PhD Position in Humanoid Locomanipulation

Exercises

Shivesh Kumar, Assistant Professor, Chalmers University of Technology, Gothenburg, Sweden

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Abstract

This document contains some basic exercises that evaluate your suitability for working in my research group at Chalmers. Please provide your solutions in the form of PDF (we highly appreciate LateX skills) and also working code (wherever asked) in a language of your choice (preferably C++, Matlab, Ruby, Julia or Python[1]).

1 Mechanics

This section contains exercises from the area of mathematical modeling of robotic systems. These are further divided into topological analysis, geometric & kinematic modeling, dynamic modeling etc.

1.1 Topology and Mobility Analysis

A robot is mechanically constructed using a set of links and joints. Topology of a mechanism describes the existence and arrangement of different rigid links and joints. Topology of a mechanism can be captured by the mathematical notion of a *graph* where links are denoted as nodes and joints are denoted as edges.

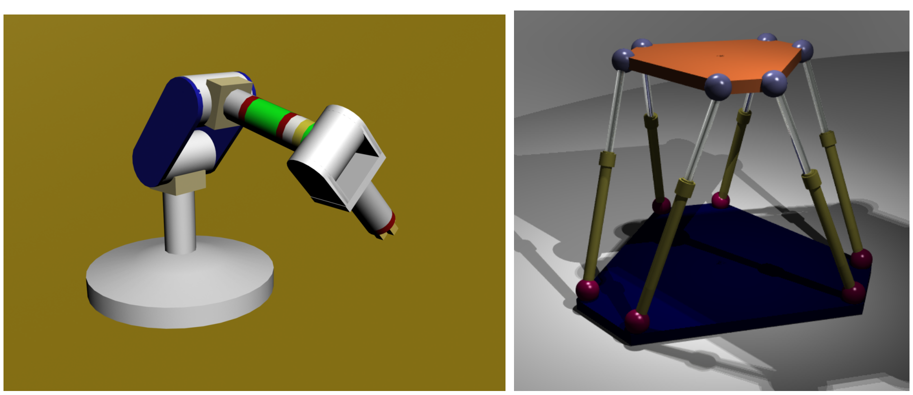

Figure 1: Left: Serial Robot, Right: Parallel Robot (Source: Wikipedia)

Exercise 1 *The most typical robot architectures of serial and parallel types are shown in Figure 1. The serial robot on the left has six revolute joints and six moving links. The parallel robot on the right has six spherical joints each on the base and end effector of the robot connected by a prismatic joint. Draw their topological graphs.*

The Chebychev–Grübler–Kutzbach criterion estimates the degree of freedom (DOF) of a kinematic chain, that is, a coupling of rigid bodies by means of mechanical constraints. The general mobility of a robot can be estimated by the following criteria:

$$m_s(\mathcal{M}) = s(N - j - 1) + f = s(-c) + f \ , \tag{1}$$

[1]If you are a Python or Julia user, you can provide your solutions in Jupyter notebook.

where

- s – motion parameter (= 3 for planar and spherical mechanisms, = 6 for spatial mechanisms)
- N – Number of links in the mechanism
- j – Total number of joints
- c – Number of independent closed loops
- f – sum of DOF of each joint.

Exercise 2 *Using Equation 1, compute the mobility of two robots shown in Figure 1. Additionally,*

1. *Provide the simplified version of the formula in Equation 1 for serial robots.*
2. *Does the formula work for redundant degrees of freedom when applied to parallel mechanisms? If not, what simplification one needs to make for using it to compute the mobility of the parallel robot in Figure 1.*

Exercise 3 *It seems almost magical that a simple formula like Chebychev–Grübler–Kutzbach criterion can be used to estimate the general mobility of a system. Does it always work? Can you find some counter examples where this formula fails?*

1.2 Geometry and Kinematics

Kinematics is referred to as the *geometry* of motion. The two main problems in geometric modeling or finite kinematics are: computation of end-effector pose from the joint coordinates (forward kinematics) and computation of joint coordinates from the end-effector pose (inverse kinematics). These could be expressed at the velocity and acceleration levels by means of 1st and 2nd order time differentiation of the kinematics expressions.

