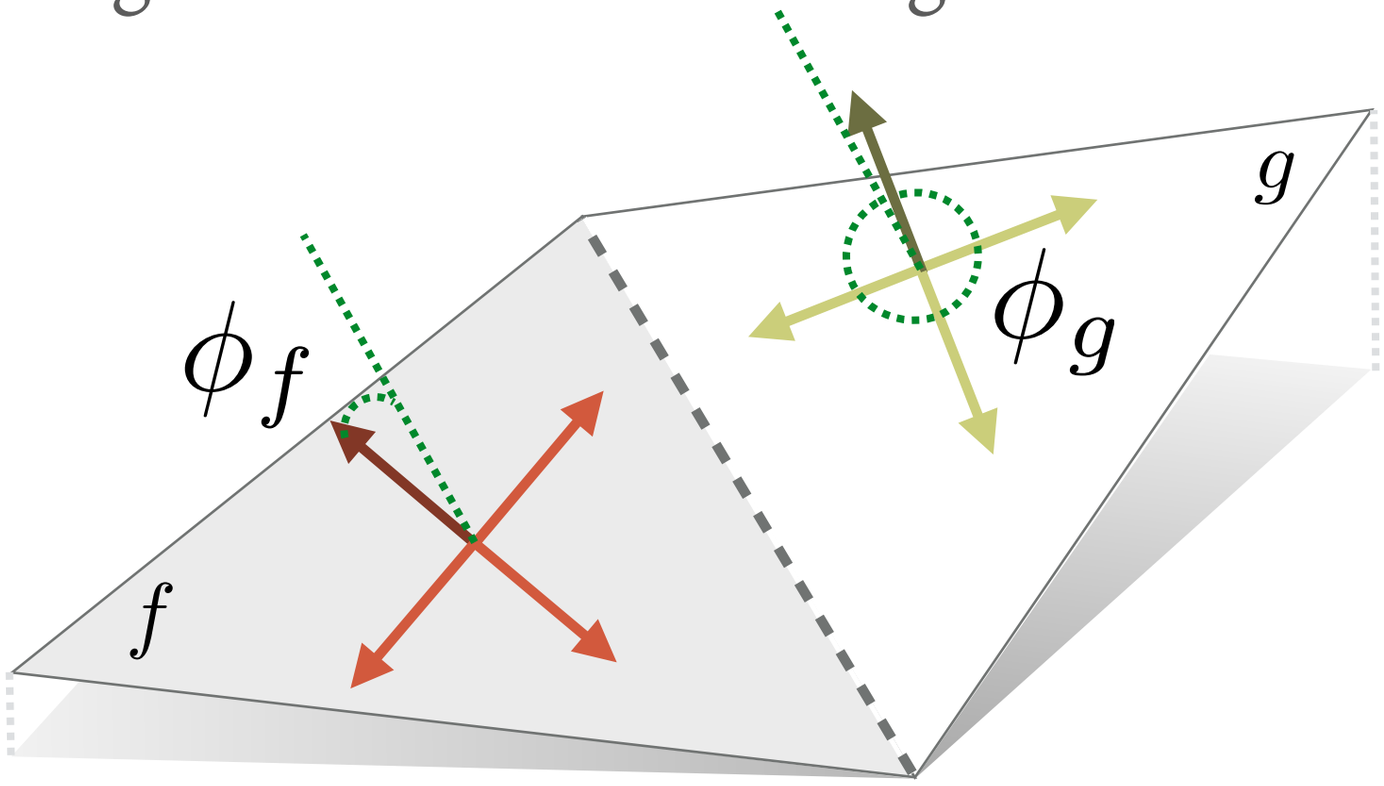


angle-based representation of a 4-direction field

CASE STUDY: N-DIRECTIONAL FIELD, FIXED TOPOLOGY

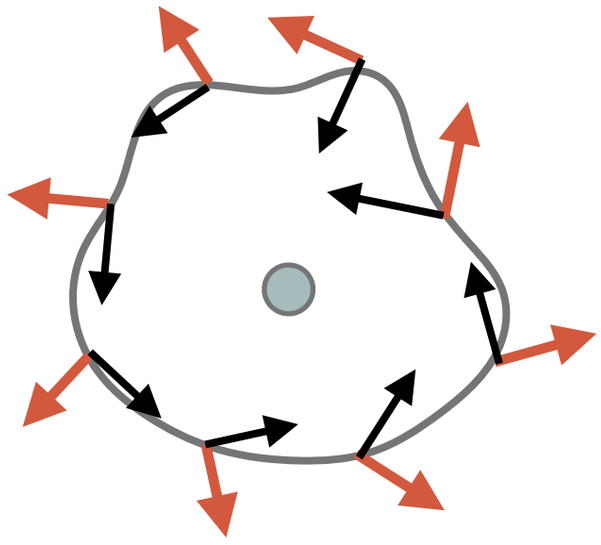


- Angle-based: solve for angles

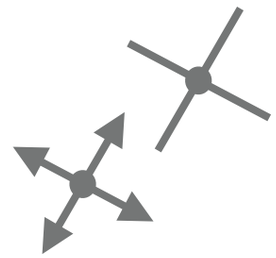


[Ray et al. 2008]
[Crane et al. 2010]

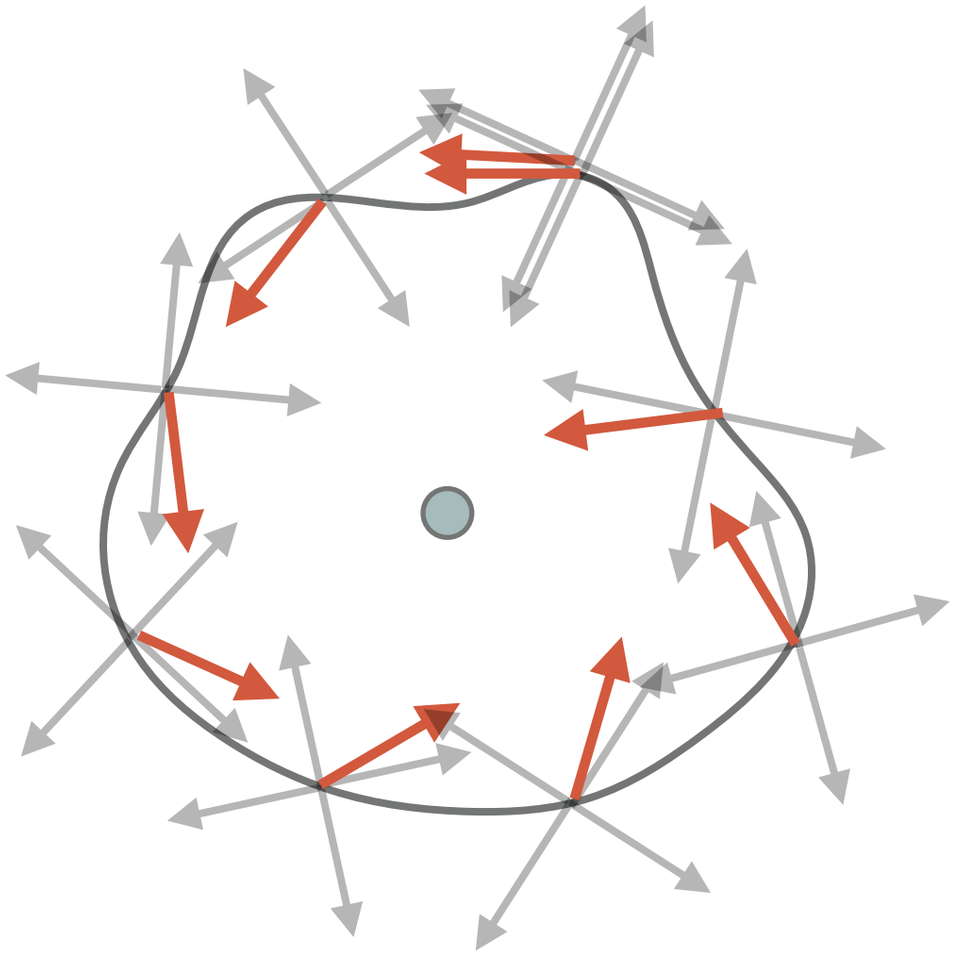
- *linear system*
- *at least one constraint needed*



CASE STUDY: N-DIRECTIONAL FIELD, FIXED TOPOLOGY

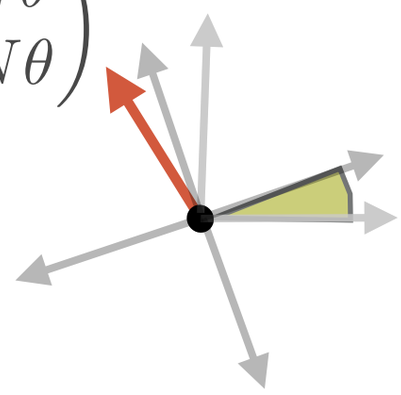


- Objective: “As-parallel-as-possible”
- Cartesian/Complex representative insensitive to 2π rotations



desired singularity index = 5/4

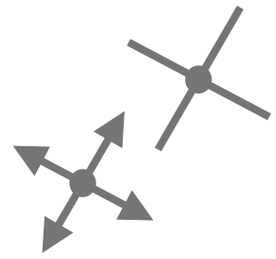
$$\begin{pmatrix} \sin N\theta \\ \cos N\theta \end{pmatrix}$$



