

PSTAT 171. HW 7 Solution (Winter 2021)

Instruction: Review textbook chapter 7 and sections 8.1-8.3. first. Multiple reading might help. Then try to solve the homework problems quickly.

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1. At the beginning of a quarter, Bridget Dubois purchases 200 shares of preferred stock for a price to provide her with a yield rate equal to 6.2%. Assuming the stipulated dividends of \$.28 per share are paid quarterly, what is Bridget's per share price?

The annual yield rate is 6.2% and the quarterly effective yield is $J = (1.062)^{1/4} - 1$. Since the dividend is 0.28 per share, the per share price is

$$\frac{0.28}{J} = \frac{0.28}{(1.062)^{1/4} - 1} \approx 18.48.$$

Note that the number of shares is not used to calculate the per share price. Since Bridget buys 200 shares of stock and each has \$ 0.28 dividends, Bridget receives $200 \cdot 0.28 = 56$ every quarter. The price of 200 shares is $56 / J \approx 3695.85$. The per share price is $3695.85 / 200 \approx 18.48$.

2. Dr. Mary Rogowski opens a new margin account at Robertson and Hendricks brokerage firm, which requires a 65% initial margin and a 45% maintenance margin. She purchases \$13,000 of stock in Bell Foods using her margin account to the full extent allowable. Prior to any interest being credited on the margin account, by what percentage may the stock fall without Dr. Rogowski's account violating the maintenance margin? Assume that she does not use the margin account for any other purpose and that no interest is credited.

The initial margin is 65% of 13000, that is, $13000 \cdot 0.65 = 8450$. Dr. Rogowski borrows $13000 \cdot 0.35 = 4550$. Suppose that the market value is x . The maintenance margin requirement of 45% implies

$$x - 4550 \geq 0.45x \quad \text{or equivalently} \quad x \geq \frac{4550}{0.55} \approx 8272.7273.$$

If $x < 8272.7273$, i.e., the price goes down by $(13000 - 8272.7273)/13000 \approx 0.363 = 36.3\%$, then there is a margin call.

3. Eleanor Michaels believes that the price of Continental Metals stock will go up. Its current price is \$35.45 per share and she has \$3,000 to invest. How many shares may she purchase with a cash brokerage account? How many may she purchase using a margin brokerage account with a 60% margin requirement? If the per share price is \$ 38.45 three months later when she sells the shares, what is her quarterly yield in each case? Assume that there are no dividends and that in the case of the margin account, Eleanor deposits exactly \$ 3,000 and interest charged to Eleanor for the three months reduces the funds received by \$ 30.

Since Eleanor has \$ 3000 cash to purchase the stock (\$ 35.45 per share), she may purchase with a cash brokerage account,

$$\frac{3000}{35.45} \approx 84 \text{ shares}.$$

For the 60 % margin requirement, she may purchase up to $3000/0.6 = 5000$, that is,

$$\frac{5000}{35.45} \approx 141 \text{ shares}.$$

If the per share price is \$ 38.45 and she uses the cash brokerage account, the investment of $35.45 \cdot 84$ becomes $38.45 \cdot 84$. The quarterly yield rate is $38.45/35.45 - 1 \approx 0.08463$, 8.463%.

If the per share price is \$ 38.45 and she uses the margin account to purchase 141 shares, then since she pays the interest \$30, she gains

$$(38.45 - 35.45) \cdot 141 - 30 = 393.$$

Here quarterly yield rate is $393/3000 \approx 0.131$, 13.1%.

4. Gregory sells a stock short at a price P and buys it back one year later for $.9P$. The required margin is 50% and interest on the margin deposit is paid at 5%. Gregory's yield for the year is 14%. Dividends are paid out at the end of the year. Find the amount of these dividends as a percent of Gregory's investment.

Let x denote the amount of the dividends. The yield rate is represented by

$$0.14 = \frac{P - 0.9P + 0.05 \cdot 0.5P - x}{0.5P}.$$

Solving for x , we obtain

$$\frac{x}{P} = 0.055.$$

Since the original investment is $0.5P$, the dividends are 11 % of the original investment.

