Extended-State-Observer-Based Control of Flexible-Joint System With Experimental Validation

Sanjay E. Talole, Jayawant P. Kolhe, and Srivijay B. Phadke

***Abstract*—In this paper, a feedback linearization (FL)-based control law made implementable using an extended state observer (ESO) is proposed for the trajectory tracking control of a flexible-joint robotic system. The FL-based controller cannot be implemented unless the full transformed state vector is available. The design also requires exact knowledge of the system model making the controller performance sensitive to uncertainties. To address these issues, an ESO is designed, which estimates the state vector, as well as the uncertainties in an integrated manner. The FL controller uses the states estimated by ESO, and the effect of uncertainties is compensated by augmenting the FL controller with the ESO-estimated uncertainties. The closed-loop stability of the system under the proposed observer–controller structure is established. The effectiveness of the ESO in the estimation of the states and uncertainties and the effectiveness of the FL + ESO controller in tracking are demonstrated through simulations. Lastly, the efficacy of the proposed approach is validated through experimentation on Quanser's flexible-joint module.**

***Index Terms*—Extended state observer (ESO), feedback linearization (FL), flexible-joint system.**

I. Introduction

THE TRAJECTORY tracking control of robotic manipulators with joint flexibility has received considerable attention, owing to the complexity of the problem. Many robots incorporate harmonic drives for speed reduction, and it is known that such drives introduce torsional elasticity into the joints [1]. Industrial robots generally have elastic elements in the transmission systems, which may result in the occurrence of torsional vibrations when a fast response is required. For many manipulators, joint elasticity may arise from several sources, such as elasticity in gears, belts, tendons, bearings, hydraulic lines, etc., and may limit the speed and dynamic accuracy achievable by control algorithms designed assuming perfect rigidity at joints. A proper choice of mathematical model for a control system design is a crucial stage in the development of control strategies for any system. This is particularly true for robotic manipulators due to their complicated dynamics. Experimental evidence suggests that joint flexibility should be taken into account in both modeling and control of manipulators if high performance is to be achieved. To model this elastic behavior in the joints, the link is considered as connected to rotor through a torsional spring of stiffness K. The introduction of joint flexibility in the robot model considerably complicates the equations of motion. In particular, the order of the related dynamics becomes twice that of the rigid robots, and the number of degrees of freedom is larger than the number of inputs, making the control task difficult.

In literature, a number of feedback control schemes have been proposed to address the issue of joint flexibility. In [2], a sliding-mode-control-based strategy is proposed. The design needs knowledge of the bounds of uncertainty and also the complete state vector for its implementation. In [3], a dynamic feedback controller is proposed for the trajectory tracking control problem of robotic manipulators with flexible joints. The design requires position measurements on the link, as well as the motor side, and the velocities required in the controller are estimated through a reduced-order observer. Furthermore, the robustness of the closed-loop system is established by assuming that the uncertainties satisfy certain conditions. In [4], a singular perturbation approach is employed for the same task wherein the controller needs measurements of position and elastic force. A nonlinear sliding state observer is used for estimating the link velocities and elastic force time derivatives. A controller design based on the integral manifold formulation [5], adaptive control [6], and a back-stepping approach [7] are some other approaches reported in the literature. Most of the schemes that appeared in literature have certain issues that require attention. First, many of them require measurements of all state variables or at least the position variables on link and motor side. Next, robustness, wherever guaranteed, is often highly model dependent. Moreover, some need knowledge of some characteristics of the uncertainties such as its bounds.

Owing to the highly nonlinear dynamics, a feedback linearization (FL) approach has also been proposed for designing the trajectory tracking controllers for flexible-joint robots [1], [2], [5]. It has been shown that the flexible-joint robot model is globally feedback linearizable through nonlinear coordinate transformation and static state feedback, and therefore, globally stable controllers can be derived using the classical geometric techniques. While the FL-based controller offers highly satisfactory performance in ideal scenario, it may suffer from certain drawbacks when put into practice. In general, the FL requires exact cancellation of nonlinearities. It offers asymptotic tracking of the reference trajectory only when the models are known exactly and the fed back states are measured without any error. In reality, these conditions are hard to meet, and therefore, the FL control laws may not offer satisfactory performance.

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Another important consideration in FL-based control law is its implementation. In general, the control law requires knowledge of all states, i.e., link position, its velocity, acceleration, and jerk. While position may be measured accurately, the joint velocity is often contaminated by noise. Furthermore, obtaining the remaining unmeasured states by numerical differentiation can be a source of difficulty when the measurements are noisy.

The aforementioned stated issues in FL-based controllers require some remedies to recover its performance. One approach to address the issue of degradation in the performance of FL controller in the presence of uncertainties is to robustify it, and to this end, various approaches have been proposed in the literature. In [8], a Lyapunov function-based robust controller is incorporated with an FL controller. Similarly, robust outer loop design based on the Lyapunov's second method is presented in [9] for the design of an autopilot for a bank-to-turn missile. In [10], [11], a robust FL has been achieved by employing the sliding-mode control (SMC) technique.

Alternatively, the performance of the FL controller can be recovered in the presence of uncertainties by estimating the uncertainties and then compensating the same by augmenting the FL controller with the estimate of uncertainties. As real plants are usually affected by significant uncertainties and unmeasurable external disturbances, disturbance rejection or compensation has become an important problem for a high-performance control system design. A great deal of effort has been devoted to address this issue, and consequently, a number of methods/approaches have been proposed to robustify systems in the presence of uncertainties and disturbances. For example, in [12], the theory of time-delay control (TDC) is presented, wherein a function representing the effect of uncertainties and external disturbances is estimated directly by using information in the recent past and, then, a control is designed using this estimate in such a way as to cancel out the effect of uncertainties and external disturbances. An application of the TDC to estimate the target acceleration in missile guidance scheme can be found in [13]. In [14], an input–output linearizing controller is robustified by estimating the uncertainties using a recently developed uncertainty and disturbance estimation (UDE). An application of the UDE in robustifying a feedback linearizing control law for a robot having joint flexibility is presented in [15], wherein the effect of joint flexibility is treated as a disturbance. The estimation of uncertainties can also be used to overcome some well-known drawbacks in SMC, as shown in [16]. As one more example, in [17], a sliding mode disturbance observer is designed to estimate the uncertainties/disturbances acting on the system. The estimated uncertainties are then used in the controller to compensate for them.

