Summer School
Advanced Structural Dynamics

June 16th-19th 2025, Copenhagen (Denmark)

Fast-slow motion analysis for friction-related problems
Lecture 1

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Outline of the lectures

- **Lecture 1 – 9:00-10:00**
  - Personal background
  - Case studies of fast-slow motion analysis for friction-related problems
  - Simplified and intuitive example of fast-slow motion analysis for sliding friction

- Lecture 2 – 13:10-14:00
  - Mass-spring system influenced by friction for a general frequency of excitation
  - Worked-out example of fast-slow motion analysis for a pile driving system

- Lecture 3 – 14:00-15:00
  - Follow-up: worked-out example of fast-slow motion analysis for a pile driving system
  - Example of results of the friction reduction of SDOF moving on an elastic rod
  - Discussion on hidden (fast) motion effect on stability and dynamic friction laws

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Personal background

- PhD in Structural Engineering (2010-2014): Vibration-based damage detection on Civil Infrastructures
- Several Post-doctoral positions (2014-2020): Variety of projects on nonlinear dynamics between mechanical and civil engineering.
- Assistant Professor at TU Delft (Civil Engineering), since 2020.

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Department of Engineering Structures (TU Delft)

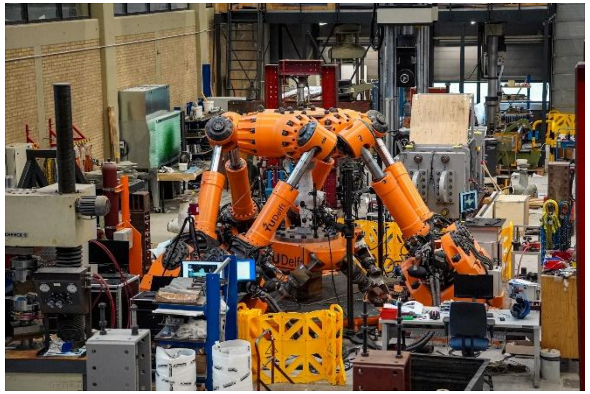
- Part of the Faculty of Civil Engineering and Geosciences (it consists of 7 Departments)
- Department of Engineering Structures: it consists of 8 sections

Steel and Composites Structures

Biobased Materials and Structures

Dynamics of Solids and Structures

Railway Engineering



Pavement Engineering

Resources and Recycling

Concrete Structures

Mechanics and Physics of Structures

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What do we do in Mechanics and Physics of Structures ?

A. Cabboi

M. Steenbergen

Group of PhD candidates and Post-docs

E. Sulollari

Y. Zheng

S. Zhou

S. Song

Y. Zheng

Main areas of research

- Vibration propagation
- Dynamic characterization of systems
- Contact dynamics and tribology
- Nonlinear vibration and damping

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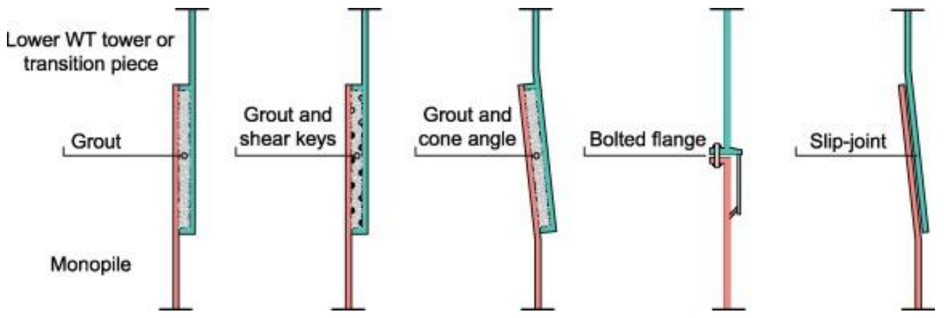
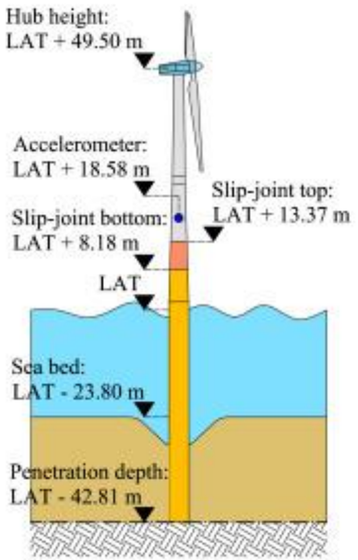
How did I end up working on slow-fast motions?

