

Latex Homework 9th Grade

Unit 1 - Methods of Proof - Formal Style of a Proof

Week 3 - Logic

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1

Form a logical expression using the logical statements A, B, C and the operators \wedge, \vee, \neg , which is true if at least two of the statements are true, but false otherwise. Note: not every operator must be used, but you can use no others.

| $(A \wedge B) \vee \neg(\neg B \vee \neg C) \vee (A \wedge C)$ | | | | | | | |
|--|-----|-----|--------------|----------------------------|--------------|--|--|
| A | B | C | $A \wedge B$ | $\neg(\neg B \vee \neg C)$ | $A \wedge C$ | $(A \wedge B) \vee \neg(\neg B \vee \neg C)$ | $(A \wedge B) \vee \neg(\neg B \vee \neg C) \vee (A \wedge C)$ |
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | T | T |
| T | F | F | F | F | F | F | F |
| F | T | T | F | T | F | T | T |
| F | F | T | F | F | F | F | F |
| F | T | F | F | F | F | F | F |
| T | F | T | F | F | T | F | T |
| F | F | F | F | F | F | F | F |

2

Simplify $\neg(A \wedge \neg(B \wedge \neg(A \wedge \neg B)))$. In particular, your answer should only have negations applied to logical variables and not compound statements.

Step 1: $\neg(A \wedge \neg(B \wedge \neg(A \wedge \neg B)))$

Step 2: $\equiv \neg(A \wedge \neg(B \wedge (\neg A \vee B)))$

Step 3: $\equiv \neg(A \wedge (\neg B \vee (A \wedge \neg B)))$

Step 4: $\equiv A \vee (\neg B \wedge (\neg A \vee B))$

Step 5: $\equiv A \vee ((\neg B \wedge \neg A) \vee (\neg B \wedge B))$

Step 6: $\equiv A \vee ((\neg B \wedge \neg A) \vee c)$

Step 7: $\equiv A \vee (\neg B \wedge \neg A)$

Step 8: $\equiv (A \vee \neg B) \wedge (A \vee \neg A)$

Step 9: $\equiv (A \vee \neg B) \wedge A$

Step 10: $\equiv (A \wedge A) \vee (A \wedge \neg B)$

Step 11: $\equiv A \vee (A \wedge \neg B)$

Step 12: $\equiv (A \vee A) \wedge (A \vee \neg B)$

Step 13: $\equiv A \wedge (A \vee \neg B)$

Step 14: $\equiv (A \wedge A) \vee (A \wedge \neg B)$

Step 15: $\equiv A \vee (A \wedge \neg B)$

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Prove that $A \Rightarrow (B \wedge C)$ is equivalent to $(A \Rightarrow B) \wedge (A \Rightarrow C)$.