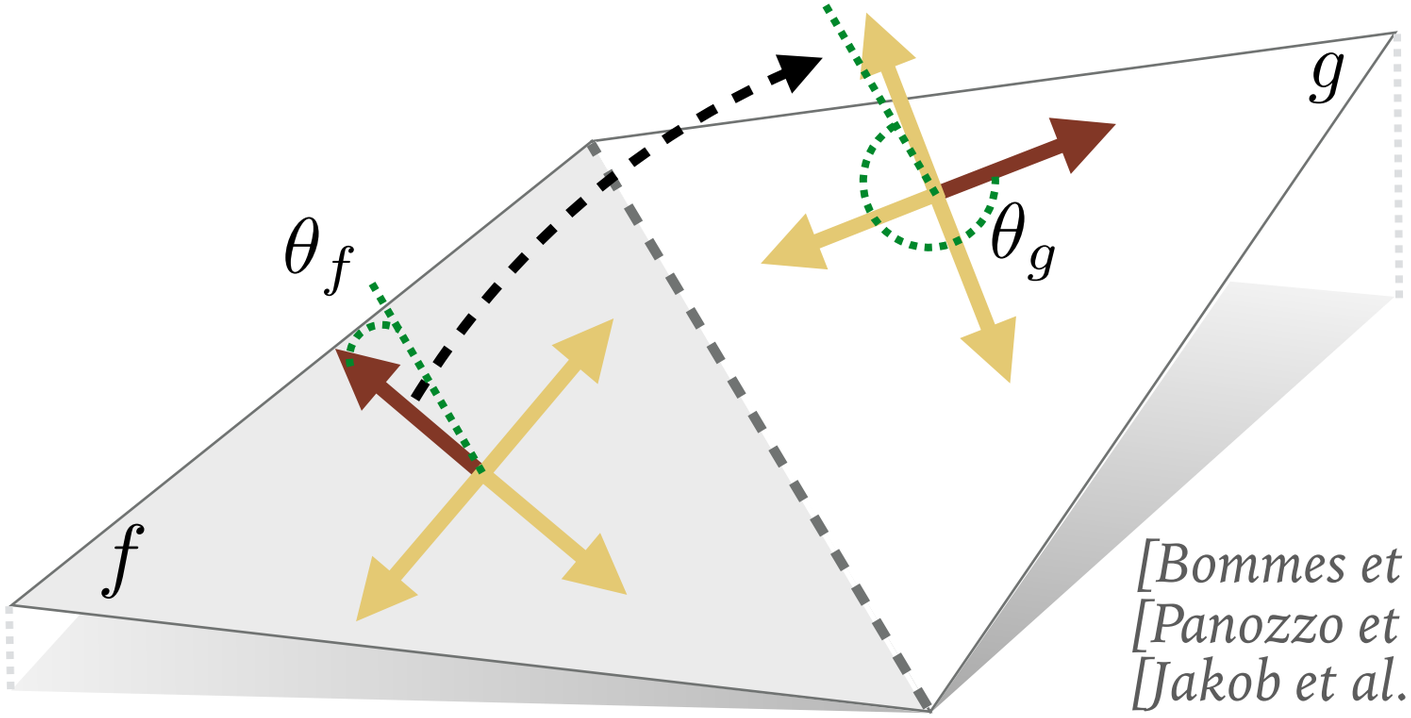
cartesian representation of a 4-direction field

How to prescribe angle difference $2\pi+\pi/4$ with principal rotation/ matching ?

CASE STUDY: N-DIRECTIONAL FIELD, FREE TOPOLOGY



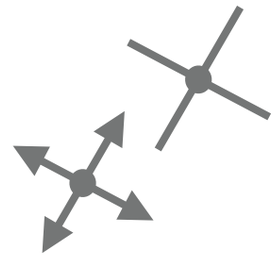
- Objective: “As-parallel-as-possible”
- Explicitly model topology (typically angle-based)
    - Matchings are explicitly modeled
    - Mixed Integer Optimization
    - Local minima



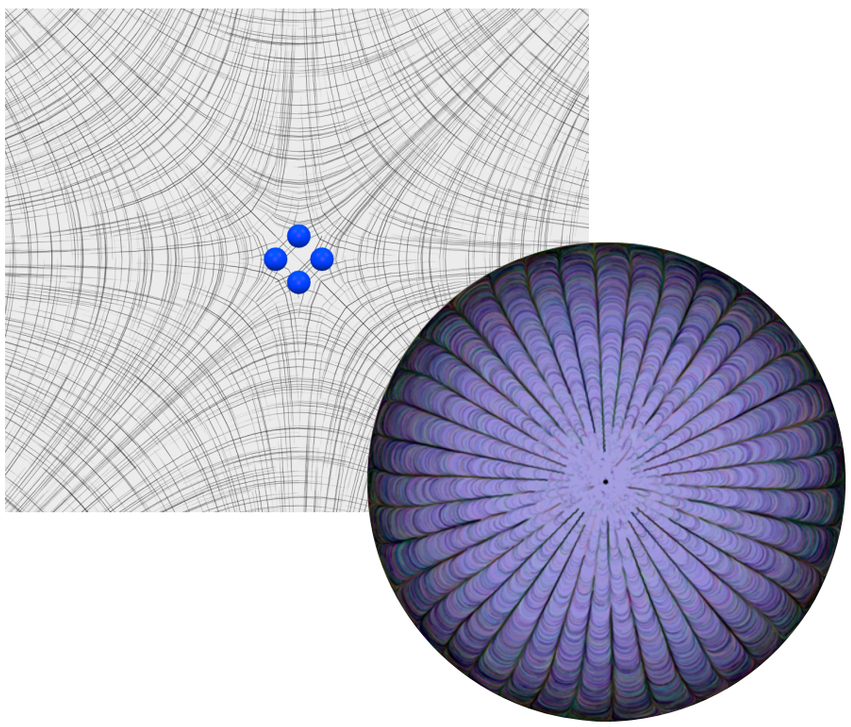
[Bommes et al. 2009]
[Panozzo et al. 2012]
[Jakob et al. 2015]

$$(\theta_f + \rho_{fg}\frac{\pi}{2} - \theta_g)^2 \quad \rho_{fg} \in \mathbb{I}$$

CASE STUDY: N-DIRECTIONAL FIELD, FREE TOPOLOGY

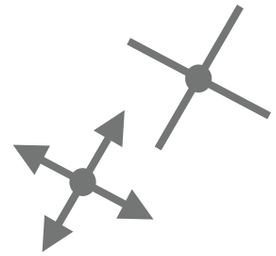


- Objective: “As-parallel-as-possible”
- Implicit topology with principal matchings (typically cartesian/complex)
    - Linear problem
    - Global optimum
    - Limited expressivity

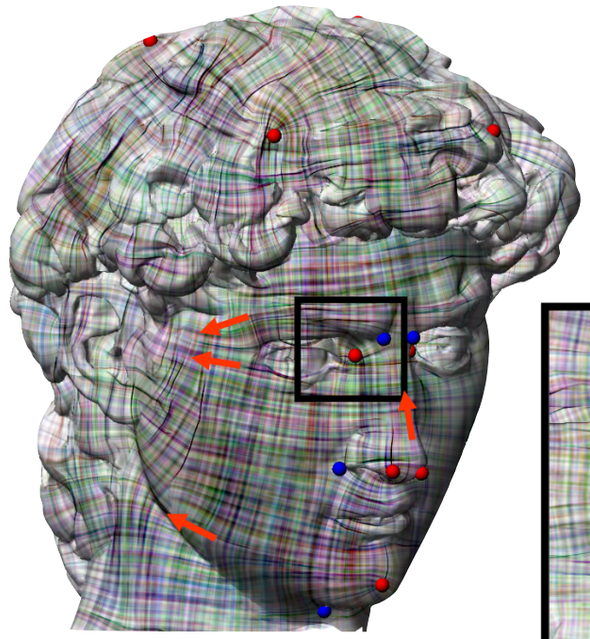


[Palacios et al. 2007]
[Knöppel et al. 2013]
[Diamanti et al. 2014]

HOW ABOUT CONSTRAINTS?



