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A swarm intelligence graph-based pathfinding algorithm (SIGPA) for multi-objective route planning

Charis Ntakolia [*], Dimitris K. Iakovidis

Department of Computer Science and Biomedical Informatics, School of Science, University of Thessaly, Greece

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ABSTRACT

Personalized tourist route planning (TRP) and navigation are online or real-time applications whose mathematical modeling leads to complex optimization problems. These problems are usually formulated with mathematical programming and can be described as NP hard problems. Moreover, the state-of-the-art (SOA) path search algorithms do not perform efficiently in solving multi-objective optimization (MO) problems making them inappropriate for real-time processing. To address the above limitations and the need for online processing, a swarm intelligence graph-based pathfinding algorithm (SIGPA) for MO route planning was developed. SIGPA generates a population whose individuals move in a greedy approach based on A^* algorithm to search the solution space from different directions. It can be used to find an optimal path for every graph-based problem under various objectives. To test SIGPA, a generic MOTRP formulation is proposed. A generic TRP formulation remains a challenge since it has not been studied thoroughly in the literature. To this end, a novel mixed binary quadratic programming model is proposed for generating personalized TRP based on multi-objective criteria and user preferences, supporting, also, electric vehicles or sensitive social groups in outdoor cultural environments. The model targets to optimize the route under various factors that the user can choose, such as travelled distance, smoothness of route without multiple deviations, safety and cultural interest. The proposed model was compared to five SOA models for addressing TRP problems in 120 various scenarios solved with CPLEX solver and SIGPA. SIGPA was also tested in real scenarios with A* algorithm. The results proved the effectiveness of our model in terms of optimality but also the efficiency of SIGPA in terms of computing time. The convergence and the fitness landscape analysis showed that SIGPA achieved quality solutions with stable convergence.

1. Introduction

Combinatorial optimization and multicriteria decision theory are employed when mathematical programming optimization models involve decision making over various attributes. Route planning and, especially, tourist route planning (TRP), recently, is considered as a multicriteria decision problem, targeting to integrate more influential factors in the decision making process, thus adding to the complexity of the methodology. Traditional approaches of TRP incorporate only hard factors expressed as single objectives, such as time, cost or distance. The evolution of TRP demands multi-objective formulations and the consideration of soft factors, such as tourist's safety, preferences, satisfaction and characteristics, *e.g.* impairment or age group (Duleba and Moslem, 2019; Gavalas et al., 2014). Indeed, visiting unfamiliar places can labor communication issues and navigation difficulties in tourist areas which make the tourist visit challenging. The situation becomes even more critical in case of individuals with disabilities, such as the visually impaired individuals (VIIs), who need to get familiarized with the surrounding environment in order to walk safely and with confidence in the visiting area. Therefore, TRP has been proven essential for providing a personalized guiding map that will take into account even the particular needs of elderly or individuals with disabilities, guiding them through routes with special installations such as guiding pavements and wheelchair bars.

In the context of route planning, modern tourism offers a variety of services to satisfy the demands of different types of tourists. Apart from personalization, new trends involve ecofriendly means of transport, such as electric vehicles for tourists (Karbowska-Chilinska and Zabielski, 2017; Ramos et al., 2018; Li et al., 2018), tourism for third age (Klimova, 2017; Furukawa and Wang, 2019) and even services for individuals with disabilities (Iakovidis et al., 2020; Lehto et al., 2018; Devile and Kastenholz, 2018; Ntakolia et al., 2020). Traditional route planning

* Corresponding author.
E-mail address: cntakolia@uth.gr (C. Ntakolia).

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approaches focus on calculating the shortest path (Bozyigit et al., 2017; Idri et al., 2017). These approaches have been a key functionality of GPS-enabled navigation devices. Alternative approaches have been developed to find the fastest (Song et al., 2017), the safest (Xu et al., 2016), the most attractive (Sacharidis et al., 2017; Liu et al., 2016), or the easiest route (Krisp and Keler, 2015). A more accurate personalization can be achieved when user preferences and user constraints are integrated in the route planning process and particularly to tourist route planning for enhancing the tourist experience of the trip for the users.

In the literature various TRP models have been proposed based on Geographical Information Systems (GIS) for retrieving geospatial information (Xiao et al., 2017; Du and Hu, 2018; Ayala et al., 2017; Cenamor et al., 2017), and on mathematical programming and optimization algorithms (Xu et al., 2016, 2017; Gavalas et al., 2017; Li et al., 2019; Yuan and Shi, 2019; Zhang, 2019; Lee and Yu, 2018; Ntakolia and Iakovidis, 2021), such as heuristics and metaheuristics, for formulating and generating the optimal tourist route. In TRP three typical route planning scenarios are observed (Table 1). However, most of the studies in the literature (Table 2) are either based on prefixed routes, while others fail to invoke, in an effective way, user requirements, user disabilities and road data. The lack of such valuable auxiliary information leads to the development of inaccurate tourist routes. Moreover, most existing route planning strategies focus only on a particular travel route planning scenario which make them unable to be directly applied to other route planning scenarios (Table 1). Thus, the need for a generic formulation of TRP that will combine technical factors and subjective factors is highlighted (Duleba and Moslem, 2019).

In our study a generic binary nonlinear model and optimization algorithm are developed for addressing the TRP problem in various scenarios (Table 1). The multi-objective model will contribute to the existing ones by incorporating the optimization of five main cultural and human attributes such as the minimization of traveled distance, walking duration, collision risk, abrupt turns and route deviations (especially in case of individuals with disabilities or electric vehicles), and multiple detours or crossovers from the same areas, under user preference constraints such as the preferred POIs to be visited and the preferred tour duration. Furthermore, the model is based on incident intervals compared to the most models in the literature which are based on time intervals, a fact that decrease significantly the computing time needed for TRP. For solving the proposed Mixed 0–1 Integer Quadratic Programming (MIQP) model a metaheuristic approach is proposed based on a swarm intelligence graph-based pathfinding algorithm (SIGPA). The SIGPA algorithm is a swarm intelligence algorithm that selects randomly a specified number of points from the initial solution for altering paths and searching the solution space in a greedy approach to find an optimal solution in a constraint solution space. The MIQLP formulation is compared with other mixed integer and linear models and solved initially with IBM ILOG CPLEX solver (CPLEX). Lastly, the proposed metaheuristic SIGPA, which is the main theoretical contribution of our study, is compared with CPLEX solver for solving the proposed MIQLP mathematical formulation for several instances, as well as, with popular

Table 1
Typical tourist route planning scenarios.

Scenario Category	Scenario Description
General Route Planning	Propose a tourist route plan including the most popular and cultural sights by minimizing the distance within a given time frame and budget
Next-Point Recommendations	For a given set of POIs that the tourist has already visited, the tourist route plan indicates the next POI to be visited based on tourist preferences, POIs popularity and time frame
Must-Visiting Planning	For a given set of POIs that the tourist must visit, an optimal tourist route with minimum detours is proposed so the tourist will pass from all the selected POIs within a specific time frame

A* algorithm for solving real scenarios of RTP. Contributions of this study include:

- The development of a generic mathematical formulation for tourist route planning that can also be extended to address robotic and other similar routing problems.
- The development of a novel metaheuristic algorithm based on swarm intelligence and a graph-based pathfinding approach for solving graph-based MO problems.
- A novel multiple-criteria decision analysis based on normalized root mean square error in order to avoid Pareto optimality approaches for finding the optimal route. This approach contributes to the development of routes that balances among the objective function terms.
- A thorough evaluation of the proposed methodology by comparing various TRP formulations and by convergence analysis and fitness landscape analysis to prove the efficiency of the novel metaheuristic algorithm.

The rest of this paper consists of five sections. In section 2, the related background is investigated with a review of the related works. A concise definition and formulation of the problem under investigation is provided in section 3. The proposed algorithm is described in detail in section 4, and the results from its evaluation are presented in section 5. The conclusions that can be derived from this study are summarized in the last section.

2. Related work

The baseline combinatorial optimization problem for TRP is commonly addressed in the literature as an orienteering problem (OP) or as an Arc Routing Problem (ARP). In the OP, a set of vertices is given. Each vertex in the set is linked with a score. The target is to find the shortest path composed of vertices that optimize the score (Vansteenwegen et al., 2011). Hence, most of the literature studies (Table 2), that address the TRP problem, are based on combinatorial optimization such as mathematical programming, heuristic algorithms and MO decision making.

2.1. Personalized TRP

A common approach to TRP is the generation of personalized routes. An iterated local search metaheuristic algorithm was proposed for implementing a context-aware mobile city guide for Athens (Greece) for tourists (Gavalas et al., 2017). The methodology provides a near-optimal sequencing of POIs with recommended tours based on travel restrictions and POI properties such as visit time availability and safety of visited areas. An improved personalized route recommendation algorithm was proposed in (Xu et al., 2016), taking into account the real-time congestion situation and the preference level of spots. An improved Ant Colony optimization (ACO) algorithm was proposed for personalized route management (Yuan and Shi, 2019) and for finding the optimal tourism route in coastal areas (Zhang, 2019). The Quota Travelling Salesman Problem with passengers was addressed in (Silva et al., 2020) and solved with ACO algorithm. The model aims to minimize travel costs taking into account passengers, requirements, vehicle capacity and travel time among other criteria. Another heuristic approach based on hybrid evolution for personalized tours was proposed for solving the day tour route problem in a time dependent stochastic environment (Liao and Zheng, 2018).

2.2. TRP to areas with specific characteristics

TRP is applied to areas with specific characteristics. A decision making model for guiding tourists through marketplaces was implemented (Sirirak and Pitakaso, 2018) based on marketplace and cultural sights location. In this study, the Adaptive Large Neighborhood Search

Table 2
Related work summarization.

TRP models	Targets	Solving approach	Studies
Personalized TRP	• tour based on travel restrictions and POI properties • safe route planning • route based on real-time congestion • preference level of spots • optimal tourism route in coastal areas • multi-objective route planning • satisfying user preferences and requirements	• Iterated Local Search metaheuristic algorithm • Ant Colony Optimization algorithm • Differential Evolution algorithm • Pareto optimality • Evolutionary Algorithm	(Xu et al., 2016; Gavalas et al., 2017; Yuan and Shi, 2019; Zhang, 2019; Silva et al., 2020; Liao and Zheng, 2018)
TRP to areas with specific characteristics	• maximize economic benefit • evaluate the interest of geological areas • forest wetlands exploration based on GIS • efficient routes of ecotourism • maximize the cultural experience • probe sightseeing routes in Asian cities • optimal route with hotel selection	• Adaptive Large Neighborhood Search method • Genetic Particle Swarm Optimization algorithm • algorithm based on the regional comprehensive destiny • cosine similarity algorithm • improved Floyd-Warshall algorithm • greedy randomized adaptive search	(Du and Hu, 2018; Xu et al., 2017; Sohrabi et al., 2020; Sirirak and Pitakaso, 2018; Zhen and Gao, 2017; Barrena et al., 2016; Yuan and Uehara, 2019)
Multi-objective TRP	• maximize profit and POI visits • minimize travel distance and costs • maximize profit, minimize routes • minimize traveled time, maximize touristic value • minimize travel distance and costs and maximize POI visits • minimize difficult routes for vehicles, maximize tourist satisfaction • maximize safety for tourists and minimize travel time and/or costs	• Genetic Algorithm • Particle Swarm Optimization • artificial bee colony • Nondominated Sorting Genetic Algorithm II • Ant Colony Optimization • Memetic Algorithm • A* • evolutionary algorithm • Pareto optimality based method	(Huang et al., 2006; Beed et al., 2020; Martín-Moreno and Vega-Rodríguez, 2018; Jin et al., 2019; Zheng and Liao, 2019; Chen et al., 2015; Mei et al., 2016; Fournier et al., 2019; De Falco et al., 2015; Zhu, 2020)

(ALNS) method was used to retrieve routes to maximize the economic benefit. Another approach based on geographical areas was proposed (Zhen and Gao, 2017) to evaluate the geological tourist value of route planning. An optimal tourism route planning algorithm was proposed (Du and Hu, 2018) for forest wetland based on Geographical Information Systems (GIS) and hybrid Genetic Particle Swarm Optimization (GA-PSO) algorithm. The TRP was formulated with different MILP models (Barrena et al., 2016) for determining efficient routes of ecotourism targeting to maximize the cultural experience of the visitors through the traveled path and solved with the use of the cosine similarity algorithm (Yuan and Uehara, 2019) in a case study in Japan. An improved Floyd algorithm was proposed (Xu et al., 2017) for computing the shortest route among 12 scenic spots in the city of Yangzhou. Also, two improvements of the original Floyd-Warshall algorithm (Floyd, 1962) were presented for reducing the computational complexities in solving the shortest travel route path problem. A greedy randomized adaptive search approach was proposed for solving the orienteering problem with hotel selection (Sohrabi et al., 2020).

