Linear Statistical Models — Quiz 06 — Full Name ______________________

1. We have a dataset measuring the lifetimes of three different types of batteries at three different temperatures. For each of the 9 combinations of type and temperature, we have 4 observations, for $n = 36$ lifetime observations total. Let $y_1, \ldots, y_n$ be the lifetimes in hours, $j(i)$ (1, 2, or 3) indicate the battery type, and $k(i)$ indicate the level of temperature (1 = 15 degrees, 2 = 70 degrees, 3 = 125 degrees).

   Below is model for y_i, and below that is the fitted model in R:

$$Y_i = b_0 + b_{j(i)} + c_{k(i)} + (bc)_{j(i),k(i)} + \varepsilon_i, \quad \varepsilon_i \overset{ind}{\sim} N(0, \sigma^2)$$

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                           Estimate Std. Error t value Pr(>|t|)
b_0      - (Intercept)       134.75      12.99  10.371 6.46e-11 ***
b_2      - type2              21.00      18.37   1.143 0.263107
b_3      - type3               9.25      18.37   0.503 0.618747
c_2      - temp70            -77.50      18.37  -4.218 0.000248 ***
c_3      - temp125           -77.25      18.37  -4.204 0.000257 ***
(bc)22 - type2:temp70         41.50      25.98   1.597 0.121886
(bc)32 - type3:temp70         79.25      25.98   3.050 0.005083 **
(bc)23 - type2:temp125       -29.00      25.98  -1.116 0.274242
(bc)33 - type3:temp125        18.75      25.98   0.722 0.476759

Residual standard error: 25.98 on 27 degrees of freedom
F-statistic:    11 on 8 and 27 DF,  p-value: 9.426e-07
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   (i) (3) Which parameters did R set to zero?

   $b_1, c_1, (bc)_{11}, (bc)_{12}, (bc)_{21}, (bc)_{13}, (bc)_{31}$

   (ii) (3) How do you interpret the intercept?

   Expected lifetime for battery type 1 at 15 degrees

   (iii) (3) What is the estimated expected lifetime for battery type 2 at 15 degrees? (Give numbers)

   $\hat{b}_0 + \hat{b}_2 + \hat{c}_1 + \widehat{(bc)}_{21}$

   $= 134.75 + 21.00 + 0 + 0$

(iv) (3) What is the estimated expected lifetime for battery type 3 at 70 degrees?

$\hat{b}_0 + \hat{b}_3 + \hat{c}_2 + (\hat{bc})_{32}$

$= 134.75 + 9.25 - 77.50 + 79.25$

(v) (3) How do you interpret the fac_temp70 estimate (-77.50)?

Expected lifetime for battery type 1 at 70 degrees minus expected lifetime for battery type 1 at 15 degrees.

2. A veterinarian is studying the growth rates of three species of felines: The bobcat ($j = 1$), cougar ($j = 2$), and jaguar ($j = 3$). She obtains a sample of 10 cats from each species (30 total), and measures their weights (in pounds) at 3 months (x_i) and 6 months of age (y_i). Here is a model for y_i:

$$Y_i = b_0 + b_{j(i)} + (c_0 + c_{j(i)})x_i + \varepsilon_i, \quad \varepsilon_i \overset{ind}{\sim} N(0, \sigma^2)$$

(i) (3) According to the model, complete this sentence: $b_0 + b_1$ is the expected . . .

6 month weight of a bobcat who weighs 0 pounds at 3 months.

(ii) (3) According to the model, complete this sentence: $c_0 + c_2$ is the expected . . .

Increase in 6 month weight for a cougar per 1 pound increase of 3 month weight

(iii) (3) What is the expected 6 month weight of a bobcat whose 3 month weight is 4 pounds?

$b_0 + b_1 + (c_0 + c_1)4$

(iv) (3) What is the expected difference in 6 month weight of a cougar and a jaguar who are both 6 pounds at 3 months? Jaguar minus cougar → should instead be cougar − jaguar

cougar: $b_0 + b_2 + (c_0 + c_2)6$

jaguar: $b_0 + b_3 + (c_0 + c_3)6$

difference: $b_3 - b_2 + (c_3 - c_2)6$

$b_2 - b_3 + (c_2 - c_3)6$

(v) (3) Explain in words why your previous answer indicates that this is an interaction model.

The diffence in six month weight between cougar and jaguar above depends on what the 3 month weight is.

OR

Since the effect of "species" depends on the value of the weights at 3 months, which is 6 at $b_2 - b_3 + (c_2 - c_3)6$

(vi) (3) Bonus: We said that the researcher is interested in growth rates. Using the same model, give a formula for how much we expect each species to grow in weight from 3 months to 6 months, and explain what parts of the model this growth depends on.

growth from 3 months to 6 months:

$$E(Y_i) - x_i = b_0 + b_{j(i)} + (c_0 + c_{j(i)})x_i - x_i$$

$$= b_0 + b_{j(i)} + (c_0 + c_{j(i)} - 1)x_i$$

Depends on both the intercepts and the slopes w.r.t 3 month weight