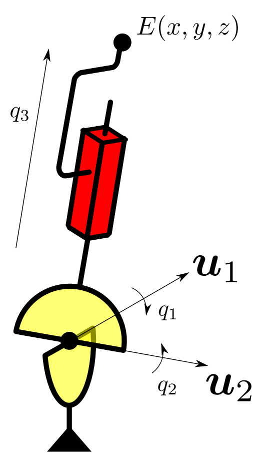

Figure 2: A 3 DOF robotic leg with 2 revolute and 1 prismatic joints

Exercise 4 *A serially connected 3 DOF robotic leg with 2 orthogonally intersecting revolute joints and a 1 DOF prismatic joint is shown in Figure 2. Given q_1 and q_2 denote the joint angles in the first two revolute joints and q_3 denote the linear displacement in the prismatic joint,*

1. *name the geometric object where the end-effector point E lives*
2. *provide the expression for forward kinematics i.e. $(x, y, z) = f(q_1, q_2, q_3)$*
3. *provide the expression for inverse kinematics i.e. $(q_1, q_2, q_3) = f^{-1}(x, y, z)$*
4. *write a program (in a language of your choice) to verify your forward and inverse kinematics for a few randomly selected poses. Also, provide a simple visualization with your program.*
5. *plot the workspace of the robot using your code.*

Exercise 5 *Consider a slider-crank linkage of type 1-RR$\underline{P}$R shown in Figure 3 which involves a linear actuator and three revolute joints. Given that q_1 denote the output joint angle and q_3 denote the input prismatic joint displacement,*

1. *provide the expression for forward kinematics i.e. $q_1 = f(q_3)$*
2. *provide the expression for inverse kinematics i.e. $q_3 = f^{-1}(q_1)$*

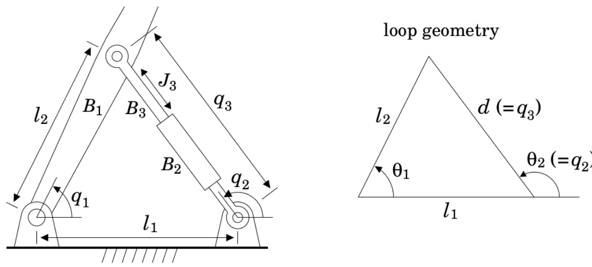

Figure 3: Slider crank linkage (1-RRPR)

3. *write a program (in a language of your choice) to verify your forward and inverse kinematics for a few randomly selected poses, also provide a simple visualization with your program*
4. *assume that maximum velocity available at the actuator is $\dot{d}_{max}$, find an expression for the maximum output angular velocity $\dot{q}_{1max}$*
5. *assume that maximum force available at the actuator is f_{max}, find an expression for the maximum output torque τ_{max}*
6. *if exists, list any singular configurations of this mechanism.*

Exercise 6 *Donald is from the United States of America and Angela comes from Europe. They have to work together on a robotics project and they don't always agree with each other. While Donald is obsessed with Imperial system of units, Angela likes to use the modern metric system. They are together working on a 6 DOF robotic manipulator (with 3 translational and 3 rotational DOFs) mounted on a 3 DOF gantry crane which can place the base of the robot in any 3D position $(x_B, y_B, z_B) \in \mathbb{R}^3$. Angela develops the software for the robot manipulator and Donald develops the software for the 3 DOF gantry crane. They are exchanging black-box models and are not allowed to modify each others code.*