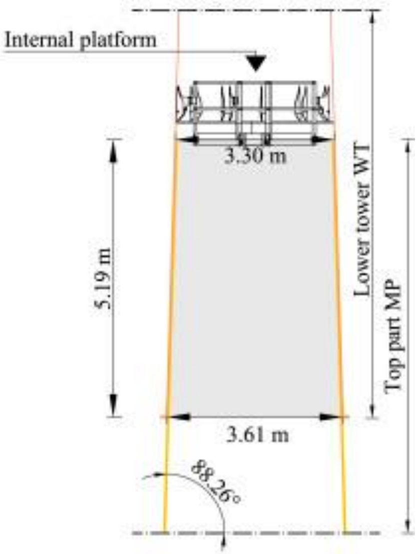
One of my post-doctoral projects in 2018 was about the installation and decommissioning of a slip joint for wind turbine connections.



(a) Picture of the DOT wind turbine.



(b) Schematic layout of the structure.



(c) Detailed drawing of the slip-joint.

- The concept of a slip joint is still not used as a standard design concept. The two tasks I had to work on at TUD were: **can we exploit an induced fast vibration to** (a) create a sound contact during installation and (b) disconnect the slip joint in case of decommissioning of the wind turbine?

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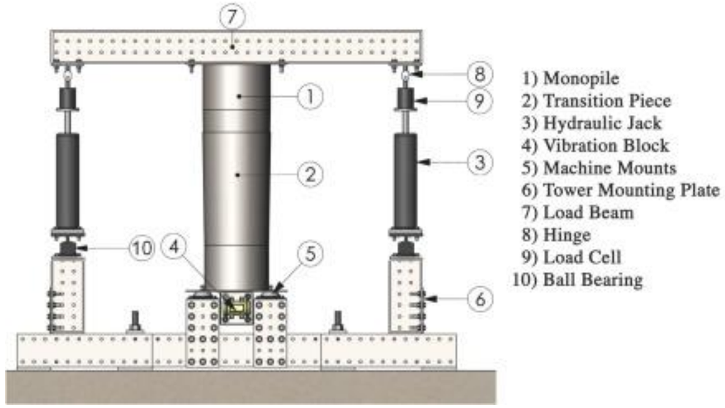
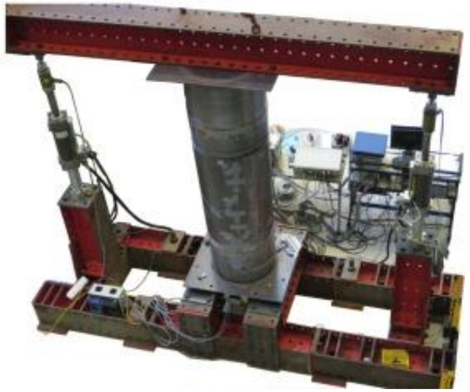
The two images from this slide and from the next slide are taken from:

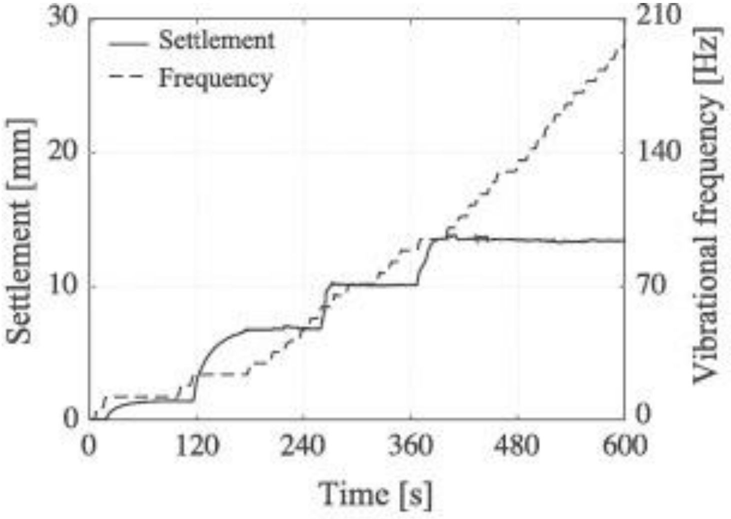
- Alessandro Cabboi, Thijs Kamphuis, Evert van Veldhuizen, Maxim Segeren, Hayo Hendrikse, Vibration-assisted decommissioning of a slip joint: Application to an offshore wind turbine, Marine Structures, Volume 76, 2021.
- Alessandro Cabboi, Maxim Segeren, Hayo Hendrikse, Andrei Metrikine, Vibration-assisted installation and decommissioning of a slip-joint, Engineering Structures, Volume 209, 2020.