- Sparse “Hard”

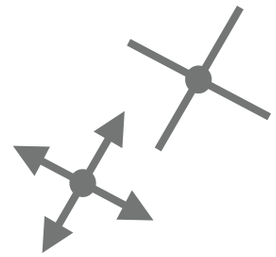


[Crane et al. 2010]
[Bommes et al. 2009]
[Hertzmann et al. 2000]
[Diamanti et al. 2014]

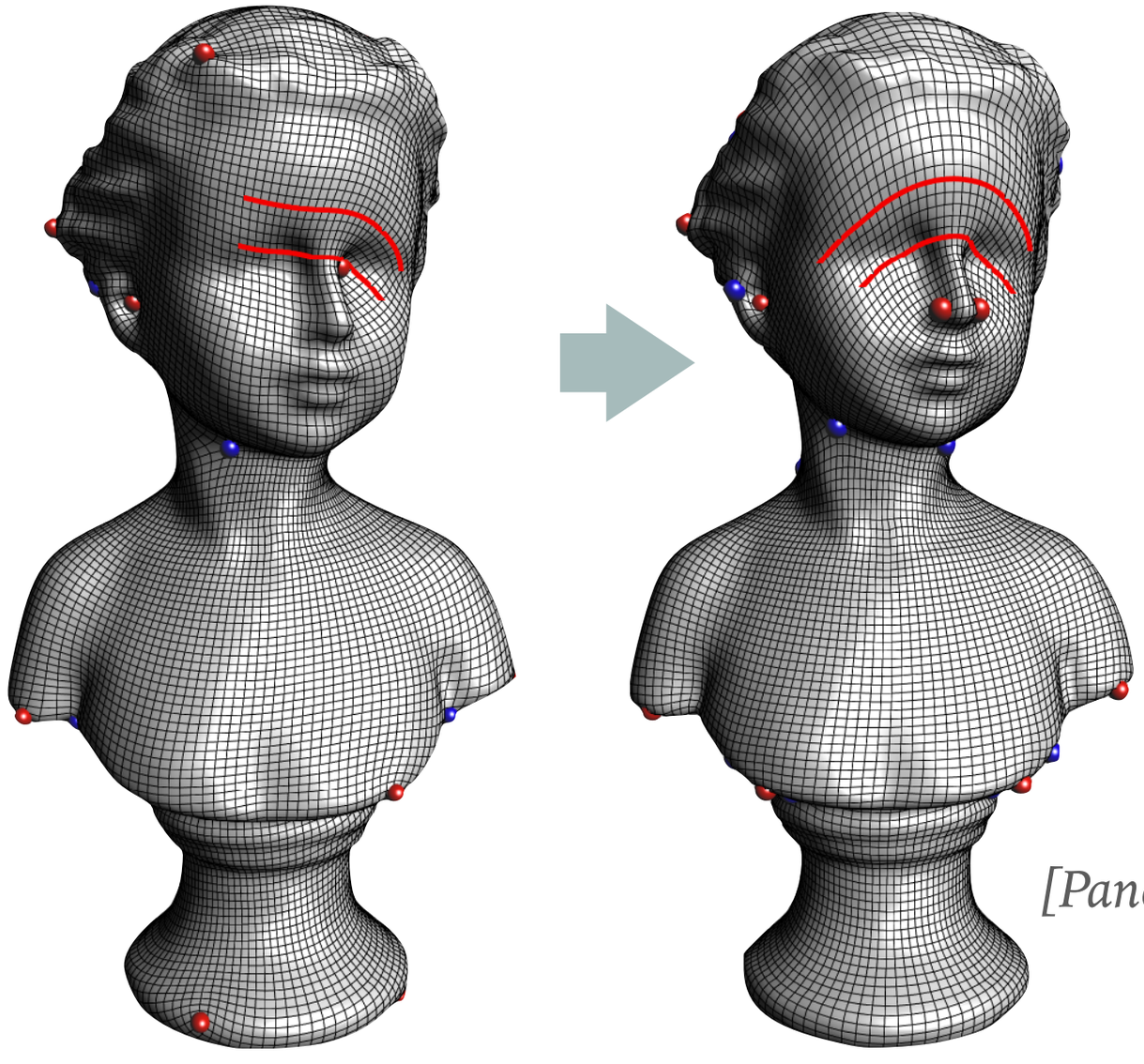
[Palacios et al. 2007]
[Ray et al. 2008]
[Jakob et al. 2015]

- Also partial constraints! [Iarussi et al. 2015] [Diamanti et al. 2015]

HOW ABOUT CONSTRAINTS?

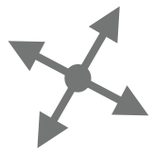


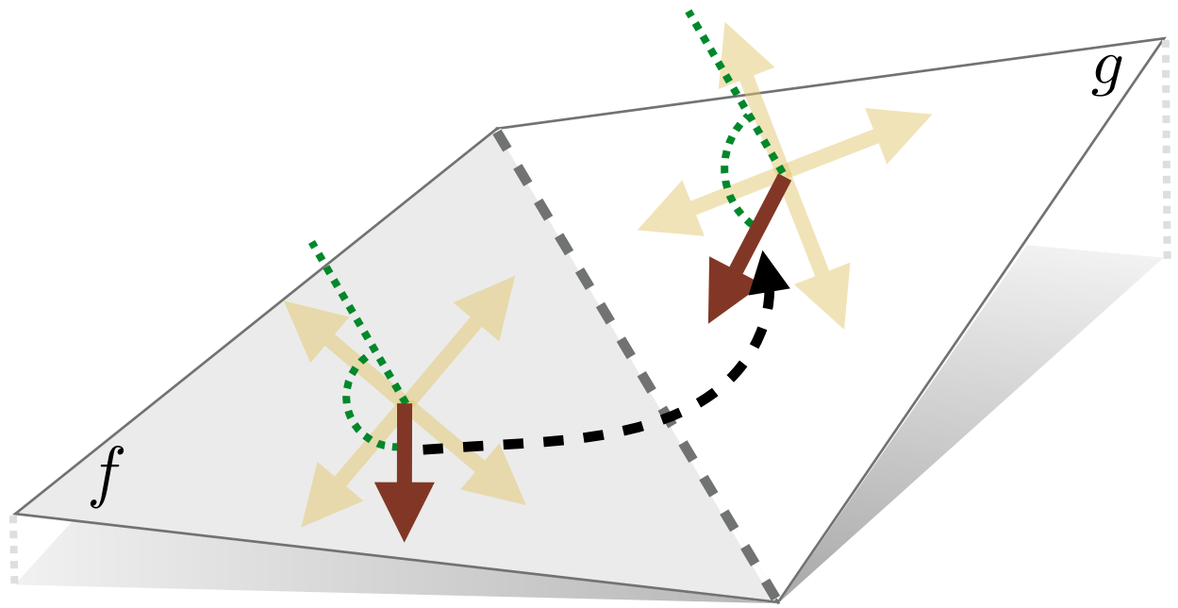
- Soft constraints



[Panozzo et al. 2012]

AVOIDING THE TRIVIAL SOLUTION

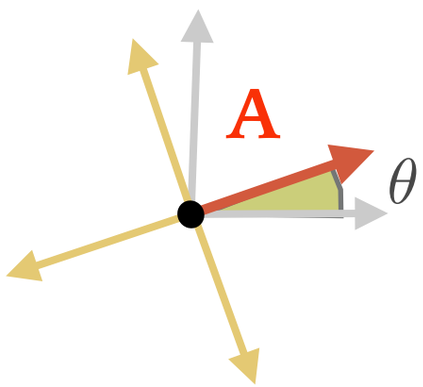
- Eg. N-Vector Fields  with a cartesian/complex representation
- Zero Field is perfectly smooth!
- Constraints are necessary
    - Per-vector unit-norm constraint [Palacios et al. 2007]
    - Integrated norm constraint [Knöppel et al. 2013]
    - Enough hard constraints [Diamanti et al. 2014]



COMPARATIVE ANALYSIS

Olga Diamanti
Geometric Computation Group
Stanford University

COMPARISONS AND DEMOS

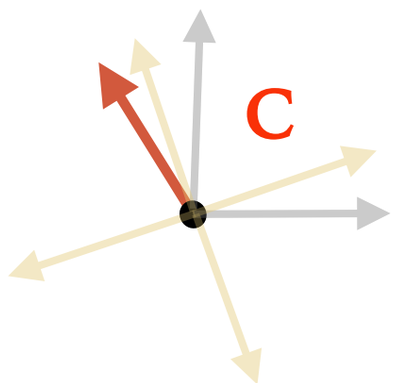


angle-based approaches

[Bommes et al. 2009]

[Panozzo et al. 2014]

[Jakob et al. 2015]



