LATEX HOMEWORK 9TH GRADE
UNIT 1 - METHODS OF PROOF - FORMAL STYLE OF A PROOF
WEEK 2 - STRUCTURE AND STYLE OF PROOF

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1

Explain what is wrong with the following proof:
Theorem: $2 = 1$
Proof: Let $a = b$. Then $a^2 = ab$ so $a^2 - b^2 = ab - b^2$ which we can factor as $(a - b)(a + b) = (a - b)b$. Canceling gives $a + b = b$ and since $a = b$ we get $b + b = b$. Dividing both sides by b gives $2 = 1$.

If $a = b$, then $(a - b)$ is 0. And we know that $\frac{0}{0}$ is indeterminate. Therefore, you can't divide it by 0.

2

Prove that for any natural numbers a, b, there exists an n with $an + b$ composite.

Proof. Theorem: $\forall a, b \in \mathbb{N}$, there exists an $n \in \mathbb{N}$ with $an + b$ composite. Proof: Without loss of generality, assume that a is odd and b is even. Every even number has a factor of 2. Assume n is even, then $a * n$ will be even. An even number plus an even number is also an even number. Assume now, without loss of generality, that b is odd. If n is odd, the $a * n$ will be odd. An odd number plus an odd number is an even number. Now assume that without loss of generality, that a is even and b is odd. If n is odd, then $a * n$ will also be odd. An odd plus an odd number is an even number. Finally, assume that without loss of generality, that a and b are both even. Then, if n is even, then $a * n$ will also be even. An even number plus an even number will also be an even number. Therefore, for any natural numbers a, b, there exists an n with $an + b$ composite. □

3

For each of the following, give an example and a counterexample:

- $n! - 1$ is prime for $n \geq 3$
- Any 3 distinct lines separate the plane into seven regions. What additional assumptions are needed in order for this to be a true statement?
- If a rational function is bounded, then it is constant.

Claim 1: $n! - 1$ is prime for $n >= 3$

- Example: $n = 5$, $n! - 1 = 119$
- Counterexample: $n = 8$, $n! - 1 = 40319 = 23 * 1753$

Claim 2: Any 3 distinct lines separate the plane into seven regions. What additional assumptions are needed in order for this to be a true statement?

Answer: None of the lines can be parallel.