Some results from the slip joint project

Lab-scale tests

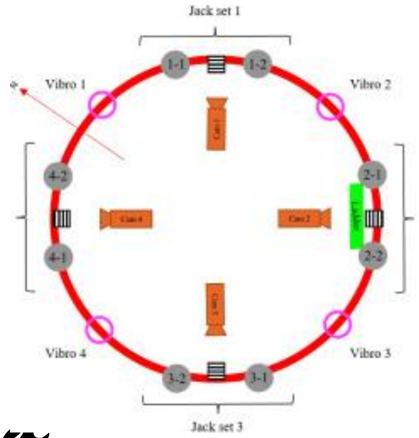


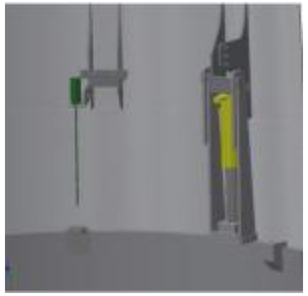


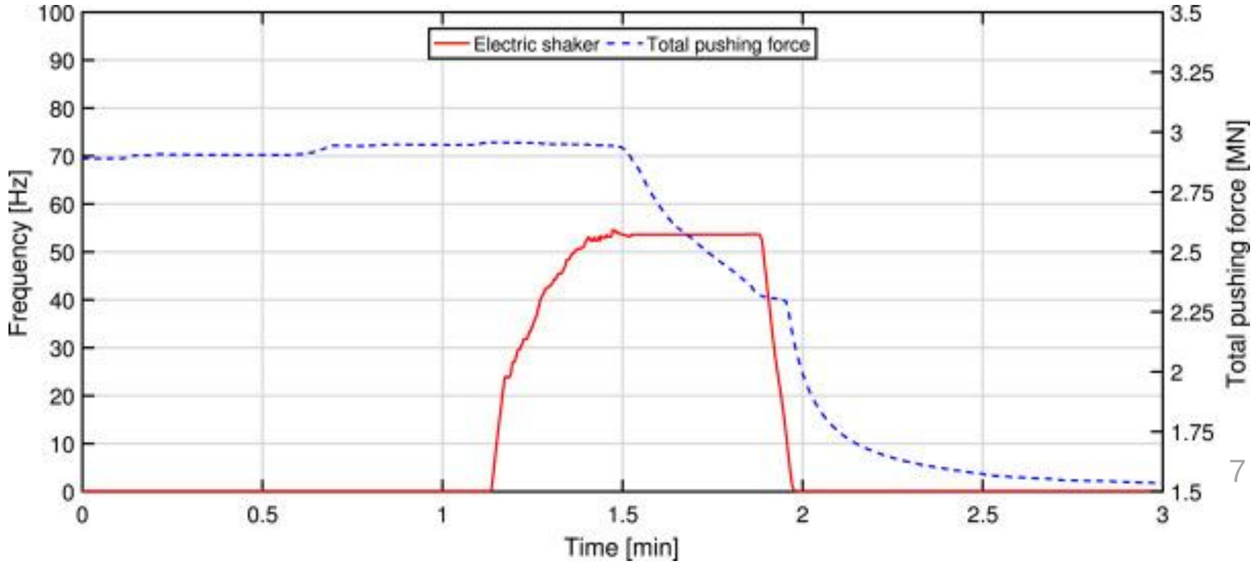


(a) The settlement of the upper cone and the vibrational frequency, Test 1.

Offshore tests







Are there other applications in which we can alter the friction behaviour through vibration? Part 1

Pile driving for monopiles

An alternative **installation** method was proposed by **TU Delft:** the **Gentle Driving** of **Piles** (**GDP).**

Idea consists to combine:

a) low-frequency/axial &

b) high-frequency/torsional loading

GDP method is envisaged to:

- improve **installation** performance
- reduce **noise emissions**

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This slide is a courtesy from: **Dr. Athanasios Tsetas.** More details on this work can be found in his recent papers, specifically: Tsetas A., Tsouvalas A., Metrikine, **The mechanics of the Gentle Driving of Piles** (2023) International Journal of Solids and Structures.

Are there other applications in which we can alter the friction behaviour through vibration? Part 2

Vibrational displacement or vibration separation of particles

- In recycling processes, vibrating conveyors are more often used to even distribute materials that need to be separated and recycled
- Applications span from chemical industry, food industry and to mining

https://www.youtube.com/shorts/bCSknafmmOU

What are the current challenges concerning the interaction of friction and vibration ?

- Commonly, you see studies focused on this type of question:

  How does friction influence the vibration response?

- However, what if we want to exploit vibration to alter/control the friction process?

Open challenges (many actually, here are just some of them):

- In general, we never know the exact friction/contact law governing the material pair.
- For now, one mechanism is known the can govern the vibration-induced friction modulation: **redirection of the friction force** throughout one cycle of vibration. However, **are there other mechanisms at play**? If so, are they relevant?
- Which is the most optimal/efficient modelling/predictive strategy to control this process?

