Design and Tuning of Dampers and Vibration Absorbers

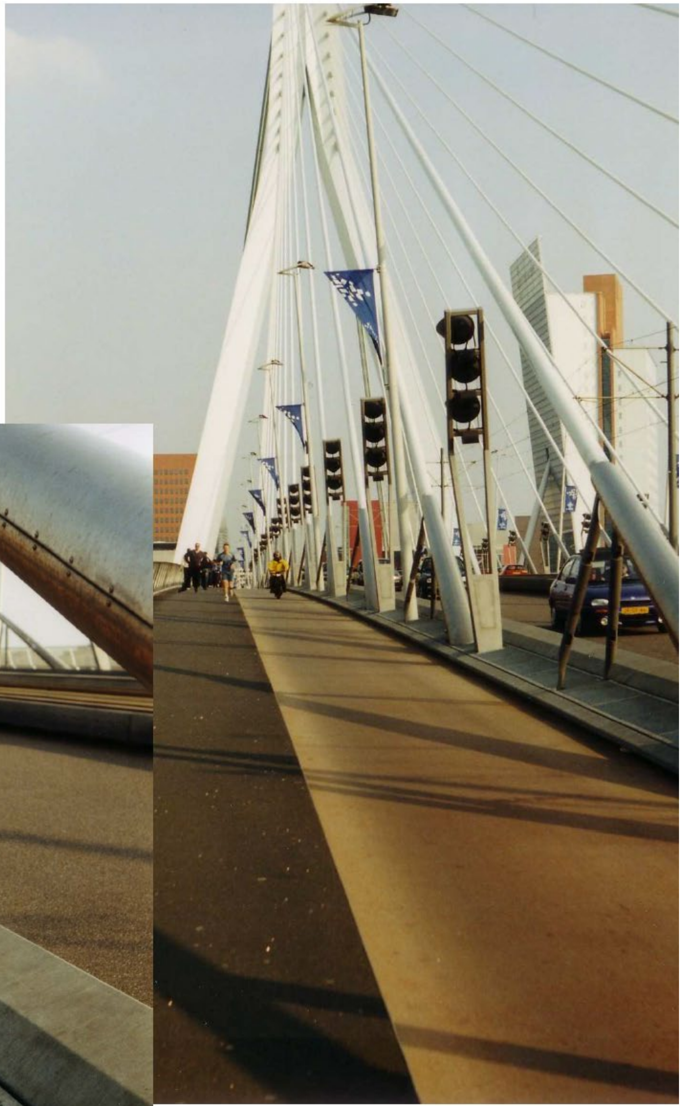
Jan Høgsberg

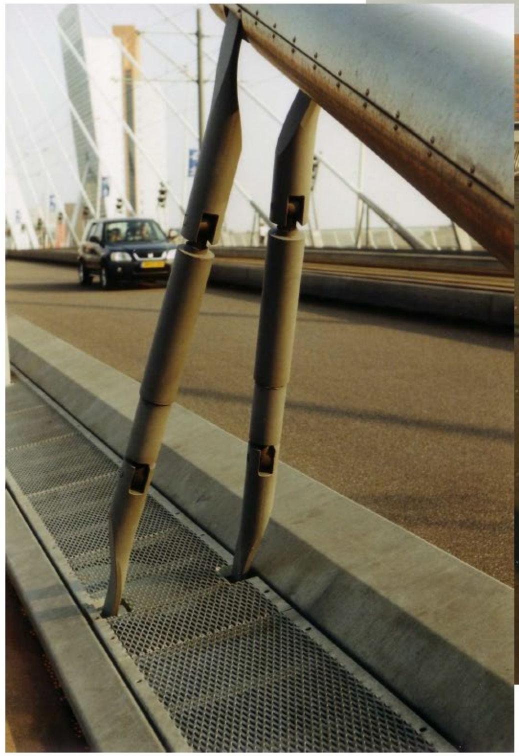
Technical University of Denmark

Local dampers on flexible structures

Cable vibrations

Erasmus Bridge

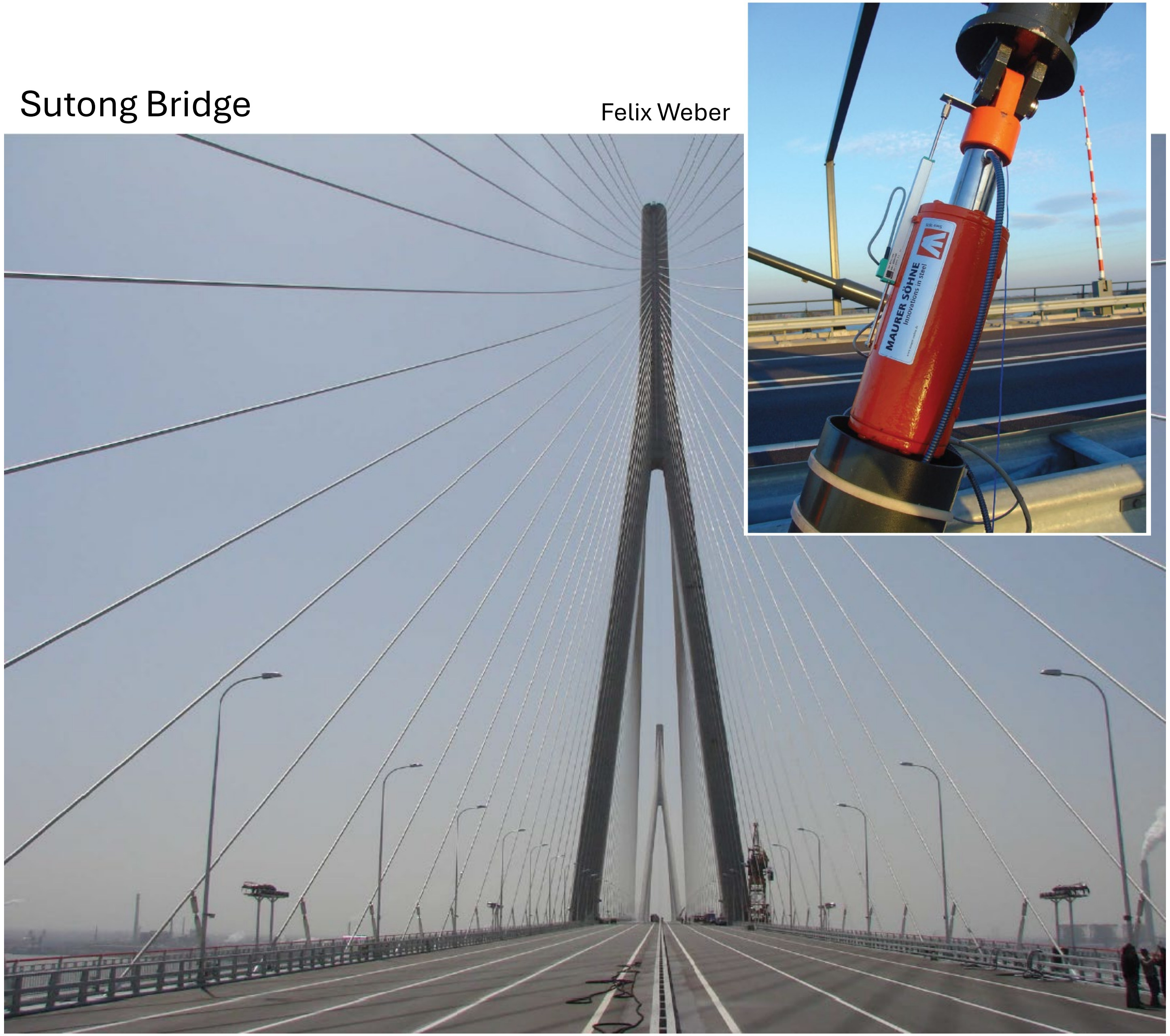




Sutong Bridge

Felix Weber

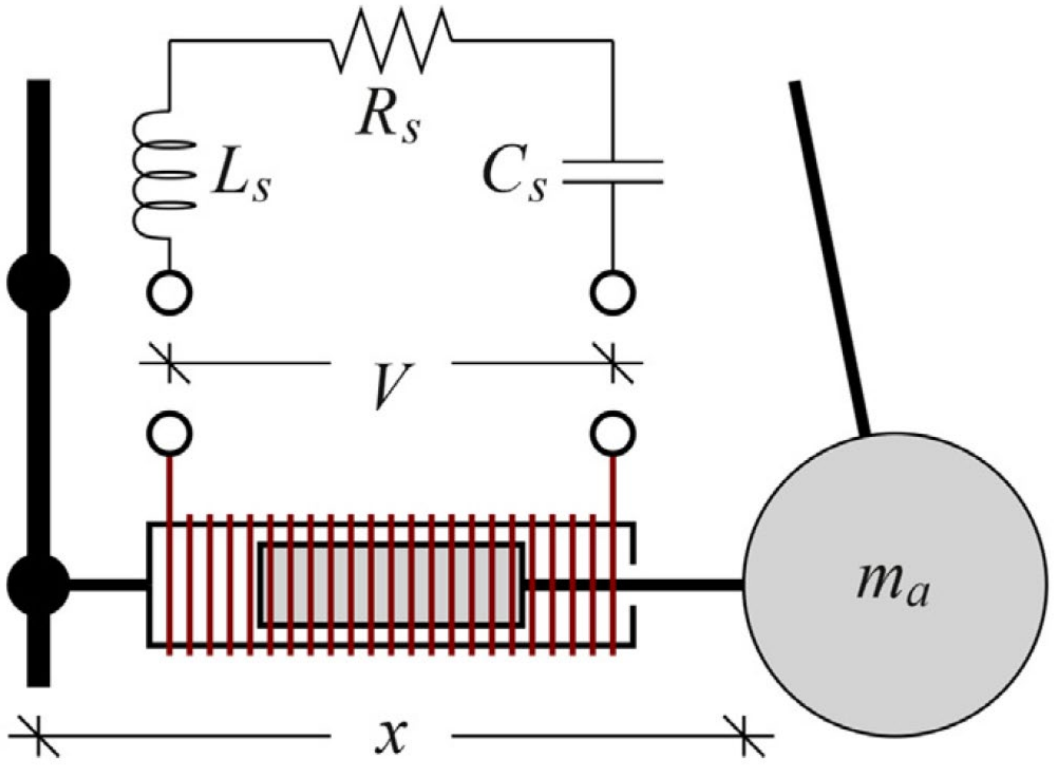
Eiland Bridge



Local dampers on flexible structures

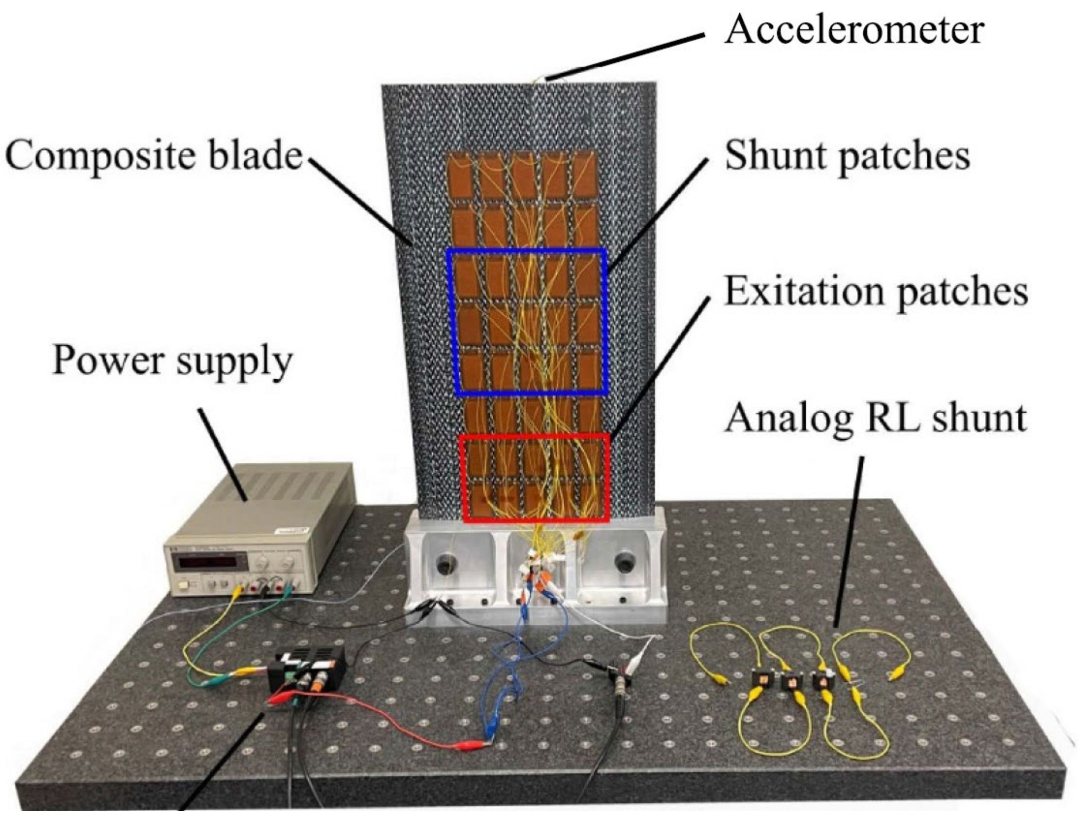
Smart Structures Technology

Shunted Electromagnets



Larsen, Zhang & Høgsberg

Shunted Piezo-electrics

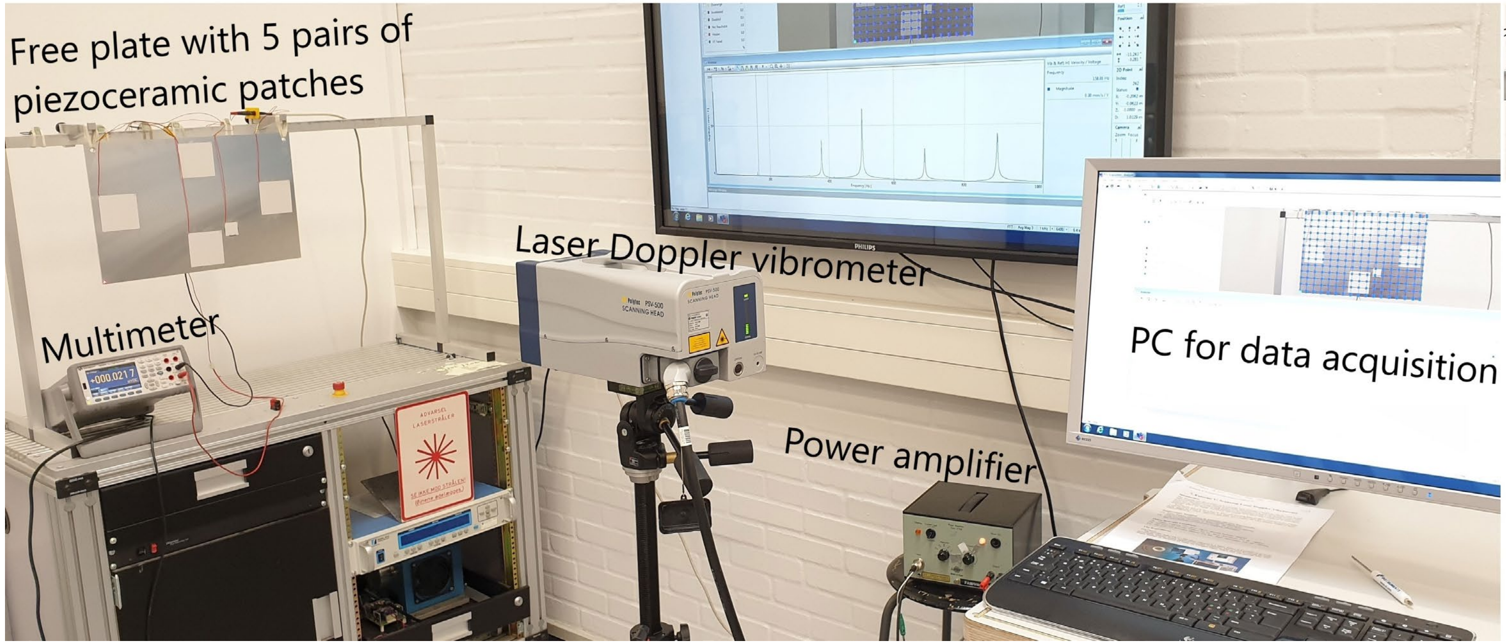


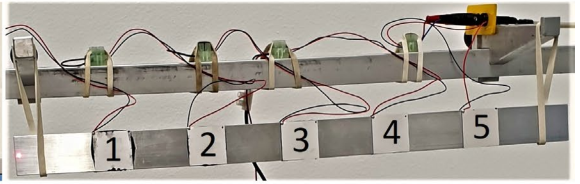
Richardt, Lossouarn, Høgsberg & Deü

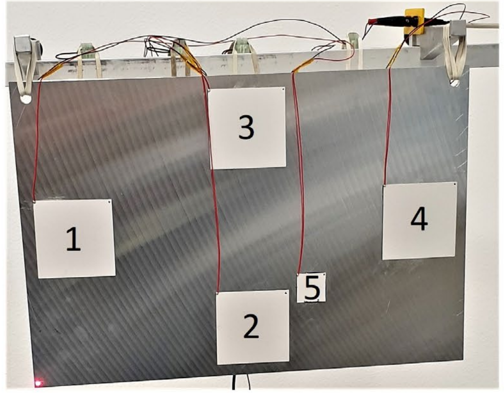
Local dampers on flexible structures

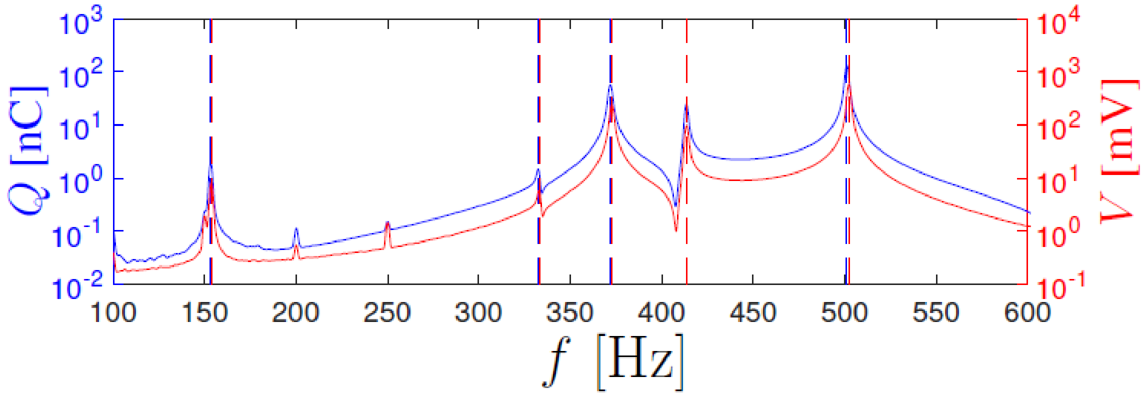
Smart Structures Technology

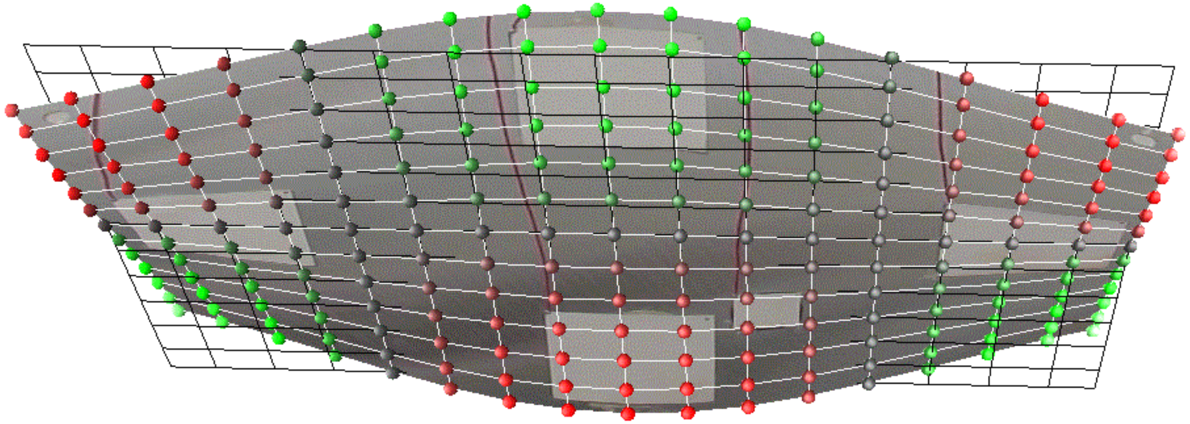
Toftekær & Høgsberg











Local dampers on flexible structures

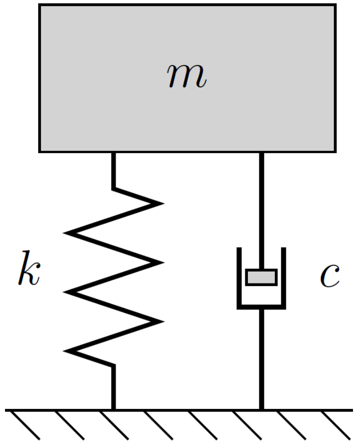
Smart Structures Technology

Toftekær & Høgsberg

Local dampers on flexible structures

The SDOF model

Single degree-of-freedom (SDOF) structure:



- *m, k, c* = modal mass, stiffness, damping
- Mode shape normalized to ?
- Modal damping *c = 0* in tuning!

What approximations are made?

... and how valid are they?

