vac-normal ordering,

Φ-normal ordering,

& Wick's Thm.

Fock space:

Fock space:

space of candidate wave functions for indistinguishable particles

Fock space:

$1e^{\ominus}$ basis:

Fock space:

1$e^\ominus$ basis: $\{\psi_p\}$

Fock space:

$1e^{\ominus}$ basis: $\{\psi_p\}$

n-$e^{\ominus}$ basis:

Fock space:

1e$^\ominus$ basis: $\{\psi_p\}$

n-e$^\ominus$ basis:

$$\Phi_{(p_1 \cdots p_n)} = \frac{1}{\sqrt{n!}} \sum_{\pi}^{S_n} \varepsilon_\pi \, \psi_{p_{\pi(1)}} \otimes \cdots \otimes \psi_{p_{\pi(n)}}$$

$e^{\ominus}$ Fock space:

the union of antisymmetric n-particle states occupation ≠ representation

occupation # representation

$$|0\ 1\ 1\ 0\ 0\ \cdots\ n_p\ \cdots\rangle$$

occupation # representation

$$|1\,0\,1\,1\,0\,0\,\cdots\,n_p\,\cdots\rangle$$

particles in ψ_p

particle-hole operators:

particle-hole operators:

$$a_1 |1\,0\,0 \cdots\rangle = |0\,0\,0\,0 \cdots\rangle$$

particle-hole operators:

$$a_1 |100\cdots\rangle = |000\cdots\rangle$$

$$a_1^{\dagger} |000\cdots\rangle = |100\cdots\rangle$$

particle-hole operators:

annihilator

$$a_1 |100\cdots\rangle = |000\cdots\rangle$$

$$a_1^\dagger |000\cdots\rangle = |100\cdots\rangle$$

particle-hole operators:

annihilator

$$a_1 |100\cdots\rangle = |000\cdots\rangle$$

creator

$$a_1^{\dagger} |000\cdots\rangle = |100\cdots\rangle$$

particle-hole operators:

$$|(p_1 \cdots p_n)\rangle$$

particle-hole operators:

$$|(p_1 \cdots p_n)\rangle = a^\dagger_{p_1} \cdots a^\dagger_{p_n} |vac\rangle$$

particle-hole operators:

$$|(p_1 \cdots p_n)\rangle = a^\dagger_{p_1} \cdots a^\dagger_{p_n} |vac\rangle$$

$$|0000\cdots\rangle$$

particle-hole operators:

$$|(p_1 \cdots p_n)\rangle = a^\dagger_{p_1} \cdots a^\dagger_{p_n} |\mathrm{vac}\rangle$$

$$\langle (p_1 \cdots p_n)| = \langle \mathrm{vac}| a_{p_n} \cdots a_{p_1}$$

particle-hole operators:

$$|(p_1 \cdots p_n)\rangle = a^\dagger_{p_1} \cdots a^\dagger_{p_n} |vac\rangle$$

$$\langle (p_1 \cdots p_n)| = \langle vac| a_{p_n} \cdots a_{p_1}$$

$$H_e = \sum_{pq} h_p^q a_p^\dagger a_q + \frac{1}{2} \sum_{pqrs} g_{pq}^{rs} a_p^\dagger a_q^\dagger a_s a_r$$

Fock space algebra boils down to

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$$\langle \mathrm{vac} | q_1 q_2 \cdots q_n | \mathrm{vac} \rangle$$

Fock space algebra boils down to

$$\langle \text{vac} | q_1 \, q_2 \cdots q_n | \text{vac} \rangle$$

$$\downarrow$$

a_p or $a_p^\dagger$

Fock space algebra boils down to

$$\langle \text{vac} | q_1 q_2 \cdots q_n | \text{vac} \rangle$$

using anticommutators

Fock space algebra boils down to

$$\langle vac | q_1 q_2 \cdots q_n | vac \rangle$$

using anticommutators

$$[a_p, a_q]_+ = [a_p^\dagger, a_q^\dagger]_+ = 0$$

Fock space algebra boils down to

$$\langle vac | q_1 q_2 \cdots q_n | vac \rangle$$

using anticommutators

$$[a_p, a_q]_+ = [a_p^\dagger, a_q^\dagger]_+ = 0$$

$$[a_p, a_q^\dagger]_+ = \delta_{pq}$$

example:

$$\langle \text{vac} | a_p a_q^\dagger | \text{vac} \rangle$$

example:

$$\langle vac | a_p a_q^\dagger | vac \rangle$$

$$= - \langle vac | a_q^\dagger a_p | vac \rangle + \delta_{pq}$$

example:

$$\langle vac | a_p a_q^\dagger | vac \rangle$$

$$0$$

$$= - \langle vac | a_q^\dagger a_p | vac \rangle + \delta_{pq}$$

example:

$$\langle vac | a_p a_q^\dagger | vac \rangle$$

$$= - \overbrace{\langle vac | a_q^\dagger a_p | vac \rangle}^{0} + \delta_{pq}$$

$$= \delta_{pq}$$

normal order

normal order $\quad a^\dagger_{p_1} \cdots a^\dagger_{p_m} a_{q_1} \cdots a_{q_n}$

normal order $a^\dagger_{p_1} \cdots a^\dagger_{p_m} a_{q_1} \cdots a_{q_n}$

vanishing vac expectation value

normal order $\quad a^\dagger_{p_1} \cdots a^\dagger_{p_m} a_{q_1} \cdots a_{q_n}$

vanishing vac expectation value

normal <u>ordered</u>

normal order $a^\dagger_{p_1} \cdots a^\dagger_{p_m} a_{q_1} \cdots a_{q_n}$

vanishing vac expectation value

normal ordered

$$:q_1 \cdots q_n: = \varepsilon_\pi \, q_{\pi(1)} \cdots q_{\pi(n)}$$

normal order $\quad a^\dagger_{p_1} \cdots a^\dagger_{p_m} a_{q_1} \cdots a_{q_n}$

