PSTAT 171. List of Review Exercises (Winter 2021)

Instruction: Review your notes from the lectures and textbook before the midterm exam. Multiple reading might help. Then try to take a bird's eye view and solve some problems below quickly, in order to find out what kind of problems you need to review or more time, i.e., your weak points. These problems come from the examples in the lectures and textbook as well as homework problems and discussion sections. You can find some hints and full solutions from there (the last 8 problems are from sections 3.7–3.12). After that, practice the problems you need for a couple of more times.

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1. Consider an investment of $500 for 5 years and suppose it earns simple interest at a rate of 10% per year. a) What is its accumulated value at time 4.25? b) What if the interest is added only at the end of year?

2. Compare the following two investments by analyzing their accumulation at the end of 3 years: [A:] Invest $100 for 3 years to earn simple interest at a rate of 8% per year; [B:] Invest $100 for 1 year to earn simple interest at a rate of 8% per year; then withdraw the proceeds and reinvest it for 2 more years.

3. Brad borrows $5000 from Bank on 1998/10/14 at 8% simple interest rate effective annually and agree to repay the loan on 1999/05/07. What is the amount of Brad's required payment using the three different methods (Actual/actual, 30/360, Actual/360)?

4. Deposit $1,000 at an account which earns interest at annual rate of 10% in first year and 15% in the second year. What's the balance at the end of 2 years?

5. A savings account earns compound interest at an annual effective interest rate i. Given that $i_{[2,4.5]} = 20\%$, find $d_{[1,3]}$.

6. Alice owes $1,000 due in 3.5 year. How much should she invest now (at time 0) to accumulate to exactly enough to pay her debt at time 3.5? Assume the effective annual rate of interest is i.

7. If $i = 10\%$, what is $i^{(2)}$?

8. Find the accumulated value of $5,000 invested for 5 years at a rate of $i^{(4)} = 5\%$.

9. Suppose interest is credited at an annual discount rate of 8% compounded quarterly. • Find Accumulation Value (AV) of $5,000 invested for 5 years. • Find the Present Value (PV) of $1,000 due in 3 years.

10. Given $i^{(4)} = 8\%$, find the equivalent i, d and $d^{(4)}$.

11. (Unknown Principal) Anita's aunt opens an account and deposits $C for her. The account earns compound interest with an annual effective interest rate of 8%. Five years after the account was open, Anita closes the account and received $3,673.32. Find C.

12. (Unknown interest rate) Anita deposits $400 in an account that grows by compound interest. After 5 years the balance has grown to $463.71. What's the annual effective interest rate this account earns?

13. Chen opened an account with $ 1,000 and $i = 0.04$ T years ago. Currently, the balance is $ 1342 . What is T?

14. John borrows $1,000. The loan is governed by compound interest at an annual effective interest rate of 10%. John repays $600 at the end of the one year and plan to complete repayment of the loan with a payment of $P at the end of the second year. Find P.
(*Consider time 0, time 1 and time 2 equations of value respectively.*)

15. Caitlin opens a savings account with a deposit of $5000. She deposits $3,000 a year later and $2,000 a year after that. The account grows by compound interest at a constant annual effective rate i. Find i in the following situations. [1] Just after Caitlin's $2,000 deposit, her balance is $11,000. [2] Three years after Caitlin opened the account, it has a balance of $11,000.

16. Franklin borrows $ 1000 with annual effective interest rate $i = 0.1$. He agrees to repay the loan by two equally spaced payments of $ 525. When should he make these payments.

17. In return for an immediate payment of $1,000, a borrower agrees to pay $600 after 2 years and $500 after 4 years. What is the yield rate of this loan? What if the second payment is due after 3 years, instead of 4 years?

18. On January 1, 1995, Siobhan's investment account balance was $ 8,412; She deposited $ 1000 on Nov. 1, 1995; She withdrew $ 600 on Nov. 1, 1997; Her balance on July 1, 1998 was $ 9,620. What is the approximate annual yield for the whole period?

