PSTAT 171. HW 5 (Winter 2021)

`Instruction: Review textbook chapter 5 first. Multiple reading might help. Then try to solve the homework problems quickly.`

Instructor: Tomoyuki Ichiba

1. An amortized loan is repaid with annual payments which start at \$ 400 at the end of the first year and increase by \$ 45 each year until a payment of \$1,480 is made, after which they cease. If interest is 4% effective, find the amount of principal in the fourteenth payment.

2. Let r and k denote positive integers, and set $n = rk$. An amortized loan lasting n interest periods has a payment of P at the end of each k interest periods. The effective interest rate per interest period is i.
(a) Explain why the outstanding loan balance at time jk, just after the payment of P is equal to $P \cdot a_{\overline{n-jk}|i} \,/\, s_{\overline{k}|i}$ for $j = 0, 1, 2, \ldots$ [HINT: Look at Section (4.2).]
(b) Use the result of (a) to establish that the interest in the payment at time $(j+1)k$ is $P(1 - v^{n-jk})$.

3. An amortized loan lasting n interest periods has a level payment of P at the end of each m-th of an interest period. The effective interest rate per interest period is i.
(a) For $k = 1, 2, \ldots, mn$ define $a^{(m)}_{\overline{n-(k/m)}|i} = (1 - v^{(n-(k/m))}) \,/\, i^{(m)}$. Explain why the outstanding loan balance at time k/m, just after the payment P, is $m\,P\,a^{(m)}_{\overline{n-(k/m)}|i}$. [HINT: You may find it helpful to look at Section (3.11).]
(b) Use the result of (a) to show that the interest at time $(k+1)\,/\,m$ is $P(1 - v^{m/k})$ for $k = 0, 1, \ldots, nm - 1$.

4. Alan borrows \$18,000 for eight years and agrees to make quarterly payments of \$770. Each of these payments consists of interest for the just completed quarter and a deposit to a sinking fund that has a nominal interest rate of 6% convertible quarterly. For the first six years, each year the lender receives 8% nominal interest convertible quarterly. For the remaining two years, the lender receives 12% nominal interest convertible quarterly. Find the amount by which the sinking fund is short of repaying the loan at the end of the eight years.

5. Bob and Barbara are friends. Bob takes out a \$10,000 loan and agrees to repay it over twelve years by making annual level payments at an effective rate of 5.62499%. At the same time, Barbara takes out a \$10,000 loan and agrees to repay it by making annual interest payments at an annual effective interest rate of i. She also agrees to make annual level deposits into a sinking fund that earns 4% annual effective interest so as to accumulate \$10,000 at the end of the twelve years. Bob and Barbara discover they have the same total annual expenditures resulting from their loans. Find the rate i.

6. A loan of \$39,999.85 is to be repaid by payments at the end of each quarter for eight years. Each payment is 2% higher than its predecessor. The loan is made at a nominal rate of discount of 4% payable quarterly. Find the balance just after the 20th payment, the amount of interest in the twentieth payment, and the amount of principal in the twentieth payment.

7. Mr. Beltram takes out a \$100,000 loan for twelve years. The applicable annual effective interest rate is a promotional rate of 2% for the first two years and 6% for the remainder of the loan term. Mr. Beltram's payments increase by 10% each year. Find the balance on the loan immediately following his fifth payment.

8. A bank makes a package of three loans to a small business.

(a) \$120,000 amortized monthly for ten years at a nominal discount rate of 6.8% convertible monthly.

(b) \$100,000 to be repaid by monthly sinking fund payments for ten years where interest is assessed at a rate of 5.4% nominal convertible monthly and the sinking fund earns 4% nominal interest convertible monthly. The bank receives the sinking fund deposits.

(c) \$200,000 to be repaid with interest at the end of ten years with an effective rate of discount of 8.2% throughout the ten years.

Find the bank's annual effective yield on each of these loans individually and on the package of loans over the ten-year period.

9. A loan of \$10,000 is negotiated, with the borrower agreeing to repay the principal over ten years as well as to make annual end-of-year payments of interest at 4% effective per annum. A \$1,100 total payment will be due at the end of each year during the first five years, and a higher level end-of-year payment will be required during the second five years.

The lender will replace his capital by means of a sinking fund earning 5% per annum. Each time he receives a payment from the borrower, he will deposit that portion representing principal into the sinking fund.

What will be the lender's yield on the whole transaction, assuming all payments are made as scheduled?