vanishing vac expectation value

$\underline{\underline{\text{normal ordered}}}$

$$:q_1 \cdots q_n: \; = \varepsilon_\pi \, q_{\pi(1)} \cdots q_{\pi(n)}$$

where π puts string in normal order

$$:a_p a_q: = a_p a_q$$

$$:a_p a_q: = a_p a_q$$

$$:a_p^\dagger a_q^\dagger: = a_p^\dagger a_q^\dagger$$

$$:a_p a_q: = a_p a_q$$

$$:a_p^\dagger a_q^\dagger: = a_p^\dagger a_q^\dagger$$

$$:a_p^\dagger a_q: = a_p^\dagger a_q$$

$$:a_p a_q: = a_p a_q$$

$$:a_p^\dagger a_q^\dagger: = a_p^\dagger a_q^\dagger$$

$$:a_p^\dagger a_q: = a_p^\dagger a_q$$

$$:a_p a_q^\dagger: = -a_q^\dagger a_p$$

contraction:

contraction:

$$\contraction{}{q}{_1}{q} q_1 q_2$$

contraction:

$$\overline{q_1 q_2} \equiv q_1 q_2 - :q_1 q_2:$$

contraction:

$$\overbracket{q_1 q_2} \equiv q_1 q_2 - :q_1 q_2:$$

$$\overbracket{a_p a_q}$$

contraction:

$$\overbracket{q_1 q_2} \equiv q_1 q_2 - :q_1 q_2:$$

$$\overbracket{a_p a_q} = a_p a_q - a_p a_q = 0$$

contraction:

$$\overbracket{q_1 q_2} \equiv q_1 q_2 - :q_1 q_2:$$

$$\overbracket{a_p a_q} = a_p a_q - a_p a_q = 0$$

$$\overbracket{a_p^\dagger a_q^\dagger} = 0$$

contraction:

$$\overbracket{q_1 \, q_2} \equiv q_1 q_2 - :q_1 q_2:$$

$$\overbracket{a_p \, a_q} = a_p a_q - a_p a_q = 0$$

$$\overbracket{a_p^{+} \, a_q^{+}} = 0$$

$$\overbracket{a_p^{+} \, a_q} = 0$$

contraction:

$$\overbracket{q_1 q_2} \equiv q_1 q_2 - :q_1 q_2:$$

$$\overbracket{a_p a_q} = a_p a_q - a_p a_q = 0$$

$$\overbracket{a_p^\dagger a_q^\dagger} = 0$$

$$\overbracket{a_p^\dagger a_q} = 0$$

$$\overbracket{a_p a_q^\dagger}$$

contraction:

$$\overbracket{q_1 q_2} \equiv q_1 q_2 - :q_1 q_2:$$

$$\overbracket{a_p a_q} = a_p a_q - a_p a_q = 0$$

$$\overbracket{a_p^\dagger a_q^\dagger} = 0$$

$$\overbracket{a_p^\dagger a_q} = 0$$

$$\overbracket{a_p a_q^\dagger} = a_p a_q^\dagger - \left(-a_q^\dagger a_p\right)$$

contraction:

$$\wick{\c q_1 \c q_2} \equiv q_1 q_2 - :q_1 q_2:$$

$$\wick{\c a_p \c a_q} = a_p a_q - a_p a_q = 0$$

$$\wick{\c a_p^\dagger \c a_q^\dagger} = 0$$

$$\wick{\c a_p^\dagger \c a_q} = 0$$

$$\wick{\c a_p \c a_q^\dagger} = a_p a_q^\dagger - (-a_q^\dagger a_p) = [a_p, a_q^\dagger]_+$$

contraction:

$$\overbracket{q_1 q_2} \equiv q_1 q_2 - :q_1 q_2:$$

$$\overbracket{a_p a_q} = a_p a_q - a_p a_q = 0$$

$$\overbracket{a_p^\dagger a_q^\dagger} = 0$$

$$\overbracket{a_p^\dagger a_q} = 0$$

$$\overbracket{a_p a_q^\dagger} = a_p a_q^\dagger - (-a_q^\dagger a_p) = [a_p, a_q^\dagger]_+$$

$$= \delta_{pq}$$

note:

note:

$$\overbracket{q_1 q_2} = q_1 q_2 - :q_1 q_2:$$

note:

$$\overbracket{q_1 q_2} = q_1 q_2 - :q_1 q_2:$$

$$\Longrightarrow \quad q_1 q_2 = :q_1 q_2: + \overbracket{q_1 q_2}$$

note:

$$\overline{q_1 q_2} = q_1 q_2 - :q_1 q_2:$$

$$\Longrightarrow \quad q_1 q_2 = \underbrace{:q_1 q_2:}_{\text{normal-ordered}} + \underbrace{\overline{q_1 q_2}}_{\text{contraction}}$$

normal-ordered with contraction

normal-ordered with contraction

$$:q_1 \cdots \overbracket{q_i \cdots q_j} \cdots q_n:$$

normal-ordered with contraction

$$:q_1 \cdots \overbracket{q_i \cdots q_j} \cdots q_n:$$

$$\equiv (-)^{j-i+1} \, \overbracket{q_i q_j} \, :q_1 \cdots \cancel{q_i} \cdots \cancel{q_j} \cdots q_n:$$

Wick's theorem:

Wick's theorem:

$$q_1 q_2 = :q_1 q_2: + \wick{\c q_1 \c q_2}$$

Wick's theorem:

$$q_1 q_2 = \underbrace{:q_1 q_2:}_{\text{normal-ordered}} + \underbrace{\overbracket{q_1 q_2}}_{\text{contraction}}$$

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$$q_1 q_2 = \underbrace{:q_1 q_2:}_{\text{normal-ordered}} + \underbrace{\overbracket{q_1 q_2}}_{\text{contraction}}$$

