Summer School
Advanced Structural Dynamics

June 16th-19th 2025, Copenhagen (Denmark)

Fast-slow motion analysis for friction-related problems
Lecture 3

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Outline of the lectures

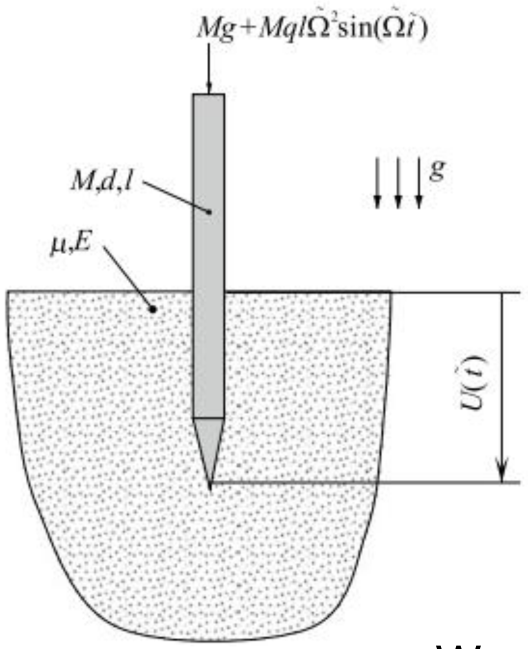
- Lecture 1 – 9:00-10:00
  - Personal background
  - Case studies of fast-slow motion analysis for friction-related problems
  - Simplified and intuitive example of fast-slow motion analysis for sliding friction

- Lecture 2 – 13:10-14:00
  - Mass-spring system influenced by friction for a general frequency of excitation
  - Worked-out example of fast-slow motion analysis for a pile driving system

- **Lecture 3 – 14:00-15:00**
  - Follow-up: worked-out example of fast-slow motion analysis for a pile driving system
  - Example of results of the friction reduction of a SDOF moving on an elastic rod
  - Discussion on hidden (fast) motion effect on stability and dynamic friction laws

Example 2: application on a vibration-assisted pile driving system

Suggested exercise from J.J. Thomsen's book (see problem 7.3, 3rd edition).



(a) Set up an equation governing the slow (i.e. average) component of the penetration displacement.
(b) Simplify this equation for the case of a relatively small average piling speed.
(c) Derive and discuss an expression for the vibrational force acting on the pile (i.e. the static force equivalencing the average effect of the fast vibrations).

We use this adimensional equation of motion:

$$\ddot{u} + \underbrace{\gamma u \,\mathrm{sign}(\dot{u})}_{} = 1 + q\Omega^2 \sin(\tau)$$

This term links the friction force to the lateral normal force exerted by the soil on the pile, as a linear function with respect to penetration depth

$$u = \frac{U}{l} \qquad \gamma = \frac{\mu E \pi d^2}{Mg} \qquad t = \omega \tilde{t}$$

$$\omega^2 = \frac{g}{l} \qquad \Omega = \frac{\tilde{\Omega}}{\omega} \qquad \tau = \Omega t$$

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Example 2: application on a vibration-assisted pile driving system

- Original equation: $\ddot{u}+\gamma u\,\mathrm{sign}(\dot{u})=1+q\Omega^2\sin(\tau)$
- Equation which accounts for the averaged effect induced by the fast motion, on the slow motion:

$$\ddot{z}+z\dot{z}\alpha=1$$

The original equation can be re-rewritten in a different form, without the external forcing, but adding a so-called **vibrational force** that includes the averaged effect of the fast excitation

$$\ddot{z}+z\dot{z}\alpha+\gamma z\,\mathrm{sign}(\dot{z})=1+\gamma z\,\mathrm{sign}(\dot{z}) \Rightarrow \ddot{z}+\gamma z\,\mathrm{sign}(\dot{z})=1+z\left(\gamma\,\mathrm{sign}(\dot{z})-\dot{z}\alpha\right)$$

$$\ddot{z}+\gamma z\,\mathrm{sign}(\dot{z})=1+\underbrace{V(z,\dot{z})}_{}\longrightarrow \textbf{Vibrational force}$$

To an observer, who is “blind to the applied excitation”, it will appear that the pile’s motion is influenced by a “mysterious” force V.

