Latex Homework 9th Grade
Unit 3 - Methods of Proof - Techniques
Week 1 - Induction

Dr. Chapman and Dr. Rupel

Edited October 27, 2023

1

Explain what is wrong with the following proof:

Theorem: Any collection of points in the plane lie along a line.

Proof: Let $P(n)$ be the statement "any collection of n points in the plane lie along a line". Every point sits on a line, so $P(1)$ is true. Assume $P(n)$ is true and consider a collection of $n+1$ points in the plane. The first n of these points and the last n of these points each lie along a line by assumption. But these share the middle $n-1$ points which lie on both lines and so the two lines must be the same. It follows that $P(n+1)$ is true. By the Principle of Mathematical Induction, $P(n)$ is true for all $n \geq 1$.

2

Let $p(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ be a degree 5 polynomial.

(a) Write down a system of equations that states $\sum\limits_{k=1}^{n} k^4 = p(n)$ for $n = 0, 1, 2, 3, 4, 5$.

(b) Solve the system of equations from (a) to find the polynomial $p(x)$ explicitly.

(c) Prove by induction that $\sum\limits_{k=1}^{n} k^4 = p(n)$ for all $n \geq 0$.
Hint: It might simplify your proof a little if you factor $p(n)$ first.

3

Define the sequence f_n for $n \geq 1$ by the recursion $f_{n+1} = f_n + f_{n-1}$ with $f_1 = f_2 = 1$. Prove by strong induction that $f_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$.
Hint: $\frac{1+\sqrt{5}}{2} = 1 - \left(\frac{1-\sqrt{5}}{2}\right)$ *and* $\left(\frac{1+\sqrt{5}}{2}\right)\left(\frac{1-\sqrt{5}}{2}\right) = -1$

4

Consider the statement $P(n)$ given by "$12|(n^4 - n^2)$".

(a) Write the beginning steps of an induction proof. What expression would you have to show is divisible by 12 in order to complete the proof? Observe that this is not obvious and would require deeper considerations and possible casework.

(b) Show instead that $P(n)$ being true implies $P(n+6)$ is true. Use this to construct a proof that $P(n)$ is true for all $n \geq 1$ by strong induction.