

## PSTAT 171. HW 1 (Winter 2021)

Instruction: Review textbook chapter 1 first. Multiple reading might help. Then try to solve the homework problems quickly.

1. Let  $A_K(t) = 3t^2 + 2t + 800$ . Show that the sequence of interest rate  $i_n$  is decreasing for  $n \geq 17$ .

With  $A_K(\cdot)$ , we compute the effective interest rate  $i_n$  at the  $n$ -th period

$$\begin{aligned} i_n &= \frac{A_K(n) - A_K(n-1)}{A_K(n-1)} = \frac{3n^2 + 2n + 800 - (3(n-1)^2 + 2(n-1) + 800)}{3(n-1)^2 + 2(n-1) + 800} \\ &= \frac{6n-1}{(n-1)(3n-1) + 800}; \quad n \geq 1. \end{aligned}$$

In order to compare  $i_n$  with  $i_{n+1}$ , we compute the difference  $i_{n+1} - i_n$  as a ratio

$$\begin{aligned} i_{n+1} - i_n &= \frac{6(n+1)-1}{n(3n+2)+800} - \frac{6n-1}{(n-1)(3n-1)+800} \\ &= \frac{-18n^2 - 12n + 4805}{(n(3n+2)+800)((n-1)(3n-1)+800)} \end{aligned}$$

for  $n \geq 1$ , where the denominator is positive for every  $n \geq 1$ , and the numerator

$$-18n^2 - 12n + 4805 = -18\left(n + \frac{1}{3}\right)^2 + 4807$$

is monotonically decreasing for  $n \geq 1$ , and if  $n \geq 17$ , then

$$-18n^2 - 12n + 4805 \leq -18(17)^2 - 12 \cdot 17 + 4805 < 0.$$

This implies that  $i_{n+1} - i_n < 0$  for  $n \geq 17$ . Therefore, we conclude that  $i_n$  is decreasing for  $n \geq 17$ .

If you like calculus, you can consider the differentiation  $f'(x)$  of

$$f(x) = \frac{6x-1}{(x-1)(3x-1)+800}; \quad x > 0,$$

in order to show that  $f'(x) < 0$  for  $x \geq 17$  and hence, the map  $n \mapsto i_n = f(n)$  is decreasing for  $n \geq 17$ .

Also, one can look at the ratio

$$\frac{i_{n+1}}{i_n} = \frac{18n^3 + 9n^2 + 4786n + 4005}{18n^3 - 9n^2 + 4798n - 800}; \quad n \geq 1,$$

and show that  $i_{n+1}/i_n < 1$  for  $n \geq 17$ .

2. The monthly simple interest rate is .5 %. What is the yearly simple interest rate?

The yearly simple interest rate is  $.5 \cdot 12 = 6$  %, because for the simple interest rate with annual rate  $i$ , the accumulation function is  $a(t) = 1 + it$ . When  $t = 1/12$  (year), we have the monthly rate .5 %, that is,

$$a\left(\frac{1}{12}\right) = 1 + \frac{i}{12} = 1.005,$$

and hence, solving it for  $i$ , we obtain  $i = 0.06$ .

Can you check the same problem but with the compound interest rate, replacing the simple interest by the compound interest?

3. Elliott received an inheritance from his Aunt Ruth when she died on his fifth birthday. On his eighteenth birthday, the inheritance has grown to \$ 32168. If the money has been growing by compound interest at an annual effective interest rate of 6.2 %, find the amount of money Aunt Ruth left to Elliott.

It was compounded with annual effective interest rate 6.2% between the 5th year to the 18th year for 13 years. Let  $K$  be the amount of money Aunt Ruth left to Elliott when she passed away. Then we have an equation of value for a single contribution:

$$K(1.062)^{13} = 32168.$$

Solving it for  $K$ , we obtain  $K = 32168/(1.062)^{13} = 14716.53$ .

If you use a calculator, please check how to calculate  $K$  quickly and practice it. It is important because this is an easy question, as you see, however, it also yields crucial mistakes (for exams) if you mistakenly touch different keys of your calculator.

4. A savings account earns compound interest at an annual effective interest rate  $i$ . Given that  $i_{[2,4.5]} = 20\%$ , find  $d_{[1,3]}$ .

With the accumulation function  $a(t) = (1+i)^t$ , we have

$$0.2 = i_{[2,4.5]} = \frac{a(4.5) - a(2)}{a(2)} = \frac{a(4.5)}{a(2)} - 1 = (1+i)^{2.5} - 1,$$

and hence,  $1+i = 1.2^{0.4}$ .

Thus, we have

$$d_{[1,3]} = \frac{a(3) - a(1)}{a(3)} = 1 - \frac{a(1)}{a(3)} = 1 - \frac{1}{(1+i)^2} = 1 - 1.2^{-0.8} = 0.13572.$$

We can solve the same problem in another way: it follows from

$$(1 - d_{[1,3]})(1 + i_{[1,3]}) = 1$$

that

$$d_{[1,3]} = 1 - \frac{1}{1 + i_{[1,3]}} = 1 - \frac{1}{(1+i)^2} = 1 - 1.2^{-0.8} = 0.13572.$$

This is the homework submission problem. The problem in Flipgrid is slightly modified from the original problem.

## Flipgrid Homework Submission

Here is the instruction how to upload your solution.

1. Find below the submission link to Flipgrid of your section (also listed in Week 1 in GauchoSpace). There are two Fligprid groups for different sections. Please choose the correct one and login with your UCSB google email.

- Group: Y. Feng sections F 11AM/F 1PM

<https://flipgrid.com/209700b1>

- Group: J.K. Chen section M 11 AM + M. Becerra section T 3 PM

<https://flipgrid.com/1e8543e6>

- If it is your first time, then your account is automatically created.

- If you have login problems, please let the instructor know.

2. There are two topics (two video clips) you are asked to submit:

(1) Upload your video “Hello, I am...” to introduce yourself: how to pronounce your name, and something we should know about you.

(2) Upload your video HW 1 Problem 4. Please read the problem carefully, since the problem is slightly different.

That is it for HW 1! We will be having online quiz next week (Monday - Tuesday) based on the homework problems.