Example 2: application on a vibration-assisted pile driving system

$$\ddot{z} + \gamma z \operatorname{sign}(\dot{z}) = 1 + z\gamma\left(\operatorname{sign}(\dot{z}) - \dot{z}\frac{2}{q\Omega\pi}\right)$$

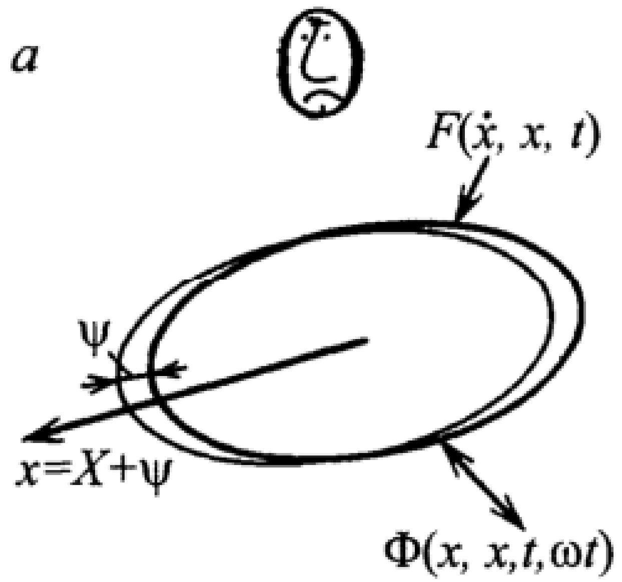
The **vibrational force** has two terms in this case:

1) A restoring force which is pointing in the direction of pile penetration
2) A term which dissipates energy. The dissipation increase with z, but decreases if the externally applied excitation is increased

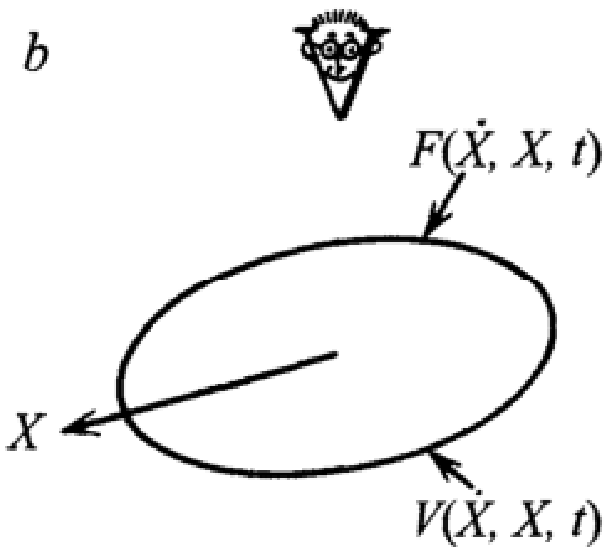
Important take-away: Not being aware of the applied excitation, may lead to the conclusion that the observed pile behaviour, during pile-driving, is governed by a complex "force", that seems to be proportional to the displacement and the velocity

Overview of the notion of Vibrational forces according to Blekhman

The world/observer of
”mechanics”



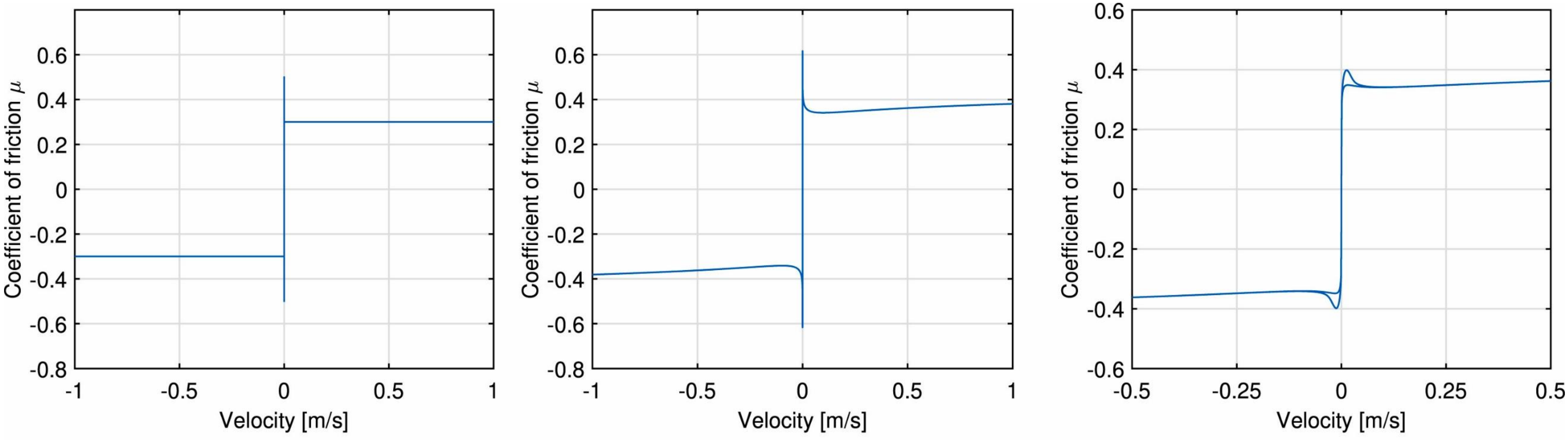
The world/observer of
” vibrational mechanics”



