1. Assignment

- Read the following document
- Understand the model
- Design a heuristic algorithm for solving the problem
- Implement the heuristic
- Test the heuristic on the test set
- Compare the results with those reported in the excel file ”Results”. (The column OptGap reporte the optimality gap of the solution obtained by CPLEX. If it is zero the OF value is optimal)
- Evaluate the gap of the heuristic solution and the gain in the computational time
- Write a brief report explaining the heuristic approach and the obtained results

2. Introduction and model formulation

Nowadays, the urban distribution has to cope with the continuous growth of e-commerce and the changing of customers’ behaviour. Indeed, customers have raised their expectations for fast (usually within very limited time slots as 2 hours), and cheap deliveries of purchased goods and they are connected, informed and empowered, and continually demand more choice and flexibility in delivery options.

To try to answer them and provide a faster service, some logistic platforms, called Satellites, are rented in a strategic node of the city. In each satellite, the items to be delivered are consolidated into vehicles with a given capacity. A mixed-fleet of vehicles is considered, composed by traditional vehicles, low-environmental vehicles (i.e., electric vans and cargo bikes) or traditional vehicles owned by unprofessional users. Notice that according to [7], different vehicles types are characterized by different costs. We assume that the orders are split in different timeslots, which are characterized by different capacity (i.e., vehicles), and costs. All the orders must be delivered by one of the vehicles or it can be delivered by using an external service of express delivery, whose cost is higher then the service cost of the vehicles .

We model this problem by proposing a new variant of bin packing problem with time-dependent cost, in which we consider a objective function, composed by the cost of warehouses and the delivery cost represented by the routing costs of an electric vehicles at a specific time slot.

To the best of our knowledge, in the literature of Bin Packing problems, the time-dependent constraints have treated rare except [2] that considered arrival time of items in discrete probability distributions. Friesen and Langston first introduced VSBPP presenting one online and two off-line algorithms with their worst-case ratio [3]. An algorithm with upper bounds for some fixed size bin presented in [8] and [6] presented method with heuristics and exact solution by correlating bin volume and cost. The VSBPP [9] where unit cost does not increase with the size of bin in the case of the sizes of items and the sizes of bins are divisible, the sizes of Bins are not divisible. Also in [5] addressed the deterministic one-dimensional bin-packing problem with unequal bin sizes and costs. A variation of the bin packing problem in which bins of different types have different costs and capacities where each bin has to be filled at least to a certain level, depending on bin size discussed in [4]. Several studies have been dedicated to the VCSBPP [1] explained the problem assuming that the cost of the unit size of each bin does not increase as the bin size increases.

Now, we present the model formulation that takes the form of a bin packing problem. in which there are sets of orders, timeslots, vehicles types different in terms of characteristics (e.g., size and volume) where

Let $\mathcal{H}$ be the time horizon composed by different time slots h. Each time slot h has a tariff for renting a unitary space unit T^h. Let $\mathcal{I}$ be the set of orders. We define d_i the demand of an order $i \in \mathcal{I}$.

Let $\mathcal{K}$ be the set of vehicles types. Each vehicle type contains a certain number of vehicles. For each vehicle $k \in \mathcal{K}$, let V_k^h and δ_k^h be respectively the capacity and the fixed cost associated to the vehicle k at the time slot h. Let c_k^h be the cost per stop.

We define C_h the capacity of the satellite at the time slot h, and c_a the unit cost for an express delivery.

Let the decision variables be the following:

- x_{ik}^h equal to 1 if an order $i \in \mathcal{I}$ is delivered at the timeslot $h \in \mathcal{H}$ by the vehicle $k \in \mathcal{K}$, and 0 otherwise.
- X_i^h equal to 1 if an order $i \in \mathcal{I}$ is delivered at the timeslot $h \in \mathcal{H}$ and 0 otherwise.
- y_k^h equal to 1 if a vehicle $k \in \mathcal{K}$ is used at the timeslot $h \in \mathcal{H}$, and 0 otherwise.
- Z_i equal to 1 if an order $i \in \mathcal{I}$ is delivered as an express delivery and 0 otherwise.

The VCSBPP with time-dependent cost and tariff may then be formulated as follows:

$$\min \quad \sum_{k\in K}\sum_{i\in I}\sum_{h\in H}(d_i T^h)x_{ik}^h + \sum_{k\in K}\sum_{i\in I}\sum_{h\in H}(c_k^h x_{ik}^h) + \sum_{k\in K}\sum_{h\in H}(\delta_k^h y_k^h) + \sum_{i\in I}(c_a Z_i) \tag{1}$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{I}}(d_i X_i^h) \leq C_h, \qquad \forall\, h \in \mathcal{H}, \tag{2}$$

$$\sum_{i\in\mathcal{I}}(d_i x_{ik}^h) \leq V_k y_k^h \qquad \forall\, h \in \mathcal{H}, k \in \mathcal{K}, \tag{3}$$

$$\sum_{h\in\mathcal{H}} X_i^h + Z_i = 1 \qquad \forall\, i \in \mathcal{I}, \tag{4}$$