2.3. Multi-objective TRP

MO route planning is commonly adopted to automated robotics, such as Unmanned Aircraft Vehicles or Unmanned Surface Vehicles, for mission planning (Ma et al., 2018; Yin et al., 2018; Nazarahari et al., 2019; Hidalgo-Paniagua et al., 2017) under static or dynamic environments. However, TRP based on MO route planning is mostly applied to static environments for finding an optimal route based on time and travel costs and/or surrounding view quality. Specifically, Genetic Algorithm (Huang et al., 2006), Particle Swarm Optimization and artificial bee colony (Beed et al., 2020; Martín-Moreno and Vega-Rodríguez, 2018), Nondominated Sorting Genetic Algorithm II (Jin et al., 2019), Ant Colony Optimization (Zheng and Liao, 2019; Chen et al., 2015), Memetic Algorithm (Mei et al., 2016) have been employed to this type of MO problem. Regarding MO TRP with tourist vehicles, such as buses and cycles, the standard aim is to maximize passengers' satisfaction or preferences and sightseeing visits in an optimal schedule. Such approached include A* (Fournier et al., 2019), evolutionary algorithm (De Falco et al., 2015) and Pareto optimality based method (Zhu, 2020).

The above studies approach the TRP problem partially by providing the shortest, fastest, safest or most attractive routes failing to propose a generic formulation that will incorporate user constraints derived from disabilities or means of transport, such as electric bicycles or scooters. Indeed, the need for MOTRP is emphasized in various studies (Ntakolia and Iakovidis, 2021; De Falco et al., 2015; Zhu, 2020; Jamal et al., 2017). However, the majority of MOTRP problems are addressed with computationally demanding approaches, such as global optimization techniques and software for linear/integer programming, *e.g.* CPLEX and path-search algorithms, *e.g.* Dijkstra or A*. To find an optimal solution under the various conflicting objectives, a percentage of the aforementioned studies adopt the conventional strategy of weighted sum method (WSM) or Pareto optimality. In WSM approach, the objectives are combined into one single objective scalar function. This generates an efficient solution for the MO problem, but the results may vary significantly depending the selection of the weights which can lead to local optima traps. The Pareto optimality approach overcomes the above limitations by providing multiple solutions to the decision maker, which make its application most popular. Most of the presented heuristic algorithms for solving the MOTRP are Pareto optimality-based approaches which aim to converge to Parent front. In that cases, the number of improved criteria (objectives), the extent of these improvements and, also, the preference information of each criterion (objective) are not considered during the calculation procedure. Furthermore, these limitations are crucial in real applications since only one 'compromise' Pareto solution is the preferable. Consequently, an additional multi-criteria decision making method is necessary for avoiding subjective or undesirable choice of a solution (Deb and Sundar, 2006; Deb et al., 2011; Hasuike et al., 2013; Shen and Ge, 2019).

Hence, for a more substantial approach to modern and personalized TRP a multi-objective decision-making model (MIQLP) for TRP is proposed. To this end, the users will be able to develop a route based on their needs and personal preferences. To generate the optimal route and to avoid the conventional approaches, a swarm intelligence graph-based

Table 3
The sets, parameters and decision variables of the model.

Sets	Parameters
$\mathscr{N}$: the set of nodes in the graph	$risk_{(i,j)}$: the collision risk factor of $arc(i,j)$
$\mathscr{A} = \{(i,j) : i,j \in \mathscr{N}\}$: the set of permitted arcs/edges in the graph (paths, pavements, roads, etc.)	$turn_{(i,j,k)}$: the turn penalty factor of the consecutive arcs $arc(i,j)$ and $cult(i,j)$: the cultural value of $arc(i,j)$
$\mathscr{N}^m = \{i : i \in \mathscr{N}\}$: the set of nodes in the graph that corresponds to POI	$loss(i,j)$: the probability of losing cultural interest while walking the $arc(i,j)$
$\mathscr{M} = \{i : i \in \mathscr{N}^m\}$: the set of POI that the user wants to visit	$d_{(i,j)}$: the distance from node i to node$j$
$\mathscr{T} = \{0, \cdots, D_{max}\}$: the set of time periods (time frame) in which the tour should finish. Every time period represents an event such as moving from one node to another.	s: the entrance node of the network
	s_0: the entrance area of the entrance that forms the entrance $arc(s_0, s)$ from where the user enters the network
	e: the exit node of the network
	e_0: the exit area of the exit that forms the exit $arc(e, e_0)$ from where the user exits the network
	D_{min}, D_{max}: the minimum and maximum desired duration of the tour by the user, respectively (in time periods)
	$T_{(i,j)}$:the duration (in periods) for the user to travel an $arc(i,j)$

Decision Variables

$$R^t_{(i,j)} = \begin{cases} 1, \textit{if the user is travelling } arc(i,j) \textit{ at time period } t \\ 0, \textit{otherwise} \end{cases}, \forall (i,j) \in \mathscr{A}, t \in \mathscr{T}$$

Artificial Variables

The Artificial Arc Variables $C_{(i,j)} = \begin{cases} 1, \textit{if the user has travelled } arc(i,j) \\ 0, \textit{otherwise} \end{cases}, \forall (i,j) \in \mathscr{A}$

The Artificial POI Variables $P_m = \begin{cases} 1, \textit{if the user has visited POI } m \\ 0, \textit{otherwise} \end{cases}, \forall m \in \mathscr{M}$

pathfinding algorithm (SIGPA) is designed. SIGPA is capable of solving graph-based multi-objective problems, such as the proposed TRP problem by using a normalized root mean square error as an evaluation criterion of the generated solutions.

The proposed mathematical formulation and metaheuristic algorithm can also be extended to robotic route planning with mandatory visits in time windows where there is a need for safe and smooth paths (Mantha et al., 2020). The POIs can play the role of mandatory destinations and the factor of penalized multiple crossovers contribute to the generation of efficient paths without unnecessary loops. In the following section the MIQLP formulation is presented. In section 4 the SIGPA is analyzed and in sections 5 and 6 the experimental evaluation and the conclusions are presented, respectively.

3. Problem definition and formulation

3.1. Model scope for tourist route planning

TRP is modeled as an arc-route planning and therefore it is defined on an undirected $graph G = (\mathscr{N}, \mathscr{A})$, where $\mathscr{N} = \{0, 1, 2, ..., n\}$ is the node set and $\mathscr{A} = \{(i,j) : i,j \in \mathscr{N}, i \neq j\}$ is the edge/arc set. Each $arc(i,j)$ is associated with costs based on the minimization criteria (see Definitions 1–4). Each route can be represented by a sequence of required arcs. Table 3 shows the sets, parameters and decision variables adopted for the generic formulation of the TRP problem in this study.

The tourist route planning is highly dependent on the purpose of the trip, user preferences and mean of transport. In this study a generic formulation is proposed to propose tourist routes for individuals that are walking around an outdoor cultural environment, on foot or by using bicycles or even electric vehicles, which is a modern and popular mean of transport within a cultural area. Also, the model can generate routes suitable for individuals with disabilities, such as visually impaired individuals. To this end, a multi-objective binary quadratic programming model is proposed for addressing the TRP problem with a generic formulation for various types of tourists. Multiple objectives are considered in the model so the users will be able to choose the attributes that their personalized tourist route will have. So, the model targets to minimize the following terms:

- The safety navigation term that minimizes the difficulty of the user to walk through the route. It takes into account the obstacles in the arcs and the existence of special installations such as wheelchair bars or guided pavements to prioritize the selection of arcs with low collision risk factor.
- The shortest path term that minimizes the distance traveled by the user in order to visit the preferred POIs in the area. In this way, the model prioritizes the pleasure over the physical fatigue of the user during his visit, an important factor especially for individuals with disabilities that would not prefer to wonder around needlessly.
- The route deviation term that minimizes the number of consecutive changes in the route in order to construct a smooth route. This term is particularly useful for individuals with disabilities such as visually impaired individuals who prefer to walk as straight as possible so they will retain their orientation.
- The cultural interest term that minimizes the multiple crossovers from the same arcs so the tourist will explore untraveled arcs with cultural value.

By minimizing these terms, the users, who are people that would like to navigate in outdoor spaces of cultural interest, will be able to explore as many cultural sites as possible, both safely and efficiently. The consideration of these terms is based on recent studies. For instance, recent surveys regarding tourist trip design problems highlights the need for generic multi-objective formulation that could cover various and more realistic TRP problems satisfying tourists modern preferences, such as using tourist electric vehicles, and variety of tourist groups, such as visually impaired individuals (Gavalas et al., 2014, 2015; Zhu, 2020; Lim et al., 2019; Eby and Molnar, 2002). The information about the area required for the parameterization of the model can be provided from Internet resources, such as Google Maps® application, and tour providers in collaboration with domain experts and local authorities.

Definition 1 – Collision risk factor

The difficulty of the tourist/user to travel a specific arc is given by the collision risk factor:

$$risk_{(i,j)} \in [0, 1], \forall (i,j) \in \mathscr{A}$$

This objective term is very important in case of children, elderly, individuals with disabilities or tourists who want to use electric vehicles or bicycles to visit point of interest. The collision risk factor is evaluated based on the 'probability' of a collision to be occurred with a static obstacle or of the difficulty of user to walk through $arc(i,j)$ due to the terrain type, terrain condition, stairs, or even due to lack of guided pavements and wheel chair bars in case of individuals with special needs or elderly individuals. In practice, the collision risk factor for each arc in the graph is set during the onsite investigation and mapping of the area by the tour provider. A high value of the factor indicates a difficult or dangerous arc to be traversed by the user. A factor that is highly linked with and dependent on the travel duration of an arc (see Definition 2).

Definition 2 – Arc travel duration

Following the approach that is adopted for travel estimation in IP models (Ntakolia et al., 2020; Mitici and Blom, 2019), the travel duration required by the user to travel a specific arc, $T_{(i,j)}$, is defined by a linear piecewise function that determines the duration (in time periods) for the user to travel an $arc(i,j)$ based on the collision risk of the arc:

$$T_{(i,j)} = \begin{cases} T_{low}, 0 \le o_{(i,j)} \le a \\ T_{med}, a < o_{(i,j)} \le b \text{ }, \forall (i,j) \in \mathscr{A} \\ T_{max}, b < o_{(i,j)} \le 1 \end{cases}$$

where $T_{max}, T_{low}, T_{med}$ are the duration (in time periods) for the user to travel an $arc(i,j)$ with high collision risk, low collision risk and medium collision risk, respectively, and a, b are the parameters of the piecewise function with $a, b \in (0,1) : a < b$. This term provides an estimation on the total duration of the visit by the user based on the selected route. The collision risk of an arc reflects the difficulty of the user to traverse it. This difficulty can affect the time needed by the user to traverse an arc and consecutively to implement the propose route. Hence, to roughly estimate the travel duration the linear piecewise function is adopted. In practice, the durations for travelling the different arcs can be initially estimated by averaging the respective durations from an initial set of users and be improved as the population of users increases over time.

Definition 3 – Turn penalty factorFig. 1

The turn penalty factor ($turn_{(i,j,k)}$) indicates the brusque change in the route and the turn that the user need to perform in order to walk from the $arc(i,j)$ to the consecutive $arc(j,k)$. The value of the turn penalty factor is based on the angle that the consecutive arcs form (Fig. 2) and for the consecutive arcs $arc(i,j)$ and $arc(j,k)$ is given by the following expression:

$$turn_{(i,j,k)} = \frac{|\widehat{\varphi}|}{180} \in [0,1] \forall (i,j),(j,k) \in \mathscr{A}$$

This factor can have significant impact to the route generation in case of electric vehicles and visually impaired individuals. In both cases, smooth routes with minimum deviations in the direction of the user are requested so the users will not lose their balance or orientation, respectively.