In the context of the present application of trajectory control of flexible-joint robotic system, any of the approaches presented before can be used for addressing the issue of uncertainties. However, as stated earlier, the implementation of the FL-based controller also needs information of the complete state vector. In this circumstance, it is desired to have an estimation of uncertainties, as well as the complete state vector. This requirement of obtaining the estimate of uncertainty, as well as states in an integrated manner, is met by the extended state observer (ESO) [18]–[21]. The ESO can estimate the uncertainties along with the states of the system, enabling disturbance rejection or compensation. Unlike traditional (linear or nonlinear) observers, the ESO estimates the effect of uncertainties, unmodeled dynamics, and external disturbances acting on the system as an extended state of the original system. The ESO can be integrated with FL-based controller to provide an estimate of uncertainties in the robotic dynamics so that the same can be compensated. Furthermore, since the ESO is designed for robotic dynamics, it also provides an estimate of the complete state vector.

In this paper, the FL-based controller is used for the trajectory tracking problem of flexible-joint robotic system. To address the issues of uncertainties and the estimation of state, an ESO is designed and integrated with the FL controller. This paper proposes a controller that robustifies FL-based controllers and makes them implementable by providing an estimate of states. This is accomplished by designing an ESO that provides estimates of uncertainty and states simultaneously and integrating it with the FL controller. The resulting controller is designated as the FL + ESO controller. The design requires measurement of position on the link side only, unlike many schemes that need more measurements. As the effect of uncertainties is compensated by estimating the same, the design does not need knowledge of any characteristic such as the bound of the uncertainties. Lastly, the design is far less model dependent in comparison with many control schemes that appear in literature. Simulations are carried out by considering significant uncertainties in robot dynamics to demonstrate the effectiveness of ESO in the estimation of states and uncertainties and the effectiveness of the FL + ESO controller in tracking. The closed-loop stability of the system under the proposed observer–controller structure is established. Finally, experimental results validating the efficacy of the proposed approach are given.

The remainder of this paper is organized as follows. Section II presents the FL controller as applied to the flexible-joint robot. In Section III, the ESO theory is briefly reviewed. The application of ESO for state and uncertainty estimation of the present problem, closed-loop stability results of the system under the proposed design, and simulation results demonstrating the effectiveness of the proposed design are given in Section IV. The results of the experimental work validating the proposed approach are presented in Section V, and finally, Section VI concludes this paper.

II. FL-Based Control

A. Mathematical Model

In this paper, we consider a single-link manipulator with revolute joint actuated by a dc motor and model the elasticity of the joint as a linear torsional spring with stiffness K, as shown in Fig. 1.

The equations of motion for this system, as taken from [1], are

$$\begin{aligned} I\ddot{q}_1 + MgL\sin(q_1) + K(q_1 - q_2) &= 0 \\ J\ddot{q}_2 - K(q_1 - q_2) &= u \end{aligned} \tag{1}$$

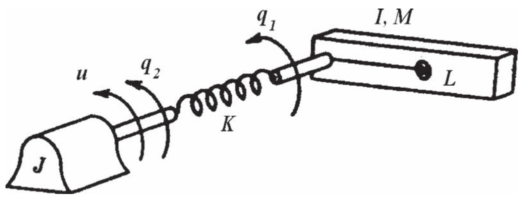

Fig. 1. Schematic of flexible-joint manipulator modeled by torsional spring.

where q_1 and q_2 are the link and motor angles, respectively, I is the link inertia, J being the inertia of motor, K is the spring stiffness, u is the input torque, and M and L are the mass and length of link, respectively. The tracking problem for the system of (1) is to find a control which ensures $q_1^\star(t) - q_1(t) = 0$ for given initial states, where $q_1^\star(t)$ is the desired trajectory for $q_1(t)$.

B. FL Controller

As stated earlier, one of the well-researched strategies for designing trajectory tracking controllers for robotic manipulators is the geometric-control-theory-based FL approach [1], [22]. In general, a single-input nonlinear dynamical system $\dot{x} = f(x) + g(x)u$ is said to be feedback linearizable if there exist a nonlinear state coordinate transformation of the form $z = T(x)$ and nonlinear feedback $u = (1/b(z))(-a(z) + \nu)$ with $b(z) \neq 0$ such that the transformed state variables $z = T(x)$ and the new input ν satisfy a linear time invariant relation $\dot{z} = Az + B\nu$.

In the present context, by defining the state variables as $x_1 = q_1$, $x_2 = \dot{q}_1 = \dot{x}_1$, $x_3 = q_2$, and $x_4 = \dot{q}_2 = \dot{x}_3$, the nonlinear dynamics (1) can be written in state-space model as $\dot{x} = f(x) + g(x)u$, where $f(x) = [x_2 \quad -(MgL/I)\sin(x_1) - K/I(x_1 - x_3) \quad x_4 \quad (K/J)(x_1 - x_3)]^{\mathrm{T}}$ and $g(x) = [0 \quad 0 \quad 0 \quad 1/J]^{\mathrm{T}}$ with output $y = x_1$. It has been shown that the state-space model is globally linearizable by diffeomorphic coordinate transformation and nonlinear static state feedback [5]. To this end, following the nonlinear state coordinate transformation as presented in [23], the original dynamics can be rewritten in terms of the new coordinates as

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= z_4 \\ \dot{z}_4 &= a(z) + bu \\ y &= z_1 \end{aligned} \tag{2}$$

where

$$a(z) = \frac{MgL}{I}\sin(z_1)\left(z_2^2 - \frac{K}{J}\right) - \left(\frac{MgL}{I}\cos(z_1) + \frac{K}{J} + \frac{K}{I}\right) z_3$$

$$b = \frac{K}{IJ}.$$

The FL control law is

$$u = \frac{1}{b}\left(-a(z) + \nu\right). \tag{3}$$

Applying the FL control law (3) to the robot dynamics (2) results into the following linear relationship between the states and new input ν:

$$\dot{z} = Az + B\nu \tag{4}$$

where the matrices A and B are defined obviously. Now, taking the new input ν as

$$\nu = \dot{z}_4^\star + k_1\left(z_1^\star - z_1\right) + k_2\left(z_2^\star - z_2\right) + k_3\left(z_3^\star - z_3\right) + k_4\left(z_4^\star - z_4\right) \tag{5}$$

where the starred quantities represent the reference values of the corresponding variables. Applying this control law to the linear system (4), one obtains the tracking error dynamics as

$$\frac{d^4 e_{c_1}}{dt^4} + k_4\frac{d^3 e_{c_1}}{dt^3} + k_3\frac{d^2 e_{c_1}}{dt^2} + k_2\frac{de_{c_1}}{dt} + k_1 e_{c_1} = 0 \tag{6}$$

where $e_{c_1}(t) = z_1^\star(t) - z_1(t)$ is the tracking error. The values of k_i being the design parameters are required to be chosen such that the desired tracking performance is achieved.

