

# PSTAT 174 Lab Assignment 3

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1 Consider the AR(2) process below:  $X_t = 0.8X_{t-1} - 0.12X_{t-2} + Z_t$  iid  $\sim N(0,1)$

(a) Express the processes in terms of the back shift operator, B:  $BX_t = X_{t-1}$   $X_t = (1 + 0.8B)Z_t - 0.12Z_{t-2}$

(b) Determine whether each process is causal and/or invertible. (Hint: use `polyroot()`).

```
polyroot(c(1,-0.8,0.12))
```

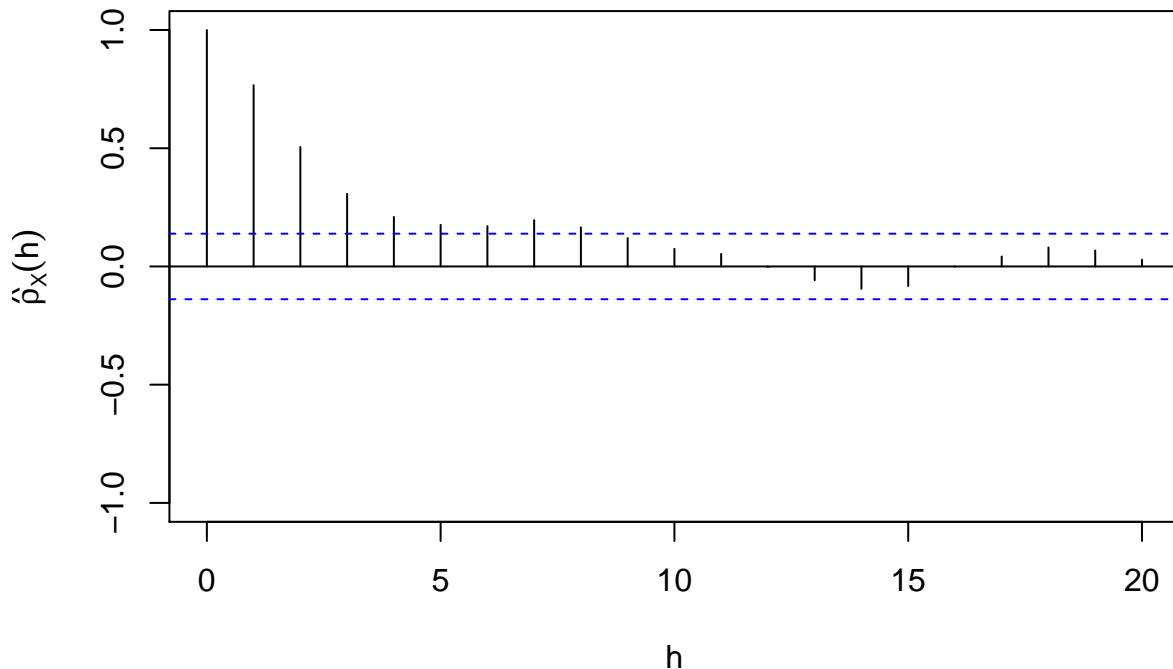
```
## [1] 1.666667+0i 5.000000+0i
```

AR(p) is always invertible by its construction:  $Z_t = \phi(B)X_t$ . AR(2) is stationary and causal because the roots of the polynomial are complex and are outside of the unit circle.

(c) We simulate 200 observations from this AR(2) model with the following code: Plot the sample ACF and PACF.

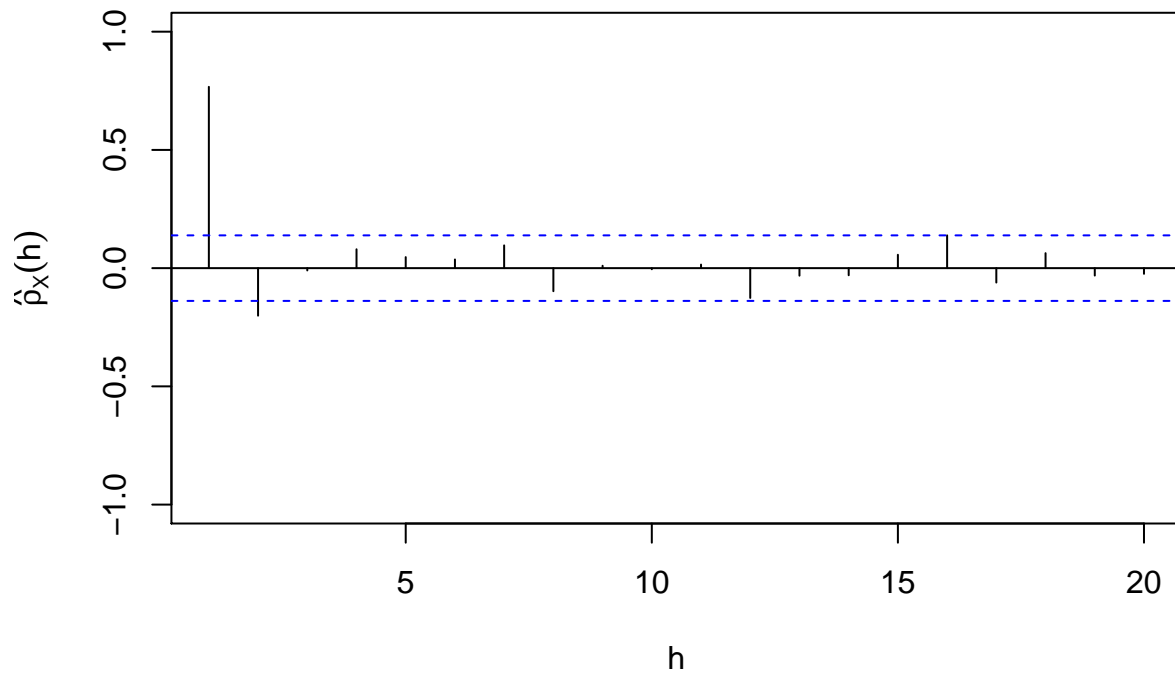
```
set.seed(1234)
ar2 <- arima.sim(model = list(ar = c(0.8,-0.12),sd = 1),n = 200)
acf(ar2,lag.max = 20,
    main = "Sample ACF",
    ylim = c(-1,1),
    xlab = "h",
    ylab = expression(hat(rho)[X](h)))
```

### Sample ACF



```
pacf(ar2,lag.max = 20,
    main = "Sample ACF",
    ylim = c(-1,1),
    xlab = "h",
    ylab = expression(hat(rho)[X](h)))
```

## Sample ACF



- (d) Use the above simulation to manually construct the Yule Walker estimates  $\phi_1$ ,  $\phi_2$ , and  $\phi^2$ . Also, use the pre-installed function `ar.yw()` for estimation.

```
#Estimation with Yul-Walker eqns
acv_ar <- acf(ar2,type = "covariance",main = "Sample ACF",plot = F)
Rho <- toeplitz(acv_ar$acf[c(1,2)]/acv_ar$acf[1])
rho <- acv_ar$acf[c(2,3)]/acv_ar$acf[1]
phi_hat <- solve(Rho) %*% rho
phi_hat
```

```
##           [,1]
## [1,]  0.9210879
## [2,] -0.2011451
```

```
yw <- ar.yw(ar2,order = 2)
# mean estimate
yw$x.mean
```

```
## [1] -0.01351863
```