19. On Jan. 1, 1995 Saul's investment account balance was $ 7688; Every month he deposited $ 100 starting from Feb. 1, 1995 to Dec. 1, 1997. On Jan. 1, 1998 the balance was 10830. What is the approximate annual yield for the three-year period?

20. Sandra invests $10,832 in the Wise Investment Fund. Three months later her balance has grown to $11,902 and she deposits $2,000. Then, two months later her fund holdings are $14,308 and she withdraws $7,000. Two years after her initial investment, she learns that her holdings are worth $12,566.

    [(a)] Write an equation of value involving the exact dollar-weighted annual yield i over the two-year period. [(b)] Compute the approximate dollar-weighted annual yield over the investment period using the simple interest approximate formula and again using the basic midpoint approximate formula.

21. (The average date κ for contributions) The balance on Jan. 1, 2002 was $ 320K and the balance on Jan. 1, 2003 was $ 374K. The amount of interest earned was $ 14K. The approximate annual yield rate was 4 %. What is the dollar-weighted average date of contributions?

22. Franklin's investment fund had a balance of $290,000 on January 1, 1995 and a balance of $448,000 two years later. The amount of interest earned during the two years was $34,000, and the annual yield rate on the fund was 5.4%. Estimate the (dollar-weighted) average date of contributions to the account.

23. Recall lecture 4 and compare the performance of the following two funds:

    [A:] $B_0 = \$15,000$, $C_{0.25} = \$20,000$, $B_1 = \$38,865$, $B_{0.25-} = \$15,408$.

    [B:] $B_0 = \$15,000$, $C_{0.25} = -\$10,000$, $B_1 = \$5,936$, $B_{0.25-} = \$15,408$.

24. Arthur buys $2,000 worth of stock. Six months later, the value of the stock has risen to $2,200 and Arthur buys another $1,000 worth of stock. After another eight months, Arthur's holdings are worth $2,700 and he sells off $800 of them. Ten months later, Arthur finds that his stock has a value of $2,100.

    (a) Compute the annual time-weighted yield rate of the stock over the two-year period.

    (b) Compute the annual dollar-weighted yield for Arthur over the two-year period.

25. Margarette receives an allowance of $ 20 at the end of each month. Find the value on Jan. 1 of her allowances for the three month period consisting of January, February and March, using a monthly effective interest rate of 0.5 %.

26. Candace wishes to buy a new car. Her budget allows her to make monthly payments of $ 500, and she has saved $ 1800 for a down payment. Candace qualifies for a 36 month auto loan with payments at the end of each month which has a nominal interest rate of 4.8% convertible monthly. How expensive a car can Candace buy?

27. Seth wishes to borrow $ 4400 so he can pay his tuition. He qualifies for a two-year loan with a monthly effective interest rate of .25% and level payments. Find the amount of Seth's monthly payments under the loan.

28. Liang wishes to accumulate 200,000 in a college fund for his new born daughter. he wishes the money to be available to her on her 18th birthday, and he is prepared to make level contributions on her firth through eighteenth birthdays with annual effective discount rate of 5 %. How large will the contributions need to be?

29. Hyun wishes to contribute $ 100 at the end of each month to his savings account until the account has a balance of at least 3000 at which time he will buy his grandmother's piano. His saving account has annual interest rate of 5.4 % convertible monthly. How many months it will take Hyun to accumulate the needed money, and how much money will he then have saved?

30. You borrow $ 10,000 and agree to repay $ 1000 at the end of each year for n years and a final smaller payment of $ X (≤ 1000) at the end of $n+1$ year. Find n, assuming the effective rate of interest of 4 % per year.

31. Finn wants to buy a car. He can afford repayments of $600 per month. The loan compony advertises an effective rate of interest of 6% per year on a 3-year loan (payable at the end of each month). The car he wants costs $35,000 today. How much would the down payment required today?