Wick's theorem:

$$q_1 q_2 = :q_1 q_2: + \contraction{}{q}{{}_1}{q} q_1 q_2$$

generalizes to

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$$q_1 q_2 = :q_1 q_2: + \contraction{}{q_1}{}{q_2}q_1 q_2$$

generalizes to

$$q_1 \cdots q_n =$$

Wick's theorem:

$$q_1 q_2 = :q_1 q_2: + \contraction{}{q_1}{}{q_2} q_1 q_2$$

generalizes to

$$q_1 \cdots q_n = :q_1 \cdots q_n:$$

Wick's theorem:

$$q_1 q_2 = :q_1 q_2: + \overbracket{q_1 q_2}$$

generalizes to

$$q_1 \cdots q_n = :q_1 \cdots q_n:$$

$$+ \sum :q_1 \cdots \overbracket{q_i \cdots q_j} \cdots q_n:$$

Wick's theorem:

$$q_1 q_2 = :q_1 q_2: + \wick{\c q_1 \c q_2}$$

generalizes to

$$\begin{aligned} q_1 \cdots q_n = {} & :q_1 \cdots q_n: \\ & + \sum :\wick{q_1 \cdots \c1 q_i \cdots \c1 q_j \cdots q_n}: \\ & + \sum :\wick{q_1 \cdots \c1 q_i \cdots \c2 q_k \cdots \c1 q_j \cdots \c2 q_l \cdots q_n}: \end{aligned}$$

Wick's theorem:

$$q_1 q_2 = :q_1 q_2: + \overline{q_1 q_2}$$

generalizes to

$$q_1 \cdots q_n = :q_1 \cdots q_n:$$

$$+ \sum :q_1 \cdots \overline{q_i \cdots q_j} \cdots q_n:$$

$$+ \sum :q_1 \cdots q_i \cdots q_k \cdots q_j \cdots q_l \cdots q_n:$$

$$+ \cdots$$

Wick's theorem:

$$Q = :Q: + :\overline{Q}:$$

Wick's theorem:

$$Q = :Q: + :\overline{Q}:$$

$\downarrow$

sum of unique single, double, triple, etc. contractions

Wick's theorem:

$$Q = :Q: + :\overline{Q}:$$

Corollary 1.

Wick's theorem:

$$Q = :Q: + :\overline{Q}:$$

Corollary 1.

$$:Q::Q': = :QQ': + :\overline{QQ'}:$$

Wick's theorem:

$$Q = :Q: + :\overline{Q}:$$

Corollary 1.

cross-contractions

$$:Q::Q': = :QQ': + :\overline{QQ'}:$$

Wick's theorem:

$$Q = :Q: + :\overline{Q}:$$

Corollary 1.

$$:Q::Q': = :QQ': + :\overline{QQ'}:$$

Corollary 2.

Wick's theorem:

$$Q = :Q: + :\overline{Q}:$$

Corollary 1.

$$:Q::Q': = :QQ': + :\overline{QQ'}:$$

Corollary 2.

$$\langle vac|Q|vac\rangle = :\overline{\overline{Q}}:$$

Wick's theorem:

$$Q = :Q: + :\overline{Q}:$$

Corollary 1.

$$:Q::Q': = :QQ': + :\overbracket{QQ'}:$$

Corollary 2. complete contractions

$$\langle vac|Q|vac\rangle = :\overline{\overline{Q}}:$$

$$Q = :Q: + :\overline{Q}:$$

Corollary 1.

$$:Q::Q': = :QQ': + :\overline{QQ'}:$$

Corollary 2.

$$\langle vac|Q|vac\rangle = :\overline{\overline{Q}}:$$

$$Q = :Q: + :\overline{Q}:$$

Corollary 1.

$$:Q::Q': = :QQ': + :\overline{Q\,Q'}:$$

Corollary 2.

$$\langle vac|Q|vac\rangle = :\overline{\overline{Q}}:$$

Corollary 3.

$$Q = :Q: + :\overline{Q}:$$

Corollary 1.

$$:Q::Q': = :QQ': + :\overbracket{QQ'}:$$

Corollary 2.

$$\langle vac|Q|vac\rangle = :\overline{\overline{Q}}:$$

Corollary 3.

$$\langle vac|:Q::Q':|vac\rangle = :\overbracket{Q::Q'}:$$

$$Q = :Q: + :\overline{Q}:$$

Corollary 1.

$$:Q::Q': = :QQ': + :\overbracket{QQ'}:$$

Corollary 2.

$$\langle vac|Q|vac\rangle = :\overline{\overline{Q}}:$$

Corollary 3.

complete cross-contractions

$$\langle vac|:Q::Q':|vac\rangle = :\overbracket{\overbracket{Q::Q'}}:$$

$$|\Phi_{(p_1 \cdots p_n)}\rangle = a^{\dagger}_{p_1} \cdots a^{\dagger}_{p_n} |vac\rangle$$

$$\langle \Phi_{(q_1 \cdots q_n)}| = \langle vac| a_{q_n} \cdots a_{q_1}$$

$$H_e = \sum_{pq} h_p^q \, a_p^{\dagger} a_q + \frac{1}{2} \sum_{pqrs} g_{pq}^{rs} \, a_p^{\dagger} a_q^{\dagger} a_s a_r$$

$$|\Phi_{(p_1 \cdots p_n)}\rangle = a_{p_1}^\dagger \cdots a_{p_n}^\dagger |vac\rangle$$

$$\langle \Phi_{(q_1 \cdots q_n)}| = \langle vac| a_{q_n} \cdots a_{q_1}$$

$$H_e = \sum_{pq} h_p^q a_p^\dagger a_q + \frac{1}{2} \sum_{pqrs} g_{pq}^{rs} a_p^\dagger a_q^\dagger a_s a_r$$