It is important to note that the transformed coordinates are themselves physically meaningful, as z_1, z_2, z_3, and z_4 are the link position, velocity, acceleration, and jerk, respectively. Since the motion trajectory is defined in terms of these variables, they are natural variables to use for control. From (3), one can note that the FL controller requires exact cancellation of nonlinearities to achieve the desired performance. In the presence of uncertainties, the nonlinearities may not get canceled exactly, which may result in poor performance, and thus, it is necessary to compensate for the effects of the uncertainties. Moreover, the feedback linearizing control requires full state measurements, which, in the present case, are the link position, velocity, acceleration, and jerk. The coordinates z_i must be either physically measurable or they need to be estimated by using an observer. The transformed states can also be computed from the measured states x_i; however, for computing z_i via x_i, one needs accurate estimates of the parameters in the manipulator model and, hence, are worth avoiding.

III. ESO

As stated earlier, the ESO is an observer which can estimate the uncertainties along with the states of the system, enabling disturbance rejection or compensation. The ESO regards all factors affecting the plant, including the nonlinear dynamics, uncertainties, and the external disturbances as a total disturbance (extended state) to be observed. Since the observer estimates the uncertainties as an extended state, it is designated as ESO. Its merit is that it is relatively independent of the mathematical model of the plant, performs better, and is simpler to implement. The robustness is inherent in its structure, as will be obvious in the next section. In [19], a comparison study of the performances and characteristics of three advanced state observers, namely, high-gain observer, ESO, and sliding mode observer, is presented, and it is shown that, overall, the ESO is superior in dealing with the uncertainties, disturbances, and sensor noise.

Several diverse applications of ESO-based control strategies have appeared in literature. The application of ESO in disturbance rejection control for uncertain multivariable systems with time delay [24], a DSP-based active disturbance rejection control design for a power converter [25], robotic hand–eye coordination [26], control of flexible-joint system [23], precision motion control [27], aircraft attitude control [28], and torsional vibration control of the main drive system of a rolling mill [29] are some examples to mention. In this section, a brief review of this approach and its application to state and uncertainty estimation for the present problem are presented.

A. Concept of ESO

Consider an nth order single-input–single-output nonlinear dynamical system described by

$$\overset{(n)}{z} = a\left(z, \dot{z}, \ldots, \overset{(n-1)}{z}, w\right) + bu \tag{7}$$

where $a(.)$ represents the dynamics of the plant and the disturbance, $w(t)$ is an unknown disturbance, u is the control signal, and z is the measured output. Let $b = b_o + \Delta b$, where b_o is the best available estimates of b and Δb is its associated uncertainties. Defining the uncertainty to be determined as $d \overset{\Delta}{=} a + \Delta b u$ and designating it as an extended state z_{n+1}, the dynamics (7) can be written in a state-space form as

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\vdots \\ \dot{z}_n &= z_{n+1} + b_o u \\ \dot{z}_{n+1} &= h \\ y &= z_1 \end{aligned} \tag{8}$$

where h is the rate of change of the uncertainty, i.e., $h = \dot{d}$, and is assumed to be an unknown but bounded function. By making d a state, however, it is now possible to estimate it by using a state estimator. To this end, consider a nonlinear observer of the form

$$\begin{aligned} \dot{\hat{z}}_1 &= \hat{z}_2 + \beta_1 g_1(e_{o_1}) \\ \dot{\hat{z}}_2 &= \hat{z}_3 + \beta_2 g_2(e_{o_1}) \\ &\vdots \\ \dot{\hat{z}}_n &= \hat{z}_{n+1} + \beta_n g_n(e_{o_1}) + b_o u \\ \dot{\hat{z}}_{n+1} &= \beta_{n+1} g_{n+1}(e_{o_1}) \end{aligned} \tag{9}$$

where $e_{o_1} = y - \hat{z}_1 = z_1 - \hat{z}_1$, and $\hat{z}_{n+1}$ is an estimate of the uncertainty. The quantities β_i are the observer gains, while $g_i(.)$ denotes the set of suitably constructed nonlinear gain functions satisfying $e_{o_1} g_i(e_{o_1}) > 0 \quad \forall\, e_{o_1} \neq 0$ and $g_i(0) = 0$. If one chooses the nonlinear functions $g_i(.)$ and their related parameters properly, the estimated state variables $\hat{z}_i$ are expected to converge to the respective states of the system z_i, i.e., $\hat{z}_i \to z_i$, where $i = 1, 2, \ldots, n+1$.

The choice of the nonlinear function is an important aspect in the ESO design. The general expression for these functions, selected heuristically based on experimental results, [19] is

$$g_i(e_{o_1}, \alpha_i, \delta) = \begin{cases} |e_{o_1}|^{\alpha_i} \operatorname{sign}(e_{o_1}), & |e_{o_1}| > \delta \\ \frac{e_{o_1}}{\delta^{1-\alpha_i}}, & |e_{o_1}| \leq \delta \end{cases}; \quad i = 1, 2, \ldots, n+1 \tag{10}$$

where $\delta > 0$. An important property of these functions is that for, $0 < \alpha_i < 1$, $g_i(.)$ yields a relatively high gain when the error is small and a small gain when the error is large. The constant δ is a small number used to limit the gain in the neighborhood of the origin and defines the range of the error corresponding to high gain.

