

# Engineering Calculus III

MATH 251 at Texas A&M

Using *Calculus: Early Transcendentals, 8<sup>th</sup> Edition*

Brandon Nguyen

Spring 2020

# Lecture 1

## 14 January 2020

### 1.1 Three-Dimensional Coordinate System

#### Definition 1.1 Three Dimensional Coordinate System

A point in three dimensions is represented by a triple,  $(x, y, z)$ , of numbers. The set of all triples is denoted by  $\mathbb{R}^3$ .

#### Definition 1.2 Right Hand Rule

The **right hand rule** describes the relationship between the three axes. To use the rule, curl the fingers of your right hand around the  $z$  axis.

#### Example 1.1

Plot the point  $(-3, 4, 2)$  in the coordinate system.

### Definition 1.3 Coordinate Planes

In  $\mathbb{R}^3$  there exists the following coordinate planes:

- $xy$ -plane defined by  $(x, y, 0)$ .
- $xz$ -plane defined by  $(x, 0, z)$ .
- $yz$ -plane defined by  $(0, y, z)$ .

These coordinate planes split  $\mathbb{R}^3$  into **octants**. Octant 1 is defined to be where the  $x, y, z$  are positive.

### Definition 1.4 Projection onto Coordinate Planes

Given a point  $P(a, b, c)$ :

- $Q(a, b, 0)$  is the **projection** of  $P$  onto the  $xy$ -plane
- $R(0, b, c)$  is the projection of  $P$  onto the  $yz$ -plane
- $S(a, 0, c)$  is the projection of  $P$  onto the  $xz$ -plane

### Example 1.2

Find the projections of  $(3, -1, 4)$  onto the coordinate planes.

## 1.2 Surfaces

### Definition 1.5 Surfaces

An equation given in  $x, y, z$  gives a **surface** in  $\mathbb{R}^3$ .

The coordinate planes can be represented by:

- $yz$ -plane  $\iff x = 0$ , or all points  $(0, y, z)$
- $xz$ -plane  $\iff y = 0$ , or all points  $(x, 0, z)$
- $xy$ -plane  $\iff z = 0$ , or all points  $(x, y, 0)$

The equations in the form of  $x = c, y = c, z = c$  in  $\mathbb{R}^3$  give planes.

### Example 1.3

Describe and sketch the following planes

1.  $x = 5$ , a plane 5 units ahead and parallel to  $yz$ -plane
2.  $z = 2$ , a plane 2 units above and parallel to  $xy$ -plane

**Example 1.4**

How do the planes  $x = 5, z = 2$  intersect?

$$(5, y, z) \cap (x, y, 2) = (5, y, 2) \quad (1.1)$$

$(5, y, 2)$  represents a line parallel to the  $y$ -axis.

**Method 1.1 Two-Variable Equations**

A surface formed by a two-variable equation can be graphed by first creating the graph in  $\mathbb{R}^2$  corresponding to the two variables. This can then be extended to  $\mathbb{R}^3$  w.r.t. the missing axis.

**Example 1.5**

Sketch and describe the following surfaces in  $\mathbb{R}^3$

1.  $y + x = 2$
2.  $x^2 + y^2 = 9$
3.  $x^2 + (z - 4)^2 = 4$

## 1.3 Spheres

**Definition 1.6 Equation of a Sphere**

The equation of a sphere with a center  $C(h, k, l)$  and radius  $r$  is given by

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2 \quad (1.2)$$

**Example 1.6**

Find an equation of the sphere with center  $(5, 3, 2)$  and radius 3. Find and describe the intersection of this sphere and the coordinate planes.

# Lecture 2

## 21 January 2020

### 2.1 Spheres (cont.)

#### Example 2.1

Show that

$$x^2 + y^2 + z^2 - 6x + 8y - 4z = 20 \quad (2.1)$$

represents a sphere, find the centre and radius.

#### Example 2.2

What is the equation of the sphere with center  $(1, 2, 3)$  that touches the  $xy$ -plane

### 2.2 Formulae

#### Definition 2.1 Distance Formula

The distance between points  $P(a, b, c)$  and  $Q(x, y, z)$  is given by

$$|PQ| = \sqrt{(x - a)^2 + (y - b)^2 + (z - c)^2} \quad (2.2)$$

#### Example 2.3

Consider the points  $P(5, 5, 1)$ ,  $Q(3, 3, 2)$ ,  $R(1, 4, 4)$

1. Find the lengths of the triangle
2. What type of triangle is it?

**Definition 2.2 Midpoint Formula**

The midpoint of  $P(a, b, c)$  and  $Q(x, y, z)$  is given by

$$\left( \frac{x+a}{2}, \frac{y+b}{2}, \frac{z+c}{2} \right) \quad (2.3)$$

**Example 2.4**

Find an equation of a sphere if one of its diameters has endpoints  $(2, 1, 4)$ , and  $(4, 3, 10)$ .

## 2.3 Solids

**Definition 2.3 Solids**

Inequalities involving  $x, y, z$  gives solids in  $\mathbb{R}^3$ .

Like a circle is just a line outlining the boundary and a disk is the circle and everything inside. A sphere refers to the shell, while a ball refers to both the sphere and everything inside.

**Example 2.5**

Sketch and describe the following regions in  $\mathbb{R}^3$

1.  $x^2 + y^2 \leq 25$
2.  $1 < x^2 + y^2 + z^2 < 16$

## 2.4 Vectors

### Definition 2.4 Vectors

A **vector** has both magnitude and direction.

Algebraically:

In three dimensions, a vector is given by

$$\vec{a} = \langle a_1, a_2, a_3 \rangle \quad (2.4)$$

where  $a_1, a_2, a_3$  are the components of  $\vec{a}$ , each corresponding to the  $x, y, z$  respectively.

Geometrically:

A vector has initial and terminal points. For example, a vector starting at the origin  $O$ , and terminal point  $P(a_1, a_2, a_3)$  is referred to the position vector of  $P$ .

Given the points  $A(x_1, y_1, z_1), B(x_2, y_2, z_2)$ , the vector with initial point  $A$  and terminal point  $B$  is given by

$$\vec{ab} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle \quad (2.5)$$

### Definition 2.5 Magnitude

The magnitude or length of a vector

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} \quad (2.6)$$

### 2.4.1 Vector Algebra

Addition

$$\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle \quad (2.7)$$

Scalar Multiplication

$$c\vec{a} = \langle ca_1, ca_2, ca_3 \rangle \quad (2.8)$$

Difference

$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b}) = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle \quad (2.9)$$

### Definition 2.6 Standard Basis Vectors