

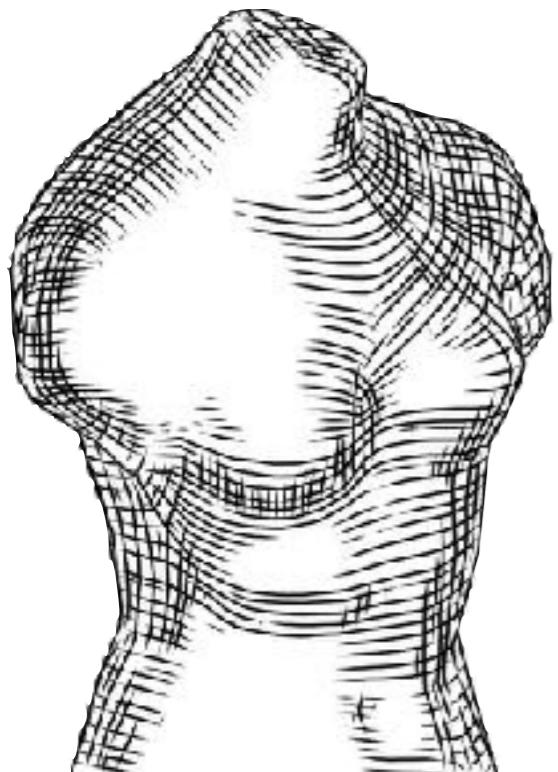
OBJECTIVES

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OBJECTIVES - FIELD FAIRNESS

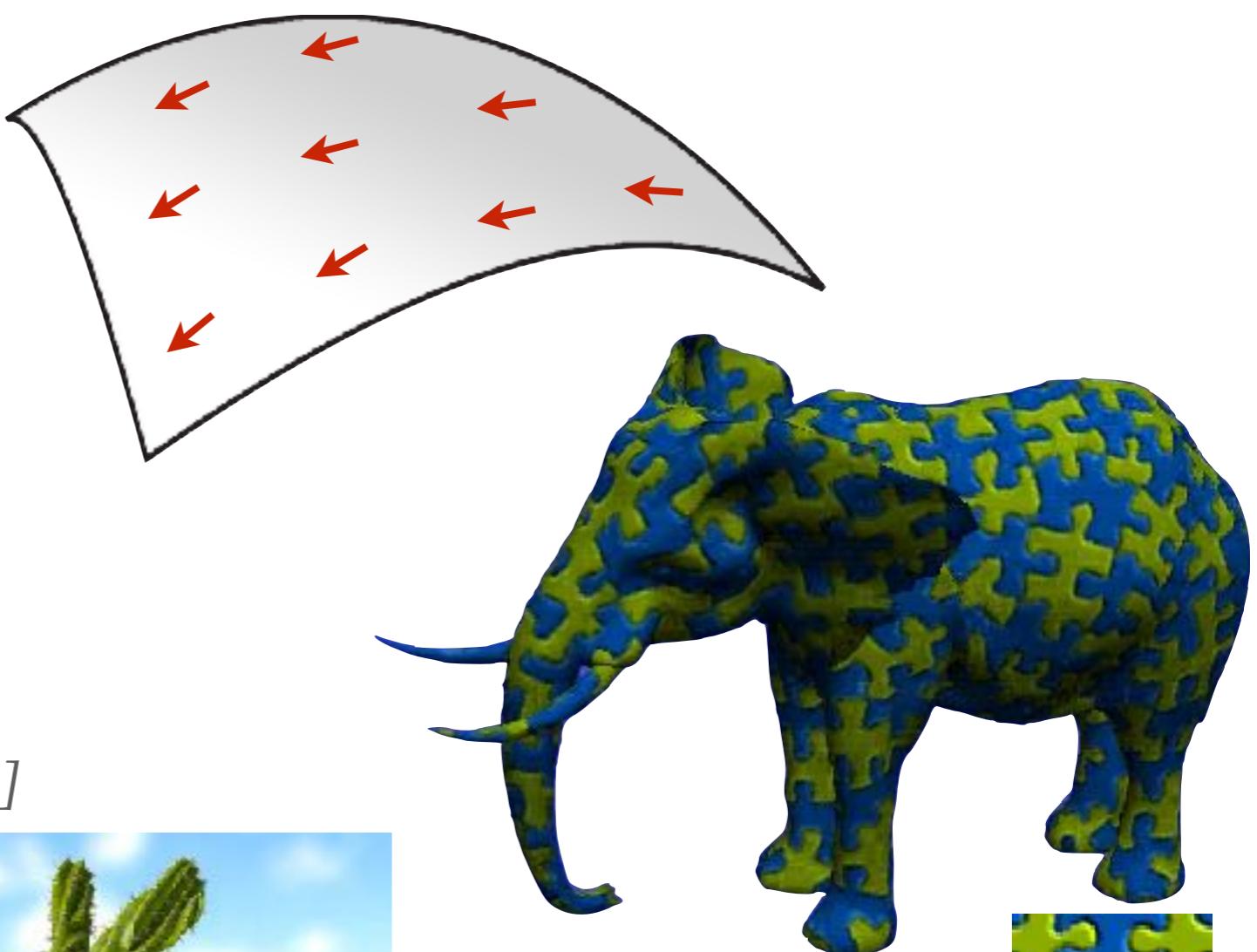
- “As-parallel-as-possible”



[Palacios et al. 2007]



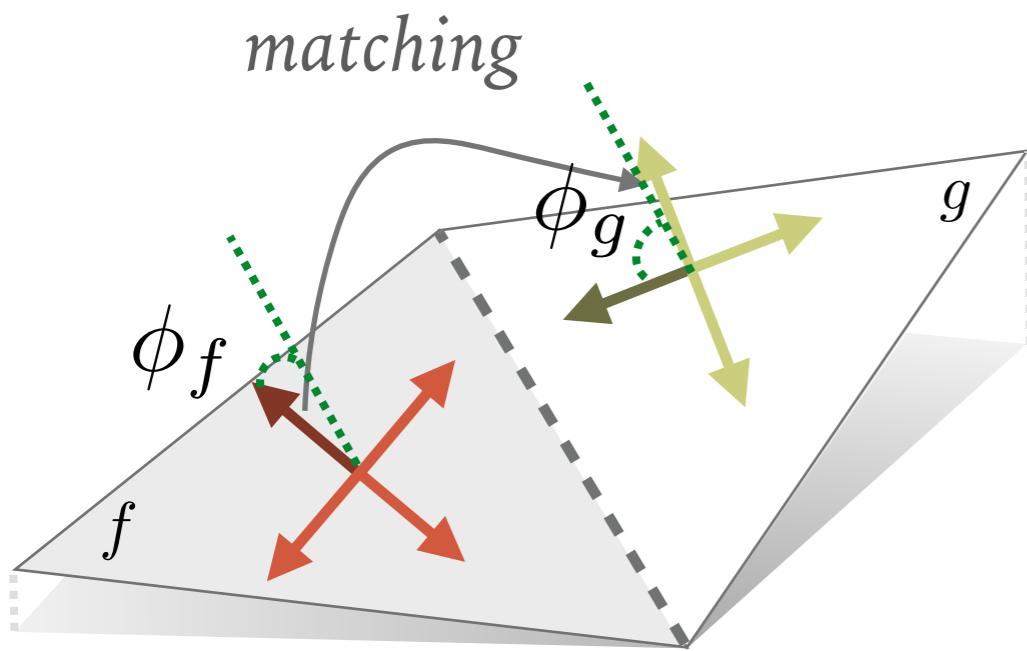
[Ray et al. 2006]
[Knöppel et al. 2015]



[Turk et al. 2001]

OBJECTIVES - FIELD FAIRNESS

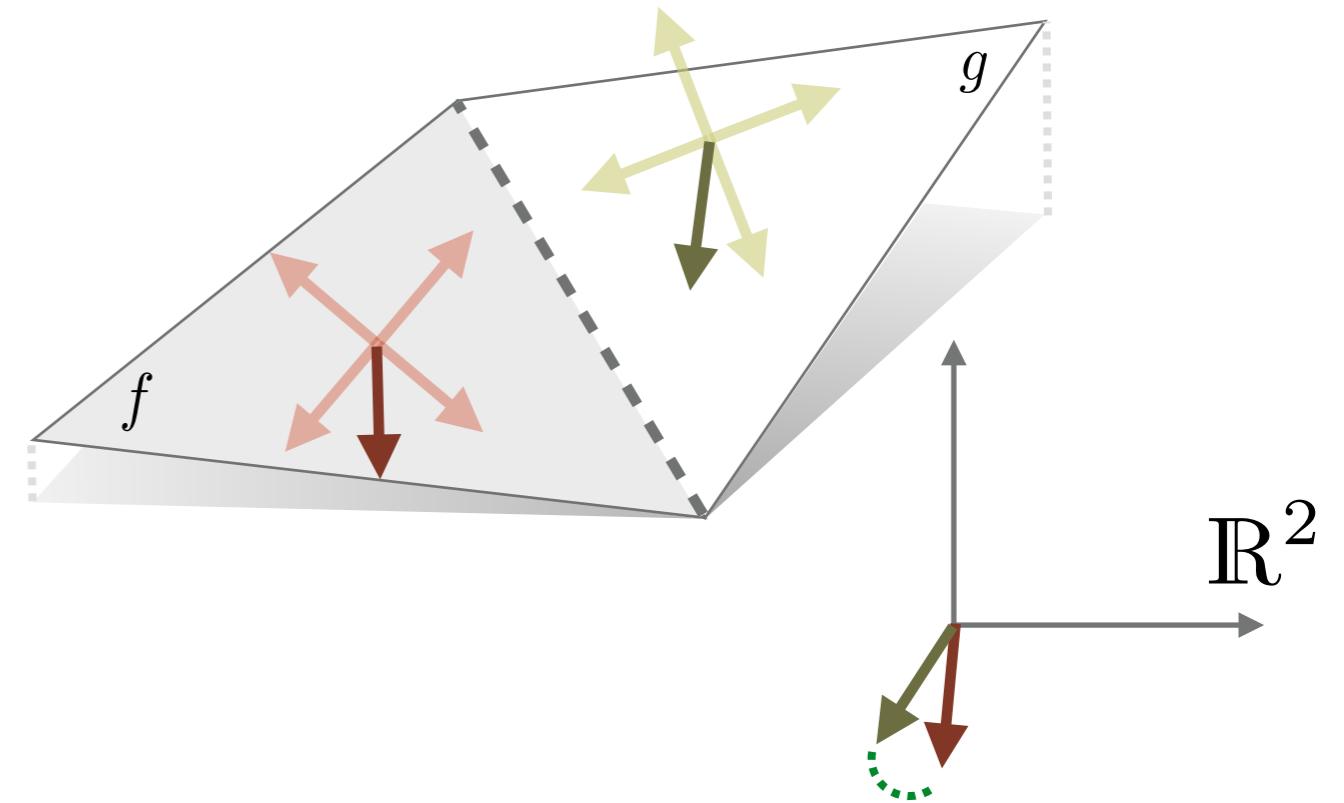
- “As-parallel-as-possible”



angle-based approaches

compare matching angles

[Hertzmann et al. 2000] [Crane et al. 2010]
[Ray et al. 2008] [Ray et al. 2009]
[Bommes et al. 2009] [Jakob et al. 2015]



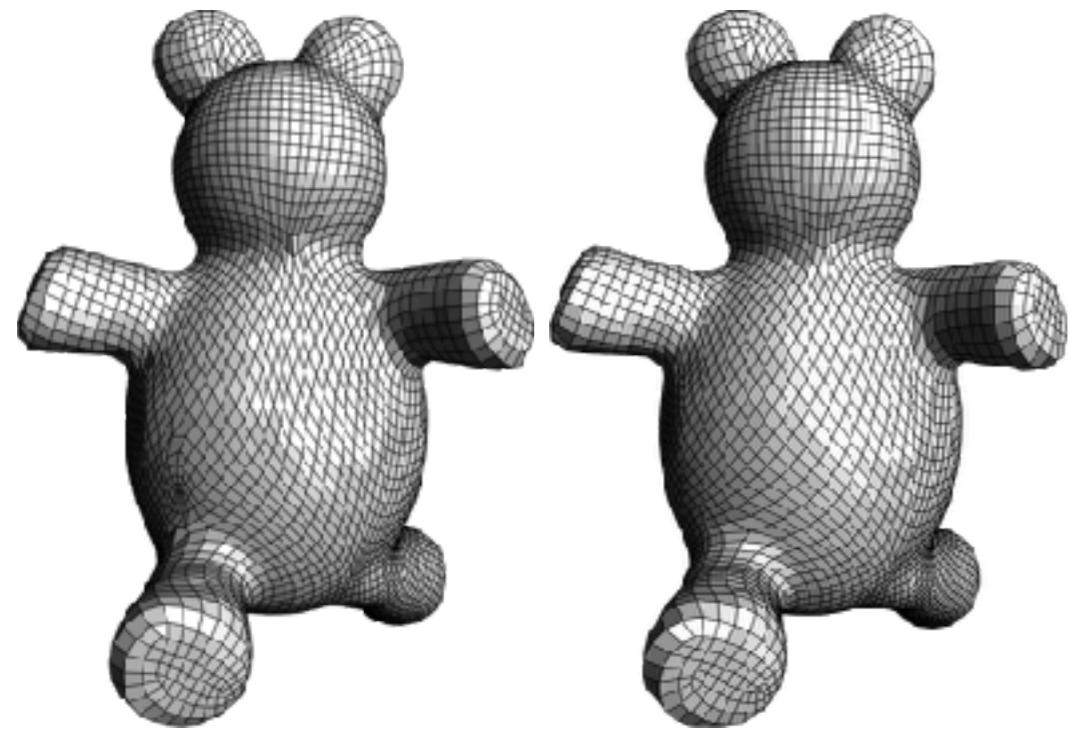
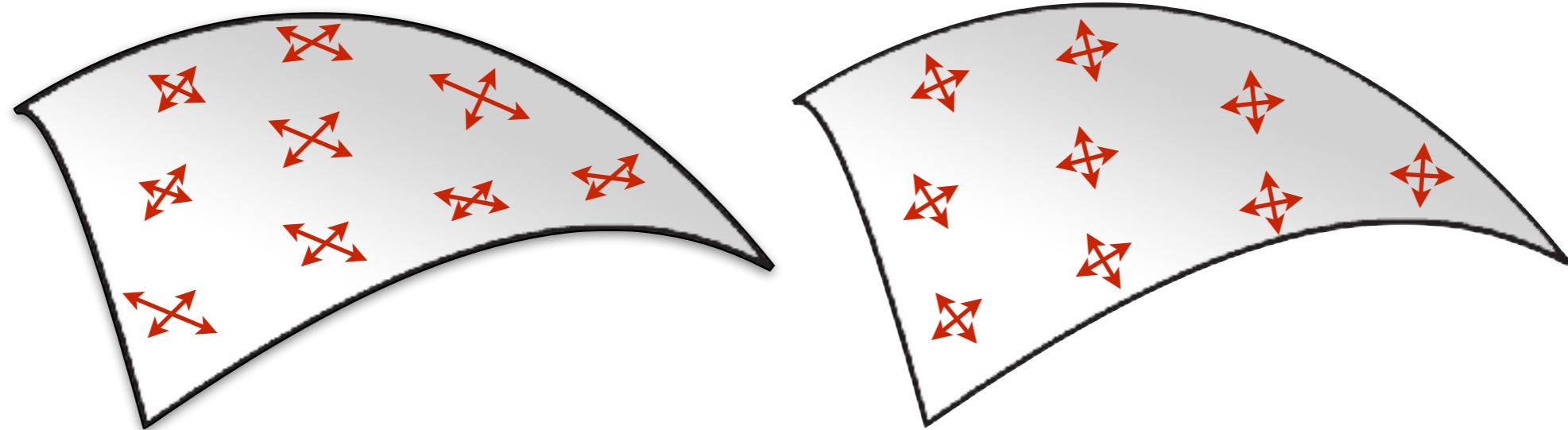
cartesian/complex approaches

compare representative vectors

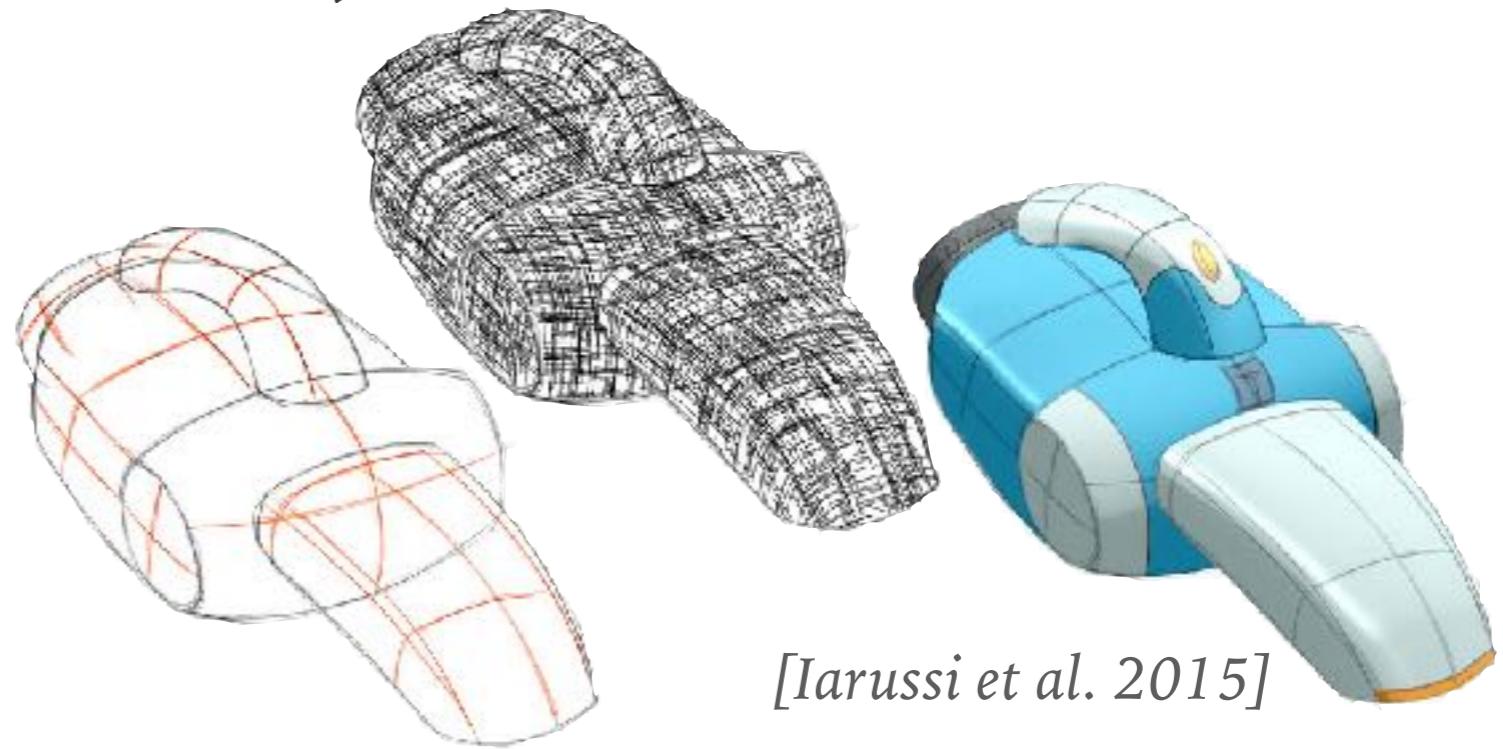
[Ray et al. 2006] [Knöppel et al. 2013]
[Palacios et al. 2007] [Diamanti et al. 2014]

OBJECTIVES - FIELD FAIRNESS

- Orthogonality

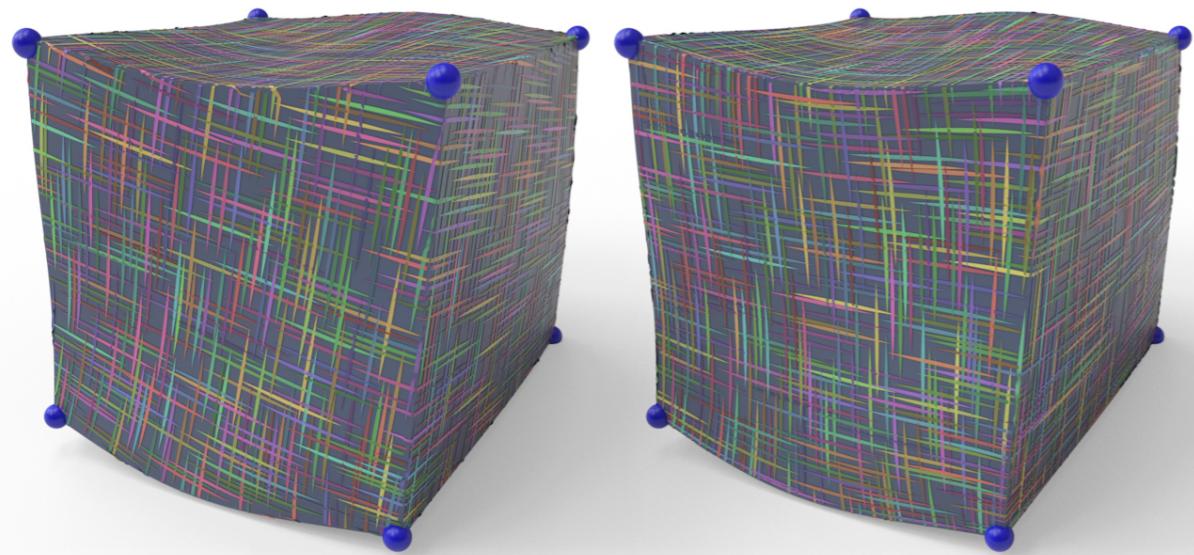


[Diamanti et al. 2014]