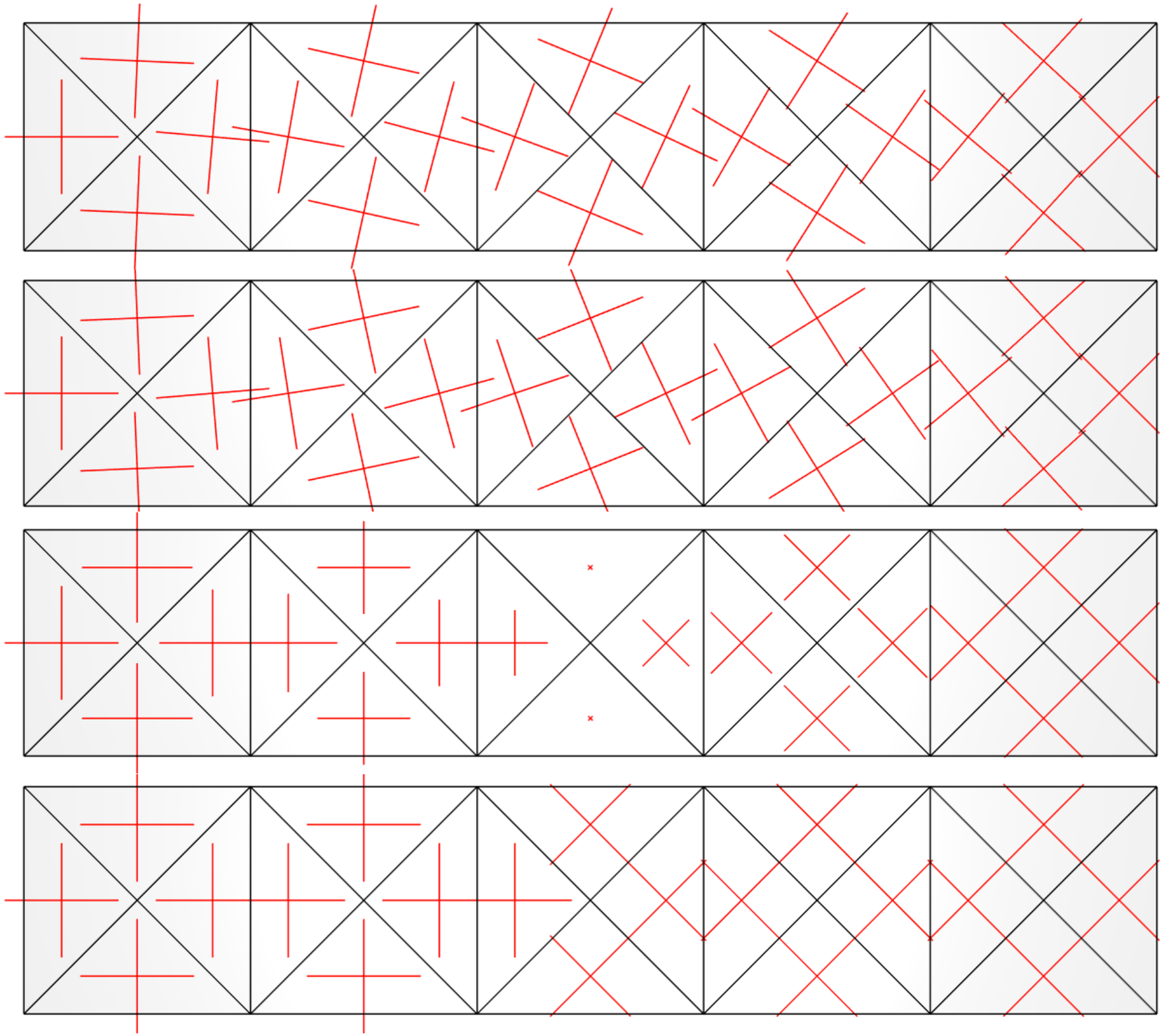
cartesian/complex approaches

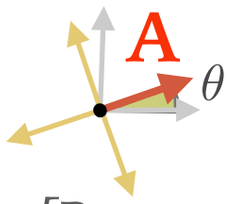
[Knöppel et al. 2013]

[Diamanti et al. 2014]

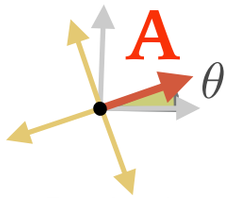
[Diamanti et al. 2015]

4–DIRECTIONAL FIELD

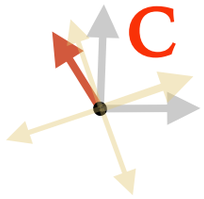




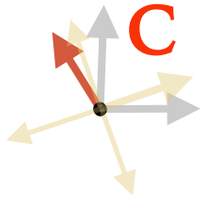
[Bommes et al. 2009]



[Jakob et al. 2015]

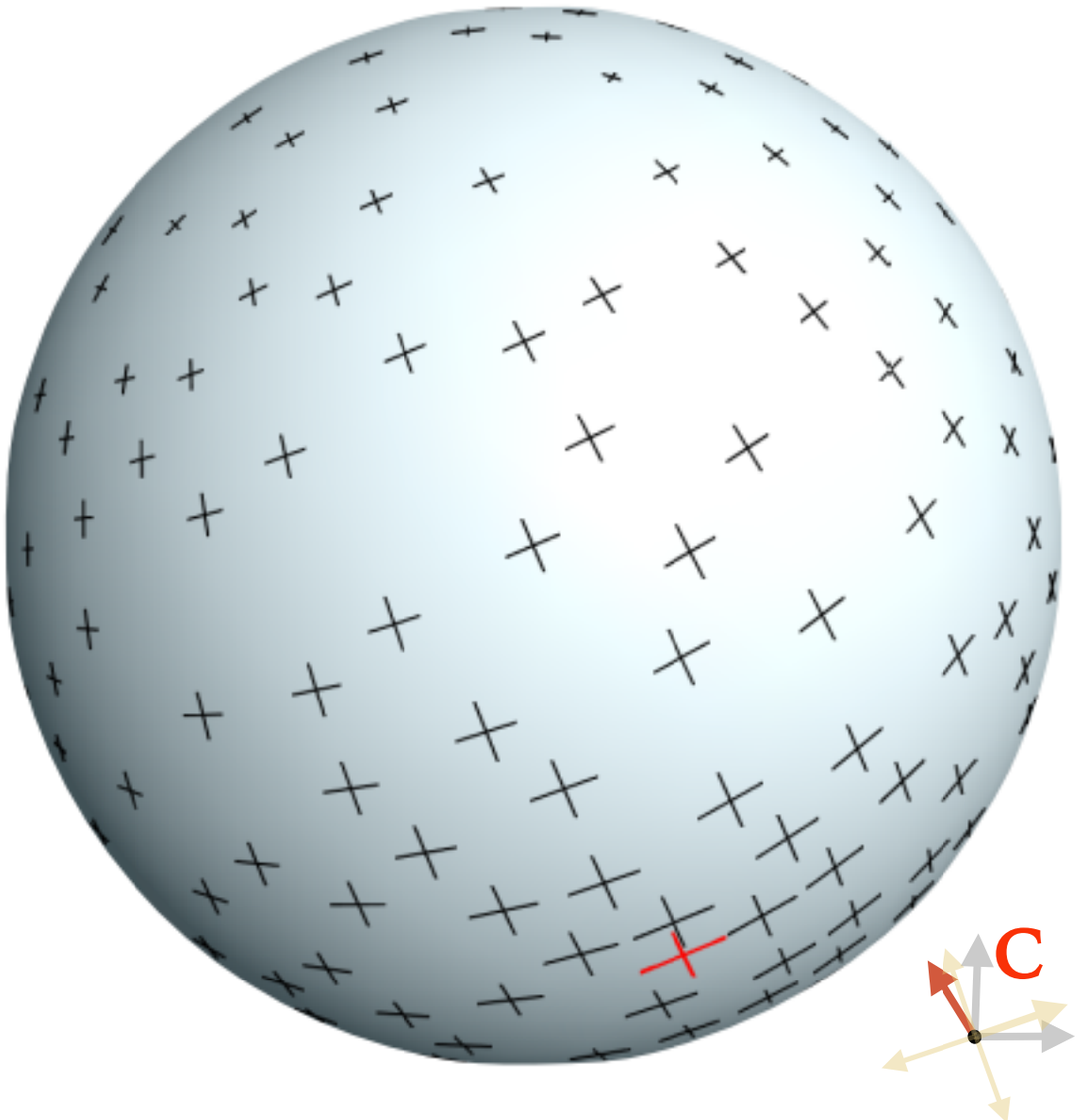


[Diamanti et al. 2014]