General overview on fast-slow motion induced effects

Good references:

1) Vibrations and stability: Advanced theory, Analysis, and Tools , J. J. Thomsen (3rd edition)
2) Vibrational mechanics: nonlinear dynamic effects, general approach, applications, I. I. Blekhman

General effects

Stiffening

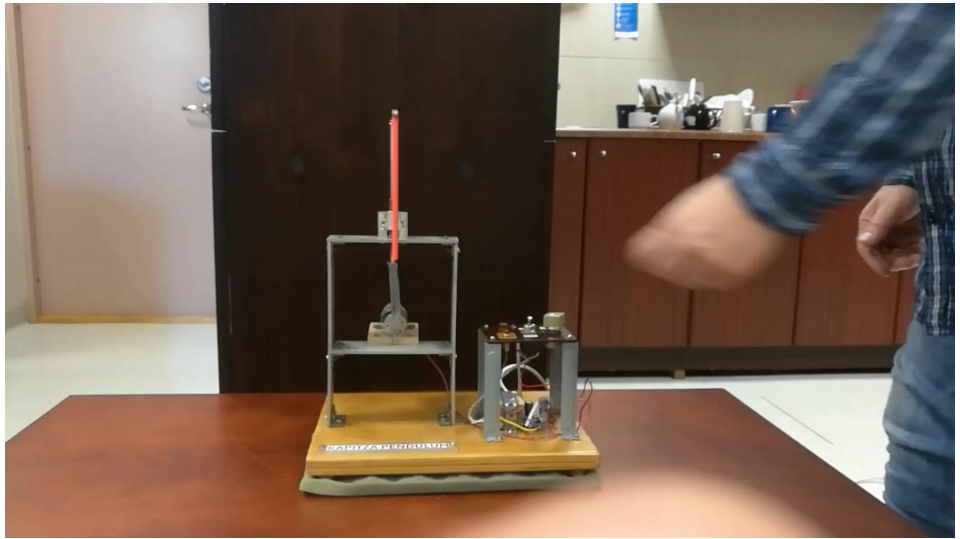
Parametrically-induced change of stiffness
(e.g. Kapitsa's pendulum)

Biasing

Change of static equilibrium position
(e.g. Kapitsa's pendulum, Brumberg's pipe)

Smoothening

Smoothening discontinuities
(e.g.dry friction into "viscous friction")

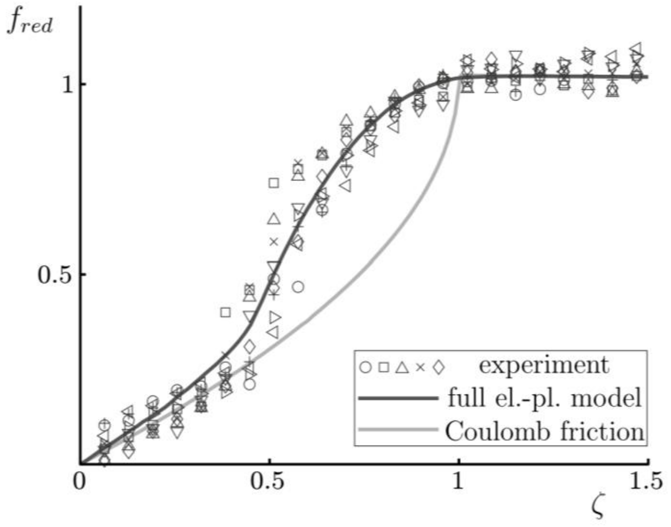


https://www.youtube.com/watch?v=RJid-eCtLdQ

Intuitive example of smoothening: transforming dry friction into “viscous friction”

Experiments date back to the late ’60s, however, compared to other areas, it is still a niche topic: not so many experiments available (often very ideal ones) to fully understand the mechanisms at place.

Kapelke, S. et al (2018)