Definition 4 – Cultural experience factor

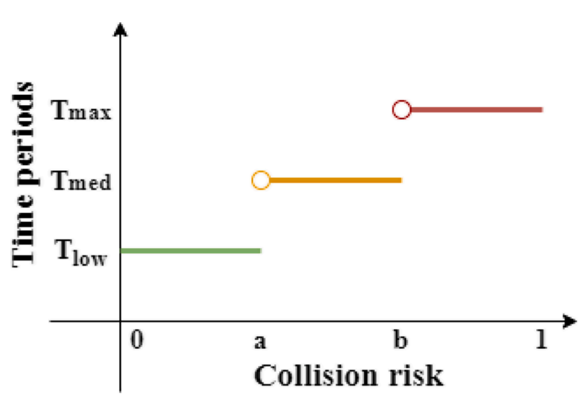

Fig. 1. The arc travel linear piecewise function.

The cultural experience factor is measured with the probability of the user to lose its interest in the cultural context while walking a specific arc $loss(i,j) \in [0,1], \forall (i,j) \in \mathscr{A}$. This cost of $arc(i,j)$ penalizes multiple crossovers from the same arc in order the tourist will be able to explore more cultural areas and therefore maintaining a high level of cultural experience. Hence, the higher the cultural value of the arc due to the presence of POIs, the lower the penalty of the arc. This factor is very important in case of tourist route planning because it controls the amount of valuable information and knowledge the tourist will gain by visiting multiple POIs. In practice, the cultural value of POIs can be defined based on the information provided by experts (*e.g.*, by historians and tourism experts), well-known databases with social feedback for tourist rankings, such as in (Ntakolia et al., 2020; Cenamor et al., 2017; Wang et al., 2019; Korakakis et al., 2017), or based on a combination of such information.

3.2. A mixed binary quadratic programming model with discrete time formulation

The sets, the input parameters, the decision variables, the objective function and the constraints of the proposed mathematical modeling are presented below:

The Artificial POI Variables

$$P_m = \begin{cases} 1, iftheuserhasvisitedPOIm \\ \quad 0, otherwise \end{cases} \forall m \in \mathscr{M}$$

Objective Function

The objective function $(o.f.)$ is a multi-criteria binary nonlinear function consisting of the following terms that need to be minimized:

$$z_1 = \sum_{t \in \mathscr{T}} \sum_{i \in \mathscr{N}} \sum_{\substack{j \in N: \\ (i,j) \in A}} \left(risk_{(i,j)} R^t_{(i,j)} \right) (o.f.1)$$

The safe navigation term is expressed in a linear form so the model will penalize the arcs with higher collision risk factor. Therefore, the model tries to optimize the tour plan by minimizing the total collision risk among the selected arcs.

$$z_2 = \sum_{t \in \mathscr{T}} \sum_{i \in \mathscr{N}} \sum_{\substack{j \in N: \\ (i,j) \in A}} \left(d_{(i,j)} R^t_{(i,j)} \right) (o.f.2)$$

The shortest path term is expressed in a linear form so the model will minimize the distance that user needs to travel in order to visit the selected POIs.

$$z_3 = \sum_{t=1}^{D_{max}} \sum_{j \in N} \left(\sum_{\substack{i \in N: \\ (i,j) \in A}} R^{t-1}_{(i,j)} \left(\sum_{\substack{k \in N: \\ (j,k) \in A}} turn_{(i,j,k)} R^t_{(j,k)} \right) \right) (o.f.3)$$

The quadradic term minimizes the route deviations so the VI user will walk a smooth route will minimum number of turns.

$$z_4 = \sum_{i \in \mathscr{N}} \sum_{\substack{j \in N: \\ (i,j) \in A}} \left(cult(i,j) C_{(i,j)} - \left(loss(i,j) \left(\sum_{t \in \mathscr{T}} \left(R^t_{(i,j)} \right) - 1 \right) \right) \right) (o.f.4)$$

The linear term of the objective function minimizes the multiple crossovers from arcs that have low cultural interest. Therefore, it maintains the cultural experience of the tourist in a high level.

The model is imposed in the following constraints:

3.2.1. Route planning constraints

$$R^0_{(s_0,s)} = 1 (c.1)$$

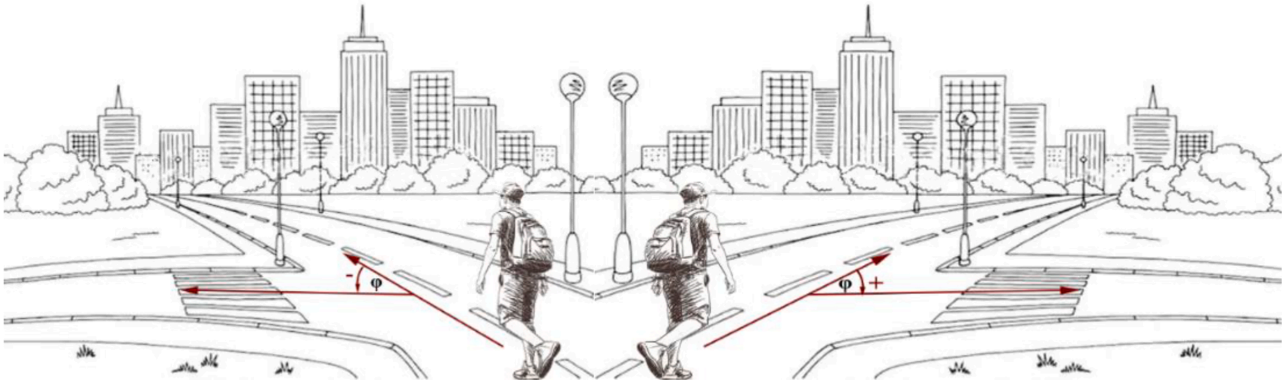

Fig. 2. Negative and positive rotate angle between paths.

$$\sum_{t=1}^{D_{max}} R^t_{(s_0,s)} = 0 (c.2)$$

$$\sum_{t=0}^{D_{max}} R^t_{(e,e_0)} = 1 (c.3)$$

$$\sum_{(i,j)\in E} R^t_{(i,j)} = \sum_{r=0}^{t} R^t_{(s_0,s)} - \sum_{r=0}^{t-1} R^r_{(e,e_0)}, \forall t \geq t_0 (c.4)$$

$$\sum_{\substack{i\in N:\\(i,j)\in A}} R^t_{(i,j)} - \sum_{\substack{k\in N:\\(j,k)\in A}} R^{t+1}_{(j,k)} = 0, \forall j \in \mathscr{N}, \forall t \geq t_0 (c.5)$$

3.2.2. User preferences constraints

$$\sum_{t=0}^{D_{max}} \sum_{\substack{i\in N:\\(i,m)\in E}} R^t_{(i,m)} \geq 1, \forall m \in M (c.6)$$

$$D_{min} \leq \sum_{t\in\mathscr{T}} \sum_{(i,j)\in\mathscr{A}} T_{(i,j)} R^t_{(i,j)} \leq D_{max} (c.7)$$

3.2.3. Artificial variable assignment constraints

$$\frac{1}{D_{max}} \sum_{t\in\mathscr{T}} R^t_{(i,j)} \leq C_{(i,j)} \leq \sum_{t\in\mathscr{T}} R^t_{(i,j)}, \forall (i,j) \in \mathscr{A} (c.8)$$

$$\frac{1}{D_{max}} \sum_{t\in\mathscr{T}} \sum_{\substack{i\in N:\\(i,m)\in E}} R^t_{(i,m)} \leq P_m \leq \sum_{t\in\mathscr{T}} \sum_{\substack{i\in N:\\(i,m)\in E}} R^t_{(i,m)}, \forall m \in \mathscr{M} (c.9)$$

3.2.4. Variable constraints

$$R^t_{(i,j)} \in \{0,1\}, \forall (i,j) \in \mathscr{A}, \forall t \in \mathscr{T} (c.10)$$

$$C_{(i,j)} \in \{0,1\}, \forall (i,j) \in \mathscr{A} (c.11)$$

$$P_m \in \{0,1\}, \forall m \in \mathscr{M} (c.12)$$

The above constraints form the field of feasible solutions of the problem. Particularly, the constraints (1) force the user to start the tour. The entrance $arc(s_0, s)$ is forced to be traversed in time period 0 in order to start counting the visiting time (in time periods). Similarly, in the constraints (3), the exit $arc(e, e_0)$ is forced to be traversed within a maximum predefined time period (D_{max}) in order to stop counting the visiting time (in time periods). The constraints (4) impose the user to keep walking to an arc until it reaches the exit. The constraints (5) assure the route continuity among arcs, while constraints 6 force the user to visit the selected POIs. Constraints (7) force the duration of the tour to be kept between the minimum and the maximum desired duration. Constraints 8 and 9 set the artificial variables to 1 or 0.

3.3. Feasibility of a solution

Given a solution $\mathscr{s}$ that consists of a sequence $\mathscr{V}$ of required arcs, the $\mathscr{s}$ is feasible if and only if for each POI selected by the tourist to be visited there is at least one arc in the solution that leads to that POI:

$$\forall m \in \mathscr{M}, \exists arc(i,m) \in \mathscr{A} : arc(i,m) \in \mathscr{V} \quad (1)$$

In Fig. 3, we present two examples of a feasible solution (Fig. 3a) and a non-feasible solution (Fig. 3b). The green node 1 is the entrance node and the red node 9 is the exit node. The orange nodes 4,6 and 7 are the POIs selected by the tourist to visit. Fig. 3a shows a solution $\mathscr{s}_1$that consists of a sequence $\mathscr{V}$ of arcs that pass from all the selected POIs by the tourist (nodes 4, 6 and 7) and therefore the solution is acceptable. On the other hand, Fig. 3b illustrates a non-feasible solution $\mathscr{s}_2$since the generated route does not pass from the POI in node 6.

4. Population-based greedy path search (SIGPA) algorithm

A metaheuristic algorithm is proposed for solving the route planning problem formulated in Section 3. The algorithm is built upon the advantages of the population-based heuristic techniques, the A^*search algorithm modified to search the solution space in a greedy approach by using the mean square error metric for evaluation criterion. Given an initial solution, the process is performed in parallel for a number of particles and repeated for a number of iterations until the termination criteria are met. Each particle searches the solution space with the greedy graph-based pathfinding algorithm (GPA) in order to find an alternative solution. In this way, the algorithm tries to improve the current found solution and avoid stationary points.

The implementation of the metaheuristic population-based greedy path search (SIGPA) algorithm for constructing a route combines the main principles of the population-based heuristics and of the well-known shortest path algorithm, A^*search algorithm. To this end, the advantages of the aforementioned approaches are exploited by the SIGPA algorithm (Table 4). The node that is selected to be included in the solution will minimize the objective and the distance function compared to the neighbor nodes. The proposed GPA algorithm uses the mean square error evaluation criterion and a distance norm for choosing an optimal route that will satisfy the user preferences such as the route to pass from the selected POIs.

An initial route $\mathscr{s}_0$ is generated. From the generated route, k nodes are randomly selected. For each selected node, the node sequence until the selected node is inserted in the new generated route. The path in the initial route that starts from the selected node is excluded from the

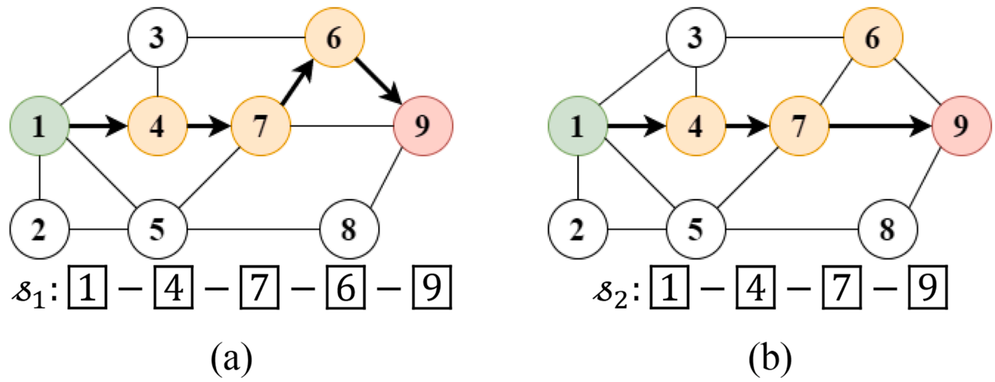

Fig. 3. Examples of (a) a feasible solution $\mathscr{s}_1$and (b) a non-feasible solution $\mathscr{s}_2$.

search space of feasible solutions forcing the algorithm to choose another direction and therefore generating a new route. The new k constructed routes and the current route are compared and the most optimal one in terms of minimizing the objective function is chosen. The process is repeated until the termination criteria are met, such as the number of iterations or the error between two consecutive iterations is smaller than a specified threshold.