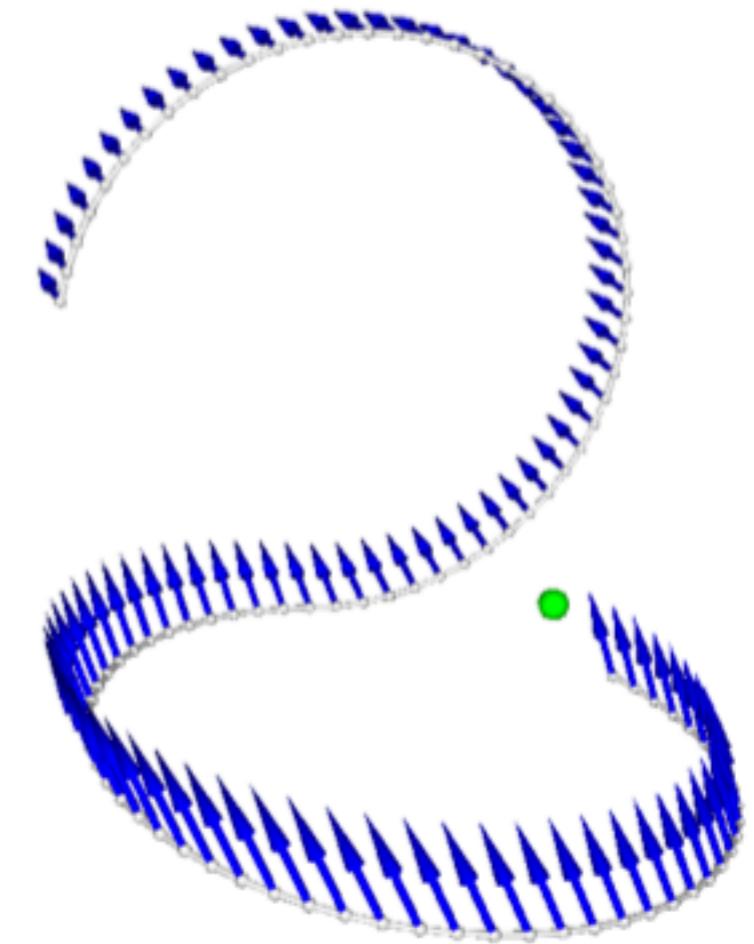


[Iarussi et al. 2015]

OBJECTIVES – ALIGNMENT TO FEATURES



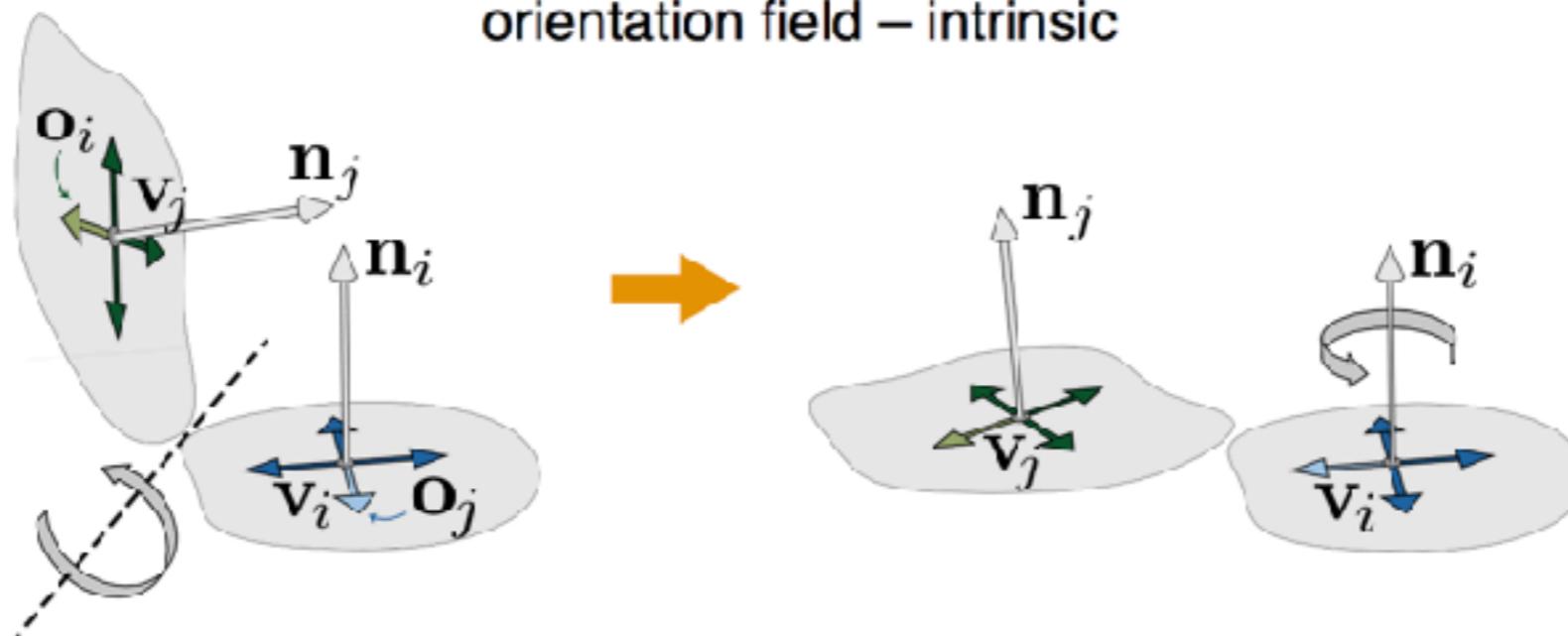
[*Jakob et al. 2015*]



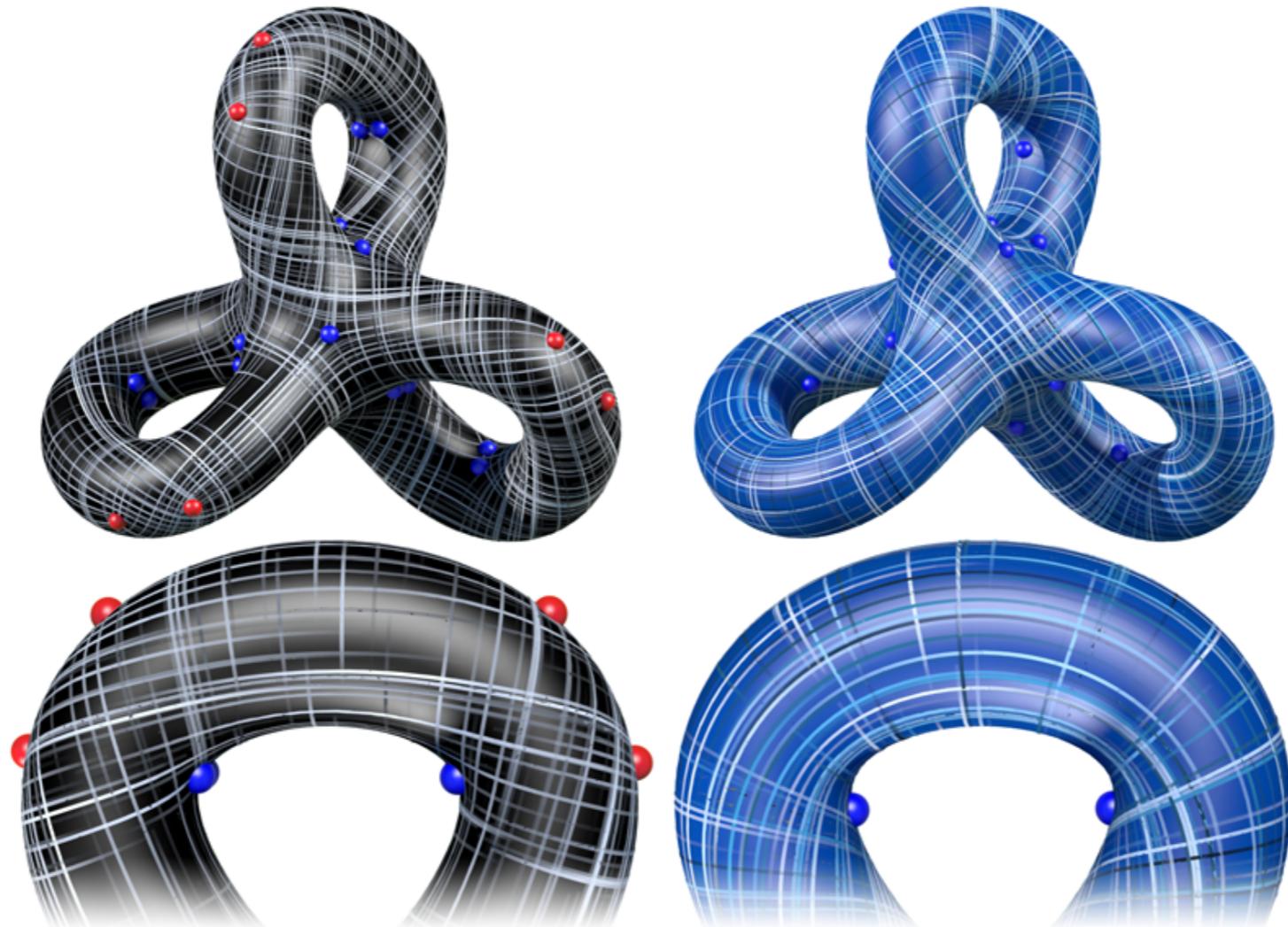
[*Huang et al. 2016*]

OBJECTIVES – ALIGNMENT TO FEATURES

orientation field – intrinsic



OBJECTIVES – SINGULARITY CONTROL



[Knöppel et al. 2013]

OBJECTIVES – ISOMETRY INDUCING

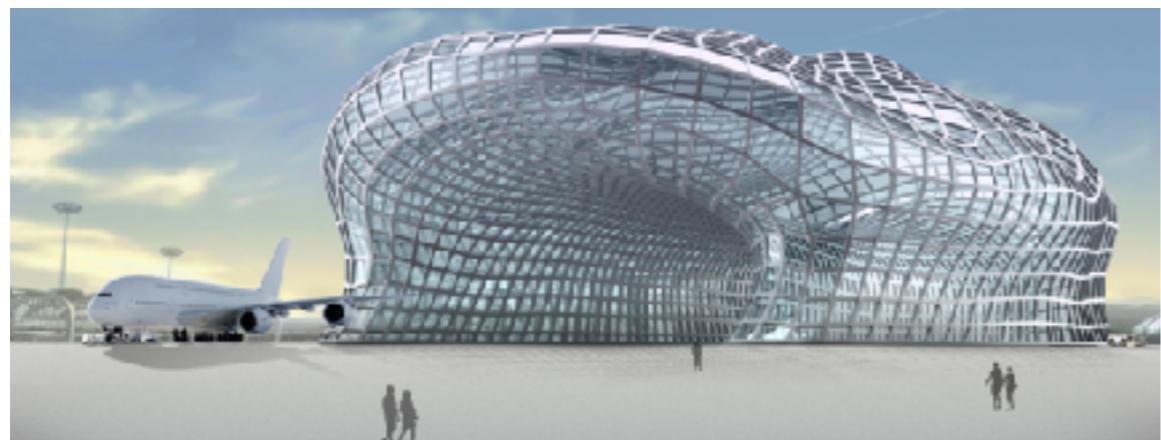


[Ben-Chen et al. 2010]

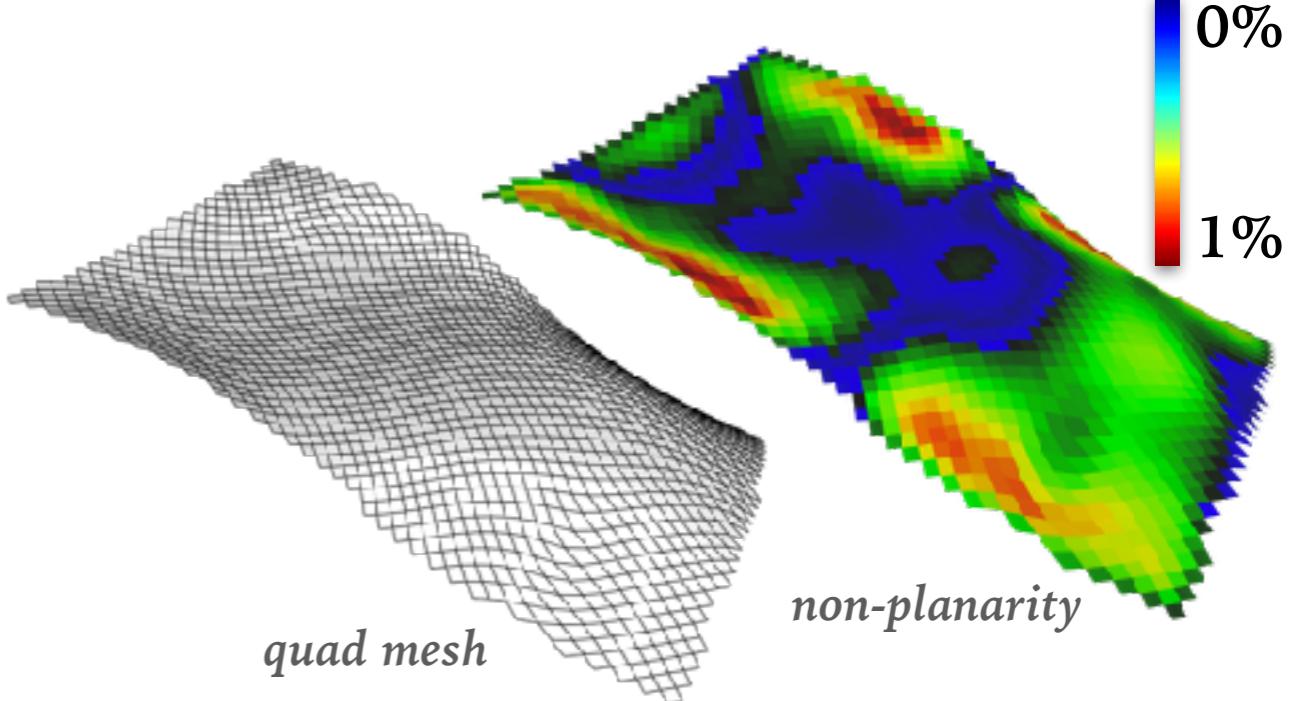


[Solomon et al. 2011]

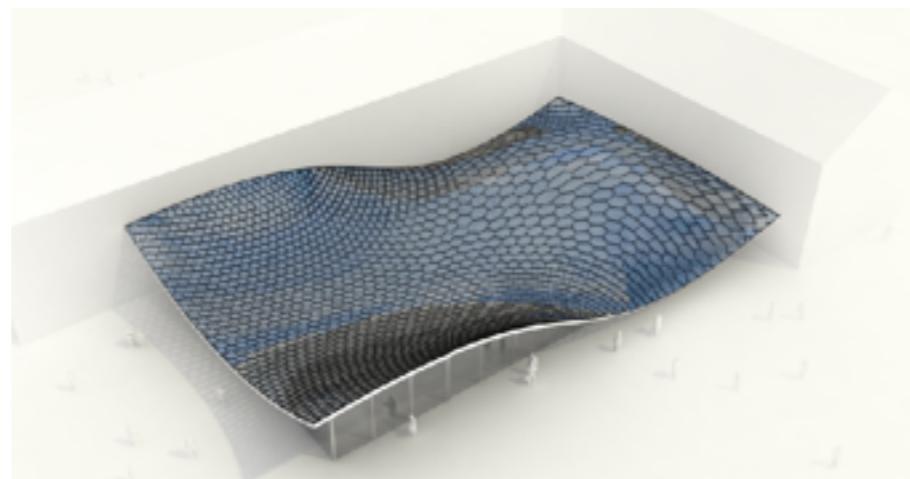
OBJECTIVES - CONJUGACY (PLANARITY)



[Liu et al. 2011]



[Diamanti et al. 2014]



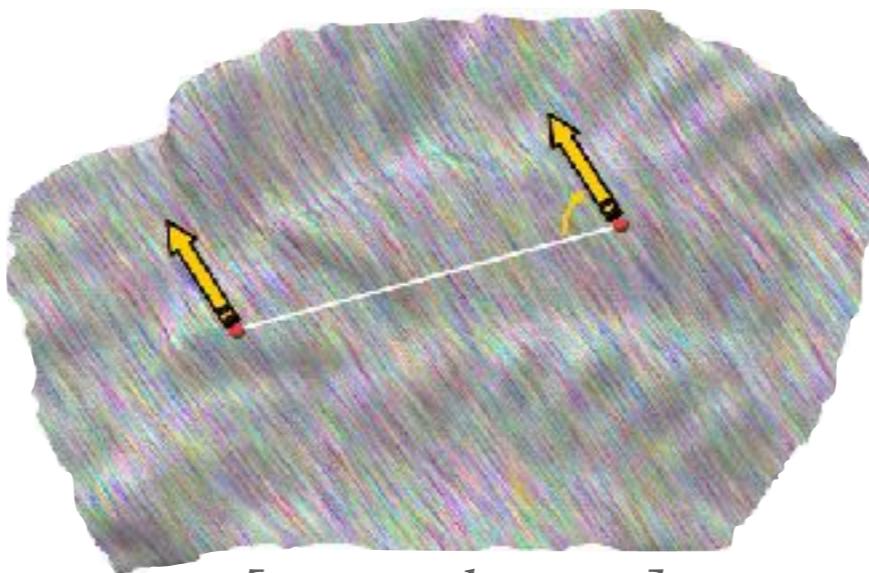
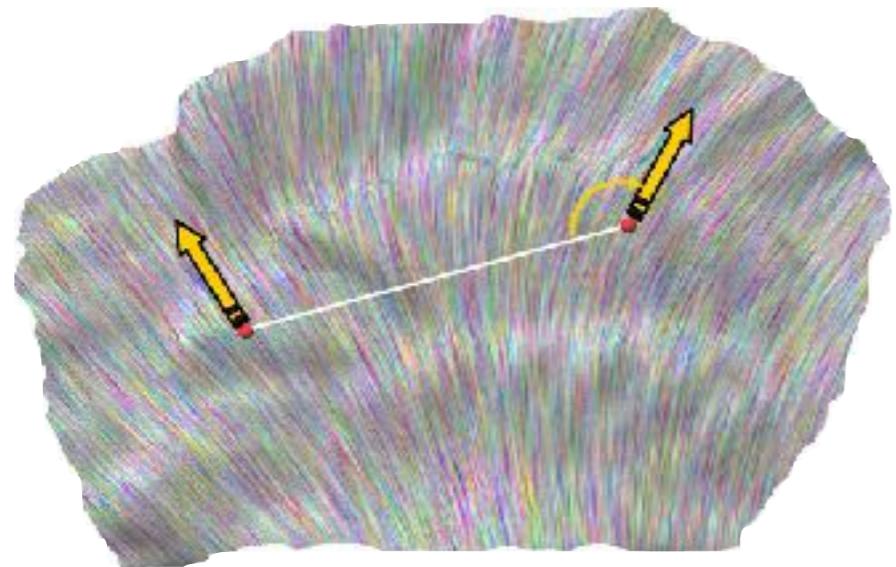
[Vaxman et al. 2015]