$\langle \Phi_{(q_1 \cdots q_n)}| H_e |\Phi_{(p_1 \cdots p_n)}\rangle$ can be evaluated using

$$|\Phi_{(p_1 \cdots p_n)}\rangle = a_{p_1}^\dagger \cdots a_{p_n}^\dagger |vac\rangle$$

$$\langle \Phi_{(q_1 \cdots q_n)}| = \langle vac| a_{q_n} \cdots a_{q_1}$$

$$H_e = \sum_{pq} h_p^q a_p^\dagger a_q + \frac{1}{2} \sum_{pqrs} g_{pq}^{rs} a_p^\dagger a_q^\dagger a_s a_r$$

$\langle \Phi_{(q_1 \cdots q_n)}| H_e |\Phi_{(p_1 \cdots p_n)}\rangle$ can be evaluated using

$$\langle vac|(a_{q_n} \cdots a_{q_1})(a_p^\dagger a_q)(a_{p_1}^\dagger \cdots a_{p_n}^\dagger)|vac\rangle$$

$$|\Phi_{(p_1\cdots p_n)}\rangle = a_{p_1}^\dagger \cdots a_{p_n}^\dagger |vac\rangle$$

$$\langle \Phi_{(q_1\cdots q_n)}| = \langle vac| a_{q_n} \cdots a_{q_1}$$

$$H_e = \sum_{pq} h_p^q a_p^\dagger a_q + \frac{1}{2} \sum_{pqrs} g_{pq}^{rs} a_p^\dagger a_q^\dagger a_s a_r$$

$\langle \Phi_{(q_1\cdots q_n)}| H_e |\Phi_{(p_1\cdots p_n)}\rangle$ can be evaluated using

$$\langle vac|(a_{q_n} \cdots a_{q_1})(a_p^\dagger a_q)(a_{p_1}^\dagger \cdots a_{p_n}^\dagger)|vac\rangle$$

and

$$|\Phi_{(p_1 \cdots p_n)}\rangle = a^\dagger_{p_1} \cdots a^\dagger_{p_n} |vac\rangle$$

$$\langle \Phi_{(q_1 \cdots q_n)}| = \langle vac| a_{q_n} \cdots a_{q_1}$$

$$H_e = \sum_{pq} h_p^q \, a_p^\dagger a_q + \frac{1}{2} \sum_{pqrs} g_{pq}^{rs} \, a_p^\dagger a_q^\dagger a_s a_r$$

$\langle \Phi_{(q_1 \cdots q_n)}| H_e |\Phi_{(p_1 \cdots p_n)}\rangle$ can be evaluated using

$$\langle vac|(a_{q_n} \cdots a_{q_1})(a_p^\dagger a_q)(a^\dagger_{p_1} \cdots a^\dagger_{p_n})|vac\rangle$$

and

$$\langle vac|(a_{q_n} \cdots a_{q_1})(a_p^\dagger a_q^\dagger a_s a_r)(a^\dagger_{p_1} \cdots a^\dagger_{p_n})|vac\rangle$$

$$\langle vac | (a_{q_n} \cdots a_{q_1})(a_p^\dagger a_q)(a_{p_1}^\dagger \cdots a_{p_n}^\dagger) | vac \rangle$$

$$\langle vac|(a_{q_n}\cdots a_{q_1})(a_p^\dagger a_q)(a_{p_1}^\dagger\cdots a_{p_n}^\dagger)|vac\rangle$$

$$= (a_{q_n}\cdots a_{q_1})(a_p^\dagger a_q)(a_{p_1}^\dagger\cdots a_{p_n}^\dagger)$$

$$\langle vac|(a_{q_n}\cdots a_{q_1})(a_p^\dagger a_q)(a_{p_1}^\dagger\cdots a_{p_n}^\dagger)|vac\rangle$$

$$= (a_{q_n}\cdots a_{q_1})(a_p^\dagger a_q)(a_{p_1}^\dagger\cdots a_{p_n}^\dagger)$$

$$\langle vac|(a_{q_n}\cdots a_{q_1})(a_p^\dagger a_q^\dagger a_s a_r)(a_{p_1}^\dagger\cdots a_{p_n}^\dagger)|vac\rangle$$

$$\langle \text{vac} | (a_{q_n} \cdots a_{q_1})(a_p^\dagger a_q)(a_{p_1}^\dagger \cdots a_{p_n}^\dagger) | \text{vac} \rangle$$

$$= (a_{q_n} \cdots a_{q_1})(a_p^\dagger a_q)(a_{p_1}^\dagger \cdots a_{p_n}^\dagger)$$

$$\langle \text{vac} | (a_{q_n} \cdots a_{q_1})(a_p^\dagger a_q^\dagger a_s a_r)(a_{p_1}^\dagger \cdots a_{p_n}^\dagger) | \text{vac} \rangle$$

$$= (a_{q_n} \cdots a_{q_1})(a_p^\dagger a_q^\dagger a_s a_r)(a_{p_1}^\dagger \cdots a_{p_n}^\dagger)$$

$$\langle vac|(a_{q_n}\cdots a_{q_1})(a_p^\dagger a_q)(a_{p_1}^\dagger \cdots a_{p_n}^\dagger)|vac\rangle$$

$$= (a_{q_n}\cdots a_{q_1})(a_p^\dagger a_q)(a_{p_1}^\dagger \cdots a_{p_n}^\dagger)$$

$$\langle vac|(a_{q_n}\cdots a_{q_1})(a_p^\dagger a_q^\dagger a_s a_r)(a_{p_1}^\dagger \cdots a_{p_n}^\dagger)|vac\rangle$$

$$= (a_{q_n}\cdots a_{q_1})(a_p^\dagger a_q^\dagger a_s a_r)(a_{p_1}^\dagger \cdots a_{p_n}^\dagger)$$

