We will study the backpropagation behavior for an exponential neuron, given by

$$f(z) = e^{2z} - 1. \tag{1}$$

Consider a two-dimensional input given by $\mathbf{x} = (x_1, x_2)^\top$. A weight vector $\mathbf{w} = (w_1, w_2)^\top$ and a bias b act on it. Thus, the output of the neuron is given by $f(x_1, x_2) = e^{2(w_1x_1+w_2x_2+b)} - 1$.

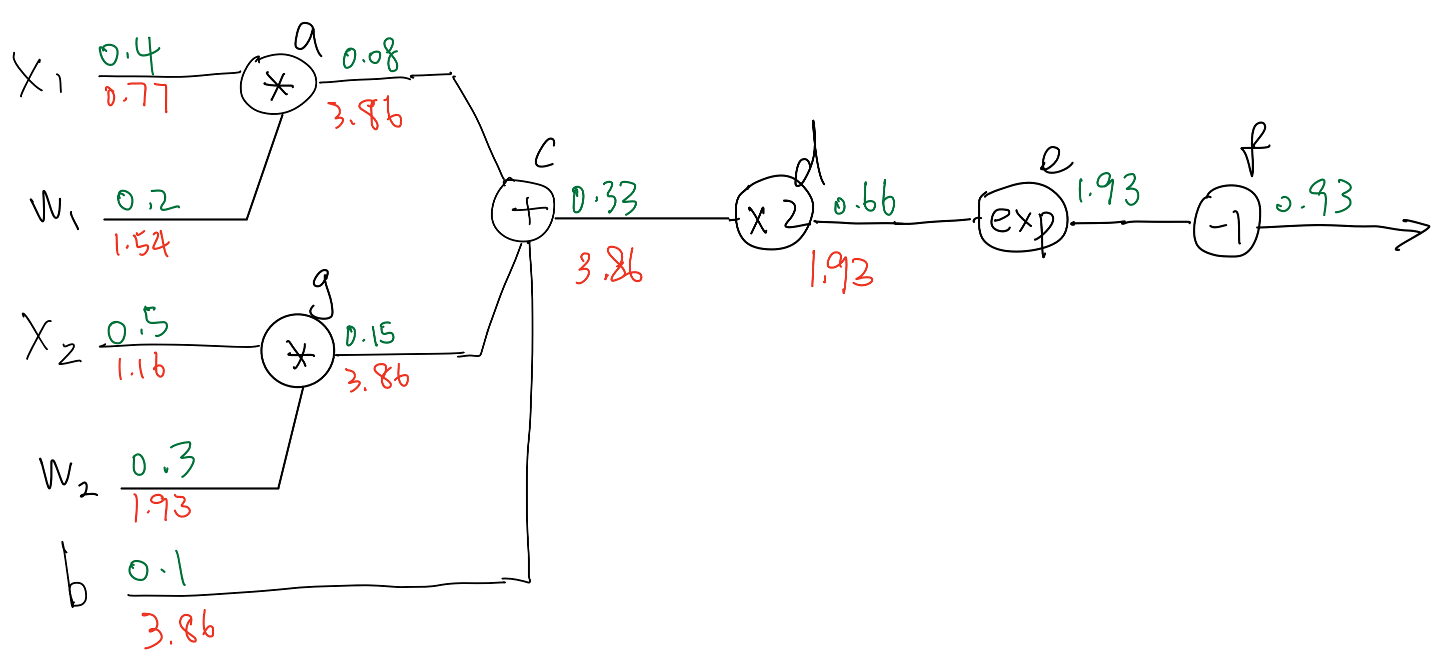
(a) Draw the computation graph for the neuron in terms of elementary operations (addition, subtraction, multiplication, division, exponentiation), as seen in class. **[2 points]**

(b) Consider inputs $x_1 = 0.4$, $x_2 = 0.5$, weights $w_1 = 0.2$, $w_2 = 0.3$ and bias $b = 0.1$. In the same figure, show the values at each node of the graph during forward propagation. **[2 points]**

(c) Use backpropagation to determine the gradients $\frac{\partial f}{\partial w_1}$, $\frac{\partial f}{\partial w_2}$ and $\frac{\partial f}{\partial b}$. Also illustrate in the same figure the intermediate gradients at each node of the computation graph. **[4 points]**

(d) Explain the process of backpropagation you used to compute partial derivatives. **[2 points]**

a)



b)

$$\frac{\partial f}{\partial w_1} = \frac{\partial f}{\partial d}\,\frac{\partial d}{\partial c}\,\frac{\partial c}{\partial a}\,\frac{\partial a}{\partial w_1}$$

$$= e^{0.66}\,(2)\,(1)\,(\overset{0.4}{\cancel{x_1}}) = 1.544$$

$$\frac{\partial f}{\partial w_2} = \frac{\partial f}{\partial d}\,\frac{\partial d}{\partial c}\,\frac{\partial c}{\partial g}\,\frac{\partial g}{\partial w_2}$$

$$= e^{0.66}(2)\ (1)\ (\overset{0.5}{\cancel{x_2}}) = 1.93$$

$$\frac{\partial f}{\partial b} = \frac{\partial f}{\partial d}\ \frac{\partial d}{\partial c}\ \frac{\partial c}{\partial b}$$

$$= e^{0.66}\ (2)\ (1) = 3.86$$

$a = x_1 * w_1$ $\qquad \frac{\partial a}{\partial w_1} = x_1$

$g = x_2 * w_2$ $\qquad \frac{\partial g}{\partial w_2} = x_2$

$c = a + b + g$ $\qquad \frac{\partial c}{\partial a} = 1 \quad \frac{\partial c}{\partial b} = 1 \quad \frac{\partial c}{\partial g} = 1$

$d = 2c$ $\qquad \frac{\partial d}{\partial c} = 2$

$f = e^d + 1$ $\qquad \frac{\partial f}{\partial d} = e^d = e^{0.66}$