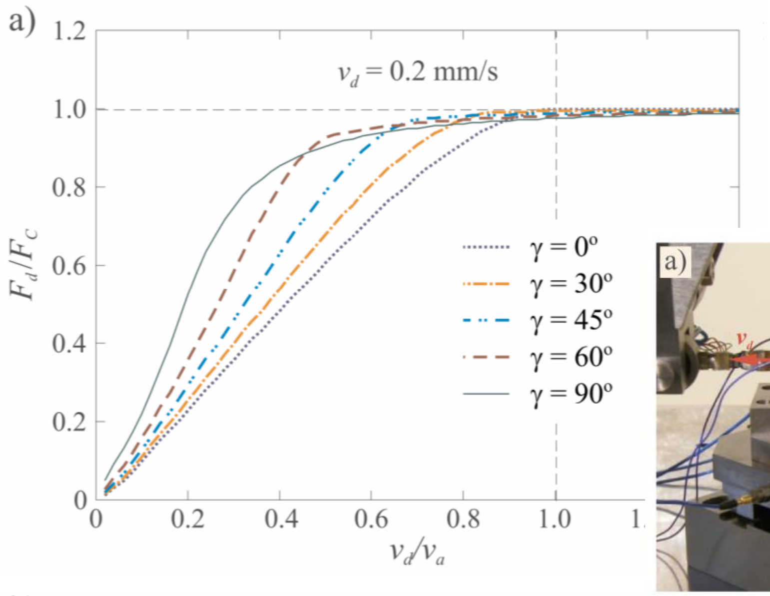


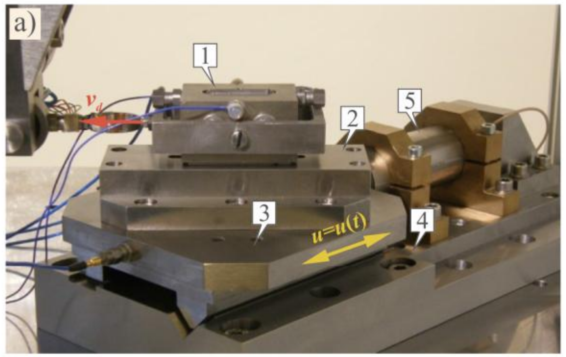
$$f_{red} = \frac{\langle F_{exc} \rangle}{\langle F_{no\_exc} \rangle}$$

$$\zeta = \frac{v_0}{a\omega}$$

Image taken from: Kapelke, S. et al, **On the effect of longitudinal vibrations on dry friction**, Tribology Letters (2018) 66:79, https://doi.org/10.1007/s11249-018-1031-0

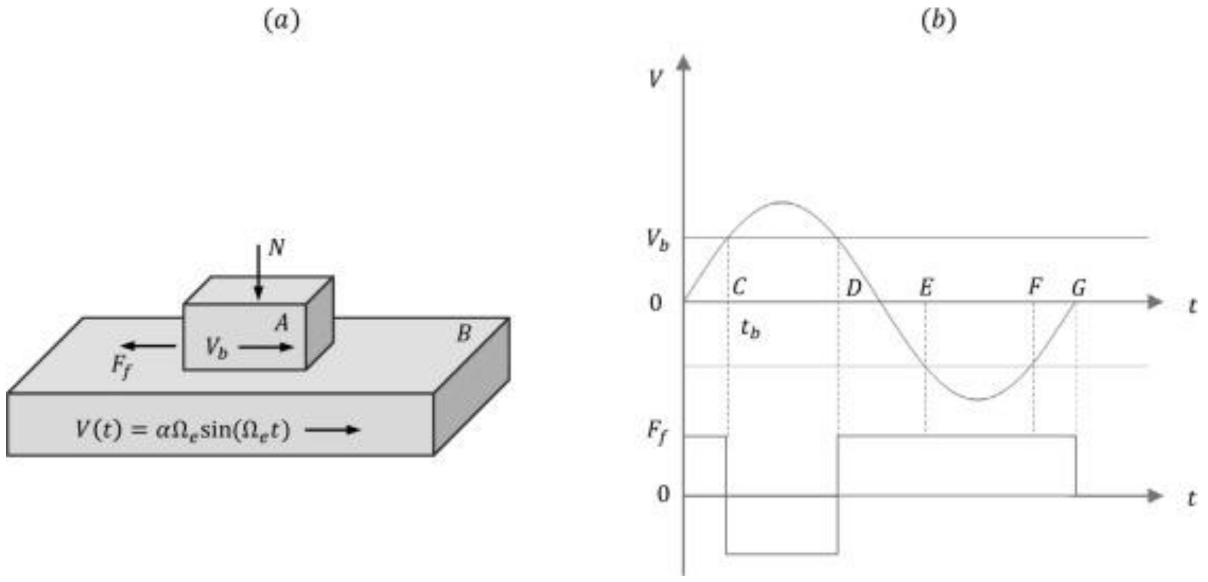
Images taken from: Gutowski, P., Leus, M. **Computational model of friction force reduction at arbitrary direction of tangential vibrations and its experimental verification**, Tribology International 143 (2020) 106065.