If you experience/observe vibrational forces, **look for hidden fast motions** first.

Images taken from: I. I. Blekhman. **Vibrational mechanics: Nonlinear dynamic effects, general approach, applications**, World Scientific Publishing, (2000)

Examples of kinetic and dynamic friction laws



(a) Amontons-Coulomb law

(b) Steady-state version of the rate-and-state friction law

(c) Rate-and-state friction law

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Image taken from: Cabboi, A., Marino, L., Cicirello, A. A comparative study between Amontons–Coulomb and Dieterich–Ruina friction laws for the cyclic response of a single degree of freedom system, European Journal of Mechanics – A/Solids, 96, 104737 (2022).

Typical examples of vibrational forces can occur for systems influenced by friction and vibration

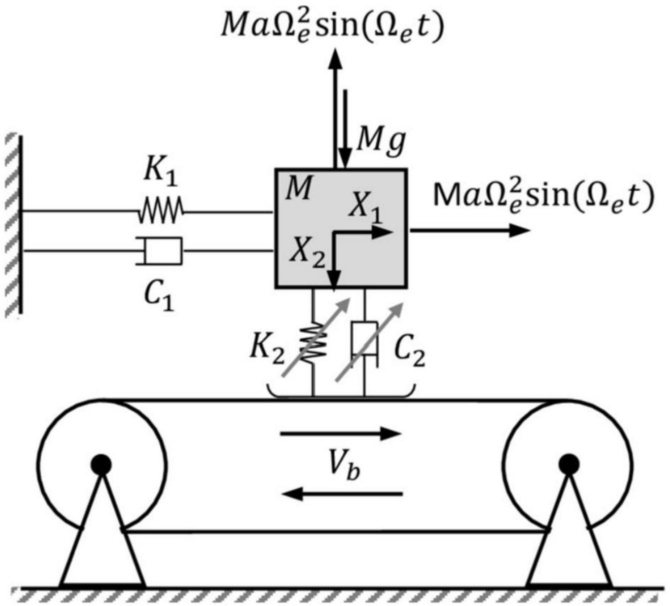
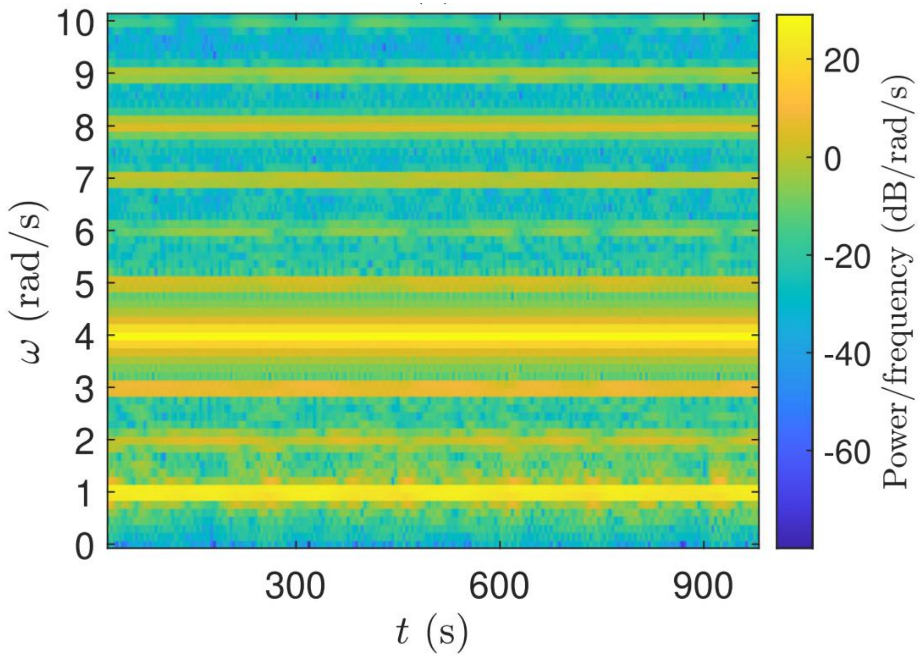