4.1. Greedy graph-based pathfinding algorithm (GPA)

4.1.1. Multiple-criteria decision analysis

Various real-world problems, such as the RP problems, are multi-objective optimization problems with conflict optimization criteria. For such problems, there is a set of solutions which is optimal. This set is referred as Pareto-optimal (PO) solutions. Conventional approaches of mathematical programming proved to be unable or inefficient in finding a desired optimal solution to this kind of multi-objective problems due to mutual restrictions or to non-convex or discrete optimal solution (Li et al., 2017). Therefore, metaheuristic, and specifically population-based, algorithms have gained a lot of attention. To overcome the limitations of Pareto-based dominant approaches (Li et al., 2017; Sharma et al., 2019; Parkes et al., 2015) applied to population-based metaheuristics an evaluation framework is proposed based on Normalized Root Mean Square Error (NRMSE) to find the most balanced Pareto optimal solution. Hence, a balanced trade-off among the optimization criteria is reassured.

Table 4
Main principles of population-based heuristics and path search algorithm adopted in SIGPA algorithm.

Population-based heuristic principles	SIGPA benefits
low-pass filter of the landscape	• ignoring local distractions
search different parts of the fitness landscape	• no impact of the initial position
focus on the search caused by crossover	• solution produced by averaging multiple solutions
A^*star search algorithm principles	SIGPA benefits
best first search	• greedy path search by holding only the best proceeding node in current position
find shortest path through a search space using heuristic function	• find the minimum cost route that is directed to target decreasing the travelled distance
no preprocessing graph procedures	• decrease significant the computing time

The evaluation of each arc contribution to the objective function will be done based on the Normalized Root Mean Square Error and a distance function in order chose the arcs with the best balance among the objective function terms.

Let $\mathscr{A}$ be the set of arcs, $\mathscr{A}_n \subset \mathscr{A}$ be the set of nodes that are neighbors with node n, $\mathscr{F}\left\{f_j, j = 1, \cdots, k | k \in \mathbb{Z}^+\right\}$ be the set of the measures, such as collision risk, loss of cultural interest, duration, distance, etc., that are used to determine the optimal route and $\mathscr{F}^a = \left\{f_j^a, j = 1, \cdots, k | k \in \mathbb{Z}^+\right\}$, $\forall a \in \mathscr{A}_n$, be the set of the values of the measures for the *arca* under examination. The contribution of the *arca* will correspond to a performance vector, $r^a \in \mathbb{R}^{+k\times 1}$, such that $r^a = [f_1^a, \cdots, f_k^a], \forall a \in \mathscr{A}_n$.

Definition 5 – Normalized Root Mean Square Error metric and metric space for the arcs under examination

Let r^* be the optimal performance vector that corresponds to the set of minimum measures $\mathscr{F}^* = \left\{f_j^* = min\left\{f_j^a | a \in \mathscr{A}_n\right\}, \forall f_j \in \mathscr{F}\right\}$. Let dr^a be the difference $r^a - r^* = \left\{f_j^a - f_j^*, j = 1, \cdots, k | k \in \mathbb{Z}^+\right\}, \forall a \in \mathscr{A}_n$and $\widehat{dr}^a$ be the normalized difference. For the normalization of the values the following equitation was used:

$$\widehat{dr}_j^a = \frac{dr_j^a - min\left\{dr_j^a | a \in \mathscr{A}_n\right\}}{max\left\{dr_j^a | a \in \mathscr{A}_n\right\} - min\left\{dr_j^a | a \in \mathscr{A}_n\right\}}, j = 1, \cdots, k | k \in \mathbb{Z}^+ \tag{2}$$

We define as the Normalized Root Mean Square Error (NRMSE) metric of arc a under examination, $\|\cdot\|_{NRMSE} : \mathbb{R}^{+n} \rightarrow \mathbb{R}^+$, the sum of the square differences between the performance vector of the arc and the optimal one:

$$NRMSE^a = \|r^a, r^*\|_{NRMSE} = \frac{1}{n}\sqrt{\sum_{j=1}^{n}\left(\widehat{dr}_j^a f_j^a - f_j^*\right)^2}, \forall a \in \mathscr{A}_n \tag{3}$$

The arc with the lowest NRMSE will be added to the arc sequence that composes the optimal route by generating a route that is closer to the ideal one that corresponds to the optimal performance vector r^*. The proof that the pair $\mathscr{M} = (\mathbb{R}^{+n}, \|\cdot\|_{NRMSE})$ is a metric space is considered trivial.

Definition 6 – Fitness function

Let $arc(i, j)$ be the arc under evaluation and e the ending node of the route. The fitness function, $f(\cdot,\cdot,\cdot):\mathbb{R}^{+3}\rightarrow\mathbb{R}^{+}$, is defined as the sum of the score based on the NRMSE evaluation criterion of an arc and a normalized distance function, $d(\cdot,\cdot):\mathbb{R}^{+2}\rightarrow\mathbb{R}^{+}$, from the ending node of the arc to the ending node of the route:

$$f(i,j,e) = NRMSE^{(i,j)} + d(j,e) \tag{4}$$

Let $\mathscr{A} = \{(1,2),\cdots,(2,3),(2,4),(2,5),\cdots\}$ be the set of the permitted arcs in a graph, node 1 be the selected node in the previous step, node 2 the current node from which an arc should be chosen, nodes 3, 4 and 5 be the neighbors nodes that are connected with node 2 and node 12 be the ending node (Fig. 4). For the arcs $(2,3)$, $(2,4)$ and $(2,5)$, we compute the evaluation score based on the evaluation framework. Also, $r^{(2,3)} = (0.13,1,0.45,0.87)$, $r^{(2,4)} = (0.72,2,0.98,0.12)$ and $r^{(2,5)} = (0.86,3,0.27,0.36)$.

Arcs	Risk factor	Arc travel duration	Turn penalty	Loss of Cultural interest
(2,3)	0.13	1	0.45	0.87
(2,4)	0.72	2	0.98	0.12
(2,5)	0.86	3	0.27	0.36

Step 1: Compute the $r^{*} = (0.13,1,0.27,0.12)$.

Step 2: Compute the normalized difference of the values of each arc with r^{*} and the NRMSE.

Arcs	Risk factor	Arc travel duration	Turn penalty	Loss of Cultural interest	NRMSE
(2,3)	0.000	0.000	0.254	1.000	0.258
(2,4)	0.808	0.500	1.000	0.000	0.345
(2,5)	1.000	1.000	0.000	0.320	0.362

Step 3: Compute the normalized distance of node 3, 4, and 5 from the ending point based on the distance function (in our example we used the Euclidean distance to compute the normalized distance).

Arcs	Distance from ending point	Normalized distance from ending point
(2,3)	3.6	1
(2,4)	2.7	0.5
(2,5)	1.8	0

Step 4: The $arc(2,5)$ with the minimum evaluation score will be inserted in the solution.

4.1.2. POI selection ranking system (POI-SRS)

In the TRP problem the tourist, starting from a starting position (starting point), should visit multiple POIs before he reaches the exit (ending point). Therefore, in order to develop a POI visit sequence, we use a POI selection ranking system for choosing the next POI to visit based on tourist's location. Our target is to choose the POI that is close to the starting point and further from the ending point.

Let $\mathscr{P}$ be the set of the POIs that need to be visited by the tourist and $\mathscr{N}$ be the set with the nodes in the graph. Let $p\in\mathscr{P}$ be a POI, $s\in\mathscr{N}$, be the starting point, $e\in\mathscr{N}$ be the ending point and $d(\cdot,\cdot):\mathbb{R}^{+2}\rightarrow\mathbb{R}^{+}$ be the distance function. The POIs are ranked based on minimizing the distance from s ($rank_s(p)$) and maximizing the distance from e ($rank_e(p)$).

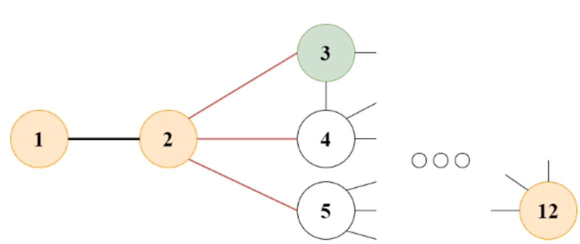

Fig. 4. Example of evaluation framework.

Hence, the POIs with the minimum distance from s or the maximum distance from e gets the lower ranks. The final rank of p, $rank(p)$, is the sum of the $rank_s(p)$ and $rank_e(p)$ of POI. The POI with the minimum rank is selected:

Algorithm 1. POI-SRS pseudoalgorithm.

POI Selection Ranking System	
1	For each p in $\mathscr{P}$
2	Compute $d(p,s)$ the distance of p from s and insert it in the matrixd_s
3	Compute $d(p,e)$the distance of p from e and insert it in the matrixd_e
4	End for each
5	order all p in $d_s$in ascending order and set $rank_s(p)$ based on their index
6	order all p in $d_e$descending order and set $rank_e(p)$ based on their index
7	For each p in $\mathscr{P}$
8	Calculate the final rank,$rank(p) = rank_s(p) + rank_e(p)$
9	End for each
10	Return $p^{*} = \underset{p\in\mathscr{P}}{\mathrm{argmin}}(rank(p))$

bellow, an example is presented for the POI selection ranking system methodology. Let $p_i\in\mathscr{P}, i\in\{1,2,3,4\}$ be the POIs. Fig. 5a shows the computed distances of each POI from the starting and ending points in metric units. From the 1–4 lines, we get that $d(p_1,s) < d(p_3,s) < d(p_2,s) < d(p_4,s)$ so $rank_s(p_1) = 1$, $rank_s(p_3) = 2$, $rank_s(p_2) = 3$ and $rank_s(p_4) = 4$. From the 5–8 lines, we get that $d(p_1,e) > d(p_3,e) > d(p_4,e) > d(p_2,e)$ so $rank_e(p_1) = 1$, $rank_e(p_2) = 2$, $rank_e(p_4) = 3$, and $rank_e(p_3) = 4$. Fig. 5b shows the final ranking derived from the 9–11 lines. Therefore, the POI that will be selected for visiting is the p_1 with the lowest rank.

4.1.3. Greedy graph-based pathfinding algorithm (GPA) design

A^{*} is an informed search or a best-first algorithm, initially designed for finding least-edge cost paths, but now it is also employed for finding optimal paths based on cost algebra conditions. Therefore, A^{*} generates a path that minimizes the fitness function: $f(x) = g(x) + h(x)$, where $x$is the next node examined, $g(x)$ is the cost of the path from the start to x and $h(x)$ is a heuristic function that estimates the cost of the cheapest path from x to the end (Zeng and Church, 2009). One major drawback of A^{*} algorithm in solving practical travel routing problems is its space complexity $O(b^{d})$ as it stores all the generated nodes in memory, where b is the average branching factor and d is the depth of the search tree. Hence, in general, algorithms with graph pre-processing and memory-bounded approaches outperform the A^{*}. However, A* still remains an effective solution in many applications (Lerner et al., 2009).

To overcome these limitations, in this section the greedy approach of the modified A^{*}search algorithm based on the NRMSE evaluation criterion is presented. This modification decreases the route query processing time by searching the solution space in a greedy approach choosing in each step the most optimal arc based on the evaluation criterion in order to minimize the fitness function. Thus, the SIGPA algorithm can effectively improve the computational efficiency and reduce the computational complexity. The proposed approach calculates the fitness function for the next candidate nodes based on the evaluation criterion and the distance from the target node (Definition – 6). The node with the lowest fitness value is selected as the next node. The algorithm does not store the other examined nodes, but it continues the process from the selected node until it reaches the target. The Algorithm 2 shows the pseudoalgorithm that implements the GPA algorithm and the Fig. 6 illustrates its flowchart.

