PSTAT 171. HW 2 (Winter 2021)

`Instruction: Review textbook chapter 2 first. Multiple reading might help. Then try to solve the homework problems quickly.`

1. Given $d^{(4)} = 0.08$, compute the equivalent rates i, d, $d^{(m)}$ and $i^{(m)}$ for each $m = 2, 6, 12$.

2. Suppose that interest is paid once every 2 years at a nominal interest rate $i^{(1/2)}$. That is, the borrower pays interest at an effective rate of $2i^{(1/2)}$ every 2 years. Find an expression for $i^{(1/2)}$ in terms of i.

3. Suppose that the accumulation function $a(\cdot)$ is given by

$$a(t) = (1.02)^t(1 + .03t)(1 - .05t)^{-1}; \quad t \geq 0.$$

Compute the force of interest δ_3 at time $t = 3$.

4. Esteban borrows \$ 20,000 and the loan is governed by compound interest at an annual effective interest rate of 6%. Esteban agrees to repay the loan by making a payment of \$ 10,000 at the end of T years and a payment of \$ 12,000 at the end of $2T$ years. Find T.

5. Suppose that there are contributions $C_{t_1}, \ldots, C_{t_n}$ at time $t_1, \ldots, t_n$, respectively. We define

$$T := \frac{1}{\log v} \log\Big(\frac{\sum_{k=1}^n C_{t_k} v^{t_k}}{\sum_{k=1}^n C_{t_k}}\Big), \quad \overline{T} := \frac{\sum_{k=1}^n t_k\, C_{t_k}}{\sum_{k=1}^n C_{t_k}}.$$

Here $v = 1/(1+i)$ and log is the natural logarithm. Verify that $\overline{T} \geq T$ for every n.

6. Define the function $f(t) = 525(1.1)^{-2t} + 525(1.1)^{-t} - 1000$, $t \geq 0$ and find the root T^*, such that $f(T^*) = 0$ by Newton's method.

7. Sandra invests \$ 10,832 in the Wise Investment Fund. Three months later her balance has grown to \$ 11,902 and she deposits \$ 2,000. Two months later her holdings are \$ 14,308 and she withdraws \$ 7,000. Two years after the initial investment, she learns that her holdings are worth \$ 12,566. Compute the approximate dollar-weighted annual yield with different methods.

8. Bright Future Investment Fund has a balance of \$ 1,205, 000 on January 1. On May 1, the balance is \$ 1,230,000. Immediately after this balance is noted, \$ 800,000 is added to the fund. If there are no further contribution to the fund for the year and the time-weighted annual yield for the fund is 16 %, what is the fund balance at the end of the year?