32. Omar had a dividend payment of $ 100 directly deposited to his savings account on the first day of each month. Find the accumulated value of the payments at the end of the year. Assume a nominal discount rate of 4.8%.

33. Dr. Street began making contributions to a new retirement account on her 30th birthday. She made a contribution of 4000 at the beginning of each year through her 64 birthday. Starting at age 65 and continuing through her 80th, she made a level withdrawal on her birthday. Find the amount of these withdrawals if they completely exhaust the balance in her account. Assume annual effective rate 6 % until she is 65, then 5 % thereafter.

34. Ms. Larson has saved 20000. On January 1, she purchases a perpetuity with annual end-of-year payments. The perpetuity price is based on annual effective interest rate of 5 %. What are the annual payments for her perpetuity?

35. Norman wishes to leave an inheritance to three charities. The total inheritance is a series of level payments at the beginning of each year forever. He wishes charities A and B to share the payments equally for 10 years, and then all payments revert to charity C. If the shares received by all three charities have the same value at the time fo the bequest, find the annual effective interest rate i.

36. A 2% Consol is a UK bond that pays £0.50 at the end of every 1/4 year infinitely (i.e., £2 per year). The price of the Consol is £35.25. What is the annual nominal rate of return convertible quarterly?

37. On her 18th birthday, Latisha receives an annuity that is to pay $ 5000 on her 25th through 39th birthdays.

Calculate the value of the annuity

(1) on her 18th birthday ($_{7|15}\ddot{a} = {}_{6|15}a$),

(2) on her 50th birthday,

(3) on her 30th birthday,

using an annual effective interest rate of 5 %.

38. Marlene received a gift of a 30 year annuity-immediate on the day she was born. The annuity pays $ 1000 on her first 10 birthdays, $ 2000 on her next 10 birthdays and $ 3000 on the following 10 birthdays.

(1) Find the value of annuity on the day she was born $i^{(4)} = 5\%$

(2) If the value of the annuity on the day she was born is 25000, then find the annual effective rate of interest.

39. Sasha repays a loan on the first of the month by paying $ 80 at the end of each of the next thirty months, including this month. The effective rate of the loan is 0.4 %. Find the loan balance immediately after the 12th payment.

40. A loan of $ 20000 is repaid by payments of $ 2500 at the end of each year and a final smaller payment one year after the last $ 2500 payment. $i = 8\%$. Find the outstanding loan balance just after the borrower has made 6 payments totaling $ 15000.

41. The Harpers purchase a home for $ 256000 with a downpayment of 40000 and finance the remainder with thirty year mortgage at $i = 6.5\%$. Just after having made their 96th end-of-month payment, the sales price is $ 282000 and the closing costs are 3 % of the selling price. The outstanding loan balance is deducted from the amount the Harpers receive and sent to the lender.

42. Waswate took out a 60-month car loan with an interest rate of 3 % convertible monthly, monthly payments of $ 252.65. He forgot to pay the 14th and 30th payments. What is the outstanding balance at the end of three years, just after he has made the payment scheduled at that time?

43. A loan is repaid by 20 end-of-quarter payments of 1000. In the first two years, the interest is 6 % convertible quarterly, and in the last three year the interest is 8 % convertible quarterly. Find OLB_6 right after the 6th payment and OLB_{15} right after the 15th payment.

44. Dimas receives 20-year annuity-due with payments of $ 1000 each July 1st and payments of $ 2000 each January 1. If the payments are all left to accumulate in a new account earning interest at an annual effective interest rate of 5 %, what is the accumulated value exactly 20 years.

45. An annuity with payments at the end of each quarter for 12 years: the first payment is 200, the second payment is 300, the third payment is 150 and fourth payment is 400. Find the accumulated value of this annuity at the end of 12th year with nominal quarterly interest rate 7.2 % .

46. Dr. Hamilton makes monthly contributions to her savings account of a nominal discount rate of 5.4 % convertible monthly. In the first year, she deposits 600 at the end of each month, and in the second year she makes the end-of-month deposits of 1000, and in the third year her contributions are

$$1200, 1400, -5000, 2000, 2200,$$

and then ends with seven deposits of 1600. Find the balance of her saving account at the end of the three years.