B. LESO

The ESO of (9) with the gains given in (10) is called the nonlinear ESO (NESO), as it employs the nonlinear gain functions. On the other hand, if one chooses $g_i(e_{o_1}) = e_{o_1}$, the ESO takes the form of the conventional Luenberger observer and is designated as the linear ESO (LESO). It can be noted that the LESO essentially represents a special case of the NESO if one chooses the NESO parameters α_i's as unity.

Writing the extended-order system (8) in state-space notation gives

$$\dot{z} = Az + Bu + Eh \tag{11}$$

where $z = [z_1 \quad z_2 \quad \cdots \quad z_n \quad z_{n+1}]^{\mathrm{T}}$ is the state vector of the extended-order system and

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & & \ddots & & \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b_o \\ 0 \end{bmatrix} \qquad E = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

and $C = [1 \quad 0 \quad 0 \quad \cdots \quad 0]$. The LESO for the system (11) is given by (9) with the gains $g(e_{o_1}) = e_{o_1}$. The state-space model of the LESO dynamics can be written as

$$\dot{\hat{z}} = A\hat{z} + Bu + LC(z - \hat{z}) \tag{12}$$

where

$$L = [\beta_1 \quad \beta_2 \quad \cdots \quad \beta_n \quad \beta_{n+1}]^{\mathrm{T}} \tag{13}$$

is the observer gain vector.

The application of NESO for the trajectory tracking problem of the flexible-joint system is presented in [23]. In [23], it was also assumed that the some estimate of the dynamics of the system is available, i.e., the estimate of $a(z)$ appearing in (7) is available. In this paper, LESO is employed instead of NESO for the same task, as the use of LESO offers certain advantages.

First, the observer gains can be chosen systematically through pole placement, whereas the various parameters appearing in NESO are usually chosen by a trial-and-error procedure. Second, the closed-loop stability for LESO can be established conclusively, as shown in the next section. Lastly, LESO is easy from hardware implementation point of view. Moreover, in this paper, the dynamics $a(z)$ is assumed to be completely unknown, and thus, it forms a part of the uncertainty to be estimated. Since this represents a more difficult case, work was progressed with this assumption.

IV. FL + ESO-Based Controller

A. Controller

Now, consider the flexible-joint robot dynamics given by (2). To account for the uncertainties in robotic parameters, we rewrite the dynamics (2) as

$$\begin{aligned}\dot{z}_1 &= z_2\\ \dot{z}_2 &= z_3\\ \dot{z}_3 &= z_4\\ \dot{z}_4 &= a(z) + bu = d + b_o u\end{aligned} \tag{14}$$

where b_o is the best available estimates of b, while Δb is its associated uncertainties. The quantity $d = a(z) + \Delta bu$ is the combined uncertainty and may also include other sources such as external disturbance acting on the system. In the presence of these uncertainties, the FL control, which will result into the same error dynamics as given by (6), is

$$u = \frac{1}{b_o}\big[\dot{z}_4^\star + k_1(z_1^\star - z_1) + k_2(z_2^\star - z_2) + k_3(z_3^\star - z_3) + k_4(z_4^\star - z_4) - d\big]. \tag{15}$$

To implement this control, one needs the value of uncertainty d apart from all the state variables. To obtain the estimates of state and uncertainty, a LESO is designed. The extended-order dynamics of (14) is given by (11) with $n = 4$, and the corresponding ESO is given by (12) with $n = 4$. With the estimates, the FL controller (15) takes a form as

$$u = \frac{1}{b_o}\big[\dot{z}_4^\star + k_1(z_1^\star - \hat{z}_1) + k_2(z_2^\star - \hat{z}_2) + k_3(z_3^\star - \hat{z}_3) + k_4(z_4^\star - \hat{z}_4) - \hat{z}_5\big] \tag{16}$$

where $\hat{z}_5$ is an estimate of the uncertainty d. The controller (16) is designated as FL + ESO controller and is implementable.

B. Closed-Loop Stability

In this section, the closed-loop stability of the system (14) under the controller–observer structure is established, wherein the controller is given by (16) and the observer is given by (12) with $n = 4$. To this end, one can rewrite (14) as

$$\dot{z}_s = A_s z_s + B_s u + B_d d \tag{17}$$

where $z_s = [z_1 \quad z_2 \quad z_3 \quad z_4]^{\mathrm{T}}$ is the state vector and

$$A_s = \begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\end{bmatrix} \qquad B_s = \begin{bmatrix}0\\0\\0\\b_o\end{bmatrix} \qquad B_d = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}.$$

Now, denoting the reference state vector $R = [z_1^\star \; z_2^\star \; z_3^\star \; z_4^\star]^{\mathrm{T}}$, the controller (16) can be rewritten as

$$u = K_s R - K_s \hat{z}_s + \frac{1}{b_o}\dot{z}_4^\star - \frac{1}{b_o}\hat{z}_5 \tag{18}$$

where $K_s = (1/b_o)[k_1 \quad k_2 \quad k_3 \quad k_4]$ is the state feedback gain vector for the system (14), and $\hat{z}_s$ is the estimate of z_s. Defining the state tracking error $e_c = R - z_s$, its dynamics is given by

$$\dot{e}_c = \dot{R} - \dot{z}_s. \tag{19}$$

The quantity $\dot{R}$ can be expressed as

$$\dot{R} = A_s R + B_d \dot{z}_4^\star. \tag{20}$$

Using (17), (18), and (20) and carrying out some simplifications lead to the following tracking error dynamics:

$$\dot{e}_c = (A_s - B_s K_s)e_c - [B_s K_s \quad B_d]e_o \tag{21}$$

where e_o is the observer estimation error vector, i.e., $e_o = z - \hat{z}$.