Intuitive example of smoothening: transforming dry friction into "viscous friction"

Case of longitudinal (tangential) excitation



Images taken from: Sulollari, E., Van Dalen, K., Cabboi, A. **Vibration-induced friction modulation for a general frequency of excitation**, Journal of Sound and Vibration, 573,118200, (2024)

- ✓ The reasoning here is to look after the averaged friction over one cycle (period T of the fast applied excitation)

$$F_{av} = \left\langle F_{F,exc} \right\rangle$$

- ✓ What's is the area equal to?

t_b is the time at which the velocity of the fast excitation equals the pulling velocity V_b

$$a\Omega_e \sin\left(\Omega_e t\right) = V_b \quad \Rightarrow \quad t_b = \frac{1}{\Omega_e}\sin^{-1}\left(\frac{V_b}{a\Omega_e}\right)$$

The area under CD cancels with the area under EF. What is left is:

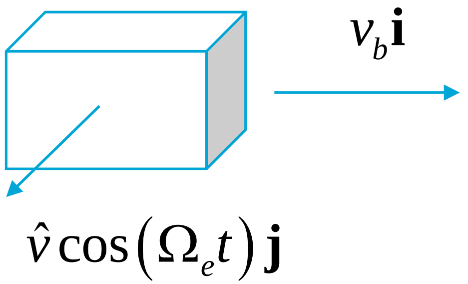
$$F_{av} = F_F 4\frac{t_b}{T} \quad \Rightarrow \quad \boxed{F_{av} = F_F \frac{2}{\pi}\sin^{-1}\left(\frac{V_b}{a\Omega_e}\right)}$$

Intuitive example of smoothening: transforming dry friction into "viscous friction"

Case of transversal excitation

$$\mathbf{v} = v_b\mathbf{i} + \hat{v}\cos\left(\Omega_e t\right)\mathbf{j}$$

$$F_F = -\mu N\frac{\mathbf{v}}{|\mathbf{v}|} = F_{F,x}\mathbf{i} + F_{F,y}\mathbf{j}$$



The component of the friction force in direction of the macroscopic movement (x-direction) is averaged over one period of vibration:

$$F_{F,x,av} = \frac{1}{T}\int_0^T F_{F,x}\left(t\right)\mathrm{d}t$$

$$F_{F,x} = -\frac{\mu N v_b}{\sqrt{v_b^2 + \left(\hat{v}\cos\left(\Omega_e t\right)\right)^2}}$$

$$\zeta = \frac{v_b}{\hat{v}}$$

$$F_{F,x} = -\frac{\mu N\zeta}{\sqrt{\zeta^2 + \left(\cos\left(\Omega_e t\right)\right)^2}}$$

$$\tau = \Omega_e t \qquad \mathrm{d}\tau = \Omega_e \mathrm{d}t$$

$$F_{F,x,av} = -\mu N\frac{1}{2\pi}\int_0^{2\pi}\frac{\zeta}{\sqrt{\zeta^2 + \left(\cos\left(\tau\right)\right)^2}}\mathrm{d}\tau$$

Use of elliptical integral identity:

$$\boxed{F_{F,x,av}\left(\zeta\right) = -\mu N\frac{2}{\pi}\frac{1}{\sqrt{1+\frac{1}{\zeta^2}}}K\left(\frac{1}{1+\zeta^2}\right)\mathrm{sign}\left(\zeta\right)}$$

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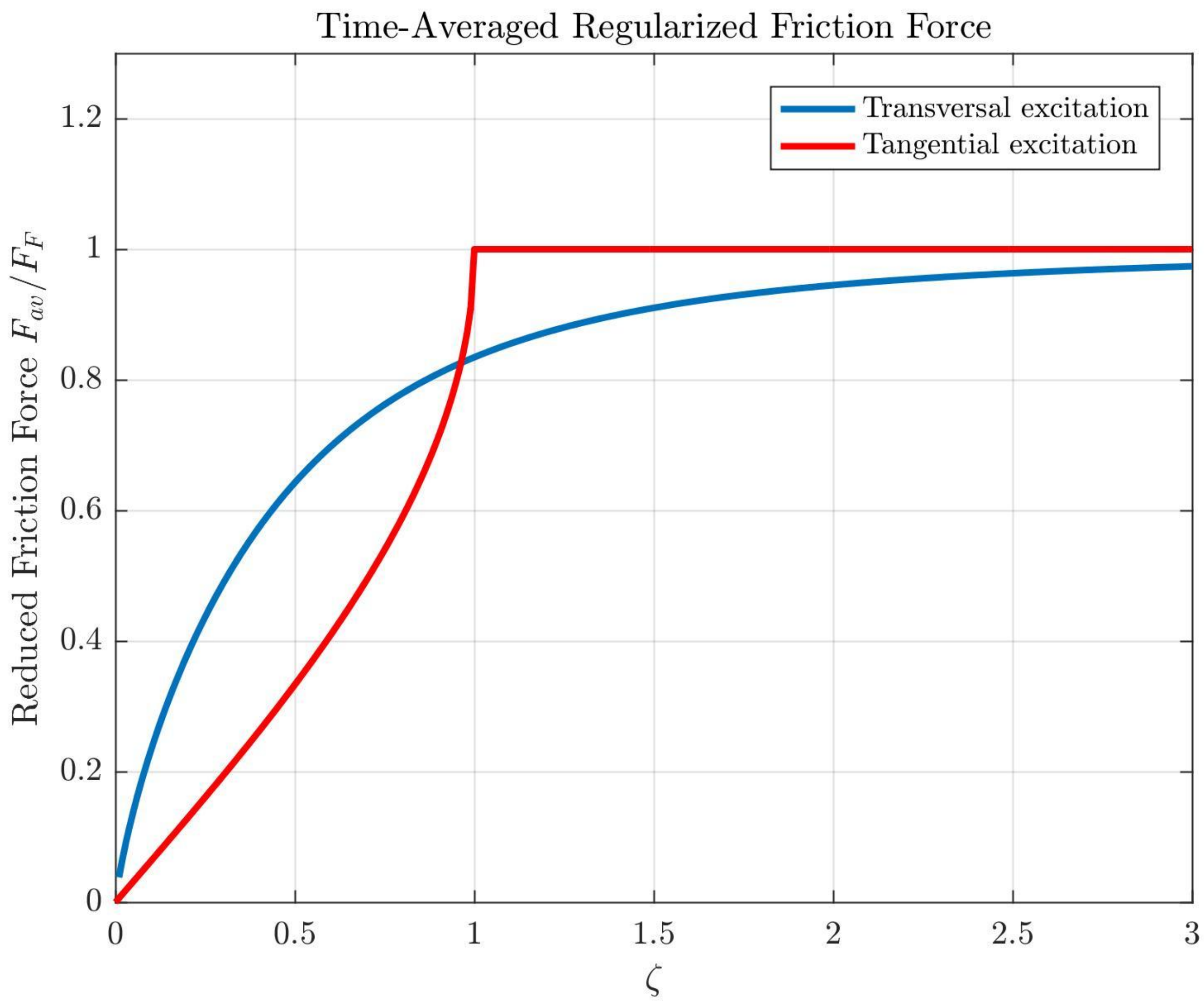
Intuitive example of smoothening: transforming dry friction into "viscous friction"