Fig. 1 Layout of the 2-DOF Hetz-Damp system subject to external harmonic loads in the normal and tangential directions

- The effective friction force will be modulated through the tangential and normal harmonic excitation
- It can lead to instability, due to the parametric excitation

Applied excitation (tang. and norm): 4 rad/s

Tang. Natural frequency: 1 rad/s



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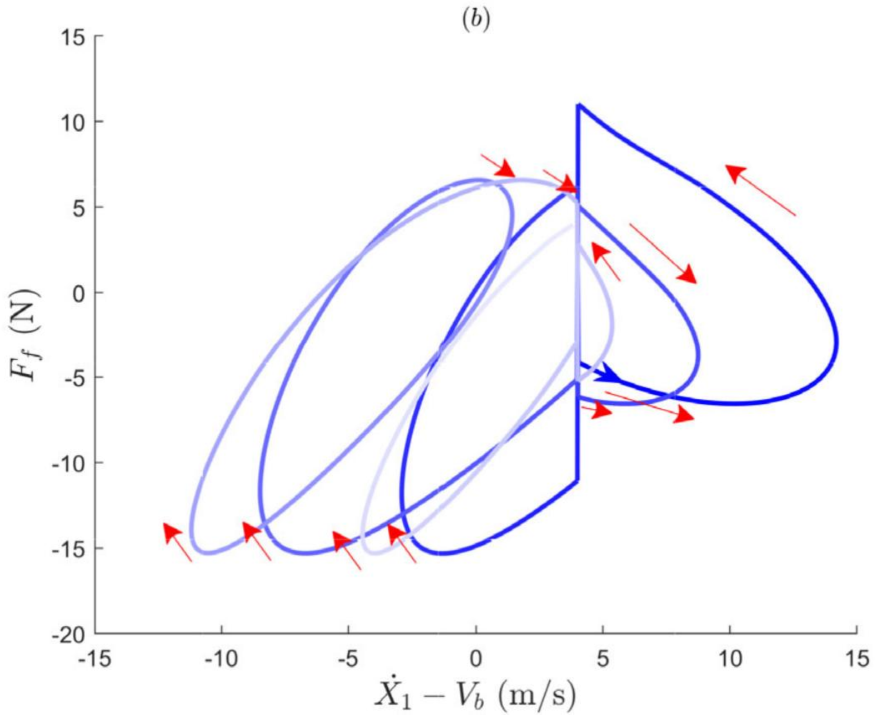
Images taken from: Sulollari, E., Van Dalen, K., Cabboi, A. Parametric excitation and friction modulation for a forced 2-DOF system, Nonlinear Dynamics, 113, (2025)

Typical examples of vibrational forces can occur for systems influenced by friction and vibration

- The effective/modulated friction force looks like:



- It exhibits the behaviour of dynamic friction laws
- It can be characterized by negative damping (negative slope of the friction force vs relative velocity)

Note: If one were to fit/develop a vibrational friction force exhibiting hysteresis and negative damping, according to the vibrational mechanic observes, the system could be solved in a simpler way. However, you may miss the underlining physics of the system

Images taken from: Sulollari, E., Van Dalen, K., Cabboi, A. Parametric excitation and friction modulation for a forced 2-DOF system, Nonlinear Dynamics, 113, (2025)

Typical examples of vibrational forces can occur for systems influenced by friction and vibration

Kapelke, S. et al (2018)