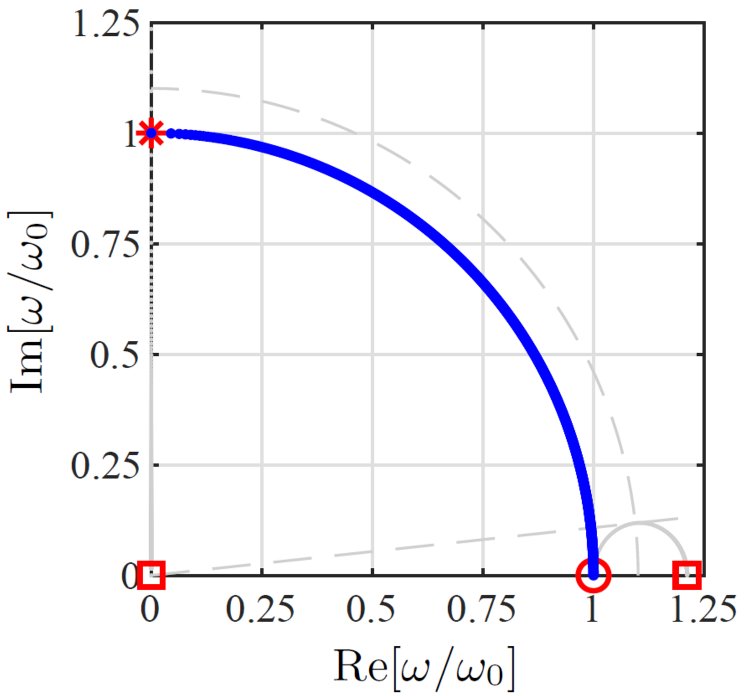
Complex natural (angular) frequency:

$$\omega = \omega_0 \left(i\zeta \pm \sqrt{1-\zeta^2} \right)$$

Damping ratio:

$$\zeta = \frac{c}{2\sqrt{km}} = \frac{c}{2m\omega_0}$$

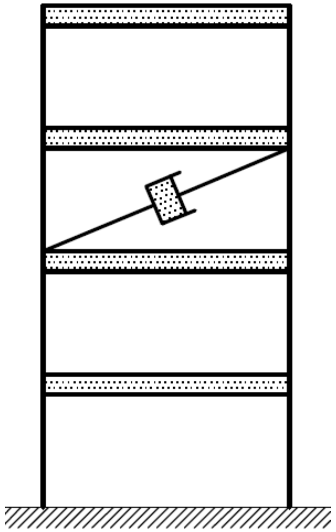
Complex root locus for increasing *c*:



Local dampers on flexible structures

The MDOF model

Multi degree-of-freedom (SDOF) structure:



Equation of motion in frequency domain:

$$\left(-\omega^2\mathbf{M}+\mathbf{K}\right)\mathbf{q}+\mathbf{w}f=\mathbf{f}_{\mathrm{ext}}$$

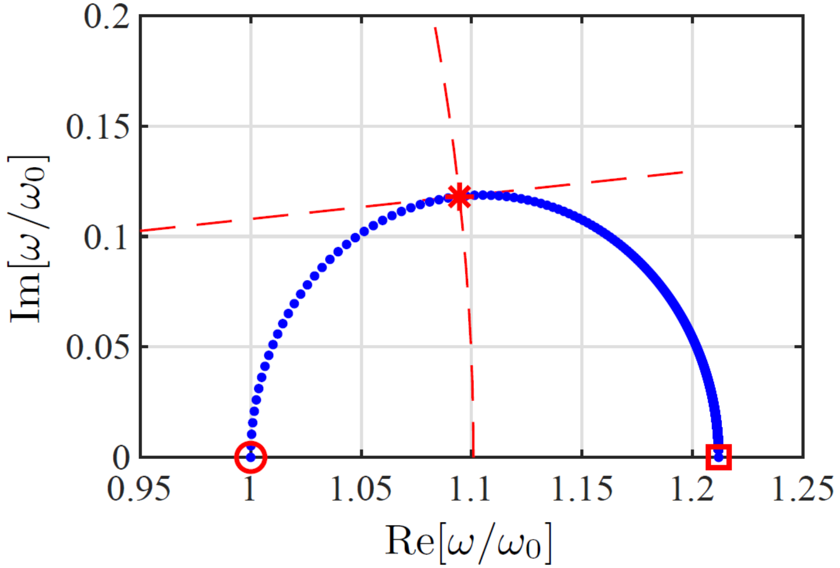
Damper motion by connectivity vector:

$$u=\mathbf{w}^T\mathbf{q}$$

Viscous damper force:

$$f \;=\; i\omega c u \;=\; i\omega c\,\mathbf{w}^T\mathbf{q}$$

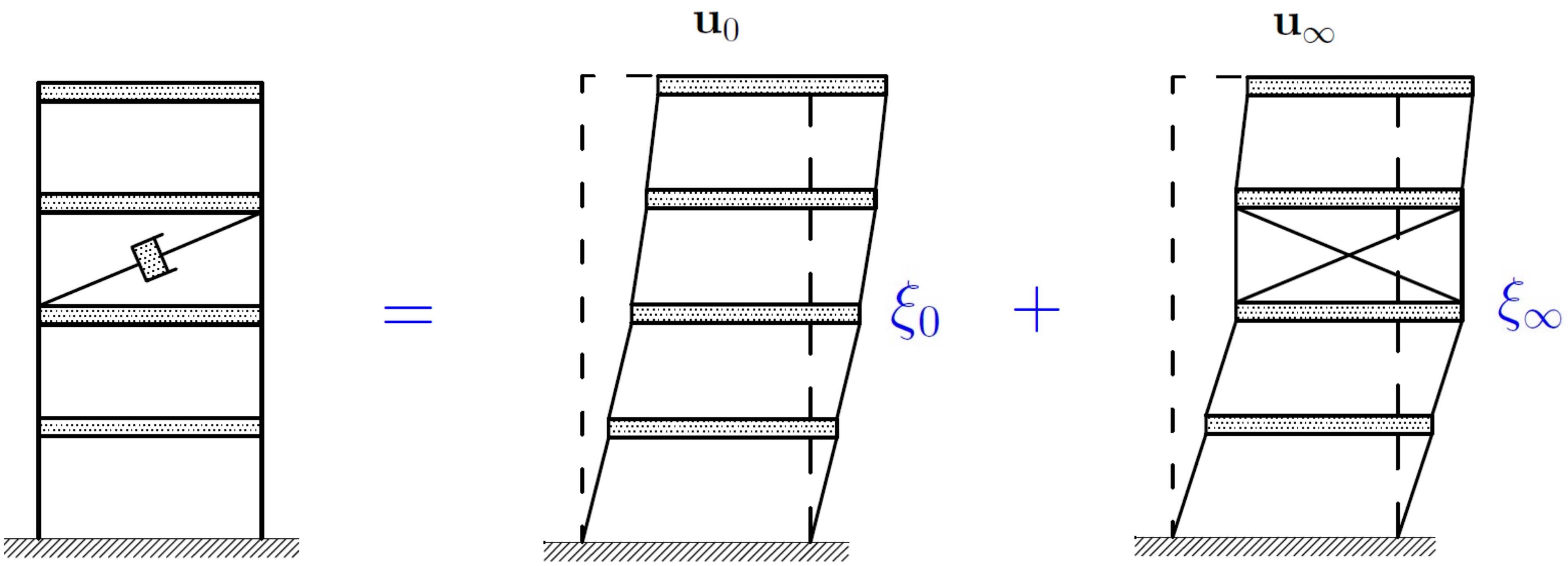
Complex root locus for increasing *c:*



- Viscous damping => Roots along *half-circles*
- Maximum damping = crossing with quarter circle (*).
- Max damping → Difference btw limit freqs → ○ to □

Local dampers on flexible structures

Two-component expansion



MDOF to 2-DOF system reduction:

$$\begin{bmatrix} \omega_0^2 - \omega^2 + H(\omega) & \kappa(\omega_0^2 - \omega^2) \\ \kappa(\omega_0^2 - \omega^2) & \omega_\infty^2 - \omega^2 \end{bmatrix} \begin{bmatrix} \xi_0 \\ \xi_\infty \end{bmatrix} = \begin{bmatrix} f_0 \\ f_\infty \end{bmatrix}$$