Let $\mathscr{U}_{n}$ be the set of the neighbor nodes that are connected with arcs with node n; $\mathscr{N}$ be the queue list with the neighbor nodes to be checked; the $\mathscr{P}$ queue list with the POI nodes to be visited; the route list $\mathscr{R}$with the visited node sequence; the starting node s and the ending node e. The graph parameters are set.

POI	$d(p_i, s)$	$rank_s(p_i)$	$d(p_i, e)$	$rank_e(p_i)$
p_1	3	1	8.1	1
p_2	5.7	3	6.7	2
p_3	4.5	2	4.5	4
p_4	6.8	4	5.2	3

(a)

POI	$rank_s(p_i)$	$rank_e(p_i)$	$rank(p_i)$
p_1	1	1	2
p_2	3	2	5
p_3	2	4	6
p_4	4	3	7

(b)

Fig. 5. (a) POI distances with ranking; (b) POI selection ranking system.

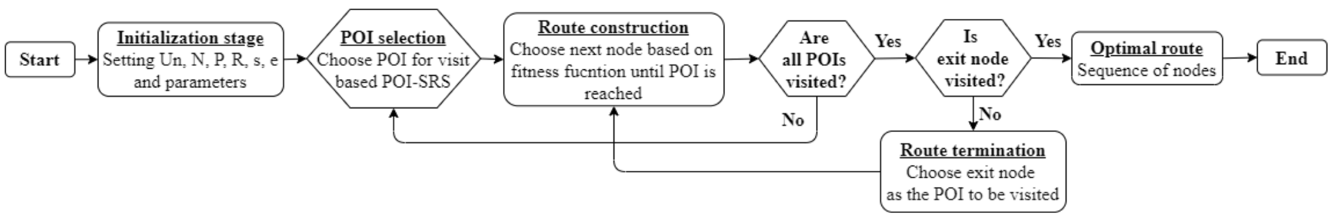

Fig. 6. The flowchart of GPA algorithm.

Algorithm 2. GPA pseudoalgorithm.

GPA algorithm

Initialization Stage
1 Initialize: $\mathscr{U}_n$, $\mathscr{N}$, $\mathscr{P}$, $\mathscr{R}$, s, e
2 Set current node $c = s$
POI Selection Stage
3 Perform POI-SRS ($\mathscr{P}, c, e$) and get $p^* \in \mathscr{P}$
4 Set the target node $t = p^*$ and update $\mathscr{P} = \mathscr{P} \backslash \{p^*\}$
Route construction
5 While $c! = t$
6 Set $\mathscr{N} = \{n : n \in \mathscr{A}_c\}$, where $\mathscr{A}_c \subset \mathscr{A}$ the set of nodes that are neighbors with nodec
7 For each n in$\mathscr{N}$
8 Calculate the evaluation score $e_n = f(c, n, t)$
9 End for
10 Let $n^* \in \mathscr{N} : e_{n^*} \leq e_n, \forall n \in \mathscr{N}$
11 Set $c = n$, update the$\mathscr{R} = \mathscr{R} \bigcup \{n^*\}$
12 End while
Route termination stage
13 Check if $\nexists p \in \mathscr{P} : p \notin \mathscr{R}$, update $\mathscr{P}$ and If $\mathscr{P}! = \varnothing$
14 Repeat POI selection (steps 3–4) and route construction (steps 5–12) stages
15 Else if $c! = e$
16 Set $t = e$, and repeat route construction stage (steps 9–18)
17 Else
18 Return route list $\mathscr{R}$

An example is used to portrait the operation of the GPA algorithm. Let be a graph with 9 nodes with node 1 be the starting node and the node 9 be the ending / target node, which means the node that we want to visit. Starting from node 1 we calculate the fitness function f for each node that is connected with an arc with node 1: $f(1,2,9), f(1,3,9), f(1,4,9)$ and $f(1,5,9)$. The arc with the lowest score will be chosen to be inserted in the route, without loss of generality let $arc(1,4)$ be the one with the lower score. Then, the $arc(1,4)$ is inserted in the route sequence and the process is repeated from node 4 until we reach the node 9 (Fig. 7).

4.2. SIGPA metaheuristic algorithm

4.2.1. Solution selection criterion

To avoid limiting the search of the solution space towards a specific direction or the swarm to get trapped on local optimal solutions, a probabilistic approach is employed in the evaluation process. In SIGPA, a swarm searches the solution space to find alternative paths. Specifically, SIGPA uses a generated path to find alternatives routes in order to search for the optimal path. The best path found from the swarm is then compared to the current optimal path in order to choose the path that will be used for the next swarm generation. To this end, in case where the best swarm solution is not the optimal one, we adopt a probabilistic decision making in the evaluation stage of SIGPA in order to accept or reject the best swarm solution instead of the current optimal solution, since a better path can also be derived from a non-optimal path. This also enables the search of the solution space from various directions.

Let $\mathscr{s}^*$ be the best solution found with evaluation score e^*, $\mathscr{s}_c$ the current solution and $\mathscr{s}$ be the new solution with evaluation score e derived from a particle over $\mathscr{s}_c$. We set the following evaluation rules for accepting a new solution as the current or optimal solution:

Rule 1. If $e < e^*$, then $\mathscr{s}^* = \mathscr{s}_c = s$ and $e^* = e$ and we continue the process with the new current solution.

Which means that if the new solution has a lower evaluation score than the current best one, we set the new solution as the best and we continue the process with the new solution as the current one.

If $e \geq e^*$, the new solution is not accepted as the best one but there are two events that can occur if we accept it as the current solution: (i) to lead to a better solution than the current best one; or (ii) to lead to a worse solution. Therefore, we can state that it follows the Bernoulli distribution, and for large numbers of iterations this can lead to the Gaussian or Standard Normal Distribution. So, let $\mathscr{p}$ be the minimum probability that we set in order to accept the new solution as current and let the probability to lead to an optimal solution be $P(Z < e^*) = \mathscr{r}$ based on the Gaussian distribution, then:

Rule 2. If $\mathscr{r} < \mathscr{p}$, the new solution is withdrawn, and we continue the process with the same current solution.

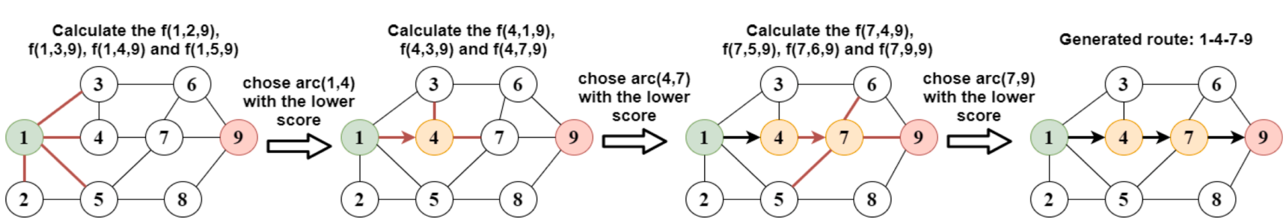

Fig. 7. Example of GPA algorithm execution in a simple graph.

Rule 3. If $r \geq p$, then $s_c = s$ and we continue the process with the new current solution.

4.2.2. SIGPA design

Almost all conventual optimization techniques search from a single point (centralized). In the proposed SIGPA algorithm, population approach is employed for searching the solution space by multiple points (distributed). Hence, in this way the chance of reaching a global optimum is improved and also local stationary points are avoided. For each particle the GPA algorithm (Section 4.1) is employed for generating new solutions. In Algorithm 3 the pseudoalgortihm of the SIGPA algorithm is presented, while in Fig. 8 the flowchart is illustrated.

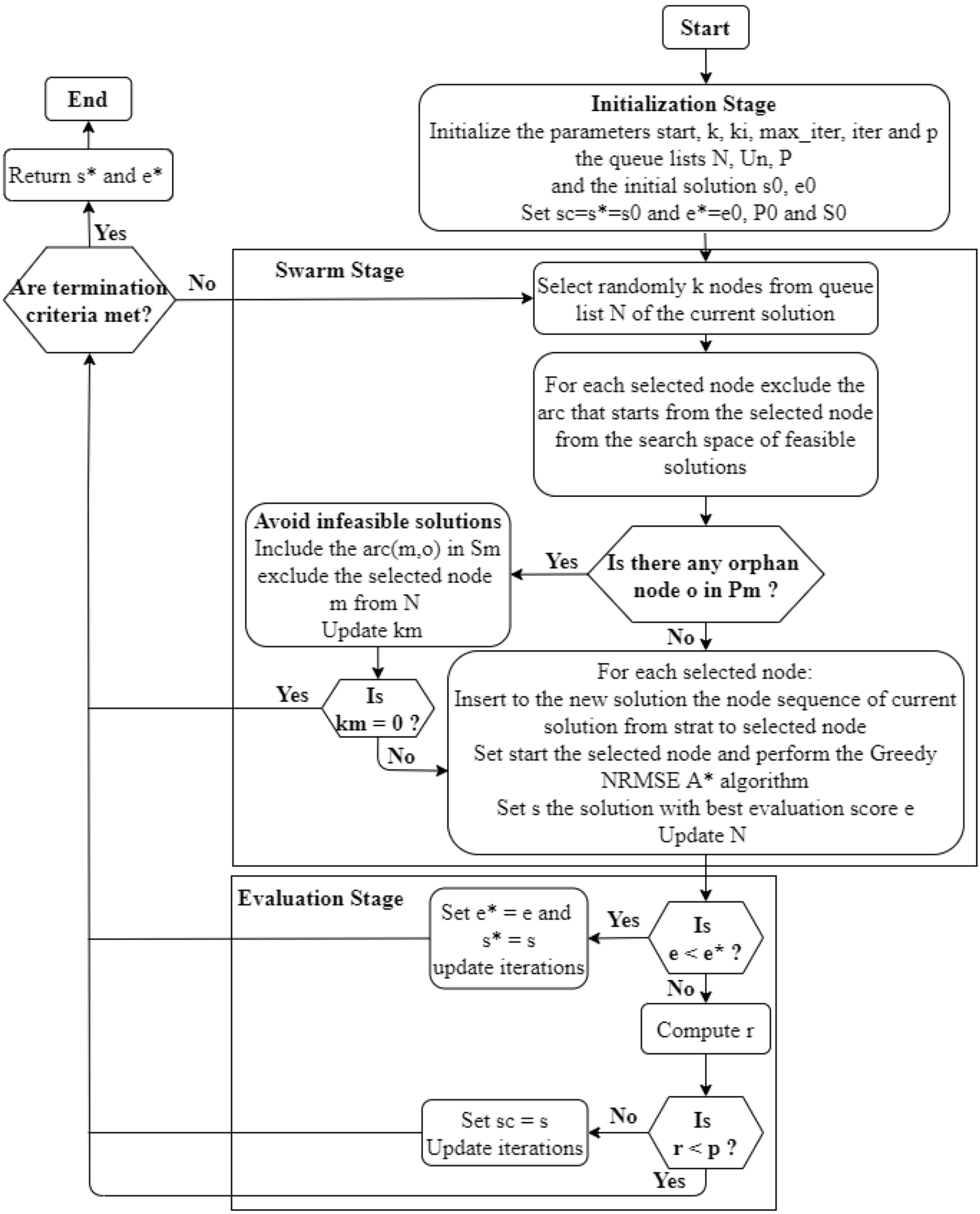

Fig. 8. SIGPA algorithm flow chart.

Let $\mathscr{N}$ be the route nodes that are not yet selected from the current solution $\mathscr{s}_c$, $\mathscr{U}_{\mathscr{n}}$ be the neighbor nodes that are connected with arcs with node $\mathscr{n}$, $\mathscr{P}$ be the selected POIs, $\mathscr{P}_{\mathscr{m}}$ be the selected POIs that should be visited by each particle-node $\mathscr{m}$ and $\mathscr{S}_{\mathscr{m}}$ be the search space of feasible solutions of $\mathscr{m}$. Let *start* and *end* be starting and ending nodes, $\mathscr{k}$ be the number of population – nodes to be randomly selected from the current route solution, $\mathfrak{W}$ be the maximum number of iterations, $\mathscr{p}$ be the acceptance probability of a solution, $\mathscr{k}_i$ be the number of population after excluding infeasible solutions in i^{th} iteration, *iter* be the current iteration and *tol* be the acceptable error for convergence. Each path is referred as a particle.

