PARTICLE REINFORCED PLASTICITY

A project report submitted by

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ABSTRACT

This theory is developed to determine the elasto plastic behaviour of particle reinforced composites. The considered material is a composite with elastic spherical particles dispersed uniformly in a ductile work hardening matrix. This theory combines the Mori Tanaka's average stress method and Hill's(1965) discovery of decreasing constraint power of matrix in polycrystal plasticity. It is approximated that the decreasing constraint power of matrix can be characterized using the secant Elastic moduli of the matrix. The stress and strain of the matrix is assumed to follow the modified Ludwik equation. In a composite usually the hard particles are the load carrying members and reduce the stress carried by the matrix. We have assumed that the plastic strain is carried only by the matrix and hence its magnitude depends on the volume fraction of the particles. Based on the Eshelby's (1957) solution of an ellipsoidal inclusion and Mori Tanaka's(1973) concept of average stress in the matrix, the average stress and strain in the particles are determined using Eshelby's equivalence principle. Computation of the effective properties by means of MATLAB and also verification with Abaqus.

CONTENTS

INTRODUCTION

Hard particles such as oxides and carbides can only deform elastically. Their constraint power is much larger than that of the matrix which is typically some epoxy resin. When subjected to a load, due to their relatively higher strength, their load sharing capability is also enhanced. The reduction of the load carried by the matrix results in an overall reduction in the plastic strain in the matrix. Moreover, the overall plastic strain in the specimen is reduced because of the reduction in the volume fraction of the matrix in comparison to the overall composite. These effects increase the effective strain hardening parameters of the system.

There are basically two approaches to this problem. Both of them deal with using the solution of Eshelby's ellipsoidal inclusion problem (1957) and Mori-Tanaka's concept of average stress in the RVE. The earlier Mori-Tanaka's problem only dealt with elastic transformation for inclusion. This was modified for the inhomogeneity by the pioneering works of Chow (1977), Taya and Chou (1981) and Taya and Mura (1981). This was done by means of using Eshelby's "equivalence principle" (ie) the stress in the inhomogeneity is same as the stress in an equivalent inclusion by means of an additional "eigenstrain". The problem of both the inclusion and inhomogeneity transforming plastically is also solved by means of the Mori-Tanaka approach. But the case that is considered here is the case where the particle is rigid and the matrix deforms plastically.

Then, in that case, the secant modulus of the matrix continues to decrease as the composite deforms plastically, but the modulus of the inclusion remains unchanged. The weakening power of the matrix was discovered by Hill (1965), and this is more complex than the elastic results produced by Kroner (1961) and Budiansky and Wu(1962). The constraint power predicted by the K-B-W was shown to be higher than what was predicted by Hill. A simpler and more direct approach is by Berveiller and Zaoui(1979) for monotonic and proportional formulations whereas Hill method is for incremental formulations. Both these methods adopt self-consistent schemes for formulation of the effective properties of a composite, as is relevant to the cases considered in those publications. Here, however, it seems that it leads to wrongful estimation of the deformation behaviour of the particle hardened composite, hence the Mori-Tanaka model will be adopted.

THEORY OF PARTICLE REINFORCED PLASTICITY

The uniaxial stress and plastic strain of the matrix are assumed to follow strain-hardening law (ie)

$$\sigma = \sigma_y + h(\epsilon_p)^n \rightarrow [1]$$

where 'h' and 'n' are the strain hardening coefficient and exponent, respectively.

The secant modulus is the total stress in the specimen divided by overall strain.

$$E_o^s = \frac{\sigma}{\epsilon^e + \epsilon^p} = \frac{1}{\frac{1}{E_o} + \frac{\epsilon^p}{\sigma_y + \mathrm{h}.(\epsilon^p)^n}}$$

Where ϵ^e is the elastic strain and E_o is the elastic modulus.

w.r.t. to the un-deformed state, the secant bulk and shear modulus are written as

$$k_o^s = \frac{\frac{E_o^s}{3}}{1 - 2 * \vartheta_o^s}$$

$$G_o^s = \frac{\frac{E_o^s}{2}}{1 + \vartheta_o^s}$$

The poisson's ratio can be obtained as

$$\vartheta_o^s = 0.5 - (0.5 - \vartheta_o) * \frac{E_o^s}{E_o}$$

Because, the bulk modulus doesn't change owing to plastic incompressibility.

If in any case, the stress state is triaxial, then the von-Mises effective stress can be used in favour of σ in eqn[1].