1. *Donald provided Angela a model which converts the base of the robot manipulator expressed in inches into actuator commands for the crane. Angela prefers to work with meters. Can she find a linear scaling factor which will help her convert the 3D position in $\in \mathbb{R}^3$ from meters to inches? Explain your answer.*
2. *Angela provided Donald a inverse kinematics model which converts the pose of the robot manipulator (translational position is expressed in meters and rotational pose is expressed in quaternion) into actuator commands for the crane. Donald prefers to work with inches for representing the translational part $(x_E, y_E, z_E) \in \mathbb{R}^3$ and degrees for representing the rotational part (roll, pitch, yaw angles). Can he find a linear scaling factor which will help him convert the 6D pose from metric system to imperial system? Explain your answer.*
3. *Is it possible for them to use each other models at all? If yes, how would they do it?*

1.3 Dynamics

Dynamics involves the study of forces and moments acting on the system and their acceleration response. The two main problems in dynamics are: computing the forces required to produce a desired acceleration (inverse dynamics) and computing the acceleration response of a system when subjected to some input forces (forward dynamics). While inverse dynamics is useful for the control of a robot, forward dynamics is used to develop simulation model.

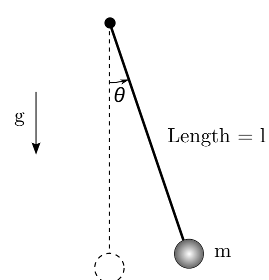

Figure 4: Simple pendulum

Exercise 7 *Consider a pendulum robot with a bob of mass m connected to the ground link by a massless link of length l in Figure 4. Denote with θ the angle made by the bob with the vertical axis in the anti-clockwise sense.*

1. *Provide the expression for torque acting at the revolute joint (τ) in terms of $(\theta, \dot{\theta}, \ddot{\theta})$ [inverse dynamics].*
2. *Provide the expression for acceleration at the revolute joint ($\ddot{\theta}$) in terms of (τ) [forward dynamics].*
3. *Write a program to simulate the free-fall of the pendulum i.e. $\tau = 0.0$ by combining the expression in (2) and a simple Newton-Euler numerical integration scheme. Also, provide a simple visualization with your program.*

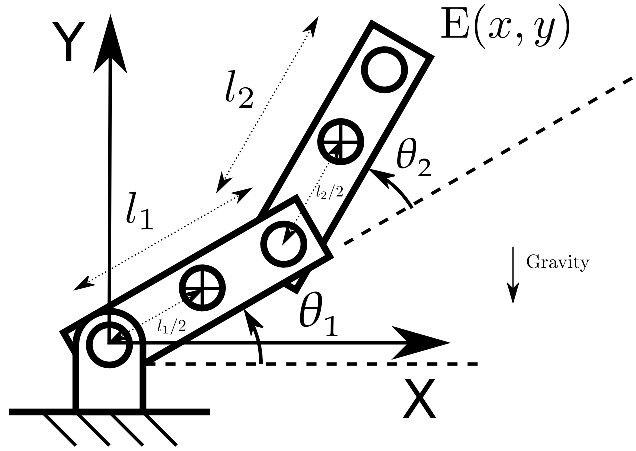

Figure 5: A planar 2R robot arm with two degrees of freedom

Exercise 8 *Consider a planar 2R robot with two degrees of freedom in Figure 5. The link lengths are denoted as l_1 and l_2 and the center of mass lies exactly at the center of the links i.e. at a distance of $l_1/2$ and $l_2/2$ from their parent joint axes. Denote with $\boldsymbol{q} = (\theta_1, \theta_2)$ the joint coordinates of the robot.*

1. *Using the principle of Lagrangian mechanics, derive the equations of motion for the system in the form $\boldsymbol{\tau} = \boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{G}$ where $\boldsymbol{M}$ denotes the joint space mass-inertia matrix of the system, $\boldsymbol{C}$ denotes the coriolis-centrifugal matrix, $\boldsymbol{G}$ is the vector of gravity forces and $\boldsymbol{\tau}$ is joint space torques vector.*
2. *Write a symbolic program using Matlab symbolic toolbox/Sympy (open source option) or Maple to derive the above expressions and verify your answer.*
3. *Derive the operational space formulation of the robot dynamics in the following form: $\boldsymbol{\tau} = \boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{G}$ where $\boldsymbol{x} = (x, y)$ is the task space coordinates of the end-effector.*

