Math 270
Midterm 1
On 1.1 - 1.9

Name: ______________________

Score: ______/ 105= ______%

Rules: You may use a calculator but no notes. Do not share calculators, or look in the general direction of other's tests. Smartphones and smartwatches must be out of sight at all times. I can answer questions about the meaning of English words, and you should ask me if you think a question has a typo. I will *not* give you hints, or comment on your answers, or explain math terminology.

Instructions: Box your answers. Show any work that there is to be shown.

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1. [18 pts] Is each statement True or False? No need to justify answers.
   (a) ________ Given two vectors $\mathbf{v}$ and $\mathbf{p}$, the set $\{\mathbf{p} + t\mathbf{v} \ : t \in \mathbb{R}\}$ describes a line through $\mathbf{v}$ parallel to $\mathbf{p}$.
   (b) ________ If S is a linearly dependent set, then *each* vector is a linear combination of the other vectors in S.
   (c) ________ If $A = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, then for every $\mathbf{b}$ in $\mathbb{R}^3$, the equation $A\mathbf{x} = \mathbf{b}$ has a solution.
   (d) ________ Every homogeneous system of equations has at least one solution.
   (e) ________ If $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^2$, then $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a linearly dependent set.
   (f) ________ If a set $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ of two or more vectors is linearly dependent, then at least one vector in the set can be written as a linear combination of other vectors in the set.

2. [6 pts] Let A be a 6×5 matrix. What must a and b be in order to define a transformation $T \ : \ \mathbb{R}^a \ \to \ \mathbb{R}^b$ by the rule $T(\mathbf{x}) = A\mathbf{x}$?

3. [10 pts] Row reduce the matrix to reduced echelon form by hand (and show work). $\begin{bmatrix} 1 & 7 & 3 & -4 \\ 1 & 7 & 4 & -2 \\ -2 & -14 & -5 & 10 \end{bmatrix}$

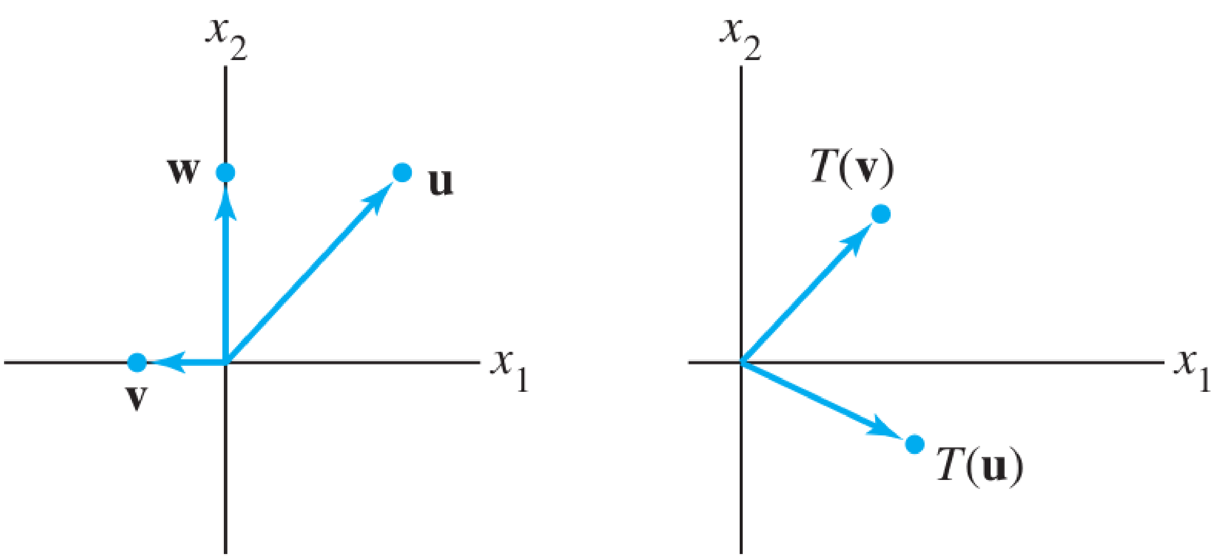
4. [10 pts] (a) Give an example of vectors two vectors, $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$, so that Span$\{\mathbf{u}, \mathbf{v}\}$ is a *plane* in $\mathbb{R}^3$.

   (b) Give an example of vectors two vectors, $\mathbf{x}$ and $\mathbf{y}$ in $\mathbb{R}^3$, so that Span$\{\mathbf{x}, \mathbf{y}\}$ is a *line* in $\mathbb{R}^3$.

5. [10 pts] (a) Write the solution set of $x_1 + 9x_2 - 4x_3 = 0$ in parametric vector form.

   (b) Write the solution set of $x_1 + 9x_2 - 4x_3 = -2$ in parametric vector form.

6. [8 pts] The figure shows vectors $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$, along with the images $T(\mathbf{u})$ and $T(\mathbf{v})$ under a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$. Draw the image $T(\mathbf{w})$ as accurately as possible. (*hint:* First, write $\mathbf{w}$ as a linear combination of $\mathbf{u}$ and $\mathbf{v}$)



7. [15 pts] State whether each set of vectors is linearly dependent or independent. Justify your answers.

   (a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

   (b) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$, a set of 4 vectors in $\mathbb{R}^3$.

   (c) The set of column vectors from the 3×3 identity matrix, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

8. [16 pts] Each part describes a linear transformation T. Find the standard matrix of each transformation.

   (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a horizontal shear transformation that leaves $\mathbf{e}_1$ unchanged, and $T(\mathbf{e}_2) = \mathbf{e}_2 + 3\mathbf{e}_1$.

   (b) $T : \mathbb{R}^3 \to \mathbb{R}^2$ projects points in $\mathbb{R}^3$ onto the x_1x_3-plane, so that $T(x_1, x_2, x_3) = (x_1, x_3)$.

   (c) (optional question for +3 extra credit pts) $T : \mathbb{R}^3 \to \mathbb{R}^3$ rotates points around the $x_2$axis at an angle $\frac{\pi}{3}$ clockwise (when looking towards the origin from the positive x_2-axis).

9. [12 pts] Large intersections in England are often one-way *roundabouts* like the one shown. (Fun Fact: In Los Angeles, you can find a roundabout at the intersection of Figueroa and San Fernando, underneath the 5/110 freeway interchange, as well as in the Burbank IKEA parking lot.) The given traffic flows are in cars per minute.

   (a) Write a system of equations describing the traffic flows $x_1, \ldots, x_6$.

   (b) Solve the system.