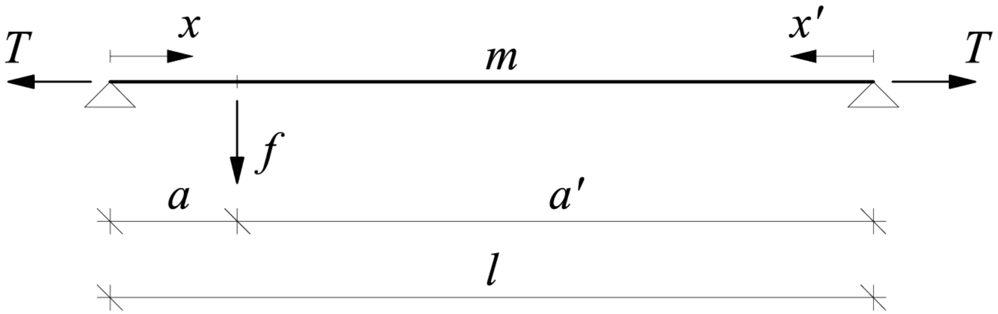
Optimal tuning:

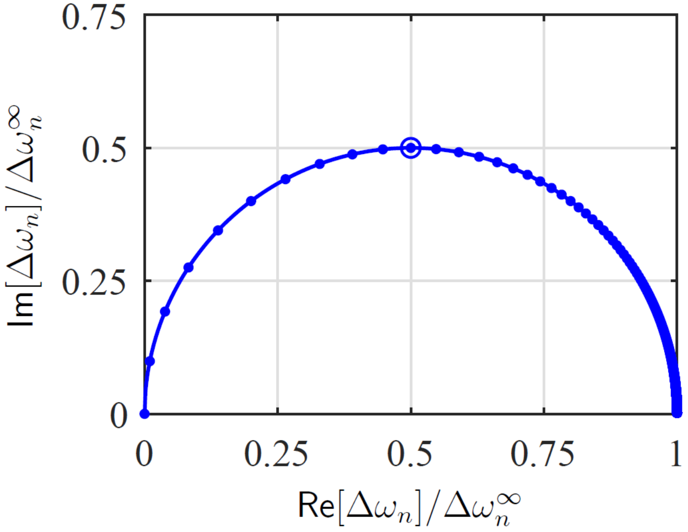
$$c^{\mathrm{opt}} = \frac{2(\omega_\infty - \omega_0)}{u^2}$$

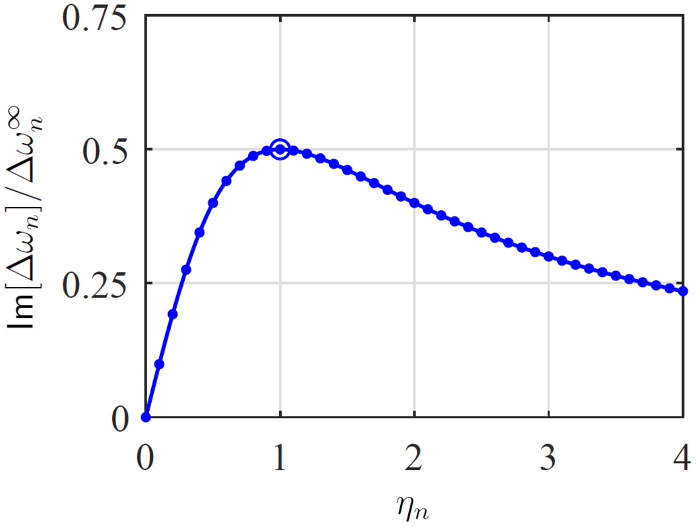
$$\zeta^{\max} \simeq \frac{\omega_\infty - \omega_0}{\omega_\infty + \omega_0}$$

Local dampers on flexible structures

Cable damper







$$\zeta_n^{\max} \simeq \frac{\Delta\omega_n^\infty}{2\,\omega_n^0} = \frac{1}{2}\,\frac{a}{\ell} \quad , \quad c^{\mathrm{opt}} \simeq \frac{T}{\omega_n^0 a}$$

Local dampers on flexible structures

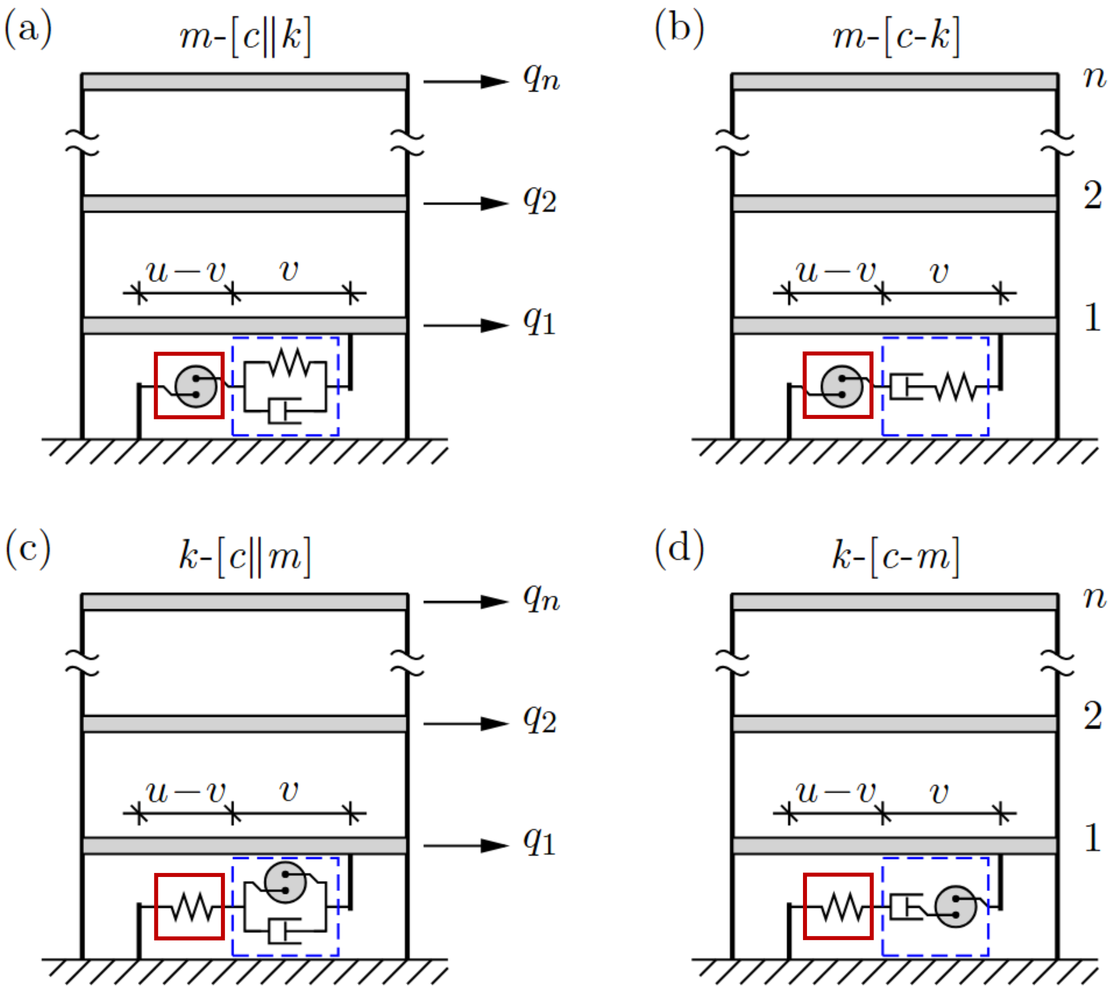
Cable damper

Optimal viscous damping

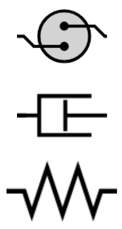
General absorber models

Inertia and stiffness absorbers

Absorbers installed in shear frame model



Elements:



- Inerter — Inertance *m*
- Dashpot — Viscous coeff *c*
- Spring — Stiffness *k*

Absorber force:

$$f = \boxed{D(\omega)v} = \boxed{G(\omega)(u-v)}$$

- Inertia-absorber: $m\text{-}[c\ldots]$
- Stiffness-absorber: $k\text{-}[c\ldots]$

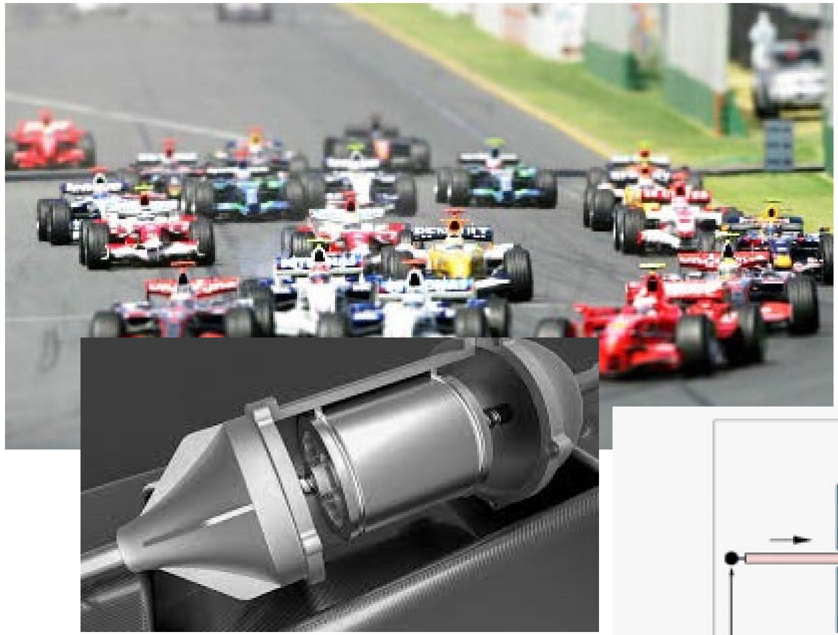
Equation of motion with *v* as absorber variable:

$$\left(-\omega^2\mathbf{M} + \mathbf{K} + G(\omega)\mathbf{w}\mathbf{w}^T\right)\mathbf{q} - G(\omega)\mathbf{w}v = \mathbf{f}_{\text{ext}}$$

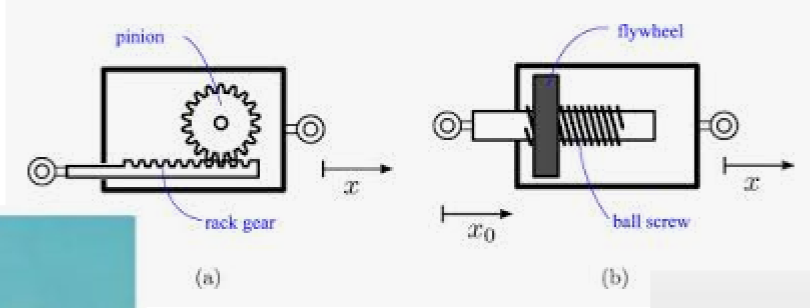
General absorber models

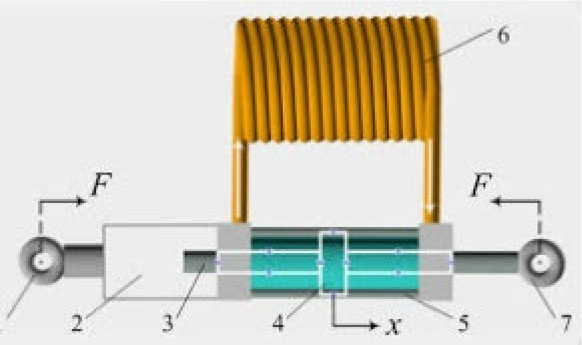
Inerters – the missing link

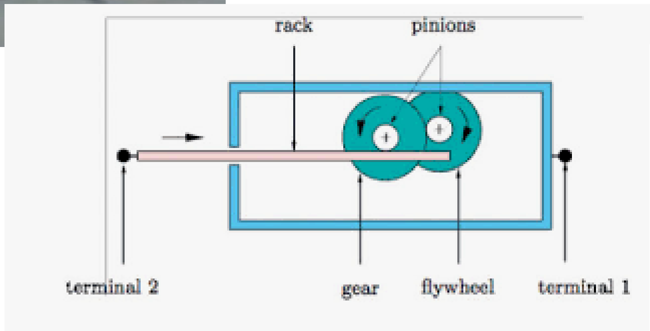
Inerter (J-damper) invented for F1 suspension