$$\sum_{k\in\mathcal{K}} x_{ik}^h = X_i^h \qquad \forall\, i \in \mathcal{I}, h \in \mathcal{H}, \tag{5}$$

$$x_{ik}^h \in \{0,1\}, \qquad \forall\, i \in \mathcal{I}, h \in \mathcal{H}, k \in \mathcal{K}, \tag{6}$$

$$y_k^h \in \{0,1\}, \qquad \forall\, h \in \mathcal{H}, k \in \mathcal{K}, \tag{7}$$

$$Z_i \in \{0,1\}, \qquad \forall\, i \in \mathcal{I}. \tag{8}$$

The objective function (1) minimizes the total cost composed by the sum of the tariff for renting the satellite, the cost per stop and the fixed cost of the vehicles for the deliveries plus the cost of the eventual express delivery. Constraints (2) ensure that at each timeslot, the capacity of the depot is not exceeded. Constraints (3) ensure that at each timeslot, the capacity of the vehicles (if used) is not exceeded. Constraint (4) ensures that each order is fulfilled in a single timeslot or by using the external express delivery service. Constraint (5 is a dummy constraint that defines appropriately the variables. Finally, constraints (6) to (8) are the integrality requirements on the decision variables.

3. Instance set

3.0.1. Set 1

Instances from [6]. Five instances were randomly generated for each combination of the following parameters:

- Number of timeslots p
- Number of orders m in the set $\{25, 100, 200, 500, 1000\}$.
- For each order $i = 1 \ldots, m$ a demand d_i (demand[orders]).
- Number of different vehicles types ($numVTypes$).
- For each vehicle type a different capacity (VeicVolume) and a different fixed cost (VeicCost)
- The total number of vehicles (o)
- For each vehicle, the type (VeicType).
- For each vehicle and time slot, a given capacity (Elect_vehicle[vehicles][timeslots])
- The time-dependent tariff for renting the satellite in each timeslot (Tarif[timeslots])
- The satellite capacity in each timeslot (Depot_Cap[timeslots])
- Assume $c_a = 10000$

orders=1..m;
vehicles=1..o;
timeslots=1..p;
VType=1..numVTypes;
demand[orders]=...;
Tarif[timeslots]=...;
Depot_Cap[timeslots]=...;
VeicVolume[VType]=...;
VeicCost[VType]=...;
VeicType[vehicles]=...;
Elect_vehicle[vehicles][timeslots]=...;

References

[1] A. Bettinelli, A. Ceselli, and G. Righini. A branch-and-price algorithm for the variable size bin packing problem with minimum filling constraint. *Annals of Operations Research*, 179(1):221–241, Sep 2010. ISSN 1572-9338. doi: 10.1007/s10479-008-0452-9. URL `https://doi.org/10.1007/s10479-008-0452-9`.

[2] S. Fazi, T. V. Woensel, and J. C. Fransoo. A stochastic variable size bin packing problem with time constraints. 2012.

[3] D. Friesen and M. Langston. Variable sized bin packing. *SIAM Journal on Computing*, 15(1):222–230, 1986. doi: 10.1137/0215016. URL `https://doi.org/10.1137/0215016`.

[4] V. Hemmelmayr, V. Schmid, and C. Blum. Variable neighbourhood search for the variable sized bin packing problem. *Comput. Oper. Res.*, 39(5):1097–1108, May 2012. ISSN 0305-0548. doi: 10.1016/j.cor.2011.07.003. URL `http://dx.doi.org/10.1016/j.cor.2011.07.003`.

[5] J. Kang and S. Park. Algorithms for the variable sized bin packing problem. *European Journal of Operational Research*, 147(2):365 – 372, 2003. ISSN 0377-2217. doi: https://doi.org/10.1016/S0377-2217(02)00247-3. URL `http://www.sciencedirect.com/science/article/pii/S0377221702002473`. Fuzzy Sets in Scheduling and Planning.

[6] M. Monaci. *Algorithms for Packing and scheduling Problem*. PhD thesis, Universitá di Bologn, Italy, 2002.

[7] G. Perboli, M. Rosano, M. Saint-Guillain, and P. Rizzo. A simulation-optimization framework for City Logistics. an application on multimodal last-mile delivery. *IET Intelligent Transport Systems*, 12(4):262–269, 2018.

[8] S. S. Seiden, R. v. Stee, and L. Epstein. New bounds for variable-sized online bin packing. *SIAM J. Comput.*, 32(2):455–469, Feb. 2003. ISSN 0097-5397. doi: 10.1137/S0097539702412908. URL `https://doi.org/10.1137/S0097539702412908`.

[9] G. Zhang. A new version of on-line variable-sized bin packing. *Discrete Applied Mathematics*, 72 (3):193 – 197, 1997. ISSN 0166-218X. doi: https://doi.org/10.1016/S0166-218X(96)00018-2. URL `http://www.sciencedirect.com/science/article/pii/S0166218X96000182`.