Exercise 9 *Consider a rigid body in n-dimensional Euclidean space with a known fixed pose.*

1. *What is the minimum number of scalar parameters to describe its dynamics (i.e. mass-inertia matrix) when $n = 1$, $n = 2$ and $n = 3$?*
2. *What is the main property of mass-inertia matrix? Explain.*
3. *In Exercise 6, suppose Donald and Angela measure the kinetic energy of the system in their respective imperial and metric unit systems. Would they be able to convert each other's energy computation by a linear scaling factor?*
4. *The kinetic energy of a system is given by $K = \frac{1}{2}\boldsymbol{V}^T\boldsymbol{M}\boldsymbol{V}$. In Exercise 6, suppose that Donald likes to express the mass-inertia matrix of the robot in the base frame ($\boldsymbol{M}_B$) and Angela likes to express the mass-inertia matrix in the end effector frame ($\boldsymbol{M}_E$) of the robot. They use them to compute the kinetic energy of the system. Will they get the same values?*

2 Control

The essence of control is to get a desired behavior from the complicated robot and environment dynamics that the robot is subjected to. Control systems usually have to work within the constraints given by the dynamics, the actuator limits, and the environment itself. Furthermore, some of the goals of the control system within these constraints could be to use as little energy as possible to reach the desired goal in minimum time. Such control problems can then be written as optimization problems which then allows us to use a rich toolbox of optimization methods to solve constrained control problems in robotics. In the following exercise, we look at the simplest form of such problems.

Exercise 10 *In Exercise 7, we created a dynamic simulator for the simple pendulum. In this exercise, we will aim at controlling the dynamic pendulum by applying torques at the motor to achieve a desired goal. One of the classical problems in control is to get the pendulum to* ***swing-up****. Swing-up refers to moving the pendulum from its lower position to an upright position ($\theta = 0 \to \pi$). The swing-up can have various constraints (such as limited torque, velocity, etc..) or various goals (such as minimum time). In this exercise, we will be creating a swing-up controller with some of these constraints and goals.*

For this exercise, use $m = 0.5$, $l = 0.5$, $g = 9.81$, and damping $b = 0.1$.

1. *From Exercise 7, we have a simulator for the pendulum dynamics. Assuming the actuation motor can apply any commanded torque, make a controller to go from $(\theta, \dot{\theta}) = (0, 0)$ to $(\theta, \dot{\theta}) = (\pi, 0)$. Prove that PD setpoint control is asymptotically stable from any initial state (Hint: use Krasovskii-LaSalle invariance principle).*
2. *Now, consider the torque limit to be 1 Nm, try the same controller with this torque limit. Does it still work? If not, explain why and reason if a swing-up can be performed with a torque limit of 1 Nm.*
3. *Write a controller that can perform a swing-up with 1 Nm torque limit and verify using the simulator written in Exercise 7.*
   (Hint: Think about the energy that can be given to the system during each direction of the swing for e.g. swing at a park, gymnastic acrobats, etc.)
4. *Run the simulation for twice the time required for the 1 Nm torque limit controller to do the swing-up. What do you observe? Can this controller maintain the upright position? How could the performance be improved?*

Exercise 11 *In the previous exercises, we have learned a bit about the kinematics, dynamics, and control of robotic systems. However, there is yet another step to bring these ideas to real robots. This difference between simulations and real robots has traditionally been referred to as* ***sim-to-real gap****. Here, we will be looking at two places where the sim-to-real gap can be reduced: better simulation and better control. These tasks respectively focus on one aspect of simulation which can be improved to get a better understanding of the dynamics and create robust controllers to handle the uncertainties in the real-system to a certain extent.*