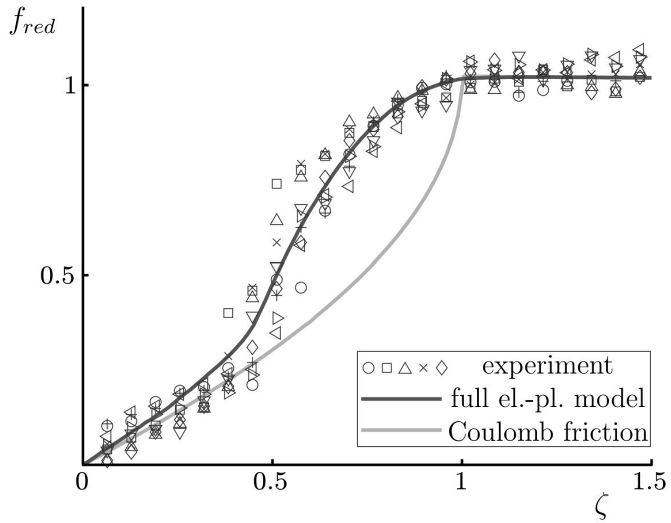
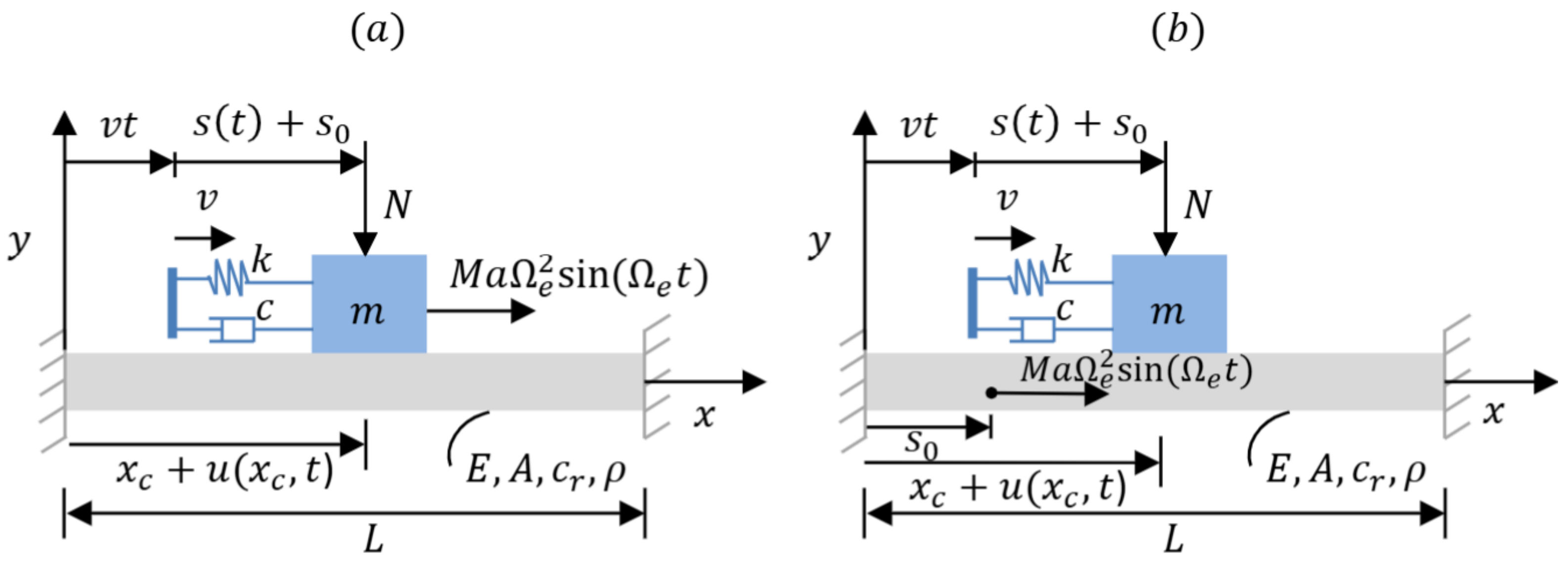


Image taken from: Kapelke, S. et al, **On the effect of longitudinal vibrations on dry friction**, Tribology Letters (2018) 66:79, https://doi.org/10.1007/s11249-018-1031-0

Kapelke et. al, had to use an Elasto-plastic friction (Dupont model), to fit the observed friction reduction, due to fast excitation.

- Can the observed behaviour of the friction reduction be explained in a different way?
- What if the fast-external excitation was triggering a system dynamic response, which was not accounted for?