but there is a better way…

Φ-normal ordering

Φ-normal ordering

$$|\Phi\rangle = |11\cdots1000\cdots\rangle$$

Φ-normal ordering

$$|\Phi\rangle = |11 \cdots 1000 \cdots\rangle$$

$$b_i^\dagger \equiv a_i$$

Φ-normal ordering

$$|\Phi\rangle = |11\cdots1000\cdots\rangle$$

$$b_i^\dagger \equiv a_i$$

$$b_1^\dagger |11\cdots1000\cdots\rangle$$

$$= |01\cdots1000\cdots\rangle$$

Φ-normal ordering

$$|\Phi\rangle = |11\cdots1000\cdots\rangle$$

$$b_i^\dagger \equiv a_i$$

$$b_1^\dagger |11\cdots1000\cdots\rangle$$

$$= |01\cdots1000\cdots\rangle$$

↳ look Ma! I made a hole!

Φ-normal ordering

$$|\Phi\rangle = |11\cdots1000\cdots\rangle$$

$$b_i^\dagger \equiv a_i$$

$$b_1^\dagger |11\cdots1000\cdots\rangle$$

$$= |01\cdots1000\cdots\rangle$$

$$b_1 |11\cdots1000\cdots\rangle = 0$$

Φ-normal ordering

$$|\Phi\rangle = |11\cdots 1000\cdots\rangle$$

$$b_i^\dagger \equiv a_i \qquad b_i = a_i^\dagger$$

$$b_a^\dagger \equiv a_a^\dagger \qquad b_a = a_a$$

Φ-normal ordering

$$|\Phi\rangle = |11\cdots 1000\cdots\rangle$$

$$b_i^\dagger \equiv a_i \qquad b_i = a_i^\dagger$$

$$b_a^\dagger \equiv a_a^\dagger \qquad b_a = a_a$$

hole/particle creators

Φ-normal ordering

$$|\Phi\rangle = |11\cdots1000\cdots\rangle$$

$b_i^\dagger \equiv a_i$

$b_a^\dagger \equiv a_a^\dagger$

hole/particle creators

$b_i = a_i^\dagger$

$b_a = a_a$

hole/particle annihilators

isomorphism

isomorphism

$$|n_1 \cdots n_i \cdots n_m \, n_{m+1} \cdots n_a \cdots\rangle$$

$$\downarrow$$

$$|\bar{n}_1 \cdots \bar{n}_i \cdots \bar{n}_m \, n_{m+1} \cdots n_a \cdots\rangle$$

isomorphism

$$|n_1 \cdots n_i \cdots n_m \, n_{m+1} \cdots n_a \cdots\rangle$$

$$\downarrow$$

$$|\bar{n}_1 \cdots \bar{n}_i \cdots \bar{n}_m \, n_{m+1} \cdots n_a \cdots\rangle$$

flip the bits occupied in Φ

isomorphism

$$|n_1 \cdots n_i \cdots n_m \, n_{m+1} \cdots n_a \cdots \rangle$$

$$\downarrow$$

$$|\bar{n}_1 \cdots \bar{n}_i \cdots \bar{n}_m \, n_{m+1} \cdots n_a \cdots \rangle$$

$$|vac\rangle \longrightarrow |TT \cdots T\,0\,0\,0 \cdots \rangle$$

isomorphism

$$|n_1 \cdots n_i \cdots n_m\, n_{m+1} \cdots n_a \cdots\rangle$$

$$\downarrow$$

$$|\overline{n}_1 \cdots \overline{n}_i \cdots \overline{n}_m\, n_{m+1} \cdots n_a \cdots\rangle$$

$$|\mathrm{vac}\rangle \longrightarrow |\overline{1}\,\overline{1} \cdots \overline{1}\, 0\, 0\, 0 \cdots\rangle$$

$$|\Phi\rangle \longrightarrow |\overline{0}\,\overline{0} \cdots \overline{0}\, 0\, 0\, 0 \cdots\rangle$$

Φ-normal order

Φ-normal order $\quad b^\dagger_{p_1} \cdots b^\dagger_{p_m} b_{q_1} \cdots b_{q_n}$

Φ-normal order $\quad b^{\dagger}_{p_1} \cdots b^{\dagger}_{p_m} b_{q_1} \cdots b_{q_n}$

vanishing Φ expectation value

Φ-normal order $\quad b^{+}_{p_1} \cdots b^{+}_{p_m} b_{q_1} \cdots b_{q_n}$

$\downarrow$

vanishing Φ expectation
value

Φ-normal ordered

Φ-normal order $\quad b^\dagger_{p_1} \cdots b^\dagger_{p_m} b_{q_1} \cdots b_{q_n}$

vanishing Φ expectation value

Φ-normal ordered

$$:q_1 \cdots q_n: \;=\; \varepsilon_\pi \, q_{\pi(1)} \cdots q_{\pi(n)}$$

where π puts string in Φ-normal order

Φ-normal contraction

$$\overbrace{q_1 q_2}^{\sqcap} \equiv q_1 q_2 - {:}q_1 q_2{:}$$

Φ-normal contraction

$$\overbracket{q_1 q_2} \equiv q_1 q_2 - \vdots\, q_1 q_2 \,\vdots$$

$$\overbracket{b_p b_q} = 0$$

Φ-normal contraction

$$\overbracket{q_1 q_2} \equiv q_1 q_2 - :q_1 q_2:$$

$$\overbracket{b_p b_q} = 0$$

$$\overbracket{b_p^\dagger b_q^\dagger} = 0$$

Φ-normal contraction

$$\overline{q_1 q_2} \equiv q_1 q_2 - \vdots q_1 q_2 \vdots$$

$$\overline{b_p b_q} = 0$$

$$\overline{b_p^\dagger b_q^\dagger} = 0$$

$$\overline{b_p^\dagger b_q} = 0$$

Φ-normal contraction

$$\overbracket{q_1 q_2} \equiv q_1 q_2 - :q_1 q_2:$$

$$\overbracket{b_p b_q} = 0$$

$$\overbracket{b_p^\dagger b_q^\dagger} = 0$$

$$\overbracket{b_p^\dagger b_q} = 0$$

$$\overbracket{b_p b_q^\dagger} = \delta_{pq}$$

Φ-normal contraction

$$\overbracket{q_1 q_2} \equiv q_1 q_2 - :q_1 q_2:$$

$$\overbracket{b_p b_q} = 0$$

$$\overbracket{b_p^\dagger b_q^\dagger} = 0$$

$$\overbracket{b_p^\dagger b_q} = 0 \longrightarrow \overbracket{a_a^\dagger a_b} = 0$$