Algorithm 3. SIGPA algorithm.

Initialization Stage
1 Initialize: $\mathscr{N}$, $\mathscr{U}_{\mathscr{n}}$, $\mathscr{P}$, $\mathscr{P}_{\mathscr{m}}$, $\mathscr{S}_{\mathscr{m}}$, *start*, $\mathscr{k}$, $\mathfrak{W}$, $\mathscr{p}$, *tol*
2 Set $\mathscr{k}_i = \mathscr{k}$, $iter = 0$
3 Set $\mathscr{P}_0 = \mathscr{P}$, perform GPA for finding an initial route solution $\mathscr{s}_0$ from *start* to *end* and compute the evaluation score e_0
4 Set the current solution $\mathscr{s}_c = \mathscr{s}_0$ and the best solution $\mathscr{s}^* = \mathscr{s}_0$ with the best evaluation score $e^* = e_0$
5 While the termination criteria are not met ($|\mathscr{N}| < \mathscr{k}$ or $iter > \mathfrak{W}$ or $\Delta e < tol$)
Population Stage
6 Select randomly $\mathscr{k}$ nodes from $\mathscr{N}$ to form the subset $\mathscr{N}^{\mathscr{k}} \subseteq \mathscr{N}$
7 For each node $\mathscr{m}$ in $\mathscr{N}^{\mathscr{k}}$
8 if the node $\mathscr{m}$ is preceding of only one node $\mathscr{n}$
9 Exclude the $arc(\mathscr{m}, \mathscr{n})$ from $\mathscr{S}_{\mathscr{m}}$
10 Else
11 Calculate the arc evaluation criterion $NRMSE^{(\mathscr{m}, \mathscr{n}_{\mathscr{j}})}$, where $\mathscr{n}_{\mathscr{j}}, \mathscr{j} \in \aleph \leq |\mathscr{U}_{\mathscr{n}}| \subseteq \mathbb{N}$ and choose the arc with the maximum score to be excluded from $\mathscr{S}_{\mathscr{m}}$
12 End if
// Avoid infeasible solution stage
13 while there is an orphan node (a node that is not connected with any other node in the graph) and this node belongs to $\mathscr{P}_{\mathscr{m}}$
14 Include the $arc(\mathscr{m}, \mathscr{n})$ in $\mathscr{S}_{\mathscr{m}}$, exclude $\mathscr{m}$ from $\mathscr{N}$ and from the population and update $\mathscr{k}_i$
15 End while
16 If $\mathscr{k}_i = 0$
17 Update *iter* and go to step 5
18 else
19 Insert to the solution $\mathscr{s}_m$ of particle $\mathscr{m}$ the node sequence of $\mathscr{s}_c$ from *start* to node $\mathscr{m}$ and update $\mathscr{P}_{\mathscr{m}}$
20 Set $start = \mathscr{m}$ and perform GPA algorithm to calculate the $\mathscr{s}_m$ and e_m
21 Update $\mathscr{N}$ by excluding $\mathscr{m}$
22 End if
23 End for each
Evaluation Stage
24 Set the candidate solution $\mathscr{s} = \mathscr{s}_{\mathscr{j}} : e_{\mathscr{j}} = min\{e_{\mathscr{j}}, \mathscr{j} \in \{1, \cdots, \mathscr{k}_i\}\}$ and $e = min\{e_{\mathscr{j}}, \mathscr{j} \in \{1, \cdots, \mathscr{k}_i\}\}$
25 If $e < e^*$
26 set $e^* = e$ and $\mathscr{s}^* = \mathscr{s}$
27 update *iter*
28 else
29 Compute the probability of Gaussian / standard normal distribution $\mathscr{r} = P(Z < e^*)$
30 If $\mathscr{r} < \mathscr{p}$
31 update *iter*
32 Else
33 Set $\mathscr{s}_c = \mathscr{s}$, update *iter* and
34 End if
35 End while
36 Return $\mathscr{s}^*$ and e^*

For a deeper and faster comprehension of the operation of the SIGPA algorithm an example is presented below (Fig. 9). In Fig. 9, the node 1 is the entrance and the node 17 the exit denoted with green and red, respectively. The orange nodes 5, 10 and 11 are the POIs that the user wants to visit. The GPA algorithm is used for finding an initial solution $\mathscr{s}_0$ (denoted with blue line). So, the node sequence that corresponds to the initial solution $\mathscr{s}_0$ is $1-3-6-5-9-10-14-13-12-11-17$. Let the evaluation score of the route $\mathscr{s}_0$ be $e_0 \in \mathbb{R}^+$. The current solution is also the best $\mathscr{s}^* = \mathscr{s}_0$ and $e^* = e_0$.

Let k be the number of nodes from the initial solution $\mathscr{s}_0$ that will be randomly selected, $n \in \mathbb{N}^*$ be the number of iterations that will be performed, $p \in [0, 1]$ be the probability of accepting a new route. For the given example, $k = 3$, hence nodes are randomly selected from the initial route. Without loss of generality let these nodes be the nodes 3, 9 and 13.

For each node, *e.g.* for node 9, we perform the same process:

1. The $path(9, 10)$ is excluded from the feasible solution space.
2. The node sequence until node 9 from the current solution $(1-3-6-5)$ is inserted in the new solution $\mathscr{s}''$.
3. Node 5 is excluded from the POI queue list since it already belongs in the node sequence of the solution.
4. The node 9 is set to be the starting point.
5. Perform GPA algorithm for constructing the path from node 9 to node 17.
6. We get the path $9-6-7-10-14-13-12-11-17$
7. We merge the paths from steps 2 and 5, and we evaluate the score of the new solution:

$$\mathscr{s}'' = 1-3-6-5-9-6-7-10-14-13-12-11-17.$$

Let $e'' \in \mathbb{R}^+$ be the score of the $\mathscr{s}''$, e' and e''' for the $\mathscr{s}'$ and $\mathscr{s}'''$ solutions derived from the same process for node 3 and node 13, respectively. Without loss of generality let assume that e''' is the best score that minimizes the evaluation function. So, $e = min\{e', e'', e'''\} = e'''$.

8. Compare the best-found solution, $\mathscr{s}'''$, with the current best solution $\mathscr{s}^*$:
   a. If $e \leq e^*$, then $e^* = e$ and $\mathscr{s}^* = \mathscr{s}'''$.
   b. If $e > e^*$, compute the probability of the standard normal distribution $r = P(e^* < Z < e)$:
      i. If $r < p$, and there are at least n unexplored nodes form the current route solution we continue the process by randomly selecting n from them, otherwise the process is terminated, and the best solution is the $\mathscr{s}^*$.
      ii. If $r \geq p$, we continue the process by setting current route the $\mathscr{s}'''$.
9. Increase the number of iterations and check the evaluation criteria.
10. If the termination criteria are met, return $\mathscr{s}^*$, e^*, otherwise the process is repeated.

4.3. Convergence analysis and fitness landscape analysis (FLA)

SIGPA is a population-based algorithm where the collective behaviors of the individuals interact locally with the environment in a greedy search approach leading to the emergence of functional global patterns. Due to the difficulty of evaluating the performance and convergence of population-based algorithms, few studies, compared to the large number of proposed heuristic and metaheuristic algorithms, have been conducted for proving the effectiveness and the efficiency of the algorithms. In this study, a convergence velocity analysis will be performed to test the locally improvement of the SIGPA algorithm from one iteration to the next one. For this reason, we adopted the expected quality gain (EQG), φ, and the expected change (EC) in the distance to the global optimum, d, measures (Sirirak and Pitakaso, 2018):

$$\varphi = \mathrm{E}[f(\mathscr{s}_{t+1}) - f(\mathscr{s}_t)] \tag{5}$$

$$d = \mathrm{E}[\|\mathscr{s}^* - \mathscr{s}_t\| - \|\mathscr{s}^* - \mathscr{s}_{t+1}\|] \tag{6}$$

where $\mathscr{s}^*$ is the global optimal solution and $\mathscr{s}_t$ denotes the best solution found from the population at t iteration.

FLA is a common approach for studying the difficulty of metaheuristic algorithms to solve optimization problems. Hence, our analysis will be based on the dynamic fitness landscape analysis oriented for population based metaheuristic algorithms (Wang et al., 2018) to test

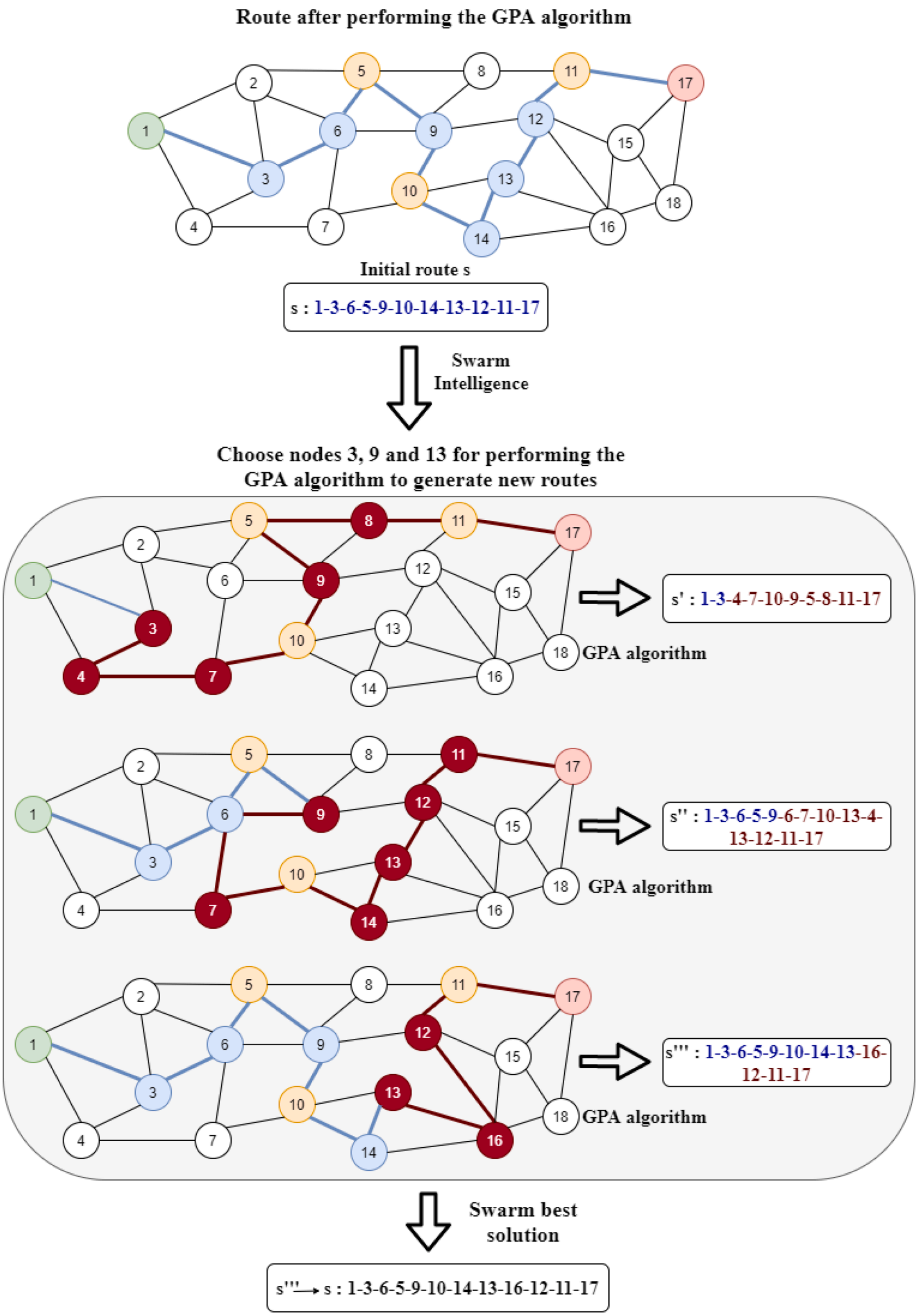

Fig. 9. Example of SIGPA metaheuristic algorithm for one iteration.

the effectiveness of the propose metaheuristic algorithm to solve route planning problems. In this context, the evolutionary probability, evolutionary ability and the evolvability measures of the population are defined. Based on Wang et al. (Wang et al., 2018) and evolutionary biology approaches that determine the evolution probability and evolvability of population, we present the definitions of the above measures in our case:

Definition 7 – Evolutionary Probability of a Population (EPP)

The EPP can be interpreted as the collective behavior of the entire population. Given an initial individual i and the generated population $\mathscr{P}_i$, define the evolutionary probability of the population to be:

$$EPP(\mathscr{P}_i) = \frac{|\mathscr{E}_i|}{|\mathscr{P}_i|} \tag{7}$$

where $\mathscr{E}_i = \{p|p \in \mathscr{P}_i : f(p) < f(i)\}$ is the set of evolved individuals in the population $\mathscr{P}_i$ for a minimization problem.