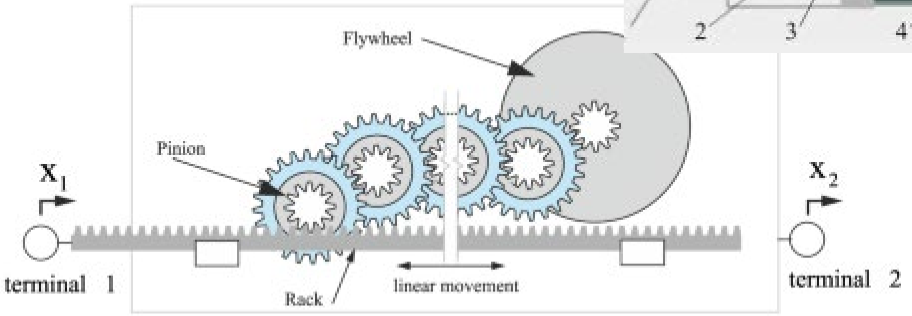
Translation into Rotation

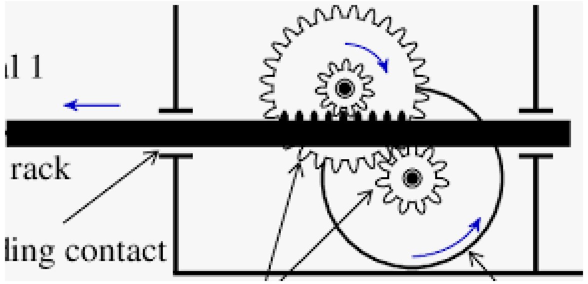




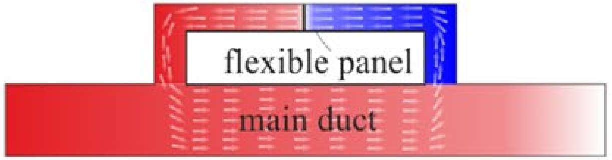


Large inertia at low mass

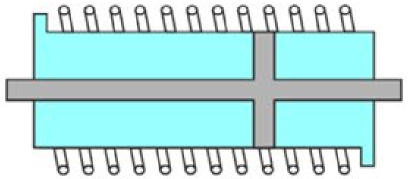




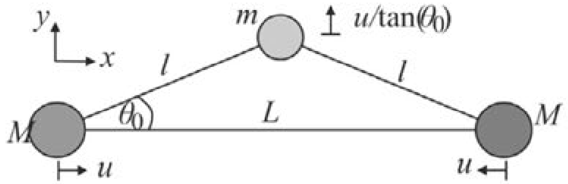
acoustic inerter



fluid inerter

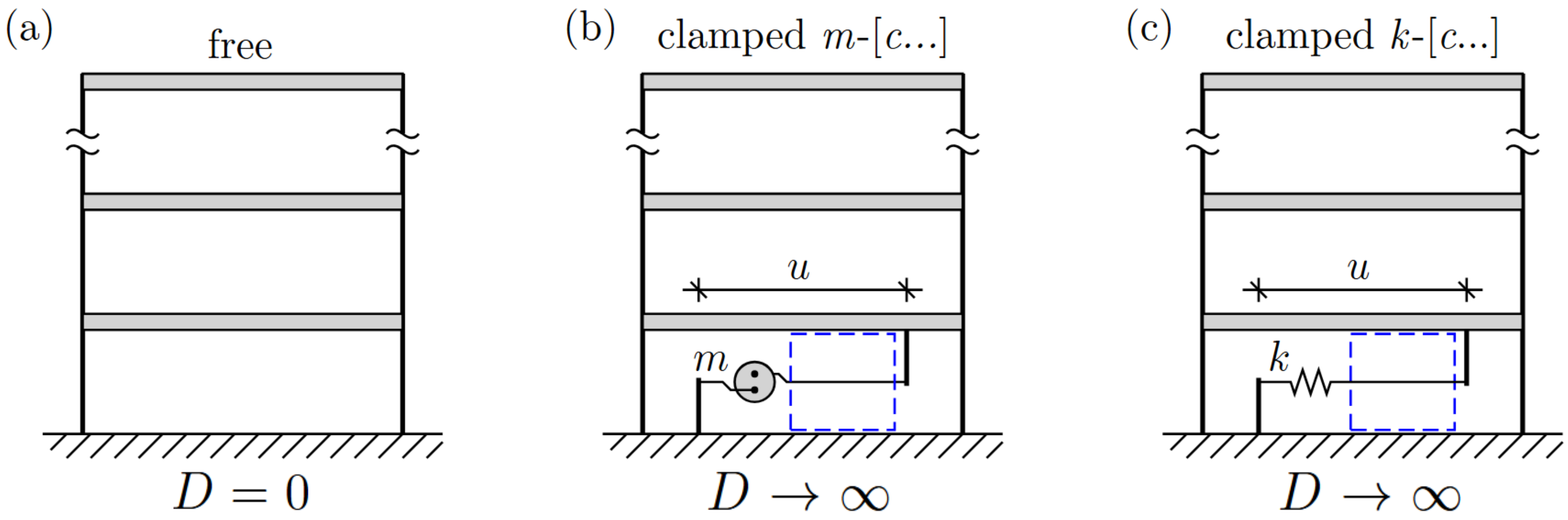


mechanical inerter



General absorber models

Limiting eigenvalue problems



Free EVP for $v = u$:

$$\left(\mathbf{K} - \omega_j^2 \mathbf{M}\right) \mathbf{u}_j = \mathbf{0}$$

Clamped EVP for $v = 0$:

$$\left(\mathbf{K} + G(\bar{\omega}_j)\mathbf{w}\mathbf{w}^T - \bar{\omega}_j^2 \mathbf{M}\right) \bar{\mathbf{u}}_j = \mathbf{0}$$

- Inertia-absorber: m-$[c...]$ $\quad \left(\mathbf{K} - \bar{\omega}_j^2 \left[\mathbf{M} + m\mathbf{w}\mathbf{w}^T\right]\right) \bar{\mathbf{u}}_j = \mathbf{0}$
- Stiffness-absorber: k-$[c...]$ $\quad \left(\mathbf{K} + k\mathbf{w}\mathbf{w}^T - \bar{\omega}_j^2 \mathbf{M}\right) \bar{\mathbf{u}}_j = \mathbf{0}$

Modal equations

Free-mode expansion

Modal representation by free modes:

$$\mathbf{q} = \sum_j \mathbf{u}_j r_j$$

Normalization to unit absorber displacement:

$$\mathbf{w}^T \mathbf{u}_j = 1$$

Modal structural equation:

$$\left(k_j - \omega^2 m_j\right) r_j + f = f_j$$

Modal mass and stiffness:

$$m_j = \mathbf{u}_j^T \mathbf{M} \mathbf{u}_j \qquad k_j = \mathbf{u}_j^T \mathbf{K} \mathbf{u}_j$$

... and load: $f_j = \mathbf{u}_j^T \mathbf{f}_{\text{ext}}$

Aorber force equation:

$$f = G(\omega)\,(u - v) = G(\omega)\mathbf{w}^T\mathbf{q} - G(\omega)v$$

Modal representation of absorber displacement:

$$\mathbf{w}^T\mathbf{q} = \sum_j \mathbf{w}^T\mathbf{u}_j r_j = \sum_j r_j$$

Split into target mode *j* = *s* and rest (residual modes):

$$\mathbf{w}^T\mathbf{q} = r_s + \sum_{j\neq s} r_j = r_s - \underbrace{\sum_{j\neq s} \frac{1}{k_j - \omega_j^2 m_s}}_{1/R_s(\omega)} f$$

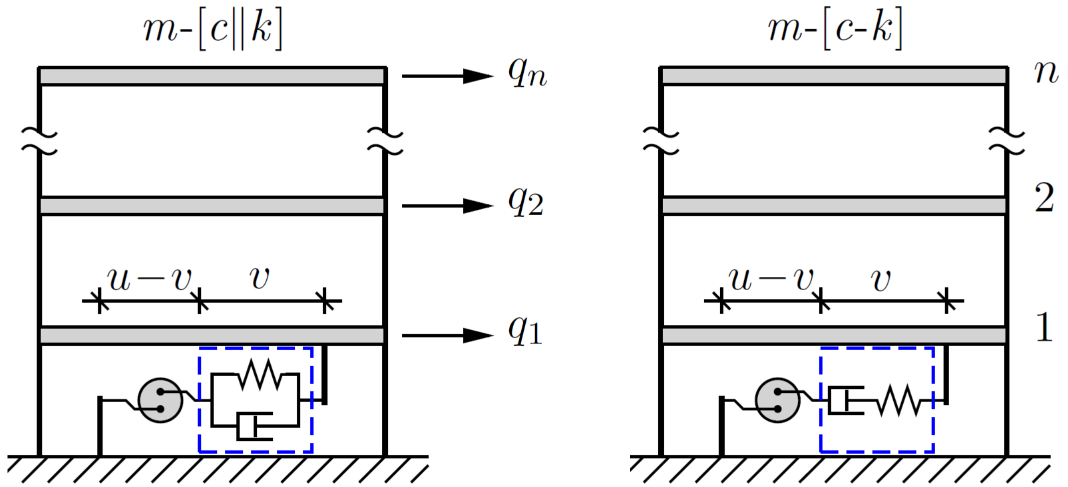
Modal absorber force equation:

$$G(\omega)v - G(\omega)r_s + \left(1 + \frac{G(\omega)}{R_s(\omega)}\right) f = 0$$

Modal equations

Effective Modal Coupling Factor (EMCF)

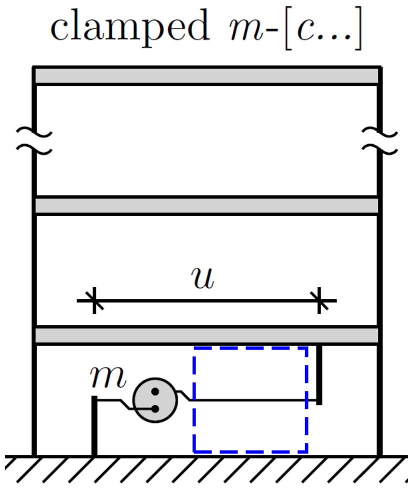
<u>Inertia</u> absorber function: $G(\omega) = -\omega^2 m$



Modal absorber force equation:

$$-\omega^2 m v + \omega^2 m r_s + \left(1 - \frac{\omega^2 m}{R_s(\omega)}\right) f = 0$$

<u>Clamped limit *v = 0*</u>



Single frequency: $\omega^2 = \bar{\omega}_s^2$

Characteristic equation for *v = 0*:

$$\omega^4 \frac{m_s m}{R_s(\omega)} - \omega^2 \left(m_s + m + \frac{k_s m}{R_s(\omega)} \right) + k_s = 0$$

Single solution requires *R* = inertia: $R_s(\omega) \simeq -\omega^2 m_s'$

Residual mode mass ratio:

$$\frac{m}{m_s'} = \frac{\mu}{\mu_*} - 1$$

Corrected absorber force equation:

$$-\omega^2 m \frac{\mu_*}{\mu} v + f + \omega^2 m \frac{\mu_*}{\mu} r_s = 0$$

EMCF replaces mass ratio !

Mass ratio:

$$\mu = \frac{m}{m_s}$$

EMCF:

$$\mu_* = \frac{\omega_s^2 - \bar{\omega}_s^2}{\bar{\omega}_s^2}$$

Modal equations

Corrected SDOF model

<u>Inertia</u> absorber function: $G(\omega) = -\omega^2 m$

Structural equation for target mode $j = s$:

$$\left(k_s - \omega_s^2 m_s\right) r_s + f = f_s$$

Modal inertia-absorber equation:

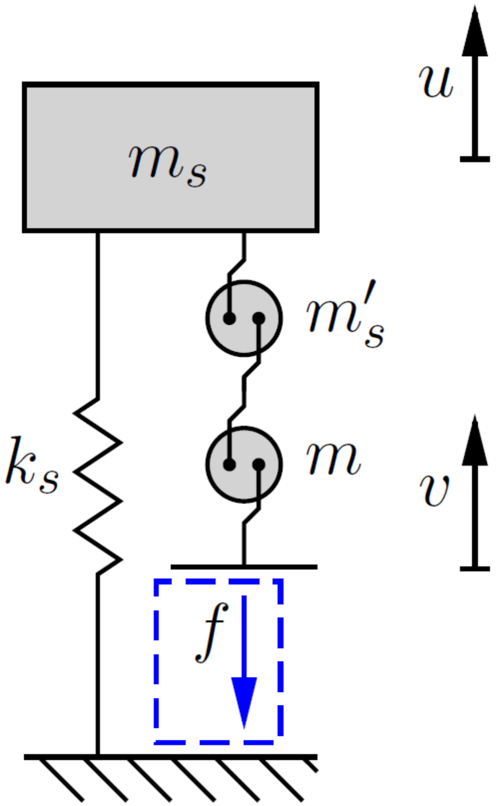
$$-\omega^2 m \frac{\mu_*}{\mu} v + f + \omega^2 m \frac{\mu_*}{\mu} r_s = 0$$

Damper function: $f = D(\omega) v$

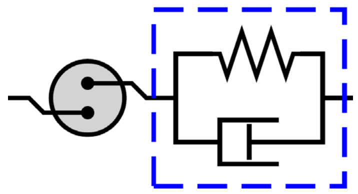
Modal frequency response function (FRF):

$$\frac{r_s k_s}{f_s} = \frac{-\dfrac{\omega^2}{\omega_s^2}\mu_* + D(\omega)/k_s}{\left(1 - \dfrac{\omega^2}{\omega_s^2}\right)\left(-\dfrac{\omega^2}{\omega_s^2}\mu_* + D(\omega)/k_s\right) - \dfrac{\omega^2}{\omega_s^2}\mu_* D(\omega)/k_s}$$

Corrected SDOF model

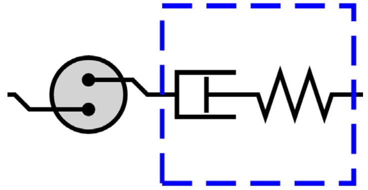


<u>Parallel</u> m-$[c \| k]$



$$D(\omega) = k + i\omega c$$

<u>Series</u> m-$[c$-$k]$

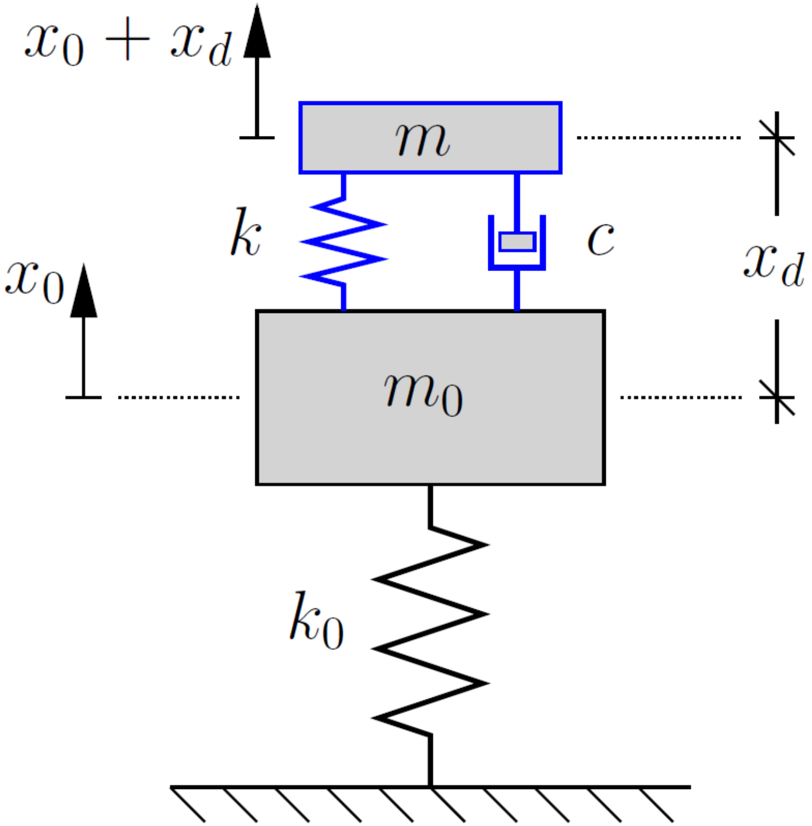


$$\frac{1}{D(\omega)} = \frac{1}{k} + \frac{1}{i\omega c}$$

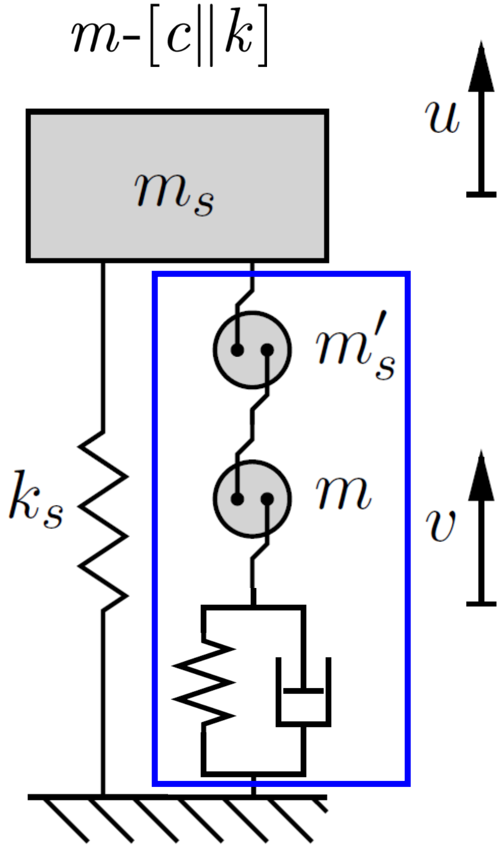
Absorber tuning

TID vs. TMD

Tuned Mass Damper (TMD)