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$$\overbracket{b_p b_q}^\dagger = \delta_{pq} \longrightarrow \overbracket{a_a a_b}^\dagger = \delta_{ab}$$

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$$\overbracket{b_p b_q^\dagger} = \delta_{pq} \longrightarrow \overbracket{a_a a_b^\dagger} = \delta_{ab} \quad \overbracket{a_i^\dagger a_j} = \delta_{ij}$$

Φ-normal contraction

$$\overbracket{a_a^\dagger\, a_b} = 0 \qquad \overbracket{a_i\, a_j^\dagger} = 0$$

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Φ-normal contraction

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$$\overbracket{q_1 q_2} \equiv q_1 q_2 - :q_1 q_2:$$

$$\Longrightarrow \overbracket{q_1 q_2} =$$

Φ-normal contraction

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$$\overbracket{q_1 q_2} \equiv q_1 q_2 - {:}q_1 q_2{:}$$

$$\Longrightarrow \overbracket{q_1 q_2} = \langle \Phi | \, q_1 q_2 - {:}q_1 q_2{:} \, | \Phi \rangle$$

Φ-normal contraction

$$\overline{a_a^\dagger a_b} = 0 \qquad \overline{a_i a_j^\dagger} = 0$$

$$\overline{a_a a_b^\dagger} = \delta_{ab} \qquad \overline{a_i^\dagger a_j} = \delta_{ij}$$

$$\overline{q_1 q_2} \equiv q_1 q_2 - :q_1 q_2:$$

$$\Longrightarrow \overline{q_1 q_2} = \langle \Phi | q_1 q_2 - \cancelto{\text{kills } \Phi}{:q_1 q_2:} | \Phi \rangle$$

Φ-normal contraction

$$\overbracket{a_a^\dagger\, a_b} = 0 \qquad \overbracket{a_i\, a_j^\dagger} = 0$$

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$$\overbracket{q_1 q_2} \equiv q_1 q_2 - :q_1 q_2:$$

kills Φ

$$\Longrightarrow \overbracket{q_1 q_2} = \langle \Phi | q_1 q_2 - :q_1 q_2: | \Phi \rangle$$

$$= \langle \Phi | q_1 q_2 | \Phi \rangle$$

one-hole density matrix

$$\overbracket{a_p\, a_q^\dagger} = \langle \Phi | a_p\, a_q^\dagger | \Phi \rangle \equiv \eta_{pq}$$

one-hole density matrix

$$\overline{a_p a_q^\dagger} = \langle \Phi | a_p a_q^\dagger | \Phi \rangle \equiv \eta_{pq}$$

$$\left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & \begin{matrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{matrix} \end{array}\right]$$

one-hole density matrix

$$a_p a_q^\dagger = \langle \Phi | a_p a_q^\dagger | \Phi \rangle \equiv \eta_{pq}$$

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$$a_i a_j^\dagger = 0 \qquad a_a a_b^\dagger = \delta_{ab}$$

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one-particle density matrix

$$\overbracket{a_p^\dagger a_q} = \langle \Phi | a_p^\dagger a_q | \Phi \rangle \equiv \kappa_{pq}$$

one-hole density matrix

$$\overline{a_p a_q^\dagger} = \langle \Phi | a_p a_q^\dagger | \Phi \rangle \equiv \eta_{pq}$$

$$\overline{a_i a_j^\dagger} = 0 \qquad \overline{a_a a_b^\dagger} = \delta_{ab}$$

$$\left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & \begin{matrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{matrix} \end{array}\right]$$

one-particle density matrix

$$\overline{a_p^\dagger a_q} = \langle \Phi | a_p^\dagger a_q | \Phi \rangle \equiv \kappa_{pq}$$

$$\left[\begin{array}{c|c} \begin{matrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{matrix} & 0 \\ \hline 0 & 0 \end{array}\right]$$

one-hole density matrix

$$\overline{a_p a_q^\dagger} = \langle \Phi | a_p a_q^\dagger | \Phi \rangle \equiv \eta_{pq}$$

$$\overline{a_i a_j^\dagger} = 0 \qquad \overline{a_a a_b^\dagger} = \delta_{ab}$$

$$\left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & \begin{matrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{matrix} \end{array}\right]$$

one-particle density matrix

$$\overline{a_p^\dagger a_q} = \langle \Phi | a_p^\dagger a_q | \Phi \rangle \equiv \kappa_{pq}$$

$$\overline{a_i^\dagger a_j} = \delta_{ij} \qquad \overline{a_a^\dagger a_b} = 0$$

$$\left[\begin{array}{c|c} \begin{matrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{matrix} & 0 \\ \hline 0 & 0 \end{array}\right]$$

Wick's thm. $Q = :Q: + :\overline{Q}:$

Corollary 1. $:Q::Q': = :QQ': + :\overbracket{QQ'}:$

Corollary 2. $\langle vac|Q|vac\rangle = :\overline{\overline{Q}}:$

Corollary 3. $\langle vac|:Q::Q':|vac\rangle = :\overbracket{\overbracket{Q::Q'}}:$

