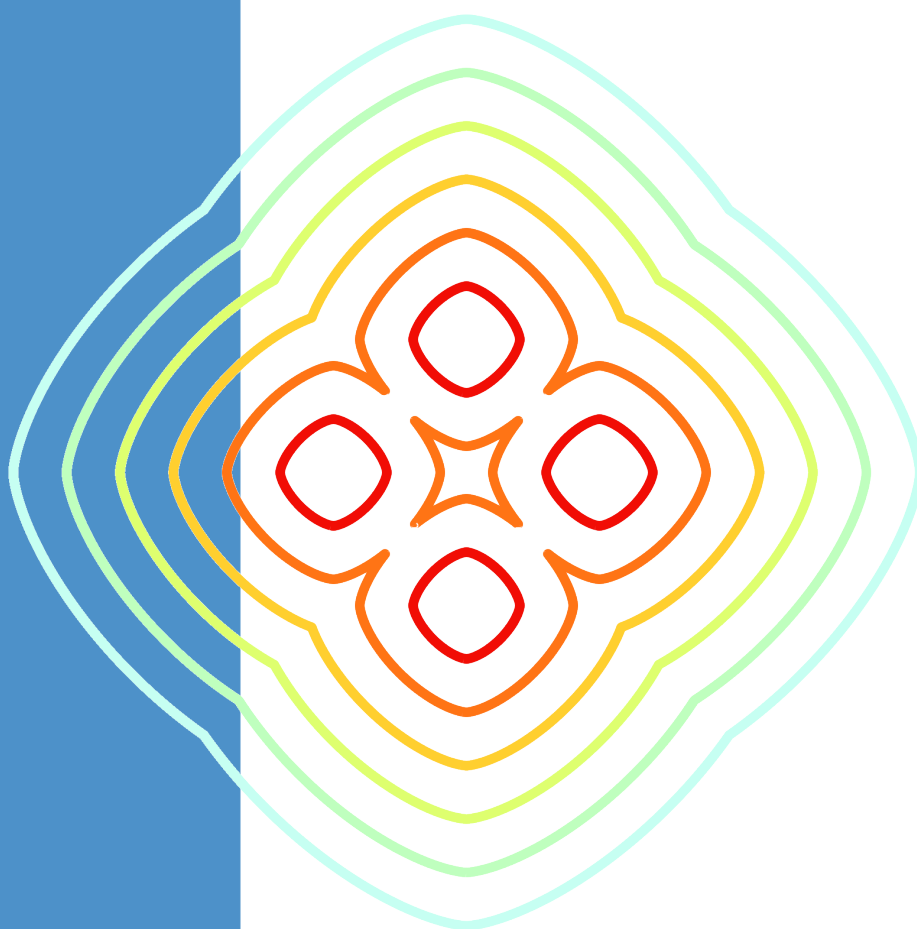


NUMERICAL METHODS FOR HAMILTON-JACOBI EQUATIONS

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As the reader may have noticed, the title of this report reads *Numerical methods for Hamilton-Jacobi equations*. It is therefore no surprise if we begin with some words on Hamilton-Jacobi equations, and then present a few numerical methods. The motivation behind this study is to solve control problems, and more precisely, optimal path problems. Practical applications immediately come to mind, as driving cargo ships towards the moon with as few fuel as possible, or finding the shortest route towards the coffee machine.

In the first chapter, we precise the definitions and class of control problem that we adress, we draw the connection with Hamilton-Jacobi equations, and we detail a representation theorem in a particular case. In the second chapter, we investigate first a class of schemes for 1D advection equations, that are, in the constant speed case, both Hamilton-Jacobi equations and scalar conservation laws. The last section turns to a Lagrangian scheme for state-constrained problems, where the focus is put on the space of controls via internal approximations.

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