Next, the observer error dynamics is obtained by subtracting (12) from (11) with $n = 4$ as

$$\dot{e}_o = (A - LC)e_o + Eh. \tag{22}$$

Combining (19) and (22) yields

$$\begin{bmatrix}\dot{e}_c\\ \dot{e}_o\end{bmatrix} = \begin{bmatrix}(A_s - B_s K_s) & -[B_s K_s \quad B_d]\\ 0 & (A - LC)\end{bmatrix}\begin{bmatrix}e_c\\ e_o\end{bmatrix} + \begin{bmatrix}0\\ E\end{bmatrix}h. \tag{23}$$

From (23), it is straightforward to verify that the eigenvalues of the system matrix of the error dynamics are given by the eigenvalues of $(A_s - B_s K_s)$ and $(A - LC)$. Since the pair (A_s, B_s) is controllable and the pair (A, C) is observable, the stability of the error dynamics (23) can always be ensured by placing the controller and observer poles appropriately. Furthermore, since (23) is stable, it is obvious that, under the assumption of boundedness of h, the bounded-input–bounded-output stability for the linear system (23) is assured. A special case arises when $h = 0$, i.e., when the rate of change of uncertainty d is zero. In this case, the error dynamics (23) is asymptotically stable. A similar result can be expected if the rate of change of the uncertainty is reasonably small.

It is in order to offer some comments on the performance of ESO when the rate of change of uncertainty $h = \dot{d}$ is not negligible. The ESO error dynamics given by (22) exhibits asymptotic stability under the assumption that the rate of change of uncertainties h is negligible. The assumption does not hold good for systems having fast varying disturbances/uncertainties or for systems having state-dependent uncertainties and is exited by fast varying reference signals. In these situations, the performance of ESO presented in Section III-B may not offer satisfactory results.

However, the issue of the fast varying uncertainty too can be dealt with in the framework of ESO by employing a higher order ESO [30]. For example, if the rate of change of uncertainty is not negligible, but its second rate is negligible, a second-order ESO can be employed. It can be proved that the estimation error dynamics for the second-order ESO will be asymptotically stable if $\ddot{d}$ is negligible. As a matter of fact, one can design an rth order ESO if the rth rate of change of uncertainty is negligible.

C. Simulation Results

Simulations are carried out to verify the performance of ESO in the estimation of states and uncertainties and to verify the tracking performance of FL + ESO controller. The nominal values of the various robotic parameters, as taken from [23], are

$$MgL = 10\ \mathrm{N\cdot m} \qquad K = 100\ \mathrm{N\cdot m/rad}$$
$$I = 1\ \mathrm{kg\cdot m^2} \qquad J = 1\ \mathrm{kg\cdot m^2}.$$

The controller gains k_i's required in (16) are obtained by placing the poles at $(1+(\tau_c/4)s)^4$ with a time constant of $\tau_c = 0.35$. The ESO gains β_i's are obtained by placing the observer poles at $(1+(\tau_o/5)s)^5$ with a time constant of $\tau_o = \tau_c/3$. Note that the observer time constant is 1/3 that of the controller. The initial conditions for the observer are taken as $\hat{z}(0) = [0\quad 0\quad 0\quad 0\quad 0]^{\mathrm{T}}$, while the initial conditions of the plant are taken as $z(0) = [0\quad 5\ \mathrm{deg}/s\quad 0\quad 0]^{\mathrm{T}}$. The initial condition for link position is taken the same, as it is a measured quantity and therefore known. In simulations, uncertainty is also introduced in b by taking K as 0.8 times, I as 1.2 times, and J as 1.2 times of their respective nominal values. For the purpose of comparison of actual uncertainty with its estimated value, actual uncertainty is computed from (14) as $d = \dot{z}_4 - b_o u$, where b_o is computed using the nominal values of the parameters. With these data, simulations are carried out by taking the reference trajectory to be tracked as $z_1 = 30\sin(t)\ \mathrm{deg}$, and the results are presented. In Fig. 2(a)–(d), the estimated states are plotted along with the actual ones, and it can be seen that the ESO estimates the states accurately. The time histories of the actual and estimated uncertainties are given in Fig. 2(e), from where it can be observed that the ESO has estimated the uncertainty quite accurately. The control input history is shown in Fig. 2(f). Fig. 2(a) also shows the time history of reference trajectory, and it can be seen that the FL + ESO controller offers highly satisfactory tracking performance in spite of the considered significant uncertainties.

D. Other ESO Implementation

For the present problem, various implementations of ESO are possible. First, one can employ NESO as given by (9) or a LESO (12). Next, if it is assumed that the some estimate of the dynamics of the flexible system is known, i.e., the estimate of

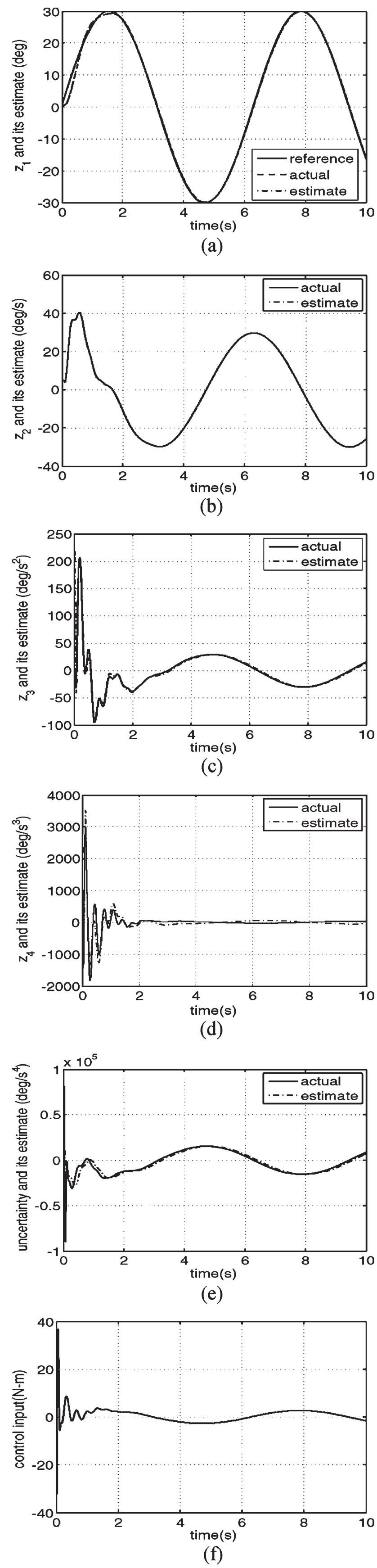

Fig. 2. State and uncertainty estimation and tracking performance.

Fig. 3. Quanser flexible-joint system—experimental setup.