Now, in the presence of an inclusion, the Eshelby's equivalence principle states,

$$\sigma^{(1)} = \bar{\sigma} + \hat{\sigma} + \sigma^{pt} = L_1 * (\epsilon^0 + \hat{\epsilon} + \epsilon^{pt}) = L^s{}_o * (\epsilon^0 + \hat{\epsilon} + \epsilon^{pt} - \epsilon^*)$$

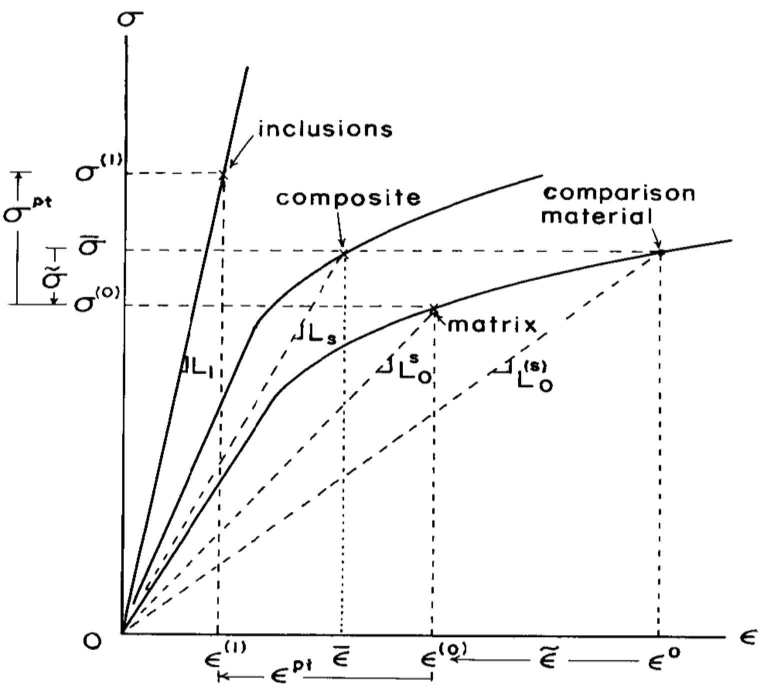
Where ϵ^* is the "eigenstrain" in the equivalent inclusion. Now, according to Eshelby's theory of ellipsoidal inclusions, we have

$$\epsilon^{pt} = S_o^s * \epsilon^*$$

Where, S_o^s is the Eshelby's tensor. It can be decomposed into hydrostatic and deviatoric parts, as $S_o^s = (\alpha_o^s, \beta_o^s)$

Where, $\alpha_o^s = \frac{1}{3} * \frac{1-\vartheta_o^s}{1+\vartheta_o^s}$

$$\beta_o^s = \frac{2}{15} * \frac{(4 - 5 * \vartheta_o^s)}{1 - \vartheta_o^s}$$



The material is assumed to be in an effective medium denoted by (s), hence, the overall mean stress can be expressed as

$$\bar{\sigma} = L_o^{(s)} * \epsilon^0$$

Now, the stress in the matrix gets perturbed by an amount, and its value is given by

$$\sigma^{(0)} = \bar{\sigma} + \hat{\sigma} = L_o^s * (\epsilon^0 + \hat{\epsilon}) \rightarrow [2]$$

The superscript 's' denotes secant moduli of the matrix. It's the same as (s) in the case of linear elasticity.

The stress and strain in the inclusion gets further perturbed by the relation

$$\sigma^{(1)} = \bar{\sigma} + \hat{\sigma} + \sigma^{pt} = L_1 * (\epsilon^0 + \hat{\epsilon} + \epsilon^{pt}) = L^s{}_o * (\epsilon^0 + \hat{\epsilon} + \epsilon^{pt} - \epsilon^*) \rightarrow [3]$$

Now, the volume average of the stresses in the composite should be equal to $\bar{\sigma}$ .This gives

$$\hat{\sigma} = -c * \sigma^{pt} \rightarrow [4]$$

From [2], we get $\hat{\sigma}$ and we substitute $\bar{\sigma} + \hat{\sigma}$ in the 2nd eqn in [3], to get

$$\sigma^{pt} = L_o^s * (S_o^s - I)$$

Putting the value of σ^{pt} in [4], we get,

$$\hat{\epsilon} = -c * (S_o^s - I)\epsilon^* - (I - (L_o^{(s)})^{-1} L_o^s)\epsilon^0$$

