PSTAT 171. HW 4 (Winter 2021)

`Instruction: Review textbook chapters 3 and 4 first. Multiple reading might help. Then try to solve the homework problems quickly.`

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1. Find the value at t = 0 of a perpetuity that pays 1,000 at the end of each year starting at t = 3 assuming that $a(t) = (t+1)(t+3)/3$.

2. Serena receives a fifty-year annuity-due that has payments that start at \$ 2,000 and increase by 3% per year through the twenty-fourth payment, then stay level at \$4,000. Find the accumulated value of this annuity at the end of fifty years if the annual effective rate of interest remains 4.2 % throughout the time of the annuity.

3. A perpetuity paying \$1,000 at the beginning of each two years has the same present value as another perpetuity with level payments, this one having payments at the end of each three years. Express the level payment amount of the second annuity as a function of the annual effective interest rate i.

4. Jason purchases a deferred perpetuity for \$ 13,520. The perpetuity has quarterly payments of \$ 750. Express the waiting time until the first payment as a function of the annual effective interest rate i.

5. Jasper is bequeathed a thirty-year deferred annuity that has a payment at the end of each third year. The first payment is for \$15,000 and is made five years after she receives the inheritance. There is always an increase of \$ 4,000 from one payment to the next. Find the value of her legacy at the time of the bequest, assuming that the annual effective interest rate remains level at 5 %.

- Do this problem twice, once using Chapter 3 methods and once using Section (4.4) techniques.

6. Bob deposits \$1,500 at the beginning of each quarter for sixteen years in a fund earning a nominal rate of interest of 6 % convertible monthly. The interest from this fund is paid out monthly and can only be reinvested at an effective annual rate of 5.2 %. Find the accumulated value of Bob's investments at the end of twentieth year.

7. An annuity lasting n interest periods has a payment at the end of each m-th of an interest period. The first payment is for an amount P. Payments are level within each interest period, and the the individual payments increases by an amount Q from one interest period to the next. Show that the accumulated value at the time of the last payment is

$$m\Big(P\, s^{(m)}_{\overline{n}|i} + \frac{Q}{i^{(m)}}(s_{\overline{n}|i} - n)\Big).$$

8. This problem concerns the annuity underlying the annuity symbols $(I^{(m)}\, a)^{(m)}_{\overline{n}|i}$.
(a) The rate of payment over an interval is the amount paid normalized by dividing by the length of the interval. Show that this annuity pays at a rate of $j\,/\,m$ over the interval $[(j-1)\,/\,m\,, j\,/\,m]$.
(b) Show that the payments of this annuity total $(n^2/2) + (n/(2m))$.
(c) Show that the limit $\lim_{m\to\infty}(n^2/2) + (n/(2m))$ of the total payment amounts found in (b) is equal to $\int_0^n t\mathrm{d}t$, the total payments of the continuously paying annuity underlying the symbol $(\overline{I}\,\overline{a})_{\overline{n}|}$.