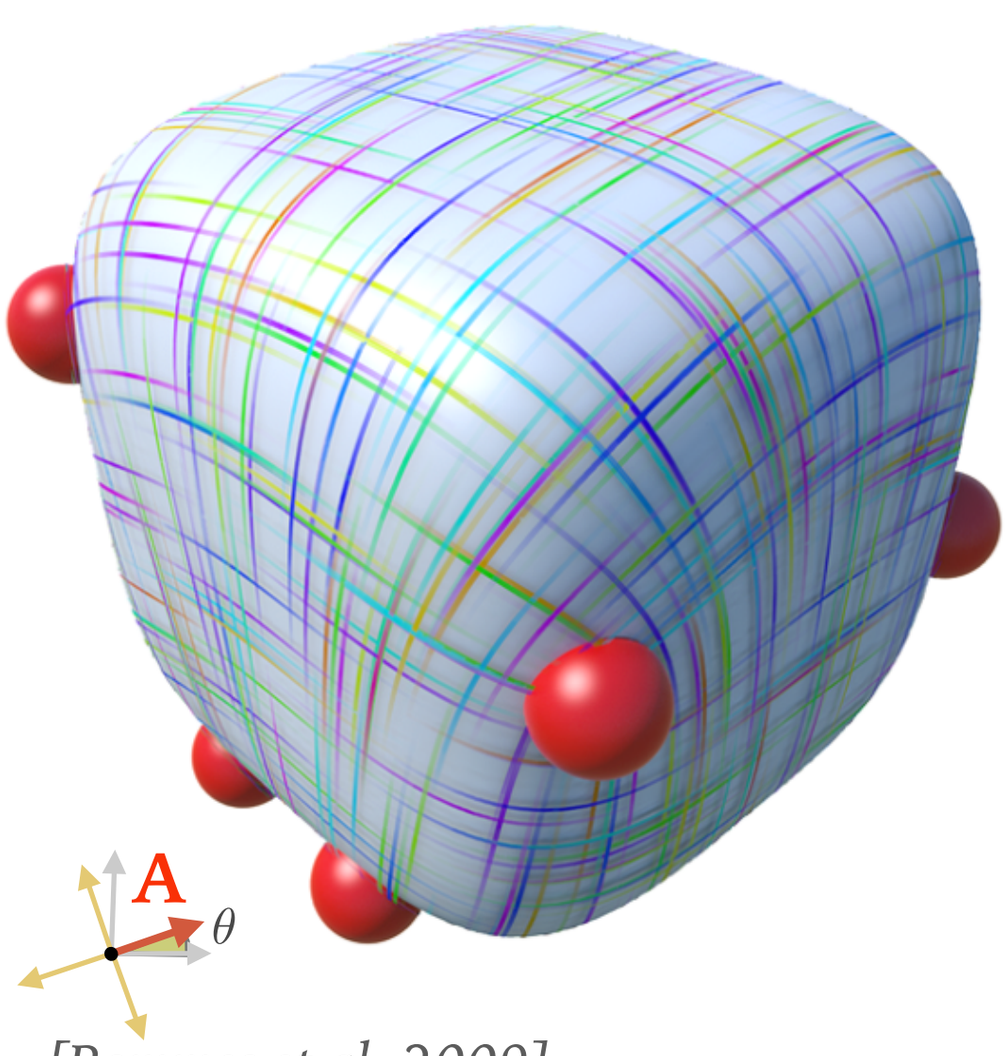
[Knöppel et al. 2013]

MAGNITUDE IN CARTESIAN REPRESENTATIONS – CONSTRAINTS

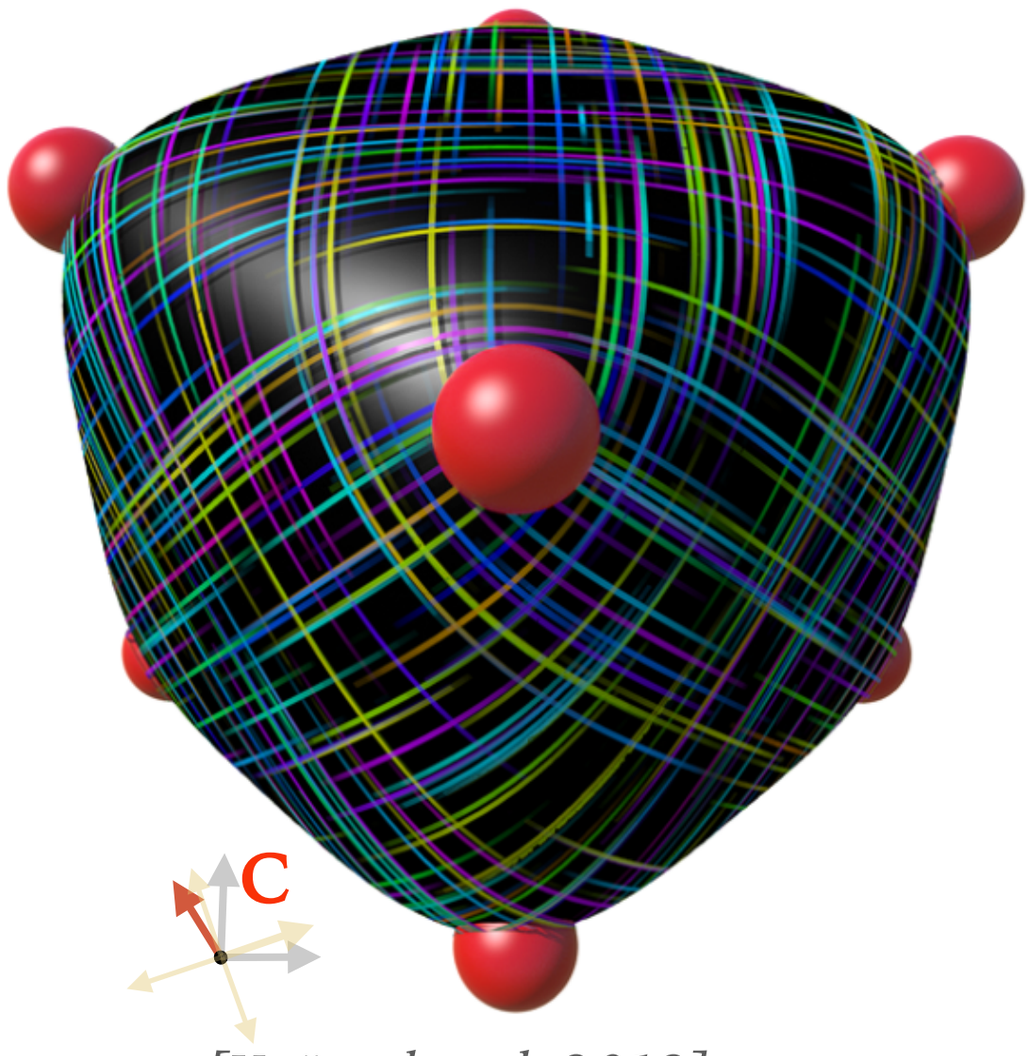


[Diamanti et al. 2014]

EFFECT OF THE FORMULATION

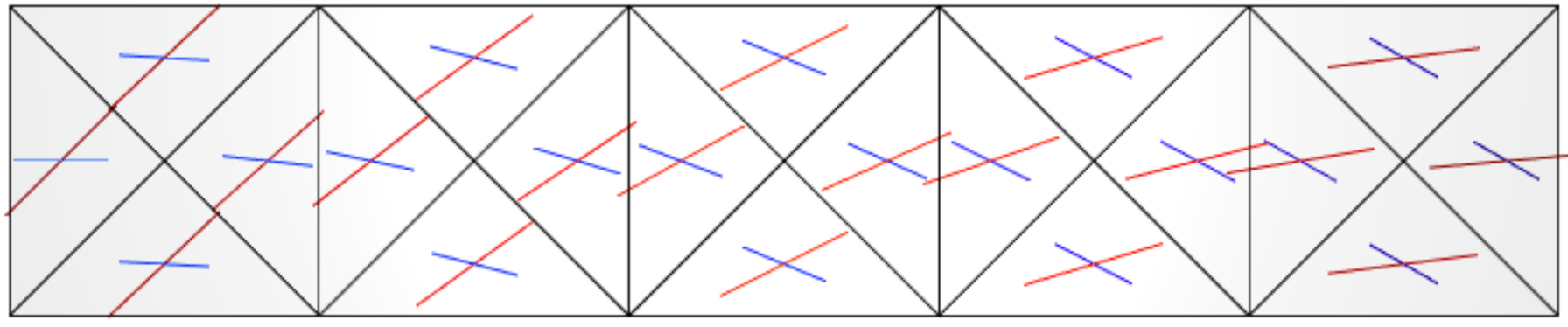


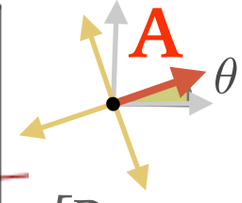
[Bommes et al. 2009]



