

Math 270
Midterm 1
On 1.1 - 1.9

Name: _____

Score: _____/ 105= _____%

Rules: You may use a calculator but no notes. Do not share calculators, or look in the general direction of other's tests. Smartphones and smartwatches must be out of sight at all times. I can answer questions about the meaning of English words, and you should ask me if you think a question has a typo. I will *not* give you hints, or comment on your answers, or explain math terminology.

Instructions: Box your answers. Show any work that there is to be shown.

1. [15 pts] State whether each set of vectors is linearly dependent or independent. Justify your answers.

(a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

(b) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$, a set of 4 vectors in \mathbb{R}^3 .

- (c) The set of column vectors from the 3×3 identity

matrix, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

2. [18 pts] Is each statement True or False? No need to justify answers.

(a) _____ Given two vectors \mathbf{v} and \mathbf{p} , the set $\{\mathbf{p} + t\mathbf{v} : t \in \mathbb{R}\}$ describes a line through \mathbf{v} parallel to \mathbf{p} .

(b) _____ If S is a linearly dependent set, then *each* vector is a linear combination of the other vectors in S .

(c) _____ If $A = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, then for every \mathbf{b} in \mathbb{R}^3 , the equation $A\mathbf{x} = \mathbf{b}$ has a solution.

(d) _____ Every homogeneous system of equations has at least one solution.

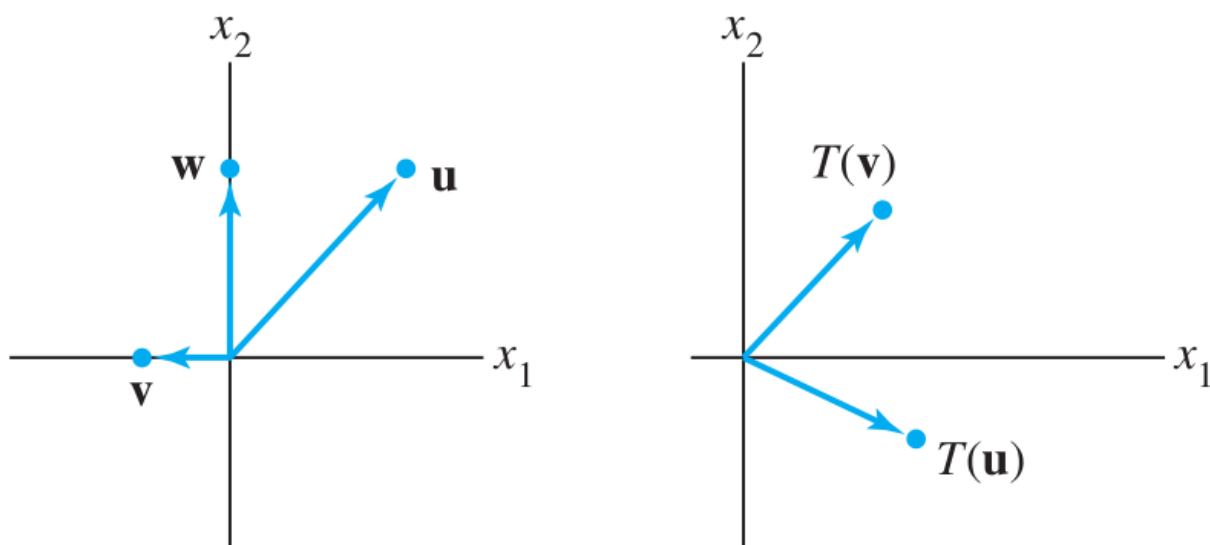
(e) _____ If \mathbf{u}, \mathbf{v} and \mathbf{w} are vectors in \mathbb{R}^2 , then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a linearly dependent set.

(f) _____ If a set $S = \{\mathbf{v}_1, \dots, \mathbf{v}_n\}$ of two or more vectors is linearly dependent, then at least one vector in the set can be written as a linear combination of other vectors in the set.

3. [6 pts] Let A be a 6×5 matrix. What must a and b be in order to define a transformation $T : \mathbb{R}^a \rightarrow \mathbb{R}^b$ by the rule $T(\mathbf{x}) = A\mathbf{x}$?

4. [10 pts] Row reduce the matrix to reduced echelon form by hand (and show work).
$$\begin{bmatrix} 1 & 7 & 3 & -4 \\ 1 & 7 & 4 & -2 \\ -2 & -14 & -5 & 10 \end{bmatrix}$$

5. [10 pts] (a) Give an example of vectors two vectors, \mathbf{u} and \mathbf{v} in \mathbb{R}^3 , so that $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ is a *plane* in \mathbb{R}^3 .
- (b) Give an example of vectors two vectors, \mathbf{x} and \mathbf{y} in \mathbb{R}^3 , so that $\text{Span}\{\mathbf{x}, \mathbf{y}\}$ is a *line* in \mathbb{R}^3 .
6. [10 pts] (a) Write the solution set of $x_1 + 9x_2 - 4x_3 = 0$ in parametric vector form.
- (b) Write the solution set of $x_1 + 9x_2 - 4x_3 = -2$ in parametric vector form.
7. [8 pts] The figure shows vectors \mathbf{u} , \mathbf{v} , and \mathbf{w} , along with the images $T(\mathbf{u})$ and $T(\mathbf{v})$ under a linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$. Draw the image $T(\mathbf{w})$ as accurately as possible. (*hint*: First, write \mathbf{w} as a linear combination of \mathbf{u} and \mathbf{v})



8. [16 pts] Each part describes a linear transformation T . Find the standard matrix of each transformation.

(a) $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a horizontal shear transformation that leaves \mathbf{e}_1 unchanged, and $T(\mathbf{e}_2) = \mathbf{e}_2 + 3\mathbf{e}_1$.

(b) $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ projects points in \mathbb{R}^3 onto the x_1x_3 -plane, so that $T(x_1, x_2, x_3) = (x_1, x_3)$.

(c) (optional question for +3 extra credit pts) $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ rotates points around the x_2 -axis at an angle $\frac{\pi}{3}$ clockwise (when looking towards the origin from the positive x_2 -axis).

9. [12 pts] Large intersections in England are often one-way *roundabouts* like the one shown. (Fun Fact: In Los Angeles, you can find a roundabout at the intersection of Figueroa and San Fernando, underneath the 5/110 freeway interchange, as well as in the Burbank IKEA parking lot.) The given traffic flows are in cars per minute.

(a) Write a system of equations describing the traffic flows x_1, \dots, x_6 .

(b) Solve the system.