Case of tangential excitation

$$\frac{F_{av}}{F_F} = \begin{cases} 1 \quad \text{for} \quad \zeta \geq 1 \\ \dfrac{2}{\pi}\sin^{-1}(\zeta) \quad \text{for} \quad 0 \leq \zeta < 1 \end{cases}$$

Case of transversal excitation

$$\frac{F_{av}}{F_F} = \frac{2}{\pi}\frac{1}{\sqrt{1+\dfrac{1}{\zeta^2}}}K\left(\frac{1}{1+\zeta^2}\right)\text{sign}(\zeta)$$



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Since we use an additional vibration, are we consuming more energy? In general, is there a benefit ?

The work done by a friction force is equal to (it corresponds to the dissipated energy):

$$U = \int F_F(x)\mathrm{d}x$$

In case of applied vibration, the argument becomes:

$$U = \int_0^T F_F \operatorname{sign}(v_{rel}) v_{rel} \mathrm{d}t \qquad v_{rel} = v_b - \hat{v}\cos(\Omega_e t)$$

Let's adimensionalize this through

$$\zeta = \frac{v_b}{\hat{v}} \qquad \tau = \Omega_e t \qquad \mathrm{d}\tau = \Omega_e \mathrm{d}t$$

$$U = \frac{1}{\Omega_e}\int_0^{2\pi} F_F \operatorname{sign}(\zeta - \cos(\tau))\hat{v}(\zeta - \cos(\tau))\mathrm{d}\tau$$

This integral is a bit tedious because of the sign term and change of sign depending on ζ, and it has to be solved for two conditions:

if $\zeta \geq 1 \Rightarrow U = F_F \dfrac{2\pi\hat{v}\zeta}{\Omega_e}$

if $0 \leq \zeta < 1 \Rightarrow U = F_F \dfrac{4\hat{v}}{\Omega_e}\left(\zeta \sin^{-1}(\zeta) + \sqrt{1-\zeta^2}\right)$

In case without vibration:

$$U_{no\_vib} = F_F v_b T = F_F \frac{2\pi\hat{v}\zeta}{\Omega_e}$$

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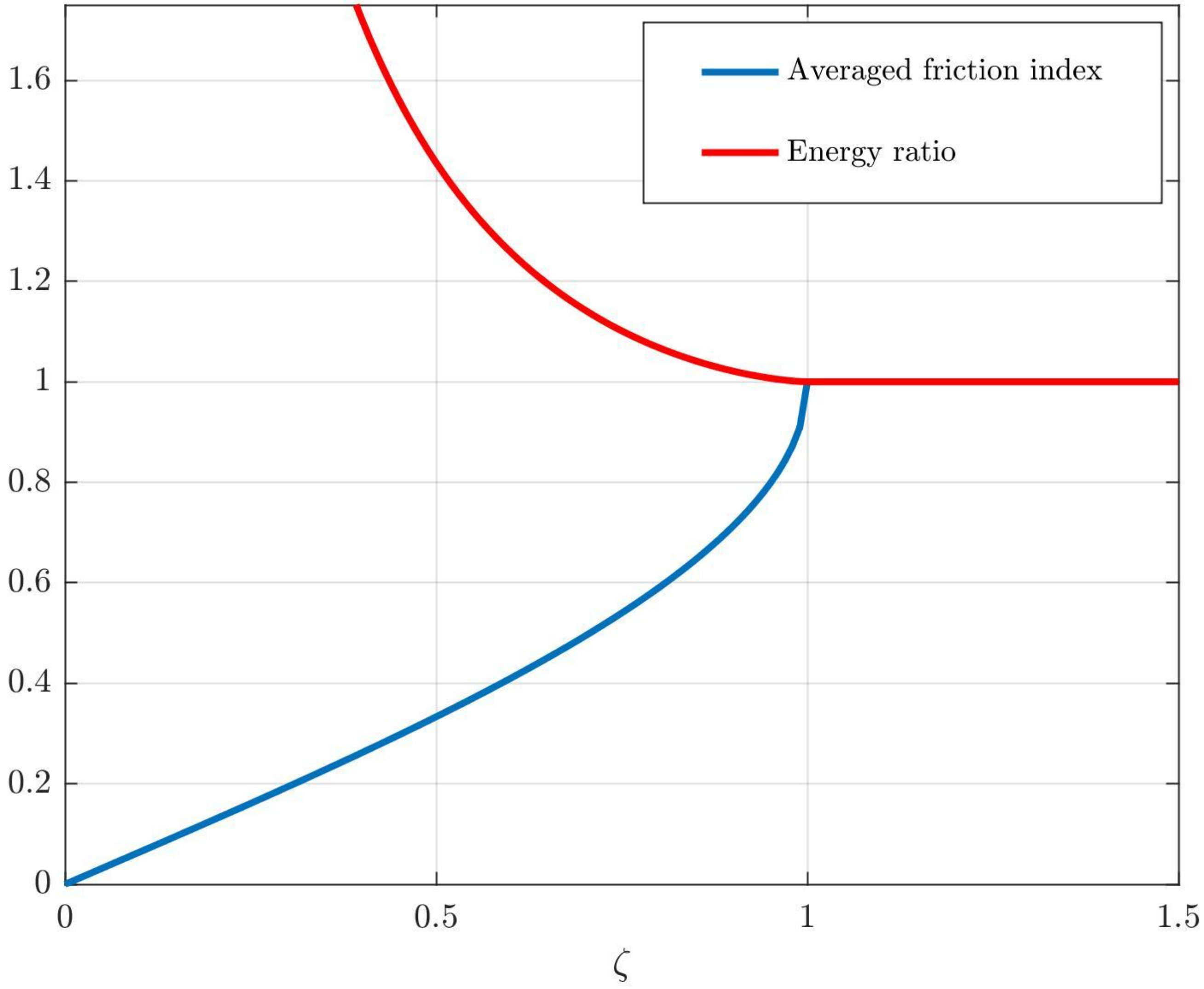
Since we use an additional vibration, are we consuming more energy? In general, is there a benefit ?

$$\varepsilon = \frac{U}{U_{no\_vib}}$$

$$\zeta \geq 1 \Rightarrow \varepsilon = 1$$

$$0 \leq \zeta < 1$$

$$\varepsilon = \frac{2}{\pi}\left(\sin^{-1}(\zeta) + \sqrt{\frac{1}{\zeta^2} - 1}\right)$$



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Where do we go from this point on?

Possible benefits from this transformation of dry (discontinuous) friction to a regularized viscous friction:

- ❑ Reduction or control of the friction force
- ❑ It can mitigate/eliminate stick-slip, chaos and friction-induced instability

Note: if such external vibration is not accounted for, and some friction force measurements are taken, there is a risk to misinterpret the measured friction force (more on this in lecture 3)

Next steps: a high frequency was mentioned, but what is meant by high? Can we generalize the outcome to a general class of dynamic systems?

Introduction to the Method of Direct Separation of Motion

General class of systems

$$m\ddot{x} = F(\dot{x}, x, t) + \Phi(\dot{x}, x, t, \omega t)$$

We assume a solution of the following type

$$x = \underbrace{X(t)}_{\text{Slow motion}} + \underbrace{\psi(t,\tau)}_{\text{Fast motion}}$$

- The method of the direct separation of motions was originated by Kapitsa in the ’60s, and the general formulation of this method was given later by Blekhman.

- The main idea here is to to find another system, which solution is close to the solutions of the original system.

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