Then, by using eshelby tensor and equivalence principle (last of eqn(3)), we obtain the concentration factors as

$$a_o = \frac{\alpha_o^s * (k1 - k0) + k0}{(c + (1-c) * \alpha_o^s)(k1 - k0) + k0}$$

$$b_o = \frac{\beta_o^s * (g1 - G_o^s) + G_o^s}{(c + (1-c) * \beta_o^s)(g1 - G_o^s) + G_o^s}$$

$$a_1 = \frac{k1}{(c + (1-c) * \alpha_o^s)(k1 - k0) + k0}$$

$$b_1 = \frac{g1}{(c + (1-c) * \beta_o^s)(g1 - G_o^s) + G_o^s}$$

Where the subscript '0' denotes the CF for the matrix and '1' for inclusion.

Now, to get the effective stress/strain response, we need average strain, which is

$$\bar{\epsilon} = \epsilon^0 + \hat{\epsilon} + c * \epsilon^{pt}$$

Now, we know $\hat{\epsilon} + c * \epsilon^{pt}$ in terms ofϵ^0, and the eshelby and secant moduli. Hence, the effective bulk and modulus and shear modulus can be calculated as

$$\frac{ks}{k0} = 1 + \frac{c(k1 - k0)}{(1-c)\alpha_o^s(k1 - k0) + k0}$$

→[5]

$$\frac{Gs}{G_o^s} = 1 + \frac{c(G1 - G_o^s)}{(1-c)\beta_o^s(G1 - G_o^s) + G_o^s}$$

→[6]

MATLAB CODES

a. Matlab code for Stress Concentration Factors

```
function [a0,b0]=find_effective(c1,et)
%c1 is the volume fraction and et is the plastic strain
E0=72E9;
E1=431E9;
v0=0.34;
v1=0.19;
k0=E0/3/(1-2*v0);
k1=E1/3/(1-2*v1);
G0=E0/2/(1+v0);
G1=E1/2/(1+v1);
S_y=269.9E6;
h=576E6;
n=0.51;
E0_s=E0*(S_y+h*(et.^n))/(S_y+(h*(et.^n)+E0*et));
v0_s=0.5-(0.5-v0)*E0_s/E0;
alpha0_s=1/3*(1-v0_s)/(1+v0_s);
a0=(alpha0_s*(k1-k0)+k0)/((c1+(1-c1)*alpha0_s)*(k1-k0)+k0);
beta0_s=2/15*(4-5*v0_s)/(1-v0_s);
G0_s=E0_s/2/(1+v0_s);
b0=(beta0_s*(G1-G0_s)+G0_s)/((c1+(1-c1)*beta0_s)*(G1-G0_s)+G0_s);
end
```