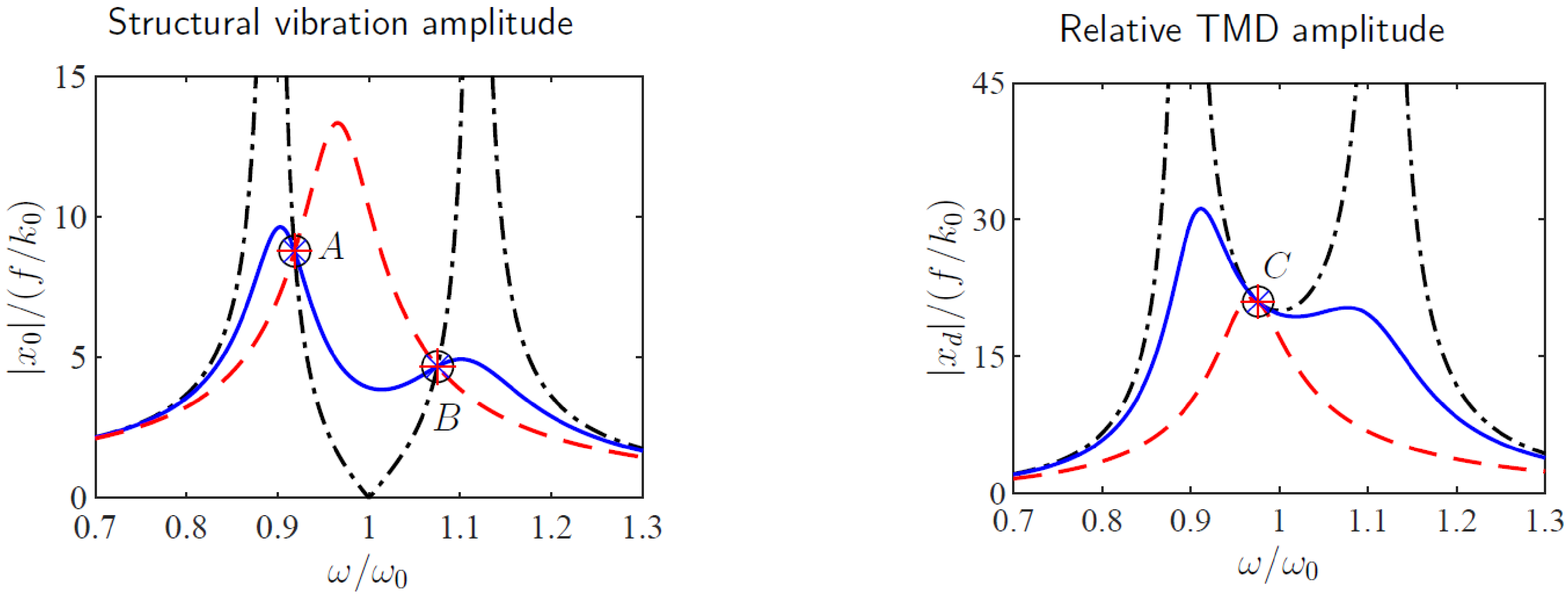
Tuned Inerter Damper (TID)



Absorber tuning

Fixed-point tuning

<u>Frequency response amplitudes</u>



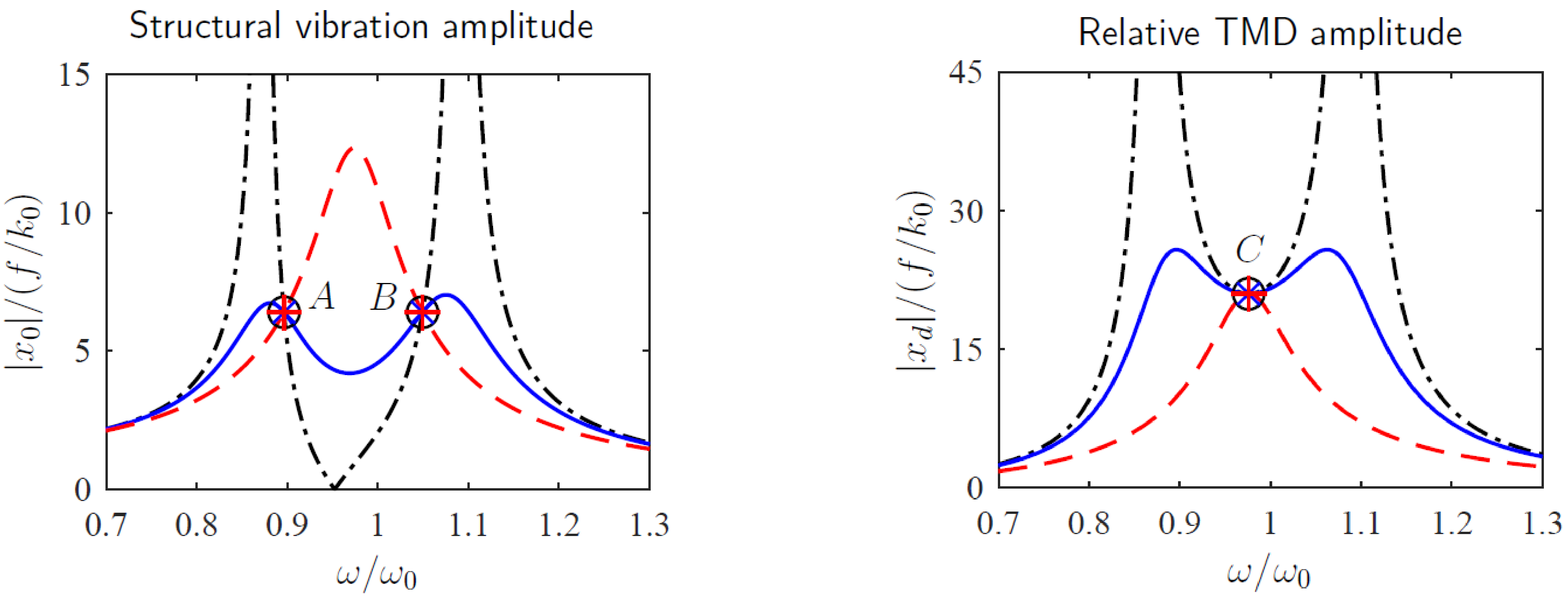
$\mu = 0.05$, $\zeta_d = 0.0$ (– · – · –) , 0.1 (——) , 0.3 (— — —)

- Response amplitude independent of TMD damping (*c*) at neutral or fixed points *A* and *B*
- Tune TMD stiffness *k* so that: $|x_0|_A = |x_0|_B$

Absorber tuning

Fixed-point tuning

<u>Frequency response amplitudes</u>



Optimal TMD stiffness *k*:

$$k_{opt} = \frac{\mu}{(1+\mu)^2} k_s$$

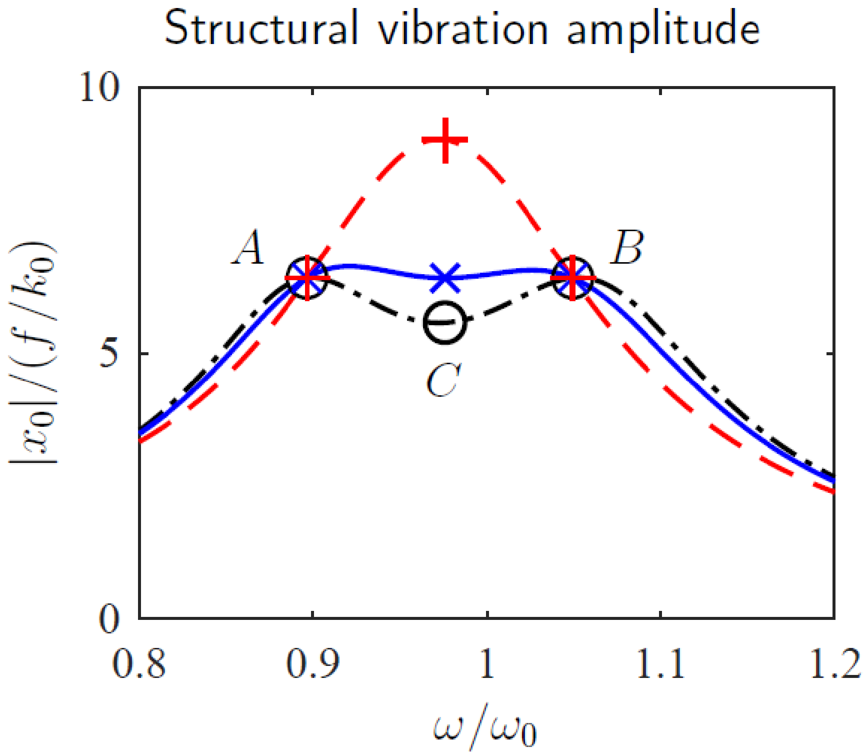
Tune TMD damping (*c*):

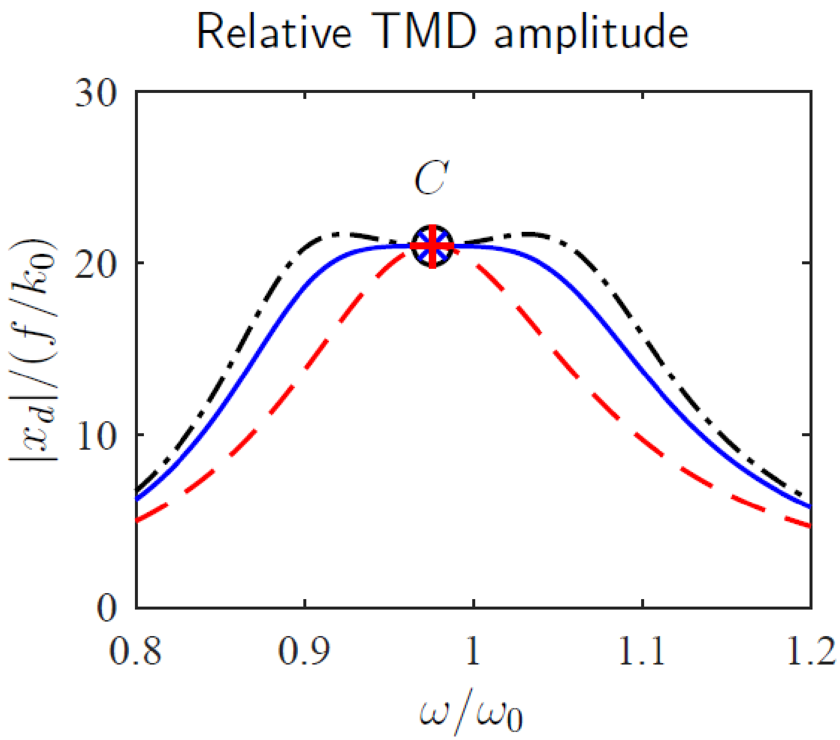
- Flat plateau at intermediate point *C*: $\omega_C = \bar{\omega}_s$
- Equal amplitudes: $|x_0|_A = |x_0|_C = |x_0|_B$

Absorber tuning

Fixed-point tuning

Frequency response amplitudes





$\mu = 0.05$, $\zeta_{classic}$ (– · – · –) , ζ_{opt} (———) , ζ_* (— — —)

Optimal TMD damper *c*:

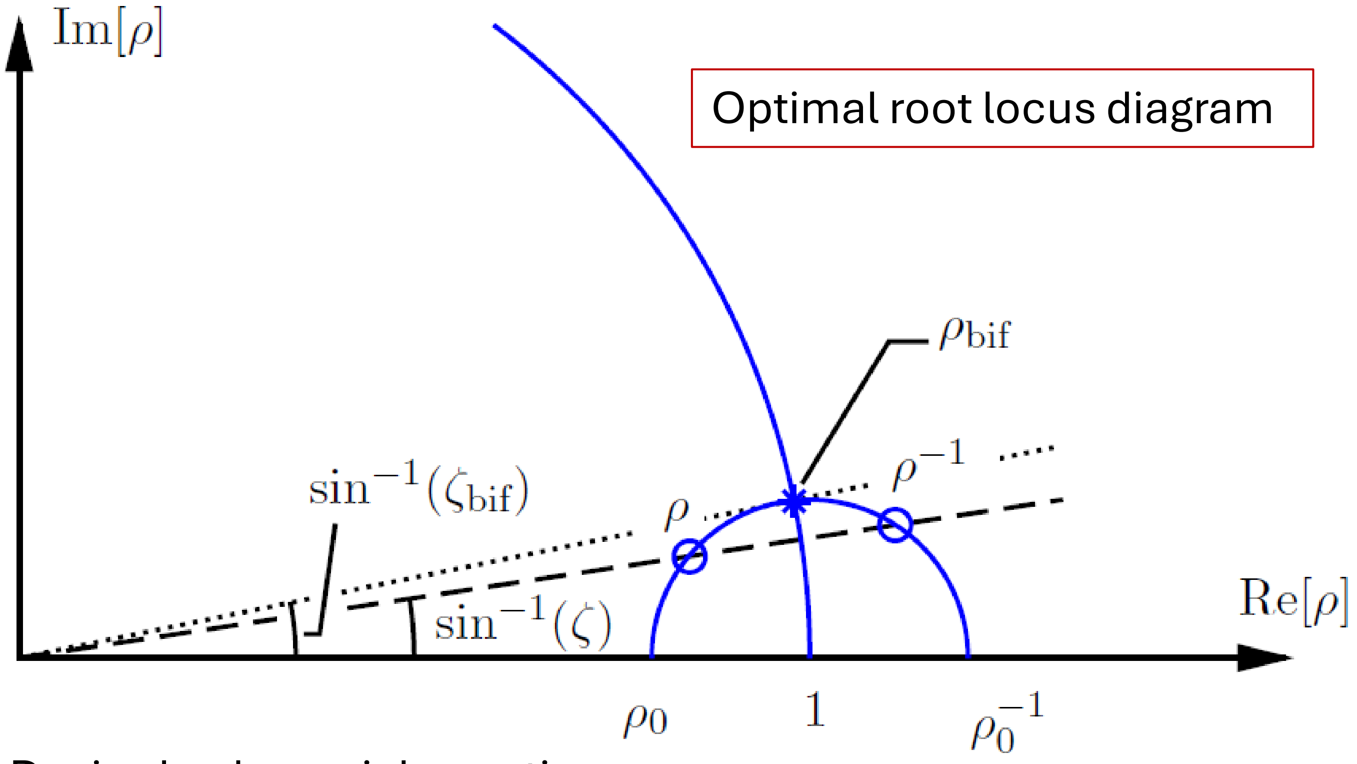
$$c_{opt} = \sqrt{\frac{2\mu^3}{(1+\mu)^3}}\sqrt{k_s m_s}$$