Definition 8 – Evolutionary Ability of a Population (EAP):

The EAP can be interpreted as the average evolutionary ability of an initial individual over its evolved population. Using the same notions and symbols as in Definition 7, the evolutionary ability of a population is defined as:

$$EAP(\mathscr{P}_i) = \begin{cases} \dfrac{\dfrac{\sum_{p \in \mathscr{E}_i} |f(i) - f(p)|}{\sigma(f(\mathscr{P}_i)) \cdot |\mathscr{P}_i|}}{|\mathscr{E}_i|}, |\mathscr{E}_i| \geq 1 \\ 0, |\mathscr{E}_i| = 0 \end{cases} \tag{8}$$

where $\sigma(f(\mathscr{P}_i))$ is the standard deviation of the fitness values of the population $\mathscr{P}_i$.

Definition 9 – Evolvability of a Population (EVP)

The EVP can be interpreted as the average evolutionary ability over the entire set of generated population. Given the EEP and the EAP, define the evolvability to be:

$$EVP(\mathscr{P}_i) = EEP(\mathscr{P}_i) \cdot EAP(\mathscr{P}_i) \tag{9}$$

5. Evaluation methodology and experimental results

5.1. Evaluation methodology

The experimental evaluation is divided in two stages. In the first evaluation process, the proposed mathematical model presented in Section 2 will be compared with mathematical models that target to minimize the traveled distance or the traveling time or maximizing the cultural experience, as they are commonly used in the literature for addressing the TRP problem. In the second stage, the SIGPA metaheuristic algorithm will be compared with commercial solver CPLEX for solving the TRP problem formulated with the proposed MIQP model.

The performance evaluation of the mathematical models under examination will be done based on the NRMSE as it is similarly defined in Section 3, Definition 5. Bellow, we give the definition of the evaluation criterion regarding the mathematical models. The model with the lowest NRMSE will be the one that performed efficiently in terms of generating a route that is closest to the ideal one that corresponds to the optimal performance vector r^*.

For the evaluation of the proposed MIQP model for TRP and the SIGPA metaheuristic algorithm, 120 validation scenarios were performed under various predefined graph sizes $G = (\mathscr{P}, \mathscr{N})$, where $\mathscr{P}$ denotes the number of POIs and $\mathscr{N}$ the number of nodes in the graph, while the number of edges, the values of the model parameters and user preferences are randomly assigned for each scenario. In total, 20 scenarios for graph size $\mathscr{G} = (2, 10)$, 20 scenarios for graph size $\mathscr{G} = (4, 20)$, 20 scenarios for graph size $\mathscr{G} = (6, 30)$, 20 scenarios for graph size $\mathscr{G} = (8, 40)$, 20 scenarios for graph size $\mathscr{G} = (10, 50)$ and 20 scenarios for graph size $\mathscr{G} = (16, 80)$. Larger graphs have not been used due to the limitations of CPLEX solver to solve the problem in the desired time limit (2 min) that thought acceptable for a near real-time navigation. Response-time requirements for typical real-time application such as control systems processes and automation is set to $100\mu s - 100ms$ (Buttazzo, 2011). The model is compared with the most frequent approaches used in the literature to solve TRP problems (Table 5), such as minimizing distance or duration; or maximizing cultural experience or safety in case of individuals with disabilities. These approaches were formulated based on the constraints and relevant objective terms of the presented formulation (Section 3). For SIGPA, apart from the maximum number of iterations, we also set as termination criterion the maximum number of consecutive iterations without solution improvement. These were set to 8000 and 80, respectively. The acceptance probability was set to 0.3.

For the evaluation of the SIGPA algorithm, a convergence analysis and a dynamic fitness landscape analysis was conducted to test the convergence velocity and efficiency of solving route planning problems based on the criteria (4) – (8). To this end, the SIGPA algorithm is compared with CPLEX solver for solving the MIQP TRP problem under the generated scenarios for proving the overall efficiency of the SIGPA algorithm for solving RP problems. This type of analysis helps to gain insights into the impact of complexity and dimensionality of the RT problem and of the parameter settings of the SIGPA algorithm on convergence to optimal solution. We refer as dimensionality of RP problem the graph size and as complexity the number of selected POIs to be visited.

To prove the effectiveness of the proposed algorithm in real case scenarios, an area (Fig. 10) based on the topology of Ancient Market in the Historical Triangle of Athens in Greece, was selected. SIGPA algorithm is compared to popular A* in computation efficiency and solution optimality. In total 150 nodes and 20 POIs have been used and various scenarios have been run for 10 (Scenario 1), 15 (Scenario 2) and 20 (Scenario 3) POIs selected by the user. Each scenario was run 20 times, from random starting point (node).

5.2. Experimental results

For the experimental validation of the proposed mathematical model and metaheuristic algorithm 120 various scenarios were randomly generated under various graph sizes. A computer with 64-bit Windows 10 operating system, AMD Ryzen 7 3800X 8-Core Processor and 32 GB RAM was used for the experiments. The models and the algorithm were coded in Java and CPLEX solver was used for solving the scenarios. The evaluation process was divided in three stages:

- Comparing the quality of the generated routes of the mathematical models (Table 5), proving that our mathematical model generates balanced Pareto optimal solutions in terms of the optimization criteria. The NRMSE evaluation criterion was used for the comparisons.
- Comparing the efficiency of the proposed SIGPA algorithm in terms of computing time and solution quality with CPLEX solver in solving

Table 5
Mathematical programming models under evaluation.

Model	Target
Model 1	Find the most culturally interesting route by minimizing the multiple crossovers from same paths
Model 2	Find the route that minimizes the number of route deviations
Model 3	Find the safest route by minimizing the collision risk among the generated route
Model 4	Find the shortest route by minimizing the distance
Model 5	Find the fastest route by minimizing the travel duration
Our Model	Find a route with the best balance of safety, shortest distance, duration, cultural experience and smooth path changes.

Fig. 10. The area used for the experimental evaluation taken by Google Earth. The yellow road corresponds to the path that are used in the designed scenarios. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

our formulation, to prove the capability of SIGPA algorithm to support real-time or online navigation applications.

- Performing convergence analysis and dynamic fitness landscape analysis to prove the stability and evolvability of SIGPA meta-heuristic algorithm under various dimensionalities of RT problem.
- Comparing SIGPA with popular A* in real application.

Table 6 summarizes the mean evaluation scores for each graph with the standard deviation. The models that achieve low evaluation score (close to zero) generate balanced routes that converge to the ideal optimal one (Section 5.1). The results showed that in average the proposed mathematical modeling, comparing to the other five approaches (Models 1 – 5) adopted in the literature for formulating TRP problems, generates more balanced routes based on the optimization criteria such as safety, distance, duration, cultural experience, route smoothness for all graph sizes and scenarios (Table 6). This is also proved by the Fig. 11, where the SIGPA algorithm has proven to be more efficient in terms of developing balanced routes as dimensionality of graph increases. Specifically, Fig. 11-a illustrates the mean collision risk value of the routes that are generated from each model for the 20 scenarios of each graph size. Fig. 11-b illustrates the mean traveled distance (in kilometers) of the routes that are generated from each model for the 20 scenarios of each graph size. Fig. 11-c illustrates the mean number of turns that the tourist need to perform during the visit from each model for the 20 scenarios of each graph size. We assume a turn if the turn degree is higher than 30°. Fig. 11-d illustrates the mean number of multiple crossovers from POIs during the visit from each model for the 20 scenarios of each graph size. As the dimensionality of the problem increases, our model proves its efficiency to generate balanced routes compared to the SOA models. However, due to global optimization solver CPLEX, the increase in the dimensionality of the TRP problems decreases significantly the accuracy of the solution achieving a relative tolerance MIP gap higher than 10%. The relative tolerance MIP gap calculates the difference between the best integer objective and the objective of the best node remaining and it is given by the expression: $\frac{|bestbound - bestinteger|}{1e{-}10 + |bestinteger|}$ (Cplex, 2010).

To compare the SIGPA algorithm with CPLEX solver we chose our mathematical model since it is a multi-objective decision-making model but also the one with the best evaluation score among the models under examination. The same scenarios were used for testing the computing time (in milliseconds) and the solution quality. The scenarios from 1 to 20 are implemented with the graph $\mathscr{G} = (2,10)$, from 21 to 40 with the graph $\mathscr{G} = (4,20)$, from 41 to 60 with the graph $\mathscr{G} = (6,30)$, from 61 to 80 with the graph $\mathscr{G} = (8,40)$, from 61 to 80 with the graph $\mathscr{G} = (8,40)$, from 81 to 100 with the graph $\mathscr{G} = (10,50)$ and from 101 to 120 with the graph $\mathscr{G} = (16,80)$. The Table 8 shows the fitness score and the computing time for CPLEX and SIGPA, respectively. We can observe that the SIGPA succeeded in finding the optimal solution within an acceptable time frame for real-time computing processing (Table 8). In Table 7 the mean results of the convergence and dynamic fitness landscape analysis for the SIGPA are presented. From the dynamic fitness landscape analysis (Table 7), the mean EEP was evaluated 0.568, the mean EAP 1090.855and the mean EVP 613.256. Thus, we can state that the dimensionality or complexity of the RTP problems do not affect the evolution probability which it is calculated round to 56%. So, in any case the half of the initial generated population is evolved and contributes to the solution improvement.

Regarding the convergence analysis and in consideration of the fact that the global optimum solution is usually unknown in real – world applications, both convergence measures EQG and EC (Table 7) were adopted and the solution with the best fitness value achieved by the best-performing model (mostly by our model) was considered as an approximation of the global optimum. It is obvious that the SIGPA succeeded in converging to the optimal solution with average convergence velocity EQG= 1.384and EC= 1.396. We can conclude that the higher EVP impacts on the acceleration of the convergence. Hence, since the SIGPA presents a stability in the population evolution at the 0.568, the convergence velocity of the algorithm is remaining in acceptable rates reaching the optimal solution in low computing time as we have already proven in all scenarios of various dimensionalities and complexities of RP problems. In general, the velocity convergence can be increased by adjusting the parameters of the SIGPA in order to reassure

Table 6
Mean evaluation score with standard deviation of each model under all sizes of graphs.

