PSTAT 171. HW 3 (Winter 2021)

`Instruction: Review textbook chapters 2 and 3 (up to section 3.5) first. Multiple reading might help. Then try to solve the homework problems quickly.`

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1. A buyer of a 2018 Toyota RAV4 has a choice of 0% financing for 60 months or a \$ 3,000 cash back incentive. He plans to make no down payment. The buyer is able to qualify for 3.9 % annual effective financing for 5-years with level end-of-month payments through his credit union and thereby take advantage of the cash back incentive. Let Y denote his negotiated price for the RAV4. Suppose $Y \ge \$32900$. Which would be preferable, the 0 % dealer financing or the \$ 3,000 cash back incentive?

We compare the monthly payments of two options: $Y/60$ (0% finance) and $(Y-3000)\,/\,a_{\overline{60}|j}$ (\$ 3,000 cash back incentive), where $j = (1.039)^{1/12} - 1$ for $Y \ge 32900$. The inequality

$$\frac{Y}{60} \le \frac{Y-3000}{a_{\overline{60}|j}}$$

is equivalent to

$$Y \ge \frac{3000}{a_{\overline{60}|j}} \Big(\frac{1}{a_{\overline{60}|j}} - \frac{1}{60} \Big)^{-1} = 32866.3 \,.$$

This means that if $Y \ge 32900$, then 0% finance is a better option.

2. Tracy receives payments of \$ X at the end of each year for n years. The present value of her annuity is \$ 493. Gary receives payments of \$ $3X$ at the end of each year for $2n$ years. The present value of his annuity is \$ 2,748. Both present values are calculated at the same annual effective interest rate. Find v^n.

We have the system of equations:

$$X a_{\overline{n}|} = 493 \,, \quad 3X a_{\overline{2n}|} = 2748 \,.$$

Dividing both sides of the second equation by those of the first equation, we obtain

$$\frac{2748}{493} = \frac{3X a_{\overline{2n}|}}{X a_{\overline{n}|}} = \frac{3(1-v^{2n})}{1-v^n} = 3(1+v^n)\,,$$

and hence,

$$v^n = \frac{2748}{493 \cdot 3} - 1 = 0.858 \,.$$

3. Given that $a_{\overline{n+1}|i} - a_{\overline{n}|i} = .177208656$ and $\ddot{a}_{\overline{n+1}|i} - \ddot{a}_{\overline{n}|i} = .185248436$, find the integer n.

It follows from the definition of the annuities that

$$v = \frac{1}{1+i} = \frac{a_{\overline{n+1}|} - a_{\overline{n}|}}{\ddot{a}_{\overline{n+1}|} - \ddot{a}_{\overline{n}|}} = \frac{.177208656}{.185248436}\,,$$

and hence, solving

$$.177208656 = a_{\overline{n+1}|} - a_{\overline{n}|} = v^{n+1}$$

for n, we obtain

$$n = \frac{\log(.177208656)}{\log(.177208656/.185248436)} - 1 = 38 \,.$$

4. Suppose that $\ddot{a}_{\overline{n}|i} = 31.61667882$ and $s_{\overline{n+1}|i} = 64024.90944$. Determine i and n.

We shall use the relationships $\ddot{s}_{\overline{n}|i} + 1 = s_{\overline{n+1}|i}$ and $\ddot{s}_{\overline{n}|i} = (1+i)^n \ddot{a}_{\overline{n}|i}$. Using these relationships, we have

$$(1+i)^n = \frac{\ddot{s}_{\overline{n}|i}}{\ddot{a}_{\overline{n}|i}} = \frac{s_{\overline{n+1}|i} - 1}{\ddot{a}_{\overline{n}|i}} = \frac{64024.90944 - 1}{31.61667882} = 2025.004 \,.$$

It follows from $\ddot{a}_{\overline{n}|i} = (1-(1+i)^{-n})/d$ that

$$d = \frac{1-(1+i)^{-n}}{\ddot{a}_{\overline{n}|i}} = \frac{1-\ddot{a}_{\overline{n}|i}/(-1+s_{\overline{n+1}|})}{\ddot{a}_{\overline{n}|i}} = \frac{1}{\ddot{a}_{\overline{n}|i}} - \frac{1}{-1+s_{\overline{n+1}|i}}$$

$$= \frac{1}{31.61667882} - \frac{1}{64024.90944-1} = 0.03161326$$

and

$$i = \frac{d}{1-d} = 0.03264528\,.$$

Thus, we conclude from $(1+i)^n = 2025.068$ and $i = 0.03264528$ that

$$n = \frac{\log 2025.068}{\log(1+0.03264528)} = 237\,.$$

5. Suppose \$ 40,000 was invested on January 1, 1980 at an annual effective interest rate of 7% in order to provide an annual (calendar-year) scholarship of \$ 5,000 each year forever, the scholarships paid out each January 1. In what year can the first \$ 5000 scholarship be made?