Classic TMD damper *c* (Den Hartog):

$$c_{classic} = \sqrt{\frac{3}{2}\frac{\mu^3}{(1+\mu)^3}}\sqrt{k_s m_s}$$

Absorber tuning

Pole placement calibration



Desired polynomial equation:

$$\rho^4 - \rho^2\left(2 + (2\zeta_{\mathrm{bif}})^2\right) + 1 + 2i\rho(2\zeta_{\mathrm{bif}})\left(-\rho^2 + 1\right) = 0$$

TID characteristic equation (denominator of FRF):

$$\rho^4 - \rho^2\left(1 + \mu_* + (1+\mu_*)^2\frac{\kappa}{\mu_*}\right) + (1+\mu_*)^2\frac{\kappa}{\mu_*} + i\rho\left(\sqrt{1+\mu_*}\right)^3\frac{\beta}{\mu_*}\left(-\rho^2+1\right) = 0$$

Tuning by comparison of terms

1) Ratio btw ρ^3 and ρ terms:

$$\rho = \frac{\omega}{\omega_s}\sqrt{1+\mu_*}$$

EMCF

$$\boxed{\mu_* = \frac{\omega_s^2 - \bar{\omega}_s^2}{\bar{\omega}_s^2}}$$

2) Constant terms:

$$\kappa = \frac{\mu_*}{(1+\mu_*)^2}$$

3) ρ^2 terms:

$$\zeta_{\mathrm{bif}} = \frac{1}{2}\sqrt{\mu_*}$$

4) ρ^3(or ρ) terms:

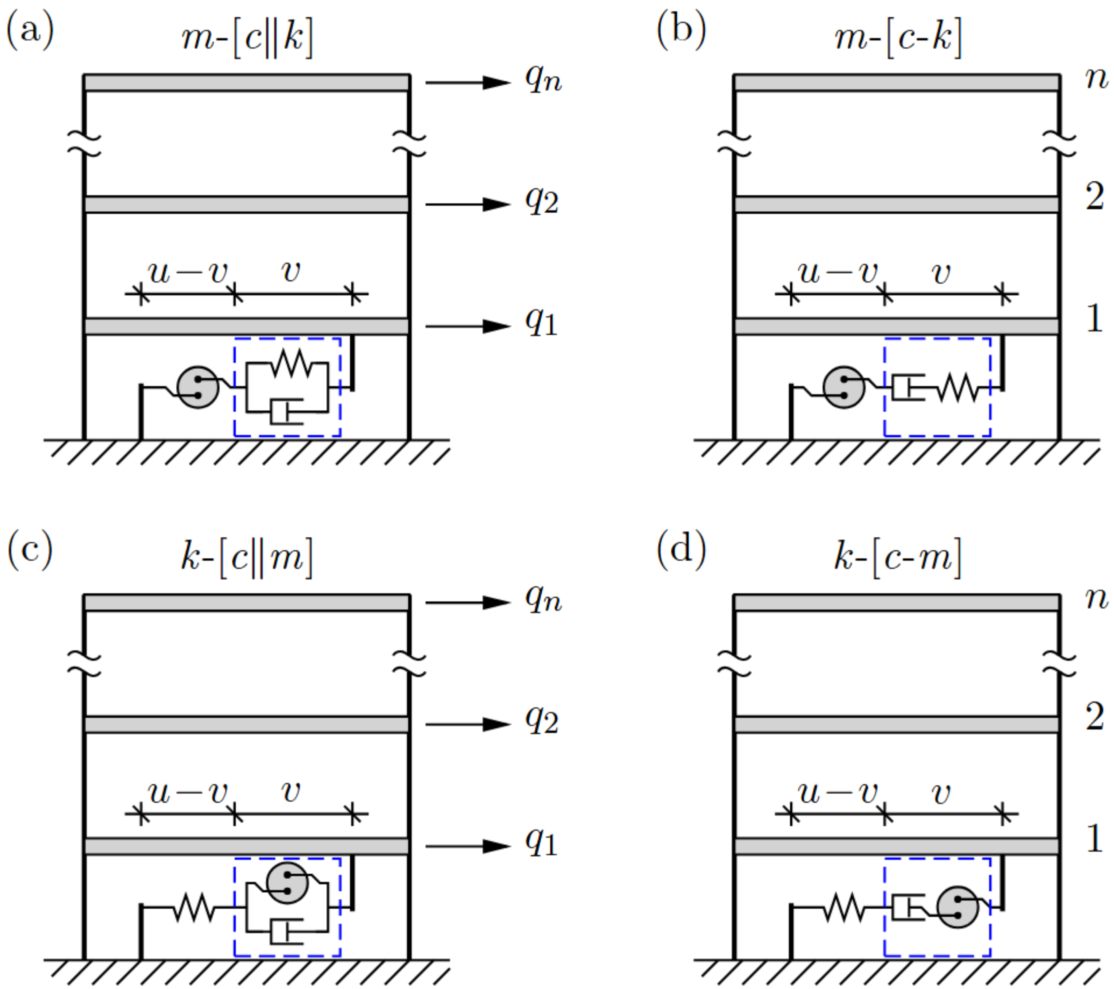
$$\beta = \frac{1}{\sqrt{2}}\beta_{\mathrm{bif}} = \sqrt{2\,\frac{\mu_*^3}{(1+\mu_*)^3}}$$

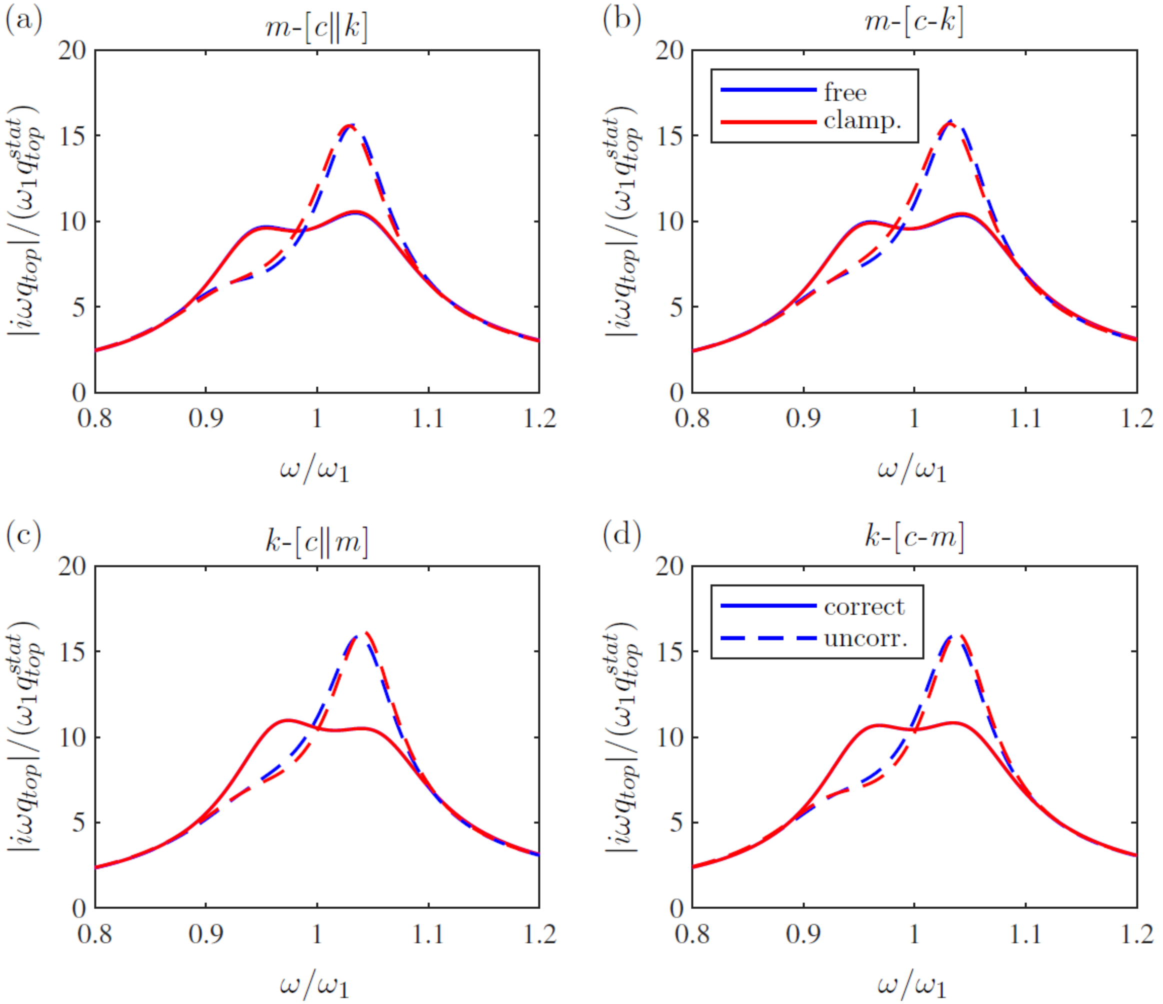
5) Absorber (TID) parameters:

$$c = \beta\sqrt{m_s k_s} \quad , \quad k = \kappa k_s$$

Damping of shear frame

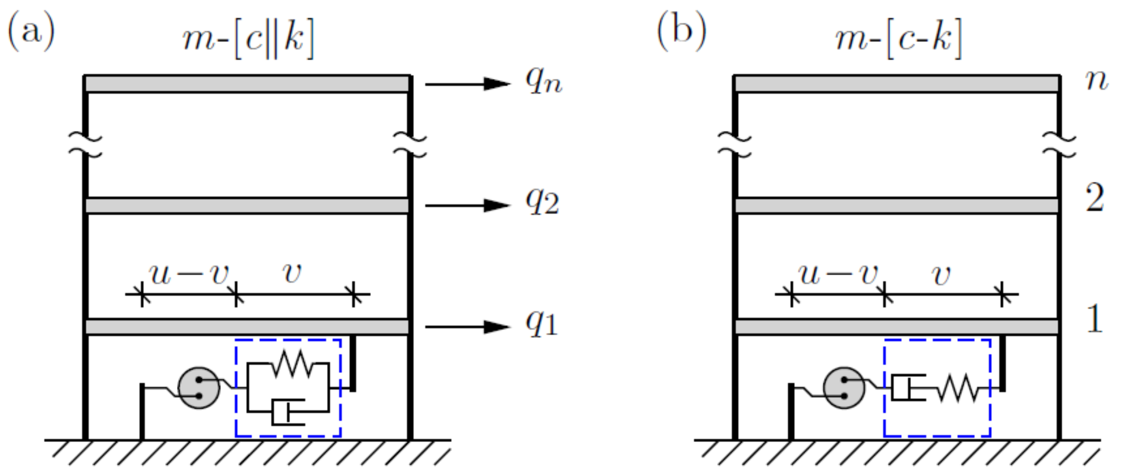
Frequency response amplitude (FRF)





Damping of shear frame

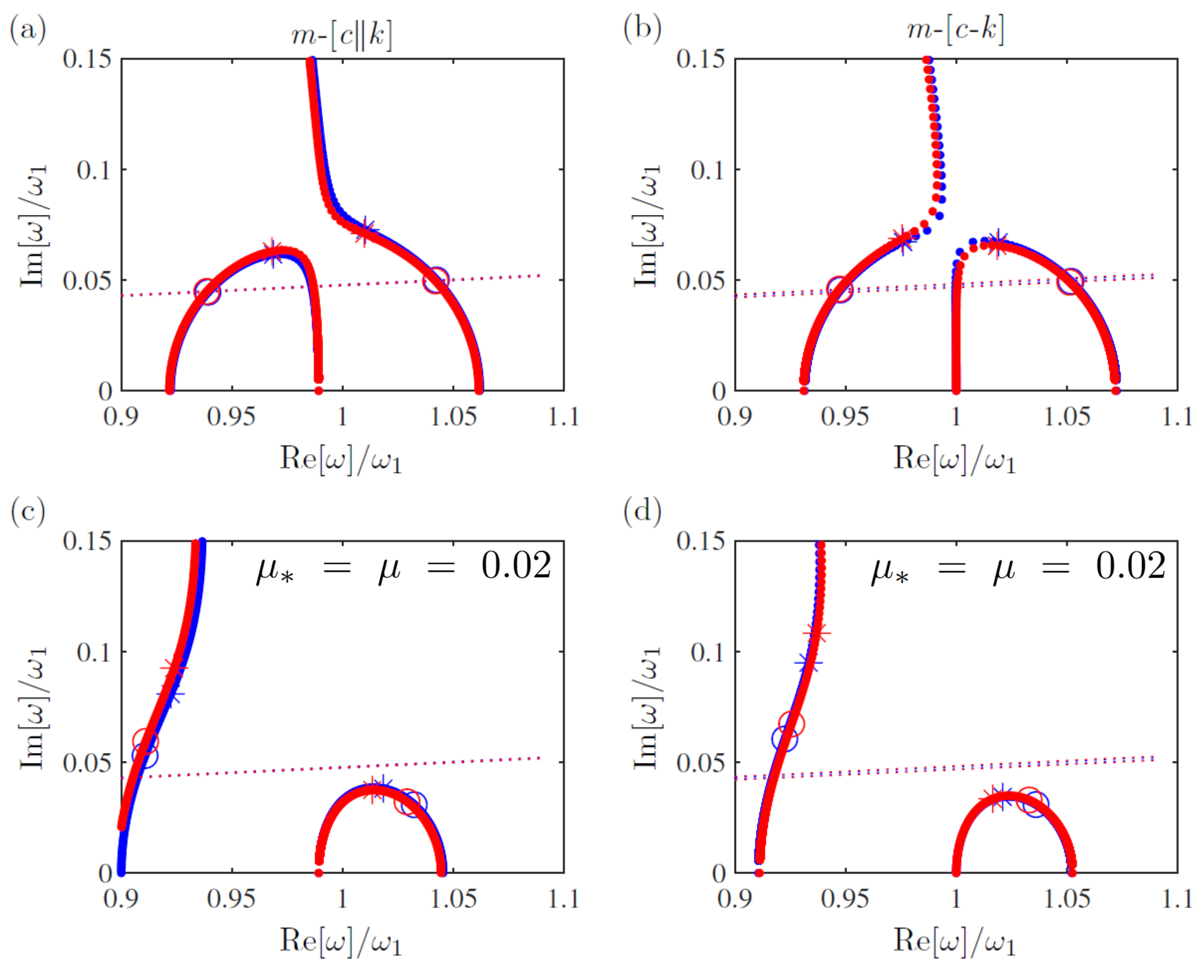
Root locus diagrams



Mode s = 1 damping ratios for $\mu = 0.02$

	free	
	$\mu_* = 0.0219$	$\mu_* = 0.0200$
m-$[c\|k]$	$\boxed{0.0473}$	0.0582
	0.0480	$\boxed{0.0299}$
m-$[c$-$k]$	0.0479	0.0655
	$\boxed{0.0470}$	$\boxed{0.0298}$

