

# **Multivariable Control Problems**

**Coursework**

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# 1 Topology and Mobility Analysis

## Exercise 1

### Serial Robot

The topology tree will contain seven (including the base) nodes depicting the links and six edges depicting the joints. The base link is depicted by B.

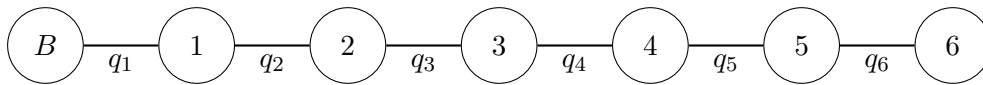


Figure 1.1: Graph Topology of Serial Robot

### Parallel Robot

The topology tree will contain fourteen (including the top-platform and base) nodes depicting the links and eighteen edges depicting the joints (six of which are the prismatic joints). The prismatic joint edges are colored green and the spherical joints are colored blue. The base link is depicted by B and the top platform is depicted by EE.

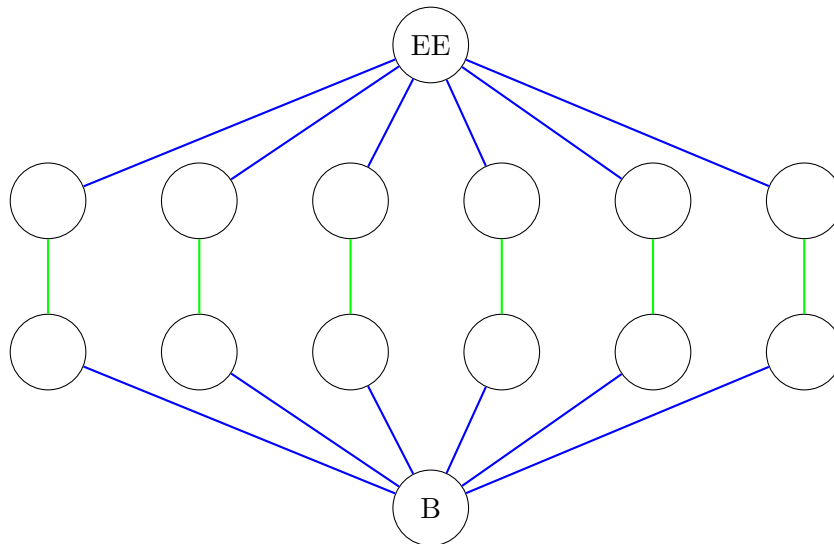


Figure 1.2: Graph Topology of Parallel Robot

## Exercise 2

Both the serial and parallel robots are considered spatial mechanisms hence for both  $s = 6$ .

For the serial robot which has six revolute joints (one degree of freedom each),

$$\begin{aligned} N &= 7, j = 6, f = 6 \\ m_s &= 6 \end{aligned}$$

The parallel robot has twelve spherical joints (three degrees of freedom each) and six prismatic joints (one degree of freedom each) with fourteen links.

$$\begin{aligned} N &= 14, j = 18, f = 42 \\ m_s &= 12 \end{aligned}$$

## Simplification for Serial Robots

The Chebychev-Grübler-Kutzbach formula can be simplified for serial manipulators which consist of no redundant degrees of freedom (eg. due to multiple closed kinematic chains). The simplification is given by

$$m_s = \sum_{i=1}^j f_i \quad (1.1)$$

where  $f_i$  is the degrees of freedom allowed by the  $i^{\text{th}}$  joint.

## Modification for Parallel Robots

It is clear that (1.1) doesn't apply to parallel mechanisms which have multiple kinematic loops and hence redundant degrees of freedom. Moreover, the original Chebychev-Grübler-Kutzbach formula also sometimes doesn't work for all parallel mechanisms. For instance, the parallel mechanism given in this question is the *Stewart Platform*, which always provides six degrees of freedom to the top platform, hence the formula should ideally give 6 as an answer (which infact happens for the configuration where universal joints connect the legs and the base). But the redundant degrees of freedom introduced due to spherical joints on both sides of the legs are not considered. This redundant degree of freedom is the rotation of the leg about its own axis, which essentially has no influence on the resultant mechanism motion. Hence, this redundancy needs to be compensated. For six legs, the redundant degrees of freedom will be six. The modified Chebychev-Grübler-Kutzbach becomes

$$m_s = s(N - j - 1) + \sum_{i=1}^j f_i - C \quad (1.2)$$

where  $C = 6$  and we get the correct answer of six for the given parallel mechanism.

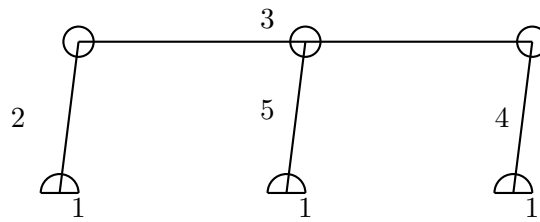
**Exercise 3**

Figure 1.3: Four-bar linkage with one additional link and two additional revolute joints

Figure 1.3 depicts a two-dimensional four-bar linkage with an additional link (5) and two additional revolute joints connecting link 5 to link 3 and the base. According to the Chebychev-Grübler-Kutzbach formula

$$N = 5, j = 6, f = 6, s = 3$$

$$m_s = 0$$

But in reality, the described mechanism will have a single degree of freedom. Hence, in this case the formula fails. The reason is that in the above mechanism, the constraint introduced by the additional joints is redundant, and the intrinsic assumption of the Chebychev-Grübler-Kutzbach formula is that the constraints provided by the joints are independent.

## 2 Geometry and Kinematics

### Exercise 4

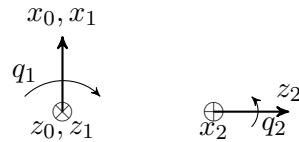
The geometric object in which the end-effector lives is called the *Workspace*.

### Forward Kinematics

The formulation of the forward kinematics is based on [?], which makes use of the *Denavit-Hartenberg* parameters.

For the given formulation, it is assumed that  $q_1 \in [0, \pi]$  and  $q_2 \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ .

For brevity, only the Z and X axes of the joint frames are drawn in 2D, axis coming out of the plane is denoted by  $\oplus$  and axis going into the plane is denoted by  $\otimes$ .



## 3 Dynamics

## 4 Control