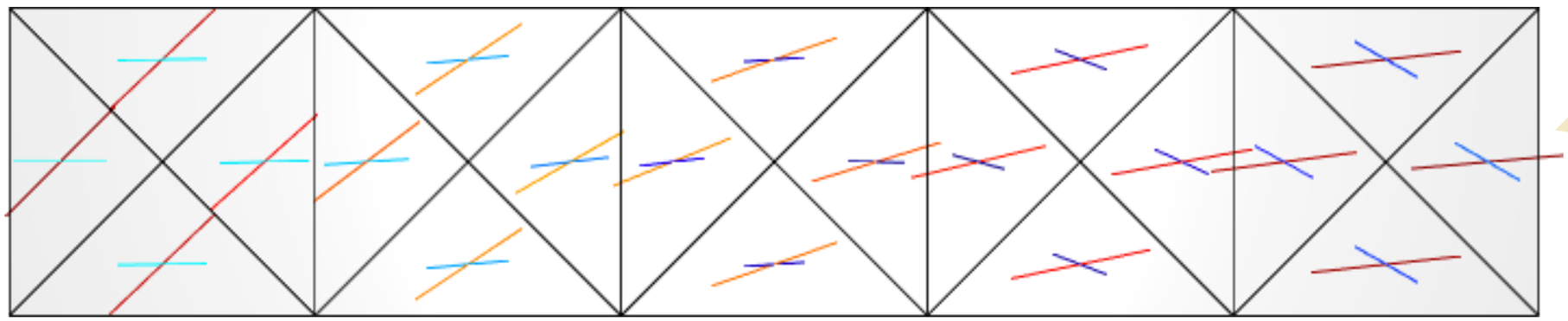
[Knöppel et al. 2013]

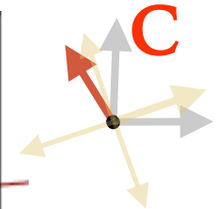
2^2-VECTOR FIELD



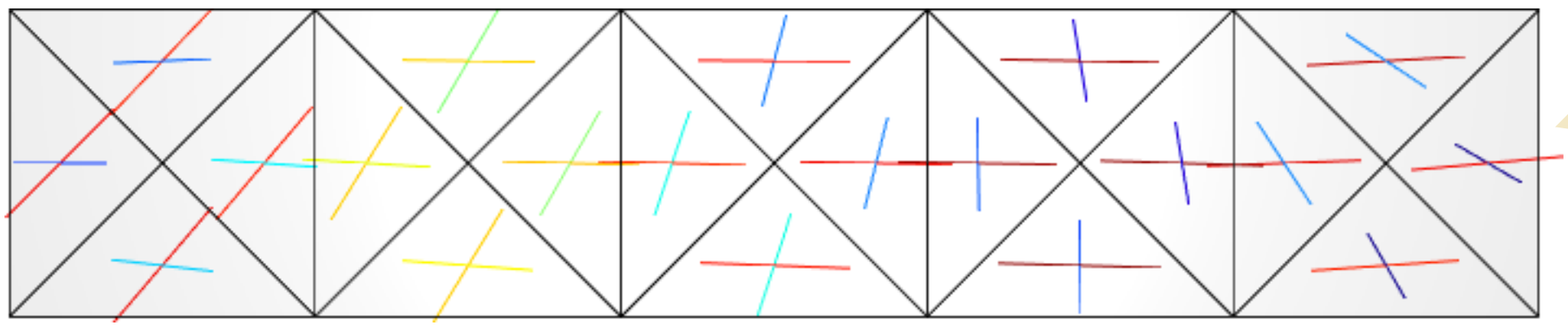


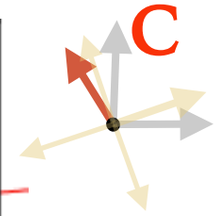
[Panozzo et al. 2014]





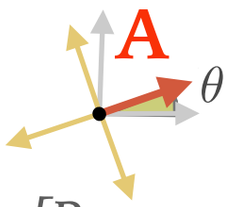
[Diamanti et al. 2014]



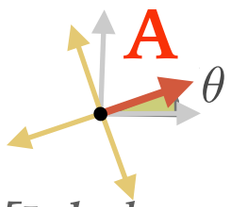


[Diamanti et al. 2015]

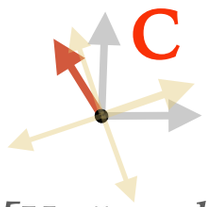
SINGULARITIES



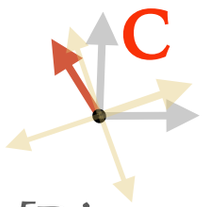
[Bommes et al. 2009]



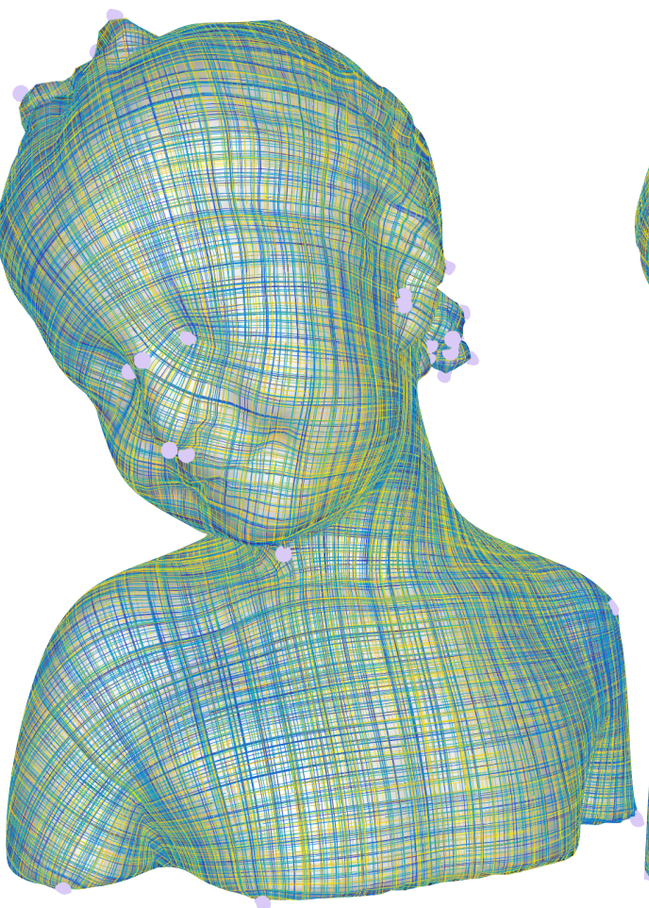
[Jakob et al. 2015]



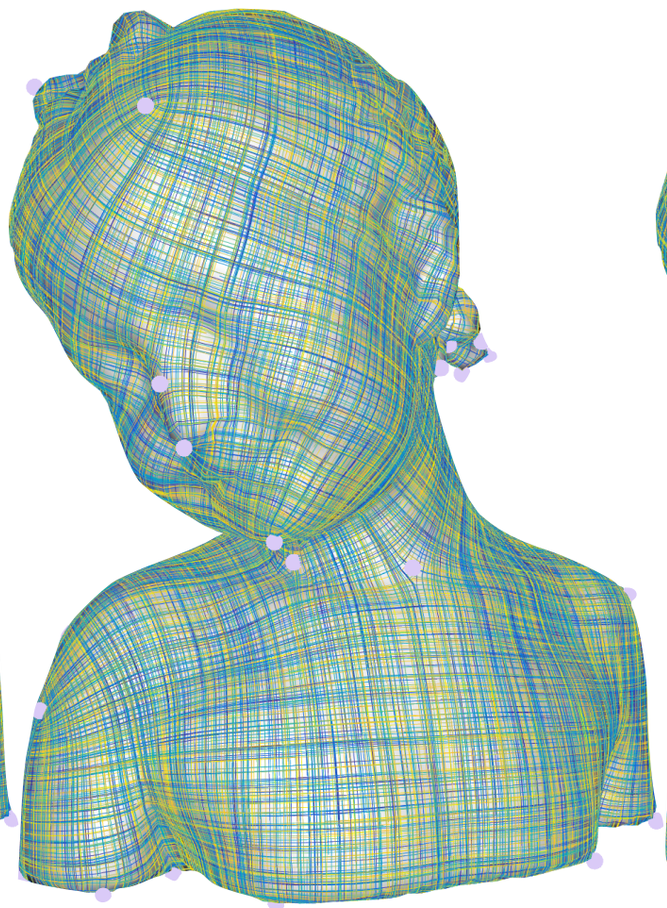
[Knöppel et al. 2013]