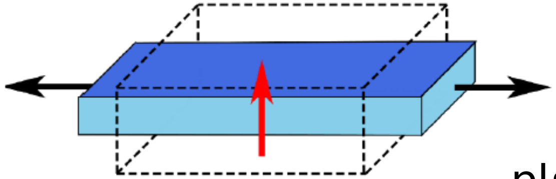
+ 30% reduction in damping ratio by $\mu_* = \mu$



Piezoelectric RL shunt damping

Mechanical equivalence

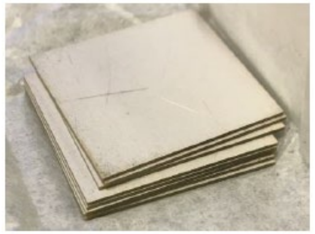
Piezoelectric effect: Deformation ↔ Voltage



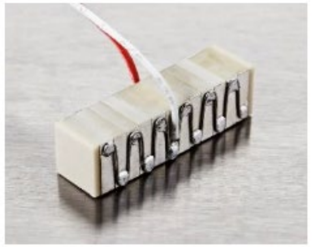
(Toftekær)

- Mechanical force ***F***
- Electric potential ***V***

plate (d31)



stack (d11)



(CTS corp)

Electric balance:

$$q(t) = -\theta u(t) + Cv(t)$$

Series Resistive-Inductive (RL) shunt:

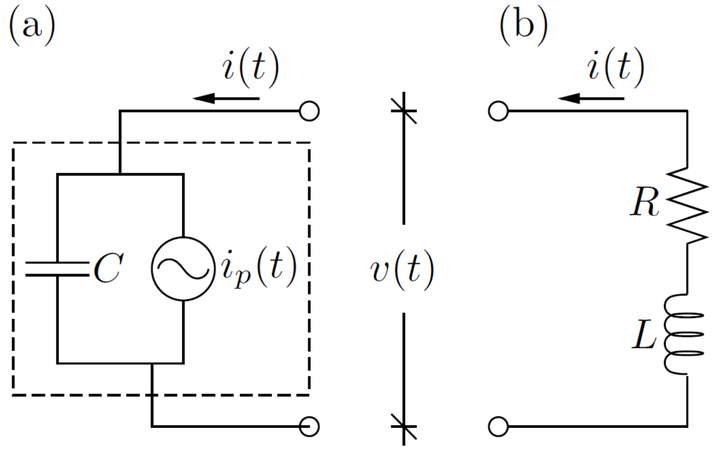
$$v(t) = -\left(R\dot{q}(t) + L\ddot{q}(t)\right)$$

Shunted piezo equation:

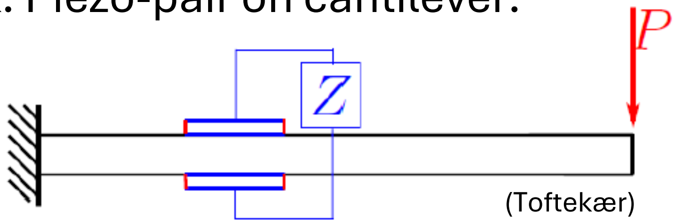
$$LC\ddot{v}(t) + RC\dot{v}(t) + v(t) = \theta\left(L\ddot{u}(t) + R\,\dot{u}(t)\right)$$

Shunt resonance at frequency: $\omega_e = 1/\sqrt{LC}$

Shunted piezo:



Benchmark: Piezo-pair on cantilever:

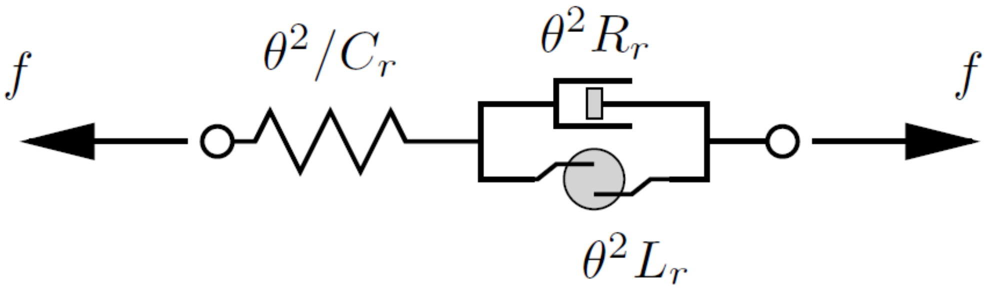


(Toftekær)

Equation of motion with piezo force $f = \theta v$:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{w}\theta v(t) + \mathbf{f}_e(t)$$

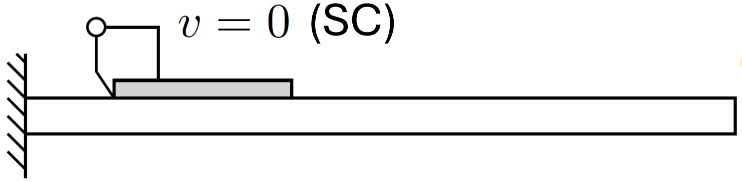
Equivalent mechanical absorber k-$[c \| m]$



Piezoelectric RL shunt damping

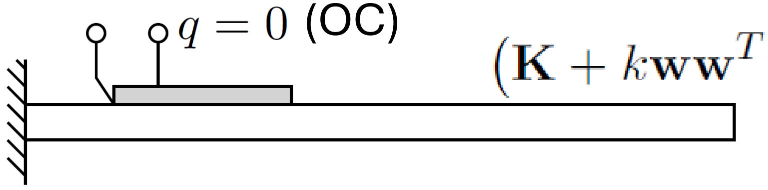
EMCF and tuning

Limiting absorbers:



free

$$(\mathbf{K} - \omega_j^2 \mathbf{M})\, \mathbf{u}_j = \mathbf{0}$$



clamped

$$(\mathbf{K} + k\mathbf{w}\mathbf{w}^T - \bar{\omega}_j^2 \mathbf{M})\, \bar{\mathbf{u}}_j = \mathbf{0}$$

$$k = \theta^2 / C$$

Effective Modal Coupling Factor EMCF:

$$\kappa_* = \frac{\bar{\omega}_s^2 - \omega_s^2}{\omega_s^2} = \frac{\omega_{oc}^2 - \omega_{sc}^2}{\omega_{sc}^2} = K^2$$

(Effective Electro-Mechanical Coupling Factor EEMCF)

Tuning by pole placement:

$$\mu = \frac{\kappa_*}{(1+\kappa_*)^2} \qquad \beta = \sqrt{2\,\frac{\kappa_*^3}{(1+\kappa_*)^3}} \qquad \zeta_{des}^2 = \tfrac{1}{8}\kappa_*$$

Shunt components:

inductor

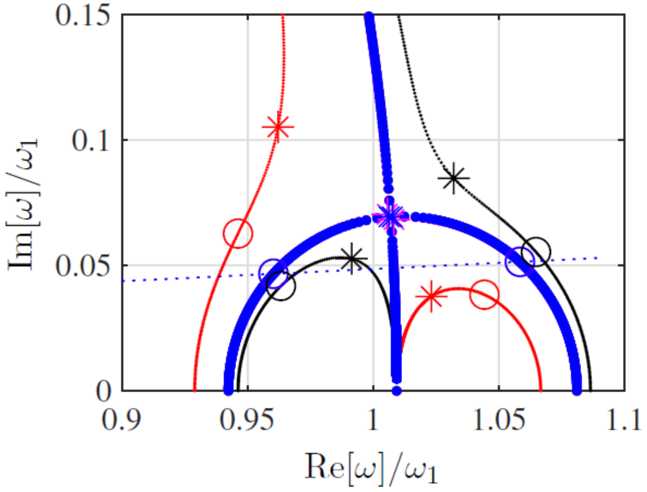
$$L = \mu \frac{m_s}{\theta^2}$$

resistor

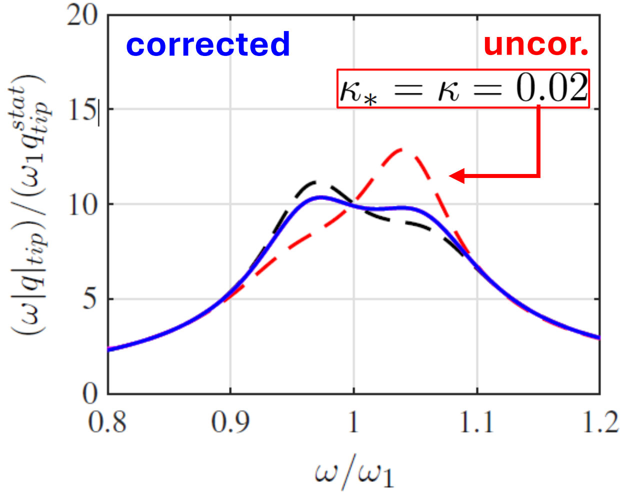
$$R = \beta \frac{\sqrt{m_s k_s}}{\theta^2}$$

Numerical analysis

Root locus



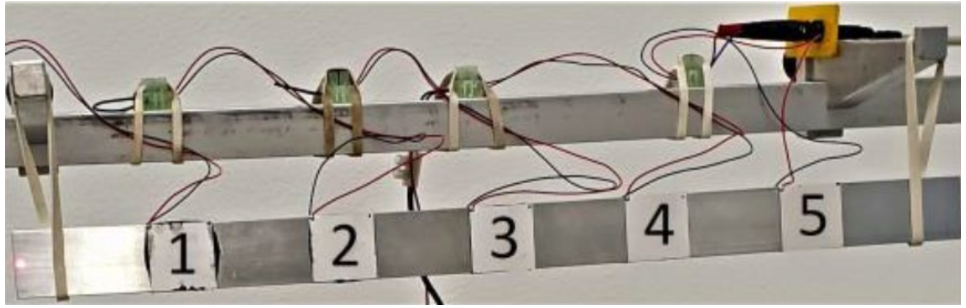
Tip FRF

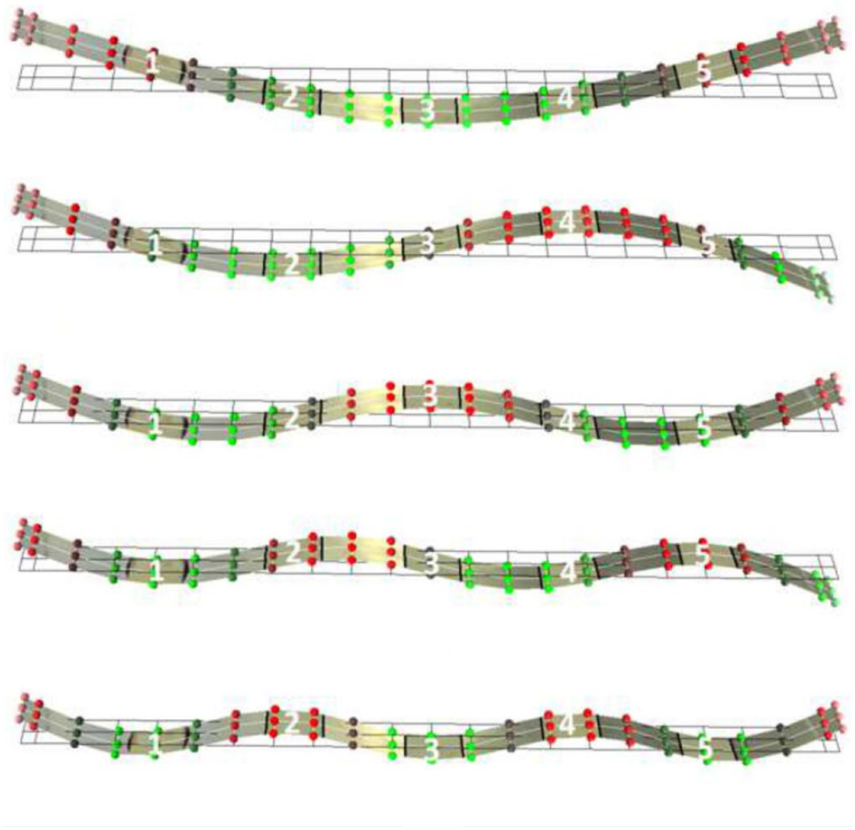


Piezoelectric RL shunt damping

Experiments – free beam

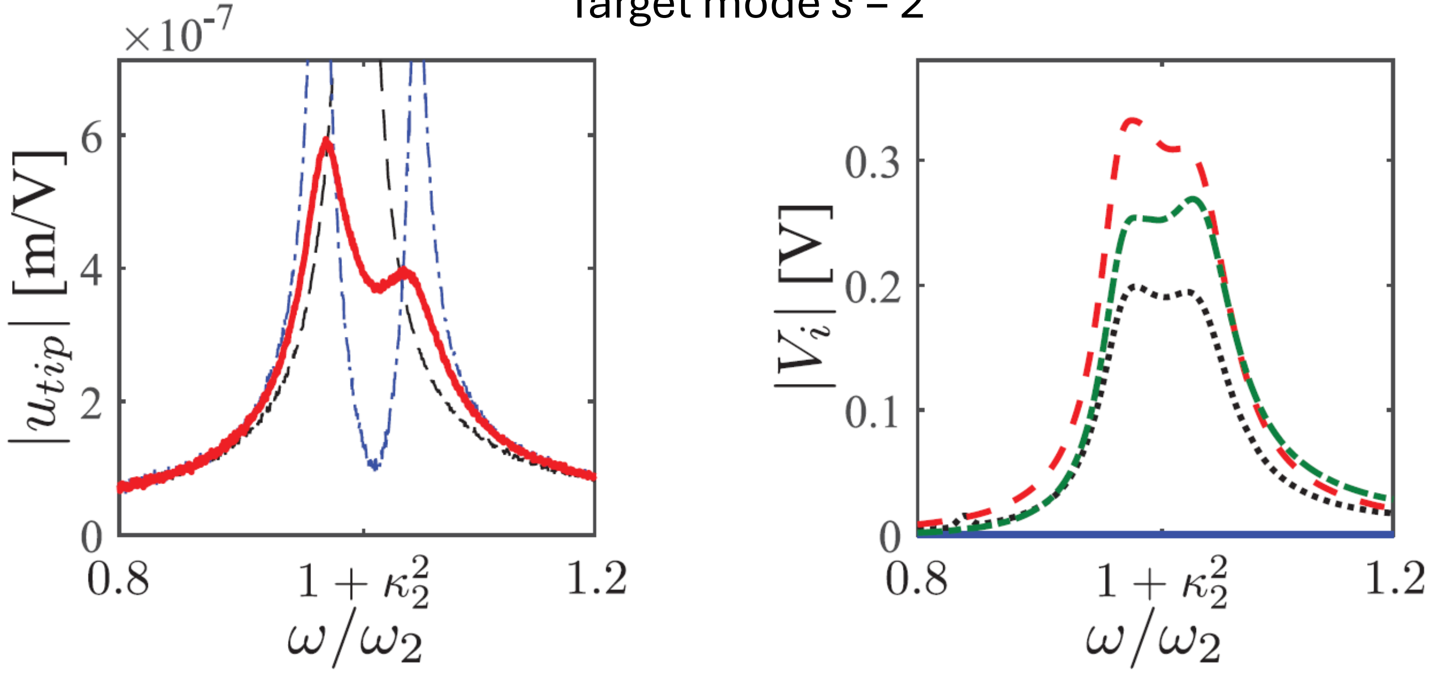
(Toftekær)