Typical examples of vibrational forces can occur for systems influenced by friction and vibration



We assumed a fast-harmonic excitation, applied on a friction oscillator, which is being pulled over an axial deformable rod (inclusion of fast motion provided by the system dynamics).

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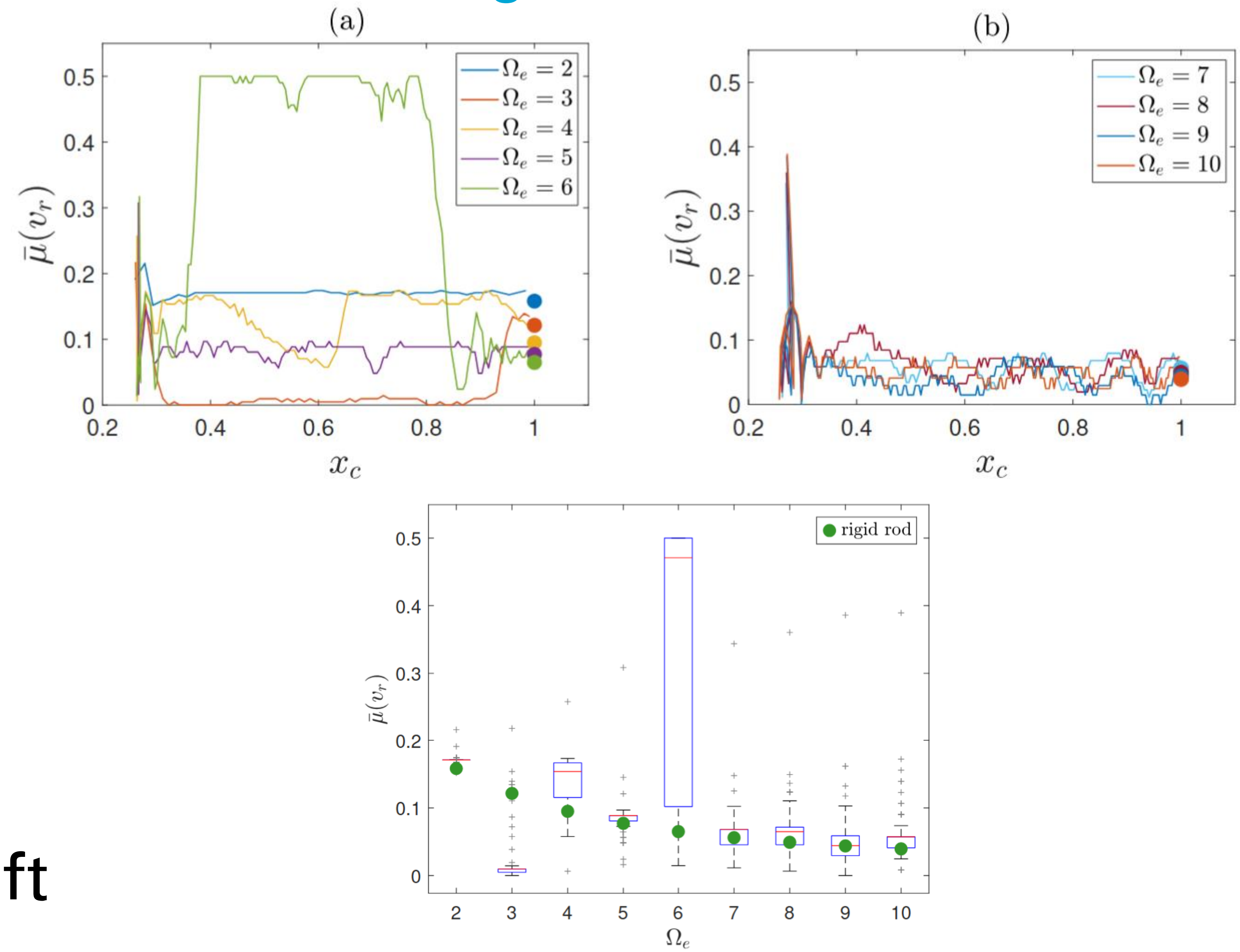
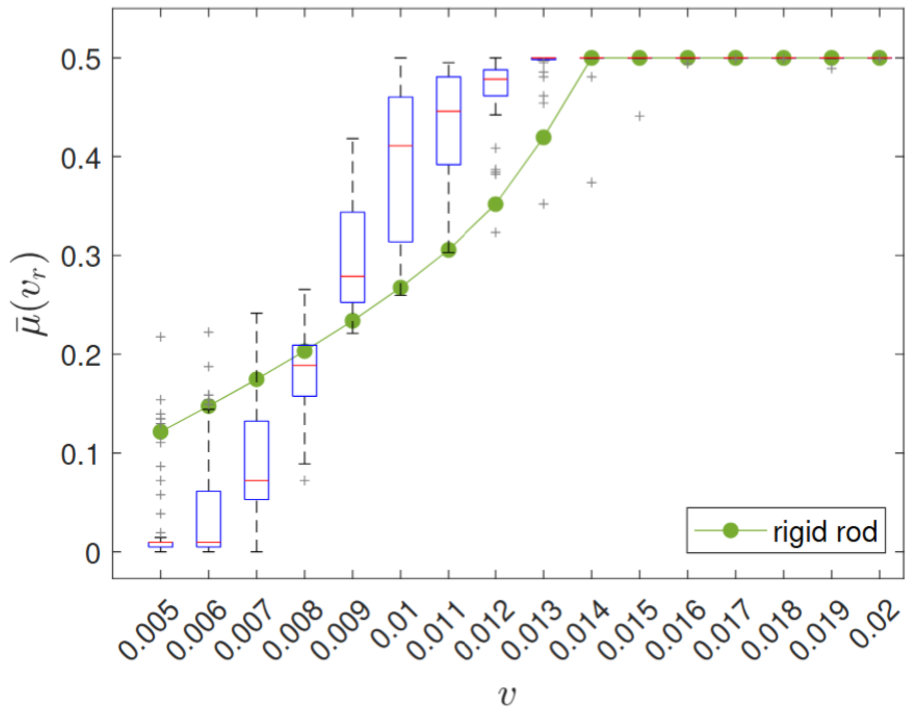
Images taken from: E. Sulollari, K.N. van Dalen, A. Cabboi, Vibration-induced friction modulation for an oscillator moving on an elastic rod, Submitted to the International Journal of Solids and Structures (currently under review)