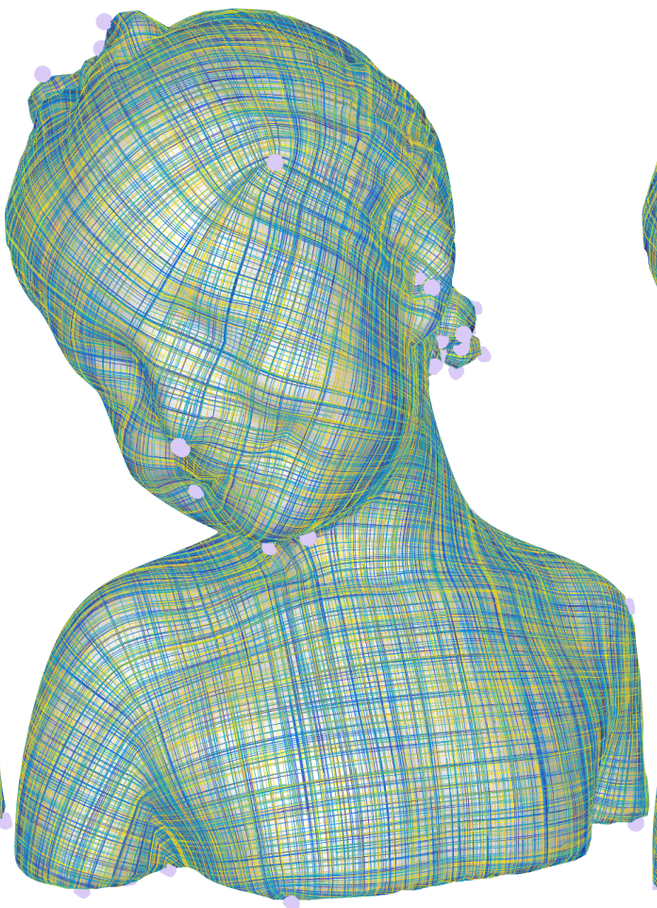
[Diamanti et al. 2015]



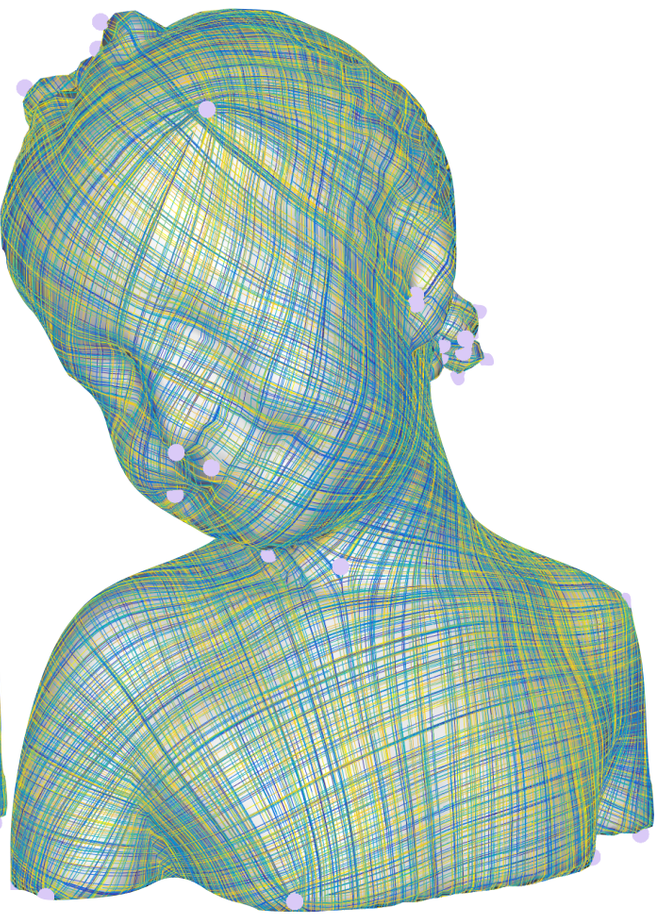
48 singularities



42 singularities

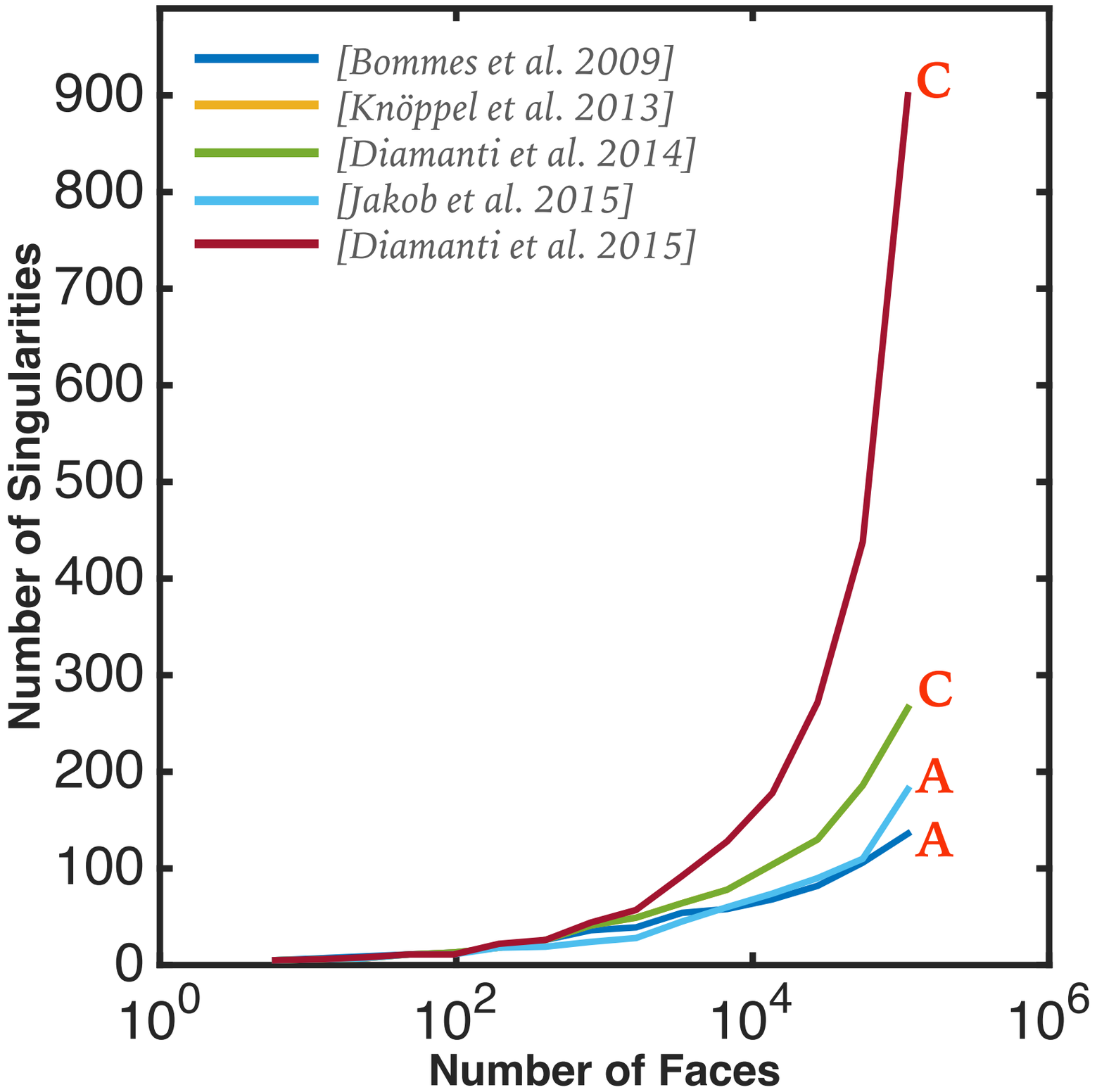


62 singularities

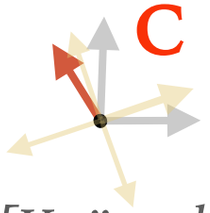


56 singularities

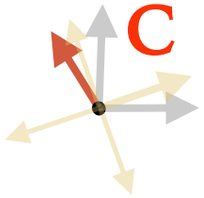
SINGULARITIES



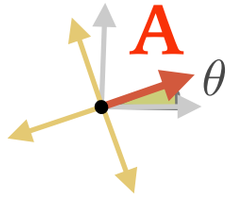
INTEGRABILITY



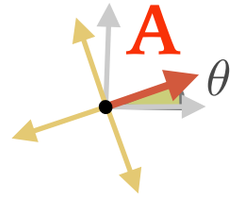
[Knöppel et al. 2013]



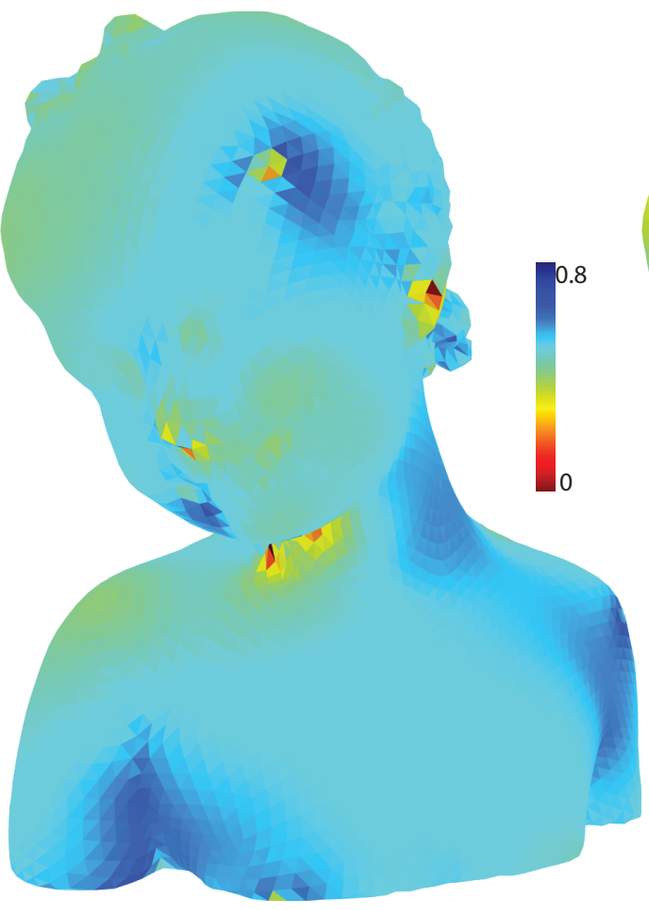
[Diamanti et al. 2014]

