2. Let r and k denote positive integers, and set $n = rk$. An amortized loan lasting n interest periods has a payment of P at the end of each k interest periods. The effective interest rate per interest period is i.
(a) Explain why the outstanding loan balance at time jk, just after the payment of P is equal to $P \cdot a_{\overline{n-jk}|i} / s_{\overline{k}|i}$ for $j = 0, 1, 2, \ldots$ [HINT: Look at Section (4.2).]
(b) Use the result of (a) to establish that the interest in the payment at time $(j+1)k$ is $P(1 - v^{n-jk})$.

OLB_j ?

a) eir: $J = (1+i)^k - 1$

disc. factor: $v_J = \frac{1}{1+J} = \frac{1}{1+(1+i)^k-1} = \frac{1}{(1+i)^k} = (1+i)^{-k} = v^k$

prospective formula:

$$B_{jk} = P \cdot a_{\overline{r-j}|J}$$

$$= P \cdot \frac{1-v_J^{\,r-j}}{J}$$

$$= P\left(\frac{1-v^{(r-j)k}}{(1+i)^k-1}\right)$$

$$= P\left(\frac{1-v^{(r-j)k}}{i}\right)\frac{i}{(1+i)^k-1}$$

$$= P\left(\frac{a_{\overline{n-jk}|i}}{s_{\overline{k}|i}}\right)$$

interest @ $T = \frac{j+1}{k}$

b) $JB_{jk} = \left[(1+i)^k - 1\right] P\left(\frac{a_{\overline{n-jk}|i}}{s_{\overline{k}|i}}\right)$

$$= \left[(1+i)^k - 1\right] P\left(\frac{(1-v)^{(r-j)k}}{(1+i)^k-1}\right)$$

$$= P(1-v)^{(r-j)k}$$

$$= P(1-v)^{rk-jk}$$ $\ast\, n = rk$

$$= P\left(1-v^{n-jk}\right)$$