CONSTRAINTS

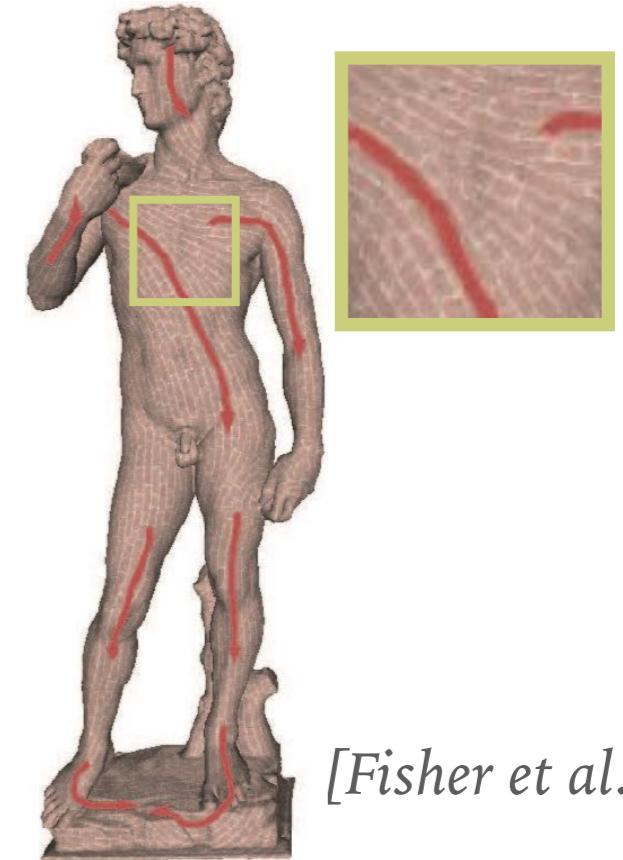
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New York University*

CONSTRAINTS - ALIGNMENT



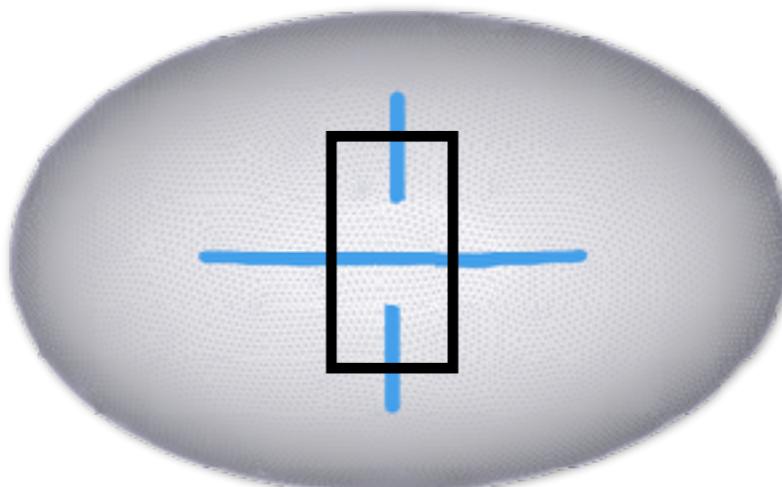
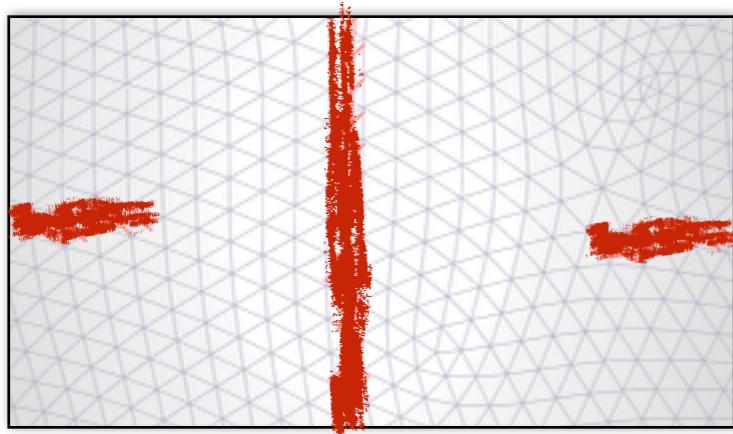
[Ray et al. 2008]



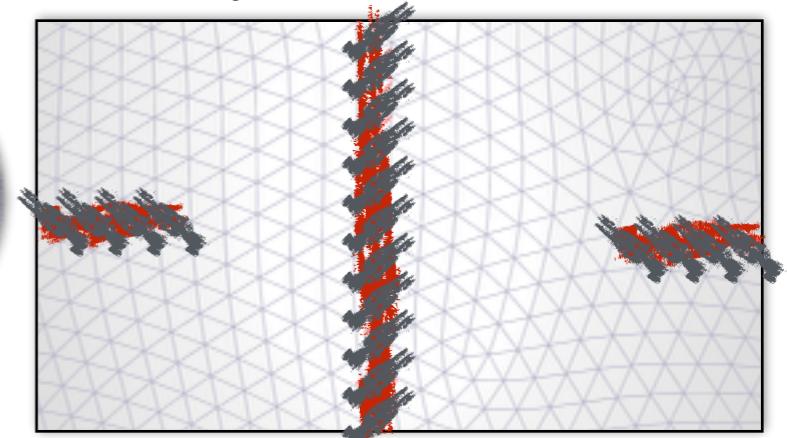
[Fisher et al. 2007]

- Complete or Partial

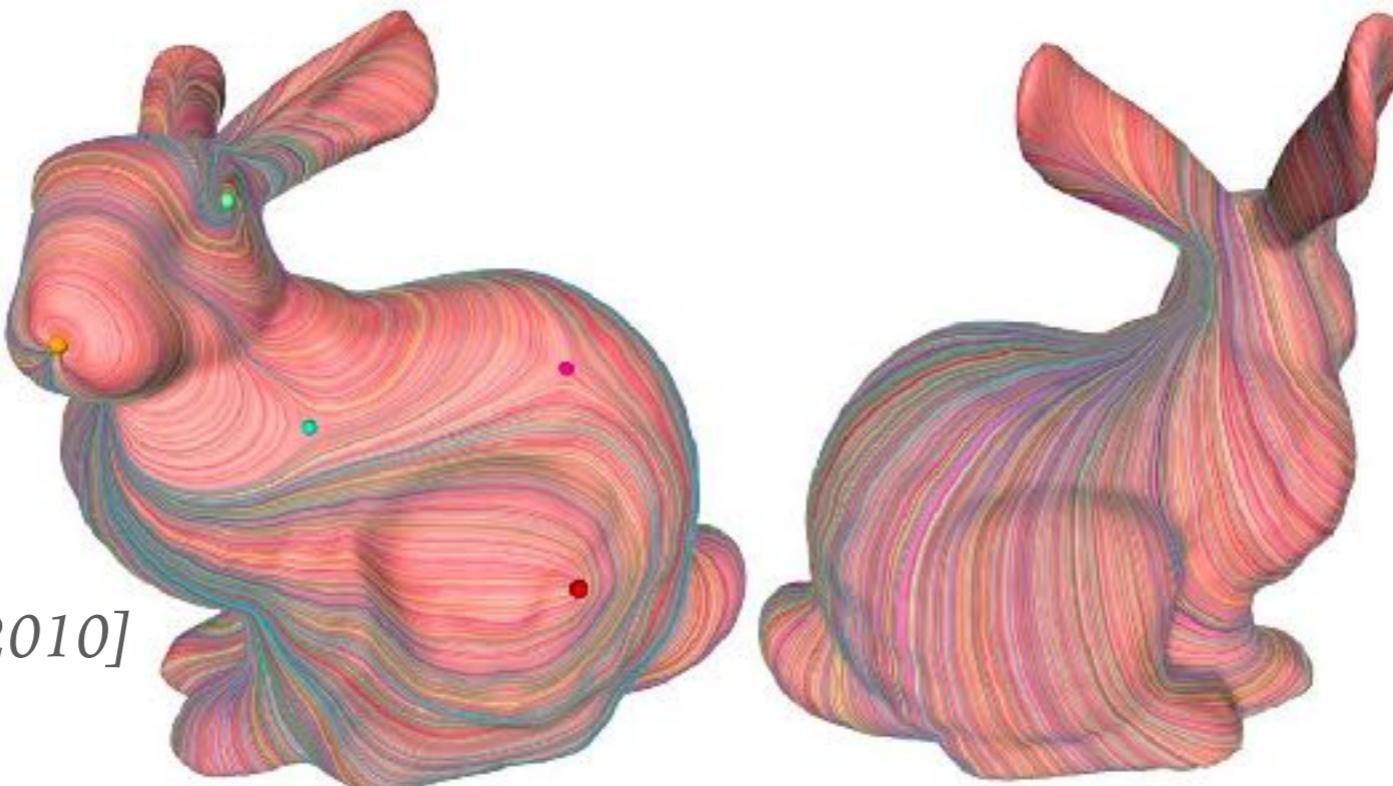
partial constraints



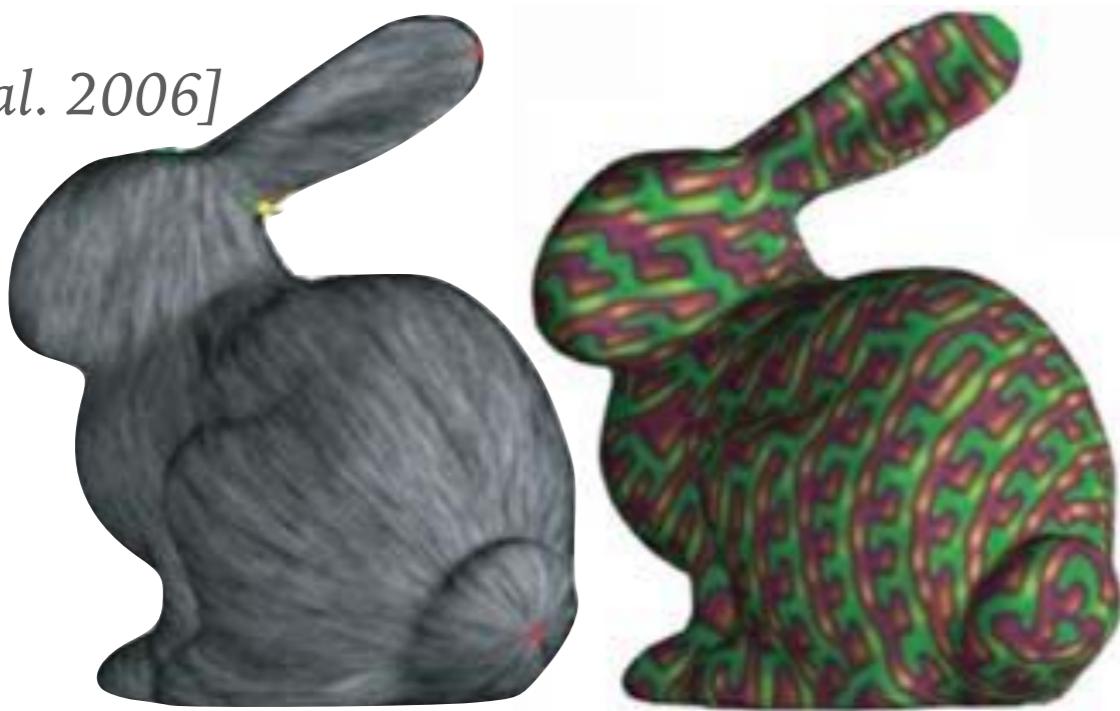
full constraints



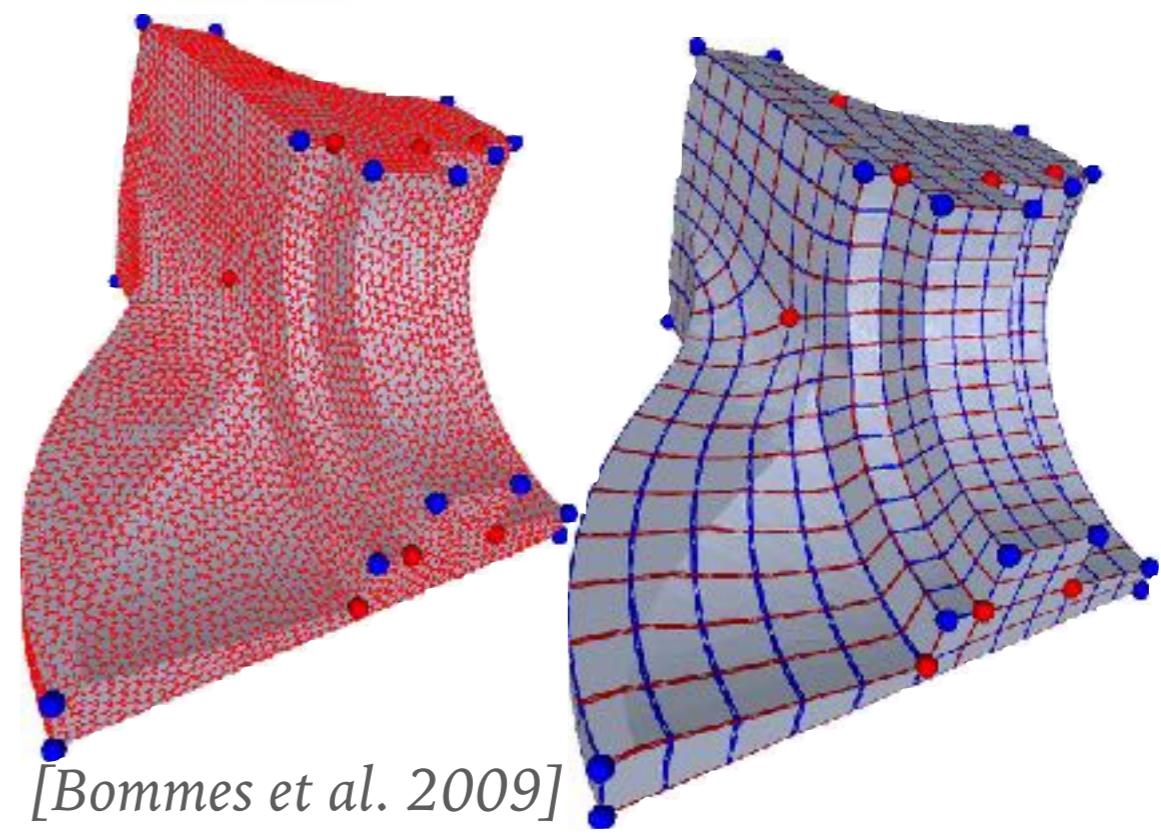
CONSTRAINTS - TOPOLOGY



[Crane et al. 2010]

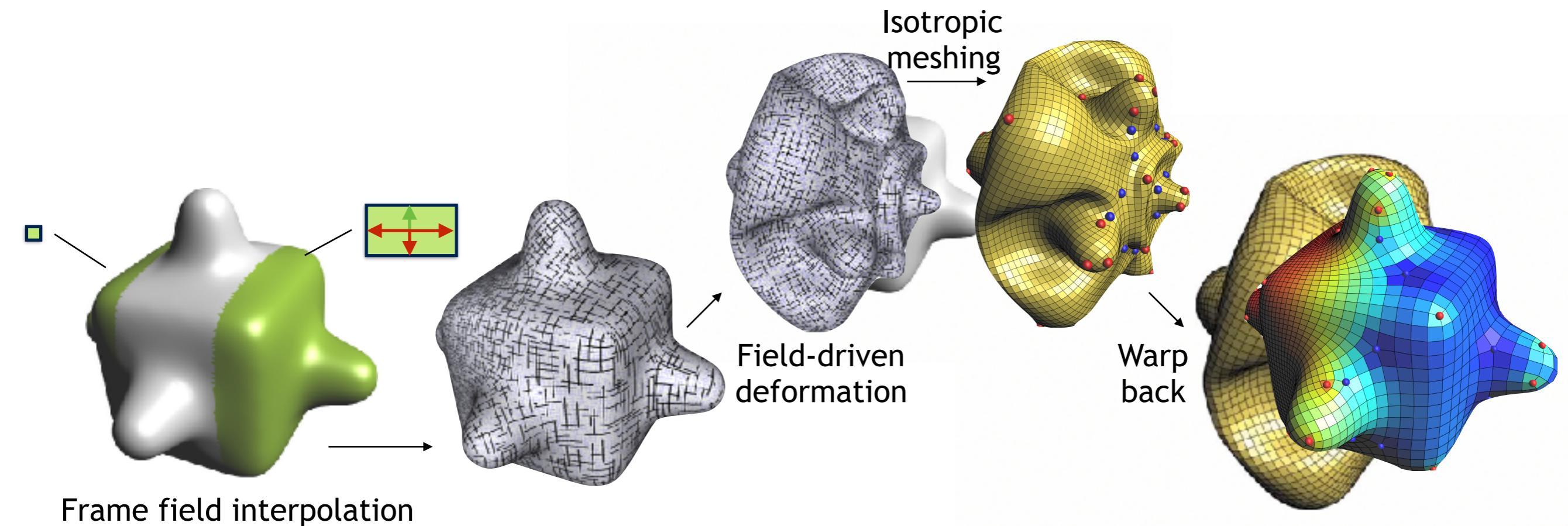


[Zhang et al. 2006]



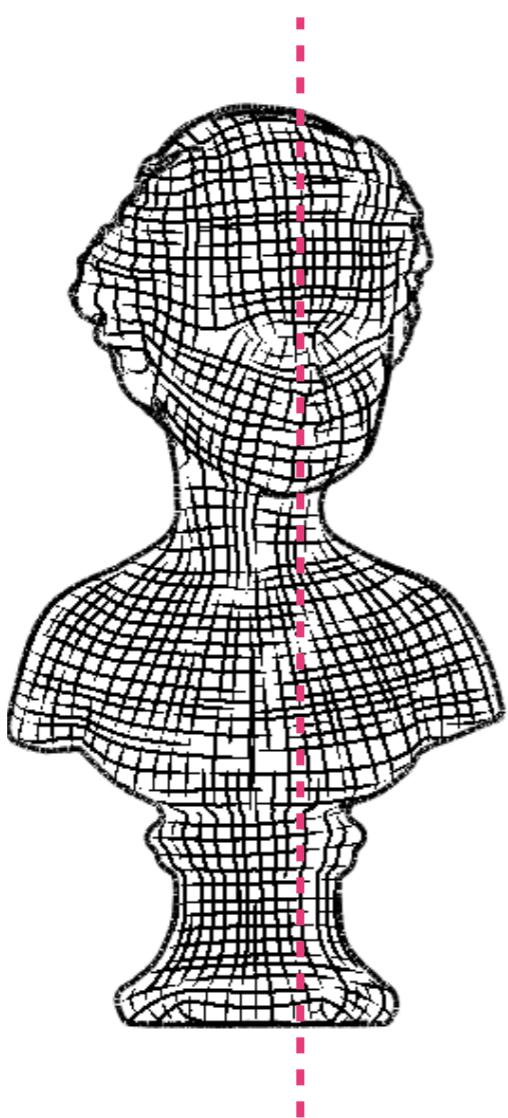
[Bommes et al. 2009]

CONSTRAINTS - SCALE

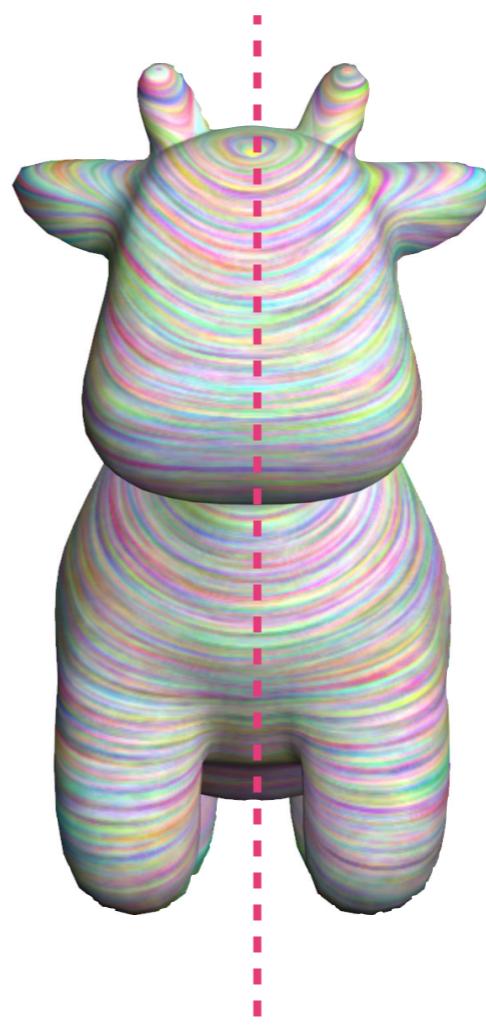


[Panizzo et al. 2014]