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Corollary 1. $:Q::Q': = :QQ': + :\overbracket{QQ'}:$

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Corollary 3. $\langle \mathrm{vac} | :Q::Q': | \mathrm{vac} \rangle = :\overbracket{\overbracket{Q::Q'}}:$

now, contractions are $\overbracket{a_p^\dagger a_q} = \kappa_{pq}$, $\overbracket{a_p a_q^\dagger} = \eta_{pq}$

Examples:

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$$a_p^\dagger a_q$$

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Examples:

$$a_p^\dagger a_q = :a_p^\dagger a_q: + :\overline{a_p^\dagger a_q}:$$

$$\langle \Phi | a_p^\dagger a_q | \Phi \rangle = :\overline{a_p^\dagger a_q}:$$

$$= K_{pq}$$

Examples:

$$a_p^\dagger a_q^\dagger a_s a_r$$

Examples:

$$a_p^\dagger a_q^\dagger a_s a_r = :a_p^\dagger a_q^\dagger a_s a_r:$$

Examples:

$$a_p^\dagger a_q^\dagger a_s a_r = \, :a_p^\dagger a_q^\dagger a_s a_r: $$

$$+ \, :a_p^\dagger a_q^\dagger a_s a_r:$$

Examples:

$$a_p^\dagger a_q^\dagger a_s a_r = :a_p^\dagger a_q^\dagger a_s a_r: + :a_p^\dagger a_q^\dagger a_s a_r: + :a_p^\dagger a_q^\dagger a_s a_r:$$

Examples:

$$a_p^\dagger a_q^\dagger a_s a_r = :a_p^\dagger a_q^\dagger a_s a_r:$$

$$+ \contraction{:}{a}{_p^\dagger a_q^\dagger}{a} :a_p^\dagger a_q^\dagger a_s a_r: + \contraction{:}{a}{_p^\dagger a_q^\dagger a_s}{a} :a_p^\dagger a_q^\dagger a_s a_r:$$

$$+ \contraction{:a_p^\dagger}{a}{_q^\dagger}{a} :a_p^\dagger a_q^\dagger a_s a_r:$$

Examples:

$$a^\dagger_p a^\dagger_q a_s a_r = :a^\dagger_p a^\dagger_q a_s a_r:$$

$$+ :\wick{\c a^\dagger_p a^\dagger_q \c a_s a_r}: + :\wick{\c a^\dagger_p a^\dagger_q a_s \c a_r}:$$

$$+ :\wick{a^\dagger_p \c a^\dagger_q \c a_s a_r}: + :\wick{a^\dagger_p \c a^\dagger_q a_s \c a_r}:$$

Examples:

$$
\begin{aligned}
a_p^\dagger a_q^\dagger a_s a_r = \; & :a_p^\dagger a_q^\dagger a_s a_r: \\
& + :a_p^\dagger a_q^\dagger a_s a_r: + :a_p^\dagger a_q^\dagger a_s a_r: \\
& + :a_p^\dagger a_q^\dagger a_s a_r: + :a_p^\dagger a_q^\dagger a_s a_r: \\
& + :a_p^\dagger a_q^\dagger a_s a_r:
\end{aligned}
$$

Examples:

$$a_p^\dagger a_q^\dagger a_s a_r = :a_p^\dagger a_q^\dagger a_s a_r:$$

$$+ :\wick{\c a_p^\dagger a_q^\dagger \c a_s a_r}: + :\wick{\c a_p^\dagger a_q^\dagger a_s \c a_r}:$$

$$+ :\wick{a_p^\dagger \c a_q^\dagger \c a_s a_r}: + :\wick{a_p^\dagger \c a_q^\dagger a_s \c a_r}:$$

$$+ :\wick{\c1 a_p^\dagger \c2 a_q^\dagger \c2 a_s \c1 a_r}: + :\wick{\c1 a_p^\dagger \c2 a_q^\dagger \c1 a_s \c2 a_r}:$$

Examples:

$$a_p^\dagger a_q^\dagger a_s a_r = :a_p^\dagger a_q^\dagger a_s a_r:$$

$$+ :a_p^\dagger a_q^\dagger a_s a_r: + :a_p^\dagger a_q^\dagger a_s a_r:$$

$$+ :a_p^\dagger a_q^\dagger a_s a_r: + :a_p^\dagger a_q^\dagger a_s a_r:$$

$$+ :a_p^\dagger a_q^\dagger a_s a_r: + :a_p^\dagger a_q^\dagger a_s a_r:$$

$$\langle \Phi | a_p^\dagger a_q^\dagger a_s a_r | \Phi \rangle = :a_p^\dagger a_q^\dagger a_s a_r: + :a_p^\dagger a_q^\dagger a_s a_r:$$

Examples:

$$a_p^\dagger a_q^\dagger a_s a_r = :a_p^\dagger a_q^\dagger a_s a_r:$$

$$+ :a_p^\dagger a_q^\dagger a_s a_r: + :a_p^\dagger a_q^\dagger a_s a_r:$$

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$$+ :a_p^\dagger a_q^\dagger a_s a_r: + :a_p^\dagger a_q^\dagger a_s a_r:$$

$$\langle \Phi | a_p^\dagger a_q^\dagger a_s a_r | \Phi \rangle = :a_p^\dagger a_q^\dagger a_s a_r: + :a_p^\dagger a_q^\dagger a_s a_r:$$

$$= K_{pr} K_{qs} - K_{ps} K_{qr}$$

$$\hat{H}_e = \sum_{pq} h_{pq}\, a_p^\dagger a_q + \frac{1}{2} \sum_{pqrs} \langle pq | rs \rangle\, a_p^\dagger a_q^\dagger a_s a_r$$

$$\hat{H}_e = \sum_{pq} h_{pq}\, a_p^\dagger a_q + \frac{1}{2}\sum_{pqrs} \langle pq|rs\rangle\, a_p^\dagger a_q^\dagger a_s a_r$$