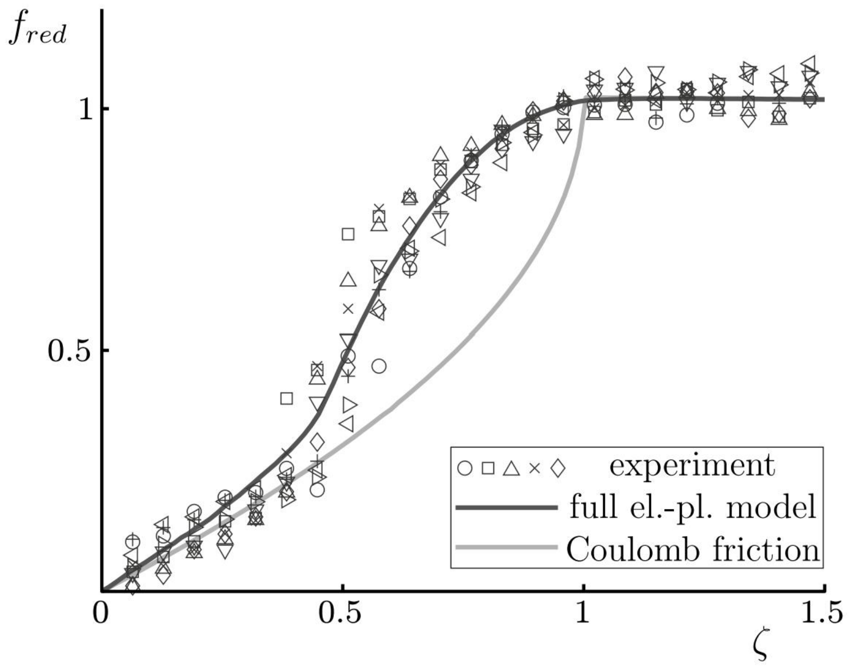
Summary of results: Averaged friction reduction throughout the length of the elastic rod



Example 1: Forced SDOF moving on a rotating belt

- We can observe deviation from the rigid case.





Last slide and main take-away

- Watch out for hidden motions, in order to explain “mysterious” behaviors of the system’s dynamic response
- The effect of fast motion, can be tackled through the method of separation of motion, applicable for a general class of systems
- Homework: feel free to come up with techniques and solutions, that can take advantage of an applied fast motion.