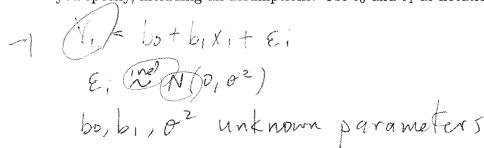
- 1. Suppose we have covariate data x_1, \ldots, x_n and response data y_1, \ldots, y_n .
 - (i) (4) Write down the simple linear statistical model for y_i , making sure to define each and every term you specify, including all assumptions. Use b_0 and b_1 as notation for the regression coefficients.



(ii) (3) Write down the formula for the sum of squares criterion.

$$rss(b_0^{\dagger},b_i^{\dagger}) = \sum_{i=1}^{n} (y_i - b_0^{\dagger} - b_i^{\dagger} x_i)^2$$

(iii) (3) Write down the formulas for the estimates of the two regression coefficients. Det me

$$\hat{b}_{i} = \frac{5}{5} \frac{xy}{5} \frac{xx}{x} = \frac{\hat{\Sigma}(x_{i} - \bar{x})^{2}}{\hat{\Sigma}(x_{i} - \bar{x})^{2}}$$

$$\frac{\hat{\Sigma}(x_{i} - \bar{x})^{2}}{\hat{\Sigma}(x_{i} - \bar{x})(y_{i} - \bar{y})}$$

$$\hat{b}_{0} = \bar{y} - \hat{b}_{i} \bar{x}$$

(iv) (3) In terms of the estimates \hat{b}_0 and \hat{b}_1 , write down the formulas for the fitted values, the residuals, and $\hat{\sigma}^2$ (the variance estimate).

$$\hat{y}_{i} = \hat{b}_{0} + \hat{b}_{i} \hat{x}_{i}$$

$$\hat{e}_{i} = \hat{y}_{i} - \hat{y}_{i}$$

$$\hat{\sigma}^{2} = \frac{1}{n-2} \sum_{i=1}^{n-2} \hat{e}_{i}^{2}$$

2. (3) Explain the difference between an estimate and an estimator.

estimate is a function of the data.
estimator is a R.V., the same function of data but with Y; replacing y;

3. (3) Name two desirable properties of estimators, and explain why they are desirable.

Unbiased sampling distribution is centered on the estimand

small variance: sampling distribution tightly centered around estimand

4. (3) In the notes, we derived the following:

$$\operatorname{Var}(\widehat{B}_0) = \sigma^2 \left(\frac{1}{n} + \frac{(\overline{x})^2}{sxx} \right), \quad \operatorname{Var}(\widehat{B}_1) = \frac{\sigma^2}{sxx}, \quad \operatorname{Cov}(\widehat{B}_0, \widehat{B}_1) = -\frac{\sigma^2 \overline{x}}{sxx}$$

Derive a simplified expression for the prediction variance $Var(\widehat{B}_0 + \widehat{B}_1x_i)$.

$$Var(\hat{\beta}_{0} + \hat{\beta}_{1}X,) = Var(\hat{\beta}_{0}) + \chi^{2}_{1}Var(\hat{\beta}_{1})$$

$$+ 2\chi_{1}Cov(\hat{\beta}_{0}, \hat{\beta}_{1})$$

$$e^{2}(\hat{x})^{2} + \chi_{1} + \chi_{1} - 2\chi_{1}X$$

$$= 5\chi_{1} + \chi_{1} - 2\chi_{1}X$$

$$= 5\chi_{1} + \chi_{1} - \chi_{1} + \chi_{2}$$

$$= o^2 \left(\frac{1}{n} + \frac{\left(x_i - \bar{x}\right)^2}{5xx} \right)$$

5. (5) The "normal equations" result from taking partial derivatives of the sum of squares criterion and setting them to zero. Here are the equations:

$$0 = \sum_{i=1}^{n} (y_i - \hat{b}_0 - \hat{b}_1 x_i) \quad \text{and} \quad 0 = \sum_{i=1}^{n} (y_i - \hat{b}_0 - \hat{b}_1 x_i) x_i$$

Solve these equations to derive expressions for \hat{b}_0 and \hat{b}_1 . You must show your work for full credit.

$$0 = \sum_{i=1}^{n} (y_{i} - \hat{b}_{0} - \hat{b}_{1}x_{i})$$

$$n\hat{b}_{0} = n\bar{y} - \hat{b}_{1}n\bar{x} \implies \hat{b}_{0} = \bar{y} - \hat{b}_{1}\bar{x}$$

$$0 = \sum_{i=1}^{n} (y_{i} - (\bar{y} - \hat{b}_{1}\bar{x}) - \hat{b}_{1}x_{i})x_{i}$$

$$0 = \sum_{i=1}^{n} (y_{i} - \bar{y})x_{i} - \hat{b}_{1}\sum_{i} (x_{i} - \bar{x})x_{i}$$

$$\sum_{i=1}^{n} (y_{i} - \bar{y})x_{i} = \hat{b}_{1}\sum_{i} (x_{i} - \bar{x})x_{i}$$

$$\sum_{i=1}^{n} (y_{i} - \bar{y})x_{i} - \sum_{i} (y_{i} - \bar{y})\bar{x} = \hat{b}_{1}\sum_{i} (x_{i} - \bar{x})x_{i}$$

$$\sum_{i=1}^{n} (y_{i} - \bar{y})x_{i} - \sum_{i} (y_{i} - \bar{y})\bar{x} = \hat{b}_{1}\sum_{i} (x_{i} - \bar{x})^{2}$$

$$\sum_{i=1}^{n} (y_{i} - \bar{y})(x_{i} - \bar{x}) = \hat{b}_{1}\sum_{i} (x_{i} - \bar{x})^{2}$$

$$\hat{b}_{1} = \sum_{i=1}^{n} (y_{i} - \bar{y})(x_{i} - \bar{x})$$

$$\sum_{i=1}^{n} (y_{i} - \bar{y})(x_{i} - \bar{x})^{2}$$