Latex Homework 9th Grade
Unit 1 - Methods of Proof - Formal Style of a Proof
Week 4 - Set Theory

Dr. Chapman and Dr. Rupel

Edited September 11, 2023

1

How many subsets of the set $\{0, 1, 2, 3, 4, 5, 6\}$ have elements which sum to 15? You should list them all.

2

Let A and B be subsets of a set Z. Prove that $(Z \setminus A) \cup (Z \setminus B) = Z \setminus (A \cap B)$.

Proof. Let $x \in (Z \setminus A) \cup (Z \setminus B)$. If $x \in Z \setminus A$, then $x \in Z$. If $x \in Z \setminus B$, then $x \in Z$. In either case, $x \in Z$. Let $x \in Z$. If $x \in (A \cap B)$, then $x \in Z$. If $x \notin (A \cap B)$, then $x \in Z$. In either case, $x \in Z$. Therefore, $(Z \setminus A) \cup (Z \setminus B) = Z \setminus (A \cap B)$. □

3

Consider sets A, B, and C. Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

Proof. Let $x \in A$, $y \in (B \cup C)$, $b \in B$, $c \in C$. $A \times (B \cup C)$ returns a set of ordered pairs (x,y). $A \times B$ gives ordered pairs (x,b), $A \times C$ gives ordered pairs (x, c). $B \cup C$ is the set of elements in B or C. This implies $b \in B \implies b \in (B \cup C)$ and $c \in C \implies c \in (B \cup C)$, meaning that $B \subseteq (B \cup C)$ and $C \subseteq (B \cup C)$. Because of this, y can be either b or c. Therefore, $(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$. It has already been shown that y can be b or c. $(A \times B) \cup (A \times C)$ will give (x,b) or (x,c). Therefore, $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$. Since $A \times (B \cup C)$ and $(A \times B) \cup (A \times C)$ are subsets of each other, they are equivalent. Hence, $A \times (B \cup C) = (A \times B) \cup (A \times C)$. □