b. Matlab code for triaxial stress state

```
function [ep0_eq,ep_mean_eq]=findPlasticStrain(c1,S1,S2,S3)
E0=72E9;
E1=431E9;
v0=0.34;
v1=0.19;
k0=E0/3/(1-2*v0);
k1=E1/3/(1-2*v1);
G0=E0/2/(1+v0);
G1=E1/2/(1+v1);
S_y=269.9E6;
h=576E6;
n=0.51;
alpha0=1/3*(1+v0)/(1-v0);
beta0=2/15*(4-5*v0)/(1-v0);
%calculating equivalent stress
S0=sqrt(0.5*((S1-S2).^2+(S2-S3).^2+(S3-S1).^2));
d=1/3*(S1+S2+S3);
S_h=[[d,0,0];[0,d,0];[0,0,d]];
S=[[S1,0,0];[0,S2,0];[0,0,S3]];
S_d=S-S_h;
syms et;
%calculating secant moduli of the matrix
E0_s=E0*(S_y+h*(et.^n))/(S_y+(h*(et.^n)+E0*et));
v0_s=0.5-(0.5-v0)*E0_s/E0;
%components of the eshelby tensor: hydrostatic(aplha0_s)and
%deviatoric(beta0_s)
alpha0_s=1/3*(1+v0_s)/(1-v0_s);
%stress concentration factors:a0,b0
```

```
a0=(alpha0_s*(k1-k0)+k0)/((c1+(1-c1)*alpha0_s)*(k1-k0)+k0);
beta0_s=2/15*(4-5*v0_s)/(1-v0_s);
G0_s=E0_s/2/(1+v0_s);
b0=(beta0_s*(G1-G0_s)+G0_s)/((c1+(1-c1)*beta0_s)*(G1-G0_s)+G0_s);
%calculation of maximum stress ; for uni-axial case, it is equal to
%CF*(Uniaxial Stress Value) and for tri-axial, it is equal to CF*equivalent
%stress
St=b0*S0;
eqn=St==S_y+h*(et).^n;
et=solve(eqn,et);
ep_star=et;
E0_s=E0*(S_y+h*(et.^n))/(S_y+(h*(et.^n)+E0*et));
v0_s=0.5-(0.5-v0)*E0_s/E0;
alpha0_s=1/3*(1+v0_s)/(1-v0_s);
a0=(alpha0_s*(k1-k0)+k0)/((c1+(1-c1)*alpha0_s)*(k1-k0)+k0);
beta0_s=2/15*(4-5*v0_s)/(1-v0_s);
G0_s=E0_s/2/(1+v0_s);
b0=(beta0_s*(G1-G0_s)+G0_s)/((c1+(1-c1)*beta0_s)*(G1-G0_s)+G0_s);
%E0_s=E0*(S_y+h*(ep_star.^n))/(S_y+(h*(ep_star.^n)+E0*ep_star));
%calculating effective plastic strain from the unloading process
ep0=1/2*(1/G0_s - 1/G0)*S_d*b0;
k_s=k0*(1+(c1*(k1-k0)/((1-c1)*alpha0_s*(k1-k0)+k0)));
G_s=G0_s*(1+(c1*(G1-G0_s)/((1-c1)*beta0_s*(G1-G0_s)+G0_s)));
k=k0*(1+(c1*(k1-k0)/((1-c1)*alpha0*(k1-k0)+k0)));
G=G0*(1+(c1*(G1-G0)/((1-c1)*beta0*(G1-G0)+G0)));
E=9*k*G/(3*k+G);
E_s=9*k_s*G_s/(3*k_s+G_s);
ep_mean2=1/3*(1/k_s-1/k)*(S1+S2+S3);
ep_mean=(1-c1)*ep0+(1-G0/G-c1*(G1-G0)/((c1+(1-c1)*beta0_s)*(G1-
G0_s)+G0_s))*S_d/2/G0+1/3*(1-k0/k-c1*(k1-k0)/((c1+(1-c1)*alpha0_s)*(k1-
k0)+k0))*S_h/3/k0;
ep0_eq=sqrt(2/3*(ep0(1,1).^2+ep0(2,2).^2+ep0(3,3).^2));
ep_mean_eq=sqrt(2/3*(ep_mean(1,1).^2+ep_mean(2,2).^2+ep_mean(3,3).^2));
end
```

c. Matlab code for triaxial displacement loading

```
function [ep_star,ep_mean]=findPlasticStrain(c1,str)
d=1/3*(str(1,1)+str(2,2)+str(3,3));
str_h=[[d,0,0];[0,d,0];[0,0,d]];
str_d=str-str_h;
str_I1=str(1,1)+str(2,2)+str(3,3);
str_I2=str(1,1)*str(2,2)+str(2,2)*str(3,3)+str(1,1)*str(3,3);
str_J2=1/3*str_I1.^2-str_I2;
str_e=sqrt(3*str_J2)
E0=72E9;
E1=431E9;
v0=0.34;
v1=0.19;
k0=E0/3/(1-2*v0);
k1=E1/3/(1-2*v1);
G0=E0/2/(1+v0)
G1=E1/2/(1+v1);
S_y=269.9E6;
h=576E6;
n=0.51;
alpha0=1/3*(1+v0)/(1-v0);
beta0=2/15*(4-5*v0)/(1-v0);
```