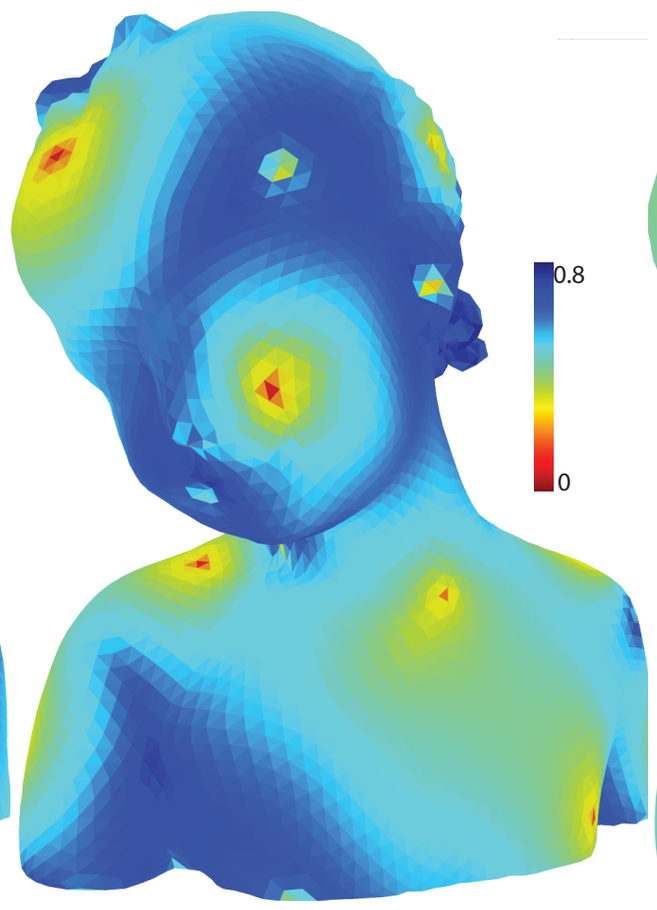


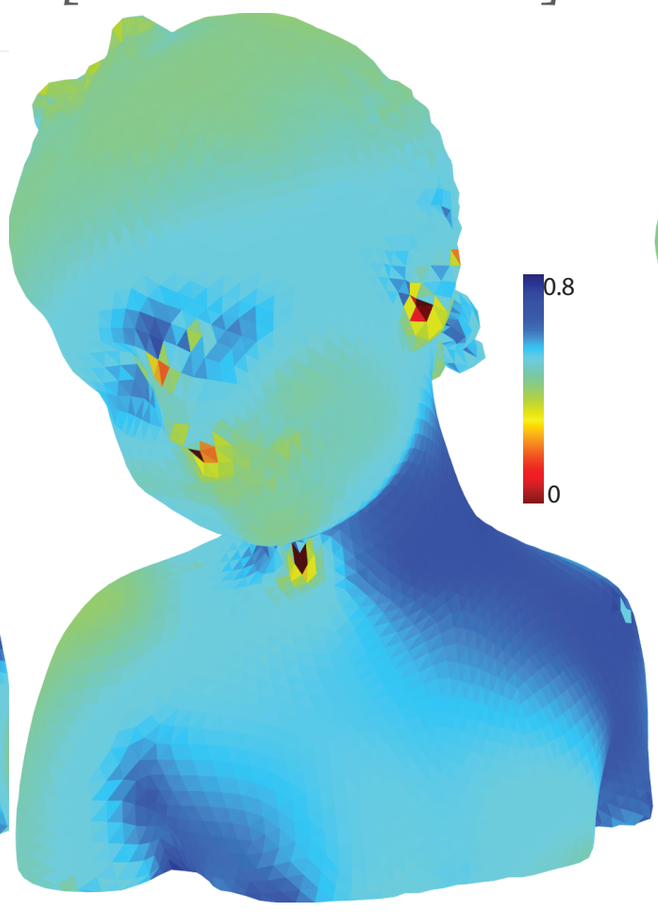
[Bommes et al. 2009]

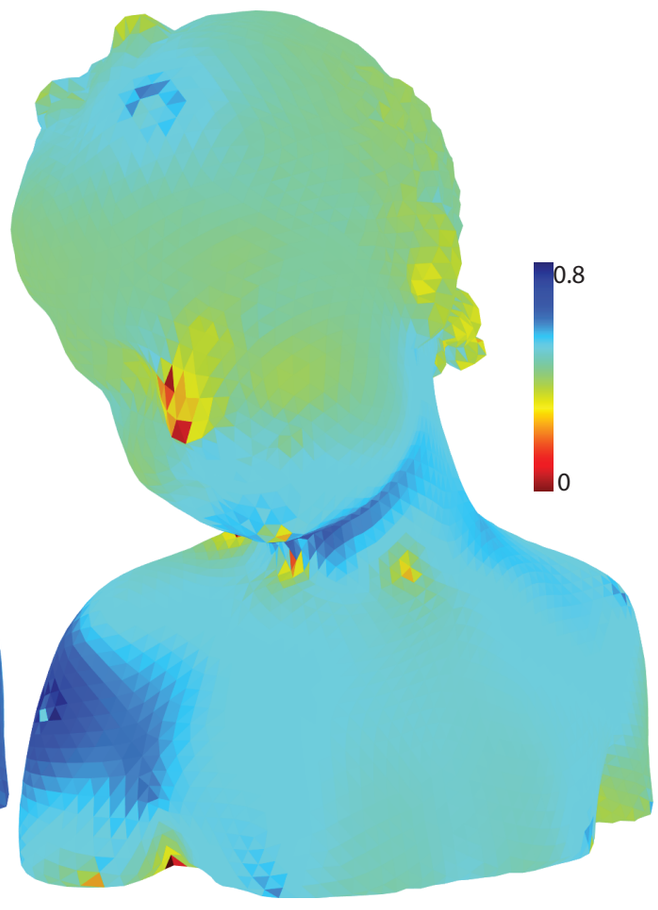


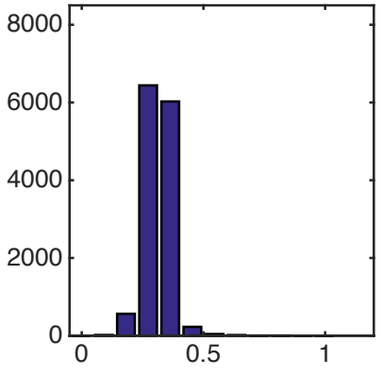
[Jakob et al. 2015]

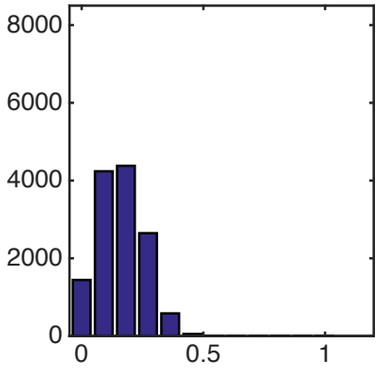


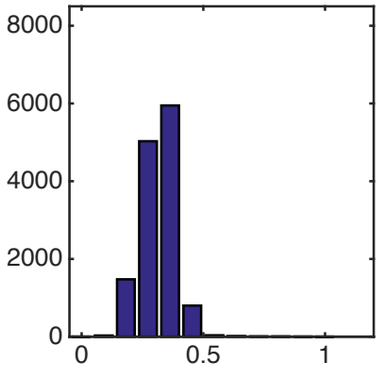


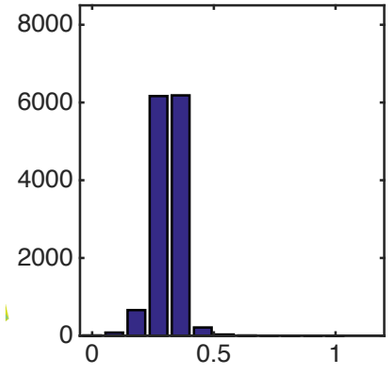












SCALABILITY