47. Consider an annuity-immediate with annual 7 payments: the first payment is 2000 and after that the payments increase by 2 % . Assume the annual effective rate of interest is 3 %. Find the present value of this annuity.

48. Susan receives an annuity for her 18th birthday. The annuity pays $ 2000 on her 19th birthday and then has an annual payment on each of her birthdays through her 35th birthday. Each year the payments increase by $ 500. Find the value of the annuity on Susan's 18th birthday, assuming the annual effective rate of interest is 4.2 %.

49. Mr. Dunn deposits $ 6000 at the end of each year for 20 years into an investment fund which pays out interests at annual effective interest rate of 6 %. He is only able to reinvest this interest at an annual effective interest rate of 4 %. What is the accumulated value of his investments at the time of the last investment.

50. Alfonso has an annuity which pays annually from Oct. 30, 2010 to Oct. 30, 2024. The payments start from $ 4000 and increase by $ 800 every year. He reinvests the payments into a savings account with a 4 % annual effective interest rate. What is the balance of his savings account on Oct. 30, 2025.

51. Rafael purchases a perpetuity-immediate that pays $ 1000 at the end of each of the first eleven years and then has increasing payments by $ 180 each year. Find the price with a 5 % annual effective interest rate.

52. Melanie purchased a fifteen-year annuity that pays $ 2100 at the end of year every year. The price was 24000. What is the yield rate?

53. Gian invests $ 58000 to buy an annuity of $ 7000 at the end of the year for thirteen years. The annuity pays $ 7000 each year. He immediately deposits $ 4000 in a savings account that earns 9%. Find the annual yield.

54. Serena invests a total of $ 10,000. She buys an annuity-due which pays $ 1000 for 10 years. The price is based on an effective interest rate of 8 %; She reinvests the payments in a savings account at an annual effective interest rate of 7 %. The balance of her $ 10,000 is invested in a 10-year certificate of deposit with a nominal annual interest rate of 9 % convertible quarterly. Find the annual effective yield.

55. Hideo deposits $ 1500 at the beginning of each year for 16 years in a fund earning an annual effective rate of interest of 6 %. The interest from this fund is paid out annually and can only be reinvested at an effective annual rate of 5.2%. At the end of twenty years, Hideo liquidates his assets. What is the yield rate for the 20 year investment?

56. Jamaar borrows $ 9600 from a bank in order to purchase a car. Jamaar can afford monthly payments of $ 345. The nominal interest rate convertible monthly is 5.4%. Explain the different payment schedules.

a) drop payment with the level payments of $ 345

b) balloon payment with the level payments of $ 345

c) balloon payment in 24 months.

57.

$$a(t) = \frac{t^2}{2} + \frac{3}{2}t + 1, \quad v(t) =$$

58. What is the accumulated value of a 5-year annuity immediate with $a(t) = 1/(1 - 0.04t)$, $t \geq 0$?

59. During the years 1994 through 1998, the interest rate are

Year	'94	'95	'96	'97	'98
Interest rate (%)	6	7	6.5	6	5.25

Bob deposited $ 10000 at the beginning of each of the years 1994 through 1998. What is the balance at the end fo 1998?

60. Derartu deposits $ 3000 in the fund at the beginning of 1966 and leaves it on deposit through 1972. What is the balance at the end of 1972?

Year of Investment	year 1	year 2	year3	year 4	ultimate rate	Year of first ultimate rate
1965	0.05	0.055	0.0475	0.05	0.055	1969
1966	0.06	0.525	0.05	0.06	0.0575	1970
1967	0.05	0.05	0.0625	0.06	0.0625	1971
1968	0.0675	0.07	0.07	0.0675	0.0615	1972

61. Consider a fund that originated in 1965.

a) What is the interest rate in effect for 1968?

b) What is the interest rate in effect for 1967?