5. Warren sells short \$3,000 of a common stock that pays dividends of \$20 after six months and \$15 after eighteen months. The margin requirement is 50%, and the annual effective interest rate on the margin account is 4%. Twenty months after he sold the stock, Warren repurchases the stock (to cover the short sale) for \$ 2,650. Find his annual yield for this twenty-month investment.

This is a short selling problem. The initial margin deposit is $3000 \cdot 0.5 = 1500$. The dividends 20 at time $6/12$ and 15 at time $18/12$ are subtracted and then adding 3000 from the selling and subtracting 2650 to buy back. The effective interest in the margin account is $i = 0.04$. Thus, with the annual yield rate j for the 20-month investment, Warren receives

$$1500(1+i)^{20/12} - 20(1+i)^{20/12-6/12} - 15(1+i)^{20/12-18/12} + 3000 - 2650 = 1500(1+j)^{20/12}$$

that is, the annual yield rate is $j \approx 0.1579$, 15.79%.

6. Yuri Popescu purchases \$16,850 of stock using his newly created margin account and borrowing to the full extent allowed. The account has a 40% maintenance margin requirement on all securities and a 50% initial margin requirement. Soon after the sale, the value of the stocks is \$18,200 and Yuri withdraws the full amount of money allowable. The price then plummets to \$13,600 and Yuri receives a margin call.

(a) How much cash would he need to deposit in order for the account to meet the maintenance margin requirement without other changes?

(b) What must be the price of marginable securities added to satisfy the 40% margin requirement? Assume no cash is added or stock sold.

(c) If the margin requirement is to be satisfied solely by liquidation of stocks, what is the market value of the stocks that must be sold?

(a) Since the initial margin requirement is 50%, the initial deposit is $16850 \cdot 0.5 = 8425$.

When the stock price goes up to 18200, then the equity becomes

$$8425 + (18200 - 16850) = 9775,$$

and so, he can withdraw the difference 675 between 9775 and $18200 \cdot 0.5 = 9100$. After the withdrawal, the cash balance is $8425 - 675 = 7750$.

When the stock price goes down to 13600. His equity becomes

$$13600 + 7750 - 16850 = 4500,$$

and he needs at least $13600 \cdot 0.4 = 5440$ for the maintenance margin. In order to meet the maintenance margin, the margin call is made for $5440 - 4500 = 940$.

(b) If he adds securities of x (instead of cash) to the margin account, then we must have

$$4500 + x \geq 0.4(13600 + x),$$

and hence, $x \geq 940/0.6 = 1566.67$. He needs to add \$ 1566.67 securities.

(c) If he liquidates y of the stock, then the value of securities is $13600 - y$. The equity is unchanged, i.e., 4500. The requirement is

$$4500 \geq 0.5(13600 - y) \quad \text{or equivalently} \quad y \geq 2600.$$

He needs to sell \$ 2600 to meet the margin requirement.

7. Leonard Talbot observes that there are three options, all with identical prices. The price of the first will increase by 10% if the market goes up, and decrease by 10% if the market goes down. The second will increase in value by 20% if the market is up, and decrease by 15% if the market is down. The third will increase by 16% if the market is up and decrease by 20% if it is down. Describe an arbitrage opportunity.

Suppose that Leonard purchases π_1 shares of the first, π_2 shares of the second and π_3 shares of the third. Assume that the short selling is allowed. Assume also that $\pi_1 + \pi_2 + \pi_3 \leq 0$ at time 0, and in the market down scenario and the market up scenario, the values of the portfolio are $0.9\pi_1 + 0.85\pi_2 + 0.8\pi_3$ and $1.1\pi_1 + 1.2\pi_2 + 1.16\pi_3$. The arbitrage opportunity means that the system of inequalities

$$\pi_1 + \pi_2 + \pi_3 \leq 0, \quad 0.9\pi_1 + 0.85\pi_2 + 0.8\pi_3 \geq 0, \quad 1.1\pi_1 + 1.2\pi_2 + 1.16\pi_3 \geq 0.$$