CONSTRAINTS - SYMMETRY

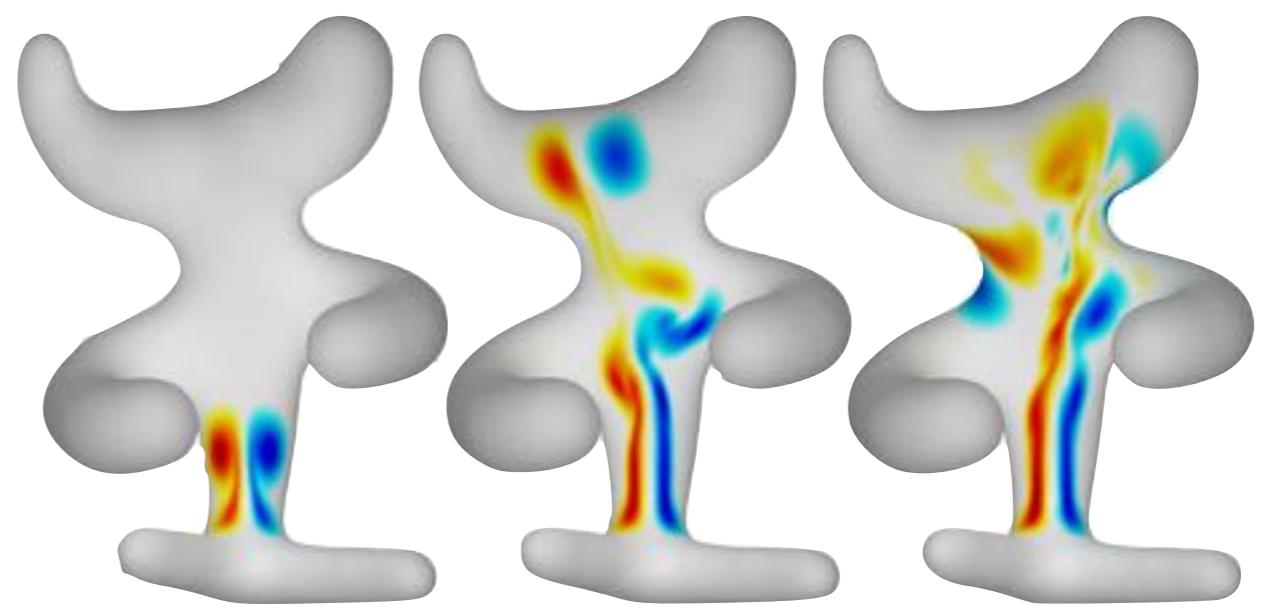
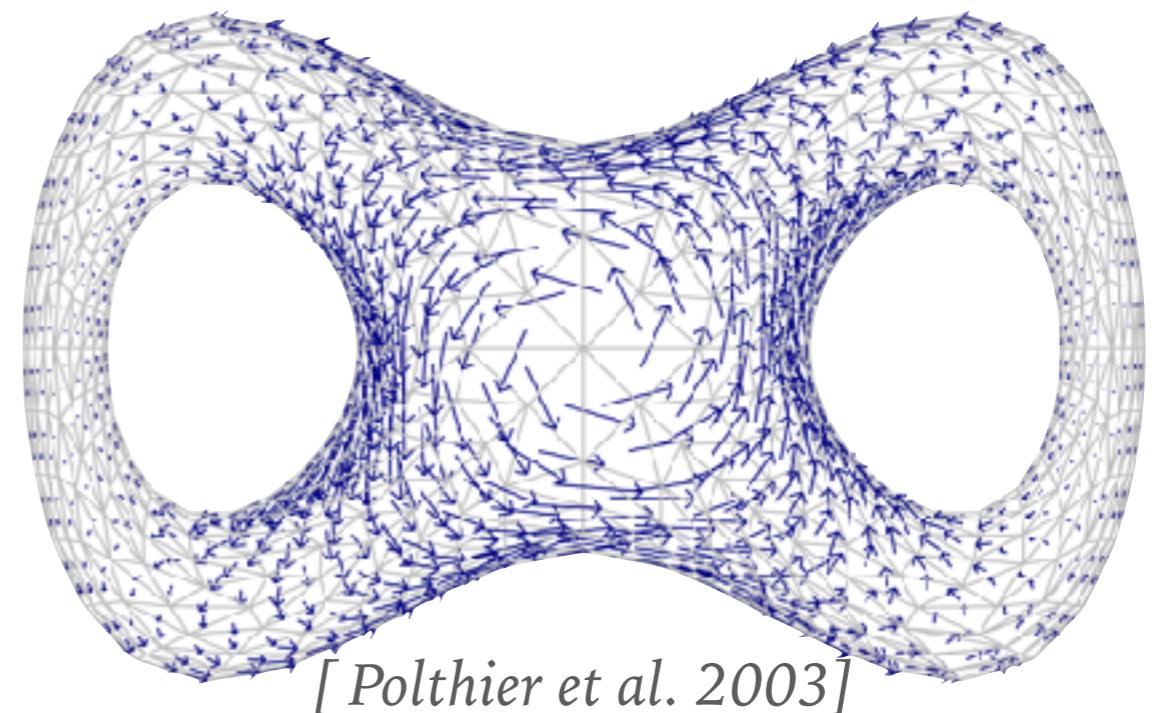
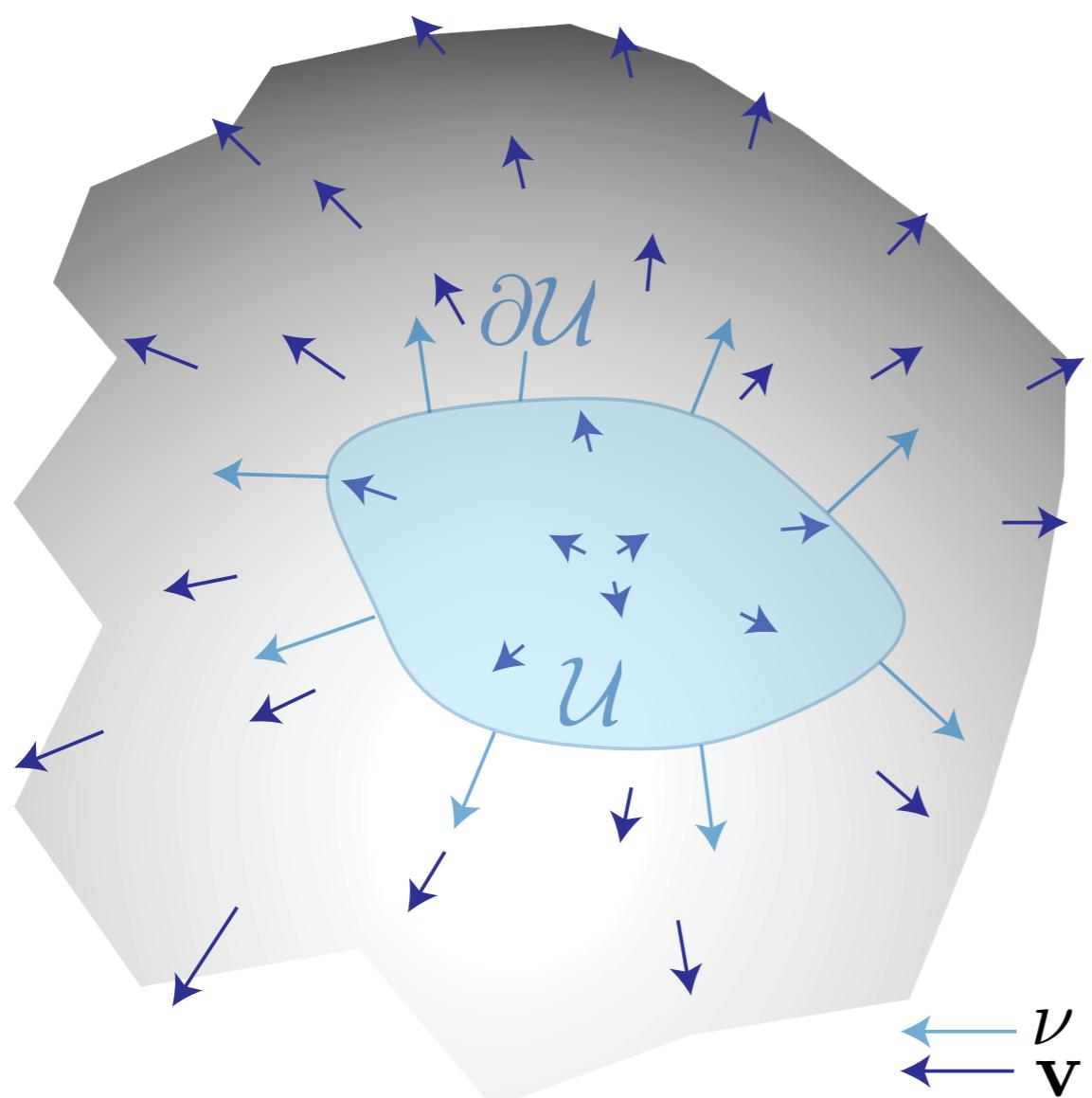


[Panozzo et al. 2012]



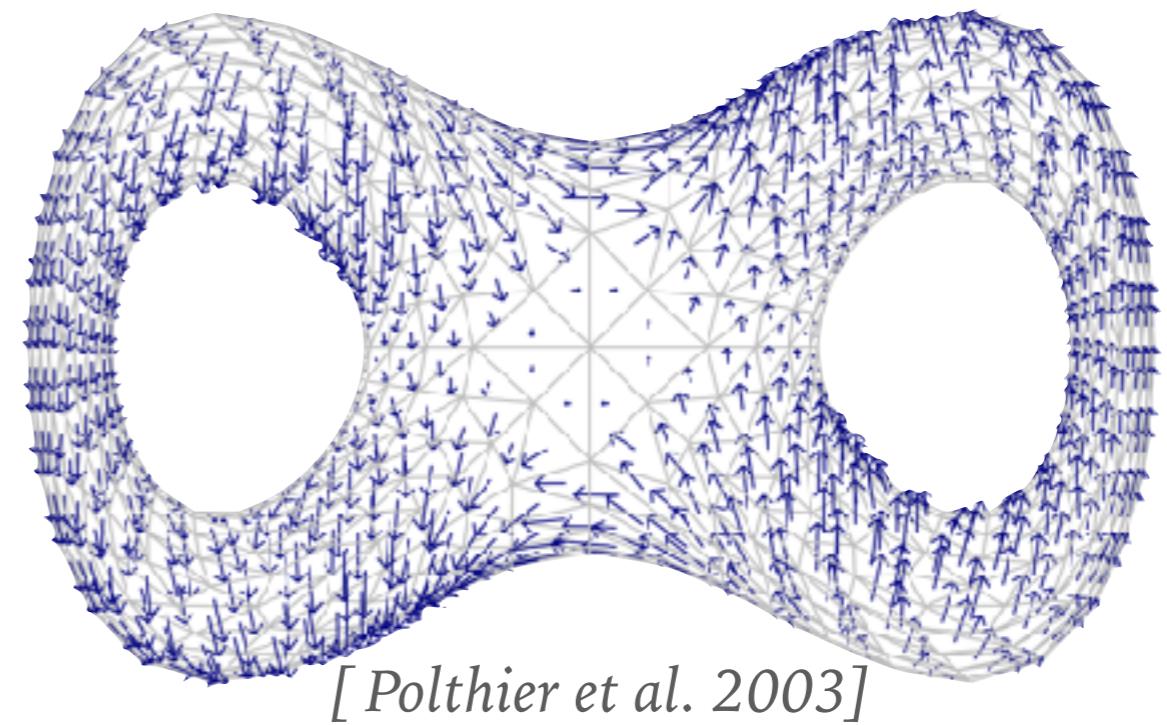
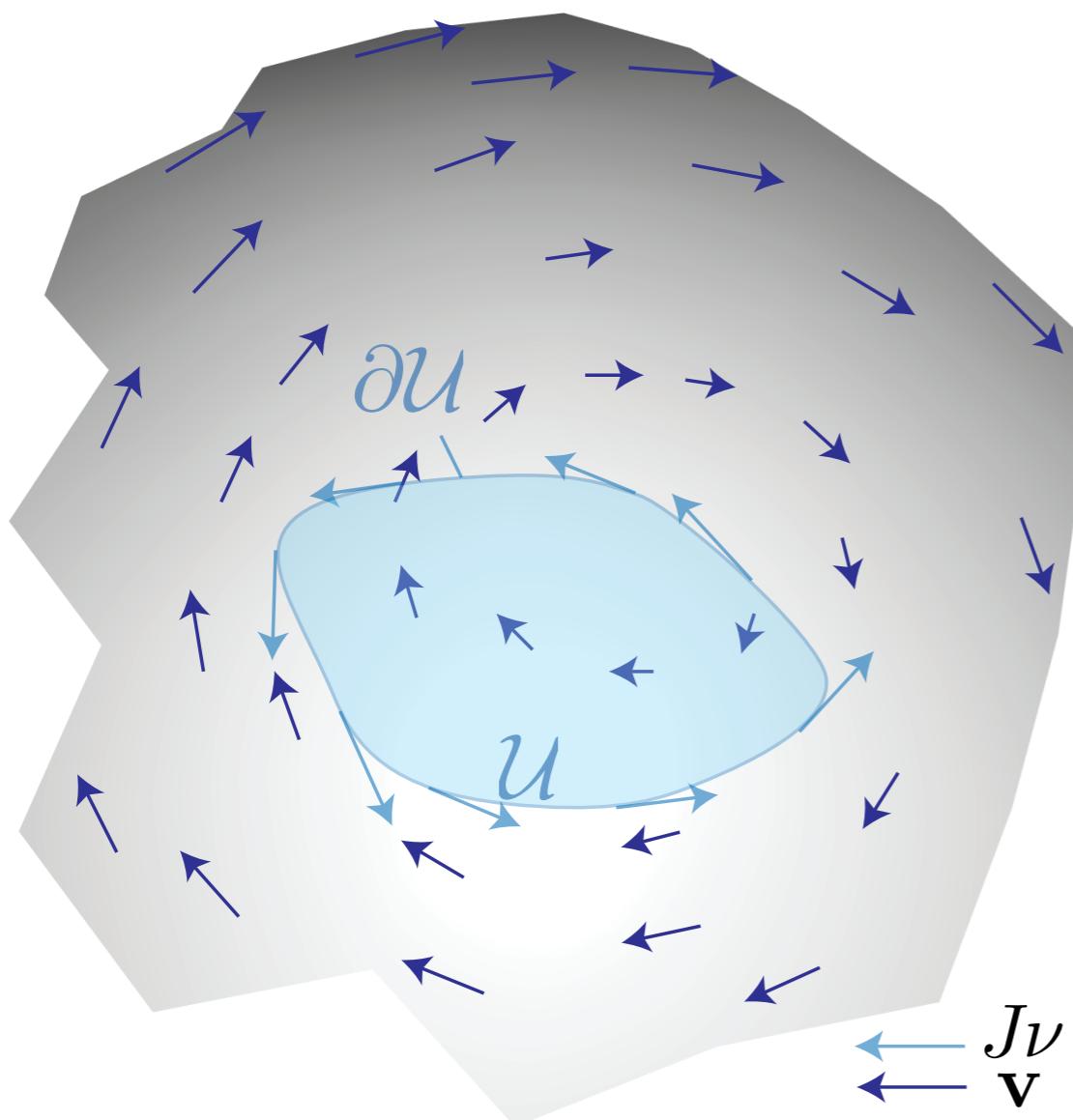
[Azencot et al. 2013]

CONSTRAINTS - DIFFERENTIAL - DIVERGENCE



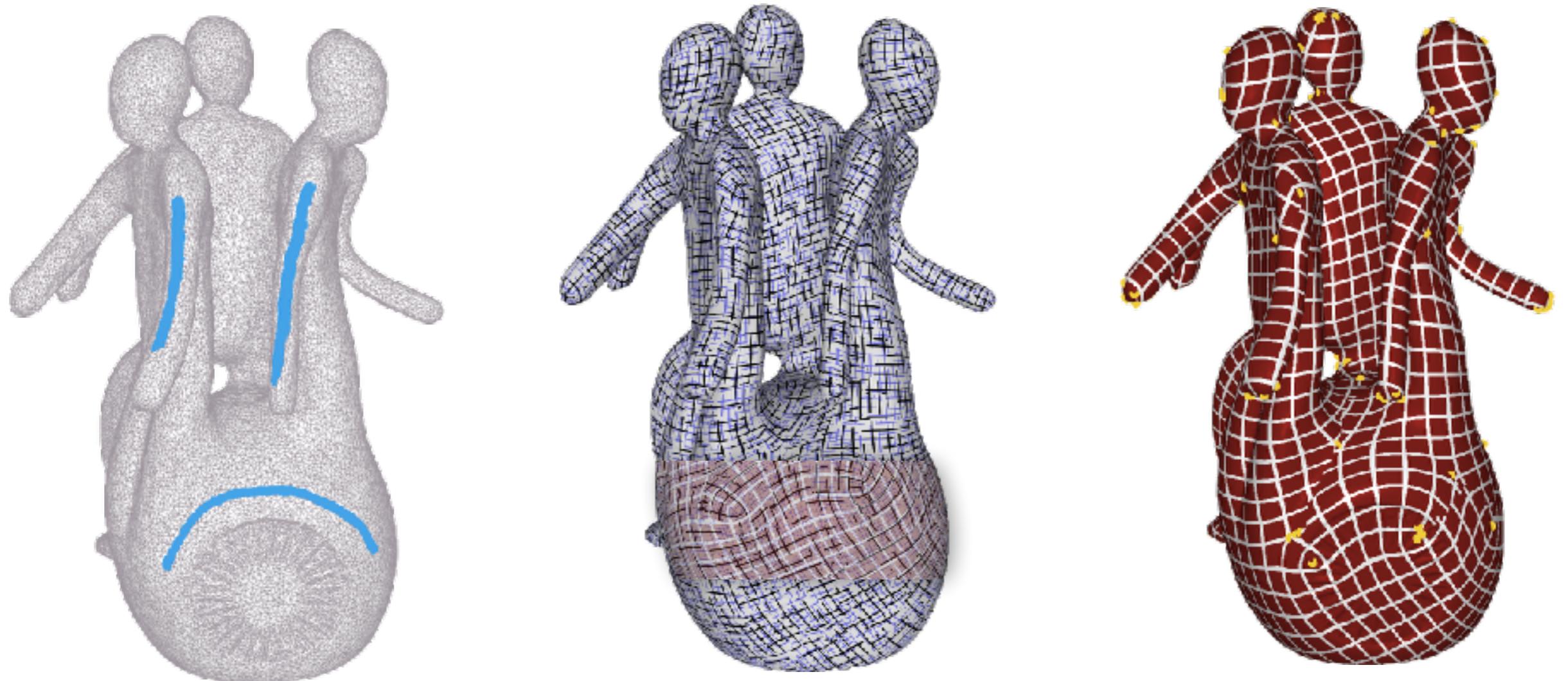
[Azencot et al. 2014]

CONSTRAINTS - DIFFERENTIAL - CURL



[Polthier et al. 2003]

CONSTRAINTS - DIFFERENTIAL - CURL



[Diamanti et al. 2015]

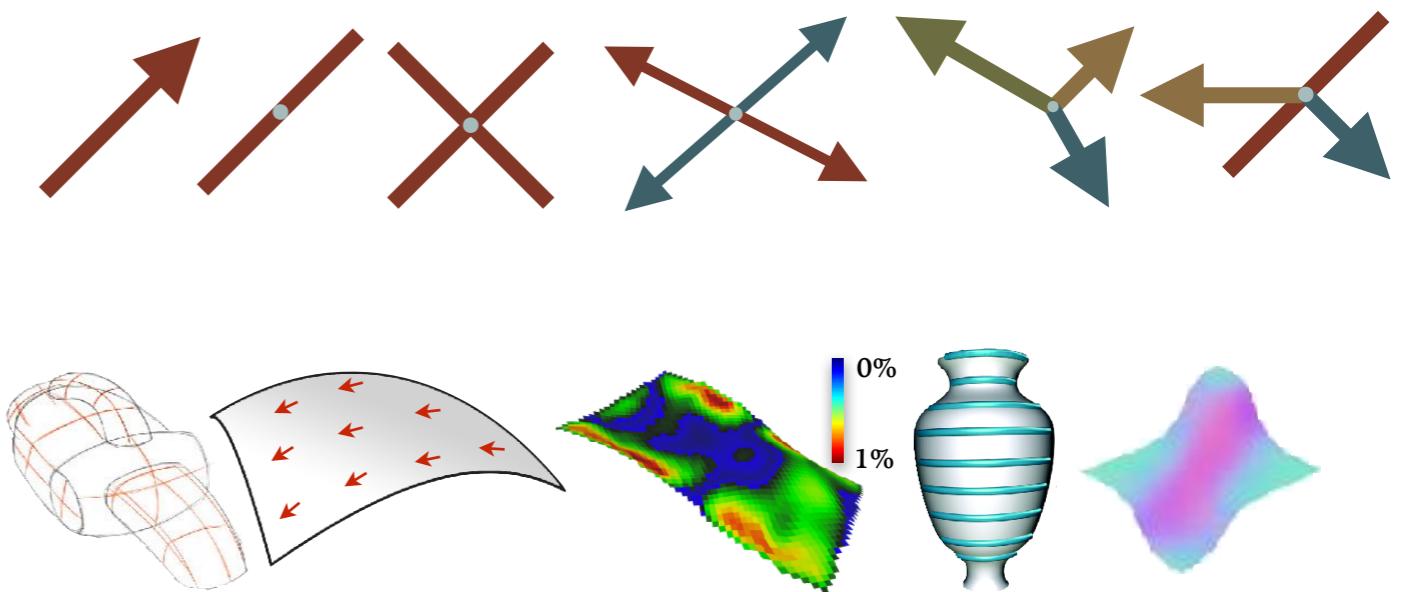
HOW TO CHOOSE THE RIGHT METHOD?

