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EE379K (Data Science Lab) – Lab #2 Written Questions

1) $Z \sim N(\mu, \sigma^2)$

$Z_{avg} = \frac{1}{n} \sum_{i=1}^{n} Z_i = \frac{1}{n}(z_1 + \dots + z_n)$

(a) $Z \sim N(0,1)$, $n = 10{,}000$

$S = Z_1 + Z_2 + \dots + Z_{10{,}000}$

$Z_{avg} = \frac{1}{n} S$

$P(Z_{avg} > 0.1) = P(\overset{X}{S} > 0.1\overset{10000}{n}) = 1 - P(X \le 1000)$

$\mu_z = 0 \rightarrow n\mu_z = 0$

$\sigma_z^2 = 1 \longrightarrow n\sigma_z^2 = 10000$

$X = \frac{S - 0}{\sqrt{10000}} = \frac{S}{100}$

$1 - P(X \le 1000) = 1 - \Phi(10) \approx \boxed{0 = P(Z_{avg} > 0.1)}$

$P(Z_{avg} > 0.01) = P(X > 100) = 1 - P(X \le 100) = 1 - \Phi(1) = \boxed{0.1587 = P(Z_{avg} > 0.01)}$

$P(Z_{avg} > 0.001) = P(X > 10) = 1 - P(X \le 10) = 1 - \Phi(.1) = \boxed{0.4602 = P(Z_{avg} > 0.001)}$

(b) $Z \sim N(\mu_z, \sigma_z^2)$, $n = n$

$X = \frac{S - n\mu_z}{\sqrt{n\sigma_z^2}}$ $\quad X = \frac{n(n^{-1/3} + \mu) - n\mu_z}{\sqrt{n\sigma^2}}$ $\quad \Phi(x) =$ uses standard normal via lookup table

$P(Z_{avg} - \mu > n^{-1/3}) = P(\frac{1}{n}X - \mu > n^{-1/3}) = P(X > n(n^{-1/3} + \mu))$

$= 1 - \Phi\left(\frac{n(n^{-1/3} + \mu) - n\mu}{\sqrt{n\sigma^2}}\right) = 1 - \Phi\left(\frac{n^{2/3}}{\sqrt{n\sigma^2}}\right)$

$P(Z_{avg} - \mu > n^{-1/2}) = 1 - \Phi\left(\frac{n^{1/2}}{\sqrt{n\sigma^2}}\right)$

$P(Z_{avg} - \mu > n^{-2/3}) = 1 - \Phi\left(\frac{n^{1/3}}{\sqrt{n\sigma^2}}\right)$

2 $\min_{B} \frac{1}{n} \sum_{i=1}^{n} (x_i B - y_i)^2 \equiv \min_{B} \frac{1}{n} \sum x_i^2 B - 2x_i y_i B + y_i^2$

a) $A = \frac{1}{n} \sum x_i^2$

$B = \frac{2}{n} \sum x_i y_i$

$C = \frac{1}{n} y_i^2$

b) $\frac{\partial B}{\partial x} = 0 \quad = \quad 2A\beta + B = 0 \quad \Rightarrow \quad \hat{B} = \frac{-B}{2A} = \frac{-\sum x_i y_i}{\sum x_i^2}$

$\frac{-\sum x_i y_i}{\sum x_i^2} \Rightarrow \frac{\sum x_i (x_i B + e)}{\sum x_i^2} \Rightarrow \frac{(B \sum x^2 + \sum x_i e)}{\sum x_i^2}$

$\Rightarrow B + \frac{\sum x_i e_i}{\sum x_i^2} = 0$

$\hat{B} = B + Ze$

$\Rightarrow B + \frac{\sum x_i e_i}{\sum x_i^2} = 0$

magnitude

know e is $p \times 1$

X is column $p \times 1$ $Z =$

need X^T $1 \times p \times p \times 1 = 1 \times 1$

$\boxed{B + \frac{X^T}{|X|^2} e}$

$Z = \frac{X^T}{|X|^2}$