PSTAT 171. HW 6 Solution (Winter 2021)

`Instruction: Review textbook chapter 6 first. Multiple reading might help. Then try to solve the homework problems quickly.`

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1. Martin Maradiaga was considering two bond offerings for purchase on March 1, 1995. Each had a purchase price of \$10,000. Bond A was an "inflation-adjusted" 4% ten-year \$ 10,000 bond with annual coupons; the coupon payments were to be based on March 1, 1995 dollars so that the inflation-adjusted coupon rate was 4% and the bond would be redeemable at an amount worth \$10,000 in March 1, 1995 dollars. Bond B was a 7% ten-year \$10,000 par-value bond with annual coupons offered by Delta Diagnostics. Which should Mr. Maradiaga purchase if he forecasts that inflation will be at a level rate of 2.75%? Why? If inflation is actually at 2.2%, find the inflation-adjusted yield on each bond.

Both bonds A and B are 10-year, at par 10000 bond with purchase price 10000.
Bond A is at par with $F = C = 10000$, annual coupon payment $m = 1$, coupon rate $r = \alpha = 0.04$, and yield rate $j = 0.04$ (inflation adjusted) in the March 1, 1995 dollar value.

Bond B is at par with $F = C = 10000$, annual coupon payment $m = 1$, coupon rate $r = \alpha = 0.07$, coupon amount $700 = 10000 \cdot 0.07$. The non-inflation adjusted yield rate $j = 0.07$. The inflation adjusted annual yield is defined by

$$i_B := \frac{1+j}{1+r} - 1 \, ,$$

where r is the annual inflation rate.
• If the inflation rate is $r = 0.0275$, then the inflation adjusted yield is $i_B = (1.07/1.0275) - 1 = 0.041363 > 0.04$, greater than the yield rate 0.04 of Bond A. Bond B is preferred.

• If the inflation rate is $r = 0.022$, then the inflation adjusted yield is $i_B = (1.07/1.022) - 1 = 0.04696673 > 0.04$, greater than the yield rate 0.04 of Bond A. Bond B is preferred.

2. Alicia bought a newly issued \$1,000 20% ten-year bond, redeemable at \$ 1,100 and having yearly coupons. It was bought at a premium with a price of \$1,400. Alicia immediately took a constant amount D from each coupon and deposited it in a savings account earning 8% effective annual interest, so as to accumulate the full amount of the premium by a moment after the final deposit. How much did Alicia deposit each year in the 8% account?

The 10-year bond is bought at premium. The premium amount $1400 - 1100 = 300$ is covered by the annual deposits D into the 8% savings account for 10 year. The equation of value at time 10 is

$$300 = D \, s_{\overline{10}|0.08} = D \cdot \frac{1.08^{10} - 1}{0.08} = 14.48656 \, D \, ,$$

and hence, $D = 20.70885 \approx 20.71$ for the first 9 payments and the last payment (i.e., the 10th payment) is

$$20.71 - (20.71 s_{\overline{10}|0.08} - 300) \approx 20.69 \, .$$

3. A \$1,000 bond with a coupon rate of 8% has quarterly coupons and is redeemable after an unspecified number of years at \$ 957. The bond is bought to yield 12 % convertible semiannually. If the present value of the redemption amount is \$ 355.40, find the purchase price using the Makeham formula. Then check your answer using another price formula.

This is the problem of the Makeham formula. The redemption amount is 957, the coupon amount is $1000 \cdot 0.08/4 = 20$, and the modified coupon rate is $g = 20/957 = 0.02089864$. The present value of the redemption amount is given $K = Cv^n = 355.40$ with unknown n. Since the semi-annual nominal rate is 12%, the effective quarterly rate is $j = 1.06^{1/2} - 1 = 0.02956301$. By the Makeham formula we obtain the purchase price P

$$P = K + \frac{g}{j}(C - K) = 780.6822 \, .$$

Without using the Makeham formula, we use

$$355.4 = 957(1.06)^{-n/2} \quad \text{and hence,} \quad n = 2 \cdot \frac{\log(957/355.4)}{\log(1.06)} \approx 34.$$

By the basic price formula we obtain

$$P = 20a_{\overline{34}|j} + 957v_j^{34} \approx 760.68.$$

4. A fifteen-year bond, which was purchased at a premium, has semiannual coupons. The amount for amortization of the premium in the second coupon is \$ 977.19 and the amount for amortization of premium in the fourth coupon is \$ 1,046.79. Find the amount of the premium.