Year of Investment	year 1	year 2	year 3	year 4	ultimate rate	Year of first ultimate rate
1965	0.05	0.055	0.0475	0.05	0.055	1969
1966	0.06	0.525	0.05	0.06	0.0575	1970
1967	0.05	0.05	0.0625	0.06	0.0625	1971
1968	0.0675	0.07	0.07	0.0675	0.0615	1972

62. If the portfolio yield method is used, then the ultimate rates are used from the start.

Assume that the ultimate rates are 5 % in 1966-1968.

What is the balance at the end of 1972?

Year of Investment	year 1	year 2	year 3	year 4	ultimate rate	Year of first ultimate rate
1965	0.05	0.055	0.0475	0.05	0.055	1969
1966	0.06	0.525	0.05	0.06	0.0575	1970
1967	0.05	0.05	0.0625	0.06	0.0625	1971
1968	0.0675	0.07	0.07	0.0675	0.0615	1972

63. Let $A_K(t) = 3t^2 + 2t + 800$,. Show that the sequence of interest rate i_n is decreasing for $n \geq 17$.

64. The monthly simple interest rate is .5 %. What is the yearly simple interest rate?

65. Elliott received an inheritance from his Aunt Ruth when she died on his fifth birthday. On his eighteenth birthday, the inheritance has grown to $ 32168. If the money has been growing by compound interest at an annual effective interest rate of 6.2 %, find the amount of money Aunt Ruth left to Elliott.

66. A savings account earns compound interest at an annual effective interest rate i. Given that $i_{[2,4.5]} = 20\%$, find $d_{[1,3]}$.

67. Given $d^{(4)} = 0.08$, compute the equivalent rates i, d, $d^{(m)}$ and $i^{(m)}$ for each $m = 2, 6, 12$.

68. Suppose that interest is paid once every 2 years at a nominal interest rate $i^{(1/2)}$. That is, the borrower pays interest at an effective rate of $2i^{(1/2)}$ every 2 years. Find an expression for $i^{(1/2)}$ in terms of i.

69. Suppose that the accumulation function $a(\cdot)$ is given by

$$a(t) = (1.02)^t (1 + .03t)(1 - .05t)^{-1}; \quad t \geq 0.$$

Compute the force of interest δ_3 at time $t = 3$.

70. Esteban borrows $ 20,000 and the loan is governed by compound interest at an annual effective interest rate of 6%. Esteban agrees to repay the loan by making a payment of $ 10,000 at the end of T years and a payment of $ 12,000 at the end of $2T$ years. Find T.

71. Suppose that there are contributions $C_{t_1}, \ldots, C_{t_n}$ at time $t_1, \ldots, t_n$, respectively. We define

$$T := \frac{1}{\log v} \log \left(\frac{\sum_{k=1}^n C_{t_k} v^{t_k}}{\sum_{k=1}^n C_{t_k}} \right), \quad \overline{T} := \frac{\sum_{k=1}^n t_k C_{t_k}}{\sum_{k=1}^n C_{t_k}}.$$

Here $v = 1/(1+i)$ and log is the natural logarithm. Verify that $\overline{T} \geq T$ for every n.

72. Define the function $f(t) = 525(1.1)^{-2t} + 525(1.1)^{-t} - 1000$, $t \geq 0$ and find the root T^*, such that $f(T^*) = 0$ by Newton's method.

73. Sandra invests $ 10,832 in the Wise Investment Fund. Three months later her balance has grown to $ 11,902 and she deposits $ 2,000. Two months later her holdings are $ 14,308 and she withdraws $ 7,000. Two years after the initial investment, she learns that her holdings are worth $ 12,566. Compute the approximate dollar-weighted annual yield with different methods.

74. Bright Future Investment Fund has a balance of $ 1,205, 000 on January 1. On May 1, the balance is $ 1,230,000. Immediately after this balance is noted, $ 800,000 is added to the fund. If there are no further contribution to the fund for the year and the time-weighted annual yield for the fund is 16 %, what is the fund balance at the end of the year?