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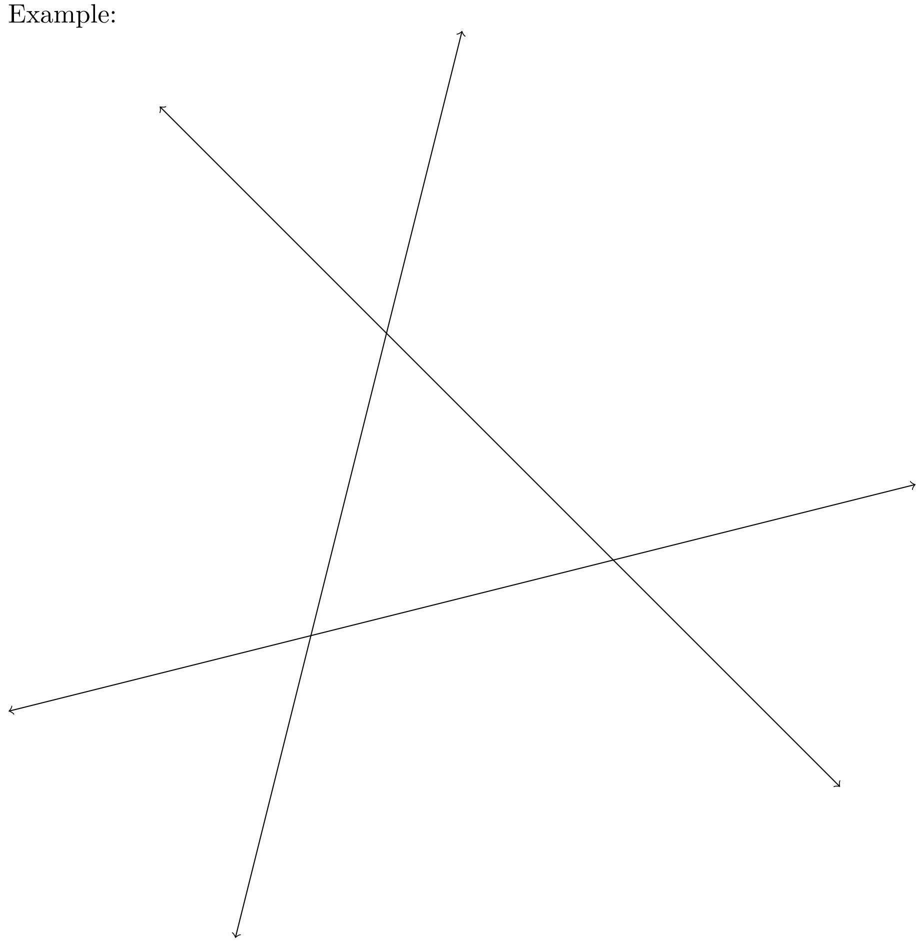
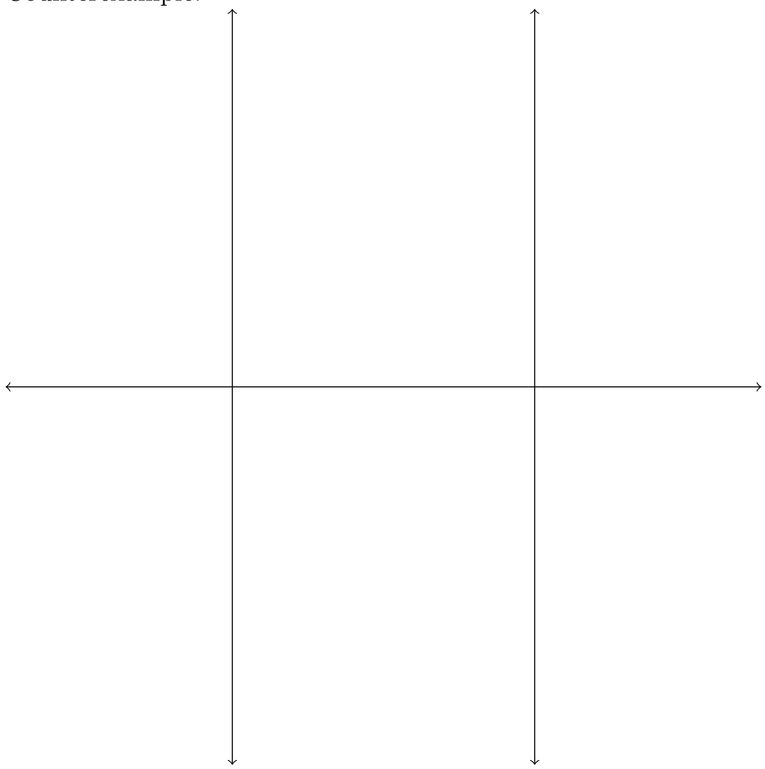
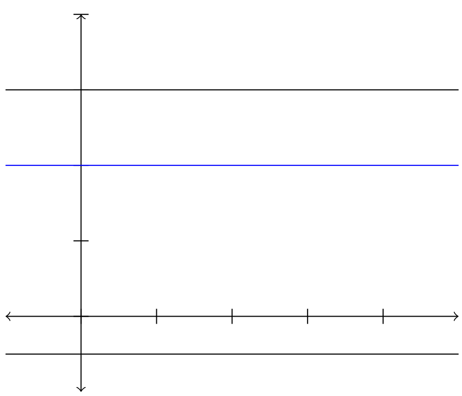
Example:



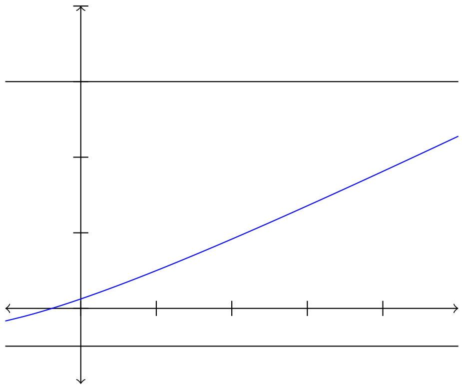
Counterexample:



Claim 3: If a rational function is bounded, then it is constant.



Example: $y = 2$



Counterexample: $y = (x^2 + 3x + 1)/(2x + 8)$