Alaris Mathematical Standard

Option Pricing and Strategy Specification

2026-01-01

Alaris Mathematical Standard

Version: 2.0
Date: January 2026
Applicability: Earnings Volatility Calendar Spread Strategy

Table of Contents

Table of contents

---

1. Rule Summary

Option Pricing

1. **American $\geq$ European**: American option price shall exceed European price.
2. **American $\geq$ Intrinsic**: American option price shall exceed intrinsic value.
3. **Regime Selection**: Use spectral collocation for $\sigma \leq 100\%$; fallback to FD otherwise.

Signal Generation

4. **IV/RV Ratio**: Signal requires IV/RV ≥ 1.25.
5. **Term Structure**: Signal requires slope ≤ -0.00406.
6. **Liquidity**: Signal requires 30-day avg volume $\geq 1,500,000$.

Position Sizing

7. **Kelly Fraction**: Use $\kappa = 0.02$ (Recommended) or 0.01 (Consider).
8. **Liquidity Limit**: Position $\leq 5\%$ of daily volume, $\leq 2\%$ of open interest.

Fault Detection

9. **Delta Drift**: Halt if $|\Delta_{\text{net}}| > 0.10$.
10. **Vega Correlation**: Reject if $|\rho_{\text{FB}}| > 0.70$.
11. **Cost Survival**: Reject if post-cost IV/RV < 1.20.

---

2. Scope

This standard defines mathematical constraints for: - American option pricing under positive and negative rate regimes - Spectral collocation method parameters and validation - Trading signal generation predicates - Risk management and fault detection

---

3. Conventions

Term	Meaning
Shall	Mandatory requirement; violation is a system error
Should	Preference; violation triggers warning
S	Spot price
K	Strike price
τ	Time to maturity (years)
r	Risk-free rate
q	Dividend yield
σ	Volatility

---

4. Option Pricing Framework

Rule 1 (American ≥ European)

The American option price shall satisfy:

$$V_{\text{American}}(S, K, \tau) \geq V_{\text{European}}(S, K, \tau)$$

Rule 2 (American ≥ Intrinsic)

The American option price shall satisfy:

$$V_{\text{Put}}(S, K, \tau) \geq (K - S)^+$$

$$V_{\text{Call}}(S, K, \tau) \geq (S - K)^+$$

Rule 3 (Early Exercise Premium)

The American value decomposes as:

$$V(\tau, S) = v(\tau, S) + \mathcal{P}(\tau, S)$$

where v is European value and $\mathcal{P}$ is early exercise premium.

---

5. Rate Regime Classification

Standard Regime

Condition: $r \geq 0$ or $r \geq q$

Boundary: Single upper boundary $B(\tau)$ satisfying:

$$\lim_{\tau \to \infty} B(\tau) = K \cdot \frac{\lambda\_}{\lambda\_ - 1}$$

where $\lambda\_$ is the negative root of:

$$\frac{1}{2}\sigma^2\lambda^2 + (r - q - \frac{\sigma^2}{2})\lambda - r = 0$$

Double Boundary Regime

Condition: $q < r < 0$

Exercise Region: $\{S : Y(\tau) \leq S \leq B(\tau)\}$

Implementation: FP-B' stabilised iteration Healy (2021).

---

6. Spectral Collocation Engine

Rule 3 (Regime Selection)

The unified engine (CREN003A) shall select method as follows:

Condition	Method	Component
$\sigma \leq 1.0$	Spectral	CREN004A
$\sigma > 1.0$	Finite Difference	CREN002A
$q < r < 0$	FD (double boundary)	CREN002A

Spectral Schemes

Scheme	Nodes	Throughput	Use Case
Fast	16	16,000/sec	Screening
Accurate	32	2,800/sec	**Default**
HighPrecision	64	42/sec	Validation

Chebyshev Nodes

Nodes distributed as:

$$\xi_j = \cos\left(\frac{(2j-1)\pi}{2N}\right), \quad j = 1, \dots, N$$

Convergence

Spectral convergence for analytic boundaries:

$$|B_N(\tau) - B(\tau)| = O(e^{-cN})$$

Error Characterisation

Moneyness	Max Deviation	Direction
OTM (S/K < 0.9)	< 0.1	Spectral lower
ATM (S/K ≈ 1)	< 0.02	Matched
ITM (S/K > 1.1)	0.2 - 1.0	Spectral higher

Note: ITM deviation is systematic (conservative for short premium).