$a(z)$ appearing in (14) is available, one can have two more ESO implementations, i.e., one with LESO and another with NESO. In [23], the application of NESO to the current problem is presented with the assumption of availability of the estimate of dynamics $a(z)$. In this paper, LESO is employed by assuming the dynamics $a(z)$ to be completely unknown and thus forming it as a part of the to be estimated uncertainty.

V. Experimental Results

In this paper, the efficacy of the proposed FL + ESO controller is demonstrated experimentally on the Quanser's rotary single-link flexible-joint module. The experimental setup consists of Quanser UPM 1503 module, Quanser Q4 data acquisition and control board, Quanser SRVO2 plant, Quanser rotary flexible-joint module, and a PC equipped with necessary software, including the Quanser WinCon. Q4 is a hardware-in-the-loop data acquisition and control board with an extensive range of input and output support. The UPM-1503 is a power amplifier to drive the motor. The WinCon software provides a hardware-in-the-loop simulation environment, i.e., a Simulink-based user-designed controller can be run in real time using the WinCon environment. The rotary single-link flexible-joint module consists of a rigid beam mounted on a flexible joint that rotates via a dc motor. The joint deflection is measured using a sensor. The experimental system is similar in nature to the control problems encountered in large geared robot joints where flexibility is exhibited in the gearbox. The objective is to design a feedback controller such that the tip of the beam tracks a desired command while minimizing joint deflection and resonance in the system. The experimental set permits one to validate a controller designed for the flexible-joint module to track a desired tip angle position. The experimental setup is as shown in Fig. 3, and more details about the same can be found in [31] and [32].

A. *Mathematical Model*

In the Quanser's rotary single-link flexible-joint module, a rigid link is mounted on a flexible joint that rotates via a dc motor. The dc servo motor load angle and joint deflection are measured using an optical encoder. As the Quanser's rotary single-link flexible-joint module operates in a horizontal plane, its mathematical model differs from the one given in (1). The difference arises due to the absence of gravity in the former case. The equations of motion for the Quanser's module, as given in [32], are

$$\begin{aligned}\ddot{\theta} + F_1\dot{\theta} - \frac{K_{\text{stiff}}}{J_{\text{eq}}}\alpha &= F_2 V_m \\ \ddot{\alpha} - F_1\dot{\theta} + \frac{K_{\text{stiff}}(J_{\text{eq}}+J_{\text{arm}})}{J_{\text{eq}}J_{\text{arm}}}\alpha &= -F_2V_m \end{aligned} \tag{24}$$

where

$$F_1 \triangleq \frac{\eta_m\eta_g K_t K_m K_g^2 + B_{\text{eq}}R_m}{J_{\text{eq}}R_m}$$
$$F_2 \triangleq \frac{\eta_m\eta_g K_t K_g}{J_{\text{eq}}R_m}.$$

The quantities appearing in these expressions are as follows: θ is the motor load angle, α is the link joint deflection, η_m is the motor efficiency, η_g is the gearbox efficiency, K_t is the motor torque constant, K_m is the back electromotive force constant, K_g is the gearbox ratio, B_{eq} is the viscous damping coefficient, R_m is the armature resistance, J_{eq} is the gear inertia, K_{stiff} is the spring stiffness, J_{arm} is the link inertia, and V_m is the motor control voltage. The aim is to design a controller that will place the tip of link $(y = \theta + \alpha)$ as desired, with minimum link deflection α. Now, taking $y = \theta + \alpha$ as the output of the system, the dynamics in terms of y and θ is rewritten as

$$\ddot{y} = \frac{K_{\text{stiff}}}{J_{\text{eq}}}F_3 y - \frac{K_{\text{stiff}}}{J_{\text{eq}}}F_3\theta \tag{25}$$

$$\ddot{\theta} = \frac{K_{\text{stiff}}}{J_{\text{eq}}}y - \frac{K_{\text{stiff}}}{J_{\text{eq}}}\theta - F_1\dot{\theta} + F_2V_m \tag{26}$$

where $F_3 \triangleq (1-(J_{\text{eq}}+J_{\text{arm}})/J_{\text{arm}})$. Defining the state variables as $x_1 = y$, $x_2 = \dot{y} = \dot{x}_1$, $x_3 = \theta$, and $x_4 = \dot{\theta} = \dot{x}_3$, the dynamics (25) and (26) can be written in state-space model as

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{K_{\text{stiff}}}{J_{\text{eq}}}F_3(x_1 - x_3) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{K_{\text{stiff}}}{J_{\text{eq}}}(x_1 - x_3) - F_1x_4 + F_2V_m.\end{aligned} \tag{27}$$

The difficulty of tracking control design can be reduced if one can find a direct and simple relation between the system output y and the control input V_m. This idea constitutes the basis for the FL approach. To this end, it can be verified that, for the considered output, the relative degree is four, i.e., y needs to be differentiated for four times for the input to appear explicitly. To this end, differentiating y four times and defining new states as $z_1 = y$, $z_2 = \dot{y}$, $z_3 = \ddot{y}$, and $z_4 = \dddot{y}$, the dynamics (27) can be rewritten in terms of the new coordinates as

$$\begin{aligned}\dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= z_4\end{aligned}$$

$$\begin{aligned}\dot{z}_4 =& -\frac{K_{\text{stiff}}F_1}{J_{\text{arm}}}z_2 - \frac{K_{\text{stiff}}(J_{\text{eq}}+J_{\text{arm}})}{J_{\text{eq}}J_{\text{arm}}}z_3\\ &- F_1 z_4 + \frac{K_{\text{stiff}}F_2}{J_{\text{arm}}}V_m\\ y =& z_1. \end{aligned} \tag{28}$$

B. FL + ESO-Based Controller

In (28), if one defines

$$d \triangleq -\frac{K_{\text{stiff}}F_1}{J_{\text{arm}}}z_2 - \frac{K_{\text{stiff}}(J_{\text{eq}}+J_{\text{arm}})}{J_{\text{eq}}J_{\text{arm}}}z_3 - F_1 z_4$$

$$b_o \triangleq \frac{K_{\text{stiff}}F_2}{J_{\text{arm}}}$$

the dynamics (28) reduces to that given in (14). For the resulting dynamics, the controller and the LESO design is then carried out in a similar manner as outlined in Section IV.