There are $n = 30$ semiannual coupons. By the premium-discount formula $P - C = C(g - j)a_{\overline{30}|j}$, we have $P_k = C(g-j)v_j^{n-k+1}$. Since $P_2 = 977.19$ and $P_4 = 1046.79$, we see

$$\frac{P_2}{P_4} = \frac{977.19}{1046.79} = v^2 \quad \text{and hence,} \quad v = \frac{1}{1+j} = \left(\frac{977.19}{1046.79}\right)^{1/2},$$

and

$$C(g-j) = P_k(1+j)^{n-k+1} = P_2(1+j)^{30-2+1} = 977.19 \cdot \left(\frac{1046.79}{977.19}\right)^{29/2} = 2650.007.$$

Thus, by the premium-discount formula $P - C = C(g-j)a_{\overline{30}|j}$, the discount $P - C$ is

$$P - C = C(g-j)a_{\overline{30}|j} = 2650.007 \cdot \frac{1 - v_j^{30}}{j} = 48739.16.$$

5. A three-year \$1,000 6 % bond with semiannual coupons has redemption amount \$ 1,040. Make amortization tables for this bond if it is bought to yield a nominal rate of 5% convertible semi-annually. Repeat for a nominal rate of 6% and then for a nominal rate of 7%, each convertible semiannually.

There are $6 = 3 \cdot 2$ coupons of $1000 \cdot 0.06/2 = 30$.
At 2.5% (= 5.0/2) effective semiannual rate, the price is

$$B_0 = P = 30a_{\overline{6}|0.025} + 1040v_{0.025}^6 = 1062.3.$$

Then we compute recursively $I_t = iB_{t-1}$, $P_t = 30 - I_t$, $B_t = B_{t-1} - P_t$ for $t = 1, 2, \ldots, 6$.

Time (semiannual)	Payment	Interest	Principal	Book value
1	30	26.55081	3.449187	1058.583
2	30	26.46458	3.535417	1055.048
3	30	26.37620	3.623803	1051.424
4	30	26.28560	3.714398	1047.710
5	30	26.19274	3.807258	1043.902
6	30	26.09756	3.902439	1040.000

Table 1: Amortization with rate 2.5% effective semiannual rate.

At 3% (= 6.0/2) effective semiannual rate, the price is

$$B_0 = P = 30a_{\overline{6}|0.03} + 1040v_{0.03}^6 = 1033.499.$$

Then we compute recursively $I_t = iB_{t-1}$, $P_t = 30 - I_t$, $B_t = B_{t-1} - P_t$ for $t = 1, 2, \ldots, 6$. The results are given below.

At 3.5% (= 7.0/2) effective semiannual rate, the price is

$$B_0 = P = 30a_{\overline{6}|0.035} + 1040v_{0.035}^6 = 1005.897.$$

Time (semiannual)	Payment	Interest	Principal	Book value
1	30	31.00498	-1.004981	1034.504
2	30	31.03513	-1.035131	1035.539
3	30	31.06618	-1.066184	1036.606
4	30	31.09817	-1.098170	1037.704
5	30	31.13112	-1.131115	1038.835
6	30	31.16505	-1.165049	1040.000

Table 2: Amortization with rate 3.0% effective semiannual rate.

Time (semiannual)	Payment	Interest	Principal	Book value
1	30	35.20640	-5.206404	1011.104
2	30	35.38863	-5.388628	1016.492
3	30	35.57723	-5.577230	1022.070
4	30	35.77243	-5.772433	1027.842
5	30	35.97447	-5.974468	1033.816
6	30	36.18357	-6.183575	1040.000

Table 3: Amortization with rate 3.5% effective semiannual rate.