Flexural mode	1	2	3	4	5
$\omega/(2\pi)$ (Hz)	152.6	415.0	806.8	1320	1958.0
$\hat{\omega}/(2\pi)$ (Hz)	153.2	416.2	808.9	1324	1968.1
κ_*	0.80	0.59	0.53	0.53	1.03

Target mode *s* = 2



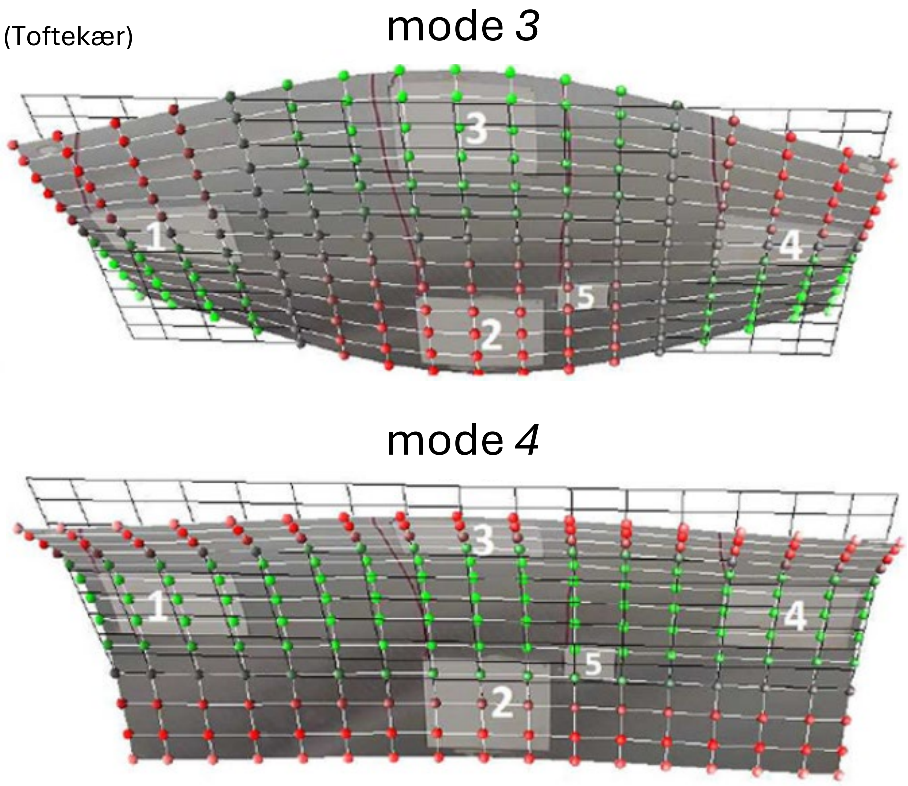
OC (- -), *L*(-·-), and *LR*(—)

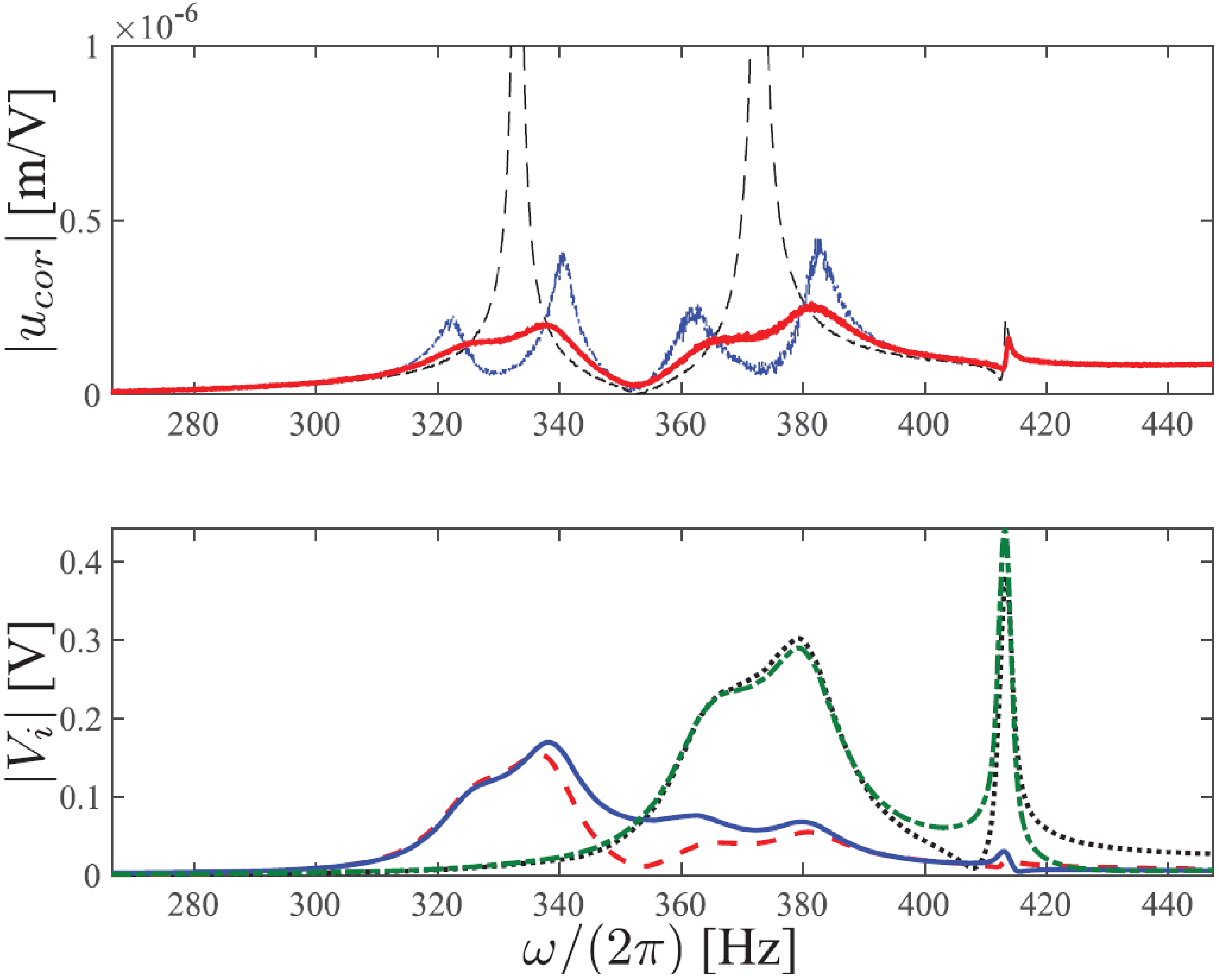
absorbers 1 (···), 2 (- -), 3 (—), and 4 (-·-)

Piezoelectric RL shunt damping

Experiments – free plate

(Toftekær)

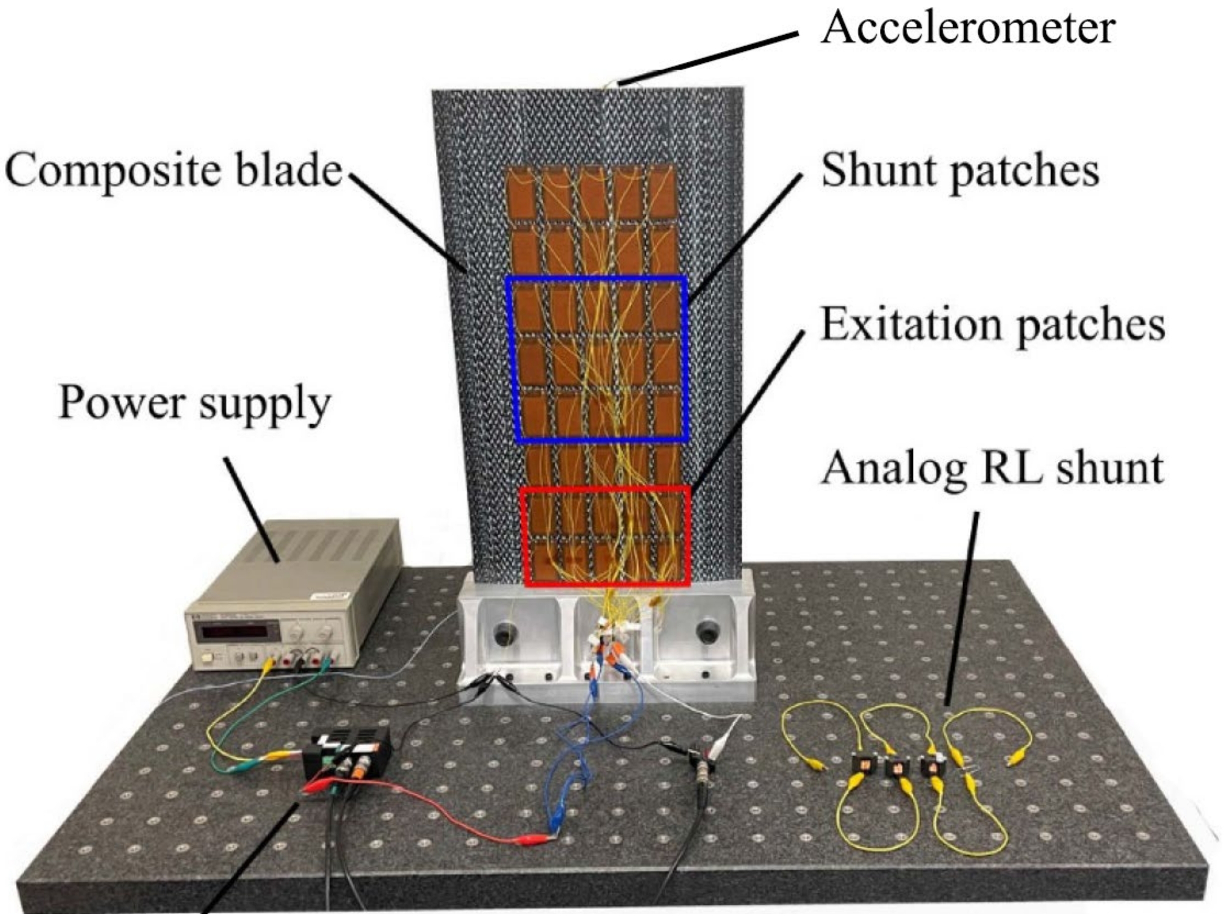


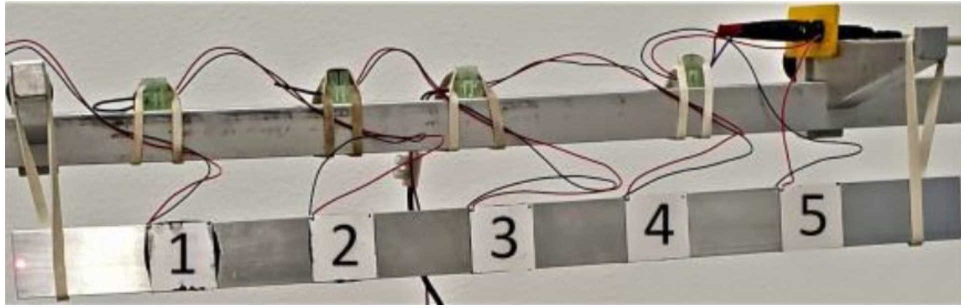


- Mode s = 3: piezos 2 and 3
- Mode s = 4: piezos 1 and 4

(Weber – Maurer.de)

That’s it!

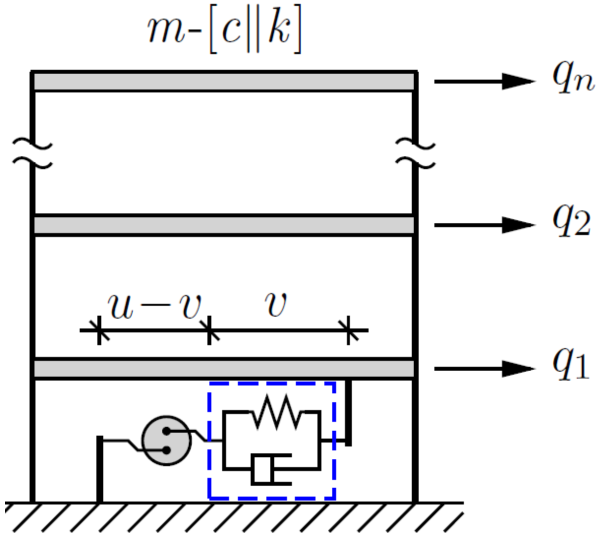




(Toftekær)

(Richardt, Lossouarn, Deü - CNAM

Exercise



n = 5 floors

Equation of motion for augmented system:

$$\tilde{\mathbf{M}}\ddot{\tilde{\mathbf{q}}}(t)+\tilde{\mathbf{C}}\dot{\tilde{\mathbf{q}}}(t)+\tilde{\mathbf{K}}\tilde{\mathbf{q}}(t) = \tilde{\mathbf{f}}(t) \qquad \tilde{\mathbf{q}}(t) = \begin{bmatrix} \mathbf{q}(t) \\ u(t)-v(t) \end{bmatrix}$$

$$\tilde{\mathbf{M}} = \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0}^T & m \end{bmatrix}, \quad \tilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}+c\mathbf{w}\mathbf{w}^T & -c\mathbf{w} \\ -c\mathbf{w}^T & c \end{bmatrix}, \quad \tilde{\mathbf{K}} = \begin{bmatrix} \mathbf{K}+k\mathbf{w}\mathbf{w}^T & -k\mathbf{w} \\ -k\mathbf{w}^T & k \end{bmatrix}$$

Tuning and time integration by python script: **shearframe_damp.py**

Exercises:

1. Download and run **shearframe_damp.py**
2. What is the modal mass *ms*? Compare with total weight of shear frame (5**mf*)
3. Compare mass ratios *mu* (μ) and *mua* (μ_*). How large is the difference in % ?
4. How large is the reduction in top-floor amplitude when installing TID?
5. Compare TID amplitude with structural amplitude.
6. Use detuned parameters (for $\mu_* = \mu$) and compare top-floor amplitude with optimal tuning