$$\langle \Phi | \hat{H}_e | \Phi \rangle = \sum_{pq} h_{pq} \langle \Phi | a_p^\dagger a_q | \Phi \rangle + \frac{1}{2}\sum_{pqrs} \langle pq|rs\rangle \langle \Phi | a_p^\dagger a_q^\dagger a_s a_r | \Phi \rangle$$

$$\hat{H}_e = \sum_{pq} h_{pq}\, a_p^{\dagger} a_q + \frac{1}{2}\sum_{pqrs} \langle pq|rs\rangle\, a_p^{\dagger} a_q^{\dagger} a_s a_r$$

$$\langle \Phi|\hat{H}_e|\Phi\rangle = \sum_{pq} h_{pq} \langle \Phi|a_p^{\dagger} a_q|\Phi\rangle + \frac{1}{2}\sum_{pqrs} \langle pq|rs\rangle \langle \Phi|a_p^{\dagger} a_q^{\dagger} a_s a_r|\Phi\rangle$$

$$= \sum_{pq} h_{pq} K_{pq} + \frac{1}{2}\sum_{pqrs} \langle pq|rs\rangle \left(K_{pr}K_{qs} - K_{ps}K_{qr}\right)$$

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$$= \sum_{pq} h_{pq}\, K_{pq} + \frac{1}{2} \sum_{pqrs} \langle pq|rs\rangle \left(K_{pr} K_{qs} - K_{ps} K_{qr} \right)$$

$$= \sum_{i} h_{ii} + \frac{1}{2} \sum_{ij} \langle ij \| ij \rangle$$

More examples:

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$$\langle \Phi | a_p^\dagger a_q | \Phi_i^a \rangle$$

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$$= h_{ia}$$

More examples:

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$$= :a_p^\dagger a_q^\dagger a_s a_r a_a^\dagger a_i:$$

More examples:

$$\langle \Phi | a_p^\dagger a_q^\dagger a_s a_r | \Phi_i^a \rangle$$

$$= : a_p^\dagger a_q^\dagger a_s a_r a_a^\dagger a_i : + : a_p^\dagger a_q^\dagger a_s a_r a_a^\dagger a_i :$$

More examples:

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$$\langle \Phi | a_p^\dagger a_q^\dagger a_s a_r | \Phi_i^a \rangle$$

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$$+ :a_p^\dagger a_q^\dagger a_s a_r a_a^\dagger a_i: + :a_p^\dagger a_q^\dagger a_s a_r a_a^\dagger a_i:$$

More examples:

$$\langle \Phi | a_p^{+} a_q^{+} a_s a_r | \Phi_i^a \rangle$$

$$= :a_p^{+} a_q^{+} a_s a_r a_a^{+} a_i: + :a_p^{+} a_q^{+} a_s a_r a_a^{+} a_i:$$

$$+ :a_p^{+} a_q^{+} a_s a_r a_a^{+} a_i: + :a_p^{+} a_q^{+} a_s a_r a_a^{+} a_i:$$

$$= \kappa_{pi} \eta_{ra} \kappa_{qs} - \kappa_{pi} \eta_{sa} \kappa_{qr}$$

$$+ \kappa_{qi} \eta_{sa} \kappa_{pr} - \kappa_{qi} \eta_{ra} \kappa_{ps}$$

$$\langle \Phi | g_{jj} | \Phi_i^a \rangle$$

$$\langle \Phi | g | \Phi_i^a \rangle$$

$$= \frac{1}{2} \sum_{pqrs} \langle pq | rs \rangle \left(\kappa_{pi} \eta_{ra} \kappa_{qs} - \kappa_{pi} \eta_{sa} \kappa_{qr} + \kappa_{qi} \eta_{sa} \kappa_{pr} - \kappa_{qi} \eta_{ra} \kappa_{ps} \right)$$

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$$= \frac{1}{2} \sum_{j} \langle ij || aj \rangle + \frac{1}{2} \sum_{j} \langle ji || ja \rangle$$

$$\langle \Phi | g | \Phi_i^a \rangle$$

$$= \frac{1}{2} \sum_{pqrs} \langle pq | rs \rangle \big(\kappa_{pi} \eta_{ra} \kappa_{qs} - \kappa_{pi} \eta_{sa} \kappa_{qr} + \kappa_{qi} \eta_{sa} \kappa_{pr} - \kappa_{qi} \eta_{ra} \kappa_{ps} \big)$$

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$$= \sum_{j} \langle ij || aj \rangle$$

$$\langle \Phi | \hat{H}_e | \Phi_i^a \rangle$$

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$$= h_{ia} + \sum_j \langle ij || aj \rangle$$

Summary & Generalization

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Φ-normal	$\contraction{}{a_p}{}{a_q^\dagger}a_p a_q^\dagger = \eta_{pq} \qquad \contraction{}{a_p^\dagger}{}{a_q}a_p^\dagger a_q = \kappa_{pq}$

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Wick's thm: $Q = :Q: + :\overline{Q}:$

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vac-normal	$\overbracket{a_p a_q^{\dagger}} = \delta_{pq}$
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vac-normal	$\overbracket{a_p\, a_q^\dagger} = \delta_{pq}$
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Ψ-normal	$\overbracket{a_p\, a_q^\dagger} = \eta_{pq} \quad \overbracket{a_p^\dagger\, a_q} = \gamma_{pq}$

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vac-normal	$\overline{a_p a_q^\dagger} = \delta_{pq}$
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Ψ-normal	$\overline{a_p a_q^\dagger} = \eta_{pq}$ $\quad \overline{a_p^\dagger a_q} = \gamma_{pq}$
	$\overline{q_1 q_2 \cdots q_m} = \lambda(q_1 q_2 \cdots q_m)$