Then we compute recursively $I_t = iB_{t-1}$, $P_t = 30 - I_t$, $B_t = B_{t-1} - P_t$ for $t = 1, 2, \ldots, 6$.

6. We are concerned with a three-year \$1,000 6% bond with semiannual coupons and a redemption amount \$1,040. Suppose that the bond was purchased on January 1, 2000. Make a chart showing the theoretical and practical dirty and clean values of the bond at the end of each quarter if the bond was purchased at a discount to yield a nominal rate of 7% convertible semiannually. Use the "30/360" basis for counting days.

There are 6 semiannual coupons of $1000 \cdot 0.06/2 = 30$ at time 0 (Jan. 1, 2000), time 1 (July 1, 2000), ... , time 5 (July 1, 2002), time 6 (Jan. 1, 2003). Here, we take 6 months as one unit of time. We shall calculate the values for each quarter (every 3 months) at time $1/2, 1, \ldots, 5/2, 6$.

We have the dirty value $\mathcal{D}_{k/2}$, the clean value $\mathcal{C}_{k/2}$, the practical dirty value $\mathcal{D}^{\text{prac}}_{k/2}$ and the practical clean value $\mathcal{C}^{\text{prac}}_{k/2}$ for $k = 1, 2, \ldots, 12$, where for odd k

$$\mathcal{D}_{k/2} = (1.035)^{1/2} B_{\lfloor k/2 \rfloor}, \quad \mathcal{C}_{k/2} = \mathcal{D}_{k/2} - 30\, s_{\overline{1/2}|0.035},$$

$$\mathcal{D}^{\text{prac}}_{k/2} = (1 + 0.035 \cdot 0.5) B_{\lfloor k/2 \rfloor}, \quad \mathcal{C}^{\text{prac}}_{k/2} = \mathcal{D}_{k/2} - 15$$

and for even k, $\mathcal{D}_{k/2} = \mathcal{D}^{\text{prac}}_{k/2} = \mathcal{C}_{k/2} = \mathcal{C}^{\text{prac}}_{k/2} = B_{k/2} = 30 a_{\overline{12-k}|0.035} + 1040 v^{12-k}_{0.035}$. Thus we obtain the following table.

k	$\mathcal{D}_{k/2}$	$\mathcal{C}_{k/2}$	$\mathcal{D}^{\text{prac}}_{k/2}$	$\mathcal{C}^{\text{prac}}_{k/2}$
1	1023.349	1008.478	1023.500	1008.500
2	1011.104	1011.104	1011.104	1011.104
3	1028.646	1013.775	1028.798	1013.798
4	1016.492	1016.492	1016.492	1016.492
5	1034.128	1019.257	1034.281	1019.281
6	1022.070	1022.070	1022.070	1022.070
7	1039.802	1024.931	1039.956	1024.956
8	1027.842	1027.842	1027.842	1027.842
9	1045.674	1030.804	1045.829	1030.829
10	1033.816	1033.816	1033.816	1033.816
11	1051.753	1036.882	1051.908	1036.908
12	1040.000	1040.000	1040.000	1040.000

7. On May 27, 1994 Jen Mago purchased a new \$18,000 fifteen-year 10% bond with annual coupons and a redemption payment of \$19,000. Jen sold the bond to Edna Wilder on December 31, 2000.