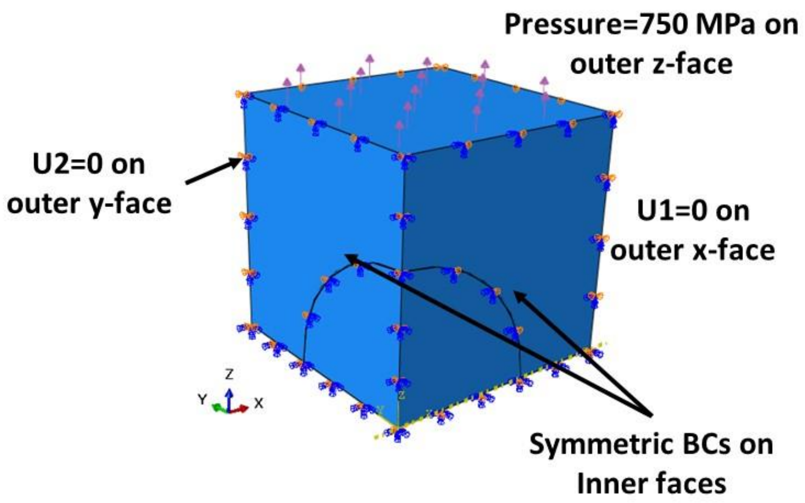
```
q0=(beta0*(G1-G0)+G0)/((1-c1)*beta0*(G1-G0)+G0);
if str_e<S_y/2/G0/q0;
    fprintf('below yield value\n');

else
    syms et;
    E0_s=E0*(S_y+h*(et.^n))/(S_y+(h*(et.^n)+E0*et));
    v0_s=0.5-(0.5-v0)*E0_s/E0;
    %components of the eshelby tensor: hydrostatic(aplha0_s)and
    %deviatoric(beta0_s)
%      alpha0_s=1/3*(1-v0_s)/(1+v0_s);
    %calculating equivalent stress
    beta0_s=2/15*(4-5*v0_s)/(1-v0_s);
    G0_s=E0_s/2/(1+v0_s);
    q0_s=(beta0_s*(G1-G0_s)+G0_s)/((1-c1)*beta0_s*(G1-G0_s)+G0_s);
    S_d=2*G0_s*q0_s*str_d;
    S_d_I2=S_d(1,1)*S_d(2,2)+S_d(3,3)*S_d(2,2)+S_d(1,1)*S_d(3,3)-
S_d(1,2).^2-S_d(2,3).^2-S_d(3,1).^2;
    S_e=sqrt(3*S_d_I2);
    eqn=S_e==S_y+h*(et).^n;
    et=solve(eqn,et)
    ep_star=et
    E0_s=E0*(S_y+h*(et.^n))/(S_y+(h*(et.^n)+E0*et));
    v0_s=0.5-(0.5-v0)*E0_s/E0;
    alpha0_s=1/3*(1+v0_s)/(1-v0_s);
    beta0_s=2/15*(4-5*v0_s)/(1-v0_s);
    G0_s=E0_s/2/(1+v0_s);
    k_s=k0*(1+(c1*(k1-k0)/((1-c1)*alpha0_s*(k1-k0)+k0)));
    G_s=G0_s*(1+(c1*(G1-G0_s)/((1-c1)*beta0_s*(G1-G0_s)+G0_s)));
    k=k0*(1+(c1*(k1-k0)/((1-c1)*alpha0*(k1-k0)+k0)));
    G=G0*(1+(c1*(G1-G0)/((1-c1)*beta0*(G1-G0)+G0)));
    ep_mean=(0.5-0.5*G_s/G)*str_d+1/9*(1-k_s/k)*str_h
end
```

ABAQUS ANALYSES

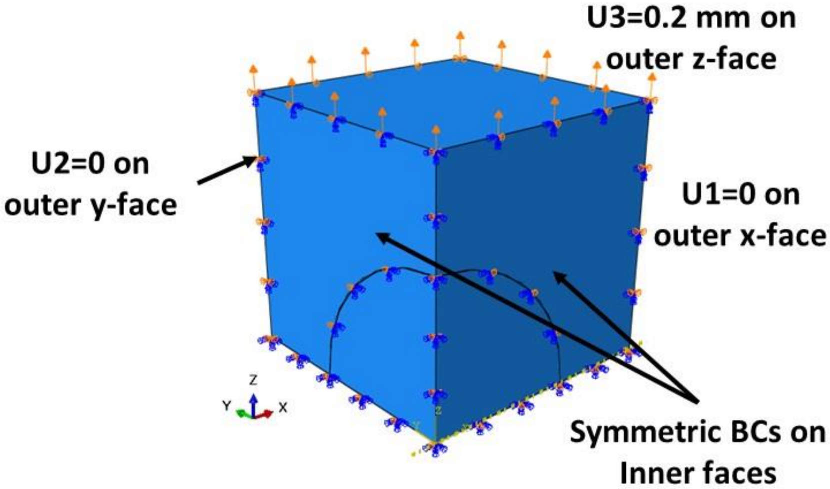
RVE is symmetric, Plastic analysis was carried out for $1/8^{th}$ model ($50x50x50\ mm^3$) of RVE and spherical particle of radius 30 mm and V_f=0.1127 . Material Properties for Matrix (Al) and Particle (SiC) are as follows:

	E (MPa)	υ
Al	72000	0.34
SiC	431000	0.19

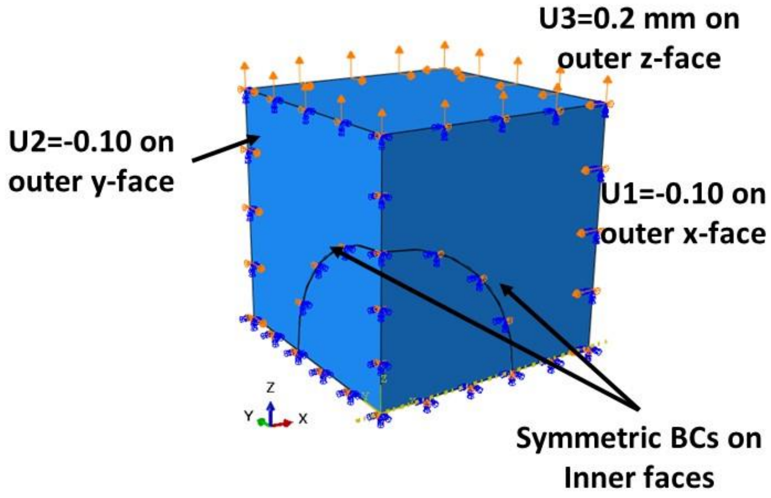
The loading and BCs for the different cases are shown.