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CHOOSING THE RIGHT METHOD

- Choose the type of object
- Determine what the ideal field is (objective)
- Consider the types of guarantees required
- Consider design strategy in terms of efficiency, convergence,...
 - this affects the choice of representation
- Consider discretization preference

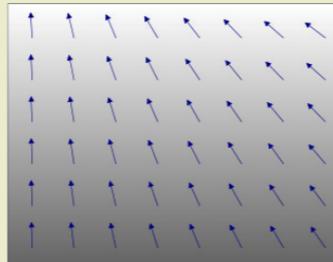
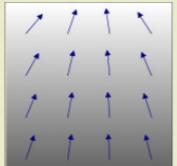


CASE STUDY: VECTOR FIELDS



Objective: Fairness

[Pedersen et al. 1995]



[Turk et al. 2001]



[Praun et al. 2000]

Objective: Other

Isometries - Killing

[Ben-Chen et al. 2010] [Azencot et al. 2013]

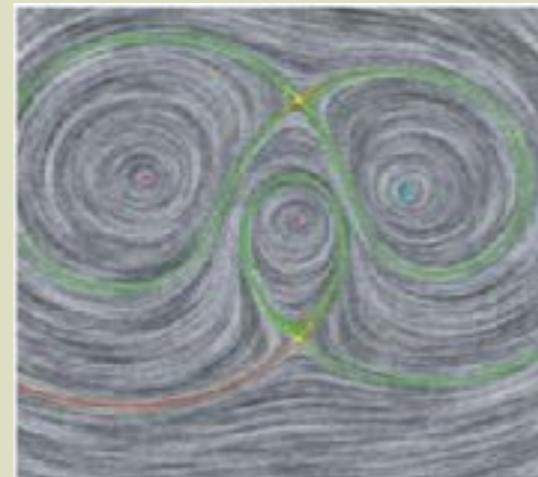
Curl / Divergence Control

[Fisher et al. 2007]

[Zhang et al. 2006]

Symmetries

[Azencot et al. 2013]



Constraints: Directional

hard

[Pedersen et al. 1995]

[Praun et al. 2000]

[Azencot et al. 2013]

soft

[Turk et al. 2001]

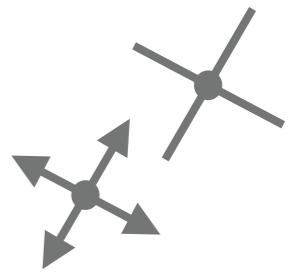
[Zhang et al. 2006]

soft + singularity control

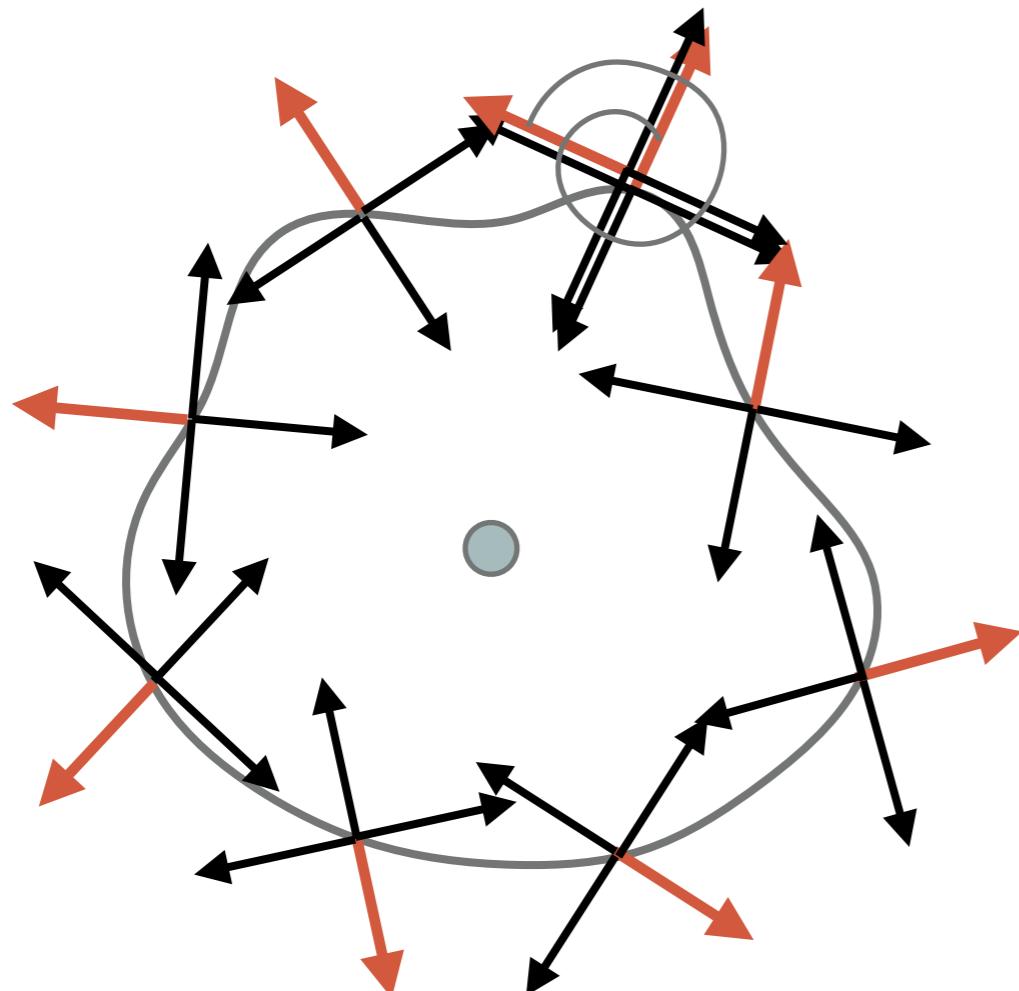
[Fisher et al. 2007]

[Zhang et al. 2006]

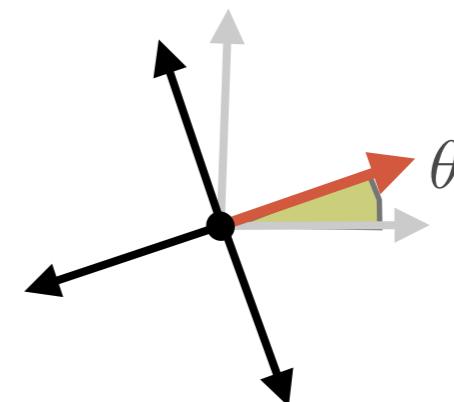
CASE STUDY: N-DIRECTIONAL FIELD, FIXED TOPOLOGY



- Objective: “As-parallel-as-possible”
- Singularity prescription straightforward with angle based representations

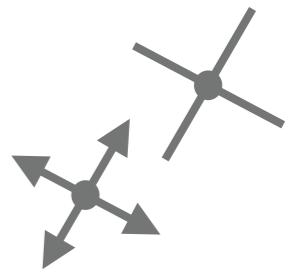


desired singularity index = 5/4

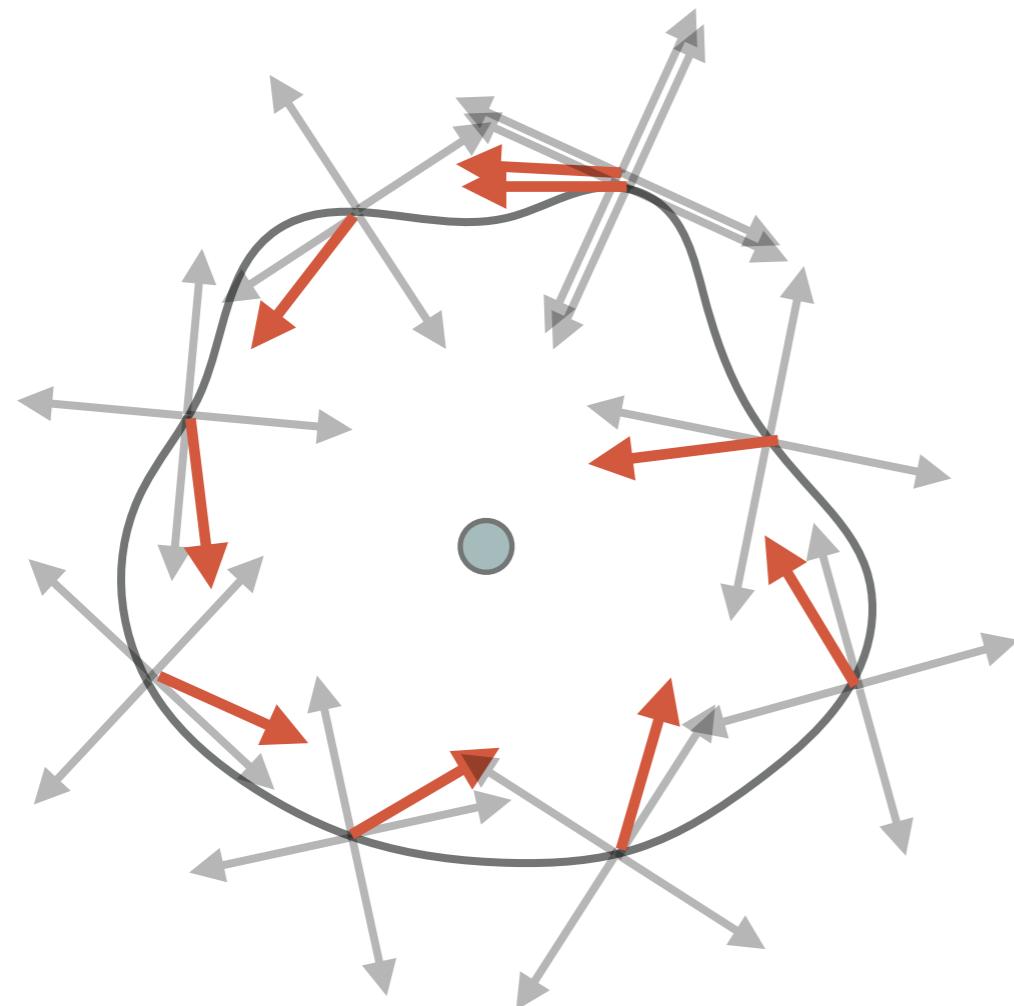


*angle-based representation
of a 4-direction field*

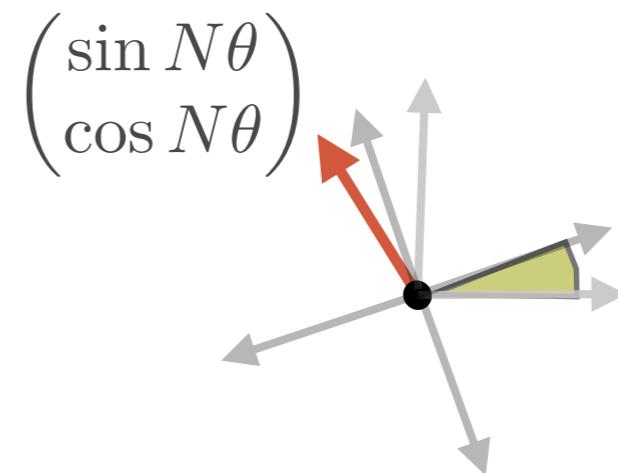
CASE STUDY: N-DIRECTIONAL FIELD, FIXED TOPOLOGY



- Objective: “As-parallel-as-possible”
- Cartesian/Complex representative insensitive to 2π rotations



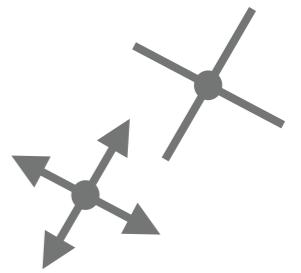
desired singularity index = 5/4



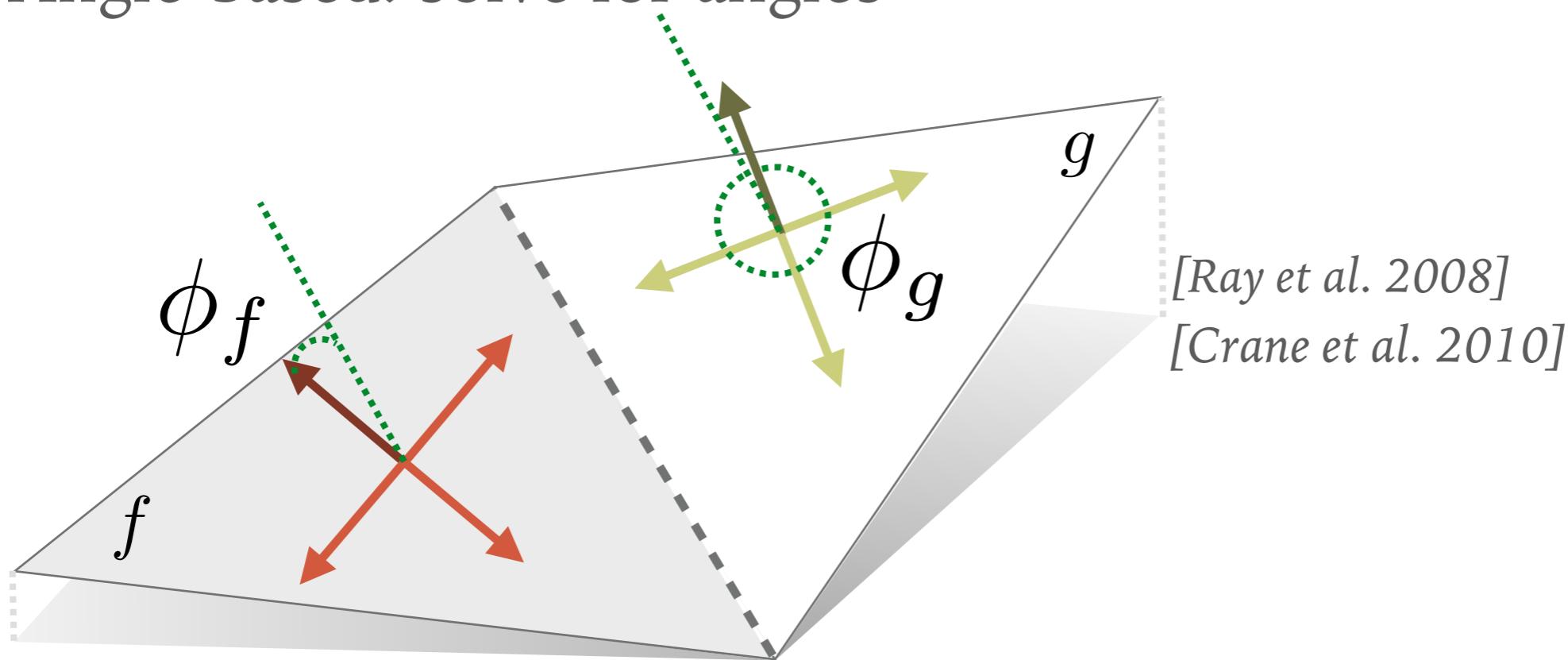
*cartesian representation of
a 4-direction field*

*How to prescribe angle difference
 $2\pi + \pi/4$ with principal rotation/
matching?*

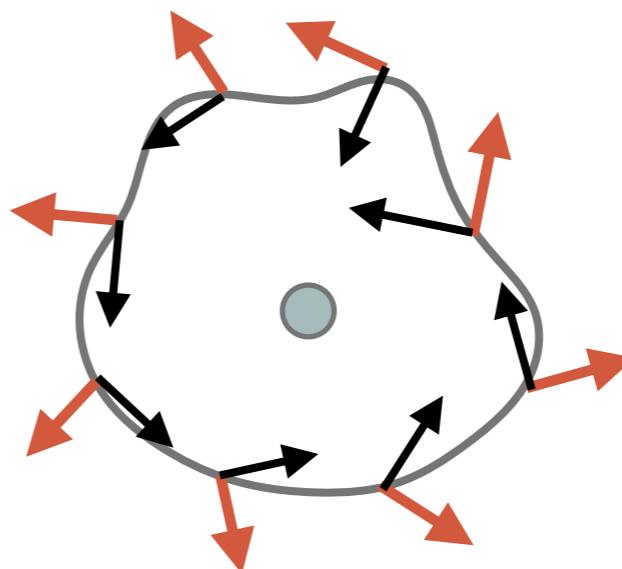
CASE STUDY: N-DIRECTIONAL FIELD, FIXED TOPOLOGY



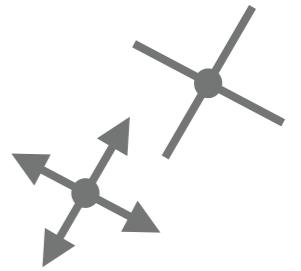
- Angle-based: solve for angles



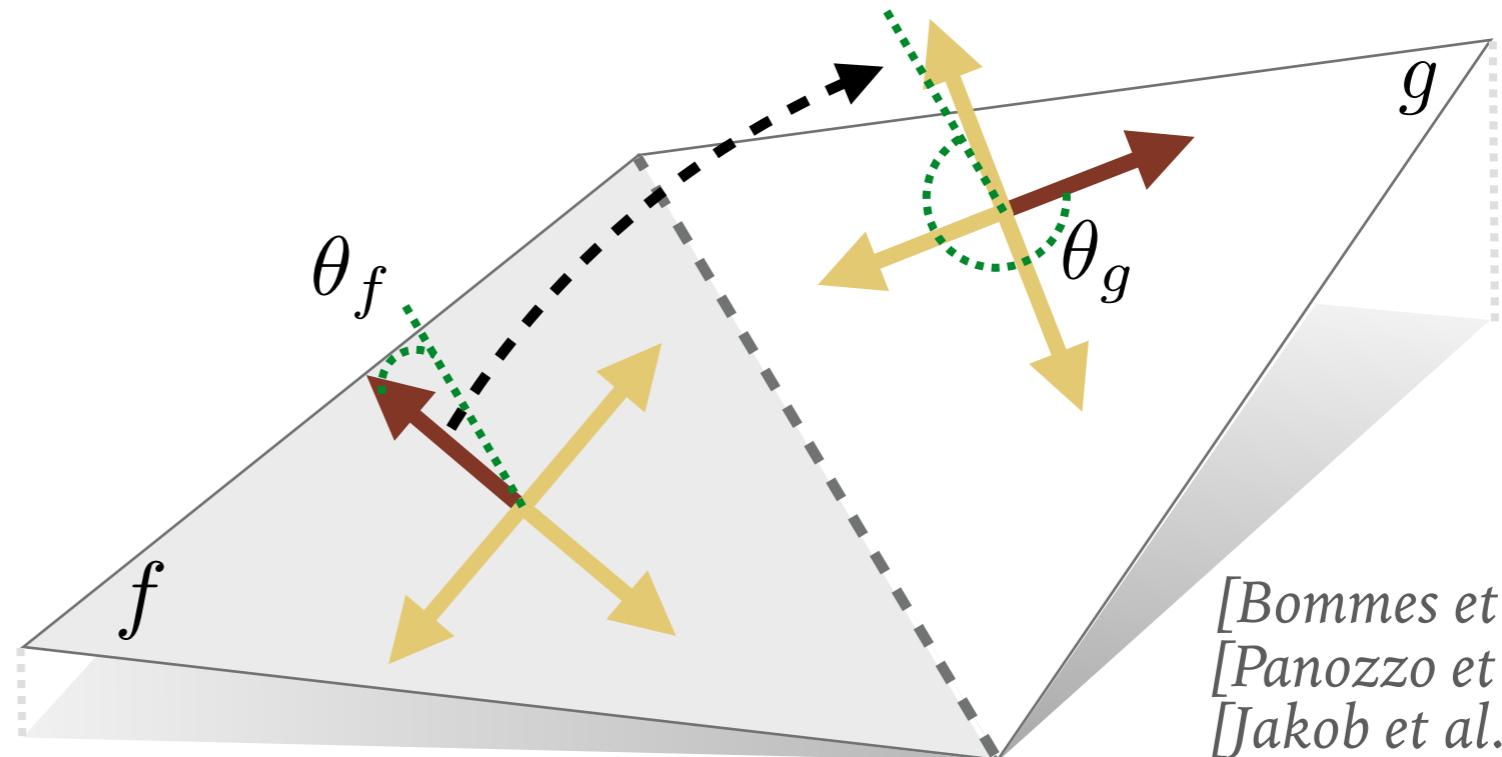
- linear system
- at least one constraint needed



CASE STUDY: N-DIRECTIONAL FIELD, FREE TOPOLOGY



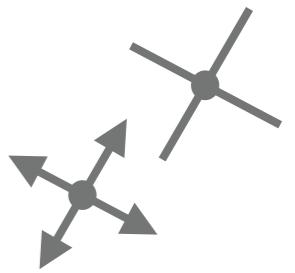
- Objective: “As-parallel-as-possible”
- Explicitly model topology (typically angle-based)
 - Matchings are explicitly modeled
 - Mixed Integer Optimization
 - Local minima