75. A buyer of a 2018 Toyota RAV4 has a choice of 0% financing for 60 months or a $ 3,000 cash back incentive. He plans to make no down payment. The buyer is able to qualify for 3.9 % annual effective financing for 5-years with level end-of-month payments through his credit union and thereby take advantage of the cash back incentive. Let Y denote his negotiated price for the RAV4. Suppose $Y \geq \$32900$. Which would be preferable, the 0 % dealer financing or the $ 3,000 cash back incentive?

76. Tracy receives payments of $ X at the end of each year for n years. The present value of her annuity is $ 493. Gary receives payments of $ $3X$ at the end of each year for $2n$ years. The present value of his annuity is $ 2,748. Both present values are calculated at the same annual effective interest rate. Find v^n.

77. Suppose that $\ddot{a}_{\overline{n}|i} = 31.61667882$ and $s_{\overline{n+1}|i} = 64024.90944$. Determine i and n.

78. Suppose $ 40,000 was invested on January 1, 1980 at an annual effective interest rate of 7% in order to provide an annual (calendar-year) scholarship of $ 5,000 each year forever, the scholarships paid out each January 1. In what year can the first $ 5000 scholarship be made?

79. Given $54\ddot{a}_{\overline{\infty}|i} = 1000$, find $s_{\overline{22}|i}$.

80. Alice owned an annuity which had level annual payments for twelve consecutive years, the first of these being in exactly twelve years. She sold it, and the selling price of $ 21,092.04 was based on a yield rate for the investor of 7.8%. What is the amount of the level payments?

81. Catfish Hunter's 1974 baseball contract with the Oakland Athletics called for half of his $100,000 salary to be paid to a life insurance company of his choice for the purchase of a deferred annuity. More precisely, there were to be semi-monthly contributions in Hunter's name to the Jefferson Insurance Company with the first payment on April 16 and the final payment on September 30. We suppose that the first eleven of these were to be for $ 4,166.67, and the final payment was to be for four cents less. ($ 4,166.67 \times 12 = $ 50,000.04.$) Using an annual effective interest rate of 6 % (a rate that figures in a six-year personal loan of $ 120,000 that Oakland's owner Charlie Finley had made to Hunter in 1969 and then promptly recalled), find the value of the specified payments to the insurance company at the scheduled time of the last payment. (Hunter wished to have such an annuity in lieu of immediate salary for tax reasons. Finley claimed that he was fulfilling the contract when he offered Hunter a $ 50,000 check at the end of the season. Finley's default on his contractual obligation led to Hunter's historic free agency. The New York Yankees signed Hunter to a five-year, $ 3,750,000 contract.)

82. Mr. Bell buys a home for an unspecified amount. He pays a down payment of $ 20,000 and finances the remainder for 15 years with level end of- month payments of $1,692. The annual effective interest rate for the first five years is 4 %, and thereafter it is 6%. Mr. Bell sells the house just after making his 100th mortgage payment. The selling price is $ 258,000. How much money will Mr. Bell get at closing? (Remember, the loan holder is paid first, and then Mr. Bell receives the balance of the inflow from the resale.)

83. Given that $A_K(t) = \frac{1000}{1-t}$ for $0 \le t < 100$, find K and $a(20)$.

84. It is known that for each positive integer k, the amount of interest earned by an investor in the k-th period is k. Find the amount of interest earned by the investor from time 0 to time n, n a fixed positive integer.

85. In how many years will $ 500 accumulate to $ 800 at 6 % simple interest?

86. At a particular rate of simple interest, $ 1200 invested at time $t = 0$ will accumulate to $ 1320 in T years. Find the accumulated value of $ 500 invested at the same rate of simple interest and again at $t = 0$, but this time for $2T$ years.