a. Uniaxial Stress loading in Z direction



b. Displacement loading in Z direction



c. **Displacement in Axial and Lateral directions**

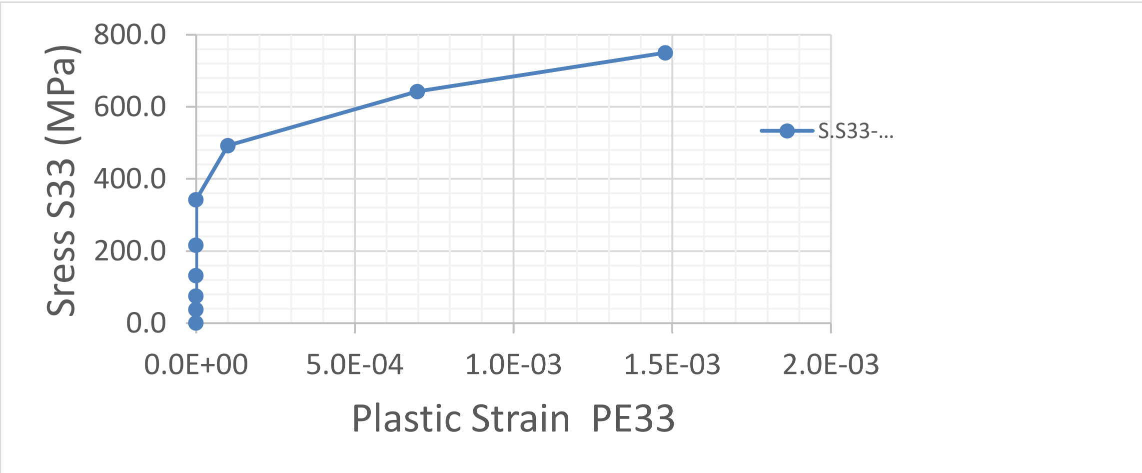
RESULTS AND DISCUSSION

Now, the overall stress in the specimen is found by the formula

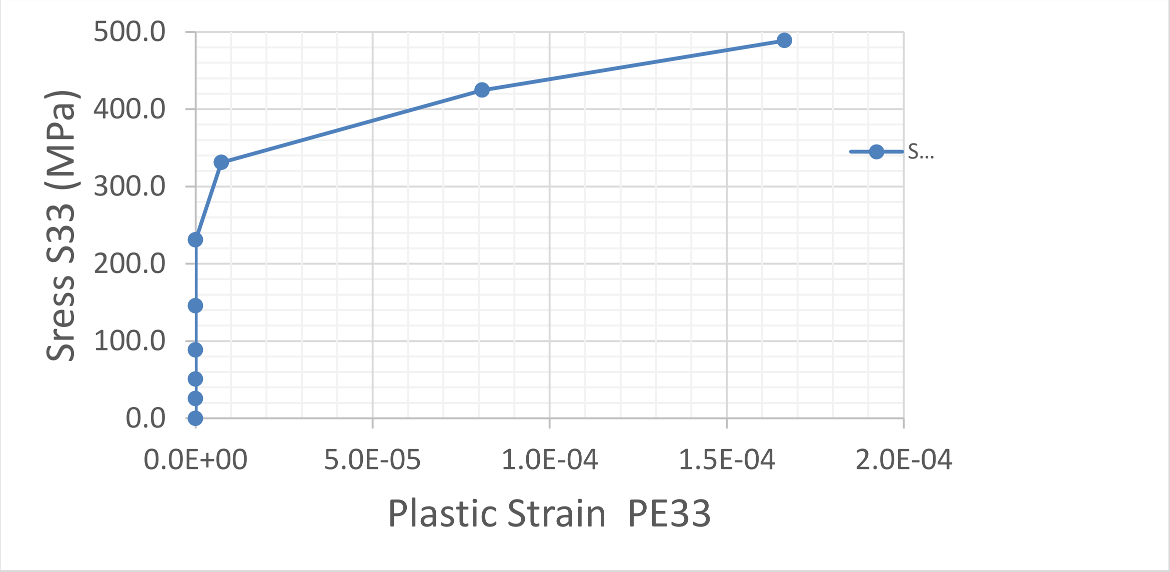
$$\bar{\sigma} = \frac{sum\big(\sigma(i) * \Delta V(i)\big)}{V}$$

Where, the elemental values were queried from Abaqus using Field output. This is the plot of S33-PE33 is shown, for

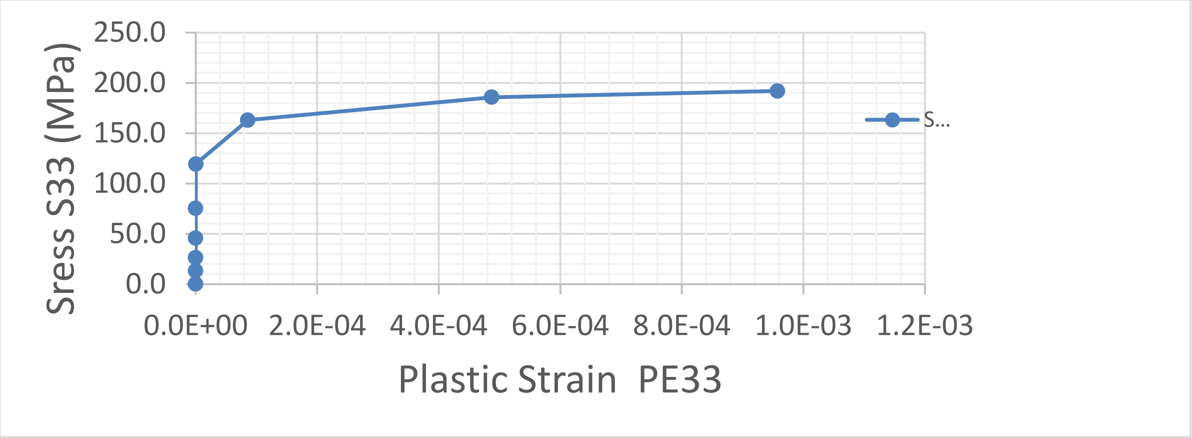
a. Uniaxial stress case:



b. Uniaxial displacement case:



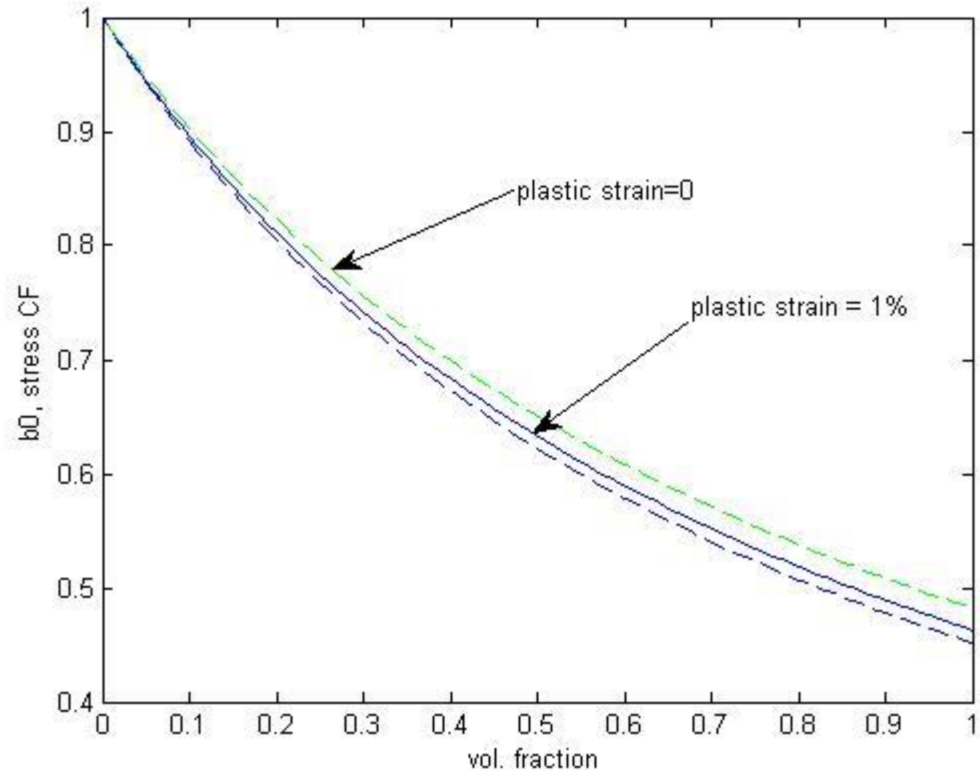
c. Tri-axial displacement case:



Stress concentration factors (or, in this case, dilution factors)

The stress dilution factors for the matrix are plotted as a function of vol. fraction for three different innate plastic strain in the specimen.

The graphs are plotted at plastic strains of 0, 1 and 2%



The value of b_0, which is vital to determining the yield criterion of the composite, is seen to decrease drastically with increasing concentration. This effect is further propounded at higher plastic strain, increasing the overall constraining power of the matrix. For the uniaxial case, the following expression is used to determine plastic strain based on unloading process.

$$\epsilon_{p(av)} = (\frac{1}{E_s} - \frac{1}{E})\sigma_{11}$$

Where, Es is the effective modulus calculated from equation [5] and [6] as

$$E_s = 9(ks)\frac{Gs}{3ks + Gs}$$

And E is the effective “elastic” moduli of the composite from Mori-Tanaka estimate. It can also be calculated from

$$\bar{\epsilon}^p_{ij} = (1 - c_1)\epsilon^p_{ij}$$

$$+ \left[1 - \frac{\mu_0}{\mu} - \frac{c_1(\mu_1 - \mu_0)}{[c_1 + (1 - c_1)\beta^s_0](\mu_1 - \mu^s_0) + \mu^s_0}\right]\frac{\bar{\sigma}'_{ij}}{2\mu_0}$$

$$+ \frac{1}{3}\delta_{ij}\left[1 - \frac{\kappa_0}{\kappa} - \frac{c_1(\kappa_1 - \kappa_0)}{[c_1 + (1 - c_1)\alpha^s_0](\kappa_1 - \kappa_0) + \kappa_0}\right]\frac{\bar{\sigma}_{kk}}{3\kappa_0}.$$

The above equation is implemented in code.

CONCLUSION

In this report we have calculated stress strain behaviour of a composite with rigid particles and work hardening matrix by two methods. We have done the modelling using ABAQUS and the corresponding stress strain graphs are plotted and we have tried to compare the stress strain graphs we get by modelling the same using MATLAB. We tried to see if atleast the trends of both the graphs are matching. There is still a mismatch between the 2 stress strain graphs. This may be due to the fault in our return mapping algorithm. We have assumed in our analysis that since the loading is monotonic there is no need to do the iterations for calculation of the plastic strain and is calculated directly. This may have resulted in the deviation of the results as the loading in ABAQUS is gradual and in MATLAB it is sudden.

REFERENCES

Tandon, G. P., and Weng, G. J., 1988,"A Theory of Particle-Reinforced Plasticity," *Journal of Appplied Mechanics,* Vol. 110, pp. 126-134.

Eshelby, J. D., 1957, "The determination of the elastic field of an ellipsoidal inclusion, and related problems", *Proceedings of the Royal Society*, Vol. A 241, pp. 376–396.

Hill, R., 1965, "Continuum micro-mechanics of elastoplastic polycrystals". *Journal of the mechanics and physics of solids*, Vol. 13 pp.89-101

Berveiller, M., and Zaoui, A., 1979, "An extension of the self-consistent Scheme to Plastically-Flowing Polycrystals," *Journal of the Mechanics and Physics of Solids,* Vol. 26 pp.325-344