C. Experimental Results

To evaluate the performance of the FL + ESO-based controller, experiments are carried out on the Quanser's flexible-joint module. The nominal values of the various flexible-joint parameters [31] are as follows: $K_{\text{stiff}} = 1.248\ \text{N}\cdot\text{m/rad}$, $\eta_m = 0.69$, $\eta_g = 0.9$, $K_t = 0.00767\ \text{N}\cdot\text{m}$, $K_g = 70$, $J_{\text{eq}} = 0.00258\ \text{kg}\cdot\text{m}^2$, $J_{\text{arm}} = 0.00352\ \text{kg}\cdot\text{m}^2$, and $R_m = 2.6\ \Omega$. The controller gains k_i's are obtained by placing the poles at $(1+(\tau_c/4)s)^4$ with a time constant of $\tau_c = 0.35$. The ESO observer gains β_i's are obtained by placing the observer poles at $(1+(\tau_o/5)s)^5$ with a time constant of $\tau_o = \tau_c/5 = 0.07$. Note that the observer time constant is 1/5 that of the controller. The initial conditions for the plant are taken as $z(0) = [0\quad 0\quad 0\quad 0]^{\text{T}}$, while the initial conditions for the observer are taken as $\hat{z}(0) = [0\quad 0\quad 0\quad 0\quad 0]^{\text{T}}$. The controller–observer was implemented on a PC having WinCon software. The desired trajectory to be tracked was taken as $30\sin(t)$ deg. The control signal generated was applied to the module through Quanser's interfacing hardware board. The experimental results are shown in Fig. 4. In Fig. 4(a), the tracking performance is presented from where one can observe that the link has tracked the reference quite accurately. The estimated uncertainty is shown in Fig. 4(b), and the control input history is shown in Fig. 4(c). It may also be noted that the effect of measurement noise is visible in the estimated uncertainty and so in the control signal, and the proposed design has offered satisfactory performance notwithstanding the noisy measurement.

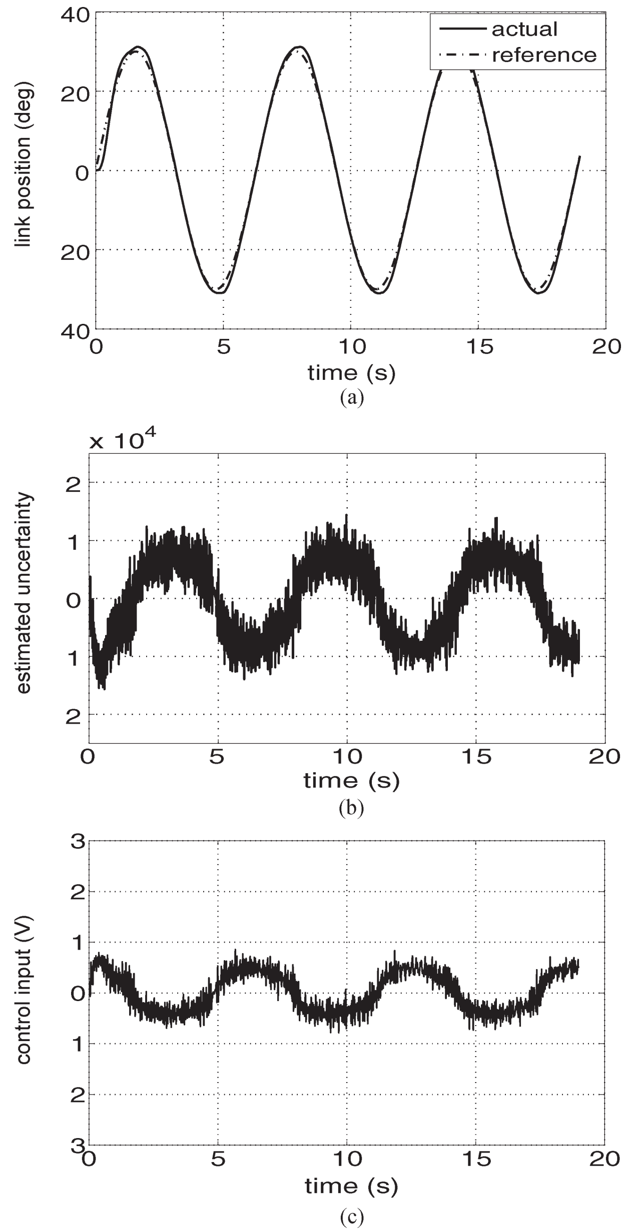

Fig. 4. Experimental tracking performance.

VI. Conclusion

In this paper, a trajectory tracking controller for flexible-joint robotic system based on FL and ESO has been presented. The well-known drawback of controller based on FL, viz., the degradation of performance in the presence of modeling uncertainties, has been removed and demonstrated through simulation and experimental validation. The proposed approach shows that the estimation of states and the uncertainties can be accomplished in an integrated manner using ESO. The simulations also show that the FL + ESO controller achieves accurate tracking of the desired trajectory. Experimental results have shown that the proposed approach is effective in practice.

References

[1] M. W. Spong and M. Vidyasagar, *Robot Dynamics and Control*. New York: Wiley, 1989.

[2] S. K. Spurgeon, L. Yao, and X. Y. Lu, "Robust tracking via sliding mode control for elastic joint manipulators," *Proc. Inst. Mech. Eng. I*, vol. 215, no. 4, pp. 405–417, 2001.

[3] Y. C. Chang, B. S. Chen, and T. C. Lee, "Tracking control of flexible joint manipulators using only position measurements," *Int. J. Control*, vol. 64, no. 4, pp. 567–593, 1996.

[4] J. Hernandez and J.-P. Barbot, "Sliding observer-based feedback control for flexible joints manipulator," *Automatica*, vol. 32, no. 9, pp. 1243–1254, Sep. 1996.

[5] M. W. Spong, "Modeling and control of elastic joint robots," *Trans. ASME, J. Dyn. Syst., Meas., Control*, vol. 109, no. 4, pp. 310–319, Dec. 1987.

[6] F. Ghorbel, J. Y. Hung, and M. W. Spong, "Adaptive control of flexible-joint manipulators," *IEEE Control Syst. Mag.*, vol. 9, no. 7, pp. 9–13, Dec. 1989.