The semi-practical clean price for the sale was \$ 18,375; this was based on her market price and the "30/360" method for counting days. Still working on a "30/360" basis, find her annual yield rate $\widetilde{j}$ and theoretical clean price $\mathcal{C}_T^{\widetilde{j}}$

There are annual coupon of 1800 every May 27 for 15 years. With the "30/360" method, the number of days from May 27, 2000 and December 31, 2000 is $31 - 27 + 30 \cdot (12 - 5) = 214$ with $f = 214/360$. The accrued interest is $1800 \cdot 214/360 = 1070$ (by practical method). The theoretical dirty price is $18375 + 1070 = 19445$. The number of days from January 1, 2001 to May 27, 2001 is $360 - 214 = 146$. Thus, we have the equation of value at time 0

$$19445 = \sum_{k=0}^{8} \frac{1800}{(1+\widetilde{j})^{k+(146/360)}} + 19000 v_{\widetilde{j}}^{8+(146/360)}.$$

Solving it for $\widetilde{j}$, we obtain $\widetilde{j} \approx 0.10052$.

Since the accrued coupon is given by

$$1800 s_{\overline{f}|\widetilde{j}} \approx 1049.16,$$

we obtain the theoretical clean price $\mathcal{C}_T^{\widetilde{j}} = 19445 - 1049.16 = 18395.84$.

8. On March 1, 1990 Juanita paid \$ 6,317 to acquire a portfolio of six \$1,000 par-value bonds. All the bonds had annual coupons. The portfolio consisted of three 12% bonds with redemption dates of March 1, 1992, 1994, and 1996 and three 10% bonds with redemption dates of March 1, 1993, 1995, and 1997. Find Juanita's yield rate.

Let us call Bond $1, 2, \ldots, 6$ with maturity March 1, 1992, 1993, ... , 1997, respectively. Then each Bond $1, 3, 5$ has annual coupons $1000 \cdot 0.12 = 120$ and each Bond $2, 4, 6$ has annual coupons $1000 \cdot 0.1 = 100$. The payments from these bonds can be tabulated as

Bond	1	2	3	4	5	6	Total
March 1, 1991	120	100	120	100	120	100	660
March 1, 1992	1120	100	120	100	120	100	1660
March 1, 1993		1100	120	100	120	100	1540
March 1, 1994			1120	100	120	100	1440
March 1, 1995				1100	120	100	1320
March 1, 1996					1120	100	1220
March 1, 1997						1100	1100

Let i denote the annual yield rate. Then the equation of value on March 1, 1990 is

$$6317 = \frac{660}{1+i} + \frac{1660}{(1+i)^2} + \frac{1540}{(1+i)^3} + \frac{1440}{(1+i)^4} + \frac{1320}{(1+i)^5} + \frac{1220}{(1+i)^6} + \frac{1100}{(1+i)^7}.$$

Solving this equation for i, we obtain $i \approx 0.094$.

9. An n-year \$ 1,000 par-value bond with 8% annual coupons has an annual effective yield of i, $1 + i > 0$. The book value of the bond at the end of the third year is \$ 990.92 and the book value of the bond at the end of the fifth year is \$995.10. Find the price of the bond.

This is (almost) Example 6.5.16. The book value B_3 at time 3 and the book value B_5 at time 5 have the relationship

$$B_5 = (1+i)B_4 - 80 = (1+i)((1+i)B_3 - 80) - 80 = (1+i)^2 B_3 - 80(1+i) - 80,$$

and hence, with $B_3 = 990.72$ and $B_5 = 995.1$, we obtain $i = 0.082775$. Hence, the price is

$$P = 80 a_{\overline{3}|i} + 990.72 v_i^3 = 985.57.$$

Flipgrid Homework Submission

`Here is the instruction how to upload your solution.`

1. Find below the submission link to Flipgrid of your section (also listed in Week 1 in GauchoSpace). There are two Fligprid groups for different sections. Please choose the correct one and login with your UCSB google email.

- Group: Y. Feng sections F 11AM/F 1PM
`https://flipgrid.com/06d874f4`
- Group: J.K. Chen section M 11 AM + M. Becerra section T 3 PM
`https://flipgrid.com/fd15312f`

- If it is your first time, then your account is automatically created.
- If you have login problems, please let the instructor know.

Upload your video HW 6 Problem 5.

♯♯ **Please read the problem carefully, since the problem is slightly different**.

That is it for HW 6! We will be having online quiz next week (Monday - Tuesday) based on the homework problems.