There are uncountably many choices of (π_1, π_2, π_3) which satisfy the arbitrage opportunity. For it, we may consider the matrix representation of the system of inequality:

$$\mathbf{A}\boldsymbol{\pi} \in \{(x_1, x_2, x_3)' \in \mathbb{R}^3 : x_1 \leq 0, x_2 \geq 0, x_3 \geq 0\},$$

where $'$ represents the matrix transpose and the matrix \mathbf{A} and the column vector $\boldsymbol{\pi}$ are defined by

$$\mathbf{A} := \begin{pmatrix} 1 & 1 & 1 \\ 0.9 & 0.85 & 0.8 \\ 1.1 & 1.2 & 1.16 \end{pmatrix}, \quad \boldsymbol{\pi} := \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}.$$

Since the matrix \mathbf{A} is nonsingular, its inverse \mathbf{A}^{-1} exists and hence, we have

$$\boldsymbol{\pi} \in \mathbf{A}^{-1}\{(x_1, x_2, x_3)' \in \mathbb{R}^3 : x_1 \leq 0, x_2 \geq 0, x_3 \geq 0\}.$$

Therefore, there are uncountably many possibility of $\boldsymbol{\pi}$.

For example, $\pi_1 = 0.2$, $\pi_2 = 0.8$ and $\pi_3 = -1$,

8. The current prices on one-year, two-year, and three-year \$10,000 zero-coupon bonds are \$9,765, \$9,428, and \$8,986.82, respectively. Find all forward rates implied by these prices.

Let r_k , $k = 1, 2, 3$ be the short rates. It follows from the zero coupon bonds, we have

$$0.9765 = \frac{1}{1 + r_1}, \quad 0.9428 = \frac{1}{(1 + r_2)^2}, \quad 0.898682 = \frac{1}{(1 + r_3)^3}.$$

Solving them for r_k , $k = 1, 2, 3$, we have

$$r_1 = 0.0241, \quad r_2 = 0.02989, \quad r_3 = 0.03625.$$

From the short rates, we compute the forward rates

$$\begin{aligned} f_{[0,1]} &= r_1 = 0.0241, \quad f_{[0,2]} = r_2 = 0.02989, \quad f_{[0,3]} = r_3 = 0.03625, \\ f_{[1,2]} &= \frac{(1 + r_2)^2}{1 + r_1} - 1 = 0.0357, \quad f_{[1,3]} = \left(\frac{(1 + r_3)^3}{1 + r_1} \right)^{1/2} - 1 = 0.0424, \\ f_{[2,3]} &= \frac{(1 + r_3)^3}{(1 + r_2)^2} - 1 = 0.0491. \end{aligned}$$

9. Calculate the total accumulated value four years from now of \$5,000 invested today and \$8,000 invested two years later, implied by the table of current spot rates below.

| Term in years | Spot rate |
|---------------|-----------|
| 1 | 2.60% |
| 2 | 3.25% |
| 3 | 3.70% |
| 4 | 4.15% |
| 5 | 4.80% |

Using the spot rates, we have the accumulation of the investment:

$$5000(1 + r_4)^4 + 8000 \cdot \frac{(1 + r_4)^4}{(1 + r_2)^2} = 14712.83.$$

Flipgrid Homework Submission

Here is the instruction how to upload your solution.

1. Find below the submission link to Flipgrid of your section (also listed in Week 1 in GauchoSpace). There are two Flipgrid groups for different sections. Please choose the correct one and login with your UCSB google email.

- Group: Y. Feng sections F 11AM/F 1PM

<https://flipgrid.com/0ec3e3ba>

- Group: J.K. Chen section M 11 AM + M. Becerra section T 3 PM

<https://flipgrid.com/25e8ce1e>

- If it is your first time, then your account is automatically created.

- If you have login problems, please let the instructor know.

Upload your video HW 7 Problem 5.

Please read the problem carefully, since the problem is slightly different.

That is it for HW 7! We will be having online quiz next week (Monday - Tuesday) based on the homework problems.