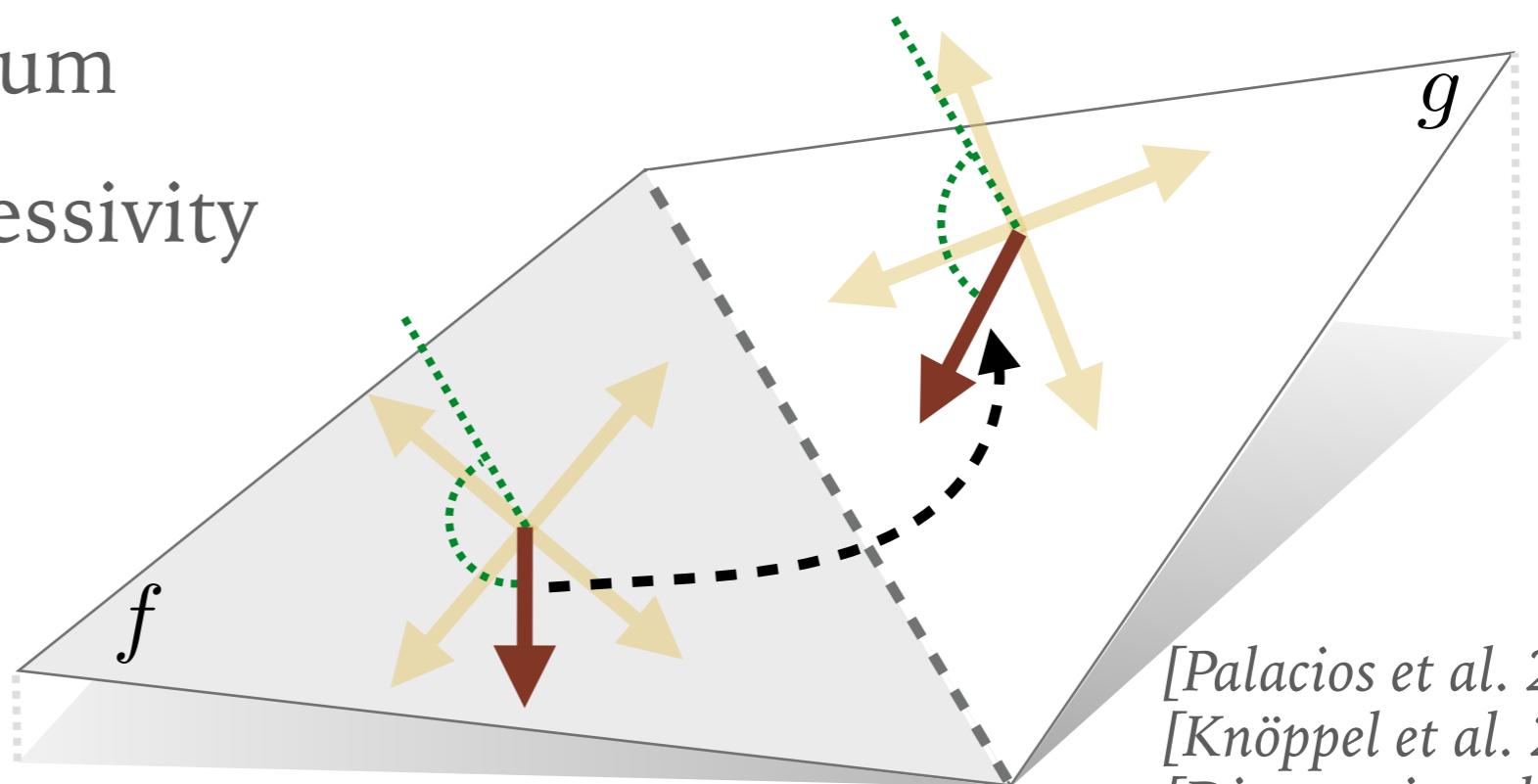
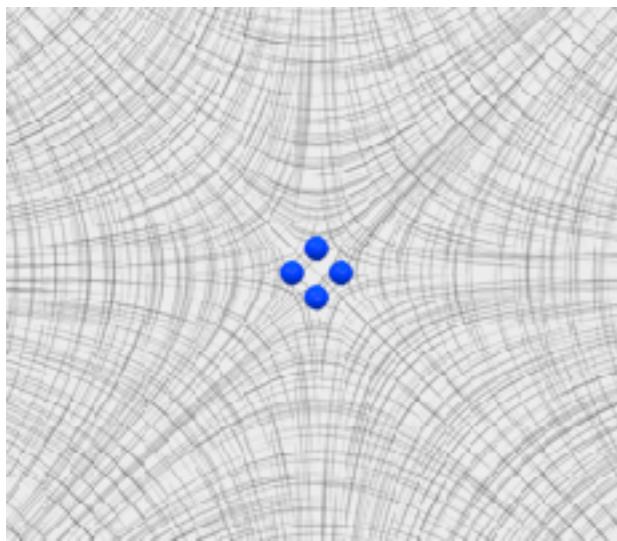
[Bommes et al. 2009]
[Panozzo et al. 2012]
[Jakob et al. 2015]

$$(\theta_f + \rho_{fg} \frac{\pi}{2} - \theta_g)^2 \quad \rho_{fg} \in \mathbb{I}$$

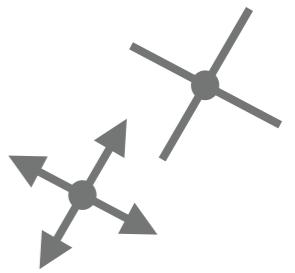
CASE STUDY: N-DIRECTIONAL FIELD, FREE TOPOLOGY



- Objective: “As-parallel-as-possible”
- Implicit topology with principal matchings (typically cartesian/complex)
 - Linear problem
 - Global optimum
 - Limited expressivity

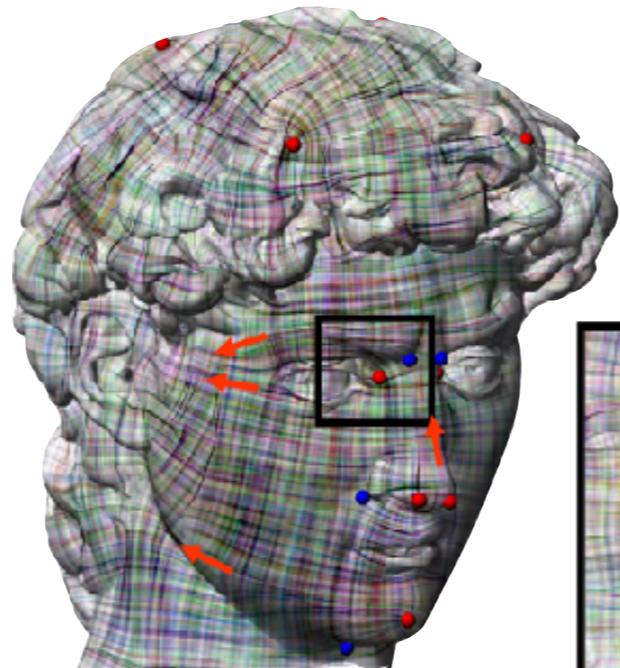


[Palacios et al. 2007]
[Knöppel et al. 2013]
[Diamanti et al. 2014]

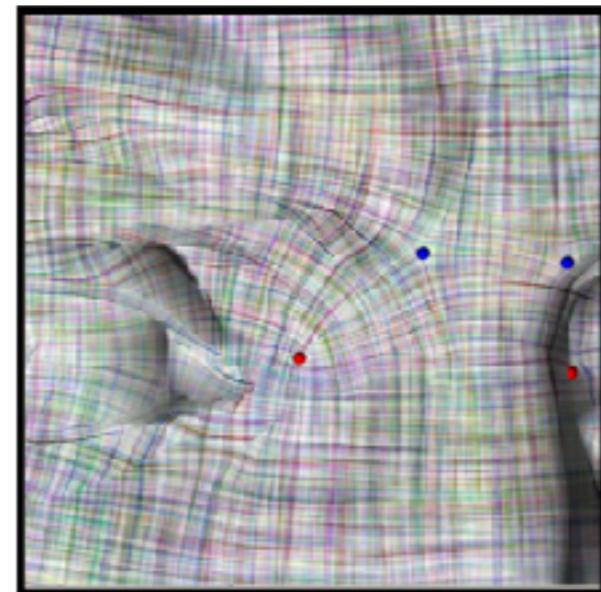


HOW ABOUT CONSTRAINTS?

- Sparse “Hard”



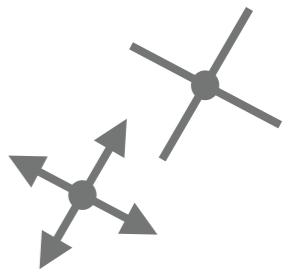
[Palacios et al. 2007]
[Ray et al. 2008]
[Jakob et al. 2015]



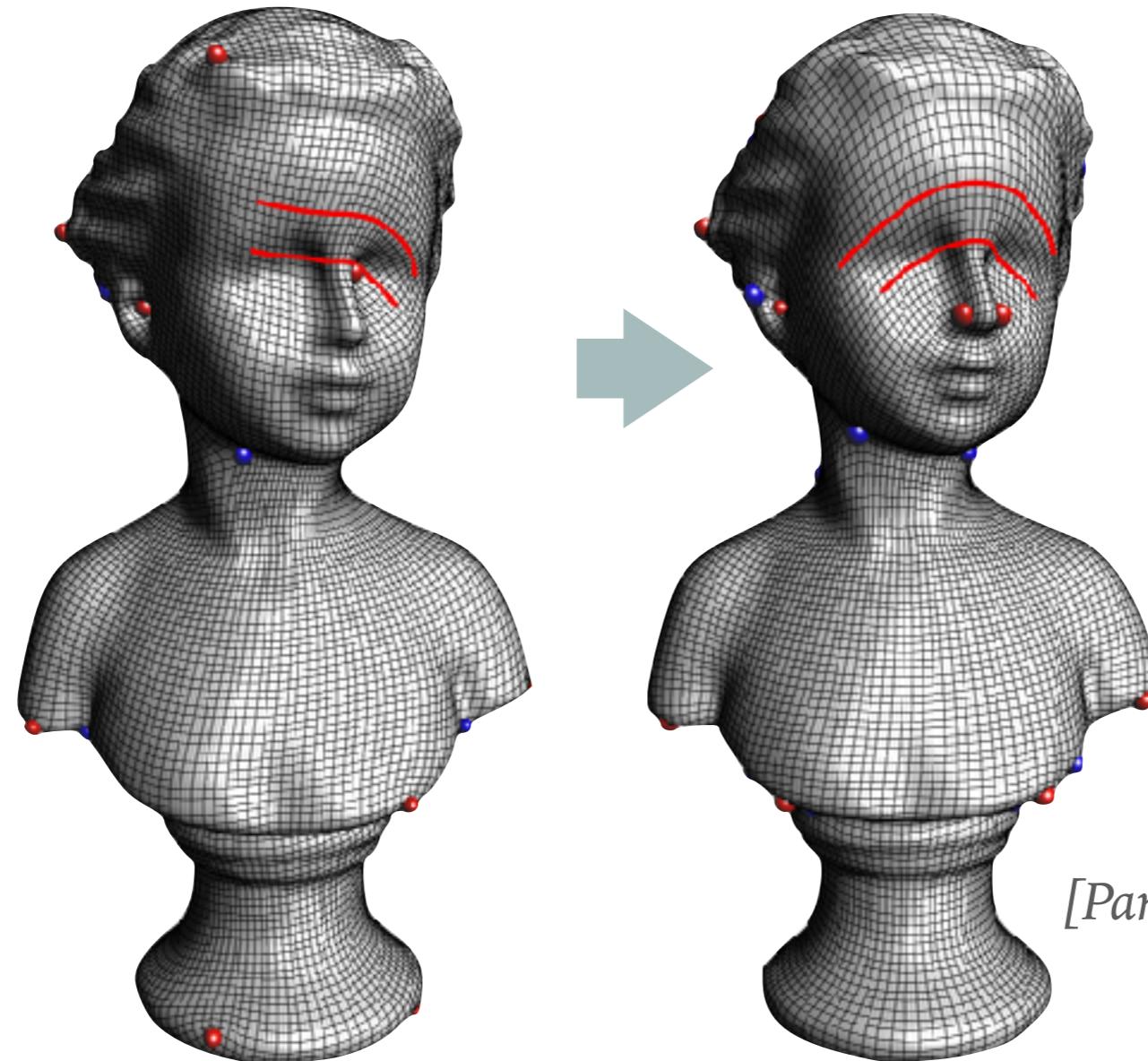
[Crane et al. 2010]
[Bommes et al. 2009]
[Hertzmann et al. 2000]
[Diamanti et al. 2014]

- Also partial constraints! [Iarussi et al. 2015] [Diamanti et al. 2015]

HOW ABOUT CONSTRAINTS?

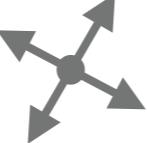


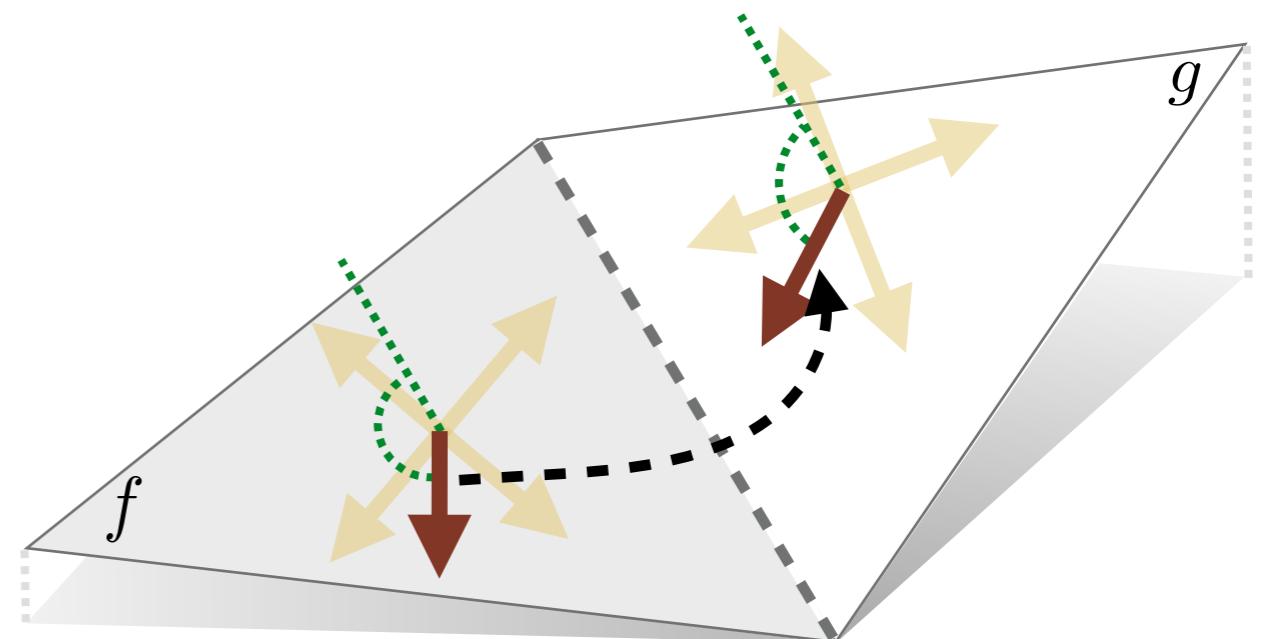
- Soft constraints



[Panozzo et al. 2012]

AVOIDING THE TRIVIAL SOLUTION

- Eg. N-Vector Fields  with a cartesian/complex representation
- Zero Field is perfectly smooth!
- Constraints are necessary
 - Per-vector unit-norm constraint [Palacios et al. 2007]
 - Integrated norm constraint [Knöppel et al. 2013]
 - Enough hard constraints [Diamanti et al. 2014]

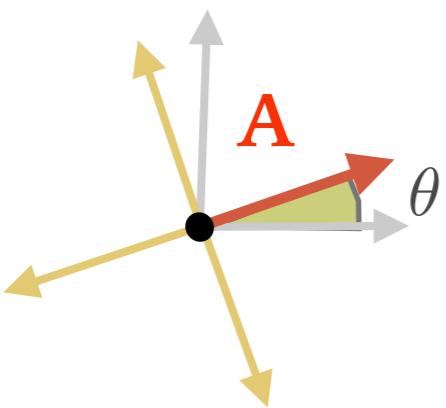


COMPARATIVE ANALYSIS

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COMPARISONS AND DEMOS

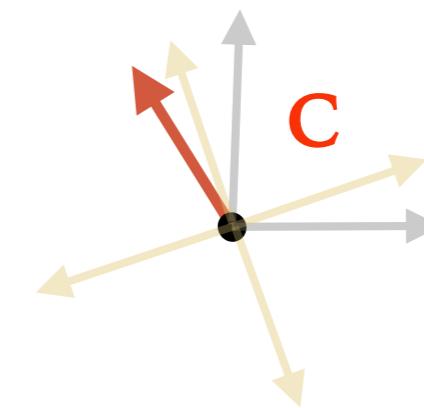


angle-based approaches

[Bommes et al. 2009]

[Panozzo et al. 2014]

[Jakob et al. 2015]



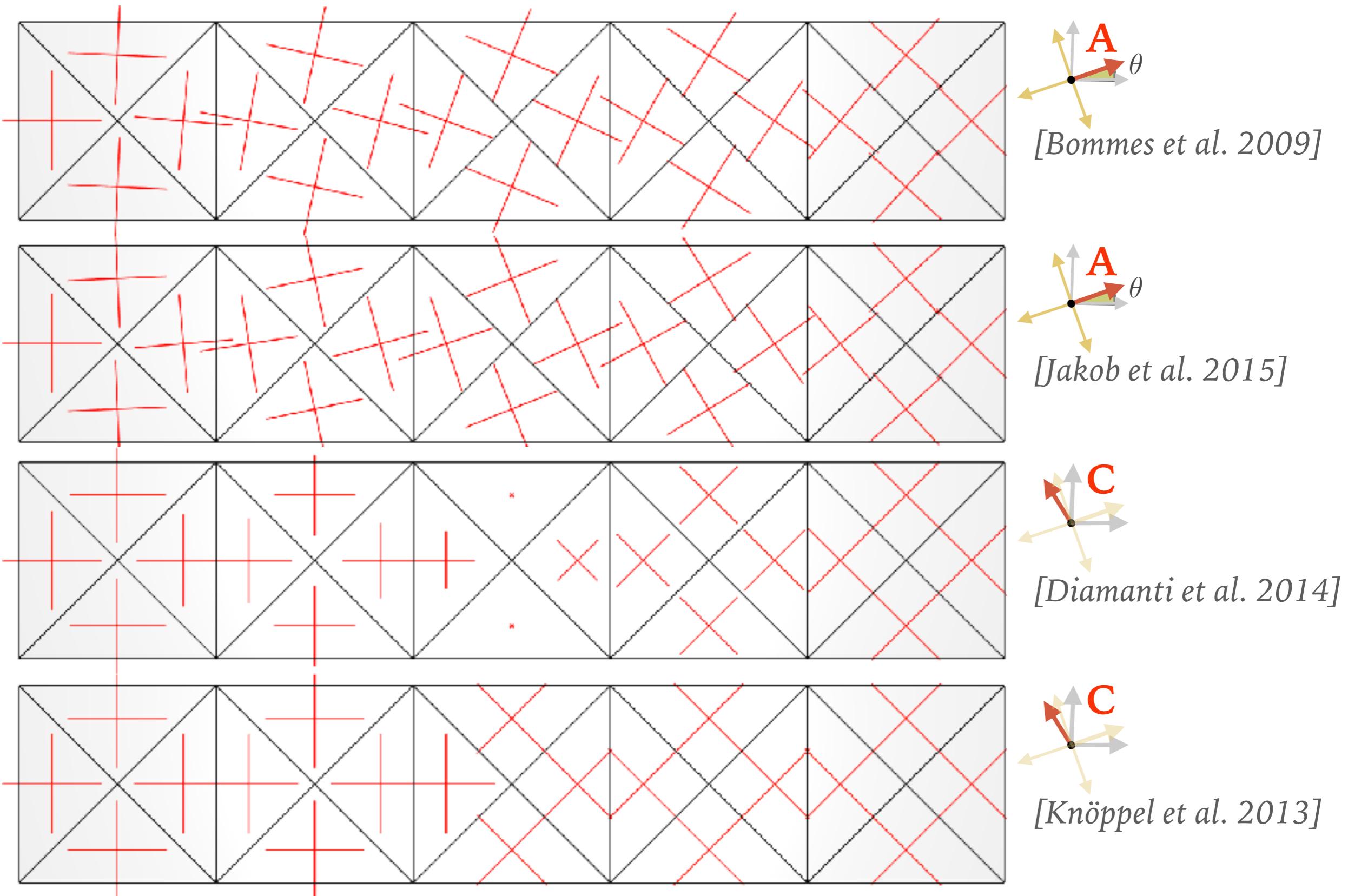
cartesian/complex approaches

[Knöppel et al. 2013]