Scenarios	Model 1	Model 2	Model 3	Model 4	Model 5	Our Model	SIGPA
$G = (2,10)$	0.119 ±0.126	0.101 ±0.217	0.115 ±0.195	0.085 ±0.139	0.032 ±0.029	**0.022 ±0.031**	**0.013 ±0.014**
$G = (4,20)$	0.474 ±0.388	0.240 ±0.218	0.339 ±0.357	0.236 ±0.245	0.136 ±0.129	**0.116 ±0.107**	**0.050 ±0.072**
$G = (6,30)$	0.834 ±0.159	0.426 ±0.238	0.717 ±0.343	0.422 ±0.273	0.332 ±0.276	**0.140 ±0.162**	**0.131 ±0.095**
$G = (8,40)$	0.670 ±0.321	0.348 ±0.245	0.437 ±0.365	0.303 ±0.225	0.290 ±0.247	**0.134 ±0.179**	**0.102 ±0.146**
$G = (10,50)$	0.610 ±0.323	0.518 ±0.357	0.698 ±0.301	0.434 ±0.329	0.253 ±0.226	**0.210 ±0.219**	**0.155 ±0.106**
$G = (16,80)$	0.302 ±0.369	0.613 ±0.344	0.432 ±0.345	0.120 ±0.143	0.166 ±0.197	**0.112 ±0.149**	**0.064 ±0.079**
Mean Evaluation Score	0.501 ±0.383	0.374 ±0.325	0.457 ±0.385	0.267 ±0.272	0.202 ±0.227	**0.122 ±0.166**	**0.086 ±0.107**

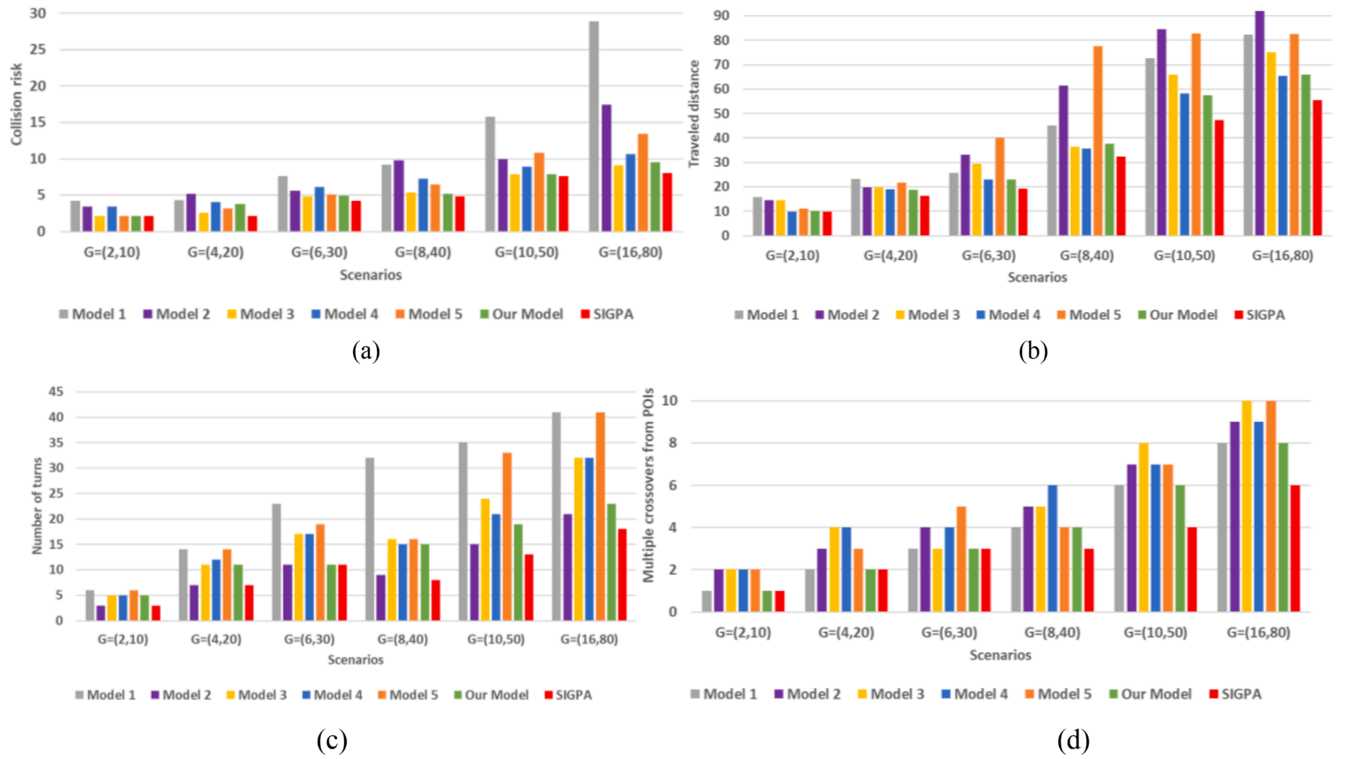

Fig. 11. Mean results of each model for the 20 scenarios of each graph size for : (a) the number of collision risk; (b) the traveled distance ; (c) the number of turns; and (d) the multiple crossovers from POIs.

Table 7
Mean values of convergence and dynamic fitness analysis.

	G(2,10)	G(4,20)	G(6,30)	G(8,40)	G(5,50)	G(16,80)	Average	Std
EQG	1.448 ±1.065	1.441 ±1.186	1.389 ±1.046	1.366 ±1.144	1.338 ±0.883	1.324 ±0.636	1.384	0.990
EC	1.448 ±1.110	1.424 ±1.084	1.405 ±1.064	1.408 ±1.117	1.382 ±0.907	1.311 ±0.588	1.396	0.976
EEP	0.531 ±0.191	0.551 ±0.211	0.579 ±0.255	0.629 ±0.247	0.553 ±0.192	0.564 ±0.233	0.568	0.220
EAP	814.563 ±616.562	835.586 ±523.703	1292.881 ±984.503	1058.882 ±622.971	1050.419 ±683.135	1492.796 ±690.919	1090.855	728.130
EVP	395.997 ±330.175	444.556 ±348.796	778.747 ±869.307	710.486 ±667.395	547.243 ±419.170	802.508 ±532.260	613.256	572.320

high EVP. The size of the initial population and the acceptance probability are two main parameters involved in the evolution and evolvability mechanism of the population in SIGPA. We should notice that as the number of the POIs in the graph is increased, the variations in the fitness function, convergence and computing time are also more obvious due to the random selection of the number of POIs to be visited. For example, in G(16, 80), scenarios from 1 to 16 selected POIs have been generated leading to larger variations in the results compared to scenarios of G(2, 10) or G(4, 20).

Regarding the real scenarios, SIGPA proved to be more computationally efficient (Table 9) compared to A* especially as the number of the POIs increase and consecutively the need for exploring a larger subarea of the graph increases, as well. Table 10 shows the comparative evaluation of SIGPA and A* on the optimality criteria for the 20 runs.

Table 9
Mean computing time in milliseconds (ms) of SIGPA and A* algorithms.

Algorithm	Scenario 1	Scenario 2	Scenario 3	Average
SIGPA	0.059±0.007	0.075±0.010	0.0101±0.030	**0.078±0.016**
A*	0.089±0.050	0.136±0.052	0.587±00.126	0.270±00.076

Table 8
Mean values of SIGPA and CPLEX algorithms.

Algorithm	Measures	G(2,10)	G(4,20)	G(6,30)	G(8,40)	G(5,50)	G(16,80)	Average
SIGPA	Computing time (ms)	0.026±0.004	0.032±0.007	0.057±0.007	0.096±0.029	0.096±0.038	0.103±0.027	**0.068±0.039**
	Fitness score	22.200±2.605	33.419±4.394	63.074±9.614	78.884±11.054	93.475±24.833	144.649±41.796	**72.617±45.479**
CPLEX	Computing time (ms)	0.317±0.252	0.793±0.496	3.979±2.810	106.235±71.120	130.479±120.987	195.644±103.406	72.908±103.378
	Fitness score	22.811±3.205	35.823±6.017	63.801±8.024	80.103±8.920	95.672±27.010	151.380±65.607	74.932±51.260

Table 10
Evaluation of SIGPA and A* on the optimality criteria.

Scenarios	Criteria	SIGPA	A*
Scenario 1	Number of obstacles	**34± 5**	57± 7
	Distance (km)	4.1± 1.1	**3.9± 2.4**
	Number of multiple crossovers	**6± 3**	8± 4
	Number of turns	**3±3**	7± 5
Scenario 2	Number of obstacles	65±**12**	73± 18
	Distance (km)	7.2± 2.3	**7.1± 4.8**
	Number of multiple crossovers	**12± 7**	36±12
	Number of turns	7± **6**	18± 9
Scenario 3	Number of obstacles	**88± 6**	124±32
	Distance (km)	9.0± 3.5	**8.5± 5.9**
	Number of multiple crossovers	**14± 5**	48± 19
	Number of turns	12± **5**	22± 8

SIGPA has proven effective on finding the route with the least number of obstacles, thus, potential collisions, number of multiple crossovers from the same POIs and number of brut turns, outperforming the A*. A* has managed to find the route with the shortest length, since it is a path search algorithm designed to find the shortest path. However, SIGPA presented a comparable performance, by generating routes with almost same distance with A* but with better trade-off among the other optimality criteria.

6. Conclusions and future work

In this study MIQP mathematical model was proposed to formulate the TRP problem in order to satisfy user preferences for modern means of tourist transport, such as electric vehicles, but also user needs in case of individuals with special needs or elderly. A metaheuristic population – based greedy path search algorithm was developed to solve the aforementioned formulation in real time applications and a new evaluation process for multi-objective optimization problems was presented for evaluating the feasible solutions in terms of the optimization criteria.

Hence, to overcome the limitations of Pareto dominance processing techniques when it comes to multi-objective optimization problems with weak distinct dominance relations in the solution space or with isolated optimal solution, a new evaluation framework was proposed:

- A multiple-criteria decision analysis based on Normalized Root Mean Square Error to rank the available potential solutions in the solution space and choose the one to be included in the solution sequence.
- An optimization balance estimator among the weights of the optimization criteria so that the chosen solution will provide the optimal tradeoff between the desired optimization terms.

The mathematical model and the metaheuristic algorithm were tested in a variety of 120 TRP scenarios and compared with the most commonly used SOA approaches for addressing this type of RP problems solved with CPLEX solver. The evaluation criterion for quality evaluation of the generated routes proved that our mathematical formulation generates more balanced Pareto optimal solutions:

- The mathematical formulation takes into account five optimization criteria for generating a route such as (i) the safety of the travelled path; (ii) the total travelled distance for visiting the selected POIs; (iii) the travelled duration; (iv) the cultural value of the route; and (v) the deviations in the orientation of the user while travelling in the route.
- The model takes into account user preferences, user special needs and modern means of transports for tourist travelling.
- Applicable to robotic and other similar routing problems.

A convergence analysis and dynamic fitness landscape analysis were, also, performed for the SIGPA to evaluate the convergence stability and the population evolvability of the proposed algorithm. These measures will show the ability and the effectiveness of the algorithm to solve RP problems. Comparing with traditional optimization techniques, almost all conventual optimization techniques search from a single point (centralized). In the proposed SIGPA algorithm:

- Population approach is employed for searching the solution space by multiple points (distributed) allowing parallelization. Hence, in this way the chance of reaching a global optimum is improved and local stationary points are avoided.
- Random selection of points to be included in the population.
- A fitness function (the NRMSE evaluation criterion in combination with distance function) is used. The lack of derivates constitutes the SIGPA algorithm suitable for any kind of discrete or continuous optimization routing problem.
- Probabilistic selection rules are applied for transition operations
- Generates a balanced route among the optimization criteria.

In general, the development of a suitable routing algorithm requires a trade-off between preprocessing time, and query processing time depending on the graph size. Literature routing algorithms range from no or limited preprocessing time, such as Dijkstra or similar algorithms, to exhaustive pre-calculation of the optimal route between all possible vertex pairs in the graph, a technique that reduces route query processing time by substituting the route queries to a simple table search. For this reason, the proposed algorithm aiming to decrease the preprocessing time adopted the A^* search algorithm and to decrease the route query processing time, it searches the solution space in a greedy approach. Also, for parallel processing the population approach is adopted trying to cover the solution space during search and escaping from stationary points.

The proposed mathematical formulation and the SIGPA algorithm were evaluated under real – world conditions in the context of the implementation of ENORASI project that is dedicated to the navigation and support of individuals with visual impairment in open outdoor cultural spaces. Tests involved the place were ENORASI project will take place in the Ancient Agora of Athens, Greece. The evaluation results and the computing time of the proposed algorithm are promising that real time processing could be achieved for navigation tasks, since the computing time satisfies the response-time requirements $(100\mu s - 100ms)$ for real-time application of control systems processes and automation (Buttazzo, 2011). Future work includes the application of the proposed formulation and metaheuristic algorithm to robotic path planning and other relevant applications with mandatory visits in time windows where the need for fast and quality solutions is of high importance.

CRediT authorship contribution statement

Charis Ntakolia: Validation, Software, Investigation, Visualization, Writing - Original draft preparation, Data curation, Software, Methodology, Conceptualization. **Dimitris K. Iakovidis:** Supervision, Writing - Reviewing and Editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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