87. Alice invests $ 2200. Her investment grows according to compound interest at an annual effective interest rate of 4 % for T years, at which time it has accumulated to $ 8000. Find T.

88. Horatio invests money in an account earning compound interest at an unknown annual effective interest rate i. His money doubles in nine years. Find i.

89. At a certain rate of compound interest, money will double in α years, money will triple in β years, and money will increase tenfold in γ years. At this same rate of compound interest, $ 5 will increase to $ 12 in n years. Find integers a, b and c so that $n = a\alpha + b\beta + c\gamma$.

90. A payment of $ X two years from now along with a payment of $ $2X$ four years from now repays a debt of $ 6000 at 6.5 % annual effective compound interest. Find X.

91. The annual effective interest rate on Mustafa's loan is 6.8 %. What is the equivalent effective quarterly discount rate on the loan?

92. Suppose we have compound interest and $d^{(4)} = 8\%$. Find equivalent rates $d^{(3)}$, i and $i^{(6)}$.

93. You have a choice of depositing your money in account A which has an annual effective interest rate of 5.2%, account B which has an effective monthly rate of .44 %, or account C that is governed by force of interest $\delta = 0.0516$. Which account should you choose? Which account would give you the lowest accumulation?

94. Given that the force of interest $\delta_t := 2t / (1 + t^2)$, find the effective rate of discount for the sixth year.

95. Marianne deposits $ 2000 in a five year certificate of deposit. At maturity the balance is $ 2580.64. Find the annual effective rate of interest governing the account.

96. Sidney borrows $ 12000. The loan is governed by compound interest and the annual effective rate of discount is 6 %. Sidney repays $ 4000 at the end of one year, X at the end of two years, and $ 3000 at the end of three years in order to exactly pay off the loan. Find X.

97. Anne and Frank Smith each borrow $ 12000 from their father. Anne and Mr. Smith have agreed that she will repay her loan in full by paying $ 6000 in two years and $ 8000 in four years. Frank prefers to make one lump payment of $ 15000 to fully repay his loan. When should he make that payment so that he and his sister will each have the same effective interest rate?

98. Sigmund, Inc. agrees to pay $ 15000 today and $ 40000 four years from today in return for $ 210000 two years from today. What is the yield rate for this four-year financial arrangement?

99. Franklin's investment fund had a balance of $ 290000 on January 1, 1995 and a balance of $ 448000 two years later. The amount of interest earned during the two years was $ 34000, and the annual yield rate on the fund was 5.4%. Estimate the (dollar-weighted) average date of contributions to the account.

100. On January 1, 1988, Antonio invests $ 9400 in an investment fund. On January 1, 1989 his balance is $ 10600 and he deposits $ 2400. On July 1, 1989 his balance is $ 14400 and he withdraws $ 1000. On January 1, 1992 his balance is $ P. Express his annual time-weighted yield as a function of P.

101. The Browns wish to accumulate at least $ 15000 at the time of their last deposit in a college fund for their daughter by contributing an amount A into the account at the end of each year for eighteen years. What is the smallest annual payment A that will suffice if the college fund earns a level annual effective interest rate of 5 %? If at the end of ten years, it is announced that the annual effective interest rate will drop to 4.5 %, how much must the Browns increase their payments in order to reach their accumulation goal? Assume that the Browns wish to continue to make level payments except for a slightly reduced final payment.

102. Sigmund and Karl each borrowed an identical amount from Ludwig at a nominal rate of discount of 5.4 % convertible quarterly. Sigmund repays his loan by making payments of $ 2000 at the end of each year for six years. Karl makes payments of $ 3200 at four equally space times $T, 2T, 3T$ and $4T$. Find T.

103. Steven Wong wishes to save for his retirement by depositing $ 1200 at the beginning of each year for thirty years. Exactly one year after his last deposit, he wishes to begin making annual level withdrawals until he has made twenty withdrawals and exhausted the savings. Find the amount of each withdrawal if the effective interest rate is 5 % during the first thirty years but only 4 % after that.