[7] J. H. Oh and J. S. Lee, "Control of flexible joint robot system by backstepping design," in *Proc. IEEE Int. Conf. Robot. Autom.*, Albuquerque, NM, Apr. 1997, pp. 3435–3440.

[8] J. S. Chen and Y. H. Chen, "Robust control of nonlinear uncertain system: A feedback linearization approach," in *Proc. 30th IEEE Conf. Decision Control*, Brighton, U.K., Dec. 1991, pp. 2515–2519.

[9] J. Huang, C. F. Lin, J. R. Cloutier, J. H. Evers, and C. D'Souza, "Robust feedback linearization approach to autopilot design," in *Proc. IEEE Conf. Control Appl.*, Sep. 13–16, 1992, vol. 1, pp. 220–225.

[10] J.-J. E. Slotine and J. K. Hedrick, "Robust input–output feedback linearization," *Int. J. Control*, vol. 57, no. 5, pp. 1133–1139, 1993.

[11] R. B. Fernandez, "Robust feedback linearization through sliding mode control," in *Proc. 29th IEEE Conf. Decision Control*, Honolulu, HI, Dec. 1990, pp. 3398–3399.

[12] K. Youcef-Toumi and O. Ito, "A time delay controller for systems with unknown dynamics," *Trans. ASME, J. Dyn. Syst., Meas., Control*, vol. 112, no. 1, pp. 133–142, Mar. 1990.

[13] S. E. Talole, A. Ghosh, and S. B. Phadke, "Proportional navigation guidance using predictive and time delay control," *Control Eng. Pract.*, vol. 14, no. 12, pp. 1445–1453, Dec. 2006.

[14] S. E. Talole and S. B. Phadke, "Robust input–output linearisation using uncertainty and disturbance estimation," *Int. J. Control*, vol. 82, no. 10, pp. 1794–1803, Oct. 2009.

[15] A. Patel, R. Neelgund, A. Wathore, J. P. Kolhe, M. M. Kuber, and S. E. Talole, "Robust control of flexible joint robot manipulator," in *Proc. IEEE ICIT*, Dec. 2006, pp. 649–653.

[16] S. E. Talole and S. B. Phadke, "Model following sliding mode control based on uncertainty and disturbance estimator," *Trans. ASME, J. Dyn. Syst., Meas., Control*, vol. 130, no. 3, p. 034 501, May 2008.

[17] C. E. Hall and Y. B. Shtessel, "Sliding mode disturbance observer-based control for a reusable launch vehicle," *J. Guid. Control Dyn.*, vol. 29, no. 6, pp. 1315–1328, Nov./Dec. 2006.

[18] J. Han, "From PID to active disturbance rejection control," *IEEE Trans. Ind. Electron.*, vol. 56, no. 3, pp. 900–906, Mar. 2009.

[19] W. Wang and Z. Gao, "A comparison study of advanced state observer design techniques," in *Proc. Amer. Control Conf.*, Denver, CO, Jun. 2003, pp. 4754–4759.

[20] Z. Gao, "Scaling and bandwidth-parameterization based controller tuning," in *Proc. Amer. Control Conf.*, Denver, CO, Jun. 4–6, 2003, pp. 4989–4996.

[21] D. Yoo, S. S.-T. Yau, and Z. Gao, "Optimal fast tracking observer bandwidth of the linear extended state observer," *Int. J. Control*, vol. 80, no. 1, pp. 102–111, Jan. 2007.

[22] H. K. Khalil, *Nonlinear Systems*. Englewood Cliffs, NJ: Prentice-Hall, 2002.

[23] S. E. Talole and S. B. Phadke, "Extended state observer based control of flexible joint system," in *Proc. IEEE ISIE*, vol. 30. Cambridge, U.K.: Univ. Cambridge, Jun. 30–Jul. 2, 2008, pp. 2514–2519.

[24] Y. Xia, P. Shi, G. P. Liu, D. Rees, and J. Han, "Active disturbance rejection control for uncertain multivariable systems with time delay," *IET Control Theory Appl.*, vol. 1, no. 1, pp. 75–81, Jan. 2007.

[25] B. Sun and Z. Gao, "A DSP-based active disturbance rejection control design for a 1-kW H-bridge DC–DC power converter," *IEEE Trans. Ind. Electron.*, vol. 52, no. 5, pp. 1271–1277, Oct. 2005.

[26] J. Su, W. Qiu, H. Ma, and P.-Y. Woo, "Calibration-free robotic eye-hand coordination based on an auto disturbance-rejection controller," *IEEE Trans. Robot.*, vol. 20, no. 5, pp. 899–907, Oct. 2004.

[27] Y. X. Su, C. H. Zheng, and B. Y. Duan, "Automatic disturbances rejection controller for precise motion control of permanent-magnet synchronous motors," *IEEE Trans. Ind. Electron.*, vol. 52, no. 3, pp. 814–823, Jun. 2005.

[28] Y. Huang, K. Xu, J. Han, and J. Lam, "Flight control design using extended state observer and non-smooth feedback," in *Proc. 40th IEEE Conf. Decision Control*, Orlando, FL, Dec. 2001, pp. 223–228.

[29] R. Zhang and C. Tong, "Torsional vibration control of the main drive system of a rolling mill based on an extended state observer and linear quadratic control," *J. Vib. Control*, vol. 12, no. 3, pp. 313–327, 2006.

[30] R. Miklosovic, A. Radke, and Z. Gao, "Discrete implementation and generalization of the extended state observer," in *Proc. Amer. Control Conf.*, Minneapolis, MN, Jun. 14–16, 2006, pp. 2209–2214.

[31] *Introduction to WinCon & SRV02*. Quanser Inc., Markham, ON, Canada, 2008.

[32] *Rotary Flexible Joint User Manual*. Quanser Inc., Markham, ON, Canada, 2008.

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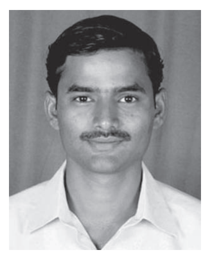

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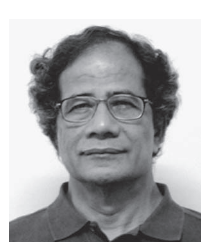

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