We wait for $(n+1)$ years to gain the sufficient amount fo fund, that is,

$$40000(1+0.07)^n \geq 5000 a_{\overline{\infty}|0.07}\,.$$

This inequality implies

$$n \geq \frac{\log(5000/(0.07\cdot 40000))}{\log(1.07)} = 8.569762\,.$$

Rounding up the value, we obtain the smallest integer 9 greater than 8.569762. Therefore, the first award can come out on January 1, 1990.

Note that here we use the perpetuity-immediate, i.e., the first award payment is one year after. This is the reason why we set $n+1$.

6. Given $54\ddot{a}_{\overline{\infty}|i} = 1000$, find $s_{\overline{22}|i}$.

Because of $\ddot{a}_{\overline{\infty}|i} = 1/d$ and $54\ddot{a}_{\overline{\infty}|i} = 1000$, we have $d = 54/1000$ and

$$i = \frac{d}{1-d} = \frac{54}{946} = \frac{27}{473}\,.$$

Therefore,

$$s_{\overline{22}|i} = \frac{(1+i)^{22}-1}{i} = 41.89597\,.$$

7. Alice owned an annuity which had level annual payments for twelve consecutive years, the first of these being in exactly twelve years. She sold it, and the selling price of \$ 21,092.04 was based on a yield rate for the investor of 7.8%. What is the amount of the level payments?

This is a deferred annuity. Let K be the level payment amount. Since at time 11, the value is $Xa_{\overline{12}|}$, the time-0 value is

$$(1.078)^{-11} X a_{\overline{12}|0.078} = 21,092.04\,.$$

Solving it, we obtain $X = 6328$.

8. Catfish Hunter's 1974 baseball contract with the Oakland Athletics called for half of his \$100,000 salary to be paid to a life insurance company of his choice for the purchase of a deferred annuity. More precisely, there were to be semi-monthly contributions in Hunter's name to the Jefferson Insurance Company with the first payment on April 16 and the final payment on September 30. We suppose that the first eleven of these were to be for \$ 4,166.67, and the final payment was to be for four cents less. (\$ 4,166.67 × 12 = \$ 50,000.04.) Using an annual effective interest rate of 6 % (a rate that figures in a six-year personal loan of \$ 120,000 that Oakland's owner Charlie Finley had made to Hunter in 1969 and then promptly recalled), find the value of the specified payments to the insurance company at the scheduled time of the last payment. (Hunter wished to have such an annuity in lieu of immediate salary for tax reasons. Finley claimed that he was fulfilling the contract when he offered Hunter a \$ 50,000 check at the end of the season. Finley's default on his contractual obligation led to Hunter's historic free agency. The New York Yankees signed Hunter to a five-year, \$ 3,750,000 contract.)

With $j = (1.06)^{1/24} - 1 = .002431$, we have

$$4166.67 s_{\overline{12}|j} - 0.04 = 50673.92 .$$

9. Mr. Bell buys a home for an unspecified amount. He pays a down payment of \$ 20,000 and finances the remainder for 15 years with level end of- month payments of \$1,692. The annual effective interest rate for the first five years is 4 %, and thereafter it is 6%. Mr. Bell sells the house just after making his 100th mortgage payment. The selling price is \$ 258,000. How much money will Mr. Bell get at closing? (Remember, the loan holder is paid first, and then Mr. Bell receives the balance of the inflow from the resale.)

There are 180 monthly payments in 15 years. The balance just after his 100th payment (still 80 payments are remained) is

$$1692 a_{\overline{80}|j} = 111894.78 , \quad \text{where} \quad j = (1.06)^{1/12} - 1 = 0.48676\%.$$

Mr. Bell gets

$$258000 - 111894.78 = 146105.22 .$$

Flipgrid Homework Submission

`Here is the instruction how to upload your solution.`

1. Find below the submission link to Flipgrid of your section (also listed in Week 1 in GauchoSpace). There are two Fligprid groups for different sections. Please choose the correct one and login with your UCSB google email.

- Group: Y. Feng sections F 11AM/F 1PM

`https://flipgrid.com/16564876`

- Group: J.K. Chen section M 11 AM + M. Becerra section T 3 PM

`https://flipgrid.com/56060440`

- If it is your first time, then your account is automatically created.
- If you have login problems, please let the instructor know.

Upload your video HW 3 Problem 4. Please read the problem carefully, since the problem is slightly different.

That is it for HW 3! We will be having online quiz next week (Monday - Tuesday) based on the homework problems.