

Notes for Algebraic Structures

Spring 2016

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Notes for Algebraic Structures, taught Spring 2016 at Carnegie Mellon University, by Professor Clinton Conley.

Administrativa

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Grading. 20% HW, $20\% \times 2$ midterms, 40% Final

Homework. Wednesday-Wednesday. Graded for completeness, one starred problem for which no collaboration of any type is allowed.
Most homework out of textbook (“D&F”).

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The Integers

Lecture 1 (2016–01–11)

NOTATION. $\mathbb{N} := \{1, 2, 3, \dots\}$ in this class.

Properties: Order, other things. Least element in a set S : $x \in S$ s.t. $\forall y \in S, x \leq y$

Addition $(\mathbb{Z}, +)$:

- Associativity $(x + y) + z = x + (y + z)$
- Identity $x + 0 = 0 + x = x$
- Inversion $x + (-x) = (-x) + x = 0$
- Commutativity $x + y = y + x$

Multiplication $(\mathbb{Z}, +, \cdot)$:

- Associative
- Distributive
- Identity (“1”)

Integer division: Assume x an integer and $y \in \mathbb{Z}^+$ then $\exists! d \in \mathbb{Z}, \exists! r \in \mathbb{Z} : 0 \leq r < y, x = d \cdot y + r$

DEFINITION 1. $y|x$ “ y divides x ” iff $\exists d \in \mathbb{Z} : x = d \cdot y$.

E.g. $3|9, 4 \nmid 7$.

DEFINITION 2. d is a gcd of x and y if

- $d|x, d|y$
- If $c|x$ and $c|y$ then $c|d$

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