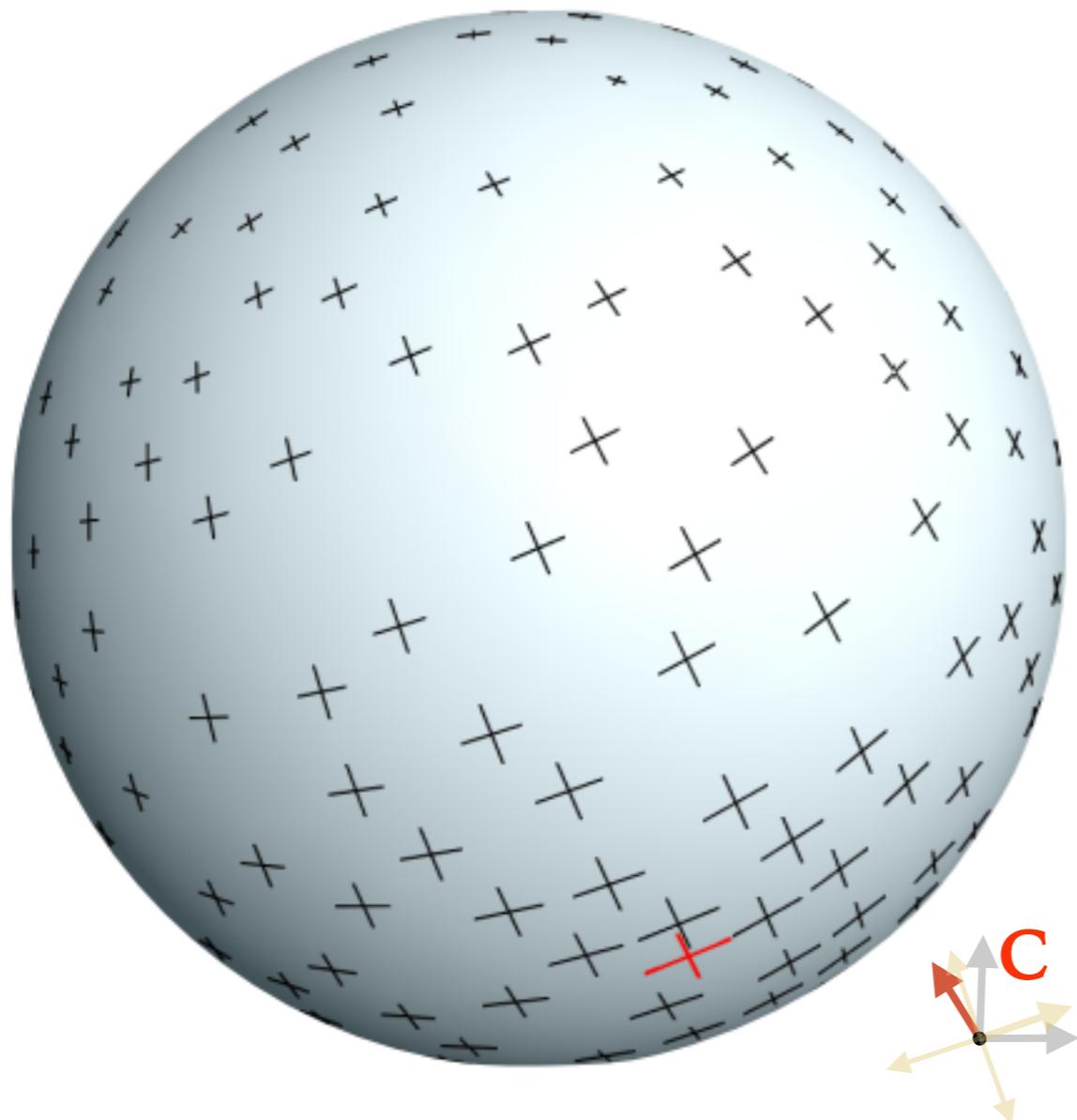
[Diamanti et al. 2014]

[Diamanti et al. 2015]

4-DIRECTIONAL FIELD

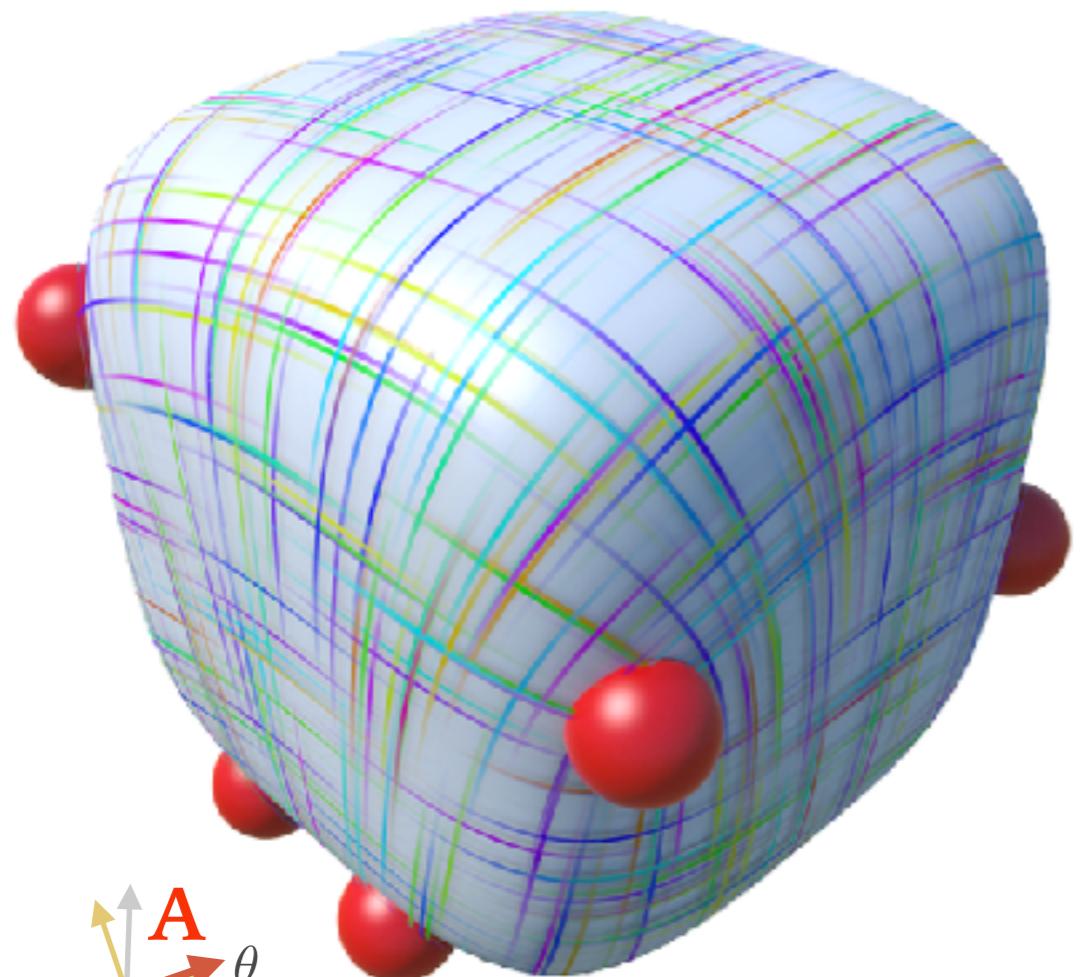


MAGNITUDE IN CARTESIAN REPRESENTATIONS - CONSTRAINTS

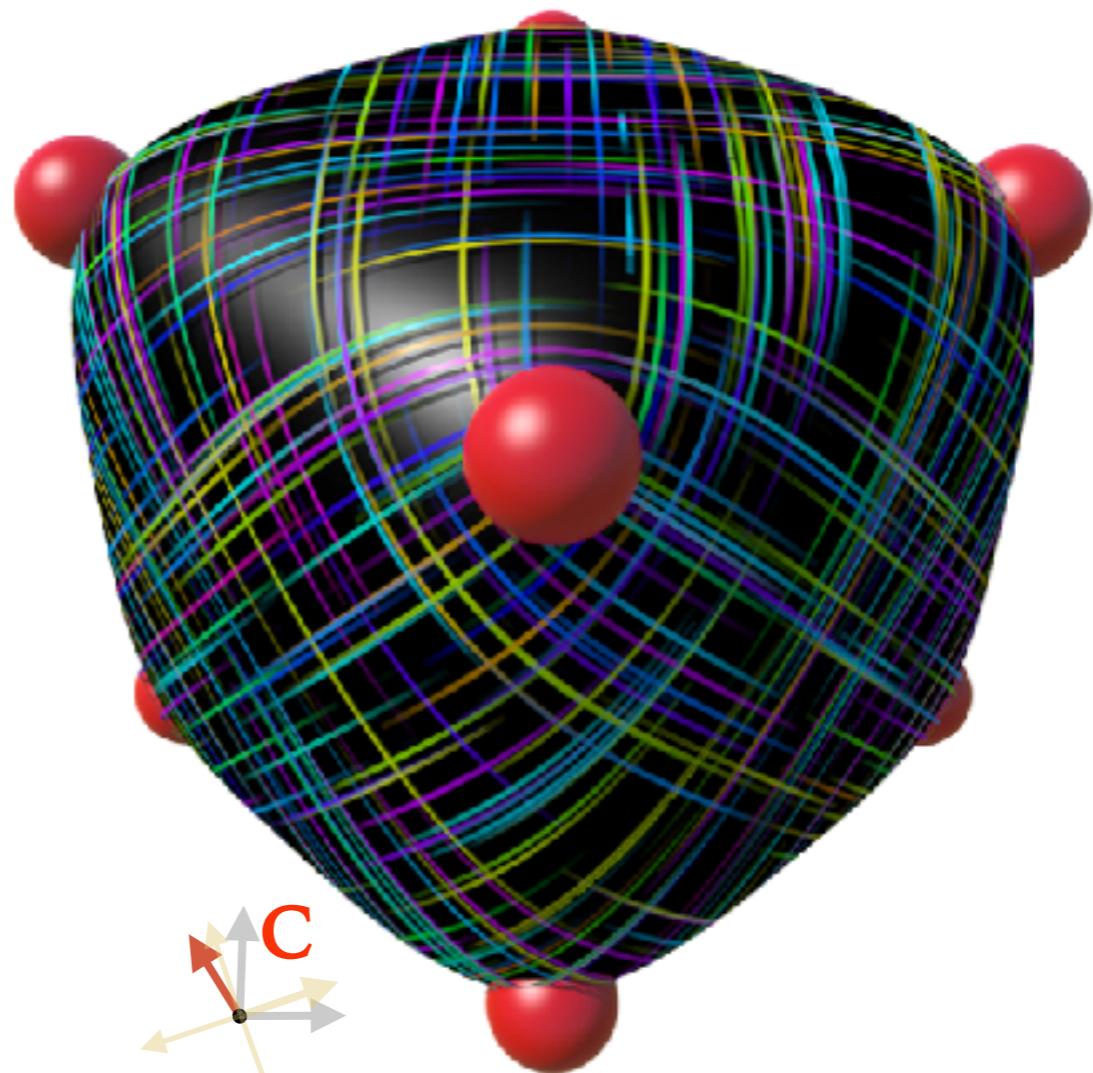


[Diamanti et al. 2014]

EFFECT OF THE FORMULATION

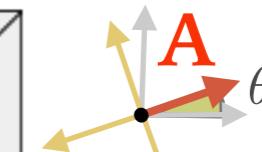
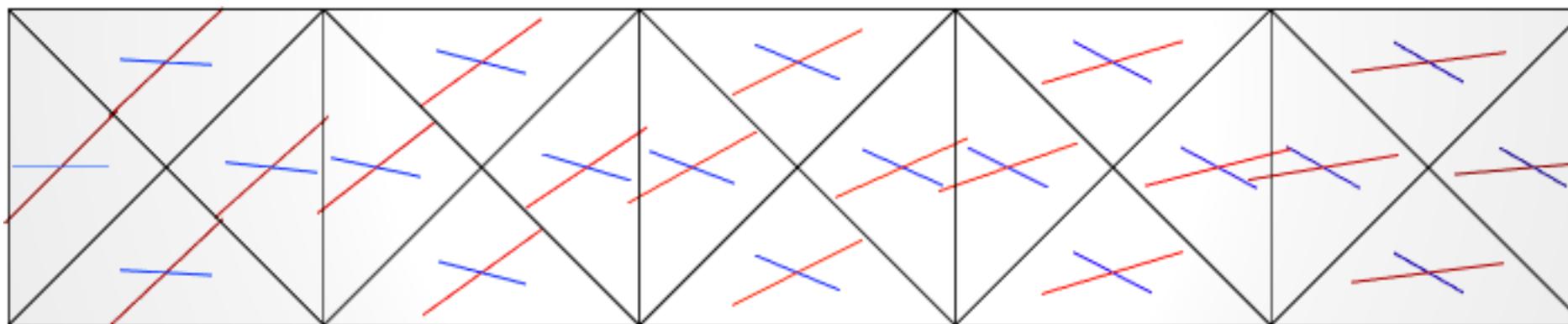


[Bommes et al. 2009]

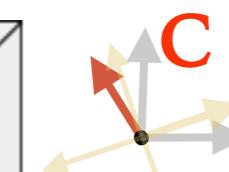
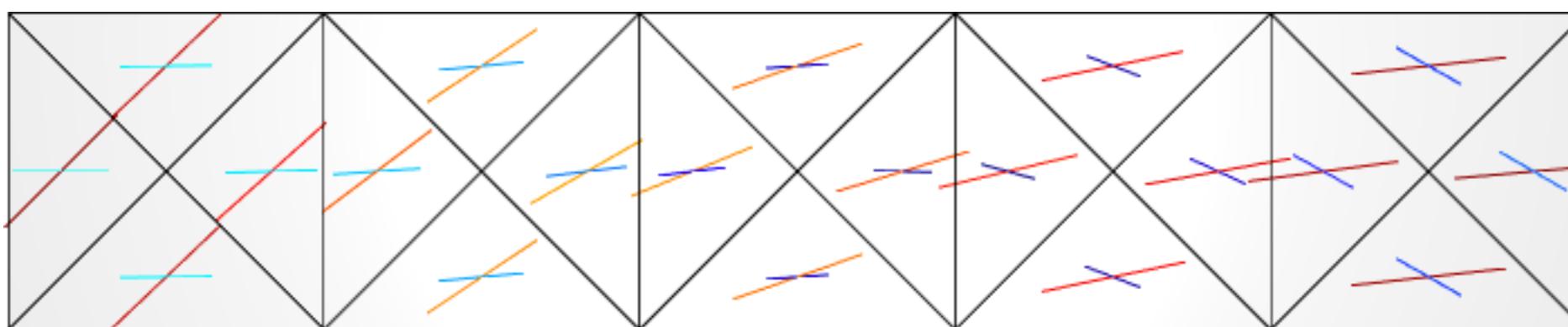


[Knöppel et al. 2013]

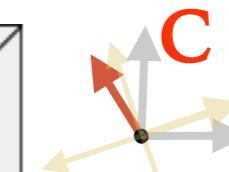
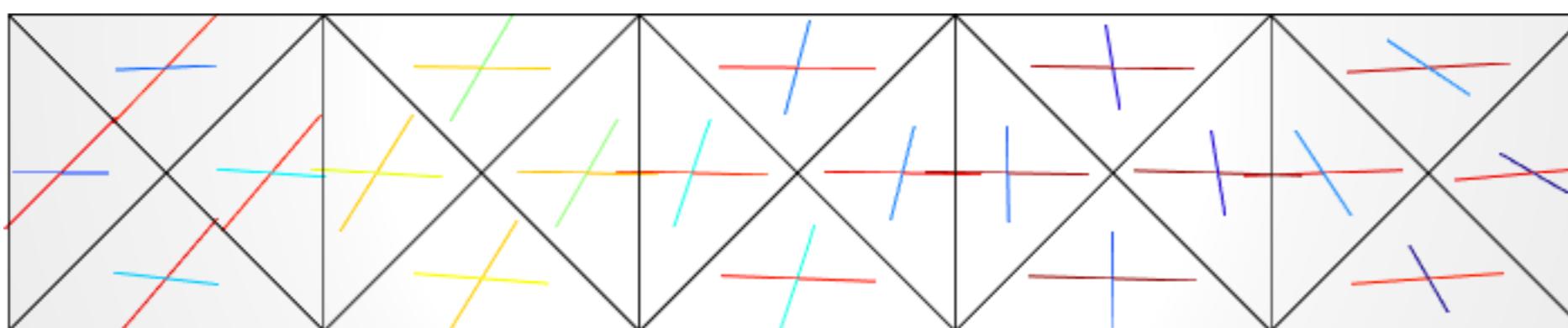
2^2 -VECTOR FIELD



[Panozzo et al. 2014]

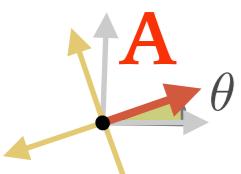


[Diamanti et al. 2014]

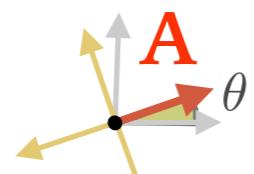


[Diamanti et al. 2015]

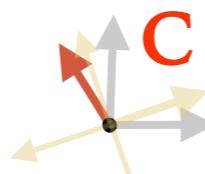
SINGULARITIES



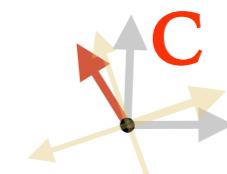
[Bommes et al. 2009]



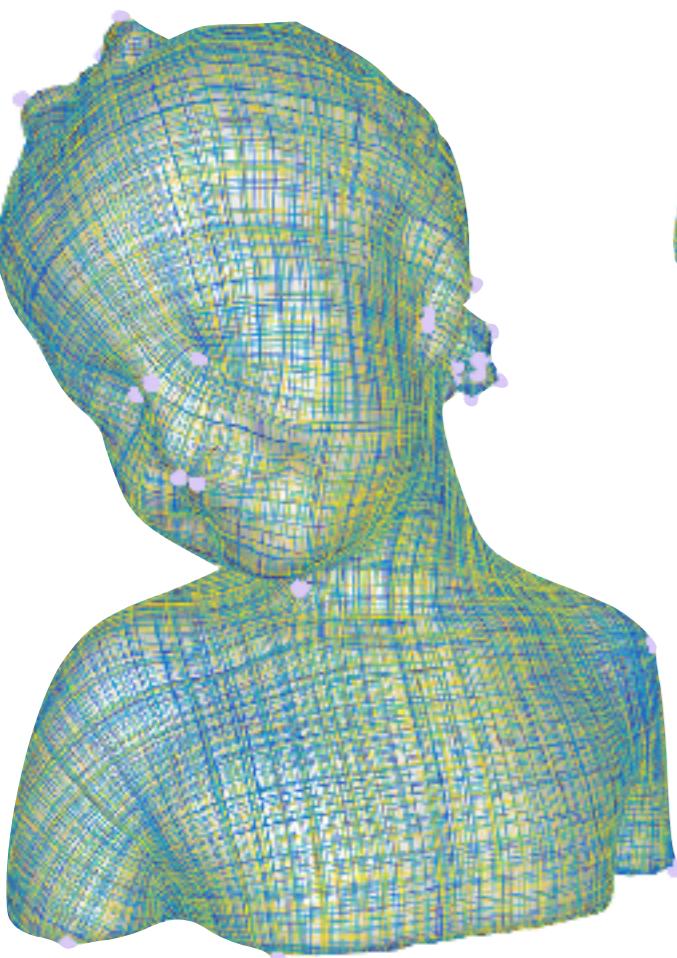
[Jakob et al. 2015]