---

7. Implied Volatility Computation

Algorithm

1. **Initial Guess:** Corrado-Miller Corrado and Miller Jr (1996) approximation
2. **Refinement:** Householder(3) iteration (cubic convergence)
3. **Fallback:** Brent's method (guaranteed convergence)

Householder(3) Formula

$$\sigma_{n+1} = \sigma_n - \frac{f(\sigma_n)}{f'(\sigma_n)} \cdot \left(1 + \frac{f \cdot f''}{2(f')^2}\right)$$

where $f(\sigma) = V_{\text{BS}}(\sigma) - V_{\text{market}}$.

Convergence

Typical: 2 iterations (vs 3-5 for Newton).

Bounds

- Minimum: 0.001 (0.1%)
- Maximum: 5.0 (500%)

---

8. Trading Signal Generation

Rule 4 (IV/RV Ratio)

A signal requires:

$$\frac{\sigma_I^{30}}{\sigma_R^{30}} \geq 1.25$$

where σ_I^{30} is 30-day implied volatility and σ_R^{30} is Yang-Zhang realised volatility.

Rule 5 (Term Structure)

A signal requires:

$$\nabla_\tau \sigma_I \leq -0.00406$$

(Negative slope indicates front-month premium.)

Rule 6 (Liquidity)

A signal requires:

$$\bar{V}^{30} \geq 1,500,000$$

where $\bar{V}^{30}$ is 30-day average daily volume.

Condition	Strength

Signal Strength

Condition	Strength
All three criteria	Recommended
Two criteria	Consider
Otherwise	Avoid

Yang-Zhang Estimator

$$\sigma_{YZ}^2 = \sigma_o^2 + k\sigma_c^2 + (1-k)\sigma_{RS}^2$$

where $k = 0.34/(1.34 + \frac{n+1}{n-1})$.

---

9. Position Sizing

Rule 7 (Kelly Fraction)

Position size shall be:

$$f_{\text{actual}} = \kappa \cdot f^*$$

where $f^* = \frac{\mu}{\sigma^2}$ is full Kelly and:

Signal	κ
Recommended	0.02
Consider	0.01
Avoid	0

Rule 8 (Liquidity Limits)

Position shall satisfy:

$$\frac{n}{V_{\text{daily}}} \leq 0.05 \quad \text{and} \quad \frac{n}{OI} \leq 0.02$$

---

10. Fault Detection

Rule 9 (Delta Drift)

Halt trading if:

$$|\Delta_{\text{net}}| > 0.10$$

Rule 10 (Vega Correlation)

Reject trade if front-back IV correlation exceeds threshold:

$$|\rho_{FB}| > 0.70$$

Rule 11 (Cost Survival)

Reject trade if post-cost ratio insufficient:

$$\frac{\sigma_I}{\sigma_R} \cdot \left(1 - \frac{C_{\text{exec}}}{V_{\text{position}}}\right) < 1.20$$

Circuit Breakers

Trigger	Threshold	Action
Daily loss	$>$ 5% capital	Halt new positions
Data stale	$>$ 15 minutes	Use cached data
VIX spike	$>$ 40	Increase cost buffers 50%
Validation fail	$>$ 10% rate	Halt signal generation

---

11. Component Mapping

Rule	Component	Method
1-2	CREN003A	`Price()`
3	CREN003A	Regime selector
4-6	STCR001A	`GenerateSignal()`
7-8	STRK001A	`ComputePositionSize()`
9	STHD003A	`AssessGammaRisk()`
10	STHD001A	`AnalyseVegaCorrelation()`
11	STCS006A	`ValidateCostSurvival()`

---

12. References

Corrado, Charles J, and Thomas W Miller Jr, 1996, A note on implied volatility estimation, *Journal of Banking & Finance* 20, 1139–1145.

Healy, Jerome V, 2021, Pricing American options under negative rates, *Journal of Derivatives* 28, 33–52.

Last Updated: 2026-01-01