104. Given $\ddot{a}_{\overline{n}|} = 12$ and $\ddot{a}_{\overline{2n}|} = 21$, find $a_{\overline{4n}|}$.

105. Svetlana won $ 1000000 in a contest, to be paid in twenty $ 50000 payments at yearly intervals, the first payment paid at the time of the contest. (Of course, the present value of her winnings is less than $ 1000000.) Svetlana decided to keep X each year to spend and deposit the remaining $ 50000 − X$ into an account earning an annual effective interest rate of 5 %. She chose the value X to be as large as possible so that, at the moment of the 20th deposit, the account would have grown to such a size that it would provide Svetlana and her heirs at least X per year in interest forever. Find X.

106. When computed using an effective interest rate of i, it is known that the present value of $\$ 2000$ at the end of each year for $2n$ years plus an additional $\$ 1000$ at the end of each of the first n years is $\$ 52800$. Using this same interest rate, the present value of an n year deferred annuity-immediate paying $\$4000$ per year for n years is $\$ 27000$. Find n.

107. Olena loans her sister Irini $\$ 8000$. The loan is to be repaid at a nominal interest of 4.8 % payable monthly. The monthly payments are to be for $\$ 100$ except for a final smaller payment. How much does Irini owe to Olena at the end of one year.

108. To accumulate $\$ 217593.30$ at the end of $5n$ months, deposits of $\$ 100$ are made at the end of the first $2n$ months and $\$ 300$ at the end of the next $3n$ months. Given that $(1+i)^n = 92.372$, find n.

109. Lucy received a gift of a twenty-one year annuity on the day she was born. The annuity pays $\$ 500$ on her odd birthdays and $\$ 700$ on her even birthdays. (a) If the nominal rate of interest is 8% payable monthly, find the value of this annuity on the day she was born. (b) If the value of the annuity at the time of her birth was $\$ 6000$, find the annual effective discount rate as a percent, correct to the nearest one-hundredth of a percent.

110. Al and Sal are twins. Al is given a fifteen-year annuity with end-of-year payments. The first payment Al receives, precisely one year from the date he is given the annuity, is for $\$ 100$, and then subsequent payments decrease by 4 % annually. Sal is given an n-year level annuity that has the same present value as Al's when the present values are calculated using $i = 5\%$. Again calculated using $i = 5\%$, the accumulated value at the end of n years of Sal's annuity is $\$ 1626.29$. Find the common present value of the two annuities and then find n.

111. Bob deposits $\$ 11000$ at the beginning of each year for six years in a fund earning an effective annual rate of 7.5 %. The interest from this fund can only be reinvested at an effective annual rate of 5 %. Find the accumulated value of Bob's investments at the end of thirteen years.

112. An n-year annuity-immediate has a payment of k^2 at time k for $k = 12, \ldots, n$. Derive a formula for the present value of this annuity in terms of the symbol $(I_{3,2}\, a)_{\overline{n-2}|i}$, assuming money grows according to the accumulation function $a(t) = (1+i)^t$ for $t \geq 0$.

113. An investor pays $\$ 250$ at the beginning of years 1-6 and $\$ 750$ at the beginnings of years 13–18. In return, he gets $\$ 900$ at the beginnings of years 7–12. Find all positive yield rates for this investment.

114. Tom invests $\$ 80000$ and earns $\$ 12000$ on the first five anniversaries of his investment. Upon receipt of each $\$ 12000$ payment, he immediately deposits $\$ 5000$ into a savings account earning 5 % payable annually. If Tom withdraws the accumulated money in the savings account a year after the last payment, find the yearly yield rate earned by Tom during the six-year period. Note that this investment includes two parts, the investment of the original $\$ 80000$ and the reinvestment at 5 %. Check that your yield rate is between the rates for these two parts.

115. Find the value at $t = 0$ of a perpetuity that pays $\$ 1000$ at the end of each year starting at $t = 3$ assuming that $a(t) = (t+1)(t+3)/3$.