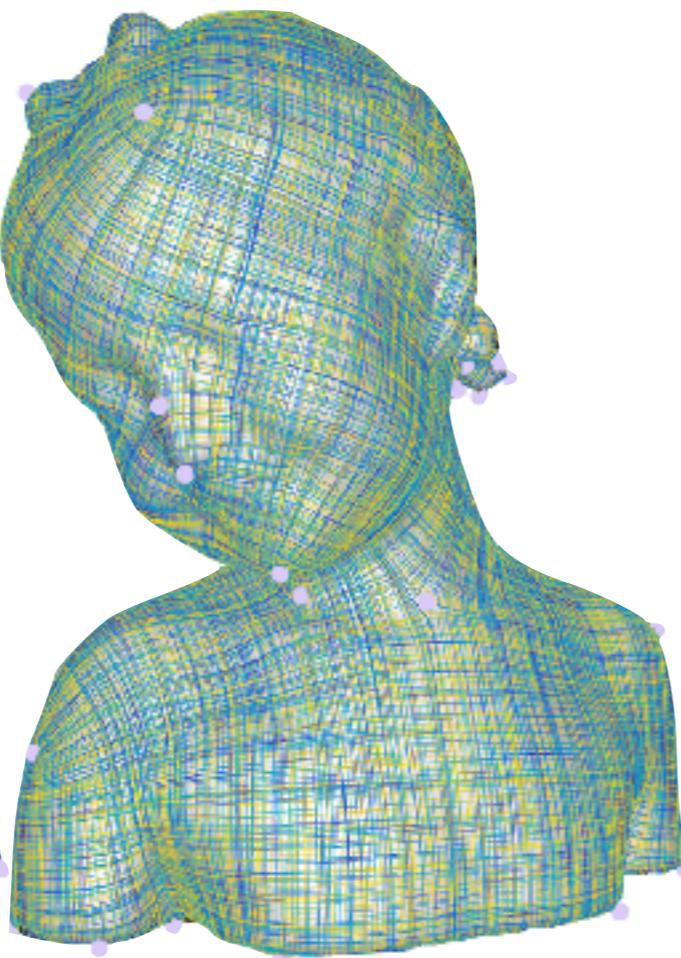
[Knöppel et al. 2013]



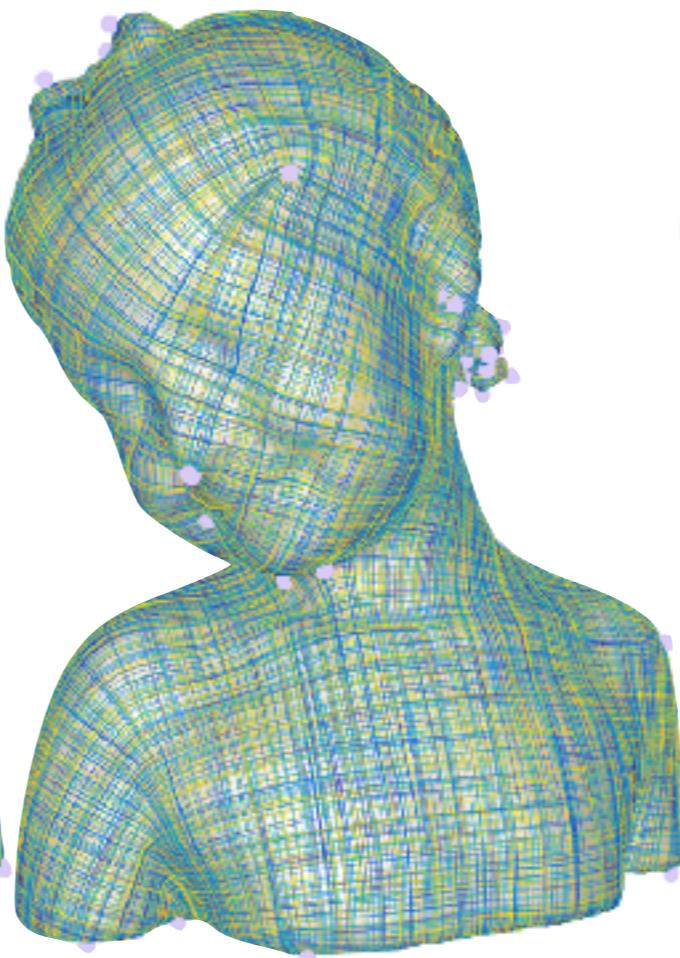
[Diamanti et al. 2015]



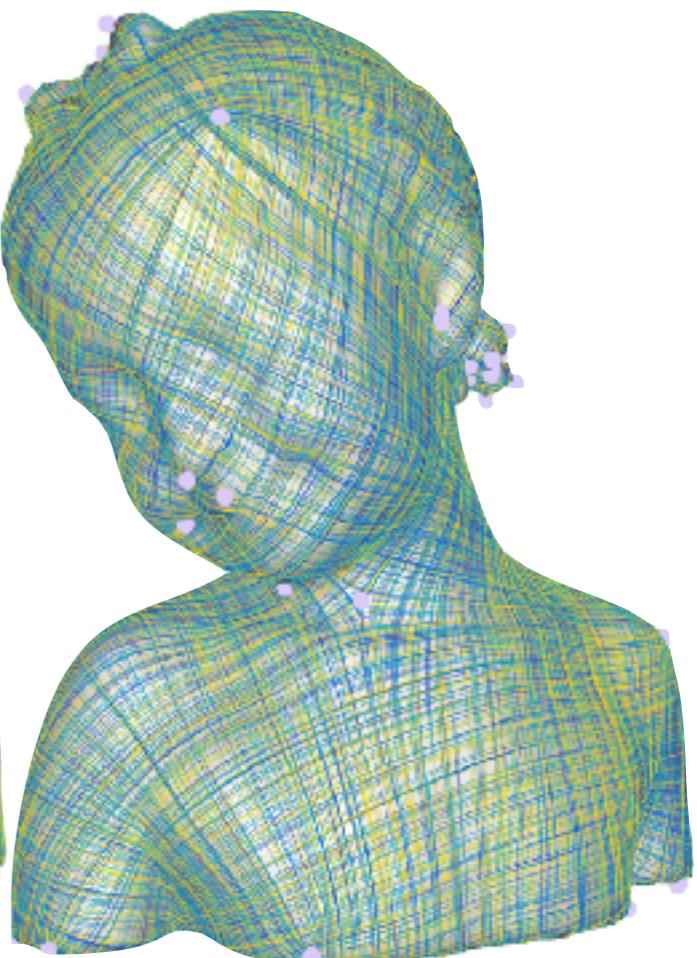
48 singularities



42 singularities

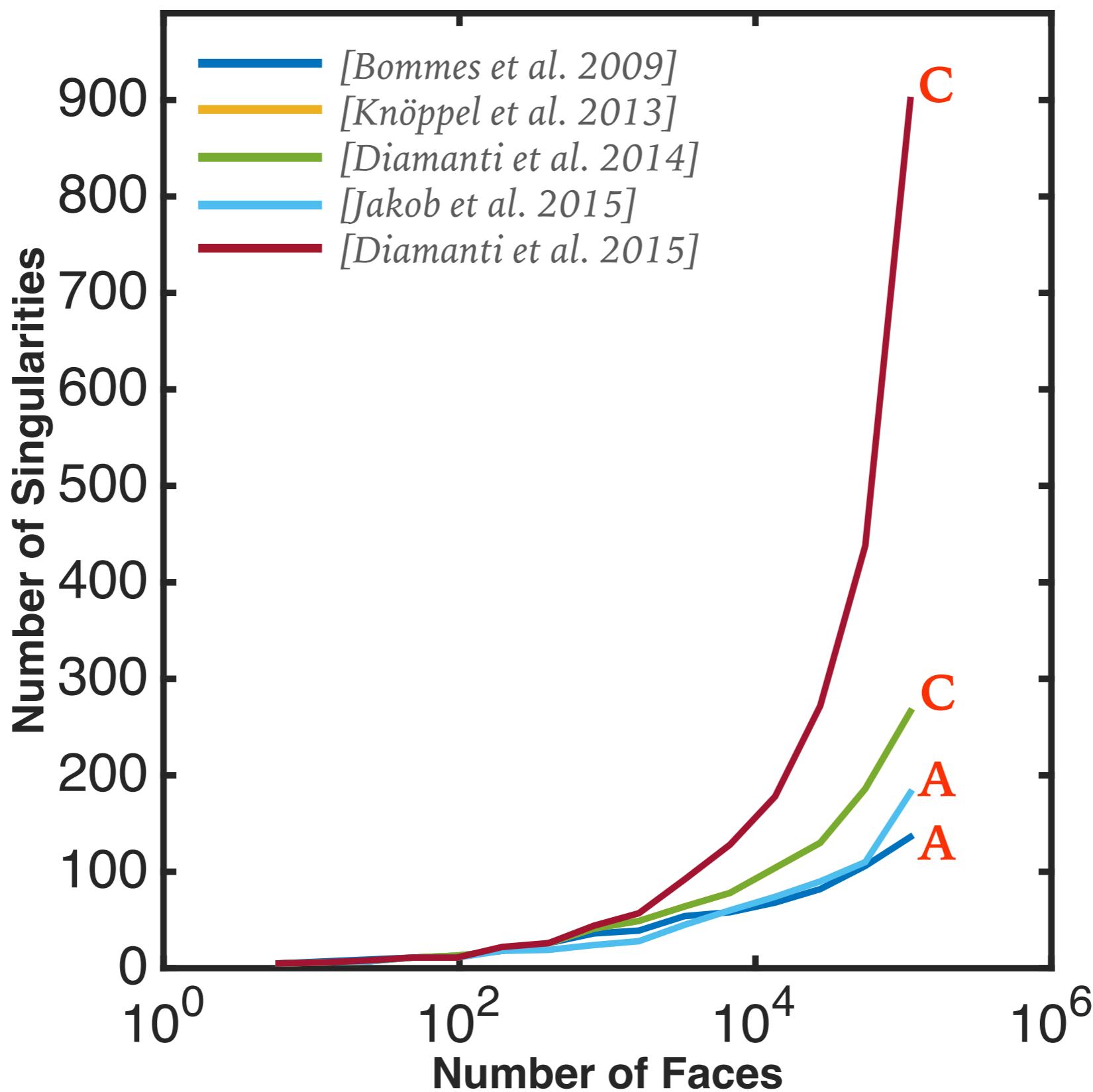


62 singularities

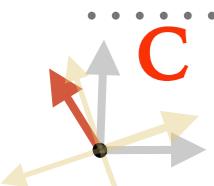


56 singularities

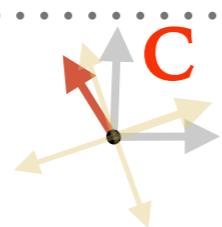
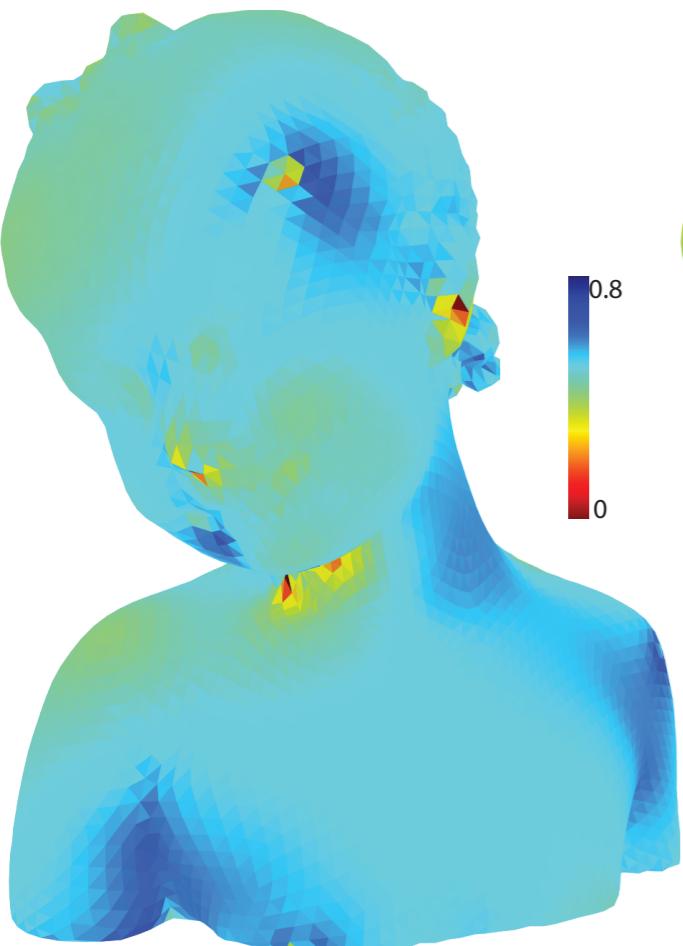
SINGULARITIES



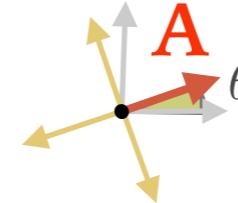
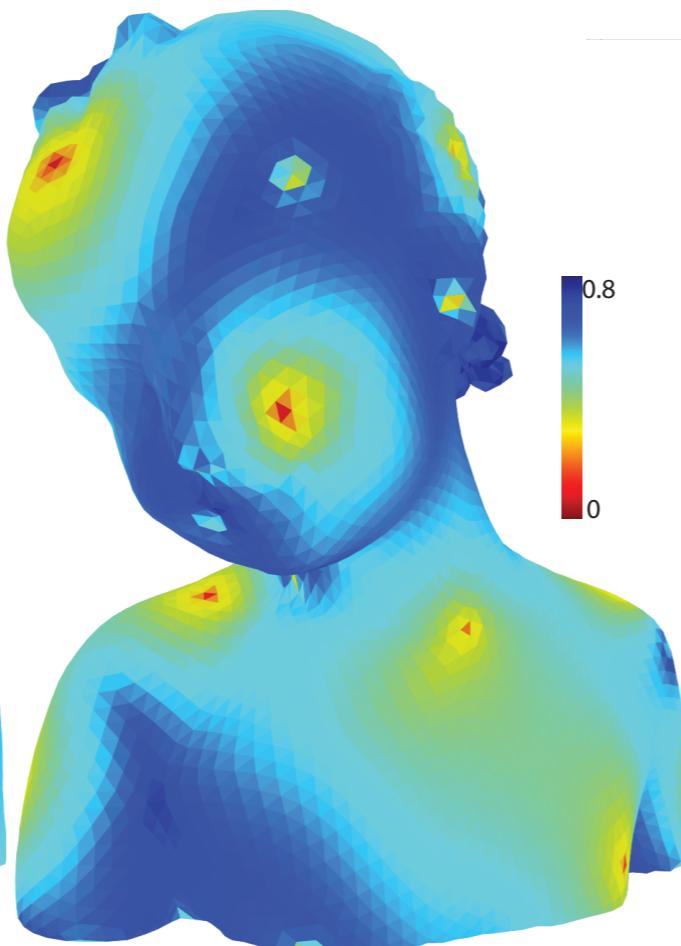
INTEGRABILITY



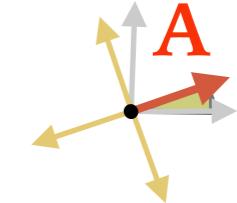
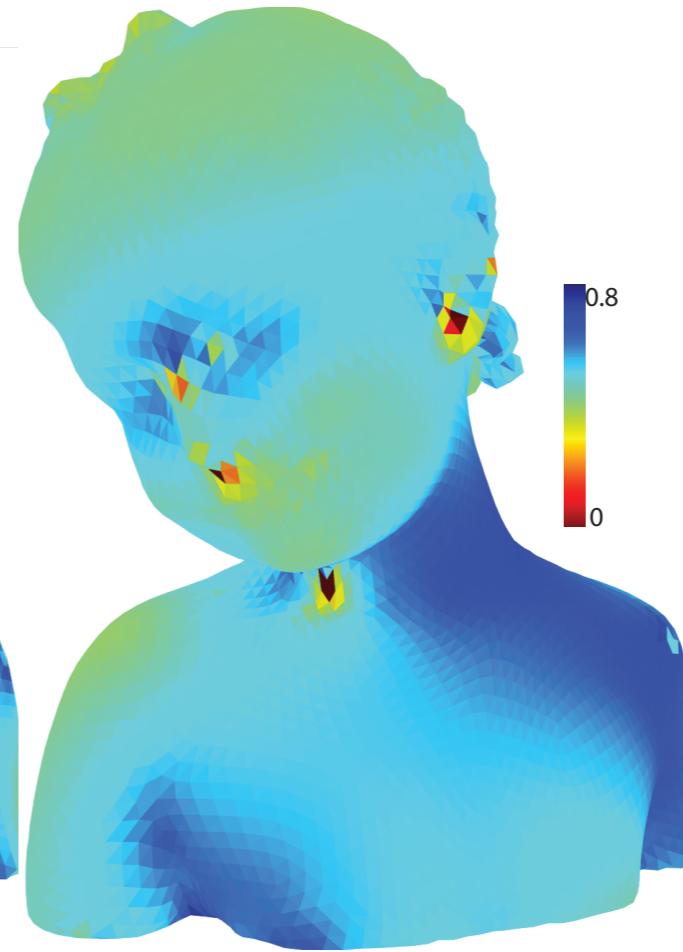
[Knöppel et al. 2013]



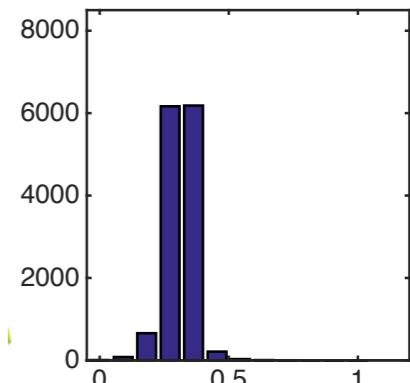
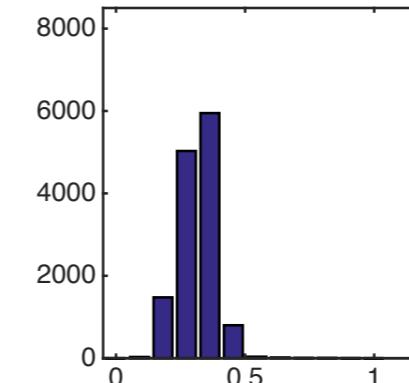
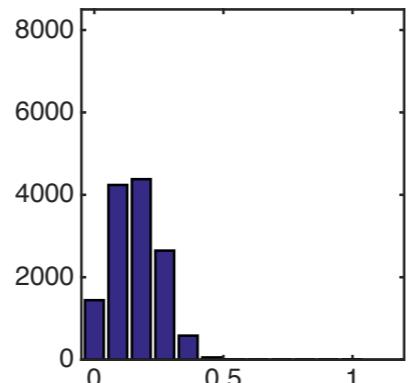
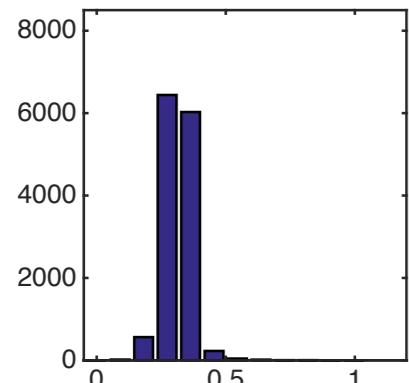
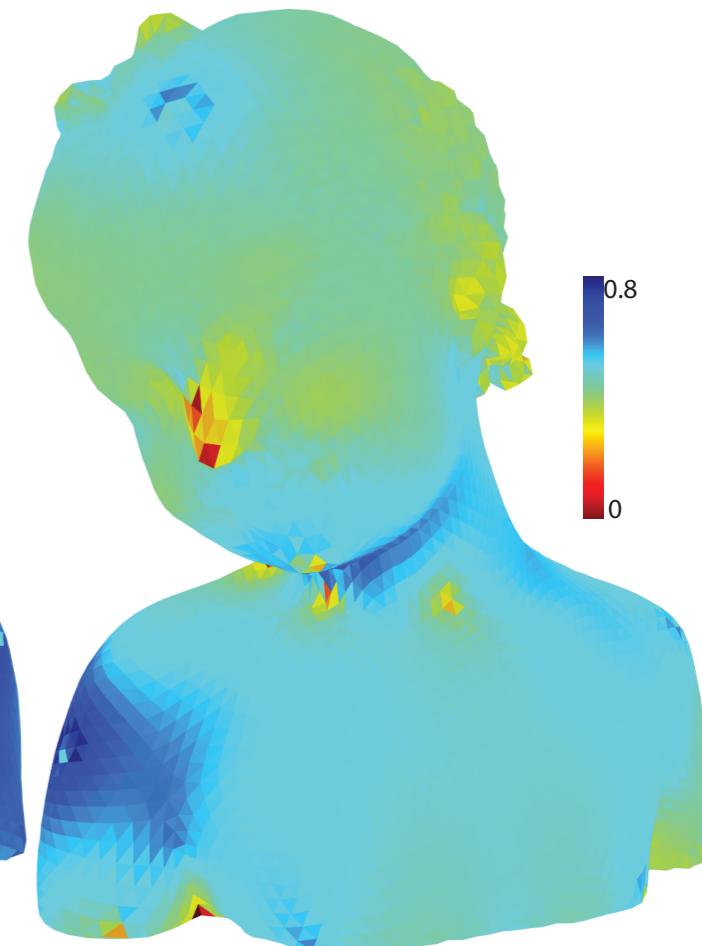
[Diamanti et al. 2014]



[Bommes et al. 2009]



[Jakob et al. 2015]



SCALABILITY