1. *Implement a Fourth order Runge-Kutta Integrator for the Pendulum Simulator and compare the difference with respect to the Euler integrator implemented in Exercise 7(3). Demonstrate the trade-offs between them.*
2. *Linearize the equations of motion around the state $(\theta, \dot{\theta}) = (\pi, 0)$ and derive the optimal controller (LQR) equations for this linear system. Improve your swing-up stability with this approach.*

Exercise 12 *Until now in the above exercises, we assumed that there is an ideal source to deliver required torques τ and provide measurements for the state feedback $(\theta, \dot{\theta})$. Electric DC motors are inarguably the most common source of actuation in robotics. These may additionally be augmented with encoders, gears, bearings, strain gauges for torque measurements, low level controller units to develop a robot joint unit.*

1. *Discuss the advantages and disadvantages of* ***direct drive actuator, geared DC actuator, series elastic actuator****. Motivate your choice for selecting an actuator for real world implementation of a torque limited simple pendulum introduced in Exercise 10.*
2. *Draw the torque (τ) vs speed (ω) diagram of a current- and voltage-limited electric DC motor and highlight the continuous operating range. Label* ***no load speed (ω_0), maximum continuous torque (τ_{cont}), rated mechanical power (P_{max}), and stall torque (τ_{stall})*** *in this diagram. How does the gear ratio affect the operating range plot?*
3. *Describe the effect of* ***friction*** *and* ***rotor inertia*** *on the robot joint control and how can they be mathematically modeled and compensated in the robot control loop.*

3 Questionnaire

1. During your studies, please select if you have taken any formal/related courses in:
   - ☐ Mechanics or Applied Mechanics
   - ☐ Mechanism Theory
   - ☐ Machine Design
   - ☐ Rigid Body Mechanics or Multi-Body Dynamics
   - ☐ Modeling and Control of Robot Manipulators
   - ☐ Humanoid Robotics
   - ☐ Biomechanics
   - ☐ Linear Control Theory
   - ☐ Non-Linear Control Theory
   - ☐ Introduction to Robotics
   - ☐ Artificial Intelligence or Machine Learning
   - ☐ Linear Algebra
   - ☐ Advanced Calculus
   - ☐ Probability Theory
   - ☐ Object Oriented Programming

   Additionally, mention your grade along with maximum possible grade in the applicable subjects.
2. How do you find the overall difficulty level of the exercises? Categorize the exercises.
   - ☐ Easy
   - ☐ Medium
   - ☐ Hard
3. What kind of additional help did you seek while solving the exercises?
   - ☐ Textbooks
   - ☐ Wikipedia
   - ☐ Research papers
   - ☐ Other online sources
   - ☐ Friends/Colleagues
4. Please categorize your attempt to solve the exercises into:
   - ☐ Solved without any external reference
   - ☐ Solved with the help of known textbooks
   - ☐ Solved with the help of online content (Wikipedia, online blogs etc.)
   - ☐ Solved after reading research papers

   List the exercise numbers in front of each option above.
5. Please select the programming languages where you have a working knowledge.
   - ☐ C/C++
   - ☐ Python
   - ☐ Matlab
   - ☐ Julia
   - ☐ Ruby
   - ☐ Mathematica
6. Please specify if you have worked with any symbolic manipulation packages.
   - ☐ Matlab Symbolic Toolbox
   - ☐ Maple
   - ☐ Singular
   - ☐ Sympy
7. Have you worked with any industrial robot platforms?
   - ☐ Universal robots
   - ☐ KUKA robots
   - ☐ Stäubli robots
   - ☐ PUMA 560
   - ☐ Others
8. Have you ever built your own robot as a hobby or group project at the University? If yes, describe your robot and your role in the project.
9. Have you worked with a version control system? If yes, which one?
   - ☐ Git
   - ☐ SVN
10. Do you have LateX skills?